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Gravitational radiation from cosmological phase transitions

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Summary. Cavitation during a first-order phase transition, which may have occurred in the early Universe as a consequence of QCD or electroweak interactions, would have produced gravitational radiation in two ways: by generating acoustic noise in relativistic plasma, and by perturbing the expansion law on large scales. Here I estimate the spectrum of the resulting stochastic background, its dependence on the parameters governing the phase transition, and the possibility of observing it above instrumental noise and other gravitational wave backgrounds of local and cosmological origin.

1 Introduction

The most frustrating property of gravitational waves – their extremely weak interaction with matter – also makes them ideal probes of the very early Universe. Gravitational waves generally propagate without attentuation throughout the relativistic era, right back to the Planck time. Some non-negligible absorption may occur, but only when two unusual conditions are met simultaneously (Vishniac 1982): (i) the wave frequency $\nu \simeq H$, the expansion rate, and (ii) there is a significant density of relativistic particles with a mean free path of order H^{-1}. Moreover, relativistic matter, with sound speed $c/\sqrt{3}$, is an excellent material for efficiently producing gravitational waves. For these reasons, the early Universe has long been conjectured to be an interesting source of gravitational radiation (Carr 1980; Rees 1983).

One particularly interesting cosmological source of gravitational waves is the relativistic bulk motion induced by first-order cosmological phase transitions. First-order transitions are tentatively predicted to occur in the standard model of strong, weak, and electromagnetic interactions. Thus the cosmological events considered here are unlike many other proposed sources of gravitational radiation, which often invoke either speculative physical theories, such as grand unification with cosmic strings (Vilenkin 1982), or speculative astrophysical scenarios, involving, for example, populations of massive black holes (Bond & Carr 1984). (Some of these other possible backgrounds, if they exist, might swamp the background discussed here.) Witten (1984) considered the possibility of detecting a signal from the QCD transition in timing measurements

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1986MNRAS.218..629H

of the millisecond pulsar. The present work extends this discussion with a detailed investigation of the processes involved, an estimate of the gravitational wave spectrum, consideration of other phase transitions and other detection techniques, the effect of the transition on any pre-existing gravitational waves, and comparison with likely sources of astrophysical noise. An important motivation for this work is to aid in design studies for spaceborne detectors.

In Section 2 the production of gravitational waves in a transition is analysed and the main features of the spectrum are derived. The effect of the phase transitions on pre-existing waves from an inflationary era via their effect on the expansion law are discussed in Section 3. The brief discussion on detectability (Section 4) will focus on instrumental noise and on the most intense astrophysical background known to exist in the relevant frequency range – waves generated by binary stars.

A plane gravitational wave has an energy density (Misner, Thorne & Wheeler 1973)

$$\varrho_g = \langle \dot{h}^2 \rangle / 16\pi \tag{1}$$

where h is the amplitude of the metric strain and we have set $G \equiv 1 \equiv c$. A stochastic background is a superposition of such waves, with Gaussian random Fourier amplitudes for each mode and polarization. The volume-averaged density of waves is most conveniently expressed in units of the critical density $\varrho_c \equiv 3H_0^2/8\pi$, where $H_0 \equiv 100\ h_{100}$ km s^{-1} Mpc^{-1} is Hubble's constant. Let $\langle h_\nu^2 \rangle$ denote the dimensionless mean square metric strain in a broad band of frequency $\delta\nu \simeq \nu$, so that h_ν is approximately $\nu^{1/2}$ times the usual spectral density. Omitting a spectral-dependent numerical factor of order unity,

$$\Omega_g(\nu) \equiv \varrho_c^{-1} \frac{d\langle \varrho_g(\nu) \rangle}{d \log \nu} \simeq \langle h_\nu^2 \rangle (\nu / H_0)^2$$

$$\simeq (h_\nu / 10^{-18})^2 (\nu / Hz)^2 h_{100}^{-2}. \tag{2}$$

In an expanding universe with scale factor a, both ν and h go like a^{-1}, so $\varrho_g \propto a^{-4}$, like the electromagnetic radiation background or relativistic matter with conserved entropy. Thus, ignoring sources of entropy increase such as out-of-equilibrium decay (Krauss 1985), Ω_g / Ω_{rad} is conserved, where Ω_{rad} ($\sim 2.4 \times 10^{-5}\ h_{100}^{-2}$ at the present epoch) is the density parameter of the photon background plus any other relativistic species that eventually annihilate or decay adiabatically into the photon background. It is particularly interesting that several detection techniques have projected sensitivies $\Omega_g \lesssim \Omega_{rad}$, which is the realm of plausible signals from the early Universe (see Fig. 1).

