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A PHOTOMETRIC AND RADIAL VELOCITY STUDY OF SIX SOUTHERN CEPHEIDS.
II. BINARITY, RADII, LUMINOSITIES, AND MASSES

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ABSTRACT

The data now available for the Galactic Cepheids AQ Car (period = $9^{d}.77$), XX Cen ($10^{d}.95$), XY Car ($12^{d}.44$), TT Aql ($13^{d}.75$), XX Car ($15^{d}.71$), and XZ Car ($16^{d}.65$) are analyzed for binarity, radius, luminosity, and mass.

There is weak evidence only for binarity of XX Cen, XX Car, and XZ Car, and we conclude that all six Cepheids may be considered to be single. The new velocity data for XX Cen suggest that it may belong to a binary system with an orbital period of 800 d.

The radii are derived by an improved version of the Baade-Wesselink technique and are found to depend upon the color index used to define the surface brightness. The $V-I$ radii, for instance, are 20%–25% larger than the $B-V$ radii, and we suggest that the Cepheid mass discrepancy problem may be alleviated by use of appropriate colors in the $B-W$ radius solutions.

Subject headings: stars: binaries — stars: Cepheids — stars: pulsation

I. INTRODUCTION

In Paper I (Coulson, Caldwell, and Gieren 1985) we presented new photometric and radial velocity data for the six southern Galactic Cepheids AQ Car, XX Cen, XY Car, TT Aql, XX Car, and XZ Car. These data constituent more than half of the data available on these stars and allowed the production of homogeneous light, color, and velocity curves. They complement similar data on 15 shorter period stars studied by Gieren (1982*b*) and on other long-period stars presently being analyzed at SAAO.

In § II of this paper we examine the data for evidence of binarity using tests like those described by Gieren (1982*a*). In § III and the Appendix we describe our method of determining radii and discuss the results, highlighting the color dependence of the radius results. We conclude by deriving luminosities and masses for these six stars and comparing them with theoretical predictions.

II. BINARITY

Several tests for the binarity of Cepheids are practicable and are applied to our sample.

a) Radial Velocity Variations

Significant, systematic, and periodic variations in the mean velocity of a star are important indications of binarity.

In Paper I an analysis of the new radial velocity data for XX Cen showed that the Cepheid may partake of orbital motion with a period of ~800 days. To seek confirmation of this and to search for similar effects among the other five stars, we examined the existing published radial velocity data and found such data for XX Cen, TT Aql, and XX Car. The sources of these data are listed in Table 1, together with a source code and corrections, $\Delta_{\rm L\,III}$, ostensibly necessary to bring measures in these sources onto the Mount Wilson standard system (Wilson 1953).

Feast (1967), who gave measures of XX Cen and XX Car, obtained the zero point of his velocity system by assuming that three of the Cepheids he observed (XX Cen, S Nor, and X Sgr) have constant mean velocities. He determined the shifts necessary to minimize the residuals of his data for these stars about Stibbs's (1955) velocity curves as -3.3, $+1.6$, and 1.6 km s^{-1}, respectively. Since we suspect that XX Cen has a variable mean velocity (Paper I), we have corrected Feast's data for XX Cen and XX Car by -4.8 and -1.6 km s^{-1}, respectively, to bring them onto a system (Stibbs's) defined by the constancy of S Nor and X Sgr alone.

For each star we put the data from the sources into sets spanning ~60 days in order to allow resolution of any orbital effects. By comparing with the velocity templates derived in Paper I, each measurement gives an estimate of the star's mean velocity at that particular epoch, and the mean value for each set is easily derived. These are shown in Table 2, together with the standard error on the mean velocity, set size, and the mean Julian date for the set. The recent data for XX Cen, derived in Paper I, and the mean velocities for TT Aql and XX Car, derived also in Paper I, are shown appropriately in Table 2 also.

XX Cen: Figure 1 shows the mean velocity data for XX Cen. While the new data suggest an 800 day periodicity, the older data do not allow confirmation of it. A recent increase in the mean velocity from about -23 km s^{-1} at JD 2,440,000 to a present value near -18 km s^{-1} is possibly indicated by Figure 1, but the effect is small enough to be accounted for by any of several observational problems. Such an effect would suggest an additional periodicity in the XX Cen system. However, while an additional periodicity was also suggested by the photometry (see Paper I), these two effects do not describe the same phenomenon. Since the photometric effects could be explained evolutionarily (Szabados 1981), we would not claim at this stage that XX Cen experiences any orbital effect other than the 800 day effect seen in the new velocity data.

