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Secular evolution of magnetic cataclysmic variables

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Summary. We show that the observed period distribution of magnetic cataclysmic variables can be easily understood on the basis of current theories of the secular evolution of these systems, and the formation of the period gap in particular, provided that the white dwarfs have magnetic moments μ *either* in the range $10^{33} \lesssim \mu \lesssim 10^{34}$ G cm^3, *or* $\mu < 10^{30}$ G cm^3, roughly as observed for isolated white dwarfs. The absence of intermediate polars with orbital periods <2 hr rules out the possibility that their fields are systematically weaker than those of the AM Her systems, unless magnetic binaries never evolve across the period gap, and the field distribution is as above for orbital periods <2 hr. We show how the spin history of EX Hya fits with this picture, and discuss other estimates giving low values of μ for the intermediate polars.

Introduction

The magnetic systems form a distinctive subclass of the cataclysmic variables (CVs). They fall naturally into two groups: the AM Herculis stars ('polars') show phase-dependent optical polarization variations on the white-dwarf spin period $P_{\rm spin}$, together with photometric, emission-line and (usually) X-ray variations at the same period. The orbital period P is not directly measured in most systems, but it is thought to be very close to $P_{\rm spin}$, presumably because the white-dwarf magnetic field exerts a synchronizing torque on the secondary star. [In fact, a small 'synodic' discrepancy between P and $P_{\rm spin}$ must exist (Campbell 1983), as the synchronizing torque would vanish if there were relative motion between the magnetic field and the secondary, and accretion torque will try to spin up the white dwarf.]* The intermediate polars, sometimes called DQ Herculis stars after a possible prototype, have $P_{\rm spin} < P$. Unlike the AM Her stars they are thought to possess accretion discs. The orbital period is detected spectroscopically, while $P_{\rm spin}$ is usually found in X-rays and/or photometrically, often accompanied in the optical by the orbital sideband periodicity $(P_{\rm spin}^{-1} - P^{-1})^{-1}$. Here the magnetic interaction with the secondary is too weak to bring about near-synchronism, as in the AM Her stars This, together with the non-detection of optical polarization variations, has led to suggestions that the white-dwarf

*See note added in proof.

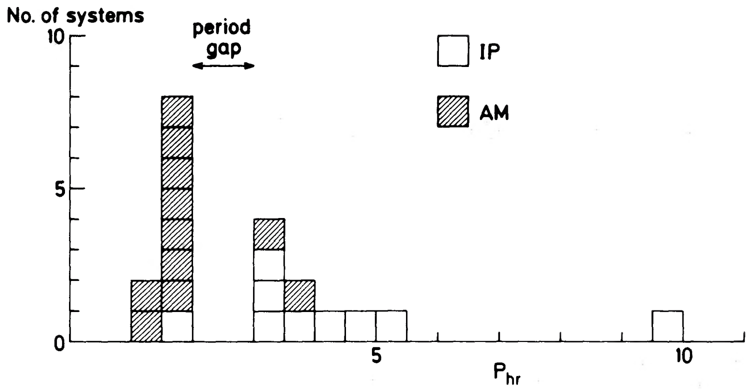

Figure 1. Histogram of magnetic CVs in half-hour period bins.

magnetic moment μ is weaker in these systems than in the AM Her stars (e.g. Lamb & Patterson 1983). However, this takes no account of the fact that the orbital period P of a CV must steadily decrease in order to maintain the mass transfer by reducing the binary separation a (for a recent review of the secular evolution of CVs see e.g. Ritter 1983). The mechanism which causes this shrinkage, and thus ultimately drives the mass transfer, must be some form of angular momentum loss from the binary. For $P \lesssim 3$ hr this is probably gravitational radiation (Kraft, Mathews & Greenstein 1962), while for long periods rotational braking via a magnetized stellar wind from the secondary (Verbunt & Zwaan 1981) is a promising candidate.

