Astron. Astrophys. 178, 283–285 (1987)

ASTRONOMY
AND
ASTROPHYSICS

The fate of the Earth in the red giant envelope of the Sun

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Received July 22, accepted September 19, 1986

Summary. A number of hydrostatic models may be found in the literature for red giants whose main sequence progenitors are solar-like stars. Of those models that are characteristic of the period beyond He core depletion when the envelope is maximally extended, some provide stellar radii just in excess of the earth-sun distance. For such a situation, if drag forces, wind ram pressure and vaporization are not too severe at the low temperature, low density surface of the solar red giant envelope, one might expect the earth to survive through the planetary nebula phase, eventually to orbit the relic solar white dwarf. Employing a 30 zone red giant model, the earth orbital decay timescale neglecting ablation/vaporization is determined to be of the order of 200 years, rendering earth survival impossible. The effects of ablation/vaporization processes are found to increase the ballistic coefficient of earth, thereby setting the 200 year decay timescale as an upper limit.

Key words: solar red giant models – hypersonic drag – orbital evolution

Star-planet orbital evolution has been discussed for the general case by Livio (1982) and Livio and Soker (1983, 1984). In the latter papers, the orbital evolution of planets of varied initial mass and with an initial location at $454R_\odot$ (normalized to present sun), was investigated employing a model stellar red giant envelope with $R/R_\odot = 417$. Livio and Soker found two distinct scenarios. Planets whose masses were less than 1% of the primary were found to evaporate during orbital decay while those greater than 1.25% accreted mass from the primary thereby providing one means of formation for low mass cataclysmic binary systems.

In this Paper, I present a specific investigation of our own star-planet system employing expected future earth-sun initial conditions and character.

Model evolutionary tracks for solar-type stars (Schönberner, 1979) provide the mass (M), luminosity (L) and surface temperature (T) expected in some $5\,10^9$ years at the point in the red giant phase when the solar envelope is maximally extended. A 30-zone red giant hydrostatic model developed by Rose and Smith (1972) with $M/M_\odot = 0.856$, $L/L_\odot = 3984$ and $T/T_\odot = 0.525$ corresponds well to the expected values and implies an envelope radius of 1.1 times the Earth-Sun distance. This hydrostatic model was adopted for the analysis and was considered non-rotating, which is a good approximation if the sun's present angular momentum is conserved.

Prior to earth entry into the model solar envelope, the earth does not experience significant mass loss via thermal processes (Jeans, 1916). For escape of an atmospheric component on a timescale of $\leq 10^8$ years (a reasonable value for the duration of the red giant phase), Jeans' theory requires that the rms thermal velocity of that component be > 0.21 times the escape velocity at the exobase. This is not even the case for monoatomic nitrogen assuming an exobase in radiative equilibrium with the solar envelope (Fig. 1). In addition, more refined thermal escape models (Spitzer, 1952), which take into account the higher atmospheric mass associated with a non-isothermal lower terrestrial atmosphere, result in an increased Jeans escape timescale. It should be noted, however, that if various non-thermal escape processes (Hunten, 1982; Kumar et al., 1983) dominate, significant mass

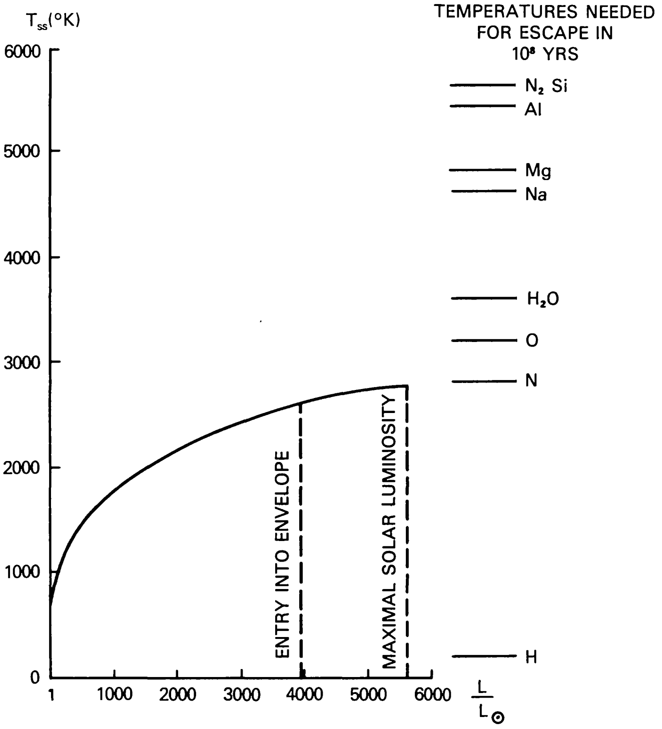

Fig. 1. The subsolar exospheric temperature of earth (T_{ss}) is plotted against solar luminosity as the Sun evolves from the present epoch to the point in the red giant phase where the solar envelope is maximally extended. Temperatures necessary for escape of various atomic and molecular species in 10^8 years are indicated at right. Jeans' escape is assumed as well as an Earth albedo of 0.5

