

CYCLOTRON LINES IN ACCRETING MAGNETIC WHITE DWARFS WITH AN APPLICATION TO VV PUPPIS

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ABSTRACT

Calculations of cyclotron line spectra for homogeneous plasmas are made for emission and absorption models. The theoretical spectra show that the visibility of cyclotron lines in the optical spectrum is limited to a small parameter space in magnetic field strength ($2\text{--}10 \times 10^7$ gauss), plasma temperature ($\lesssim 15$ keV), and the accretion column nearly perpendicular to the line of sight for extended periods of time. These model spectra also confirm the conclusion by Wickramasinghe and Meggitt that the broad lines in VV Pup are due to cyclotron emission. The best fit to the data of VV Pup results in a polar magnetic field strength of $B = 3.15 \times 10^7$ gauss, a postshock temperature $kT = 8.7$ keV, and dimensionless plasma parameter $\Lambda \approx 10^6$. Our fit is better than that of Wickramasinghe and Meggitt, whose best fit values are $B = 3.18 \times 10^7$ gauss, $kT = 10$ keV, and $\Lambda \approx 10^5$. We do not require an additional unpolarized component of radiation in the blue and UV spectral regions, as suggested by Wickramasinghe and Meggitt. We have also calculated the circular polarization using our best-fit values, and obtain reasonable agreement with the observations of Visvanathan and Wickramasinghe. The polar cap fraction is estimated to be 6×10^{-5} .

Subject headings: stars: accretion — stars: individual — stars: magnetic — stars: white dwarfs

I. INTRODUCTION

The 10 known AM Her binaries form a subclass of cataclysmic variable stars. Their distinguishing characteristic is the emission of strongly polarized light ($\sim 10\%$; Tapia 1977). Characteristics more in common with other classes of cataclysmic variables are rapid optical variability (flickering) and X-ray variability (see, e.g., the reviews of Chiappetti, Tanzi, and Treves 1980; Liebert and Stockman 1984). The standard model for these systems envisions a magnetic white dwarf accreting matter from a red dwarf companion. The strong magnetic field prevents formation of an accretion disk; instead, the accreting matter flows along the magnetic field lines and is funneled into an accretion column above one or both the magnetic poles of the white dwarf (Chanmugam and Wagner 1977). A static shock above the magnetic poles slows and heats the infalling matter, so that the hot postshock plasma (with a temperature T , where $kT \approx 10$ keV; Hoshi 1973) settles onto the white dwarf surface.

Several methods have been used to determine the magnetic fields of AM Her binaries. The first method explains the large fractional optical polarization as being caused by cyclotron emission from the hot postshock region or its vicinity at cyclotron harmonics $n \approx 5\text{--}20$. In order to place these harmonics in the optical band, one requires magnetic fields $\sim (1\text{--}4) \times 10^7$ gauss (see Chanmugam and Dulk 1981; Meggitt and Wickramasinghe 1982; Barrett and Chanmugam 1984). The second method uses Zeeman spectroscopy and has led to the identification of magnetic fields of $\sim 2 \times 10^7$ gauss in AM Her (Schmidt, Stockman, and Margon 1981; Latham, Liebert, and Steiner 1981; Hutchings, Crampton, and Cowley 1981), CW 1103+254 (Schmidt, Stockman, and Grandi 1983), and H 0139-268 (Wickramasinghe, Visvanathan, and Tuohy 1984). This method leads to the determination of the magnetic field at the surface of the white dwarf. However, it must be used when the system is in a "low" state (a state when the system is optically faint, implying low accretion rates) or a faint phase

(an orbital phase when the white dwarf eclipses the shock and postshock regions). It gives no direct information about the accretion column. The third method uses observations of cyclotron lines and is potentially the most accurate one for determining the magnetic field strength in the accretion column. However, it has been the least fruitful, since only one AM Her binary, VV Pup, during one time span, 1979 February (Visvanathan and Wickramasinghe 1979, hereafter VW; Stockman, Liebert, and Bond 1979, hereafter SLB) has shown these spectral features corresponding to a field of 3.08×10^7 gauss. The advantage of this method is that it can be used to probe the structure of the accretion column. This method was also used to obtain the first direct measurement of a magnetic field of a neutron star ($B \approx 5 \times 10^{12}$ gauss; Trümper *et al.* 1977 and Voges *et al.* 1983) and a solar flare ($B \approx 10^4$ gauss; Willson 1984).