In a recent paper Chanmugam & Ray (1984) examined the orbital evolution of magnetic CVs due to magnetic braking and gravitational radiation and concluded that DQ Herculis stars must evolve into AM Herculis stars as the system shrinks into the white-dwarf magnetosphere and synchronizes. In this paper we reconsider this problem with particular emphasis on the magnetic field distribution. We support the conclusion of Chanmugam & Ray and we suggest that the fields in AM Hers and DQ Hers are of the same order.

Since the ratio r_μ/a of the magnetospheric radius of the white dwarf to the separation goes as $\dot{M}^{-2/7}P^{-2/3}$ and $\dot{M} \sim P^{5/3\star}$ for $P \gtrsim 3$ hr (equation 2.7), it is clear that an AM Her system with $P = 3$ hr must have been an intermediate polar at longer periods. The field strengths in the two groups of magnetic CVs thus cannot in general be distinct: at least some intermediate polars must have field strengths of the same order as the AM Her systems. We can regard the field strength as constant during the lifetime of a CV, since the field decay time for white dwarfs is $\gtrsim 10^9$ yr (Chanmugam & Gabriel 1972; Fontaine, Thomas & van Horn 1973). Note that significant field decay would force us to conclude that intermediate polars had *stronger* fields than AM Hers.

Thus, the failure to detect optical polarization in intermediate polars must be due to radiative transfer effects arising from higher accretion rates or dilution by unpolarized disc radiation and magnetic field inhomogeneities (Barrett & Chanmugam 1984).

Powerful evidence in favour of these considerations is provided by the period histogram of magnetic CVs (Fig. 1). Most striking is the clustering of AM Her systems at short periods $P \sim 1.5$ hr: of the 10 systems plotted only two lie above the CV 'period gap' between 2–3 hr, whereas all except one (EX Hya) of the intermediate polars lie on the long side of the gap.

This strongly suggests that almost all magnetic CVs have magnetic moments μ lying in quite a narrow range: at a period $P \lesssim 3$ hr, the separation a and transfer rate $\dot{M}$ are both low enough to allow the synchronizing torque to turn the binary into an AM Her system, while for $P \gtrsim 3$ hr this torque is always too weak compared to accretion torques, and the system appears as an intermediate polar.

$\star$These results are somewhat dependent on the assumed secondary mass-to-radius power index α_s (see Discussion p. 183 following equation 2.4).

In the remainder of this paper we shall justify these statements quantitatively, and show that magnetic CVs have magnetic moments μ in the range $10^{33} \lesssim \mu \lesssim 10^{34}$ G cm^3, while the 'non-magnetic' majority of CVs have $\mu \lesssim 10^{30}$ G cm^3. We suggest that DQ Her and AE Aqr may actually be extreme members of the latter group, while EX Hya is at the lower edge of the 'magnetic' group. The spin history of this system supports current ideas about the origin of the period gap in the CV's.

2 Magnetic interactions

The scale of the region over which the white-dwarf's magnetic field $B \sim \mu r^{-3}$ influences the accretion flow is set by the magnetospheric radius

$$r_\mu = 2.7 \times 10^{10}\,\mathrm{cm}\,\mu_{33}^{4/7} \dot{M}_{16}^{-2/7} M_1^{-1/7} \phi. \tag{2.1}$$

Here μ_{33} is the magnetic moment $\mu = B_0 R_{\mathrm{WD}}^3$ in units of 10^{33} G cm^3, B_0 is the surface field, R_{WD} is the white-dwarf radius and M_1 is its mass in solar masses, $\dot{M}_{16}$ is the accretion rate in units of 10^{16} g s^{-1}, and we have included a dimensionless factor ϕ of order unity to allow for uncertainties in the precise value of r_μ. Equation (2.1) is obtained by balancing magnetic stresses $\sim B^2/8\pi$ against the ram pressure of the accreting matter, assumed spherical and freely falling (e.g. Elsner & Lamb 1977). Since $B^2 \sim r^{-6}$, and the free-fall and Kepler velocities of accreting material differ only by a factor $\sqrt{2}$, r_μ is a reasonable estimate also for disc accretion [see Ghosh & Lamb (1979) for the aligned rotator and Anzer & Börner (1980) for dipoles in the disc plane]. For a system to appear as 'magnetic' it must have μ large enough to disrupt the accretion flow above the white-dwarf surface, i.e.

