

THE ECLIPSES OF CATAclySMIC VARIABLES. I. HT CASSIOPEIAE

E.-H. ZHANG, EDWARD L. ROBINSON, AND R. EDWARD NATHER

Department of Astronomy and McDonald Observatory, University of Texas at Austin

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ABSTRACT

HT Cas is an eclipsing dwarf nova with an orbital period of 106 minutes. We have obtained high-speed, multicolor photometry of HT Cas on 15 nights between 1982 October and 1984 October and on another two nights in 1985 January. HT Cas did not erupt from 1982 to 1984. Our 1985 January observations were obtained during the first eruption to break this spell of quiescence. The eruption had all the characteristics of a supermaximum and, therefore, HT Cas is a member of the SU UMa subclass of dwarf novae. To analyze the photometry, we have written a program to synthesize the light curves of eclipsing cataclysmic variables. The program includes the usual physical mechanisms incorporated in programs to calculate the light curves of normal eclipsing binaries, and it includes the effects of the accretion disks found in most cataclysmic variables. Our most important results concern the accretion disk in HT Cas. While HT Cas was at minimum light, its accretion disk appeared to be optically thick and had a temperature distribution the same as theoretical models of steady state, optically thick accretion disks. The mass transfer rate through the accretion disk was $\sim 4 \times 10^{-10} M_{\odot} \text{ yr}^{-1}$. These results are consistent with recent theories of the dwarf nova eruption in which the eruption is caused by an instability in a non-steady state disk because HT Cas ceased to erupt between 1982 and 1984 and because these theories predict no eruptions at the high mass transfer rate in HT Cas. The supermaximum of 1985 January followed our last observation at minimum light by two months, suggesting that the supermaxima of dwarf novae do not require unstable accretion disks.

Subject headings: stars: accretion — stars: dwarf novae — stars: eclipsing binaries — stars: individual

1. INTRODUCTION

The cataclysmic variables are close binary systems in which a late-type star (which we shall call the secondary) fills its Roche lobe and is transferring mass through the inner Lagrangian point to a white dwarf (the primary). In most of these systems the transferred mass forms a gas disk—or ring—encircling the white dwarf; the exceptions are systems in which the white dwarf has a magnetic field strong enough to alter the gas flow and distort or disrupt the disk (see reviews by Robinson 1976; Warner 1976; Cordova and Mason 1983). Many cataclysmic variables eclipse (Ritter 1984), and, in principle, analysis of the eclipses could give much useful information. For example, eclipse analysis could give the orbital inclination, the relative dimensions of the various components, and the temperatures of the components. If the radial velocity curves of the two stars are known, the absolute masses and dimensions of the system can also be derived.

Unfortunately, the cataclysmic variables are complicated systems. There are three important contributors to their luminosity: the two stars and the accretion disk. The accretion disk may have a bright boundary layer at the surface of the white dwarf and a bright spot on its outer edge formed by the impact of the stream of gas from the secondary star. In addition, the accretion process causes rapid changes in the brightness of the system ("flickering") that mask and distort the systematic variations caused by the eclipses. The properties of the system can also vary on time scales of weeks, months, and years. Analysis of the eclipses of cataclysmic variables is, therefore, extremely difficult, and, as a result, although many attempts have been made to analyze the eclipses, the results are often incomplete and inconclusive. Furthermore, the validity of even the more careful analyses have been difficult to evaluate because questions about the uniqueness of the solutions and

the errors in the derived parameters are usually not addressed fully.

We have begun a new project to observe and analyze the eclipses of cataclysmic variables. We overcome many of the worst observational problems by carefully choosing the stars to observe and by observing many eclipses of each star so that the random variations in the light curves can be averaged out. To analyze the eclipses we calculate synthetic light curves and then fit the synthetic light curves to the observed light curves. The synthetic light curves include the effects of Roche geometry for the secondary star; an accretion disk with a bright spot on its outer edge; gravity darkening on the late-type star; limb darkening of the stars and the disk; and reflection effect. We *do not* assume that the accretion disk looks like a steady state viscous disk. We perform a complete error analysis, including an analysis of the correlations between the parameters (temperatures, radii, and so on) yielded by the fitting procedure. As a result, we can decide which of the parameters are reliable and which are merely indicative or should be ignored altogether.

In this paper we report our first results, a study of the eclipsing dwarf nova HT Cas. The eclipses of HT Cas, first discovered by Bond, were extensively observed by Patterson, who derived an orbital period of 106.05 minutes for the system (Patterson 1981). Young, Schneider, and Schectman (1981) observed HT Cas spectroscopically and found strong Balmer and He I emission lines that were 2800 km s^{-1} wide and double peaked. The *K*-velocity of the Balmer lines was $115 \pm 6 \text{ km s}^{-1}$. Using his photometry and the spectroscopy of Young *et al.*, Patterson obtained $0.70 \pm 0.07 M_{\odot}$ and $0.19 \pm 0.02 M_{\odot}$ for the masses of the white dwarf and the late-type star, respectively. Although HT Cas is faint— $V = 16.5$ out of eclipse and $V = 18.5$ in eclipse, and its eclipses last only a few minutes—we

TABLE 1
PHOTOMETRIC OBSERVATIONS OF HT CAS

Run Number	Date (UT)	HJD at Beginning (2445000+)	Integration Time (s)	Length of Run (hr)	Filter	Telescope (m)
2736.....	1982 Oct 15	257.77739	1	0.5	None	2.1
2739.....	1982 Oct 16	258.71146	1	3.3	<i>B</i> , none	2.1
2743.....	1982 Oct 18	260.74797	1	0.4	<i>B</i>	2.1
2747.....	1982 Oct 19	261.82723	1	1.2	<i>R</i>	2.1
2757.....	1982 Dec 13	316.67661	1	1.7	<i>B</i>	2.1
2763.....	1982 Dec 15	318.65694	1	1.7	<i>B</i> , <i>R</i>	2.7
2769.....	1983 Jan 13	347.57637	3, 1	4.5	<i>B</i> , <i>R</i>	2.1
2830.....	1983 Nov 6	644.71632	1	2.0	<i>B</i> , <i>V</i> , <i>R</i>	2.1
2835.....	1983 Nov 7	645.77540	1	2.2	<i>B</i> , <i>V</i> , <i>R</i>	2.1
2838.....	1983 Nov 8	646.57168	1	6.1	<i>B</i> , <i>V</i> , <i>R</i>	2.1
2843.....	1983 Nov 9	647.67792	1	2.6	None	2.1
2855.....	1983 Nov 30	668.72180	1	1.4	<i>B</i> , <i>V</i> , <i>R</i>	2.1
2858.....	1983 Dec 4	672.55371	1	2.9	<i>B</i> , <i>V</i> , <i>R</i>	2.1
2863.....	1983 Dec 5	673.60629	1	2.5	<i>B</i> , <i>V</i> , <i>R</i>	2.1
2865.....	1983 Dec 5	673.75606	1	0.6	<i>B</i> , <i>V</i> , <i>R</i>	2.1
0026.....	1984 Oct 20	993.70319	2	0.8	None	2.1

chose to observe this system because it is the only dwarf nova in the northern hemisphere in which the eclipse of the white dwarf can be clearly separated from the eclipse of the disk and its substructure.

We present the observations and the results obtained from fitting the light curves of HT Cas in the main text of this paper. A description of the light-curve synthesis program and the error analysis method is given in the Appendix.

II. PHOTOMETRIC OBSERVATIONS

We obtained high-speed photometry of HT Cas on 15 nights between 1982 October and 1984 October. There were no eruptions of HT Cas during this period of time, according to the AAVSO (Mattei, private communication). The observations were made at McDonald Observatory, mainly with the 2.1 m telescope, using the high-speed photometer described by Nather (1973) and equipped with an RCA 8850 or an RCA 9658 photomultiplier tube. We observed a total of 24 eclipses of HT Cas. Details of the observing runs can be found in Table 1. We also observed HT Cas on two nights in 1985 January when it was erupting. We delay a discussion of the eruption light curves until § V.

We used the Johnson *B*, *V*, and *R* filters for most of the

photometry. A nearby star, $\sim 2/4$ west of HT Cas, was chosen as the comparison star. On the nights of 1983 November 7, November 8, and December 5 we calibrated the *B*, *V*, and *R* magnitudes of the comparison star using standard stars selected from Moffett and Barnes (Moffett and Barnes 1979; also Landolt 1973). Its magnitude and colors are $V = 13.61 \pm 0.04$, $(B - V) = 0.839 \pm 0.006$ and $(V - R) = 0.764 \pm 0.009$.

Figure 1 shows the light curve in white light of two consecutive eclipses (run 2843) and Figure 2 shows the *B*, *V*, and *R* light curves of another two consecutive eclipses (run 2858). All the light curves have been corrected for sky background, dark noise, and atmospheric extinction. The gaps in the light curves are due to measurements of sky and the comparison star. Each point in the white-light curve shown in Figure 1 is an average of three consecutive one-second integrations. Each point in Figure 2 is also an average of three one-second integrations, but the interval between the one-second integrations was 3 seconds.

Table 2 gives the mid-eclipse times of 23 of the 24 eclipses that were observed without filters or with a *B* filter. The eclipse of cycle 26042 (run 2835) was obtained under poor seeing conditions, so its time of mid-eclipse was not extracted. Since the eclipses of HT Cas are both asymmetric and variable in shape,

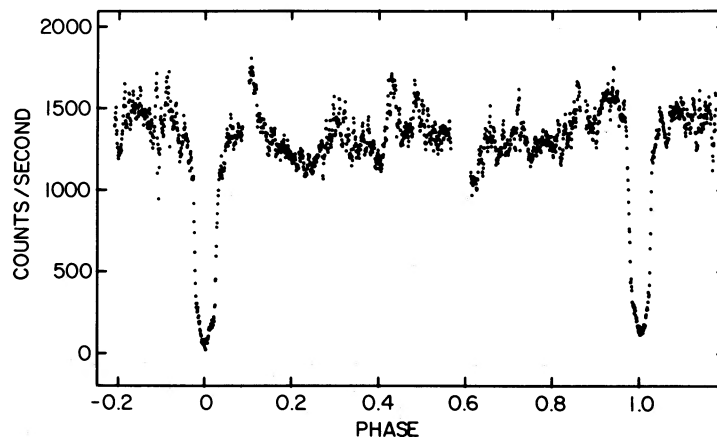


FIG. 1.—White-light curve from run 2843. Each point represents an average of three contiguous one-second integrations.

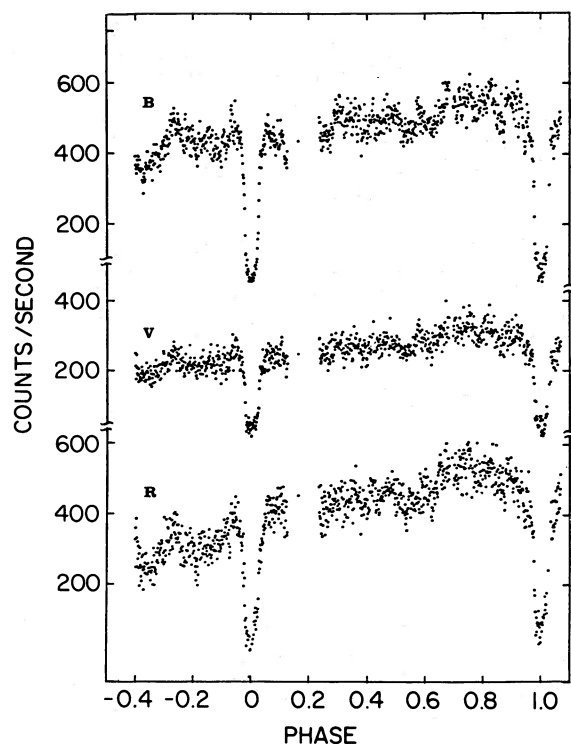


FIG. 2.—Light curves in *B*, *V*, and *R* colors, respectively, from run 2858. Each point represents an average of three one-second integrations but with three-second intervals between each integration.

we determined the mid-eclipse times by fitting straight lines to both the ingress and egress branches of the primary eclipse and then drawing four horizontal lines between the two straight lines. The time of mid-eclipse is given by the average of the times of the midpoints of the horizontal lines. The times of mid-eclipse determined this way have a precision of about ± 3 s. The times of mid-eclipse given in Table 2 are expressed in heliocentric JDs.