There are several papers on the subject of cyclotron lines in accreting magnetic white dwarfs. In particular, Mitrofanov (1980) analyzed the cyclotron line profile at the fundamental and second harmonics for different optical depths. The calculations were made using a nonrelativistic theory which took into account absorption in the ordinary and extraordinary modes. Mitrofanov made the assumption that the magnetic fields of AM Her stars were of the order of 10^8 gauss, so that the fundamental and second cyclotron harmonics were in the visible spectrum. However, the determination of $B = 3.08 \times 10^7$ gauss in VV Pup by VW and of $B = 2.9 \times 10^7$ gauss by Wickramasinghe and Visvanathan (1980) was based on the assumption that the broad spectral features, equally spaced in frequency, were absorption lines at cyclotron harmonics $n = 6, 7$, and 8. A more recent analysis of Wickramasinghe and Meggitt (1982, hereafter WM), however, concludes that the spectral features are actually cyclotron emission lines. The broad peaks correspond to cyclotron harmonics $n = 6, 7$, and 8, with the radiation emitted perpendicular to the magnetic field of strength 3.18×10^7 gauss in a plasma of characteristic

temperature $kT \approx 10$ keV, and dimensionless parameter $\Lambda = \omega_p^2 L / \omega_B c \approx 10^5$ (WM). Here, ω_p is the plasma frequency, L is the thickness of the plasma slab, and $\omega_B = eB/mc$ is the cyclotron frequency.

The purpose of this paper is to study in further detail the formation of cyclotron lines in accreting magnetic white dwarfs. In § II, the absorption coefficients for cyclotron radiation are discussed, and a table comparing the results of Chanmugam and Dulk (1981) to Meggitt and Wickramasinghe (1982) is given. Next, a general analysis of the cyclotron spectrum is presented for a homogeneous plasma slab parallel to a constant magnetic field. Several figures show how the spectrum changes with various plasma temperatures, viewing angles, optical depths, and magnetic field strengths. Finally, in this section, a simple equation relating the frequency of the cyclotron line in terms of the cyclotron frequency, harmonic number, plasma temperature, and viewing angle is presented. The homogeneous model is then applied in § III to the observations of cyclotron lines and circular polarization in VV Pup (SLB, VW). The results obtained from numerical fits to the data are compared to the results of WM. Finally, in § IV, we calculate the size of the postshock region and the distance to VV Pup using the results of § III. The orientation of the accretion column of VV Pup is also briefly discussed.

II. ABSORPTION AND EMISSION OF CYCLOTRON LINES FROM A PLASMA SLAB

To calculate the absorption or emission of radiation from a source, it is necessary to solve the equations of radiative transfer. In general, the equations are a set of four linear integro-differential equations for the Stokes's intensities I , Q , U , and V in three spatial variables (Chandrasekhar 1960, p. 24). If the effects of scattering are small, the equations simplify to a set of four linear partial differential equations. Normally, the polarizable effects of the medium are neglected, and the set of four linear partial differential equations are reduced to a single linear partial differential equation for the total radiation intensity. For a magnetoactive plasma, the effects of polarization can be quite large, and therefore should be considered (Ginzburg 1964, p.81). Two methods are frequently used to solve the equations. The first step of these methods involves reducing the equations to a set of linear differential equations in one spatial variable by using the geometry of a plane-parallel atmosphere or an infinite slab. One method presented by Pacholczyk (1977) then solves a system of four coupled equations in I , Q , U , V in the limit of large Faraday rotation (Meggitt and Wickramasinghe 1982). The other method makes the assumption of large Faraday rotation at the outset (Ramaty 1969). This allows the set of four coupled equations to be reduced to two uncoupled equations in I_o , I_x , one for each mode (ordinary and extraordinary) of wave propagation. We have used this method in the present paper (see also Chanmugam and Dulk 1981; Barrett and Chanmugam 1984). The result derived by Ramaty (1969) is

$$\frac{dI_{x,o}}{dz} + K_{x,o}(\omega, \theta)I_{x,o}(\omega, \theta) = J_{x,o}(\omega, \theta), \quad (1)$$

where $I_{x,o}$ is the specific intensity, $K_{x,o}$ the absorption coefficient, and $J_{x,o}$ the emissivity, for the extraordinary (x) mode and the ordinary (o) mode. For a Maxwellian distribution of electrons, Kirchhoff's law is applied to obtain the absorption coefficient $K_{x,o}$ in terms of the blackbody intensity and the

emissivity:

$$K_{x,o}(\omega, \theta) = J_{x,o}(\omega, \theta) / (n_{x,o} I_{RJ}), \quad (2)$$

where $n_{x,o}$ is the index of refraction for both modes, and $I_{RJ} = kT\omega^2/8\pi^3c^2$ is the Rayleigh-Jeans intensity for each mode. Therefore, a solution for $I_{x,o}$ is found in terms of just the absorption coefficient $K_{x,o}$. The absorption coefficient for cyclotron radiation depends on four independent variables: the frequency of the radiation ω , the plasma temperature kT , the angle θ between the magnetic field direction $\mathbf{B}$ and the wave vector $\mathbf{k}$, and the dimensionless constant Λ . Calculations of the total absorption coefficient ($K = 1/2[K_x + K_o]$, in units of $\omega_p^2/(\omega_B c)$), were made for various temperatures, angles θ , and ω/ω_B for both the first method of Meggitt and Wickramasinghe (1982) and the second method given by Chanmugam and Dulk (1981) (Table 1).