TABLE 1

SOURCES OF RADIAL VELOCITY DATA

Source	Code	$\Delta_{\mathrm{L\,III}}$
Joy 1937	J	−0.2
Stibbs 1955	S	0.0
Feast 1967	F	−4.8 XX Cen } see text −1.6 XX Car }
Evans 1976	E	0.0 (assumed)
Lloyd Evans 1980	L	−0.4
Paper I 1985	CCG 1	0.0

TT Aql: the radial velocity data of TT Aql show no evidence of mean velocity variations.

XX Car: the apparent variation within Feast's (1967) data is due mainly to the measure at JD 2,437,722.56; the other seven measures yield a mean of 1.0 ± 4.3 km s^{-1}. The difference of ~ 12 km s^{-1} between this and the mean velocity derived in Paper I may be significant, but will require further observations to be substantiated.

b) *Photometric tests*

i) *intrinsic Colors*

Mean intrinsic colors for the six stars in the sample are given in Table 11 of Paper I. With data for the 15 stars studied by Gieren (1981*a*, *b*, 1982*a*, *b*) we derive the following:

$$\begin{array}{ll} (B-V)_0 & = 0.45 \log P + 0.34 \ (0.05 \text{ mag}) \\ & \ \ \pm 0.05 \qquad\quad 0.04 \\ \langle B\rangle_0 - \langle V\rangle_0 & = 0.37 \log P + 0.37 \ (0.05) \quad , \\ & \ \ 0.05 \qquad\quad 0.04 \\ (V-I)_0 & = 0.32 \log P + 0.45 \ (0.03) \\ & \ \ 0.03 \qquad\quad 0.03 \end{array}$$

TABLE 2

MEAN VELOCITIES OF XX CENTAUR, TT AQUILAE, AND XX CARINAE

Cepheid	Mean Julian Date	V	(±)	Source	N
XX Cen	2,433,825	−14.4	3.8	S	7
	2,434,127	−23.9	4.2	S	10
	2,434,227	−21.3	...	S	1
	2,438,500	−22.8	3.1	F	5
	2,438,580	−24.0	3.2	F	7
	2,440,380	−19.7	2.8	L	3
	2.440.651	−23.0	...	L	1
	2,440,805	−24.9	3.5	L	2
	2,443,975	−15.6	0.6	CCG 1	8
	2,444,302	−19.9	2.3	CCG 1	1
	2,444,334	−20.6	1.4	CCG 1	4
	2,444,446	−20.2	0.5	CCG 1	5
	2,444,656	−15.5	2.3	CCG 1	1
	2,444,693	−14.9	0.8	CCG 1	4
	2,444,709	−16.3	1.0	CCG 1	10
	2,444,775	−16.9	1.2	CCG 1	4
	2,445,042	−19.1	1.0	CCG 1	6
	2,445,091	−19.6	0.7	CCG 1	3
TT Aql	2,421,826	−2.7	...	J	1
	2,422,089	−6.3	0.2	J	2
	2,425,460	16.2	...	J	1
	2,425,549	−1.6	...	J	1
	2,425,596	2.6	7.9	J	3
	2.426,135	4.4	4.4	J	2
	2,440,831	1.5:	...	E	1
	2,441,122	5.7:	6.4	E	4
	2,444,177–4,778	0.7	0.5	CCG 1	33
XX Car	2,436,977	5.6	...	F	1
	2,437,418	1.4	2.0	F	2
	2,437,743	−11.1	6.7	F	2
	2,438,472	1.6	3.1	F	3
	2,443,968–5,093	−10.8	0.4	CCG 1	53