Usually a increases more slowly than t, and the physical wavelength of a particular wave decreases with time relative to the Hubble length. A wave which had $\nu = H'$ ('entered the horizon') at a temperature T' now has a frequency $\nu_0(T') = H'a'$, where a is normalized to unity today. For an adiabatic expansion, the entropy density $\propto NT^3 \propto a^{-3}$, where N is the number of effective degrees of freedom at T, and for T in the radiation-dominated era of a Friedmann expansion, $\varrho_{rad} = \frac{1}{2} N a_{Stefan} T^4 = 3\ H^2/8\pi$. Assuming $\Omega_0 = 1$, $H_0 = 100$ and $T_0 = 2.7$ K, we derive*

$$P_0(T) \equiv \nu_0^{-1} = 6 \times 10^6 \text{ s } N^{-1/6}\ T_{GeV}^{-1}. \tag{3}$$

This gives the present period of waves which matched the Hubble length at temperature T_{GeV} GeV. Thus we find that experiments in a practical range of experimental frequencies probe very early epochs indeed; wave periods from 2 yr down to 10^4 s, accessible to astronomical detection techniques, probe temperatures of the order of 100 MeV up to 600 GeV. This is an

*The N-dependence differs from equation (B6) of Witten (1984) because we have included the effect of N on T at fixed entropy. Here, it is clear that the N-dependence can virtually be ignored.

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1986MNRAS.218..629H

interesting range of temperatures, because of the phase transitions which may occur in this range due to QCD or to symmetry breaking of the electroweak interactions.

2 Gravitational waves from phase transitions

Suppose relativistic matter undergoes a first-order phase transition at a critical temperature T_1, from a high-temperature phase A to a low-temperature phase B. The cosmological expansion supercools the matter by δT in phase A below T_1 in a metastable state. Bubbles of phase B nucleate at isolated points in the plasma and expand rapidly (near the speed of light) until they equilibrate the mean pressure in the two phases. Nucleation ceases when the bubbles have expanded enough to compress the remaining phase A back to the critical temperature and pressure T_1, P_1, where the two phases can coexist in equilibrium (see Hogan 1982, 1983; Witten 1984, De Grand & Kajantie 1984; and Applegate & Hogan 1985 for further discussion of these processes.) Although the medium is now in phase equilibrium, it is not quiescent. Pressure waves from the expanding bubbles produce a background of acoustic noise which in turn generates a gravitational wave background. In addition, the coexistence of bubbles of matter at two different densities produces large-scale density fluctuations on a time-scale H^{-1} because of fluctuations in the effective equation of state and hence in the expansion law. Let us estimate the spectrum of the background produced by these two effects.

Consider a medium with a stochastic superposition of plane-wave pressure fluctuations $(\delta p/p)$ of wavelength R. These fluctuations generate gravitational waves with frequency $\nu \simeq v_S/R$, where v_S is the sound speed. The gravitational wave energy density generated in one wave period R/v_S is a fraction of the total energy density given by

$$\Omega_g/\Omega_{rad} \cong (L/L_0) v_S^{-1} \tag{4}$$

where L is the gravitational luminosity of a typical region of size R and L_0 is the 'Dyson luminosity', $L_0 = c^5/G = 3.6 \times 10^{59}$ erg s^{-1}=1 in our units, which corresponds to radiating the entire rest mass of such a region in a light crossing time. The standard quadrupole radiation formula, valid if R/v_S is smaller than the Hubble distance H^{-1}, gives

$$L/L_0 \sim \alpha^2, \tag{5}$$

where α denotes the quadrupolar component of the internal acoustic energy flow in such a region (Misner *et al.* 1973; Thorne 1983).