Visvanathan and Wickramasinghe (1979) and SLB initially assumed that the broad features in the spectrum of VV Pup were caused by absorption in a cool magnetized plasma surrounding the postshock region of the accretion column, and determined the polar magnetic field to be 3.08×10^7 gauss. We thought it useful to include calculations of absorption in a magnetized plasma slab. Our simple model (Fig. 1) has a magnetized plasma slab with $B = 3 \times 10^7$ gauss aligned parallel to the slab's surface. A blackbody source with temperature $kT = 30$ keV is behind the plasma slab. This background source is meant to represent the postshock region shining on the cooler foreground plasma slab of temperature $kT = 10$ keV. Assuming $J_{x,o} = 0$ for the plasma slab, we obtain a solution to equation (1):

$$I(\omega, \theta) = I_{RJ}(kT_s)[\exp(-\tau_x) + \exp(-\tau_o)], \quad (3)$$

where kT_s is the temperature of the background source and $\tau_{x,o} = K_{x,o}L$ the optical depth of the plasma. Theoretical spectra (Fig. 2) for this absorption model are then calculated using equation (3) and the collisionless cyclotron absorption coefficients (Chanmugam and Dulk 1981). We note the major

TABLE 1
VALUES OF TOTAL CYCLOTRON ABSORPTION COEFFICIENT

kT (keV)	θ	ω/ω_B	$\log(K_{MW})$	$\log(K_{CD})$
1.0.....	90.0	5.93	-7.64	-7.64
1.0.....	78.5	5.93	-8.01	-8.01
1.0.....	66.4	5.93	-8.51	-8.50
1.0.....	53.1	5.93	-9.12	-9.18
1.0.....	36.9	5.93	-10.31	-10.46
5.0.....	90.0	5.67	-5.08	-5.09
5.0.....	78.5	5.67	-5.25	-5.25
5.0.....	66.4	5.67	-5.64	-5.65
5.0.....	53.1	5.67	-6.22	-6.25
5.0.....	36.9	5.67	-6.74	-6.74
10.0.....	90.0	4.00	-5.00	-5.00
10.0.....	90.0	6.00	-4.98	-5.00
10.0.....	90.0	8.00	-5.94	-5.96
10.0.....	90.0	10.00	-7.12	-7.16
20.0.....	90.0	4.00	-2.99	-2.99
20.0.....	90.0	6.00	-3.80	-3.83
20.0.....	90.0	8.00	-4.78	-4.82
20.0.....	90.0	10.0	-5.59	-5.65
30.0.....	60.0	16.00	-6.87	-7.47
30.0.....	30.0	16.00	-8.52	-8.57
30.0.....	60.0	18.00	-7.36	-8.35
30.0.....	30.0	18.00	-9.14	-9.21

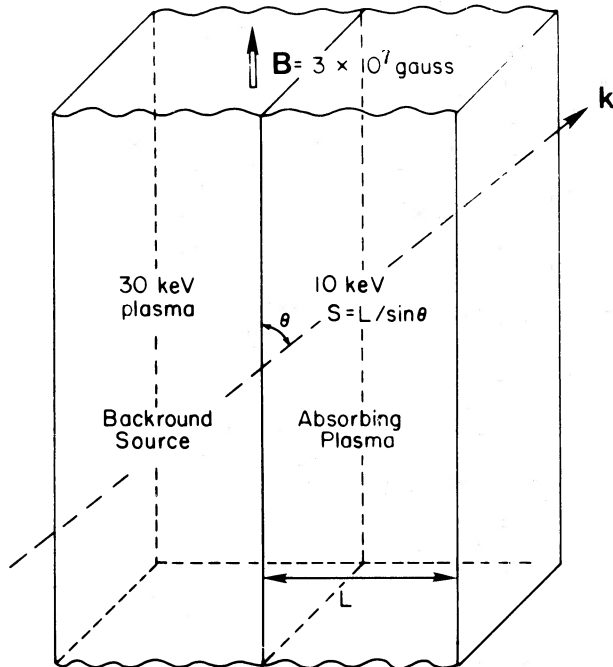


FIG. 1.—Diagram of a hot ($kT = 30$ keV) blackbody source shining on a cooler ($kT = 10$ keV) infinite plasma slab of thickness L with a magnetic field ($B = 3 \times 10^7$ gauss) parallel to the surface. The radiation, with wave vector k , propagates at an angle θ to the magnetic field. Diagram describes geometry of the absorption model. Emission model is identical to the absorption model except for omission of the background source.

features of these spectra (WM; Masters 1978): (1) the troughs are wide and rounded, (2) the crests are narrow and sharply peaked, (3) the troughs have an extended blue wing, and (4) the spectrum rises steeply toward the blue. The troughs correspond to the cyclotron lines in absorption with the profile of the lines being asymmetric. The asymmetry exists because a

cyclotron line is the sum of the absorption from that harmonic and all other cyclotron harmonics. At finite temperatures, each cyclotron harmonic is shifted in frequency by different amounts, resulting in greater absorption on the blue side of the line center than the red side (see discussion of eq. [6]). The steep rise in the blue region of the spectrum results from our assumption that the background source is an optically thick plasma at a high temperature of 30 keV. The emission can then be approximated throughout the entire optical spectrum by a Rayleigh-Jeans spectrum.