A least squares fit to our 23 mid-eclipse timings, plus 11 timings at quiescent state from Patterson (1981), yielded the following ephemeris:

$$\text{mid-eclipse} = \text{HJD } 2443727.93723 + 0^{\text{d}}0736472070E \quad (1)$$

$\pm 48 \qquad \qquad \pm 24$

TABLE 2
ECLIPSE TIMINGS FOR HT CAS

Run (1)	HJD Mid-eclipse (2445000+) (2)	Cycle <i>E</i> (3)	<i>O</i> − <i>C</i> (day) (4)
2736.....	257.81067	20773	−0.00001
2739.....	258.76811	20786	0.00004
	258.84170	20787	−0.00002
2743.....	260.75655	20813	0.00000
2747.....	261.86127	20828	0.00001
2757.....	316.72849	21573	0.00006
2763.....	318.71699	21600	0.00009
2769.....	347.58661	21992	0.00000
	347.66033	21993	0.00008
2830.....	644.75304	26027	−0.00005
2838.....	646.59428	26052	0.00001
	646.66796	26053	0.00005
	646.74161	26054	0.00005
	646.81517	26055	−0.00004
2843.....	647.69894	26067	−0.00003
	647.77268	26068	0.00006
2855.....	668.76198	26353	−0.00010
2858.....	672.59163	26405	−0.00010
	672.66540	26406	0.00002
2863.....	673.62283	26419	0.00004
	673.69641	26420	−0.00003
2865.....	673.77005	26421	−0.00004
0026.....	993.76714	30766	−0.00006

The residuals (*O* − *C*) from the above ephemeris are listed in column (4) of Table 2 and plotted in Figure 3. The standard deviation of individual timings in the (*O* − *C*) diagram is less than ± 5 seconds. A parabolic fit to the residuals in the (*O* − *C*) diagram gave

$$(O - C) = -1.447 \times 10^{-5} + 0.968 \times 10^{-8}E - 0.363 \times 10^{-12}E^2 \quad (2)$$

Although there does appear to be curvature in the (*O* − *C*) diagram, the method discussed by Pringle (1975) shows that the quadratic term is significant at a confidence level less than 96%. Furthermore, because of the large gap between our data and Patterson's, we consider the quadratic term to be unreliable. Therefore we suggest that the linear ephemeris of equation (1) should be used until further eclipse timings confirm the changing period.

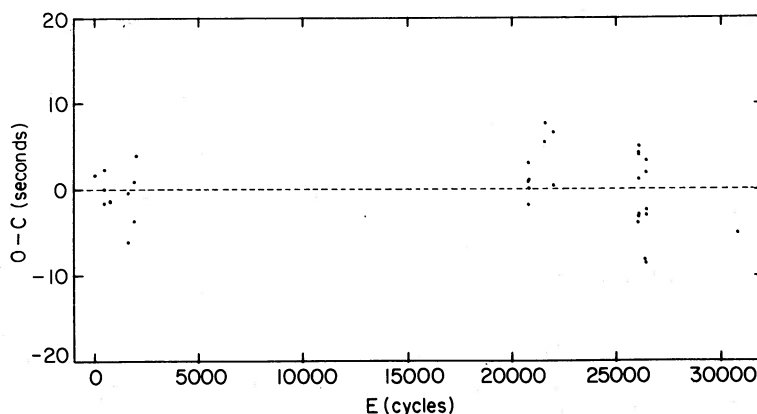


FIG. 3.—The (*O* − *C*) diagram for the eclipses of HT Cas. The first 11 points are from Patterson (1981). The ephemeris used to make the diagram is the linear ephemeris given by eq. (1).

TABLE 3
MAGNITUDES AND COLORS OF HT CAS

UT	CYCLE NUMBER	OUTSIDE ECLIPSE			PRIMARY MINIMUM		
		<i>V</i>	(<i>B</i> − <i>V</i>)	(<i>V</i> − <i>R</i>)	<i>V</i>	(<i>B</i> − <i>V</i>)	(<i>V</i> − <i>R</i>)
1983 Nov 6	26027	16.45 ± 0.13	−0.21 ± 0.10	0.56 ± 0.13	18.54 ± 0.31	+0.57 ± 0.32	0.63 ± 0.39
1983 Nov 7	26042	16.57 ± 0.16	−0.03 ± 0.12	0.68 ± 0.13	18.24 ± 0.38	+0.63 ± 0.35	1.00 ± 0.40
1983 Nov 8	26052	16.60 ± 0.14	−0.12 ± 0.09	0.63 ± 0.14	18.80 ± 0.45	+0.28 ± 0.29	1.11 ± 0.37
	26053	16.43 ± 0.17	−0.17 ± 0.11	0.61 ± 0.12	18.56 ± 0.46	+0.28 ± 0.30	0.75 ± 0.38
	26054	16.55 ± 0.18	−0.19 ± 0.11	0.71 ± 0.13	19.15 ± 0.48	−0.09 ± 0.29	1.30 ± 0.41
	26055	16.48 ± 0.15	−0.23 ± 0.13	0.68 ± 0.14	19.01 ± 0.51	−0.16 ± 0.31	1.36 ± 0.38
1983 Nov 30	26353	16.32 ± 0.18	−0.03 ± 0.12	0.60 ± 0.15	18.10 ± 0.40	+0.16 ± 0.28	0.74 ± 0.41
1983 Dec 4	26405	16.50 ± 0.13	−0.12 ± 0.11	0.49 ± 0.15	18.29 ± 0.37	+0.56 ± 0.30	0.06 ± 0.40
	26406	16.33 ± 0.13	−0.11 ± 0.11	0.62 ± 0.16	18.17 ± 0.52	+0.21 ± 0.29	0.59 ± 0.39
1983 Dec 5	26419	16.26 ± 0.15	−0.09 ± 0.12	0.60 ± 0.13	18.07 ± 0.34	+0.17 ± 0.27	0.81 ± 0.41
	26420	16.30 ± 0.17	−0.12 ± 0.11	0.51 ± 0.14	18.12 ± 0.27	−0.01 ± 0.28	0.46 ± 0.40
	26421	16.31 ± 0.13	−0.10 ± 0.12	0.57 ± 0.15	17.96 ± 0.33	+0.43 ± 0.30	0.58 ± 0.37

NOTE.—Mean Values: Outside Eclipse: $V = 16.43 \pm 0.11$; ($B - V$) = -0.13 ± 0.06 ; ($V - R$) = $+0.60 \pm 0.06$. Minimum light: $V = 18.42 \pm 0.38$; ($B - V$) = $+0.25 \pm 0.25$; ($V - R$) = $+0.78 \pm 0.35$.

III. MORPHOLOGY OF THE LIGHT CURVES

Table 3 lists the *B*, *V*, and *R* magnitudes and colors for the 12 eclipses of HT Cas for which all three colors are available. The magnitudes outside eclipse were obtained by averaging all data points outside the phase range from -0.05 to $+0.05$ and those at the bottom of eclipse by averaging all data points from phase -0.013 to $+0.013$. The estimated errors in the magnitudes outside eclipse are due mainly to the intrinsic flickering of the system and the estimated errors in the magnitudes at mid-eclipse are due to photon counting noise.

Figures 4a, 4b, and 4c show the 10 eclipses in *B*, *V*, and *R* colors, respectively, from runs 2838–2865. Each point in the light curves represents an average of three one-second integrations. The brightness of HT Cas fluctuates from orbit to orbit. For the 12 eclipses in Table 3 the ranges in magnitude are 0.37 in *B*, 0.34 in *V*, and 0.35 in *R* for the light outside eclipse. For most cycles, the eclipses are deeper in *B* than in *V* and *R*. The average depths of eclipses of HT Cas are 2.26 mag in *B*, 1.97 mag in *V*, and 1.79 mag in *R*. When HT Cas became brighter, the factor by which the brightness increased was larger at mid-eclipse than outside eclipse. The eclipse solutions described in § IV demonstrate that the white dwarf and the inner disk are blocked by the secondary star at mid-eclipse. Therefore, the outer part of the disk or the secondary star, or both, are responsible for the change in brightness.

The light curve of HT Cas is complex. A primary eclipse with steep ingress and egress branches can be seen in every cycle and is caused by the eclipse of the white dwarf and the central region of the disk by the secondary star. Before the primary eclipse there is a gradual fading caused by the partial eclipse of the outer region of the accretion disk. A short halt after the primary ingress and a standstill after the primary egress can be seen in some eclipses and are due to the eclipse of the bright spot on the outer edge of the disk. Except for the primary eclipse, the various features are not present in every eclipse, and even when they are present, the features are not as pronounced as in other similar systems, e.g., Z Cha (Cook and Warner 1984). Thus, for example, Z Cha has a pronounced hump in its light curve just prior to the primary eclipse, but the equivalent hump in HT Cas is always weak and often absent.

IV. LIGHT CURVE SYNTHESIS

In this section we derive the geometric and physical parameters of HT Cas by analyzing its eclipse light curve. Since the

light curve of HT Cas is contaminated by flickering, we have taken the average of the 12 eclipses listed in Table 3 to reduce the random variations. Since no gross changes in the luminosity of HT Cas were observed from 1982 October to 1984 October, and since examination of the individual eclipses did not show large systematic changes of their shapes and depths with time, the average light curves should be a valid representation of the eclipses of HT Cas during these years. To show this point, we have plotted the white-light curves of run 2736 (1982 October) and run 0026 (1984 October) in Figure 5. Each point in the light curves is a mean of five two-second integrations. The shapes of the two eclipses are different, but the differences are no larger than those seen in consecutive eclipses (see, for example, Fig. 1).

The average light curves in *B*, *V*, and *R* are plotted in Figure 6 and are the light curves we used for our numerical study of HT Cas. The average light curves are normalized to 1.0 when out of eclipse (the true fluxes in each color can be retrieved using the mean magnitudes of HT Cas given in Table 3). Since we have normalized the light curves, we can only use the shapes and depths of the three eclipses to extract information about HT Cas; the photometric colors of the system are not involved in the fitting procedure. The advantage of fitting the normalized light curves is that the fitting is reddening free. If we had fitted the flux in the different colors, the uncertainty in the reddening would have dominated the fitting process and could have introduced large errors in the derived parameters. Our light-curve synthesis program does produce colors, however, and we shall show in the discussion that—with reasonable assumptions about the reddening to HT Cas—the physical parameters we derive for HT Cas do yield the observed colors of the system.

a) Basic Model

The procedure we will use to solve the light curve of HT Cas is to construct light curves from a model and adjust the parameters of the model to optimize the fit of the constructed curves to the observed light curves. A similar approach has been taken by Frank and his co-workers (Frank *et al.* 1981; Frank and King 1981) and by Schwarzenberg-Czerny (1984a). Frank *et al.* (1981) and Frank and King (1981) studied U Gem, UX UMa, and RW Tri by fitting the visual and infrared light curves. In their models, the secondary star was taken to be spherical and the light from the white dwarf was ignored. They