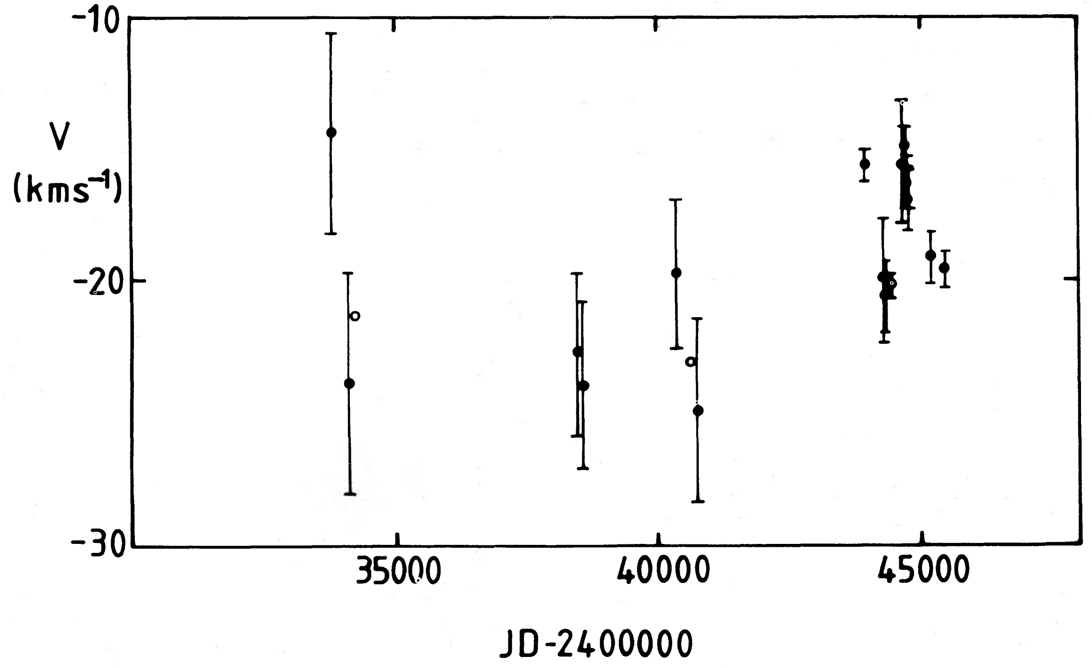

FIG. 1.—Mean velocity of XX Cen 1950–1983. Mean velocities and associated errors are given in Table 2.

TABLE 3

AREAS, LENGTHS, AND OPENNESS OF CEPHEID TWO-COLOR LOOPS

	$(U-B)/(B-V)$			$(B-V)_0/(V-I)_0$		
CEPHEID	Area (m^2)	Length (m)	Openness (m)	Area (m^2)	Length (m)	Openness (m)
AQ Car ...	0.0156	0.510	0.031	0.0036	0.427	0.008
XX Cen ...	0.0175	0.680	0.026	0.0069	0.614	0.011
XY Car ...	0.0357	0.734	0.048	0.0134	0.629	0.021
TT Aql ...	0.0341	1.015	0.034	0.0105	0.821	0.013
XX Car ...	0.0275	0.904	0.029	0.0200	0.922	0.022
XZ Car ...	...	...	...	0.0136	0.737	0.018
Mean........................			0.034			0.016
s.d...........................			0.009			0.006

where the magnitude in parentheses is the standard error per point. None of the stars in the extended sample deviates significantly from any of these relations.

ii) *Loop Openness in the Two-Color Diagrams*

The openness of loops in the $(U-B)/(B-V)$ and $(B-V)_0/(V-I)_0$ diagrams has been suggested as a test for binarity (e.g., Stobie 1970; Madore 1977). The use of this property has until now been qualitative, although Gieren (1982*a*) has estimated the openness of loops for the stars he studied. With the availability of high-quality photometry it should be possible to quantify loop openness, which we define as the ratio of the area enclosed to the length in the blue-red direction. For present purposes we shall ignore the presence or number of loop crossovers in the area calculation, and we shall define the length to be the square root of the sum of squares of the appropriate color amplitudes. The areas, lengths, and resulting openings are given in Table 3 for the $(U-B)/(B-V)$ and the $(B-V)_0/(V-I)_0$ loops. The mean openings are

$$(U-B)/(B-V) \quad 0.034 \pm 0.009 \text{ mag (s.d.)} \quad N = 5 \, ,$$

$$(B-V)_0/(V-I)_0 \; 0.016 \pm 0.006 \text{ mag} \qquad N = 6 \, .$$

These values are somewhat larger than the means for the shorter period stars studied by Gieren (1982*a*), but a full discussion of possible trends with period is left until further data are available.

TABLE 4

SLOPES OF THE $(B-V)_0/(V-I)_0$ DIAGRAMS AND PHASE SHIFT PARAMETERS

	$d(B-V)_0/d(V-I)_0$			
CEPHEID (1)	Observed (2)	Predicted (3)	$\Delta\phi_1$ (4)	$\Delta\phi_2$ (5)
AQ Car	1.46	1.46	0.060	0.121
XX Cen	1.41	1.47	0.068	0.122
XY Car	1.52	1.50	0.070	0.118
TT Aql	1.47	1.51	0.062	0.091
XX Car	1.32	1.53	0.071	0.105
XZ Car	1.47	1.54	0.16:	0.24:
s.d.	±0.03	0.07	...	...

NOTES.—Col. (3): Predicted slope is derived from

$$\frac{d(B-V)_0}{d(V-I)_0} = 0.37 \log P + 1.09 \, .$$

Col. (4): $\Delta\phi_1 = \phi_V^{\min} - \phi_{U-V}^{\min}$ (Gieren 1982*a*).
Col. (5): $\Delta\phi_2 = \Delta\phi_1/(\phi_V^{\min} - \phi_V^{\max})$ (Fernie 1980).