Next, a simple model for the emission from a plasma slab was considered. In this model, the plasma slab represents the postshock region of the accretion column. We solve equation (1) by including the emissivity of the plasma slab, assuming no incident flux on it. This is done by substituting equation (2) into equation (1) to obtain

$$I(\omega, \theta) = I_{RJ}(kT)[2 - \exp(-\tau_x) - \exp(-\tau_o)] \quad (4)$$

The theoretical spectra (Figs. 3–6) for this model show features which are the inverse of those features in the absorption model. The major spectral features present in the emission model spectra are (1) deep, narrow troughs, (2) broad, rounded crests, (3) extended red wings on the troughs, and (4) decreasing flux toward the blue for $\Lambda < 10^6$. Now, the broad peaks correspond to the cyclotron lines, and the asymmetry in the troughs is reversed. A notable feature of this model is the decrease of the flux toward the blue as the plasma slab becomes optically thin.

The four independent variables affect the appearance of cyclotron lines in several ways. The most important of these is the plasma temperature. As the plasma temperature increases, the cyclotron lines broaden and become less distinctive until they completely disappear. WM restricted the temperature for the production of cyclotron lines to $kT < 20$ keV. However, Figure 6 shows that these features are barely visible at 15 keV. We believe ~ 15 keV is probably a better upper limit on the

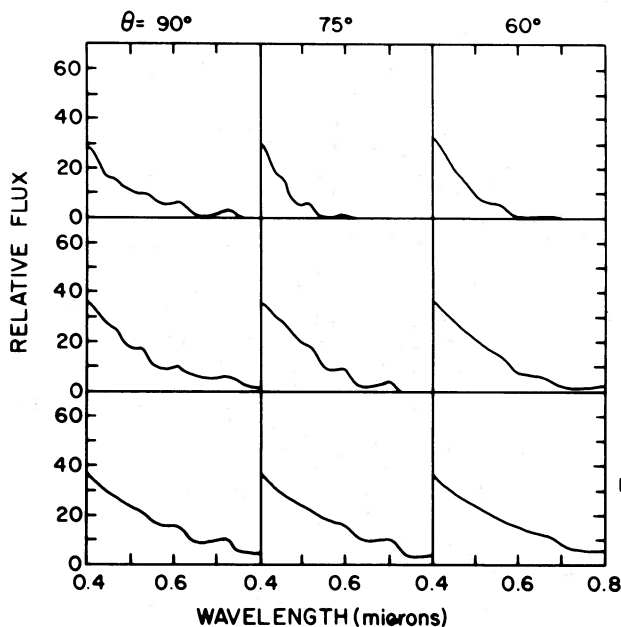


FIG. 2

FIG. 2.—Theoretical absorption spectra for θ (angle between magnetic field B and wave vector k) = 90° , 75° , 60° and $\Lambda (= \omega_p^2 / \omega_{bc}) = 10^6$, 10^5 , 10^4 , at 10 keV and $B = 3 \times 10^7$ gauss.