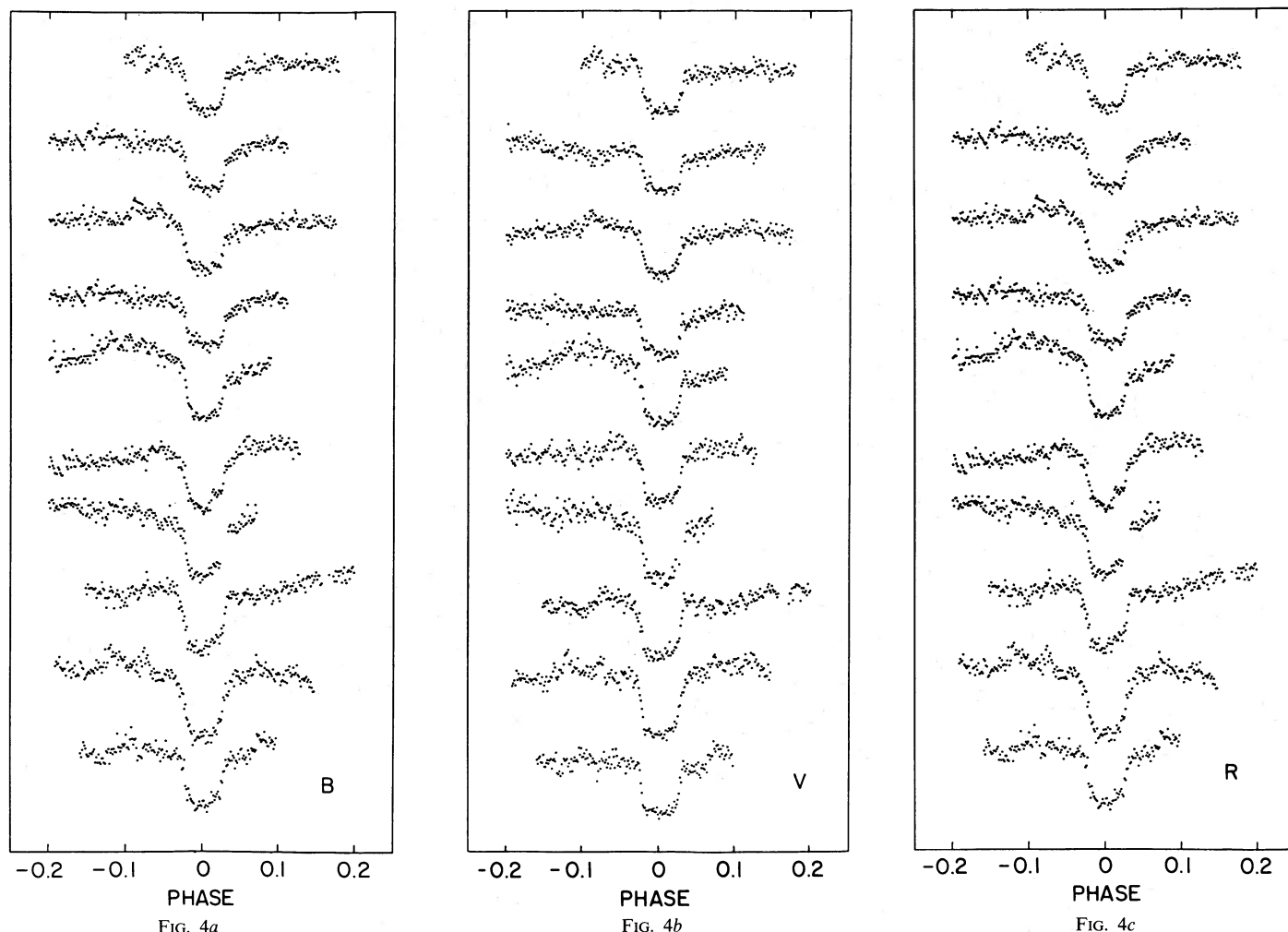


FIG. 4.—Ten eclipses of HT Cas. Each point is the intensity average of three one-second integrations. (a) Eclipses in B color. (b) Eclipses in V color. (c) Eclipses in R color. The cycle numbers of the eclipses are (from top to bottom): 26052, 26053, 26054, 26055, 26353, 26405, 26406, 26419, 26420, and 26421.

found that the light curves of UX UMa and RW Tri could be well fitted by the eclipse of an optically thick, steady state accretion disk, but no fit could be found for the eclipse of U Gem. The approximations they used could introduce errors in the derived parameters. Calculations by Schwarzenberg-Czerny (1984b) show that the spherical approximation of the secondary star may lead to a 10% error in its relative radius and a 15% error in its mass. These errors are comparable to the observational errors and therefore cannot be neglected. Schwarzenberg-Czerny (1984a) discussed the general dependence of the light curves of a disk eclipse on the parameters of the disk. As an example he studied the eclipses of LX Ser in U, B, and V colors. Although he took into account the Roche geometry of the secondary star in his model, he ignored the light from the secondary and the white dwarf.

Horne (1985a) has also developed an eclipse mapping method to study the accretion disks in cataclysmic variables. In his model, the secondary fills its Roche lobe and a flat disk lies in the orbital plane. No light from the secondary was included in the computations. Synthetic light curves were computed by numerical simulation of the accretion disk eclipse, and images of the disk were reconstructed by maximum entropy method. The primary difference between Horne's

method and ours for analyzing eclipses is in this use of the maximum entropy method by Horne. Horne and Cook (1985) analyzed Z Cha in eruption and found that the radial temperature distribution of the accretion disk in Z Cha is consistent with a steady state mass transfer.

The model of cataclysmic variables used in our computations included four components of luminosity: the primary star (a white dwarf), the secondary star which fills its Roche lobe, an accretion disk around the primary star, and a bright spot on the edge of the disk. Roche geometry is used for the secondary star to take into account the tidal distortion. Gravity darkening on the surface of the secondary star, limb darkening of the stars and the disk, and reflection effects among the components of the system are also included in our computations.

We assume that the disk is geometrically thin and lies in the orbital plane. We do not assume that the disk has the brightness distribution of an optically thick, steady state accretion disk. Instead, we adopt a power-law distribution of the temperature with radius:

$$T(R) = T_d(R/R_d)^{\xi}, \quad (3)$$

where R is the distance from the center of the disk, R_d and T_d are the radius and temperature at the outer edge of the disk,

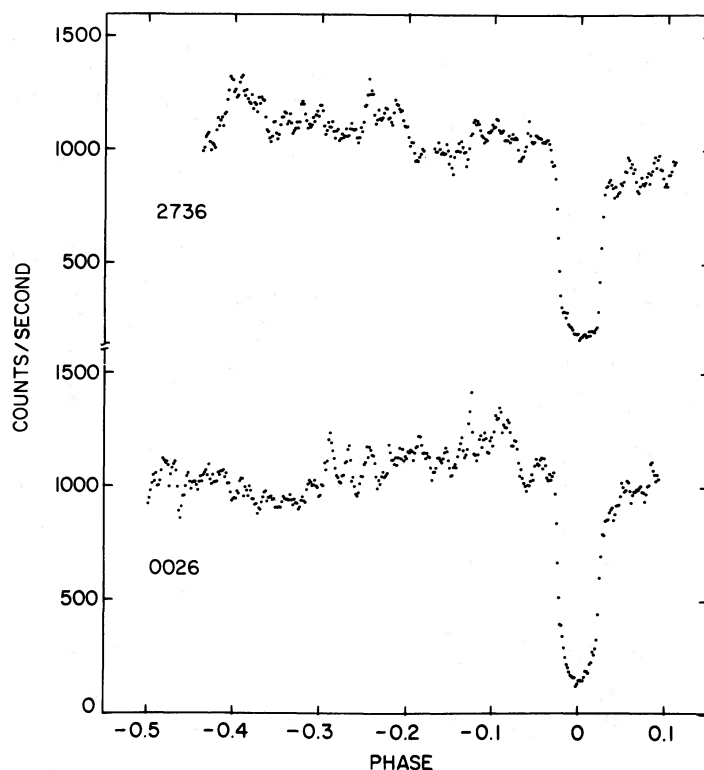


FIG. 5.—White-light curves of run 2736 (1982 October) and run 0026 (1984 October). Each point is a mean of five two-second contiguous integrations.

respectively, and ξ is the exponent of the distribution. The values of ξ , T_d , and R_d are determined from the observations. The monochromatic radiation from the surface of the accre-

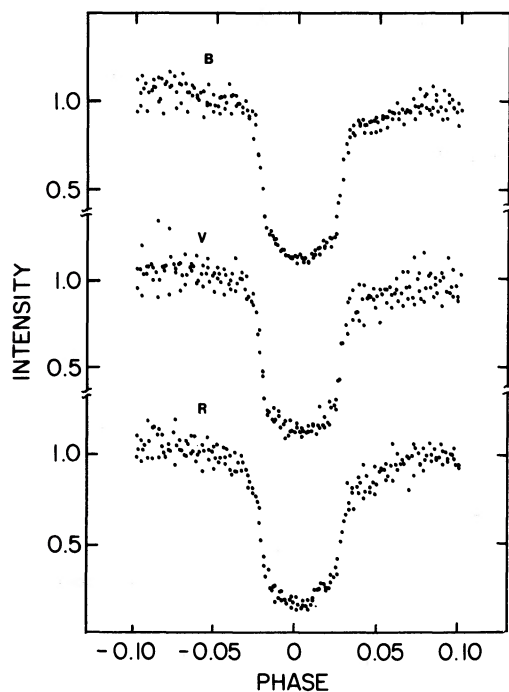


FIG. 6.—Averaged light curves from the 12 eclipses in B, V, and R colors, respectively. The phases are from -0.10 to $+0.10$. Each point is the mean light curve during a phase interval of 0.001 corresponding to a time interval of 6.363 s.

tion disk is calculated from the Planck function. This use of blackbody radiation assumes that the disk is optically thick and, consequently, the derived temperature and brightness distribution will be incorrect for any parts of the disk that are optically thin. As we shall show in § VI, we have two external tests of the validity of this assumption and both are satisfied. The position of the bright spot on the outer edge of the disk is specified by R_s , the distance of its inner edge from the center of the disk, α_s , the angle along the azimuthal direction from the line joining the center of the two component stars, and $\Delta\alpha_s$, its spread along the azimuthal direction. A temperature T_s is assigned to the bright spot.

Computation of the light curve of a cataclysmic variable involves many parameters, both geometrical and physical, and, for clarity, we list the main parameters in Table 4. A detailed description of our model, the light-curve synthesis program, and the algorithm is given in the Appendix.

b) Initial Values of the Parameters

Our fitting procedure requires initial values for all the parameters listed in Table 4. Some of the parameters will cause only second-order effects in our results. These include the limb-darkening coefficients for the two stars and the disk and the albedo of the photosphere of the secondary star. Because they have only second-order effects, we have fixed the values of these parameters at reasonable *a priori* values and we do not attempt to determine them from the fitted light curves. We have chosen $u_1 = 0.36$ (Warner, Robinson, and Nather 1971) and $u_2 = u_3 = 0.4$ for the limb-darkening coefficients. Since the secondary stars in cataclysmic variables are late-type stars and should have convective envelopes, we have chosen the albedo of the secondary stars to be 0.5 (Rucinski 1969).

The white-light curves of HT Cas have a higher signal-to-

TABLE 4
INPUT PARAMETERS FOR THE SYNTHESIS OF LIGHT CURVES OF
CATAclysmic VARIABLES

q	Mass ratio M_1/M_2
i	Orbital inclination
R_1	Radius of the primary star
R_d	Outer radius of the accretion disk
u_1	Limb-darkening coefficient of the primary star
u_2	Limb-darkening coefficient of the secondary star
u_3	Limb-darkening coefficient of the disk
W	Albedo of the secondary star
T_1	Temperature of the primary star
T_2	Temperature on the pole of the secondary star
T_d	Temperature at the outer edge of the disk
T_s	Temperature of the bright spot
α_s	Angle from the line between the centers of the stars to the line between the centers of the bright spot and the disk
$\Delta\alpha_s$	Angular extension of the bright spot
R_s	Distance from the disk center to the inner edge of the spot (outer edge of the spot is at R_d)
β	Gravity darkening coefficient
ξ	Exponent used for the temperature distribution of the disk

noise ratio that the color light curves so we have used them to make the preliminary estimates of the geometric parameters of the system, including the orbital inclination i , the radius of the white dwarf R_1 , and the outer radius of the disk R_d .

Since the light curves are flat bottomed, it is clear that the eclipse of the white dwarf or the central part of the accretion disk is total. The phases of the first and second contacts of the eclipse, ϕ_1 and ϕ_2 , are both functions of the mass ratio $q = M_1/M_2$, the orbital inclination i , and the radius of the white dwarf R_1 , where M_1 and M_2 are masses of the primary and the secondary stars, respectively. If the mass ratio q is given and the phases of ϕ_1 and ϕ_2 are determined from light curves, the two functions yield i and R_1 . Because of the flickering in the light curves it is difficult to determine the two phases accurately. Therefore, instead of determining ϕ_1 and ϕ_2 directly, we use our light-curve synthesis program to fit the observed ingress branch of the primary eclipse approximately by assuming that a single spherical object (the white dwarf) is being eclipsed by a Roche lobe of mass ratio q to find out the values of i and R_1 for a given mass ratio q .

The gradual decrease in luminosity before the primary eclipse is caused by the partial eclipse of the accretion disk. The first contact of this partial eclipse in our white-light curves is at about phase $\phi_d = -0.065$ in agreement with the value, -0.063 , given by Sulkanen, Brasure, and Patterson (1981). The phase ϕ_d is a function of q , i , and R_d . Since we have already determined i as a function of q , this function will yield the radius R_d as a function of q .

We chose the initial value of the surface temperature of the secondary star as follows. Patterson (1984) gives a relation between the orbital period and absolute magnitude of the secondary in cataclysmic variable: $M_v = 17.7 - 11.00 \log P(\text{hr})$ for $0.05 < \log P < 0.7$. According to this relation the absolute visual magnitude of the secondary star in HT Cas is 14.9 mag, which implies a spectral type of M6 or M7 if it is a main-sequence star. This spectral type is internally consistent because, if we choose $M_2 = 0.19 M_\odot$ (Patterson 1981), then $\log (M_2/M_\odot) = -0.72$. For M6 main-sequence stars, Allen (1973) gives a value of $\log (M_2/M_\odot) = -0.80$. Johnson (1966) gives an effective surface temperature of 3000 K for M6 main-sequence star, so we have chosen 3000 K as the initial value for the surface temperature.