$$r_\mu > R_{\mathrm{WD}} = 10^9 R_9\,\mathrm{cm}.$$

Hence the minimum μ required for an intermediate polar is

$$\mu_{33}(\mathrm{IP}) = 3.1 \times 10^{-3} \dot{M}_{16}^{1/2} M_1^{1/4} R_9^{7/4} \phi^{-7/4}. \tag{2.2}$$

To give an AM Her system r_μ must be comparable to the separation a, and almost certainly it must exceed the primary Roche radius. The latter is a slowly varying fraction of a which depends on the mass ratio $q = M_2/M_1$ ($M_2 M_\odot$=secondary mass). Using Kepler's law for a and standard approximations for the Roche radius, we find the minimum μ required for an AM Her system:

$$\mu_{33}(\mathrm{AM}) \simeq 0.53\, M_{16}^{1/2} \dot{M}_1^{5/6} P_{\mathrm{hr}}^{7/6} \phi^{-7/4} \tag{2.3}$$

with an error in the coefficient $\lesssim 12$ per cent in the range $0.2 \lesssim q \lesssim 1$. Here P_{hr} is the period measured in hours.

We can transform (2.2), (2.3) to equations of the form $\mu = \mu(P, M_1)$ by using the period–mass relation for a Roche-lobe-fitting secondary star near the lower main sequence

$$M_2 = 0.11\, P_{\mathrm{hr}} \tag{2.4}$$

(e.g. Warner 1976). This relation is sensitive to the assumed main-sequence radius-to-mass dependence. The lower main sequence is adequately described by $R_2 = R_\odot M_2^{\alpha_2}$ with $0.8 \lesssim \alpha_2 \lesssim 1.0$ (e.g. Patterson 1984) and (2.4) holds for $\alpha_2 = 1$. In addition we need equations relating M to P and these are also somewhat dependent on α_2. For simplicity we state in the paper only the results for $\alpha_2 = 1$ but we plot results for other values of α_2 in Figs 2 and 3. For $P_{\mathrm{hr}} < 3$ the mass transfer is probably driven by gravitational radiation at a rate (e.g. Savonije 1983)

$$\dot{M}_{\mathrm{GR},16} = 0.71\, q^{-2/3}[(1+q)^{1/3}(4-3q)]^{-1}. \tag{2.5}$$