Excluding XZ Car, which was omitted from the $(U-B)/(B-V)$ mean above because of incomplete phase coverage in U, none of the stars in our sample appears deviant by this test.

iii) *Slopes of the $(B-V)_0/(V-I)_0$ Diagrams*

Although the data in this diagram follow the quadratic form of the intrinsic line (Dean, Warren, and Cousins 1978), a straight line fit should reflect the slope of the intrinsic line at the mean colors of the Cepheid. The slope predicted from the intrinsic line and from the $(V-I)_0$–log P relation derived earlier is

$$d(B-V)_0/d(V-I)_0 = \underset{\pm 0.03}{0.37} \log P + \underset{\pm 0.03}{1.9} \, .$$

The observed and predicted slopes are shown in column (3) of Table 4. None of the observed values is significantly deviant.

iv) *Amplitudes and Ratios*

In Table 5 we show U, B, V, and I amplitudes and some of their ratios and the color amplitudes of $U-B$, $B-V$, and $V-I$ and their ratios. These results are derived from the data in Table 9 of Paper I.

While amplitude ratios have been considered as tests for binarity (Fernie 1979; Gieren 1982*a*), the intrinsic scatters of these ratios prove to be quite substantial, however. From the data of Schaltenbrand and Tammann (1971), we estimate that the ratio of amplitudes $(U-B)/(B-V)$ has a standard deviation of ~20%–25% at any period between 3 and 15 days. In this light the data in Table 5 show no obvious deviants, although the $(U-B)/(V-I)$ ratio for XX Car appears suspiciously low, indicating a possible blue companion.

v) *Phase Shifts*

The presence of a blue companion has been suggested to move the minimum phase of the $U-V$ curve to an earlier phase than the V minimum. Table 4 shows two parameters proposed to quantify this effect; column (4) shows values of

$$\Delta\phi_1 = \phi_V^{\min} - \phi_{U-V}^{\min} \, ,$$

as used by Gieren (1982*a*), and column (5) shows

$$\Delta\phi_2 = \Delta\phi_1/(\phi_V^{\min} - \phi_V^{\max}) \, ,$$

as proposed by Fernie (1980). Omitting XZ Car we find

$$\langle\Delta\phi_1\rangle = 0.066 \pm 0.005 \text{ (s.d.)} \, ,$$

$$\langle\Delta\phi_2\rangle = 0.111 + 0.013 \text{ (s.d.)} \, .$$

TABLE 5

PHOTOMETRIC AMPLITUDES

Color	AQ Car	XX Cen	XY Car	TT Aql	XX Car	XZ Car
U	1.28	1.84	1.88	2.45	2.69	2.21
B	0.96	1.40	1.37	1.76	2.02	1.65
V	0.61	0.90	0.88	1.10	1.30	1.07
I	0.40	0.58	0.52	0.65	0.75	0.66
U/V	2.09	2.03	2.14	2.22	2.06	2.06
B/V	1.56	1.55	1.56	1.60	1.55	1.54
I/V	0.64	0.64	0.59	0.59	0.58	0.62
$U-B$	0.37	0.46	0.55	0.78	0.66	0.68:
$B-V$	0.35	0.50	0.50	0.65	0.71	0.58
$V-I$	0.23	0.34	0.35	0.46	0.56	0.43
$U-B/B-V$	1.04	0.93	1.09	1.14	0.93	1.17:
$U-B/V-I$	1.59	1.36	1.57	1.69	1.17	1.59:
$B-V/V-I$	1.53	1.47	1.44	1.41	1.26	1.36

These results confirm previous observations that even solitary Cepheids show nonzero values of these parameters. The value of $\langle \phi_1 \rangle$ agrees with that found by Gieren for his shorter period stars. None of our sample (XZ Car omitted) appears deviant in this regard.

b) Summary of Tests

The usefulness of the tests described above has been discussed by Gieren (1982*a*). We repeat that evidence from more than one test seems necessary before asserting binarity and note only weak evidence for three of the stars in our sample:

XX Cen (radial velocities);
XX Car (amplitude ratios);
XZ Car (loop openness).

We conclude, therefore, that each of these six stars may be assumed to be single.

III. RADII

We have used the maximum likelihood version of the Baade-Wesselink method described by Balona (1977), modified in three ways.