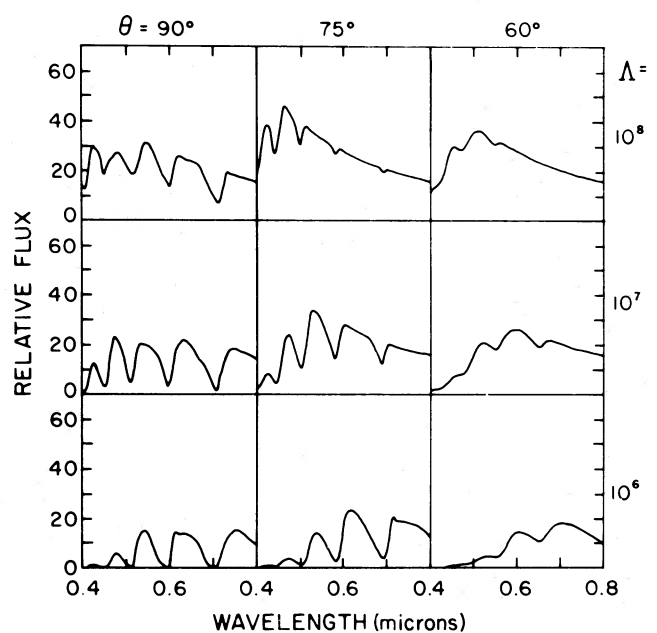
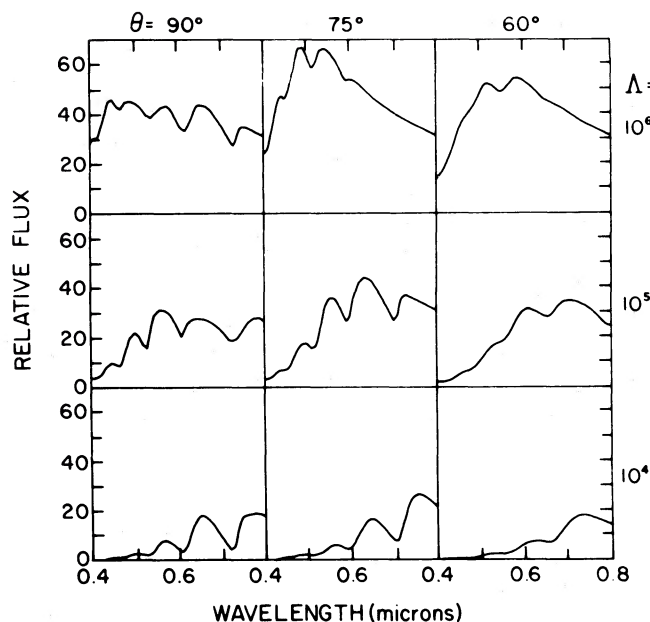
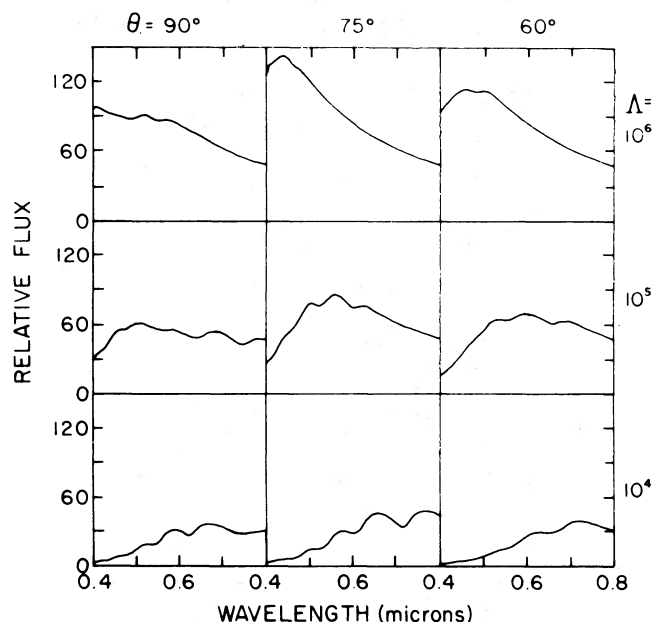


FIG. 3

FIG. 3.—Theoretical emission spectra for $\theta = 90^\circ$, 75° , 60° , and $\Lambda = 10^8$, 10^7 , 10^6 at $kT = 5$ keV and $B = 3 \times 10^7$ gauss.

FIG. 4.—Same as Fig. 3, except $\Lambda = 10^6, 10^5, 10^4$, and $kT = 10$ keVFIG. 5.—Same as Fig. 3, except $\Lambda = 10^6, 10^5, 10^4$, and $kT = 15$ keV

plasma temperature for $\Lambda \geq 10^5$, if distinct cyclotron lines are to be seen from a hot plasma. Many other contributing factors, such as background noise, the shift in the frequency of the lines with θ , and the inhomogeneous temperature distribution, will degrade the spectral quality even further. Another important limiting factor to the visibility of cyclotron lines is the strength of the magnetic field. For $B < 1.5 \times 10^7$ gauss, one requires $n > 10$ in the visible spectrum. At these high harmonics, the lines become broader and closer to one another. Eventually, the individual harmonics become indistinguishable, and the spectrum forms a continuum. The harmonic at which the spectrum becomes a continuum, decreases with increasing temperature. Finally, the cyclotron lines become smeared together because of Doppler broadening when $\theta < 60^\circ$ and $kT > 10$ keV. For $kT < 10$ keV, the cyclotron lines become smeared together at smaller angles because Doppler broadening is less important. Most AM Her binaries have $kT > 10$ keV, so Doppler broadening is significant. Therefore, for cyclotron lines to be observable in these systems, the accretion column probably needs to be approximately perpendicular to the line of sight ($\theta \approx 90^\circ$) for large fractions of the orbital period with the postshock temperature $\lesssim 15$ keV.