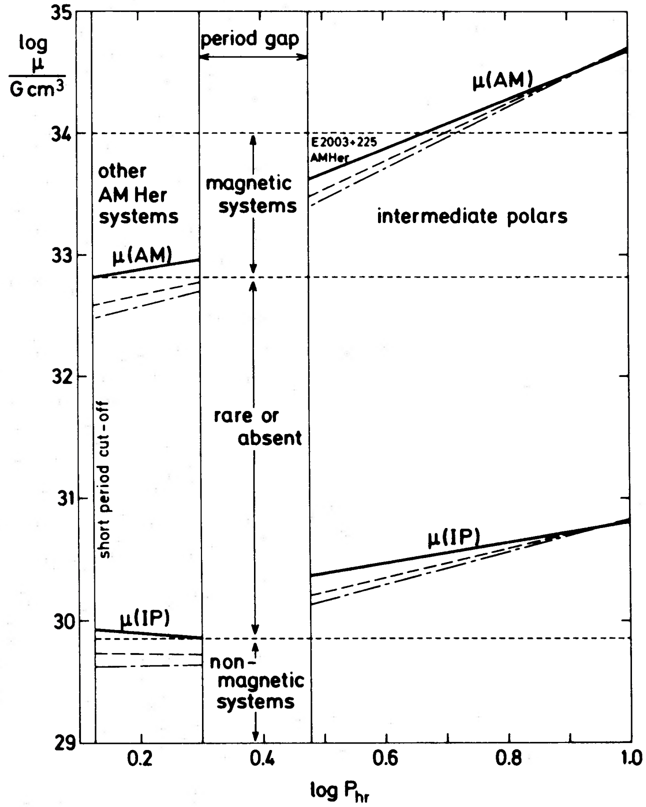

Figure 2. Magnetic moment $\mu=B_0R_{\rm WD}^3$ versus binary period. Systems evolve from right to left along $\mu\simeq$const lines. Most begin their CV life as an intermediate polar with $\mu(\text{IP})\lesssim\mu\lesssim\mu(\text{AM})$. When they cross the boundary $\mu(\text{AM})$ they become an AM Her type. The μ-boundaries shown correspond to different main sequence mass-to-radius power indexes: $\alpha_2=1$, 0.85 and 0.8 are indicated as solid, dashed and dot-dashed lines. Systems below the lower dotted line never become 'magnetic'. The relative distribution of the two types of CV's shown in Fig. 1 is explained if most magnetic systems fall in the range few$\times10^{32}\lesssim\mu\lesssim10^{34}$ G cm^3.

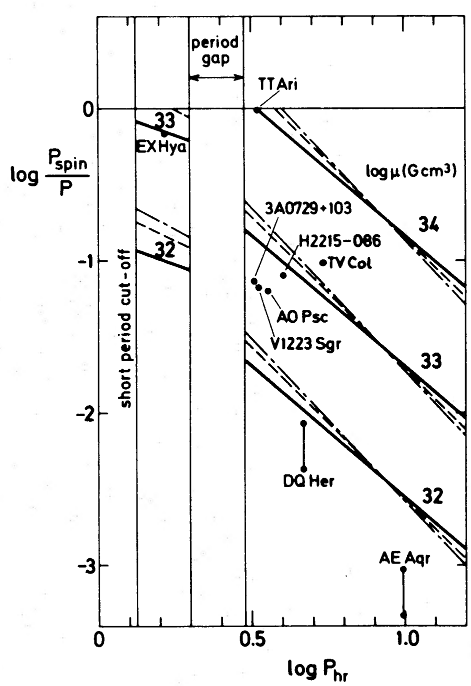

Figure 3. The ratio $P_{\rm spin}/P$ for the eight intermediate polars of Fig. 1 as a function of the orbital period. The expected equilibrium ratios [*cf.* equation (2.11)] for $\mu_{33}=0.1$, 1 and 10 are shown for the same values of α_2 as in Fig. 2.

For $P_{hr}>3$ we assume that magnetic braking drives the mass transfer at a rate (Verbunt & Zwaan 1981)

$$\dot{M}_{MB,16}=76.5(f/1.78)^{-2}M_1q^{5/3}(1+q)^{1/3}(4-3q)^{-1} \tag{2.6}$$

where f is a dimensionless parameter in the range 0.73–1.78. Evolutionary considerations related to the CV period gap suggest that f is in the lower end of this range (Ritter 1984a). the rate of angular momentum loss by magnetic braking depends on the moment of inertia of the secondary, usually described by the 'radius of gyration' r_{g2}. We have taken $r_{g2}=0.45$ in deriving (2.6), which is appropriate for stars with deep convective envelopes.

Expressing the leading powers of q in (2.5) and (2.6) in terms of the period using (2.4) we get

$$\begin{aligned}\dot{M}_{16}&=\dot{M}_{GR,16}\simeq 3.1\, P_{hr}^{-2/3}M_1^{2/3}[(1+q)^{1/3}(4-3q)]^{-1} && P_{hr}<3,\\ \dot{M}_{16}&=\dot{M}_{MB,16}\simeq 1.9\,(f/1.78)^{-2}\,P_{hr}^{5/3}M_1^{-2/3}\,(1+q)^{1/3}(4-3q)^{-1} && P_{hr}>3.\end{aligned} \tag{2.7}$$

(Note that the changeover from magnetic braking to gravitational radiation is unlikely to occur smoothly: this is the currently most promising explanation for the period gap between 2–3 hr – see below.)

