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RATE OF ENERGY GAIN AND MAXIMUM ENERGY IN DIFFUSIVE SHOCK ACCELERATION

J. R. JOKIPII
University of Arizona
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ABSTRACT

The problem of diffusive shock acceleration of fast charged particles is reexamined with emphasis on the rate of energy gain, and the maximum energy which can be attained in a given circumstance. The direction of the average magnetic field at the shock is shown to have a large effect. If the perpendicular diffusion coefficient is much smaller than the parallel coefficient, particles can gain much more energy if the shock is quasi-perpendicular than if it is quasi-parallel. Many of the published discussions of this problem are applicable only to the quasi-parallel case, so the maximum energy attainable can be substantially higher (by a factor of 100 or more) than previous discussions would predict, in cases where the shock is quasi-perpendicular. The energy gain increases as $\kappa_\perp$ decreases. The principal limitation comes from the requirement that diffusion be a valid approximation to the particle motion, and that the particle be able to diffuse fast enough to encounter the shock many times.

Subject headings: diffusion — particle acceleration — shock waves

I. INTRODUCTION

The importance of shock acceleration of charged particles in astrophysics is underscored by the number of recent papers presenting models of shock acceleration. Most of the discussions consider transport in the *diffusive* approximation, where the particles are scattered by magnetic irregularities and relax to a nearly isotropic angular distribution. In this approach, the acceleration is caused in part by the large relative motion between the scattering centers causing the diffusion in front of the shock and those behind the shock, and, if the magnetic field has a component normal to the direction of propagation, in part from the drift along the shock front. However, most discussions have neglected the magnetic field change and the resulting particle drifts, effectively restricting consideration to quasi-parallel shocks. In some cases possible diffusion normal to the ambient magnetic field is explicitly rejected (e.g., Axford 1980). A number of authors (see, e.g., Pesses, Decker, and Armstrong 1982 for a review) have considered drifts and quasi-perpendicular shocks in another limit, for the most part neglecting scattering and diffusion (the "shock-drift" mechanism). Decker and Vlahos (1986) considered scattering in numerical simulations of individual particle motion at oblique shocks, in an application to solar flares. For one discussion which explicitly includes the magnetic field in the *diffusive* theory with finite perpendicular diffusion, and which establishes the connection between diffusive theory and the shock-drift mechanism, see Jokipii (1982). The rest of the present paper will build on these earlier results including magnetic fields and perpendicular diffusion, in the diffusion approximation.

Since the charged particles gain only a small amount of energy in each traversal of the shock front, the rate of energy increase depends on the rate at which particles scatter back and forth across the shock. Furthermore, since the maximum energy attainable in most situations is limited by the time available for acceleration due to a finite lifetime of the shock itself, escape of the particles from the vicinity of the shock, or to losses caused by collisions or synchrotron losses), the rate of energy gain also determines the maximum energy attainable. Previous discussions of the maximum attainable energy in diffusive shock acceleration have been based on the concept that the scattering mean free path determines the rate at which particles cross the shock. Hence the maximum energy gain rate occurs for the smallest scattering mean free path, which can not reasonably be smaller than the gyroradius r_g. This would then set an upper limit on the energies which can be attained, indepenent of the scattering mechanism. However, if the shock is such that the average magnetic field has a component normal to the propagation direction, and if $\kappa_\parallel > \kappa_\perp$, the gyromotion of the particle can carry it across the shock many times between each scattering, with the consequence that the energy gain rate can be much larger than previous discussions would allow. In the process, the particle drifts along the shock, as discussed previously (Jokipii 1982). This effect also occurs in the shock-drift mechanism discussed above, although in most discussions of this mechanism the diffusion approximation is not valid. The goal of the present paper is to study the consequences of the magnetic field for energy gain in the diffusion approximation, and to make revised estimates of the maximum energies attainable in a variety of situations.