Consider first the effect of acoustic perturbations. Expanding bubbles nucleated at random locations produce shot noise pressure fluctuations, which leads to acoustic white noise on scales much larger than the typical bubble separation R_n. Thus pressure fluctuations averaged over regions of size R have mean square amplitude

$$\langle (\delta p/p)^2 \rangle_R \simeq (R_n/R)^3 \delta^2 \tag{6}$$

where δ is the fractional supercooling $(\delta T/T_1)$ undergone by the medium before nucleation. Stochastic pressure fluctuations on scale R lead to

$$\alpha \simeq \left(\frac{M v_S^2}{R/v_S} \right) \left(\frac{\delta p}{p} \right) \tag{7}$$

where M is the mass-energy in a sphere of radius R. Using the Friedmann equations, one can show that $M/R \simeq (RH)^2$. If we suppose that there is no damping of sound waves, so that waves undergo

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1986MNRAS.218..629H

$\simeq \nu/H$ oscillations in a Hubble time, we find from these formulae that

$$\Omega_g/\Omega_{rad} \simeq \delta^2 (R_n H)^3 v_S^6, \qquad (H \lesssim \nu \lesssim v_S/R_n), \tag{8}$$

independent of ν. In the case of maximal damping, where waves undergo just one oscillation before being damped out (by streaming of collisionless relativistic particles, say), Ω_g drops off as ν^{-1} at high frequencies. For $\nu < v_S/R_n$, the spectrum cannot be calculated statistically, as it depends on the details of the plasma motion on scales $\lesssim R_n$. However, even an optimal scenario (with collisions of coherent planar slabs) would lead to a fall-off at least as rapid as $\Omega_g/\Omega_{rad} \propto \nu^{-1}$ at high frequencies.

Another effect generates gravitational waves with $\nu \lesssim H$ with an amplitude which is insensitive to the supercooling before nucleation. During the period when the two phases are in approximate pressure equilibrium, fluctuations are produced because the universe is expanding with a spatially varying effective equation of state (Press & Vishniac 1980). The pressure of a region of pure phase B decreases like a^{-4}, but if it contains a lump of phase A material it decreases more slowly. Thus, fluctuations in the mixture of phases A and B lead to large-scale matter flows to equilibrate pressure. The movement of material takes place on a time-scale H^{-1} on all length scales, so very little power appears in high-frequency waves. The typical fluctuation on the scale $R \sim v_S/H$ for which the acoustic frequency matches the expansion rate $\nu = H$ is $\delta p/p \sim (R_n H/v_S)^{3/2}$. On this scale, $\alpha \sim v_S^5(\delta p/p)$, leading to waves with amplitude

$$\Omega_g/\Omega_{rad} \cong (R_n H)^3 v_S^6 \qquad (\nu \sim H). \tag{9}$$

For $\nu < H$, gravitational waves are outside the horizon at T_1. They have amplitude h_{enter} when they enter the horizon approximately equal to their *initial* rms pressure perturbation $(\delta p/p)$ (Bardeen 1980; Press & Vishniac 1980). This leads to a spectrum

$$\Omega_g/\Omega_{rad} \propto h_{enter}^2 \propto \nu^3 \qquad (\nu < H). \tag{10}$$

Thus we arrive at the spectral shape shown as QCD and EW in Fig. 1. The solid portions of the spectra QCD and EW in Fig. 1 represent definite predictions for the spectral shape, while the dashed portions represent an upper envelope, the shallowest spectrum that could be produced if the acoustical generation operated with maximal effectiveness.

Two uncertain parameters have entered this discussion – the nucleation scale R_n and the supercooling before nucleation δ. The first of these affects the normalization of the entire spectrum, while the second affects the acoustically generated part, $\nu > H$. Let $W(t)$ be the probability of a nucleation occurring before time t in a microscopic volume T^3 (we now set $\hbar \equiv 1$). If a relaxation wave propagates out from each site at velocity v_S which turns off further nucleation, then a good estimate of the mean separation between bubbles when most of the volume has relaxed is

$$R_n \simeq (W/\dot{W}) v_S. \tag{11}$$

Thus R_n is the mean separation of nucleated sites when the mean separation is changing at a rate equal to v_S. Before this time, new sites proliferate faster than the influence of existing sites can propagate; after this time, compression waves from any new bubbles can easily grow to meet their neighbours, so all of the out-of-equilibrium phase A material is quickly compressed. For material free of impurities, $W(t)$ and δ are determined by the surface tension between the two phases. In the case of QCD, this can in principle be calculated from first principles, using lattice techniques. The spectrum of the gravitational wave background could thus become a fairly firm prediction of the standard cosmological model with standard QCD. In the case of electroweak theory, the nature of the transition depends on the symmetry-breaking mechanism, which is poorly understood. Experimental data (e.g. from SLC) on the Higgs sector should improve this situation.