Substituting (2.7) into (2.2) and (2.3) and replacing the scarcely varying functions of q by suitable averages for the range $0.2\leqslant q\leqslant 1$, we obtain

$$\mu_{33}(\mathrm{IP})=\begin{cases}3.09\times10^{-3}\,\phi^{-7/4}\,M_1^{7/12}\,R_9^{7/4}\,P_{hr}^{-1/3} & P_{hr}<3,\\ 3.15\times10^{-3}\,(f/1.78)^{-1}\phi^{-7/4}\,M_1^{-1/12}\,R_9^{7/4}\,P_{hr}^{5/6} & P_{hr}>3,\end{cases} \tag{2.8}$$

$$\mu^{33}(\mathrm{AM})=\begin{cases}0.51\,\phi^{-7/4}\,M_1^{7/6}\,P_{hr}^{5/6} & P_{hr}<3,\\ 0.46\,(f/1.78)^{-1}\phi^{-7/4}\,M_1^{1/2}\,P_{hr}^{2} & P_{hr}>3.\end{cases} \tag{2.9}$$

Fig. 2 plots μ(IP), μ(AM) against P_{hr} for the case $M_1=1$, $R_9=0.5$, $\phi=1$ and $f=1.78$, for three different values of the main-sequence mass–radius exponent α_2. As $\mu\simeq$const during the lifetime of a CV, the systems evolve along horizontal lines form right to left in this diagram. The orbital periods evolve on a time-scale of the order of the mass-transfer time-scale*

$$t_{\dot{M}}\equiv\frac{M_2M_\odot}{\dot{M}}=\begin{cases}7.0\times10^{8}\,M_1^{-2/3}P_{hr}^{5/3}\,\mathrm{yr} & P_{hr}<3,\\ 9.1\times10^{8}(f/1.78)^2\,M_1^{2/3}\,P_{hr}^{-2/3}\,\mathrm{yr} & P_{hr}>3.\end{cases} \tag{2.10}$$

If, in the course of its evolution, a system crosses into the region $\mu>\mu$(IP), it will appear as an intermediate polar. To become an AM system it must cross the curve $\mu=\mu$(AM). At every stage the spin period must be longer than the 'equilibrium period' (e.g. Ghosh & Lamb 1979) as r_μ cannot exceed the 'corotation radius' r_{co} at which centrifugal forces on matter corotating with the white-dwarf balance gravity. This requires $P_{spin}>(4\pi^2r_\mu^3/GM_1M_\odot)^{1/2}$ which, using Kepler's law, yields

$$\frac{P_{spin}}{P}\gtrsim\left(\frac{r_\mu}{a}\right)^{3/2}(1+q)^{1/3}. \tag{2.11}$$

In fact it is mostly likely that P_{spin} will always be close to the equilibrium value given by (2.11). If the white dwarf is rotating more rapidly (slowly) than this, spin-down (spin-up) will occur on a time-scale (for $r_\mu>R_{WD}$)

$$t_{spin}\sim\frac{M_1M_\odot R_{WD}^2r_{g1}^2}{\dot{M}r_\mu^2}\sim\frac{R_{WD}^2r_{g1}^2}{qr_\mu^2}t_{\dot{M}} \tag{2.12}$$

*For conservative mass-transfer $P/\dot{P}\simeq 2t_{\dot{M}}/(3\alpha_2-1)$.