Hirshfield, Baldwin, and Brown (1961) presented a temperature-dependent formula for the frequency of cyclotron lines:

$$\omega/\omega_B = n(1 - n/\mu), \quad (5)$$

where n is the harmonic number and $\mu = mc^2/kT$. They obtained this formula by considering a two-dimensional Maxwellian distribution of electrons and propagation of radiation perpendicular to the magnetic field. We have generalized the equation of Hirshfield *et al.* (1961) by considering a three-dimensional Maxwellian distribution for the electrons and propagation at an angle θ to the magnetic field (see Appendix for derivation) to obtain

$$\omega/\omega_B = [-1 + (1 + 8n \sin^2 \theta/\mu)^{1/2}]/(4n \sin^2 \theta/\mu). \quad (6)$$

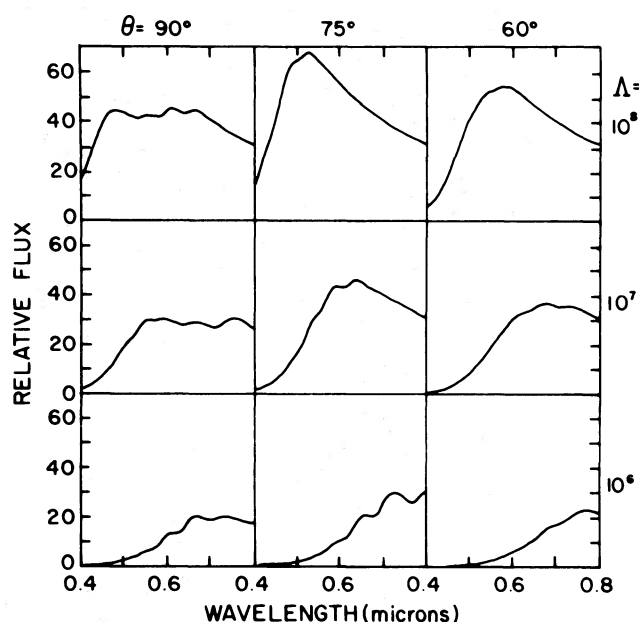
Our equation could be very useful in determining the strength

of the magnetic field, the temperature, and the geometry of the cyclotron emitting region. We will later apply equation (6) in § III.

III. FIT TO OBSERVATIONS OF VV PUPPIS

a) Cyclotron Lines

The brightness of AM Her binaries alternates between “high” and “low” states. The high state is normally 2–3 mag brighter than the low state and may last from several months to many years. The high state is the result of high accretion rates onto one or both magnetic poles, whereas the low state corresponds to low accretion rates onto either pole. VV Pup has been observed in both states. Normally, the high state has

FIG. 6.—Same as Fig. 3, except $B = 2 \times 10^7$ gauss, and $kT = 10$ keV

$m_{\max} \approx m_{\min} \approx 15$ (WM) (although $m_{\max} = 13.9$ has been observed; Visvanathan and Wickramasinghe 1981), where $m_{\max}$ corresponds to the maximum and $m_{\min}$ to the minimum brightness of the system during an orbital period. Both poles were believed to be accreting at that time because circular polarization measurements showed a constant negative polarization outside of the bright phase (Fig. 1 of Liebert and Stockman 1979; Visvanathan and Wickramasinghe 1981). During the low state, the system's brightness drops to $m_{\max} \approx m_{\min} \approx 17.8$ with an observed lack of significant H and He emission and flickering (Liebert *et al.* 1978). These observations indicate little accretion during the low state. In 1979 February, VV Pup was between high and low states. The brightness of the system varied from $m_{\max} \approx 15.7$ – 16.5 , to $m_{\min} \approx 16.9$ to $m_{\min} \approx 17.8$ at the end of the month (SLB). Presumably, only one pole was accreting during this time span because of the large change in brightness between bright and faint phases and the lack of negative circular polarization during the faint phase (WM). This period of time was when the cyclotron lines were observed by VW and SLB.

We first study the spectrophotometric data of SLB (specifically their Fig. 2) instead of VW's data, for two reasons: (1) the phase dependence of their spectra is better, and (2) the frequency range of their spectra is wider. SLB's data cover an entire orbital period ($P \approx 100$ min) broken up into nine spectral frames of 8 minutes each. Frames 6, 7, and 8 are the observations which contain the bright phase of the orbital period and exhibit the cyclotron lines. A comparison of the absorption model calculations (Fig. 2), the emission model calculations (Figs. 3–6), and the data of SLB (frames 6–8 of their Fig. 2) shows that the emission model represents the data much better than the absorption model. Thus, we confirm the conclusion of WM that the cyclotron lines are a result of cyclotron emission from a single accreting magnetic pole, and not the result of absorption through a cooler plasma boundary layer as proposed earlier by VW and SLB.

By making a detailed examination of SLB's frames 6–8, we find that the minimum of the trough, at a wavelength $\lambda = 0.57 \mu\text{m}$, is displaced by $\sim 0.01 \mu\text{m}$ in frame 7 from its value in frames 6 and 8. This shift in wavelength (or frequency) of the cyclotron lines could be a result of changes in the magnetic field strength, the plasma temperature, or the orientation of the accretion column. We believe a change in the orientation of the accretion column, with the magnetic field strength and plasma temperature remaining constant, the most likely explanation. In the succeeding few paragraphs, we will show, using equation (6), how one may place limits on the orientation of the accretion column by observing the frequency shift of the cyclotron lines at various angles θ .