1. We deredden the photometric data before solving for the radius, taking into account the phase dependence of reddening as given by Dean, Warren, and Cousins (1978). The relative formulae may be found in § IX of Paper I. Following Balona we then have

$$V_0 = A(B-V)_0 - 5 \log (R_0 + r) + \text{constant} \qquad (1)$$

at all phases.

2. Balona assumed that the radius displacement r is small compared with the mean radius R_0 and so could express $\log_{10} (R_0 + r)$ as $\mu r/R_0$, where $\mu = \log_{10} e$. Equation (1) could be solved for R_0, A, and the constant by the maximum-likelihood technique. This approximation is not strictly tenable for the longer period Cepheids, since often $r/R_0 > 0.1$. We have thus modified Balona's method, as described in the Appendix.

3. The use of V and $B-V$ in equation (1) arises from the traditional expressions of effective temperature and bolometric correction (see Balona 1977, eq. [1.2]):

$$\log T_{\text{eff}} = a_0 + a_1(B-V)_0 \,,$$
$$M_{\text{bol}} - M_v = b_0 + b_1(B-V)_0 \,.$$

Any magnitude and color may suffice, however, provided that the relations in that system are linear, as above. Thus we have determined the radii for magnitudes B, V, R, and I and for colors $B-V$, $V-R$, $V-I$, and $R-I$, not just for the traditional V, $B-V$ combination. The photometry, radial velocities, and associated errors are as described in Paper I. Note that the error on each evaluation of the radius displacement calculated by integration of the radial velocity curve is related to the typical error of a velocity observation by $\sigma_r = 0.0808P\, \sigma_{RV}/\sqrt{N}$, where N is the number of velocity observations, and P, σ_{RV}, and σ_r are in units of days, km s^{-1}, and solar radii, respectively.

Solutions using the various possible choices for magnitudes and colors are given in Table 6 for TT Aql. In this table, N is the number of simultaneous photometric data in the chosen color-magnitude system, R is the derived radius in solar radii, A the surface brightness coefficient, and σ the standard deviation in solar radii of the radius displacements calculated from the photometry with respect to the radius displacement curve calculated by integrating the radial velocity curve. The geometric projection factor used is $p = 1.31$, and the errors are calculated using the sinusoidal approximations of Balona and Stobie (1979). The results in Table 6 for TT Aql typify those for the other stars, in that the derived radii appear insensitive to the choice of magnitude but frequently show significant dependence upon the choice of color. The procedure of dereddening the photometry before obtaining the solution has negligible effect upon R but a small effect upon A. And for these six stars the procedure of solving iteratively to account for non-negligible $\Delta R/R$ produces radii only ~2% larger than Balona's (1977) original formulation. The new approach is expected to show its full usefulness for Cepheids of still longer period.

TABLE 6

RADIUS OF TT AQUILAE

Magnitude	Color	N	R ($R_\odot$)	± (%)	A	± (%)	σ ($R_\odot$)
B	$B-V$	148	68	5	3.07	3	1.0
	$V-R$	90	107	9	7.32	3	3.6
	$V-I$	93	98	8	4.27	3	2.9
	$R-I$	88	86	10	10.28	4	4.2
V	$B-V$	148	68	5	2.07	3	1.0
	$V-R$	90	89	6	4.94	3	2.0
	$V-I$	93	85	6	2.88	3	1.7
	$R-I$	88	79	7	6.93	4	2.5
R	$B-V$	90	70	5	1.66	3	0.9
	$V-R$	90	89	6	3.96	3	2.0
	$V-I$	88	85	5	2.30	3	1.6
	$R-I$	88	80	6	5.54	4	2.1
I	$B-V$	93	72	5	1.36	3	0.9
	$V-R$	88	87	5	3.24	3	1.8
	$V-I$	93	84	5	1.89	3	1.7
	$R-I$	88	79	6	4.58	3	2.1
Weighted means........	$B-V$		$70 \pm 4\ R_\odot$				
	$V-R$		$90 \pm 5\ R_\odot$				
	$V-I$		$86 \pm 5\ R_\odot$				
	$R-I$		$80 \pm 5\ R_\odot$				

The solutions are also extremely insensitive to the adopted errors on the observational data.

TABLE 7

MEAN RADII BY COLOR INDEX[a]

Cepheid	R_{B-V}	R_{V-R}	R_{V-I}	R_{R-I}	Δ[b]
AQ Car	57	77	67	54	0.071
	±3	6	5	6	40
XX Cen	52	52	60	68	0.070
	3	4	4	7	37
XY Car	63	88	94	100	0.171
	3	7	6	11	35
TT Aql	70	90	86	80	0.094
	4	5	5	5	34
XX Car	75	108	91	72	0.083
	3	6	5	6	30
XZ Car	75	95	91	83	0.082
	4	6	6	10	39

[a] Solar units.