II. THE BASIC PROBLEM

Consider first the idealized case of a locally plane shock wave propagating in the minus x-direction with speed V_{sh} in a homogeneous plasma in which there is an average magnetic field B which is uniform both behind and in front of the shock. Let the projection of $\boldsymbol{B}$ onto the shock define the z-direction. We work in the shock frame, in which the shock is stationary and the flow velocity $\boldsymbol{V}_1$ (whose magnitude is V_{sh}) is normal to the shock front (for the present discussion, we assume that the field is such that it has negligible effect on the flow, but merely acts as a passive additive). The parameters in front of the shock are given a subscript 1 and those behind a subscript 2. The fluctuating magnetic field irregularities are assumed to result in isotropization and consequent diffusion of energetic particles. We will come back in § IV to discuss the validity of the diffusion approximation.

Energetic particles then propagate according to the standard transport theory (e.g., see Jokipii 1971 for a review). If κ_{ij} is the diffusion tensor, f is the (nearly isotropic) phase space density, and $\boldsymbol{V}$ is the plasma velocity, the transport equation may be written in terms of position x_i, time t, and particle momentum p, as (Parker 1965; Axford 1965)

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x_i}\left(\kappa_{ij}^{(S)} \frac{\partial f}{\partial x_j}\right) - (V_i + V_{d,\,i}) \frac{\partial f}{\partial x_i} + \frac{1}{3} \nabla \cdot \boldsymbol{V}\left(\frac{\partial f}{\partial \ln p}\right) + Q(x_i, t, p) , \tag{1}$$

where, for particles of velocity w, momentum p, and charge q, $V_d = (pcw/3q) \nabla \times (\boldsymbol{B}/B^2)$, c is the speed of light, and where $Q(x_i, t, p)$ gives the local source strength.

The diffusion tensor in equation (1) has in general an antisymmetric part, $\kappa_{ij}^{(A)}$, in addition to the symmetric part, $\kappa_{ij}^{(S)}$. The antisymmetric part contains the effects of the Alfvénic drifts (Forman, Jokipii, and Owens 1974; Jokipii, Levy, and Hubbard 1977). The divergence of the antisymmetric part of the tensor is the usual drift velocity $\boldsymbol{V}_d$ averaged over the nearly isotropic pitch-angle distribution and this has been absorbed in the term containing $\boldsymbol{V}_d$ in equation (1). Note that in the diffusion limit the above expression is true even at the shock, where the flow velocity and magnetic field are in general discontinuous.

The term proportional to the divergence of the plasma flow velocity in equations (1) and (2) corresponds to the energy change of the particles due to the expansion or compression of the plasma. At the shock, this term is very large (in the limit, a delta function) and gives the net acceleration in the diffusion models. Note also, however, that the drift velocity is also very large at the shock if the magnetic field changes, as it will in all but purely parallel shocks.

The value of the drift velocity may be obtained in terms of the magnetic field values on the two sides of the shock. If we regard the shock as infinitesimally thin, and using the fact that B_z is the transverse component of the magnetic field, the drift velocity may be written in terms of the ratio $r = V_1/V_2$ (note: $1 < r < 4$), and the angle θ_1 between $\boldsymbol{B}_1$ and the x-direction as (Jokipii 1982)

$$V_d = \hat{e}_y \frac{pcw}{2qB_1} \frac{(r-1)[1-(r+1)\sin^2(\theta_1)]}{[\cos^2(\theta_1) + r^2 \sin^2(\theta_1)]} \sin(\theta_1)\delta(x - x_{\text{shock}}) . \tag{2}$$

It is of interest to note that for a given value of r the drift velocity in equation (2) changes sign as θ_1 changes smoothly from 0 to $\pi/2$, reflecting the changing contribution of the gradient drift (which dominates for large θ_1) and the curvature drift (which is in the opposite direction and which dominates for small θ_1).