If we assume that the orientation of the accretion column during frames 6 and 8 was perpendicular to the line of sight, then $\theta = 90^\circ$. Equation (6) can then be used to determine the strength of the magnetic field. The wavelengths for the maxima of the broad peaks are $\lambda = 0.60, 0.525$, and $0.47 \mu\text{m}$ for frames 6 and 8 of SLB and frame 1, 2, 9–11 of VW. Since the cyclotron lines are nearly equally spaced in frequency, it is not obvious which set of cyclotron harmonics at a specific temperature fits the data correctly. The correct set of cyclotron lines can only be found by trial and error, as shown in Table 2. We see that at $kT = 0$ keV, a magnetic field strength of 2.55×10^7 gauss is consistent for cyclotron harmonics $n = 7, 8$, and 9, and, at 10 keV, a magnetic field strength of 3.56×10^7 gauss is consistent with $n = 6, 7$, and 8. We may determine which of the two

TABLE 2
MAGNETIC FIELD STRENGTH FOR CYCLOTRON LINES^a

n	Group 1	Group 2	Group 3
At $kT = 0$ keV			
5.....	35.70	...	...
6.....	34.00	29.75	...
7.....	32.55	29.14	25.50
8.....	...	28.48	25.50
9.....	...	...	25.32
At $kT = 5$ keV			
5.....	38.85	...	...
6.....	37.57	32.88	...
7.....	36.49	32.67	28.59
8.....	...	32.37	28.98
9.....	...	...	29.16
At $kT = 10$ keV			
5.....	41.67	...	...
6.....	40.60	35.52	...
7.....	39.80	35.63	31.18
8.....	...	35.57	31.85
9.....	...	...	32.29

NOTE.—The observational spectra give wavelengths of three distinct cyclotron lines which are to be associated with three cyclotron harmonics. If the lines are taken to be cyclotron harmonics $n = 5, 6$, and 7 (group 1), for example, then a magnetic field strength may be calculated using eq. (6) for each harmonic. A probable solution is found when each cyclotron harmonic in the group has the same magnetic field strength.

^a $B \times 10^{-6}$ gauss.

solutions is more correct by also considering the width of the cyclotron lines. The $kT = 0$ keV solution is clearly inconsistent with the observed broad-line width, unless this is due to other inhomogeneities. We are then left with the $kT = 10$ keV solution and a magnetic field strength of 3.56×10^7 gauss.

Now equation (6) can be used to find the wavelength of the cyclotron harmonics for angles other than 90° . As an example, the wavelength for the center of the trough between cyclotron harmonics $n = 7$ and 8 is $0.575 \mu\text{m}$ at $\theta = 90^\circ$. Table 3 shows the wavelengths of this trough for other angles. The shift of $\sim 0.01 \mu\text{m}$ between frame 7, and frames 6 and 8 corresponds to a change in the angle θ from 90° to $\sim 70^\circ$.

We also fitted the emission model in § II, using a χ^2 minimization analysis, to the observations in frames 6, 7, and 8. The free parameters used were the magnetic field, the plasma temperature, the angle θ , and the dimensionless parameter Λ . As stated earlier in this section, we decided to fit frames 6, 7, and 8, although the noise in these spectra is larger than in Figure 3

TABLE 3
WAVELENGTH OF TROUGH AT
VARIOUS VALUES OF θ

θ ($^\circ$)	λ (μm)
90.....	0.575
80.....	0.573
70.....	0.565
60.....	0.554

TABLE 4
BEST-FIT PARAMETERS TO FIGURE 2 OF SLB

Frame	$B/10^6$ (G)	kT (keV)	θ	$\Lambda/10^5$
6.....	31.8	8.8	90°0	7.7
7.....	31.4	7.1	85.0	50.1
8.....	31.5	8.7	88.5	19.3

(which is a composite of frame 6, 7, and 8) of SLB, to obtain more information on the orientation of the accretion column.

The values which best fit the data are given in Table 4 and are presented in Figure 7. The value for the magnetic field strength, $B = 3.15 \times 10^7$ gauss, is consistent with the value found by WM, but significantly less than the value found by using equation (6). We note that the calculations are very sensitive to the magnetic field strength, and therefore three-digit accuracy in the results is not unreasonable. The plasma temperature found in the analysis ($kT = 8.7$ keV) is lower, but not significantly lower, than the plasma temperature indicated by the analysis of WM ($kT \geq 10$ keV). However, our values for Λ are approximately an order of magnitude greater than WMs.