[b] $\Delta = \log R_{V-I} - \log R_{B-V}$.

For all the stars in our sample, we show in Table 7 the mean

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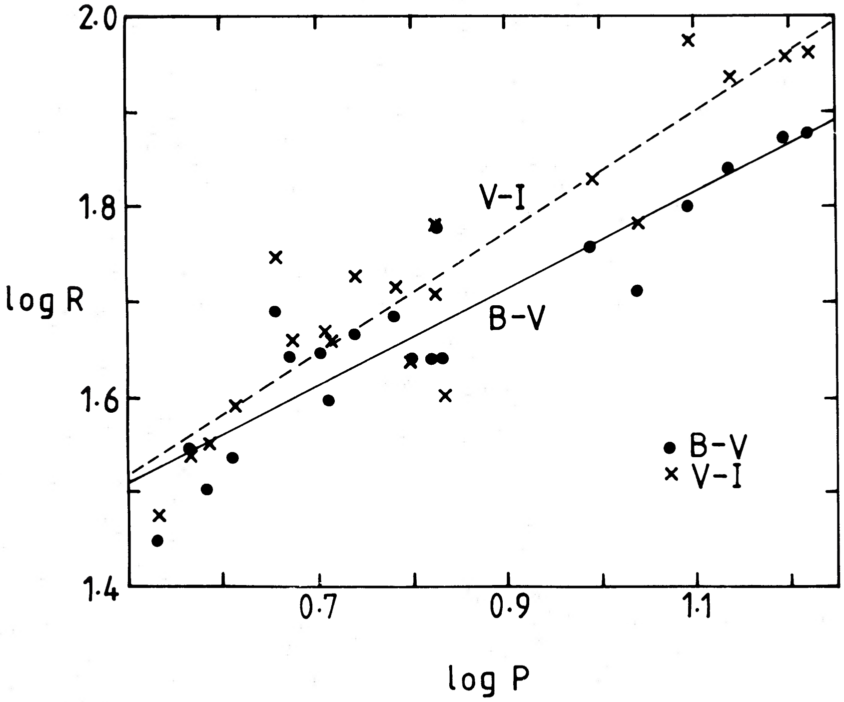

FIG. 2.—Radii of the six stars studied here, together with those given by Gieren (1982*b*), plotted against period. Filled circles are the $B-V$ radii and crosses are the $V-I$ radii (*see text*). Solid line is the best-fitting least-squares line to the $B-V$ radii, and dashed line that for the $V-I$ radii.

radii for each choice of color, being the weighted means of the radii derived using that color and each of the magnitudes. The final column shows the parameter $\Delta = \log R_{V-I} - \log R_{B-V}$, which clearly shows that the $V-I$ radii are $\sim 25 \pm 11\%$ larger than the $B-V$ radii. The shorter period stars studied by Gieren (1982*b*) show a much smaller effect $(7 \pm 8)\%$, which suggests that Δ may be period dependent.

We note also that the $V-I$ radii are in better agreement with the surface brightness radii of these stars derived by Gieren (1985) than are the $B-V$ results.

The color dependence of the derived radius is due to the imperfect description of the surface brightness by the color. Cyclic variations in gravity (e.g., Karp 1975) or microturbulence (e.g., Benz and Mayor 1982), or both cause variations in line blanketing that destroy the one-to-one relation between color and surface brightness that is a fundamental assumption of the Baade-Wesselink technique. Since the effects of blanketing decrease for redder colors, the corresponding radius may be expected to be a better estimate of the star's true radius. Model atmospheres by Kurucz (1979) indicate that $V-I$ is quite insensitive to small metallicity changes (Caldwell and Coulson 1985) and hence, presumably, to small changes in opacity or blanketing. The long wavelength base of $V-I$ and the good quality of the $V-I$ solutions above encourage us to adopt R_{V-I} as our best estimator of the radius of a Cepheid.

a) The Period-Radius Relation

The results for our six Cepheids, together with those for Gieren's (1982*b*) shorter period stars (AZ Cen excluded), yield the following $P-R$ relations:

$$\log R_{B-V} = \underset{\pm 0.052}{0.509} \log P + \underset{0.044}{1.255} \ (0.049)\ ,$$

$$\log R_{V-R} = \underset{0.069}{0.627} \log P + \underset{0.059}{1.226} \ (0.065)\ ,$$

$$\log R_{V-I} = \underset{0.063}{0.625} \log P + \underset{0.054}{1.209} \ (0.059)\ ,$$

$$\log R_{R-I} = \underset{0.092}{0.602} \log P + \underset{0.079}{1.198} \ (0.087)\ ,$$

where the magnitude in parentheses is the standard error per point. The $B-V$ relation is considerably different from the other three, we suspect, because of the effect of blanketing upon the B waveband. The adopted $V-I$ relation is also in better agreement with the theoretical $P-R$ relation, as derived, for instance, by Fernie (1984):

$$\log R = 0.694 \log P + 1.177\ ,$$

and gives hope for a resolution of the discrepancies in Cepheid sizes discussed in that paper. The $B-V$ and $V-I$ solutions are shown in Figure 2.