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1986MNRAS.218..629H

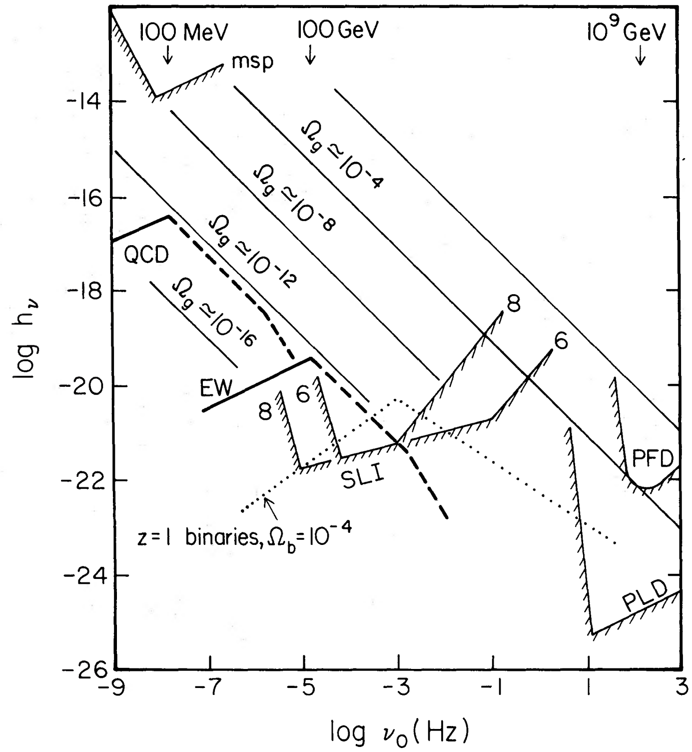

Figure 1. Spectrum of predicted gravitational wave backgrounds, and sensitivity of detection techniques. Dimensionless broad-band metric strain h_ν is plotted against frequency ν. QCD and EW represent the spectrum of the stochastic background produced by phase transitions at ~100 MeV and ~100 GeV, respectively. Lines labelled Ω_g show where the peak of a broad spectrum $\delta\nu \sim \nu$ with the indicated energy density would lie. Curve 'msp' indicates millisecond pulsar, and 'SLI' indicates spaceborne laser interferometer, with 6 and 8 referring to 10^6 and 10^8 km baselines. 'PFD' and 'PLD' are possible first and later detectors in a large ground-based interferometer. The dotted line represents a simple model of the background expected from binary stars, as described in Section 4. Energies at the top show the temperature at the time waves of the designated present-day ν had $\nu = H$.

In principle nucleation may also be catalyzed by various impurities, such as monopoles, whose abundance depends on the previous cosmic history and is therefore on a more speculative footing than the standard model. If we ignore this possibility, it is not implausible that for either transition δT is of the same order as T_1 – corresponding to a strongly first-order transition, with a large surface tension between phases. Even if δT is this large, however, the time-scale for thermal or quantum nucleation $W/\dot{W}$ does not normally exceed $\sim\{H \ln (T/H)\}^{-1} \sim 10^{-2}/H$ (Hogan 1983). This gives an estimate of R_n that is applicable to a broad class of nucleation mechanisms, but one must bear in mind that larger or smaller values can in principle occur.

The QCD and EW spectra in Fig. 1 are normalized to show the spectrum of gravitational waves which would arise from transitions with $(\delta T/T) \simeq 1$, $(R_n H) \simeq 10^{-2}$, and $v_S \sim 3^{-1/2}$, which correspond to typical values for these parameters in a strongly first-order transition. The resulting peak value of $\Omega_g/\Omega_{rad} \simeq 10^{-7.5}$ is preserved by expansion, and the frequency $\nu \sim H$ is normalized to the present by equation (3). Note that the amplitude at $\nu \lesssim H$ is not sensitive to δ. Other first-order transitions would, for the same parameters, produce a spectrum of the same shape and Ω_g, shifted to the appropriate $P_0((T_1)$.