The mass ratio of HT Cas is not well constrained by the observed light curves of the system. We have, therefore, solved the light curve for several mass ratios. The basic data for determining the mass ratio of HT Cas is the spectroscopy of Young *et al.* (1981). They found the projected orbital velocity, K , of the primary star to be $115 \pm 6 \text{ km s}^{-1}$. Kepler's law and the K velocity give

$$M_2/M_\odot = (0.0116 \pm 0.0018)(1 + q)^2/\sin^3 i. \quad (4)$$

Since the relation between q and i is already known from the numerical computations of the first and second contacts ϕ_1 and ϕ_2 of the primary eclipse, equation (4) becomes a relation between $M_2/M_\odot$ and q . This relation is plotted in Figure 7 as the three dashed lines, corresponding to $K = (115 \pm 6) \text{ km s}^{-1}$, $K = 115 \text{ km s}^{-1}$, and $K = (115 - 6) \text{ km s}^{-1}$, respectively. Using another form of Kepler's law:

$$(a/R_\odot) = 0.507(M_2/M_\odot)^{1/3}(1 + q)^{1/3}P(\text{hr})^{2/3}, \quad (5)$$

where a is the semi-major axis of the system and P is the orbital period in hours; and using the relation between q and the equivalent radius of the Roche lobe R_2 found by Eggleton (1983):

$$(R_2/a) = 0.49q^{-2/3}/[0.6q^{-2/3} + \ln(1 + q^{-1/3})]; \quad (6)$$

and using the assumption that the radius and mass of the secondary star in HT Cas satisfy the relation

$$R_2/R_\odot = \alpha(M_2/M_\odot)^\beta, \quad (7)$$

we can find an additional relation between $M_2/M_\odot$ and q . The exact values of α and β in equation (7) are controversial but, fortunately, these values are not critical for the choice of the range of q . We have chosen two sets of values: $\alpha = 1.0$ and $\beta = 1.0$ for a secondary that lies on the slightly evolved main sequence and $\alpha = 1.2$ and $\beta = 1.0$ for a secondary that lies on the more evolved main sequence. The two solid lines in Figure 7 correspond to the two sets of values. From Figure 7 we estimate that the mass ratio in HT Cas lies between 2.0 and 3.5. We have solved the eclipse light curves of HT Cas for the six mass ratios $q = 2.0, 2.5, 2.83, 3.0, 3.5$, and 4.0 . The solution for $q = 2.83$ was included to make comparison with the results of Young *et al.* (1981) easier.

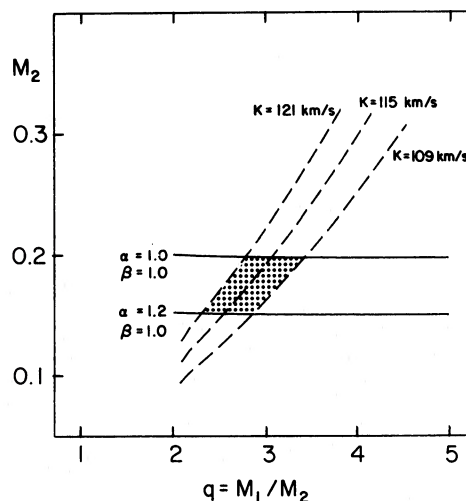


FIG. 7.—Relations between the mass of the secondary in HT Cas and the mass ratio. The most probable range of the mass ratio is $2.0 \leq q \leq 3.5$.

TABLE 5
SYSTEM PARAMETERS OF HT CAS

PARAMETER	MASS RATIO (M_1/M_2)					
	2.00	2.50	2.83	3.00	3.50	4.00
i°	74.4	75.6	76.3 ± 0.6	76.6	77.4	78.1
$M_1/M_\odot$	0.22	0.39	0.53 ± 0.07	0.60	0.88	1.24
$M_2/M_\odot$	0.11	0.16	0.19 ± 0.02	0.20	0.25	0.31
R_1/a	0.017	0.019	0.020 ± 0.003	0.020	0.021	0.021
R_2/a	0.213	0.218	0.221 ± 0.011	0.222	0.227	0.229
$T_1(K)$	39000	39000	39000 ± 3000	40000	40000	40000
$T_2(K)$	2800	2800	2800 ± 700	2800	2800	2800
$T_d(K)$	6100	6600	6800 ± 300	7000	7300	7500
ξ	-0.66	-0.66	-0.66 ± 0.05	-0.66	-0.64	-0.64
$T_s(K)$	14200	14400	14000 ± 4000	14000	13800	14000
α_s°	34	32	31 ± 11	30	28	27

c) Light-Curve Fitting and Parameter Optimization

We fit the synthetic light curve to the observed light curve by the following technique. We first ignore the light from the bright spot. We fix the values of all the parameters for which initial values have been estimated and guess the values of the three parameters, T_1 , the temperature of the primary star, T_d , the temperature at the outer edge of the disk, and ξ , the exponent of the power-law distribution of temperature across the disk. We then optimize these guesses. For example, we can fix T_1 and change the other two parameters to get the best possible fits to the light curves. Then we test whether a change in T_1 could improve the fits. If it does, we change T_1 in the correct direction until the fits can no longer be improved and then reoptimize T_d and ξ . After a few cycles, no further improvement can be obtained by changing these three parameters. We then add the bright spot and repeat the above procedure to obtain the best possible fits. To model approximately the obscuration of the spot by the disk when the spot is on the opposite side of the disk from the observer, we assume that the bright spot stays invisible after its mid-eclipse. At this stage we have preliminary fits to the light curves. We now change all the parameters, except the mass ratio, to get the best possible fits to the eclipses of all the three colors. We then repeat the entire procedure for a different mass ratio.

The criterion for the goodness of the fit is that the standard deviation, defined by formula (A12) in the Appendix, has the

smallest value for all three colors. The standard deviations are calculated only for the part of the light curves between phases -0.10 and $+0.10$, i.e., only the ingress and egress branches and the bottom of the eclipse.

The derived parameters of HT Cas are given in Table 5. The synthetic light curves in the B , V , and R colors for $q = 2.83$, together with the averaged observed light curves, are plotted in Figures 8a, 8b, and 8c, respectively.

The most interesting result of our study is that the accretion disk in HT Cas looks like a steady state, viscous accretion disk. That is, ξ is nearly equal to -0.75 . To check this specific result, we tried to force a fit to the light curves using two other temperature distributions: $\xi = -0.40$ and $\xi = +1.5$ for $q = 2.83$. The procedure to force the fits was the same as we described above except that the value of ξ was fixed. The synthetic light curves for each case are plotted in Figures 9 and 10, respectively. The synthetic light curves obviously do not fit the observed light curves. For $\xi = -0.40$ we could not obtain the correct eclipse depths in all colors and for $\xi = +1.5$ we could obtain neither the correct depths nor the correct shapes of the eclipses. The failures to fit the observed light curves for $\xi = -0.40$ and $\xi = +1.5$ add considerable confidence to the value $\xi = -0.66$ that we found.

Finally, we note that the use of multicolor photometry was crucial. The solutions for HT Cas would have been grossly underconstrained if the light curve had been measured in only

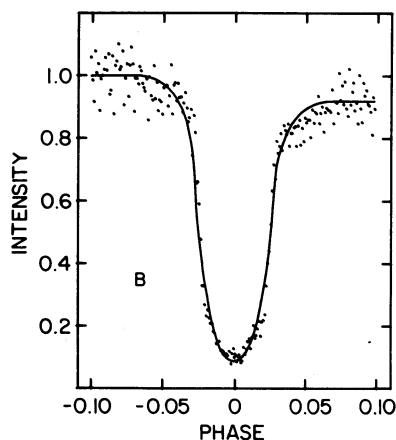


FIG. 8a

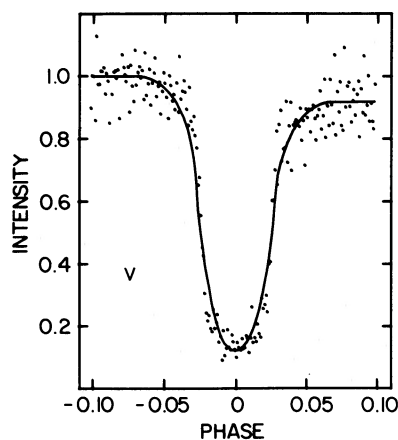


FIG. 8b

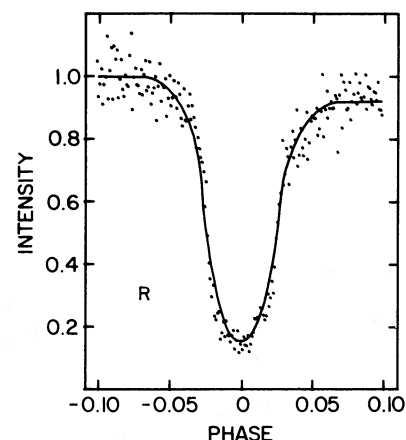


FIG. 8c

FIG. 8.—Averaged light curves and the synthetic light curves, respectively, for the case of $q = 2.83$. (a) B color. (b) V color. (c) R color.

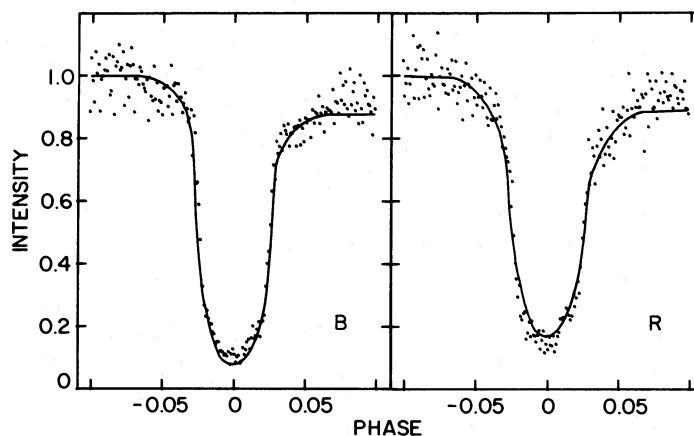


FIG. 9.—Synthetic light curves in *B* and *R* colors for $\xi = -0.40$

one color. Thus, we could have given many of the parameters, especially the temperatures, arbitrarily different values and produced fits that were as good as the fits using the correct values.

d) Error Analysis

The details of our method for estimating the errors in the derived parameters of HT Cas are described in the Appendix. Briefly, our method amounts to forcing one or two parameters to be slightly different from their best-fit values, reoptimizing the remaining parameters, and then measuring the increase of the variance of the fit that results from using the incorrect values of the parameters. The resulting standard deviations of the parameters are attached to each of the parameters for $q = 2.83$ in Table 5. The standard deviations of the parameters are nearly the same for other q 's as they are for $q = 2.83$. We believe the estimated errors of the various parameters represent a fair assessment of the true internal errors. For example, the light from the late-type star in HT Cas is a small contributor to the total luminosity and the change of its temperature does not cause a pronounced change in the synthetic light curve; hence its temperature has a relatively large standard deviation. The errors we quote are purely internal statistical errors, however, and do not include systematic errors that we have no way to assess. Possible sources of systematic errors in our results are errors in the radial velocities and

errors in the physics we have assumed for the light-curve synthesis program.

The procedure for computing the correlation coefficients between the derived parameters is also given in the Appendix. The correlation coefficients are generally small, with the following exceptions:

$$\rho \text{ between } i \text{ and } R_d = -0.25 ;$$

$$\rho \text{ between } i \text{ and } R_1 = -0.20 ;$$

and

$$\rho \text{ between } T_1 \text{ and } T_d = +0.16 ;$$

$$\rho \text{ between } T_d \text{ and } \xi = +0.24 .$$

The last two coefficients are positive because we have fitted the light curves of relative intensity instead of the absolute luminosity of the system. If we increase T_1 , we must also increase T_d to have the same depth of eclipse. The same is true for the correlation coefficient between T_d and ξ .