Further $V-I$ radii should improve the accuracy of the observed $P-R$ relation (Coulson and Caldwell 1985). It will also be interesting to obtain radii using infrared colors to compare with our $V-I$ relation and with theory.

b) The Surface Brightness Coefficient

With each radius calculation we also obtain a value of the surface brightness coefficient A, in that magnitude-color system. These are listed in Table 8 together with the formal unweighted means and standard deviations. From this sample none of the six stars appears significantly deviant.

IV. CEPHEID MASSES

The mass of a single-mode Cepheid may be determined in one of four ways, as described by Cox (1979, 1980). Adoption of larger Cepheid distances, and hence luminosities, and of lower reddenings, and hence temperatures, has led to reasonable agreement between M_{ev}, M_{th}, and M_Q: the evolutionary, theoretical, and pulsation masses, respectively. For $P < 10$ d, the

TABLE 8

SURFACE BRIGHTNESS COEFFICIENTS

Magnitude	Color	AQ Car	XX Cen	XY Car	TT Aql	XX Car	XZ Car	Mean	(±)
B	$B-V$	3.01	3.16	3.10	3.07	3.18	3.28	3.13	0.09
	$V-R$	7.27	7.50	7.17	7.32	6.78	7.44	7.25	0.26
	$V-I$	4.25	4.22	4.24	4.27	3.97	4.44	4.23	0.15
	$R-I$	10.43	9.67	10.44	10.28	9.59	11.33	10.29	0.63
V	$B-V$	2.01	2.17	2.10	2.07	2.18	2.29	2.14	0.10
	$V-R$	4.87	5.15	4.82	4.94	4.65	5.13	4.93	0.19
	$V-I$	2.84	2.89	2.86	2.88	2.71	3.10	2.88	0.13
	$R-I$	6.97	6.62	7.02	6.93	6.52	8.18	7.04	0.59
R	$B-V$	1.61	1.75	1.67	1.66	1.69	1.90	1.71	0.10
	$V-R$	3.88	4.23	3.88	3.96	3.66	4.15	3.96	0.21
	$V-I$	2.28	2.35	2.28	2.30	2.13	2.55	2.32	0.14
	$R-I$	5.49	5.34	5.58	5.54	5.12	6.57	5.61	0.50
I	$B-V$	1.31	1.41	1.38	1.36	1.36	1.54	1.39	0.08
	$V-R$	3.18	3.44	3.17	3.24	2.93	3.51	3.24	0.21
	$V-I$	1.85	1.90	1.87	1.89	1.72	2.12	1.89	0.13
	$R-I$	4.64	4.42	4.66	4.58	4.17	5.72	4.70	0.53

Wesselink masses, $M_{\rm W}$, are also in agreement with the others, whereas there remains difficulty in reconciling $M_{\rm W}$ with the others for $P > 10$ d. Typically, $\langle M_{\rm W}/M_{\rm th}\rangle \approx 0.6$ for $P > 10$ d (Cox 1979).

The use of inhomogeneous stellar models improved the agreement for the shorter period stars (Cox 1980), but only slightly alleviated the above discrepancy for the longer period stars, and, in any case, these models are generally considered inappropriate for the longer period, higher mass stars whose lifetimes are too short to allow the necessary helium enrichment of the surface.

The new data for shorter period stars obtained by Gieren (1982*b*) have improved the precision of the diagnostic mass ratios but show, in fact, slightly better agreement using the homogeneous models.

For the longer period stars in this work we shall adopt the King IVa homogeneous model (King *et al.* 1975, with the formalism of Faulkner 1977), although the results agree well (within 2%) with those using the model of Cox, King, and Stellingwerf (1972; CKS).

We use temperatures by Pel (1978, and private communication), the P-L-C relation of Feast (1984; which is identical to that by Martin, Warren, and Feast 1979), and the bolometric corrections for Iab supergiants given by Schmidt-Kaler (1982). The results are shown in Table 9, where successive columns give the star name, its fundamental period, the effective temperature, the derived theoretical luminosity and mass, the pulsational luminosity and mass, and finally, for both $B-V$ and $V-I$ radii the corresponding Wesselink luminosity and mass.