3 Primordial waves

The most widely accepted theory for the origin of cosmological structure posits primordial scale-invariant scalar metric fluctuations. Mechanisms for producing such fluctuations, such as quan-

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1986MNRAS.218..629H

tum effects in inflationary models, invariably also produce tensor fluctuations of comparable amplitude, which ultimately are interpreted as a stochastic gravitational wave background (some effects produce *only* tensor fluctuations). [Various such processes are discussed by Starobinsky (1979), Rubakov, Sazhin & Veryaskin (1982), Abbott & Wise (1984), Halliwell & Hawking (1985), Hawking (1985), and other authors referred to in these papers.] It is thus interesting to investigate the consequences of pre-existing stochastic gravitational waves having some fixed amplitude $\langle h^2\rangle^{1/2}\equiv h'$ when they cross the horizon, $\nu=H$. A scale-invariant spectrum of this type might extend to very high frequencies indeed, given by equation (3) with T set equal to the reheating temperature after inflation. This is typically $\gtrsim 10^{14}$ GeV, leading to a maximum frequency $\gtrsim 20$ MHz.

The value of h' can in principle be derived from the parameters of an inflationary model, where it can be generated in two distinct ways – via quantum fluctuations of the gravitational field itself, or via fluctuations in some scalar field or fields ϕ which control the inflation and are coupled to gravity. We have in any case an observational bound on $h'\lesssim 10^{-4}$ from the quadrupole moment of the microwave background. If the corresponding scalar perturbations are to produce galaxies and clusters, we have also a lower bound, $h'\gtrsim 10^{-6}$.

Let H', a' denote the expansion rate and scale factor when a wave of observed frequency ν_0 crosses the horizon. Then from (2), redshifting h and ν gives

$$\Omega_g(\nu_0)\cong(h'a')^2(H'a')^2=h'^2(H'a'^2)^2. \qquad (12)$$

For adiabatically expanding, relativistic plasma, Ha^2 is constant, and in fact waves which enter the horizon during the radiation dominated era have

$$\Omega_g\equiv h'^2\Omega_{rad}=\text{const }(\nu^0). \qquad (13)$$

Note that in the adiabatic limit the spectrum is not altered by annihilation of relativistic particle species (that is, reducing N). Changing N alters $H(T)$ and $P_0(T)$ (equation 3), but leaves the equation of state, $p=\varrho/3$, and the expansion law unchanged. Since gravity couples only to the energy-momentum tensor, the gravitational wave spectrum $\Omega_g(\nu)$ must be unaffected by such events. This disagrees with some conclusions of Krauss (1985).

However, features may be introduced into the spectrum in situations where the expansion law *is* altered – for example, during one of the first-order phase transitions just mentioned. In a confinement transition for example N changes discontinuously by a factor N_u/N_c. Confined and unconfined phases can coexist in equilibrium at the critical pressure and temperature P_1, T_1, where $P_1\simeq N_cT_1^4$, the thermal pressure of the confined phase. This means that $P_1<N_cT_1^4$ – the thermal pressure of the u phase is partially cancelled by a negative-pressure 'bag constant' from gluon interactions. Therefore one expects that for an expansion in volume by a factor of order N_u/N_c, the equation of state is $p<\varrho/3$ (Bonometto & Pantano 1984). In the case where $p\ll\varrho/3$, as in non-relativistic matter, $a\propto H^{-2/3}$, so the gravitational wave spectrum (10) goes like $\Omega_{gw}\sim\nu^{-2}$ over a range of ν of order $(N_u/N_c)^{1/6}$. For QCD, this is a small effect; the spectrum of primordial waves remains flat to within a factor of about 20 per cent in Ω_g.

4 Detection

Projected sensitivites to stochastic backgrounds of three observational techniques are shown in Fig. 1. These are: (i) the limit attainable in principle with about 10 yr of timing measurements on the millisecond pulsar, at frequencies $\nu_0\lesssim 10^{-7}$ Hz (Blandford, Narayan & Romani 1984); (ii) the limit reached after an integration time $\hat{t}\sim 10^6$ s with a plausible first detector, and a possible later

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1986MNRAS.218..629H

detector, in the proposed Caltech–MIT dual 5 km laser interferometer gravitational wave detection system, at $\nu_0 \gtrsim 10$ Hz; and (iii) the limit reached after $\hat{t} \sim 10^6$ s with a spaceborne laser interferometer, at $10^{-6} \lesssim \nu_0 \lesssim 10$ Hz.