V. SUPEROUTBURST OF HT CAS

HT Cas did not have any obvious outburst from 1980 March 13 to 1985 January (AAVSO, private communication). In 1985 January it had a long and bright outburst lasting 10 days. According to the AAVSO, the visual magnitude of HT Cas reached its maximum brightness (visual magnitude 10.8) between January 13 and January 15 and then dropped to its

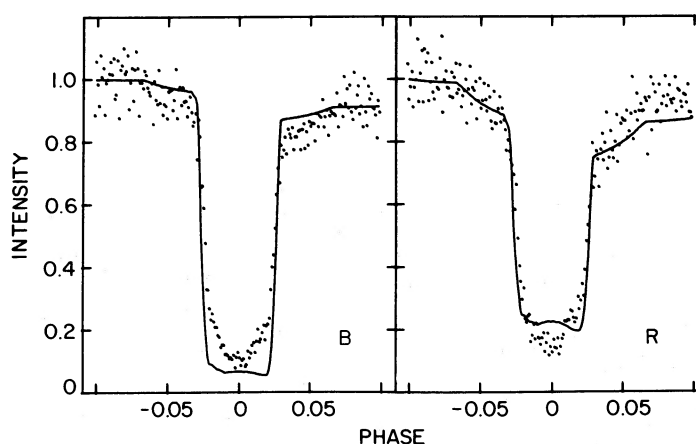


FIG. 10.—Synthetic light curves in *B* and *R* colors for $\xi = +1.50$

TABLE 6
PHOTOMETRIC OBSERVATIONS OF HT CAS IN SUPEROUTBURST

Run Number	Time of Start (UT)	Cycle Number	Mid-eclipse (HJD)	(O - C) (s)
3000.....	01:48:14 1985 Jan 19	32000	2446084.64778	-5
3005.....	02:14:43 1985 Jan 20	{32013	2446085.60540	+11
		{32014	2446085.67898	+6

quiescent state after January 28 (IAU circulars No. 4027 and No. 4028). We obtained high-speed photometry of HT Cas at McDonald Observatory on 1985 January 19 and 20 (UT) with the 2.1 m telescope, using Johnson *U* and *V* filters. Details of the observations are given in Table 6. We did not obtain photometry of the comparison star during these observations, so we cannot give the magnitudes of HT Cas on the standard *UBV* system; however, the estimated visual magnitude of HT Cas at the time of our observations was mag 13.

Three eclipses of HT Cas were obtained during the two observation runs. The cycle numbers, mid-eclipse timings, and the (*O* - *C*) residuals of the three eclipses are also given in Table 6. The (*O* - *C*)'s were calculated from the ephemeris given by equation (1). The light curves of HT Cas of run 3000 are plotted in Figure 11 and the light curves of run 3005 are in Figure 12. The light curves have been corrected for sky background, dark noise, and atmospheric extinction. Each point in the light curves is the mean of five one-second integrations.

Two superhumps are seen in the light curves of run 3000 and one in the light curves of run 3005. Superhumps are a diagnostic for the superoutbursts of SU UMa stars, one of the subclasses of dwarf novae. HT Cas is, therefore, an SU UMa star (for a review of the properties of the SU UMa stars and their

superhumps, see Vogt 1980). The period of the superhump is $P_{sh} = 0^d.076077 = 1.033P_{orb}$, and the beat period of the superhumps, P_{beat} is

$$P_{beat} = (1/P_{orb} - 1/P_{sh})^{-1} = 2.3 \text{ days} . \quad (8)$$

The beat period is comparable to the beat periods of other SU UMa stars, which typically lie between 2 and 3 days (Osaki 1985). The amplitudes of the superhumps are time and color dependent.

The mechanism causing superoutbursts of SU UMa stars is not clearly known and, as we shall argue in the next section, is different from the mechanism that causes the normal outbursts of dwarf novae (Vogt 1981; Whitehurst, Bath, and Charles 1984; Osaki 1985). The cause of the superhumps in the eruption light curves of SU UMa stars is also uncertain; we are therefore reluctant to use our synthesis program to model the light curves. Nevertheless, the three eclipses obtained during the superoutburst can provide a qualitative estimate of the size of the disk. The phase of the first contact of the partial eclipse of HT Cas in the three eclipses is not clearly shown but is somewhere between -0.085 and -0.095. Assuming that the mass ratio of HT Cas is $q = M_1/M_2 = 2.83$, using the orbital inclination we derived earlier (76°3), and adopting -0.090 ± 0.005 for the phase of the first contact, we find that $R_d = (0.34 \pm 0.07)a$. The disk lies inside the Roche lobe of the white dwarf, but it is 50% larger than it was when HT Cas was at minimum light.

VI. DISCUSSION

a) Color Indices

To derive parameters of HT Cas we fitted the shapes of the eclipse light curves but, since we normalized the light curves

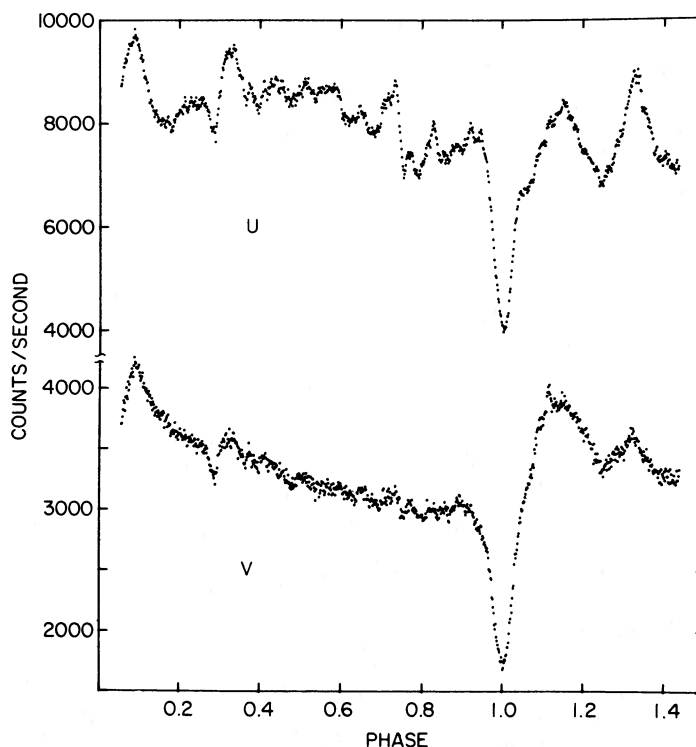


FIG. 11.—Light curves of HT Cas obtained in run 3000 while HT Cas was erupting. The upper one is in *U* and the lower one in *V*. Each point is an average of five one-second integrations.

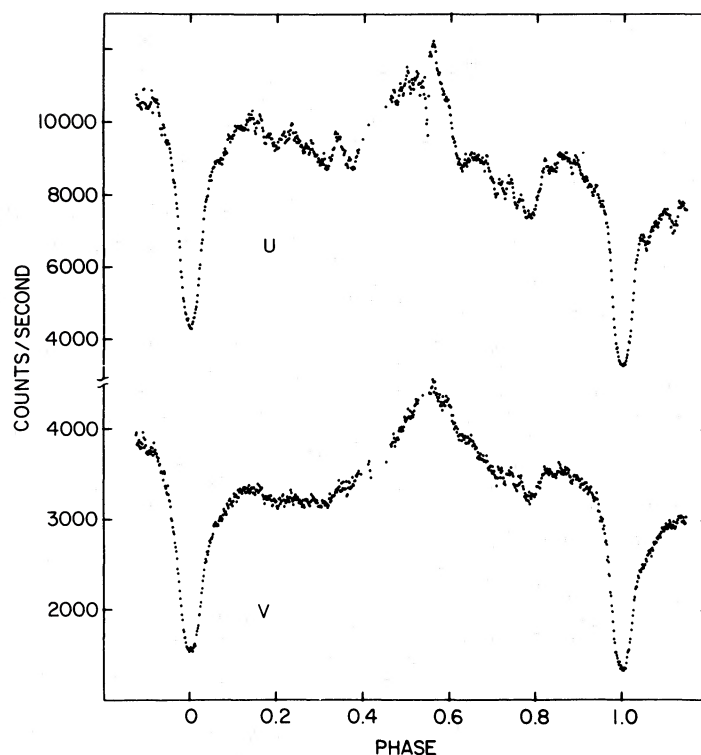


FIG. 12.—Light curves of HT Cas obtained in run 3005 while HT Cas was erupting. The upper one is in U and the lower one in V . Each point is an average of five one-second integrations.

out of eclipse to 1.0, we did not use the colors of the system. Nevertheless, our program can calculate the colors of the system, and, therefore, the colors of HT Cas are an independent check on the validity of our results. In the catalogue of Bruch (1984) the $(B-V)$ color of HT Cas lies between -0.04 and 0.09 and the $(U-B)$ color lies between -1.14 and -1.11 . The $(B-V)$ color obtained from our observations of HT Cas is -0.13 . Using the model parameters given in Table 5 (for $q = 2.83$) and using the transformations given by Matthews and Sandage (1963), we calculated synthetic colors of HT Cas to be $(B-V) = -0.11$ and $(U-B) = -1.02$, in good agreement with the observed colors. We will show in § VIc that the effect of interstellar extinction on the colors of HT Cas is small and can be ignored. The calculated colors depend directly on our assumption that the disk in HT Cas is optically thick and emits blackbody radiation. The agreement between the observed and calculated colors supports the validity of this assumption.

b) Mass Ratio

We have solved the eclipses of HT Cas for mass ratios between 2.00 and 4.00, but the range of acceptable mass ratios can now be further narrowed. Spectroscopy by Young *et al.* (1981) shows that the gas in the accretion disk around the white dwarf has velocities up to 2830 km s^{-1} in the Balmer lines. If we assume that the line broadening is due only to the Keplerian motion of the material in the disk and that the white dwarf has a carbon core, the white dwarf must have a mass greater than $0.5 M_{\odot}$ to generate an orbital velocity of 2820 km s^{-1} at its surface (Hamada and Salpeter 1961). Using the mass ratios and the masses of the white dwarf in Table 5, we find that the mass ratio of HT Cas must be greater than 2.7 to satisfy this requirement.

The secondary stars in cataclysmic variables can be on or above the main sequence but cannot lie significantly below the main sequence as long as they are burning hydrogen in their cores. Lacy (1977) gives the following empirical mass-radius relation for main-sequence stars:

$$\log R/R_{\odot} = 0.917 \log M/M_{\odot} - 0.020 ;$$

$$\text{for } -1.00 \leq \log M/M_{\odot} \leq +0.12 . \quad (9)$$

For each mass ratio and mass of the secondary given in Table 5 we can calculate the radius of the secondary, $R_2/R_{\odot}$, using equations (5) and (6), and compare the result with the radius calculated according to equation (9). We find that the secondary falls significantly below the lower main sequence if q is greater than 3.3. For $q = 3.5$, the radius of the secondary is $\sim 25\%$ less than that of a main-sequence star with the same mass. Thus, the mass ratio of HT Cas lies between 2.7 and 3.3.

The mass ratio $q = 2.83$ given by Young *et al.* (1981) is in the required range, and at $q = 2.83$, the masses and orbital inclination of HT Cas we have derived (Table 5) are the same as those of Young *et al.* The mass ratio given by Patterson (1981), $q = 3.68$, is outside the acceptable range, and the masses he finds are too small for $q = 3.68$. The major source of the difference between our results and Patterson's is that he used approximate analytic formulae instead of light-curve synthesis to obtain his results.

c) The Secondary Star and the Distance to HT Cas

Patterson (1984) derived an empirical relation between the orbital period and the effective temperature of the secondary star in cataclysmic variables:

$$T(K) = 2630 + 1380 \log P ; \quad \text{for } 0.05 < \log P < 0.70 .$$

(10)

According to this relation, the secondary star of HT Cas has a surface temperature of 2900 K, which agrees well with our result of 2800 K.

The temperature of the secondary star in HT Cas corresponds to a spectral type between M7 and M8 and an absolute visual magnitude of $M_v \approx 15$. At mid-eclipse, the apparent magnitude of HT Cas is $m_v = 18.44$. Our synthetic light curves show that the secondary star contributes $\sim 2.2\%$ of the total light at mid-eclipse, so that the visual magnitude of the red star alone is ~ 22.6 . This corresponds to a distance of 320 pc. Figure 45 of Deutschman, Davis, and Schild (1976) gives the interstellar reddening as a function of distance in the direction of Cassiopeia-Andromeda. For a distance of 320 pc this relationship gives $E(B-V) \approx 0.10$ mag, which, in turn, gives $A_v \approx 0.3$ mag. Since the standard deviation of the V magnitudes of HT Cas at mid-eclipse is ~ 0.4 mag and is larger than A_v , we have ignored the corrections to the distance and colors of HT Cas caused by extinction and reddening. However, since HT Cas is close to the galactic plane we cannot rule out the possibility that the extinction in its direction is considerably larger.

d) Mass Transfer Rate

If we assume that the total luminosity of the disk, L_d , is due to energy emitted by mass flowing through the disk and is not, for example, due to reprocessed energy from the white dwarf and the boundary layer between the white dwarf and the disk, then

$$L_d = kGM_1\dot{M}/R_1, \quad (11)$$

where M_1 and R_1 are the mass and radius of the white dwarf and k is 0.5 for an optically thick, steady state, viscous disk (Pringle 1981). Assuming the disk radiates like a blackbody, we have

$$L_d = 2 \int_{R_1}^{R_d} \int_0^{2\pi} \sigma T_d^4 (R/R_d)^{4\xi} R dR d\theta. \quad (12)$$

For HT Cas we find

$$\dot{M} = L_d R_1 / kGM_1 = 3.8 \times 10^{-10} M_\odot \text{ yr}^{-1}. \quad (13)$$

Equation (13) is the best estimate we can give for $\dot{M}$, but because of the large ranges of values in some of the parameters we derived for HT Cas, the above given mass transfer rate could be different from the real one by a factor of 10.