For these six stars, the pulsational and theoretical masses and luminosities agree reasonably well on average:

$$M_Q/M_{\rm th} = 0.85 \pm 0.09 \quad ({\rm s.d.})\ ,$$

$$L_Q/L_{\rm th} = 0.88 \pm 0.07 \quad ({\rm s.d.})\ ,$$

while the persistent Wesselink mass anomaly,

$$M_{\rm Wes}^{B-V}/M_{\rm th} = 0.44 \pm 0.07\ , \quad ({\rm s.d.})\ ,$$

is alleviated to a considerable extent by using $V-I$ radii:

$$M_{\rm Wes}^{V-I}/M_{\rm th} = 0.74 \pm 0.23 \quad ({\rm s.d.})\ .$$

Some fairly complete resolution of the Cepheid mass discrepancy problem may be forthcoming provided an appropriate photometric color is used.

V. CONCLUSION

The $V-I$ color is proposed as a more suitable indicator of surface brightness than $B-V$. The Baade-Wesselink radii derived using $V-I$ prove to be larger than their $B-V$ counterparts and to a large extent seem to ease the discrepancies between observed and theoretical period-radius relations, and hence between the various calculations of Cepheid masses.

We thank Luis Balona once again for the use of his invaluable computer programs and also for assistance in the development of the iterative routine described in the Appendix. We also thank J. W. Pel for communicating to us estimates of the mean temperatures of four of the six stars in this study.

TABLE 9

MASSES AND LUMINOSITIES

Cepheid	P (*d*)	$T_{\rm eff}$ (K)	$L_{\rm th}$	$M_{\rm th}$	L_Q	M_Q	$L_{\rm Wes}^{B-V}$	$M_{\rm Wes}^{B-V}$	$L_{\rm Wes}^{V-I}$	$M_{\rm Wes}^{V-I}$
AQ Car	9.7692	5443	4324	7.29	3701	6.00	2520	3.77	3586	5.56
XX Cen	10.9541	5575	5792	7.90	4814	6.26	2308	2.66	3176	3.82
XY Car	12.4357	5365	5733	7.88	4903	6.49	2906	3.50	6469	9.19
TT Aql	13.7549	5265	6032	7.99	4975	6.29	3327	3.92	5022	6.37
XX Car	15.7113	5545	9556	9.04	8600	7.93	4699	3.90	6918	6.09
XZ Car	16.6507	5075	6582	8.18	6718	8.38	3386	3.73	4854	5.65

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APPENDIX

A DESCRIPTION OF THE ITERATIVE APPROACH TO DETERMINING THE RADII OF PULSATING STARS

The basis is equation (1.4) of Balona (1977):

$$V - A(B-V) + 5 \log (R_0 + r) - B = 0 , \tag{A1}$$

i.e.,

$$V - A(B-V) + \mu \ln \left(1 + \frac{r}{R_0}\right) - (B - 5 \log R_0) = 0 , \tag{A2}$$

where $\mu = 5 \log_{10} e = 2.1715$. Under the assumption $r \ll R_0$, as adopted by Balona,

$$V - A(B-V) + \frac{\mu}{R_0} r - (B - 5 \log R_0) = 0 . \tag{A3}$$

This set of equations between observable variables V, $B-V$, r with associated errors σ_V, σ_{B-V}, σ_r is solvable for estimates A_1, B_1, R_1 of A, B, R_0, by the method of maximum likelihood (Balona 1977).

Let $R_0 = R_1 + \rho$; then equation (A1) becomes

$$V - A(B-V) + 5 \log (R_1 + \rho + r) - B = 0 ,$$

i.e.,

$$V + 5 \log (R_1 + r) - A(B-V) + \mu \ln \left(1 + \frac{\rho}{R_1 + r}\right) - B = 0 . \tag{A4}$$

In the case $\rho \ll (R_1 + r)$ this becomes

$$y - A(B-V) + \mu\rho\left(\frac{1}{R_1 + r}\right) - B = 0 ,$$

where $y = V + 5 \log (R_1 + r)$, $B-V$ and $(1/R_1 + r)$ are the new variables with errors $\sigma_y^2 = (\sigma_V^2 + \mu^2\sigma_r^2/R_1^2)$, σ_{B-V}, and σ_r/R_1^2, respectively. Estimates A_2, B_2, and ρ_2 may be derived as above, a new estimate of R_0, viz., $R_1 + \rho_2$, inserted into equation (A1) and the process repeated until convergence.

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