where $r_{g1} \simeq 0.2$–0.5 is the radius of gyration of the white dwarf. This time-scale is always shorter than $t_{\dot{M}}$ and becomes $\ll t_{\dot{M}}$ when $r_\mu \sim a$. As the binary evolution proceeds (r_μ/a) increases as $P_{hr}^{-8/7}$ above the gap and as $P_{hr}^{-10/21}$ below the period gap [*cf.* equations (2.1) and (2.7)]. P_{spin} will, by (2.11), approach P more and more closely, until ultimately the magnetic synchronizing torque (Campbell 1983) is strong enough to control the subsequent spin evolution of the system. Note that the latter torque is negligible in comparison with material torques until P and P_{spin} are very close, i.e. $|P-P_{spin}|/P \lesssim 10^{-1}$. Thus the approach of P_{spin} to P takes place in a fraction of $t_{\dot{M}}$ (roughly the time required to evolve from $r_\mu/a \sim 0.5$ to $r_\mu/a \sim 1$), and we can regard $\mu=\mu(\mathrm{AM})$ in Fig. 2 as the lower boundary of the region occupied by AM Her systems. Clearly TT Ari, which has $(P-P_{spin})/P \sim 3$ per cent, must be quite close to this curve, implying magnetic moment $\mu \sim 10^{34}$ G cm^3.

3 Secular evolution

We consider a cataclysmic binary evolving under the influence of magnetic braking and gravitational radiation as in equations (2.7) and (2.10). (We exclude from the discussion GK Per, which has an evolved secondary; $P_{hr}=48$.) The system is presumably 'born' at some period $P_{hr} \lesssim 12$ (in the sense of becoming semi-detached), and evolves to shorter periods because of magnetic braking. At $P_{hr} \sim 3$, the secondary star becomes fully convective and can no longer anchor the magnetic field required for rotational braking (Rappaport, Verbunt & Joss 1983; Spruit & Ritter 1983). Because its Kelvin–Helmholtz time-scale t_{KH} is now significantly longer than the mass transfer time $t_{\dot{M}}$, the secondary has been pulled out of thermal equilibrium and has a radius R_2 slightly bigger than its main-sequence radius R_{MS}. When magnetic braking stops, it therefore shrinks to this radius on a time-scale $\sim t_{KH}(R_2-R_{MS})/R_{MS}$. In doing so, it detaches and mass transfer ceases until gravitational radiation has shrunk the orbit sufficiently to bring the Roche lobe size down to R_{MS}, when the system 'turns on' again with a period ~2 hr. Evolution then proceeds via gravitational radiation until $P \sim 80$ min, when the secondary becomes degenerate and the system 'turns off' finally (Paczynski & Sinkiewicz 1981). This picture accounts plausibly for the period gap between $P_{hr}=2$ to 3.

For magnetic CV's in which the white dwarf has a magnetic moment μ three differing evolutionary sequences are predicted, depending on the value of μ.

(1) $\mu \gtrsim 10^{33}$ *G cm*^{3}. Such systems begin life as intermediate polars, as they are always above $\mu(\mathrm{IP})$ (see Fig. 2). As they approach the period gap at $P_{hr} \simeq 3$, they will either be AM Her systems already (if $\mu_{33}>3$), or be quite near synchronism as $r_\mu \sim 0.5a$. When mass transfer switches off in the period gap ($r_\mu \to \infty$ formally) the magnetic torque remains and synchronizes systems near the $\mu(\mathrm{AM})$ boundary. Furthermore, as the secondary is well within the magnetosphere, any stellar wind material with relative specific angular momentum $\sim a/(P-P_{spin})$ will be captured and accreted producing a synchronizing torque. An estimate of the time-scale shows that wind accretion rates $\sim 10^{12}$ g s^{-1} are sufficient to bring synchronism before $P_{hr}=2$ is reached. If at $P_{hr}=2$ the system is above the line $\mu=\mu(\mathrm{AM})$ it will remain synchronized and spend the rest of its 'cataclysmic' lifetime as an AM Her star. For systems at the lower end of this range of μ-values, the field may not be strong enough to retain synchronism at $P_{hr} \simeq 2$, and the white dwarf may be spun-up by accretion torques. As P decreases towards 80 min, however, the binary is likely to become an AM Her system.