III. ACCELERATION RATE

Equation (1) has been solved for many situations (Toptygin 1980). The time for acceleration of particles from an initial momentum p_0 to a momentum p_1 may be written as (see, e.g., Forman and Morfill 1979; Morfill *et al.* 1981; Drury 1983; Forman and Drury 1983)

$$\tau_a = \frac{3}{V_1 - V_2} \int_{p0}^{p_1} \left(\frac{\kappa_1}{V_1} + \frac{\kappa_2}{V_2}\right) \frac{dp}{p} , \tag{3}$$

where $\kappa_1 = \kappa_{xx}$ as a function of momentum p in the upstream region, and $\kappa_2 = \kappa_{xx}$ downstream of the shock. It is clear that the rate of acceleration is determined by κ_{xx} and the shock velocity and is otherwise independent of the direction of the magnetic field. κ_{xx} may be written in terms of the diffusion coefficients perpendicular and parallel to the magnetic field, $\kappa_\perp$ and $\kappa_\parallel$, and the angle θ between the magnetic field and the x-axis, as

$$\kappa_{xx} = \kappa_\parallel \cos^2(\theta) + \kappa_\perp \sin^2(\theta) . \tag{4}$$

Clearly, the rate of acceleration may depend in a complicated manner on the direction of the magnetic field relative to the shock normal, and on the functional form of the dependence of $\kappa_\perp$ and $\kappa_\parallel$ on particle energy. There is no general agreement on the value of $\kappa_\perp$ for particle transport in a turbulent magnetic field. The contribution of the field random walk or meandering, which arises in quasilinear theory, is complex and not fully understood (Jokipii 1971; Barge, Millet, and Pellat 1984). In view of the uncertainties, it is common to take the result from standard kinetic theory, which corresponds to ordinary collisions (see, e.g., Axford 1965), and this will also be done here. However, if field-line meandering plays a significant role, some of the following conclusions will change. Consider, then, the following specific case. Let the mean free path parallel to the magnetic field, $\lambda_\parallel$, be a constant factor η times the gyroradius r_g, so that $\kappa_\parallel = \eta r_g w/3$, and furthermore, take the kinetic theory value for the ratio of $\kappa_\perp$ to $\kappa_\parallel$, given by

$$\frac{\kappa_\perp}{\kappa_\parallel} = \frac{1}{1 + (\lambda_\parallel/r_g)^2} . \tag{5}$$

This corresponds, approximately, to a particle being shifted one gyroradius normal to the magnetic field in one scattering mean free path. Finally, let the angle of the magnetic field be θ_1 as defined above, and again consider a shock ratio r. It is then readily shown that

$$\kappa_1 = \frac{pcw\eta}{3qB_1}\left[\cos^2(\theta_1) + \frac{\sin^2(\theta_1)}{1+\eta^2}\right] \tag{6a}$$

and

$$\kappa_2 = \frac{pcw\eta}{3qB_1}\left[\frac{1}{\cos^2(\theta_1) + r^2\sin^2(\theta_1)}\right]^{3/2}\left[\cos^2(\theta_1) + r^2\frac{\sin^2(\theta_1)}{1+\eta^2}\right] . \tag{6b}$$

Then, defining the quantity $p^{-1}(db/d\tau_a)$ from equation (3) as a rate of acceleration, we may write for this case

$$\frac{1}{p}\frac{dp}{d\tau_a} = \frac{V_{sh}^2(r-1)}{3r(\kappa_1 + r\kappa_2)}, \tag{7}$$

with κ_1 and κ_2 as given above in equation (6).

In order to see the effect of the magnetic field, this can be compared with the usual result for the rate of acceleration for the quasi-parallel assumption, which has been used in prior discussions of this problem. Denoting by R_a the ratio of the present rate of acceleration to that obtained in the quasi-parallel case (where κ_1 and κ_2 are set equal to $\eta r_g w/3$), one obtains the expression

$$R_a = \{1 + r[\cos^2(\theta_1) + r^2 \sin^2(\theta_l)]^{-1/2}\} \Big/ \left\{\cos^2(\theta_1) + \frac{\sin^2(\theta_1)}{1+\eta^2} + r\left[\frac{1}{\cos^2(\theta_1) + r^2\sin^2(\theta_1)}\right]^{3/2}\left[\cos^2(\theta_1) + r^2\frac{\sin^2(\theta_1)}{1+\eta^2}\right]\right\}. \tag{8}$$