The most promising technique for detecting waves from the QCD or EW phase transitions appears to be laser interferometry in space. With a 1-mW laser and 50-cm mirrors, the spectral density of photon shot noise is expected to be (Faller *et al.* 1984)

$$|(\delta l/l)^2|^{1/2} \sim 10^{-19}/\sqrt{\mathrm{Hz}} \sim 10^{-22}/\sqrt{10^{-6}\ \mathrm{Hz}} \tag{14}$$

approximately independent of the baseline. With standard coincidence techniques this would allow detection of a stochastic background as weak as (Weiss 1979)

$$h_\nu \sim \frac{\sqrt{\nu}(\delta l/l)_{\mathrm{noise}}}{\pi(\nu\hat{t})^{1/4}} \sim 10^{-22}\left(\frac{\hat{t}}{10^6\ \mathrm{s}}\right)^{-1/2}(\nu\hat{t})^{1/4} \tag{15}$$

with an integration time of $\hat{t} \sim 10^6$ s. Photon noise is expected to be the dominant source of noise from $P \sim 10$–10^3 s up to $P \sim 10^4$–10^5 s, depending on the interferometer baseline (10^6 or 10^8 km, as shown in Fig. 1). More details on the instrumental noise can be found in Faller & Bender (1984), and Faller *et al.* (1984).

In fact these instruments are expected to be so free of local noise that the main obstacle to detecting a high-redshift background is possibly the gravitational wave background from binary star systems in our own galaxy, or in other galaxies at $z \lesssim 1$ (Rosi & Zimmerman 1976). An added difficulty is that such stars are likely to produce a background with spectral characteristics quite similar to those of the phase transitions.

The general magnitude of the background from the local Hubble volume can be estimated using a simple model. Suppose the density parameter of binaries with period P, per octave of P, is $\Omega_b(P)$. Using standard formulae (Misner *et al.* 1973, section 36.4) one finds that the fraction of rest mass radiated in a Hubble time is $(H_0 P)^{-1}(P/R_S)^{-7/3}$, where R_S is the Schwarzschild radius corresponding to the reduced mass of the binary. Thus

$$\Omega_g \sim 10^{-10}\,\Omega_b(P)(M/M_\odot)^{7/3}(P/10^4\ \mathrm{s})^{-10/3}. \tag{16}$$

Note that for $P < P_{\mathrm{Hubble}} \simeq 10^3$ s $(M/M_\odot)^{5/8}$, the orbital decay time from gravitational radiation is less than H_0^{-1}. Thus

$$\Omega_b \propto \tau_{\mathrm{dec}} H_0 \propto P^{8/3} \qquad (P < P_{\mathrm{Hubble}}) \tag{17}$$

so that $\Omega_{gw} \propto P^{-2/3}$ independently of the initial shape of $\Omega_b(P)$, as long as no other mechanism contributes to orbital decay. Fig. 1 shows the spectrum from a population of binaries at $z \lesssim 1$ set up initially with $\Omega_b = \mathrm{const} = 10^{-4}$.

A background woud also be produced by contact binaries, white-dwarf doubles, neutron star binaries, and cataclysmic variables within our own galaxy. A thorough empirical analysis of these sources by Bender, Hils & Webbink (1985) gives a spectrum similar to the above estimate, except that it peaks at a somewhat lower frequency. Here, there is hope that directional information could be used to remove some of the local background, but it is not clear whether the cosmological phase transition backgrounds would be above the confusion limit.

If no source of noise or gravitational radiation exists more intense than those discussed here, there appears to be a reasonable hope of observing the gravitational signal from a phase transition if it is strongly first order and cavitation occurs on the scale expected for homogeneous nucleation. Further progress in lattice QCD should tell us whether this is a firm prediction of the standard model, and if it is, it should probably be taken seriously enough to influence detector design considerations.

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1986MNRAS.218..629H

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