(2) $10^{30} \lesssim \mu < 10^{33}$ *G cm*^{3}. Such systems will spend most of their life as intermediate polars, possibly synchronizing by wind accretion during the passage through the period gap, but spinning up in $\lesssim 10^7$ yr when mass transfer starts again at $P_{hr} \lesssim 2$. Thus they will certainly appear as intermediate polars at periods below the gap.

(3) $\mu \lesssim 10^{30}$ *G cm³*. These systems spend all their lifetimes below the line μ(IP) and never show any magnetic behaviour. They cannot spin down in the gap because both magnetic and wind torques are negligible.

A comparison with Fig. 1 immediately reveals the complete absence of the large numbers of rapidly spinning intermediate polars with $P_{hr}<2$ which would be predicted if any systems fell under case (2) above. Although from (2.10) the systems spend a longer time below the gap than above it (see also Rappaport *et al.* 1983), there is only one intermediate polar, EX Hya, with $P_{hr} \lesssim 2$. Indeed, the unusual behaviour of EX Hya (it has $P_{spin}=67$ min$\simeq 0.7P$, and is currently spinning up on a time-scale $-P_{spin}/\dot{P}_{spin} \sim 2.8\times10^6$ yr, Gilliland 1982 and Sterken *et al.* 1983) is just what is predicted for the weakest field systems of case (1). The simplest explanation for these facts is that white dwarfs in CVs are either 'non-magnetic', with $\mu \lesssim 10^{30}$ G cm^3, or 'magnetic' with $\mu \gtrsim 10^{33}$ G cm^3): systems in the range $10^{30} \lesssim \mu \lesssim 10^{33}$ G cm^3 must be very rare. An upper limit to μ is given by the absence of near-synchronized systems at $P_{hr} \gtrsim 5$, implying $\mu \lesssim 10^{34}$ G cm^3 (see also Fig. 3 above). This is supported by the paucity of AM Her systems which are 'cyclotron dominated' (e.g. King & Lasota 1979) i.e. having fields strong enough ($>2\times10^8$ G) to suppress hard X-ray emission.

This division of white dwarfs in CVs into a 'non-magnetic' class clearly separated from a much smaller magnetic group, whose fields fall into a fairly restricted range, is consistent with the field distribution observed amongst isolated white dwarfs (Angel 1977, 1978): these are observed to have $5\times10^6 \lesssim B \lesssim 2\times10^8$ G, or $6\times10^{32} \lesssim \mu \lesssim 2.5\times10^{34}$ G cm^3 for a 1 $M_\odot$ white dwarf.

Further support for these conclusions is provided by Fig. 3 where P_{spin}/P is plotted as a function of P for the same intermediate polars as in Fig. 1. The most obvious feature of this diagram is that these systems fall on a relatively restricted band corresponding to $10^{32} \lesssim \mu \lesssim 10^{34}$. It is also consistent with the idea that most of them evolve along $\mu \simeq$const lines with P_{spin} near the equilibrium period (2.11) (see also Lamb & Patterson 1983). The apparently low value of μ implied for DQ Her and AE Aqr may be due to these systems spinning faster than (2.11) would allow. Also if $f \lesssim 1$ is more appropriate for the magnetic braking law (Ritter 1984b) it would raise the estimated μ by a factor ~2.

4 Discussion

Our conclusions above, that white dwarfs in CVs either have $10^{33} \lesssim \mu \lesssim 10^{34}$ G cm^3 or $\mu \lesssim 10^{30}$ G cm^2 conflicts with some earlier field estimates for intermediate polars (e.g. Lamb & Patterson 1983) but is consistent with recent work by Chanmugam & Ray (1984) and by Barrett & Chanmugam (1984).