Values of R_a from equation (8) for $n = 10$ and $\eta = 100$, and for a strong shock ($r = 4$) are plotted in Figure 1. It is clear that if the angle θ_1 is 60° or 70° or larger, the acceleration rate can be enhanced considerably over the values used in previous work. In the limit that the shock has an angle of 90°, the acceleration rate is increased by a factor of $1 + \eta^2$. Since the mean free path is often taken to be a factor of 10 larger than r_g, the rate of energy gain can be increased by 100.

This analysis could be used in situations where $\kappa_\perp/\kappa_\parallel$ has a form different from that in equation (5), with similar conclusions if $\kappa_\perp/\kappa_\parallel \ll 1$. Physically, the reason for the increased acceleration rate is that the particle can drift along the shock face, effectively colliding with it many times in one scattering mean free path. Since these collisions are not the result of a diffusive process, the rate of energy gain is high.

It should be noted that, for purely normal shocks, this analysis would imply that there is no limit on the energy gain, which increases as the square of the ratio of $\lambda_\parallel$ to r_g. In fact, other factors will limit the attainable acceleration rate, and these will now be considered.

IV. LIMITS

The discussion in the preceding paragraph suggests that for perpendicular shocks, as long as the diffusion approximation is valid, the rate of energy gain will increase with the ratio of mean free path to gyroradius. Two effects act to limit the maximum energy gain.

First, as discussed previously (Jokipii 1982), the finite size of the shock front (transverse to the shock propagation direction) may result in a limit to the distance a particle can drift, and hence on the energy increase. For a quasi-normal shock, if $\kappa_\perp \ll \kappa_\parallel$, the energy gain is a constant (of the order of unity) times the potential energy gained in the $\boldsymbol{V} \times \boldsymbol{B}$ electric field. The finite size of the shock then limits the energy gain to a value of the order of the potential energy gain available in drifting along the shock face.

Second, even if the shock dimension does not provide a limit, the mean free path cannot be taken arbitrarily large. If it were to be too large, the distribution function would become highly anisotropic, and the diffusion approximation would no longer be valid. Without a complete analysis of the particle trajectories, a precise evaluation of just what the limit on $\lambda_\parallel$ is appears impossible. However, some estimates can be made.

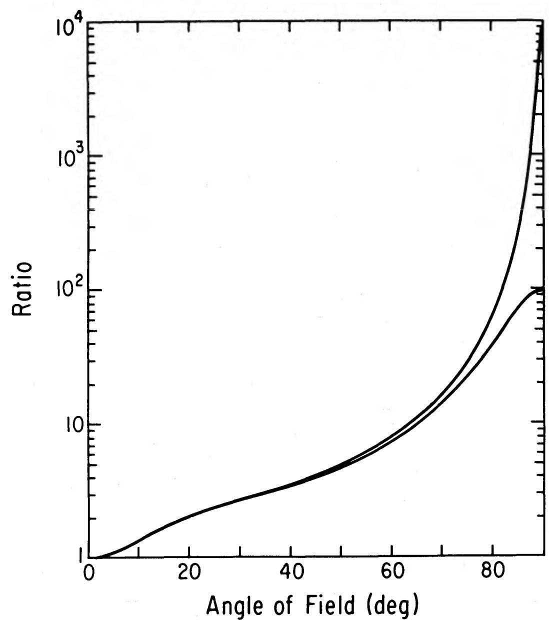

FIG. 1.—Plot of the ratio of energy gain rate with a transverse magnetic field to that neglecting the magnetic field given in eq. (8), as a function of angle between the upstream magnetic field and shock normal, θ_1. The upper curve is for a scattering mean free path $\lambda_\parallel$ equal to 100 times the gyroradius r_g, and the lower is for $\lambda_\parallel = 10\, r_g$.