Hessman (1985) has constructed theoretical models to calculate radiation from accretion disks in cataclysmic variables using a modified version of the Williams and Ferguson program (Williams and Ferguson 1982). As in Tylenda (1981), an α -model prescription for the viscosity was used, and unlike the Williams and Ferguson models, continuum scattering opacity was included self-consistently. Population I element abundances of H, He, C, N, O, Na, Mg, Ca, and Fe were used to determine the electron densities. The increased continuum optical depths (due to inclusion of continuum scattering) and the use of fully angle-averaged fluxes and normal abundances make his models somewhat cooler than the Williams and Ferguson models for the same mass transfer rate, but otherwise Hessman's results are similar to the Williams and Ferguson models. At the mass transfer rate we have derived for HT Cas Hessman's models show that the disk is optically thick in the continuum out to $r = 0.33a$ even for $\alpha = 1$ and that at the outer edge of the disk of HT Cas ($r = 0.221a$) the optical depth is ~ 100 . Our assumption that the disk in HT Cas is optically thick is, therefore, self-consistent.

TABLE 7
RADII OF THE ACCRETION DISK IN HT CAS

q	R_d/a		
	Flannery	Paczynski	Present Work
2.0.....	0.1153	0.413	0.213
2.5.....	0.1270	0.434	0.218
3.0.....	0.1402	0.451	0.222
3.5.....	0.1486	0.465	0.227
4.0.....	0.1567	0.480	0.229

e) The Structure of the Accretion Disk

Table 7 gives the values of R_d/a we have derived for HT Cas and, for comparison, the theoretical disk radii calculated according to the models of Flannery (1975) and Paczynski (1977), respectively. Flannery assumed zero viscosity in calculating the radii of his disks. Paczynski assumed that the pressure in the disk was vanishingly small. His disks correspond roughly to the maximum sizes of disks with high viscosity. The radii we derived for the disk in HT Cas are much larger than those calculated by Flannery and smaller than those calculated by Paczynski. Our results confirm the conclusion of Sulkanen *et al.* (1981) that the disk in HT Cas is viscous.

Most modern theoretical works on the causes of dwarf nova eruptions have invoked instabilities in the accretion disk (e.g., Meyer and Meyer-Hofmeister 1982; Papaloizou, Faulkner, and Lin 1983; Cannizzo and Wheeler 1984; Smak 1984). According to these models, the transferred mass from the secondary star in a dwarf nova piles up in the disk so that the distributions of mass and surface temperature in the disk differ greatly from the distributions in a steady state disk. In the most extreme cases the mass accumulates in a torus at the outer edge of the disk. When the stored material reaches a critical temperature and surface density, a thermal instability ensues. The thermal instability causes the mass stored in the disk to accrete onto the white dwarf and gives rise to the normal outbursts observed in dwarf novae.

The temperature distribution across the disk in HT Cas fits the distribution expected from a steady state, optically thick, viscous accretion disk; the disk *does not* look like a non-steady state disk. For a steady state, optically thick, viscous accretion disk, the temperature distribution could be approximated by a power law with an exponent $\xi = -0.75$ (Pringle 1981). Our eclipse solution shows that the temperature distribution across the disk in HT Cas at quiescent state follows a power law with an exponent $\xi = -0.66 \pm 0.05$. Since we have ignored some other physical processes that may be at work in the disk, such as the heating of the outer part of the disk by the central part, the distribution due only to the mass accretion could be steeper than our result, i.e., the temperature distribution due only to mass accretion would look even more like that of a steady state, optically thick accretion disk.

Perhaps surprisingly, the fact that the disk in HT Cas looks like a steady state disk *agrees* with theoretical models for dwarf novae because HT Cas had not erupted in nearly five years. Furthermore, at the mass transfer rate we have found, it should not have erupted. A general feature of the models of disk instabilities in dwarf novae is that there exists a critical mass transfer rate $\dot{M}_{\text{crit}}$, below which steady accretion is not possible. When the mass transfers from the secondary at a rate higher than $\dot{M}_{\text{crit}}$, the instabilities which cause the normal outbursts in dwarf novae do not occur. Comparing the expressions for

$\dot{M}_{\text{crit}}$ from the works of Meyer and Meyer-Hofmeister (1982), Papaloizou *et al.* (1983), Smak (1983), and Cannizzo and Wheeler (1984), Shafter, Wheeler, and Cannizzo (1986) find that all four are in remarkable agreement on the form of $\dot{M}_{\text{crit}}$, and they summarize the results for $\dot{M}_{\text{crit}}$ by

$$\dot{M}_{\text{crit}} = 10^{16} R_{10}^{2.6} M_1^{-7/8} \text{ g s}^{-1}, \quad (14)$$

where R_{10} is the radius of the disk in units of 10^{10} cm. Inserting the radius of the disk in HT Cas into equation (14), we find $\dot{M}_{\text{crit}} = 1.7 \times 10^{-10} M_{\odot} \text{ yr}^{-1}$. Thus, the observed mass transfer rate in HT Cas ($\dot{M} = 3.8 \times 10^{-10} M_{\odot} \text{ yr}^{-1}$) is greater than $\dot{M}_{\text{crit}}$, and, according to the current theories, no instabilities in the accretion disk in HT Cas should occur.

HT Cas had a superoutburst in 1985 January. The mechanism of superoutbursts of dwarf novae is not clearly known, but it may be quite different from that of normal outbursts. The eclipse of run 0026 (Fig. 5) was obtained only two months before the superoutbursts and is not noticeably different from any of the other eclipses of HT Cas, so the disk was still in a steady state at that time. Since the observed superoutburst of HT Cas just followed our observations, it is likely that the disk was still in a steady state when the superoutburst occurred. This conclusion implies that an instability other than an instability in the accretion disk should be found to explain the superoutbursts. It appears to us that instabilities of the kind discussed by Bath (1973) and Osaki (1985) in which an instability in the late-type star causes superoutbursts deserve further attention.

f) Caveat

We have assumed throughout our analysis that the accretion disk in HT Cas is optically thick and have used blackbody radiation to calculate its emission. Our model of HT Cas is not unique, however. Horne (1985b) and his colleagues have also studied multicolor light curves of HT Cas in quiescence and have concluded that the accretion disk is optically thin. Their model has almost the same geometric parameters as our model but has different thermal parameters. According to their model, the mass transfer rate from the secondary star in HT Cas is extremely low, the white dwarf has a temperature less than 10,000 K, and the distance to HT Cas is much less than 320 pc. Neither of the two models is free from problems, but there is no unequivocal way to decide which is correct with the available data.

There are two main objections to our model. An optically thick disk cannot produce the emission lines that are observed in the spectra of HT Cas at minimum light (Williams 1983). The observed emission lines could, however, come from a chromosphere on top of an optically thick disk. Thus, although the emission lines prove that there is optically thin material in HT Cas, they do not prove that the disk itself is optically thin. The synthetic light curves shown in Figure 8 are not perfect fits

to the observed light curves. We have, however, restricted ourselves to a power-law to describe the temperature distribution across the disk. The power law is only an approximation of the real temperature distribution across the disk, and better fits to the observed light curves could have been obtained by using more complicated models for the distribution.

The model of HT Cas with an optically thin accretion disk produces an extremely unusual cataclysmic variable system. We have already noted that the temperature of the white dwarf is exceptionally low. The mass transfer rate must also be extremely low. Hessman's (1985) disk models indicate that a rate of mass transfer less than $\sim 10^{-12} M_{\odot} \text{ yr}^{-1}$ is necessary to make most of the disk in HT Cas optically thin. This may be an impossibly low rate. With the masses of the two stars in HT Cas given in Table 5 for $q = 2.83$, even gravitational radiation produces a mass transfer rate of $\sim 4 \times 10^{-11} M_{\odot} \text{ yr}^{-1}$. One may ask then how it is possible to produce a mass transfer rate as low as required for an optically thin disk in HT Cas.

We prefer our model because it gives internally consistent results. The mass transfer rate derived from the model is greater than both the minimum rate set by gravitational radiation and the critical rate $\dot{M}_{\text{crit}}$, thus explaining the absence of eruptions of HT Cas during our observations. The temperature distribution across the disk follows a power law with $\zeta = -0.66$, which is close to the value -0.75 given by theoretical models of steady state, optically thick accretion disks. This agreement can hardly be coincidental. Finally, with the parameters given by this model the theoretical models of radiation from accretion disks in cataclysmic variables by Hessman (1985) do indeed produce an optically thick accretion disk in HT Cas.

HT Cas is an unusually important system, and it is worth the effort to obtain data that would positively eliminate one of the two models. Although the two models give similar results at the optical wavelengths, they should give different results at ultraviolet wavelengths. The optically thin model predicts a low ultraviolet flux; the optically thick model could give either a high or a low flux depending on the extinction to HT Cas. An ultraviolet spectrogram with a quality high enough to measure the extinction to HT Cas should, therefore, discriminate between the two models. As HT Cas is faint, it may be necessary to use the space telescope to obtain a spectrogram with suitably high quality.

We thank F. V. Hessman for calculating the α -model disks for HT Cas discussed in § IVd and W. H. Jefferys for bringing the error analysis method discussed in the Appendix to our attention. We thank Keith Horne for his helpful comments. We thank Janet A. Mattei and the observers of the AAVSO for notifying us of the 1985 January outburst of HT Cas. This research was supported in part by NSF grants AST 83-16496 and AST 85-00790.

APPENDIX

DESCRIPTION OF THE LIGHT-CURVE SYNTHESIS PROGRAM

Our program to synthesize light curves of eclipsing binaries can be operated in three modes, corresponding to three different structures of the binary star. Mode 1 is for a detached or semidetached system, or a system in marginal contact, and Roche geometry is used for both component stars. Mode 2 is also for a detached or semidetached system. Roche geometry is used for one component, but the other component is assumed to be a compact object that may be approximated by a sphere. Mode 3 is used for cataclysmic variables. The model for mode 3 is that proposed by Warner and Nather (1971) and by Smak (1971). It includes three sources of

radiation, the primary star (a white dwarf), the secondary star which fills its Roche lobe, and an accretion disk around the primary star with a bright spot at its edge.

The synthetic light curves is calculated in three steps. First, we calculate the geometry of the system. Then we compute the temperature and intensity distribution across the surface of each component, taking into account gravity darkening, limb darkening, and reflection effect. Finally, we calculate the light from each component that can be seen by an observer and add the contributions of all the components together to yield the synthetic light curve.

We have also developed a method of analyzing the errors in the parameters derived from the light-curve synthesis program. The method gives standard deviations for each parameter and correlation coefficients between the parameters.

a) The Geometry

We assume that the orbits of the stars are circular. We also assume synchronous rotation for the two stars and, hence, the geometry of the Roche zero-velocity surfaces is uniquely determined by the mass ratio $q = M_1/M_2$. Taking spherical polar coordinates centered on the center of gravity of the secondary star (M_2) and adopting M_2 as the unity of mass, a , the distance between the centers of gravity of the two stars, as the unit of length and choosing the unit of time so that the constant of gravity is $G = 1$, we have (Kopal 1959)

$$\Psi = 1/r + q/(1 - 2/r + r^2)^{1/2} - qrl + \frac{1}{2}(1 + q)(1 - n^2)r^2 + \frac{1}{2}q^2/(1 + q), \quad (\text{A1})$$

where $\Psi = a\Omega/GM_2$, Ω is the total potential due to all forces acting on a point (r, θ, ϕ) , and (l, m, n) are the direction cosines of the radius vector to the point. In mode 1 the photospheres of both stars are coincident with the Roche zero-velocity surfaces, and equation (A1) is solved numerically to give the geometry of the surfaces. In modes 2 and 3 the secondary fills its Roche lobe and the radius of the primary star (a white dwarf for cataclysmic variables) is small compared to the radius of its Roche lobe, so we assume its photosphere is spherical.