A conclusion qualitatively identical to ours will follow provided (1) $\mu \sim$const through the evolution of a CV, (2) CVs evolve from $P_{hr} \sim$ few down through the period gap, and ultimately to $P \sim 80$ min, and (3) the period histogram (Fig. 1) is not grossly distorted by selection effects. It is hard to see a way out of any of these assumptions that does not lead to greater difficulties. To evade (1), one would need *selective* field decay for $\mu_{33} \lesssim 1$ fast enough to eliminate all such fields for $P_{hr}<2$. Theories of the period gap have been constructed which do not embody (2), but they have difficulties, for example in preventing the gap from filling in due to secular evolution. A viable theory of this type would in any case have to predict that $\mu_{33} \lesssim 1$ or $\mu_{33} \lesssim 10^{-3}$ for $P_{hr}<2$ to account for the lack of intermediate polars below the gap. Finally although selection effects undoubtedly exist in Fig. 1, it is very hard to see how they could prevent one discovering most of the intermediate polars below the gap. In particular these systems are (for CVs) quite strong X-ray sources, with characteristic periodicities which pick them out very easily.

Previous estimates suggesting fields an order of magnitude less for intermediate polars than

AM Her systems fall into two classes: (1′) the lack of detectable optical polarization, and (2′) the spin-up rates $\dot{P}_{\rm spin}$ of the intermediate polars (e.g. Lamb & Patterson 1983).

It is clear that (1′) is not a particularly strong limit, for unlike in the AM Her systems, where the optical light is dominated by cyclotron radiation from the accreting polecaps, intermediate polars probably possess accretion discs which dilute any cyclotron emission with unpolarized light (see also Barrett & Chanmugam 1984). Since the cyclotron emission comes from the polecaps, it probably cannot exceed the fraction of optical light pulsed at $P_{\rm spin}$ since hard X-ray light curves show that large fractions of the polecaps are self-eclipsed. In many intermediate polars this [as opposed to pulsing at the orbital sideband $(P_{\rm spin}^{-1}-P^{-1})^{-1}$] is already $\lesssim 1$ per cent of the optical. Even in systems where this amplitude is sometimes large (e.g. in H2215−086 it occasionally reaches ~50 per cent) this may well be dominated by thermal emission from the polecap (King & Shaviv 1984). Finally, the higher accretion rates expected for $P_{\rm hr}>3$ may make polarization difficult to detect because of radiative transfer effects: if the cyclotron harmonics having $\tau_{\rm cyc}\sim 1$ (which dominate the polarization) are shifted out of the visible into the ultraviolet it is unlikely that polarizations can be measured in the optical. This would also apply to fields $\gtrsim 10^8$ G. None the less it appears worthwhile to search intensively for polarization from those systems showing large optical pulsing at $P_{\rm spin}$.

The determination of μ by measuring $\dot{P}_{\rm spin}$ [as in (2′)] is also not straightforward. First, this rate is only measured at one epoch and one has to assume that it can be extrapolated to others; studies of pulsing X-ray sources (Rappaport & Joss 1983, fig. 1.5) show that this may sometimes not be justified. Secondly, it is quite probable that other torques beside accretion torques operate on the white dwarf e.g. wind torques. Furthermore $\dot{P}_{\rm spin}<0$ is predicted (and measured in some cases) during accretion epochs but, as we have seen [equation (2.11) and Fig. 3], $P_{\rm spin}$ is expected to *increase* secularly due to orbital evolution. Until this process is understood in detail (i.e. relative duration of phases of spin-up and spin-down, magnitude of torques involved, etc.) one should be cautious with respect to results derived from the magnitude and sign of $\dot{P}_{\rm spin}$.

Our considerations here suggest a further input into the study of the formation of CVs: this should yield a white-dwarf magnetic field distribution rather similar to that of isolated white dwarfs. With this given, we have seen that the magnetic systems do fit quite well to the present picture of the secular evolution of CVs. The difference in mass transfer rates $\dot{M}$ across the gap [from (2.7) a factor ~10] accounts naturally for the peak in the distribution of AM Her systems at $P_{\rm hr}<2$.

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Note added in proof

According to Campbell (*Mon. Not. R. astr. Soc.,* **211,** 83) stable synchronous orientations exist in which material torques vanish. In such states P and $P_{\rm spin}$ remain equal even in the presence of mass transfer.