loss could result prior to earth entry. Atmospheric escape of hydrogen is possible, but of little interest for the terrestrial planets.

On entry into the solar envelope, the earth's orbital velocity corresponds to a Mach number of 7 which, from the earth frame, is roughly the ratio of directed to thermal energy in the intercepted stream. The Earth cannot gravitationally accrete significant amounts of H and He from the envelope – just the thermal contribution to the velocities associated with these components is too high to allow capture by Earth's weak gravitational field. One would expect, however, ongoing ablation/vaporization processes.

The drag force (F_D) associated with the hypersonic flight of a small gravitating mass through the extended envelope of a primary has been investigated by Alexander et al. (1976) and is given by

$$F_D = \pi R^2 \rho(r) v^2(r) [1 + \beta] \tag{1}$$

where

$$\beta = \ln (R_{\max}/R)$$

Here r is the distance between the gravitating mass and the center of the primary, $v(r)$ is the orbital velocity, $\rho(r)$ is the density of the local environment, $R_{\max}$ is the maximal extent of the planet's gravitational field usually defined as the neutral point between planet and star, and R is the radius of the interaction cross section taken as the geometric radius of earth.

The first term in Eq. (1) represents the ram pressure force on a hard sphere of radius R while the second term accounts for the long range gravitational interaction. In the hypersonic limit thermal pressure is small compared to ram pressure so that the drag effects of the former may be neglected.

With regard to dynamics, one can assume circular orbits and a virially relaxed system if the orbital energy is nearly an adiabatic invariant over one orbital period. Such turns out to be the case here. The equation of motion neglecting ablation/vaporization processes, is given by

$$[1 + \beta]\dot{m}/m = -\frac{d}{dt} \ln [r v(r)] \tag{2}$$

where $\dot{m}$ is the envelope mass intercepted per unit time and m is the mass of earth. Integration of Eq. (2) within the model solar envelope of Rose and Smith yields r as a function of time t. The decay time turns out to be remarkably short with a 99% decrease in orbital radius in less than 300 years (Fig. 2).

The specific case of Earth orbital decay within the solar red giant envelope was first discussed by Vila (1984). Assuming a constant orbital velocity throughout the decay, he defined a "mean survival time" $\tau_v = m/\dot{m}$, which in reality is the $1/e$ time for orbital radius decay. His treatment neglected the long range gravitational drag interaction β and implies a solar envelope where $\rho(r) \propto 1/r^2$. This simplistic picture, while admitting an easily obtained analytic solution, fits poorly to relevant hydrostatic models. In addition, the hydrostatic model he used to determine the maximal extent of the solar envelope is inconsistent with the evolutionary track for the Sun when the effects of mass loss due to winds are included (Schönberner, 1979). The e-folding time (τ_v) from the present treatment is only 210 years as compared with Vila's value of 5000 years.

τ_v is proportional to R^2/m where R^2/m may be taken as earth's ballistic coefficient (BC). If ablation/vaporization processes are now considered, it is clear that the possible resulting variation in the BC directly affects τ_v.

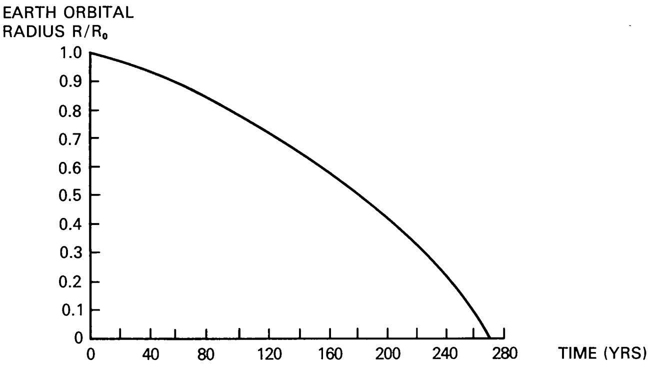

Fig. 2. Earth orbital decay. Earth orbital radius in astronomical units is plotted as a function of time. The $1/e$ timescale is only 210 years