The surface gravity at each point on the surface of the secondary is given by

$$g(x, y, z) = -[(\partial\Psi/\partial x)^2 + (\partial\Psi/\partial y)^2 + (\partial\Psi/\partial z)^2]^{1/2}, \quad (\text{A2})$$

where x, y , and z are the coordinates of the point in a Cartesian coordinate system centered on the center of mass of the secondary (Kopal 1959). The X -axis points toward the primary and the Z -axis is perpendicular to the orbital plane of the system. The three derivatives in equation (A2) can be expressed in spherical polar coordinates by

$$\begin{pmatrix} \partial\Psi/\partial x \\ \partial\Psi/\partial y \\ \partial\Psi/\partial z \end{pmatrix} = \begin{pmatrix} l & (1 - l^2) & -ln \\ m & -lm & -mn \\ n & -ln & (1 - n^2) \end{pmatrix} \begin{pmatrix} P \\ Q \\ R \end{pmatrix}, \quad (\text{A3})$$

where $P = \Psi/r$; $Q = q\{[r/(1 - 2rl + r^2)^{3/2}] - r\}$ and $R = -2an$.

We mesh the surface of each star into 31×51 grid points. Along the azimuthal direction (ϕ), the 51 points are distributed uniformly. Along the direction of increasing θ (from 0 to π), the 31 grid points are distributed at equal intervals in $\cos \theta$. This method of distributing the points along the θ direction makes the elemental areas of the star's surface more nearly equal. Equations (A1) and (A3) yield, for each grid point, the radius (r), the effective surface gravity (g), the direction cosines (l, m, n), the unit vector normal to the surface ($\mathbf{n}_s$: l_s, m_s, n_s) with components given by

$$l_s = (1/g)(\partial\Psi/\partial x), \quad m_s = (1/g)(\partial\Psi/\partial y), \quad n_s = (1/g)(\partial\Psi/\partial z), \quad (\text{A4})$$

and the element surface area,

$$\Delta\Sigma = (r^2/\cos \beta) \sin \theta \Delta\theta \Delta\phi, \quad (\text{A5})$$

where $\cos \beta = \mathbf{n} \cdot \mathbf{n}_s$.

The accretion disk around the primary star is circular, geometrically thin, and in the plane of the orbit. We use plane polar coordinates (r, α) for the disk, and mesh the grid into 31×51 points uniformly spaced in azimuth and radius. The position and size of a bright spot on the edge of the disk are specified by three grid points (n_1, n_2, n_3) such that the position and size of the spot (r_s, α_s) are given by $r(n_1) \leq r_s \leq r(n_2)$ and $\alpha(n_2) \leq \alpha_s \leq \alpha(n_3)$.

b) Photometric Parameters

The temperature across the tidally distorted surface of the secondary star is altered by gravity darkening. The temperature T at a point (r, θ, ϕ) is related to T_{pole} , the temperature on the pole of the star, by

$$T(r, \theta, \phi) = T_{\text{pole}}[g(r, \theta, \phi)/g_{\text{pole}}]^\beta, \quad (\text{A6})$$

where $g(r, \theta, \phi)$ and g_{pole} are surface gravity at the point and the pole, respectively, and β is the gravity-darkening coefficient. Von Zeipel (1924) showed that the outgoing flux was proportional to the local gravity g in a radiative atmosphere, or $\beta = 0.25$. Lucy (1967) derived a gravity-darkening coefficient of $\beta = 0.08$ for stars with convective envelopes. Since the secondary stars in cataclysmic variables are late-type stars and should have convective envelopes, we use $\beta = 0.08$ for our calculations.

Reprocessing of radiation from a companion star—usually called the reflection effect—also strongly alters the temperature distribution across the surface of a star in a close binary. Since rigorous treatments of reflection effect can consume large amounts of computer time, we have adopted a simplified treatment similar to that described by Chen and Rhein (1969). In cataclysmic variables, the temperature of the primary star and the accretion disk are generally much higher than the temperature of the secondary star, so

we only compute the change in temperature of the secondary star due to reprocessing of the radiation from the primary and the disk and ignore the changes in temperatures of the primary and the disk due to the radiation from the secondary. If $F_{\text{irr}}(r, \theta, \phi)$ is the incident flux of light from all points on the primary and the disk that can be seen from the point (r, θ, ϕ) on the surface of the secondary star, W is the bolometric albedo at the point, and if the reprocessed light is completely rethermalized, the temperature T_e at the point becomes

$$T_e^4 = T_{\text{pole}}^4 [g(r, \theta, \phi)/g_{\text{pole}}]^{4\beta} + WF_{\text{irr}}/\sigma, \quad (\text{A7})$$

where σ is the Stefan-Boltzman constant. The albedo W depends on the structure of the irradiated photosphere and is not a well-determined parameter. Following Rucinski (1969), we have assumed that half of the incident light is thermalized and reemitted and half is convected into the interior of the star, and we use $W = 0.5$. We normally ignore the reflection effect due to the bright spot on the edge of the disk because it does not change our results significantly and because inclusion of the bright spot would drastically increase the amount of time required to compute a light curve. The temperature across the disk will also be altered by reprocessing of the radiation from the white dwarf or inner boundary layer of the disk. However, we treat the temperature distribution across the disk as an unknown to be determined by fitting the synthetic light curves to real data, so that the reflection effect in the disk could be extracted from our results if desired, but it is not built into the temperature distribution.

The late-type star, the white dwarf, and the accretion disk are all limb darkened, and we compute the limb darkening from the usual linear law

$$I = I_0(1 - u + u \cos \gamma), \quad (\text{A8})$$

where u is the limb-darkening coefficient and γ is the angle between the line of sight and the normal to the surface.

Radiation from each element of area on the surfaces of the two component stars and across the disk is computed assuming the surfaces radiate as black bodies. Blackbody radiation is a crude approximation, but it is adequate for our purposes. Furthermore, it is far from clear that a more sophisticated treatment of the local spectral energy distribution is justified for the accretion disk.

The temperature distribution across the accretion disk in our model is not restricted to that of an optically thick, steady state accretion disk. We adopt a power law for the distribution:

$$T(r) = T_d(r/r_d)^\xi, \quad (\text{A9})$$

where $T(r)$ is the temperature at position (r, α) and T_d is the temperature at the outer radius r_d of the disk. The exponent ξ is determined by best fits to the observed light curves. The monochromatic radiation from the surface of the accretion disk is calculated from the Planck function. The use of blackbody radiation implies that the disk is optically thick. Consequently, the derived temperature and brightness distribution will be incorrect for the optically thin part of the accretion disk.

c) Light-Curve Synthesis

The above calculations yield the geometric and physical parameters needed to synthesize a light curve. For the eclipsing star (the secondary star) we simply add up the light from all the points that can be seen by the observers. To calculate the light from the eclipsed star (the primary) and the disk, we first project all the points on the edge of the secondary star onto the plane of the sky. The shadow of the secondary on the plane of the sky forms a polygon and the projected edge points become vertices of the polygon. Then we project all the points on the surfaces of the primary star and the disk facing the observers onto the plane of the sky. A subroutine checks each projected point of the primary star and the disk to see whether it is outside the polygon formed by the shadow of the secondary star. Then the program adds up the light from all the points that are outside the polygon. Finally, the light from the three components yields the light curve. The light curve synthesis can be performed for different colors. The response functions of U, B, V, R , and I filters of Johnson's photometric system are adopted from Johnson (1965).

The light-curve synthesis program is written in Fortran 77. The program is run on a VAX 11/780 computer with a UNIX operating system. In mode 1 the program needs 1633.4 seconds for synthesizing 50 phase points and 1680.8 seconds for 100 phase points. In mode 3 the program needs 814.5 seconds for synthesizing 50 phase points and 932.6 seconds for 100 phase points.

d) Error Analysis

Many geometric and physical parameters are involved in the light-curve synthesis. Small changes in some parameters can bring about relatively large changes in the synthetic light curve while changes in other parameters will have only a slight effect. Consequently, some of the parameters derived by light-curve fitting are reliable and some are only indicative. Furthermore, the parameters are not independent of one another; many of them are strongly correlated. Our error analysis method can give both the standard deviations for the derived parameters and the correlation coefficients between pairs of the parameters.

Suppose that the true values of a set of N quantities are $x_1, x_2, \dots, x_N$, and that the measured values of the quantities are $\chi_1, \chi_2, \dots, \chi_N$. If the errors in the measurements, $\epsilon_i = \chi_i - x_i$, have a multinormal distribution, then the probability distribution function for the errors is

$$f(\epsilon_1, \dots, \epsilon_N) = \frac{1}{(2\pi)^{N/2} \sigma_1 \dots \sigma_N} \exp \left[-\frac{1}{2} \left(\frac{\epsilon_1^2}{\sigma_1^2} + \dots + \frac{\epsilon_N^2}{\sigma_N^2} \right) \right], \quad (\text{A10})$$

where $\sigma_1, \dots, \sigma_N$ are the standard deviations of the measurements. This function is sometimes called a likelihood function since it tells what the errors in the measurements are likely to be.

A common problem in observational astronomy is to estimate the true values of x_i , given the measurements χ_i . One solution to this problem is to choose the x_i so that they maximize the likelihood function. This solution, called a maximum likelihood solution,

is the same as the standard least-squares solution if the likelihood is a multinormal distribution because, in this case, maximizing the likelihood function is equivalent to minimizing

$$S = \frac{\epsilon_1^2}{\sigma_1^2} + \cdots + \frac{\epsilon_N^2}{\sigma_N^2}. \quad (\text{A11})$$

When we fit our synthetic light curves to the observed light curves, we have N observational data points I_i^{obs} and N calculated points I_i^{cal} . We adopt a normal distribution for the errors, $\epsilon_i = (I_i^{\text{obs}} - I_i^{\text{cal}})$, and assume the σ 's are equal to one another, i.e., $\sigma_1 = \sigma_2 = \cdots = \sigma_N = W$. When we have

$$S = W^{-2} \sum (I_i^{\text{obs}} - I_i^{\text{cal}})^2. \quad (\text{A12})$$

Thus, choosing the best fit to the observational data is equivalent to minimizing S . We denote the minimum value of S by S_0 .

The calculated points will be a function of n parameters $\xi_1, \xi_2, \dots, \xi_n$. Suppose that when $\xi_1 = \xi_{10}, \xi_2 = \xi_{20}, \dots, \xi_n = \xi_{n0}$, the calculated points yield a best fit, i.e., S has its minimum value S_0 , then for small changes in the parameters we have

$$S = S_0 + C^T \sigma^{-1} C = S_0 + C^T A C, \quad (\text{A13})$$

where σ is the covariance matrix, $A = \sigma^{-1}$, C is an $n \times 1$ matrix:

$$C = \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{pmatrix} = \begin{pmatrix} \xi_1 - \xi_{10} \\ \xi_2 - \xi_{20} \\ \vdots \\ \xi_n - \xi_{n0} \end{pmatrix}, \quad (\text{A14})$$

and C^T is the transposed matrix of C . The errors and the covariances among the errors in the best-fit values of the ξ_i are given by the components of the covariance matrix.

We know that S has its minimum value S_0 when $C_1 = C_2 = \cdots = C_n = 0$. If we change C_i , the i th element of C , by a small amount d , while keeping the other elements equal to 0, then we have

$$C_i = K C = d, \quad (\text{A15})$$

where $K = (0, \dots, 0, 1, 0, \dots, 0)$, i.e., the i th element of K is equal 1 and all the other elements are zero.

We want to minimize S , subject to the condition that $C_i = d$, i.e., $K C - d = 0$. Using the method of Lagrange multipliers, we construct a new function:

$$\tilde{S} = S_0 + C^T A C - 2\lambda(C_i - d) = S_0 + C^T A C - 2\lambda(K C - d), \quad (\text{A16})$$

where λ is a Lagrange multiplier. The requirement that $\delta \tilde{S} = 0$ gives us an equation:

$$\delta \tilde{S} = 2C^T A \delta C - 2\lambda K \delta C = 0. \quad (\text{A17})$$

By solving this equation we find that

$$\begin{aligned} \lambda &= d/(K A^{-1} K^T), \\ C^T &= [d/(K A^{-1} K^T)] K A^{-1}, \\ C &= [d/(K A^{-1} K^T)] A^{-1} K^T, \\ S &= S_0 + C^T A C = S_0 + [d^2/(K A^{-1} K^T)] = S_0 + d^2/\sigma_{ii}, \end{aligned} \quad (\text{A18})$$

where $K A^{-1} K^T = K \sigma K^T = \sigma_{ii}$. Then the standard deviation of the i th parameter is given by

$$\sigma_i^2 = \sigma_{ii} = d^2/(S - S_0). \quad (\text{A19})$$

In order to get σ_{ij} , we transform to a new set of variables:

$$\begin{aligned} C'_1 &= C_1, \dots, C'_{i-1} = C_{i-1}, \\ C'_i &= (1/\sqrt{2})(C_i + C_j), \\ C'_{i+1} &= C_{i+1}, \dots, C'_{j-1} = C_{j-1}, \\ C'_j &= (1/\sqrt{2})(C_i - C_j), \\ C'_{j+1} &= C_{j+1}, \dots, C'_n = C_n. \end{aligned} \quad (\text{A20})$$

The variance S equals its minimum value, S_0 , at $C'_1 = \cdots = C'_i = \cdots = C'_j = \cdots = C'_n = 0$.