Consider, for simplicity, a perpendicular shock. First we note that as a particle interacts with the shock and drifts along its face, it gains energy (this energy is mainly in the perpendicular velocity). At the same time its gyro orbit is being engulfed by the shock at approximately the shock velocity V_{sh}. One may estimate that the perpendicular energy of the particle will increase by a factor of the order of the shock ratio r in this process. This will produce a substantial anisotropy, and hence the particle must be scattered in the time required to drift through the shock to maintain near isotropy. This leads to the criterion

$$\frac{\lambda_{\parallel}}{w} < \frac{r_g}{V_{sh}} . \tag{9}$$

It is also instructive to evaluate the streaming anisotropy parallel to the upstream magnetic field. This comes out to be of order $\eta V_{sh}/w$. Again, the requirement that this be less than unity leads to the result in equation (9).

In addition, we should guarantee that the particles can diffuse upstream ahead of the shock, which requires that the "diffusion velocity" $\kappa_{xx} f^{-1}(\partial f/\partial x)$ be equal to or of order V_{sh}. But the gradient length scale $[f^{-1}(\partial f/\partial x)]^{-1}$ cannot be less than the particle gyroradius, which then leads to the requirement

$$\kappa_{xx} > r_g V_{sh} \tag{10}$$

or, for quasi-perpendicular shocks, again using the kinetic theory expression for $\kappa_{\perp}/\kappa_{\parallel}$ for the case $\eta \gg 1$,

$$\eta < \frac{w}{V_{sh}} , \tag{11}$$

which is the same as in equation (9).

It is possible that a more detailed study of orbits may yield a more stringent condition, but for the present, it appears that w/V_{sh} is a reasonable estimate for the upper limit on $\lambda_{\parallel}/r_g$, which then gives a new upper limit on the rate of energy gain, which in many cases, for high-energy particles, may be much larger than that obtained in previous discussions. In cases where $\kappa_{\perp}/\kappa_{\parallel}$ is not given by the kinetic theory result (if, for example, field line meandering plays an important role), some other constraint would apply, with different consequences for the maximum rate of energy gain.

Applying this to equation (8) we see that the upper limit on the ratio R_a is of the order of $(w/V_{sh})^2$ if $\kappa_{\perp}$ is as in equation (5). Since typical shock speeds are of the order of 10^8 cm s^{-1}, and $w \approx c$, the rate of acceleration can be as high as 10^4 times the quasi-parallel rate.

Note that equations (11) and (9) become quite stringent constraints for low-energy particles, where w may not be much larger than V_{sh}. Hence, these considerations lead to the expectation that low-energy injection occurs more readily when the shock is quasi-parallel, as suggested previously by other authors (e.g., Pesses, Jokipii, and Eichler 1981). However, if the particle velocity is much higher than the shock velocity, very efficient diffusive acceleration can occur at perpendicular shocks.

Finally, it should be mentioned that the reduction of the effective diffusion coefficient can result, in certain cases, (such as, for example, the termination shock of a wind) in the particles being more effectively confined to the region of the shock, which also helps increase the maximum energy.

V. APPLICATIONS

Now consider briefly the application of the above results to a number of important astrophysical contexts.

a) *Termination Shocks*

The acceleration of charged particles at the termination shock of a wind (first considered by Jokipii 1968, and subsequently studied by Cassé and Paul 1980; Volk and Forman 1982; Webb, Forman, and Axford 1985; Pesses, Jokipii, and Eichler 1981; and Jokipii and Morfill 1985, 1986) is of considerable current because it may explain the anomalous component and ultra–high-energy cosmic rays. Because the magnetic field in the wind is an Archimedean spiral which is tightly wrapped near the termination shock, the shock will be nearly normal except for a small region near the rotation axis. Hence the above considerations will apply. Note that Volk and Forman (1982) and Webb, Forman, and Axford (1985) explicitly excluded the possibility of perpendicular diffusion, so their conclusions differ from those in this paper. Typical wind velocities are a few times 10^7–10^8 cm s^{-1}, so the upper limit on the mean free path is perhaps a few hundred times the gyroradius. As demonstrated by Jokipii and Morfill (1986) in detailed numerical calculations, the resulting maximum energies in the case of the Galactic wind can be as large as the highest observed energies ($>10^{19}$ eV). For the case of the solar wind and the anomalous component, as pointed out by Pesses, Jokipii, and Eichler (1981) and Jokipii (1986), the implied injection of low-energy particles over the poles of the solar wind terminal shock (where the shock is quasi-parallel) appears to be in agreement with observations.