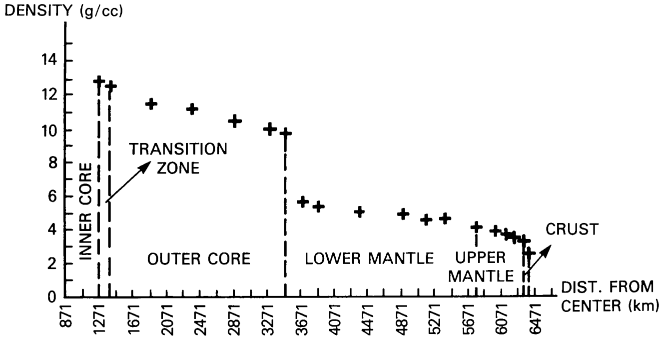

Fig. 3. A model Earth density profile (Allen, 1976)

Figure 3 represents a terrestrial density profile (Allen, 1976) from which one can obtain the BC as a function of distance from the center of Earth (Fig. 4). While remaining approximately con-

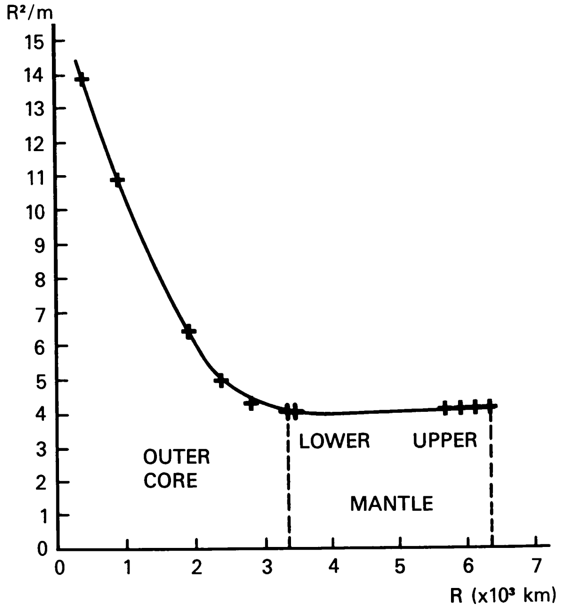

Fig. 4. Ballistic coefficient of Earth. Within the solar envelope ablation/vaporization processes act to reduce Earth's radius (R). Such processes increase the ballistic coefficient of Earth thereby enhancing orbital decay. R^2/m is in units of 10^7 km^2/Earth mass

stant as the Earth's mantle is removed through ablation and vaporization, the *BC* increases dramatically in the outer core, rendering $\tau_v = 210$ years as an upper limit.

A significant planetary bow shock, as in the case of Venus, may exist even in the absence of a strong planetary magnetic field. Only a well established ionosphere is necessary to support such a shock and should be expected for earth within the solar red giant envelope. The radius of the interaction cross section might then, in fact, be larger than the radius of solid Earth due to drag forces experienced by layers of the ionosphere. Such a condition will serve only to enhance orbital decay.

So far, the stellar wind normally associated with the red giant phase has been neglected. Due to a radially symmetric wind, the associated wind ram pressure experienced by the Earth will oppose solar gravitation tending to increase τ_v. The ratio (Φ) of the former to the latter is given by

$$\Phi = [BC][\dot{M}/M(r)][\tilde{v}/2G] \qquad (3)$$

where $\dot{M}$ is the solar mass loss rate due to the wind, $M(r)$ is the solar mass enclosed within orbital radius r and $\tilde{v}$ is the wind speed. A Φ approaching zero therefore implies a totally negligible wind effect.

IUE studies (Dupree, 1982) provide the ranges for $\dot{M}$ and $\tilde{v}$ that should characterize red giants. Typical values for such stars are $\dot{M} = 10^{-6}\,M_\odot/\mathrm{yr}$ and $\tilde{v} = 50\,\mathrm{km\,s^{-1}}$ which yield $\Phi = 0.0025$ assuming a whole Earth *BC*. Red giants whose progenitors are solar-like stars are expected to have significantly lower $\dot{M}$ so that Φ becomes effectively zero. In addition, within the solar envelope the presence of such winds is questionable.

Acknowledgements. The author wishes to thank the referee Dr. E. Schatzman and Dr. M.J. Mumma for critically reading the manuscript.

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