We change C'_i from 0 to d while setting $C'_j = 0$. Or, equivalently, we set $C_i = C_j = d/2^{1/2}$, while setting the other C to remain 0. Then we have

$$\sigma_{C'_i}^2 = d^2/(S - S_0). \quad (\text{A21})$$

TABLE 8
PHOTOMETRIC ELEMENTS OF ZZ BOO

Parameter	McNamara <i>et al.</i>	Our Calculation
Inclination i	$88^\circ 54 \pm 0.22$	$88^\circ 54 \pm 0.09$
Mass ratio M_1/M_2	1.04 ± 0.04	1.04 ± 0.04
Radius:		
R_1	0.122 ± 0.001	0.1223 ± 0.0008^a
R_2	0.119 ± 0.001	0.1189 ± 0.0008^a
Surface temperature:		
T_{e1}	6900°	$6900^\circ \pm 29^\circ$
T_{e2}	6900°	$6900^\circ \pm 27^\circ$

^a Averaged radius.

Since $\sigma_{c_i'}^2 = \frac{1}{2}(\sigma_{c_i}^2 + 2\sigma_{c_i c_j} + \sigma_{c_j}^2)$, then

$$\begin{aligned}\sigma_{ij} &= \sigma_{c_i'}^2 - \frac{1}{2}(\sigma_i^2 + \sigma_j^2) \\ &= d^2/(S - S_0) - \frac{1}{2}(\sigma_i^2 + \sigma_j^2).\end{aligned}\quad (\text{A22})$$

If desired, σ_{ij} can be converted to the correlation coefficient ρ_{ij} by

$$\rho_{ij} = \sigma_{ij}/\sqrt{\sigma_i \sigma_j}.\quad (\text{A23})$$

For computational reasons it is convenient to use a slightly different definition of C' . We set

$$C'_i = C_i/\sigma_{c_i} + C_j/\sigma_{c_j}, \quad C'_j = C_i/\sigma_{c_i} - C_j/\sigma_{c_j},\quad (\text{A24})$$

and for the other C' each is equal to its corresponding C . By setting $C_i = d\sigma_i/2$ and $C_j = d\sigma_j/2$, where d is a small value, we find that

$$\rho_{ij} = \frac{1}{2}d^2/(S - S_0) - 1.\quad (\text{A25})$$

The best fit to the observed light curves yields the minimum value of S , i.e., S_0 . To calculate the standard deviation of a parameter we change the value of the parameter by a small amount d , which we set to be 5% of the derived value, and fix this parameter at the changed value. With this parameter fixed at its new value we change other parameters to get a best possible fit for all the colors, i.e., find another minimum of S , and compute S . Inserting S , S_0 and d into equation (A19) gives the standard deviation of this parameter.

The correlation coefficients are calculated according to equations (A24) and (A25). We first choose two parameters and change them to new values according to (A24). We then find the best fits for all colors with the two parameters fixed at the new values and compute S . Inserting S , S_0 , and d into equation (A25) gives the correlation coefficient between these two parameters.

e) Tests of the Program

We checked the Roche lobe geometry computed by our program against published results (Plavec and Kratochvil 1964; Boshkarev, Karitskaya, and Shakura 1979; Mochnacki 1984). For a given mass ratio the differences between our results and the published results are 0.0001 in the Roche lobe potential and less than 0.004 in the radii. We then used our program to solve the light

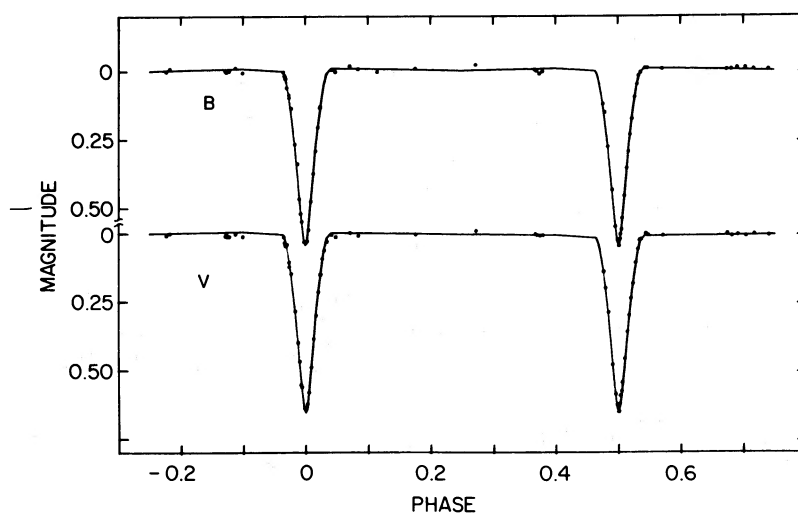


FIG. 13.—The observed light curves and the synthetic light curves of ZZ Boo in B (upper) and V (bottom) colors. The observed data are from McNamara *et al.* (1971).

TABLE 9
PHOTOMETRIC ELEMENTS OF WX CEP

Parameter	Ebbighausen <i>et al.</i>	Our Calculation
Inclination i	$85^\circ 47 \pm 0.07$	$85^\circ 40 \pm 0.12$
Mass ratio M_1/M_2	1.00	1.00 ± 0.007
Radius:		
R_1	0.2530 ± 0.0005	0.2495 ± 0.0007^a
R_2	0.1700 ± 0.0005	0.1686 ± 0.0005^a
Spectral type:		
Star 1	A5	...
Star 2	A2	...
Surface temperature (K):		
Star 1	...	$8320^\circ \pm 28^\circ$
Star 2	...	$9050^\circ \pm 30^\circ$

^a Averaged radius.

TABLE 10
PHOTOMETRIC ELEMENTS OF SX AUR

Parameter	Chambliss <i>et al.</i>	Our Calculation
Inclination i	$82^\circ 00 \pm 0.08$	$82^\circ 10 \pm 0.11$
Mass ratio M_1/M_2	0.5986 ± 0.0037	0.5986 ± 0.0027
Radius of star 1:		
$r_1(\text{pole})$	0.3969 ± 0.0011	0.3969 ± 0.0005
$r_1(\text{side})$	0.4192 ± 0.0014	0.4190 ± 0.0007
$r_1(\text{back})$	0.4477 ± 0.0019	0.4477 ± 0.0011
$r_1(\text{point})$	0.5135 ± 0.0072	0.5153 ± 0.0064
Radius of star 2:		
$r_2(\text{pole})$	0.3139 ± 0.0012	0.3139 ± 0.0010
$r_2(\text{side})$	0.3277 ± 0.0014	0.3276 ± 0.0012
$r_2(\text{back})$	0.3602 ± 0.0021	0.3602 ± 0.0020
$r_2(\text{point})$	0.4281 ± 0.0020	0.4311 ± 0.0041
Surface temperature:		
T_{e1}	19200° ^a	19200°
T_{e2}	$14804^\circ \pm 18^\circ$	$14700^\circ \pm 26^\circ$

^a Assumed temperature according to spectral type.

curves of three binary systems: ZZ Boo, WX Cep, and SX Aur, for which eclipse solutions using other computer programs have been published.

ZZ Boo is an eclipsing binary with a period of $4^d.9917$. Photometric observations of the system have been published by McNamara, Hansen, and Wilcken (1971). Observations show no evidence of ellipticity or reflection effects and indicate that ZZ Boo is a stable binary system consisting of two well-separated F0 main-sequence stars. McNamara *et al.* employed the differential least-squares method outlined by Irwin (1947) to derive the physical and geometric element of the system. Since ZZ Boo is a detached system, we have used it to check our program operated in mode 1. Table 8 gives the parameters by using our program and the parameters given by McNamara *et al.* (1971). There is no significant difference between our results and those of McNamara *et al.* Figure 13 shows the synthetic light curves using our derived parameters, together with the observed eclipse in B and V colors by McNamara *et al.*

WX Cep, also designated BD +62°2091 and HD 213631, was observed by Ebbighausen, Penegor, and Straton (1975), who obtained 3120 photoelectric magnitudes in each of the B and V filters. Ebbighausen *et al.* derived preliminary elements following the procedure given by Russell and Merrill (1952) and then corrected the elements using the method of Irwin (1947). Their results show that both the component stars are smaller than their limiting Roche surfaces. Their derived elements of the system are given in Table 9. The elements of WX Cep obtained using our program are listed in Table 9, and the synthetic light curve calculated by our program is shown in Figure 14. There is no significant difference between our results and those of Ebbighausen *et al.* (1975) for the orbital inclination, mass ratio, and temperatures of the two stars. There are, however, differences of $\sim 1\%$ between our results and those of Ebbighausen *et al.* (1975) for the radii of the two stars. The reason for these differences is that we have used equipotential surfaces for the photospheres of the two stars while the two stars in the model of Ebbighausen *et al.* were spherical.

SX Aur provides a more interesting test of our program. It is an eclipsing binary consisting of two B-type stars with a period of $1^d.210081$. Chambliss and Leung (1979) have analyzed UBV photoelectric observations using the Russell-Merrill method (Russell and Merrill 1952), the Kopal-Jurkevich method (Kopal 1959), and the Wilson-Devinney method (Wilson and Devinney 1971). The third procedure yielded the most consistent solutions. The system was found to be semidetached, with the less massive component

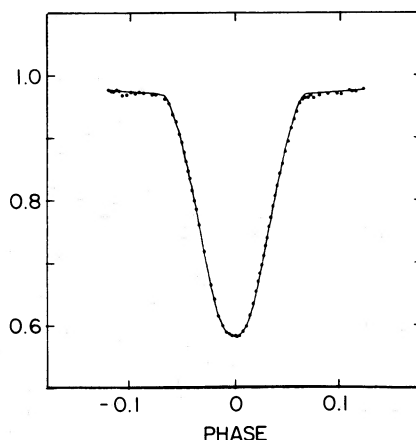


FIG. 14.—The observed light curve and the synthetic light curve in V of WX Cep. The observed data are from Ebbighausen *et al.* (1975).

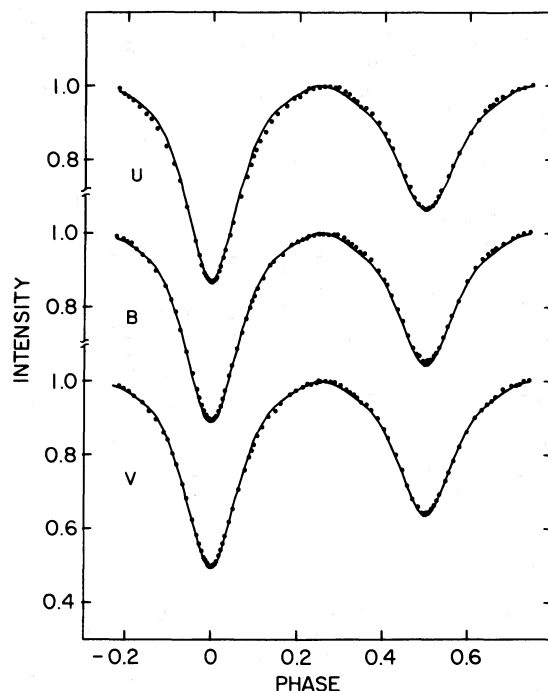


FIG. 15.—Observed light curves and the synthetic light curves in U , B , and V of SX Aur. Observed data are from Chambliss and Leung (1979).

filling its Roche lobe. The other component nearly fills its Roche lobe. SX Aur was chosen to check our program operated in mode 1 with one component filling its Roche lobe. The elements of the system of Chambliss and Leung and those derived using our program are given in Table 10. The observed light curves and our synthetic light curves in U , B , and V , respectively, are plotted in Figure 15. There is no significant difference between our results and those of Chambliss and Leung.

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R. EDWARD NATHER, EDWARD L. ROBINSON, and ER-HO ZHANG: Department of Astronomy, University of Texas at Austin, Austin, TX 78712