b) *Supernova Blast Waves*

Acceleration at supernova blast waves is generally accepted as the source of the bulk of Galactic cosmic rays. The finite lifetime of the blast wave leads to an upper limit on the energy of particles accelerated at a supernova shock. The most thorough and comprehensive discussion of this problem was published by Lagage and Cesarsky (1983), who estimated the maximum energy as $\sim 10^{14-15}$ eV. Again, if the supernova occurs in a coherent magnetic field, the blast wave will be quasi-normal over a substantial part of its surface (a band near the "equator"), and the considerations of the present paper will be applicable, although a full calculation is required to determine the effect. We expect that the maximum energy may be increased by a substantial factor.

c) Accretion Shocks

A number of authors have suggested that accretion shocks around some objects can explain the presence of the high-energy particles necessary to explain the high-energy γ-rays emitted (e.g., Kazanis and Ellison 1986; Cowsik and Lee 1982). Again, in these objects, it is possible that the magnetic field may be quasi-perpendicular to the shock normal, with the consequence that the maximum energy of the accelerated particles may be substantially higher than previously estimated.

d) Interplanetary Corotating Shocks

Again, the corotating shocks observed in the solar wind are often quasi-perpendicular, and the considerations in this paper must be applied. However, in some cases, it appears that the scattering is insufficient and the diffusion approximation is not accurate.

e) Solar Flares

Lee and Ryan (1986) have recently suggested that energetic particles associated with solar flares are accelerated by a shock wave produced by the flare explosion. The model has many attractive features, but the authors find that the acceleration times are somewhat longer than those inferred from particle observations. Again, if the magnetic field were quasi-perpendicular, the acceleration time could be substantially shortened as discussed above. Application to oblique shocks has recently been explored, from a consideration of particle trajectories by Decker and Vlahos (1986).

VI. SUMMARY AND CONCLUSIONS

The rate of acceleration and hence the maximum energy attained in diffusive acceleration is quite sensitive to the geometry of the magnetic field. If the shock is quasi-normal, the rate of acceleration may be orders of magnitude larger than previous discussions, neglecting the magnetic field, would imply. Diffusive shock acceleration therefore is potentially more efficient than previously thought.

The acceleration for a quasi-normal shock is determined primarily by the perpendicular diffusion coefficient $\kappa_\perp$, which is poorly understood. Nonetheless, the present discussion establishes the fact that if $\kappa_\perp$ is finite, the acceleration process may be significantly more efficient than previously thought, and the rate of acceleration increases as $\kappa_\perp$ decreases down to some minimum value. This minimum value of $\kappa_\perp$ is determined by the requirement that the diffusion approximation be valid and that the diffusing particles be able to diffuse upstream to encounter the shock repeatedly.

For the standard kinetic relationship between perpendicular and parallel diffusion, $\kappa_\perp/\kappa_\parallel = 1/[1 + (\lambda_\parallel/r_g)^2]$, the condition $\lambda_\parallel/r_g < w/V_{sh}$ must be satisfied. In this case the ratio of the energy gain rate to that for a quasi-parallel shock can be of order $(w/V_{sh})^2$. In cases where there are other contributions to the perpendicular diffusion (such as meandering of field lines, etc.), the quantitative conclusions will change, but the general result that acceleration can be enhanced in quasi-normal shocks will still be valid.

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J. R. Jokipii: Department of Planetary Sciences, University of Arizona, Tucson, AZ 85121