The best-fit values obtained from the analysis of SLB data were then applied to frames 2 and 9 of VW. The fit to this data is very good (see Fig. 8), when one considers that absolutely no adjustments were made to the best-fit values except for scaling

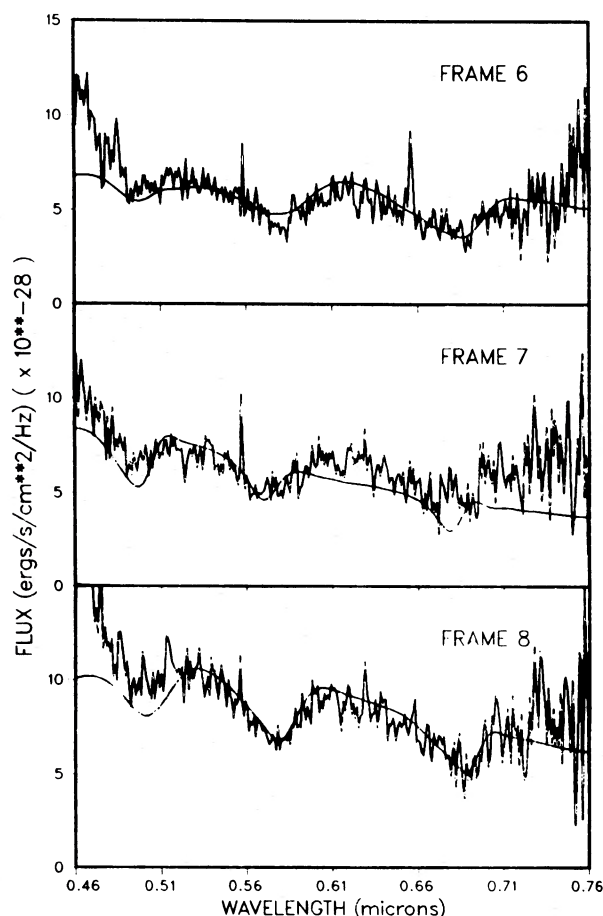


FIG. 7.—Best fit to bright phase data of frames 6, 7, and 8 of SLB. Best fit values for these frames are given in Table 4.

to the observations. In addition, we fitted the data of VW using the values of WM ($kT = 10$ keV, $B = 3.18 \times 10^7$ gauss, $\Lambda \approx 10^5$) with our model calculations. The fits to the data (Fig. 8) are not as good as the fits using our values. We note that our calculations were made using two different methods (Chanmugam and Dulk 1981; Tamor 1978) to calculate the absorption coefficient, and the results are in excellent agreement with one another.

WM fitted theoretical model intensities to data taken from Visvanathan and Wickramasinghe (1979) and SLB of the mean bright phase intensity spectrum. Theoretical models fitted to the mean bright phase intensity data will result in higher temperatures than theoretical models fitted to phase-dependent bright phase intensity data, such as that of SLB. This occurs because the frequency of the cyclotron line width is narrowest at $\theta = 90^\circ$. As θ decreases, the peak frequency of the line changes and the width of the line broadens. For example, the mean bright phase flux of WM has θ vary from 90° to $\sim 70^\circ$, whereas frame 6 of SLB is taken over a shorter interval of time, and θ probably varies from 90° to $\sim 85^\circ$. Note, however, that the width of the cyclotron line becomes broader as the temperature of the plasma increases. If theoretical models are fitted to mean bright phase data at a specific angle of θ , then the required temperature will increase to compensate for the broader line width of these observations. The higher temperature then increases the amount of absorption in the plasma, which results in a lower value for Λ . WM fitted the data of VV Pup in this manner. This explains why our value for the temperature of the plasma are lower than WM and also why we obtain a significantly larger value for Λ .

Wickramasinghe and Meggitt (1982) showed that the orientation of VV Pup did not vary by more than $\Delta\theta = 20^\circ$ during the bright phase. They determined this range by presenting theoretical spectra of a plasma at $kT = 10$ keV, $B = 3.18 \times 10^7$ gauss, and $\Lambda = 10^5$ for various values of θ . Their estimate for $\Delta\theta$ is in good agreement with our earlier analysis of the VV Pup spectra using equation (6).

b) Circular Polarization

The circular polarization (Visvanathan and Wickramasinghe 1979) was observed during two bright phases to be 4% and 6% for 4000–6000 Å. Our calculations of the circular polarization for a complete bright phase was found by averaging the polarization over 4000–6000 Å and over angles 90° , 85° , 80° , and 75° . The resulting value of 9% is in good agreement with the observations.

IV. DISCUSSION

The results from § III may also be used to determine the distance to VV Pup and the size of its postshock region. Aizu (1973) gives the temperature of the postshock plasma as

$$kT_s = (3/16)GM_p/R, \quad (7)$$

where m_p is the proton mass. For a degenerate nonrelativistic electron gas, the mass-radius relation of a white dwarf (Kittel 1969, p. 243) is

$$R \approx 7.94 \times 10^8 (M/M_\odot)^{-1/3} \text{ cm}. \quad (8)$$

Substitution for R , equation (7) gives

$$kT_s = 32.7 (M/M_\odot)^{4/3} \text{ keV}. \quad (9)$$

Using $kT_s = 8.7$ keV (Table 2), it follows that $M \approx 0.37 M_\odot$ and $R \approx 1.1 \times 10^9$ cm.

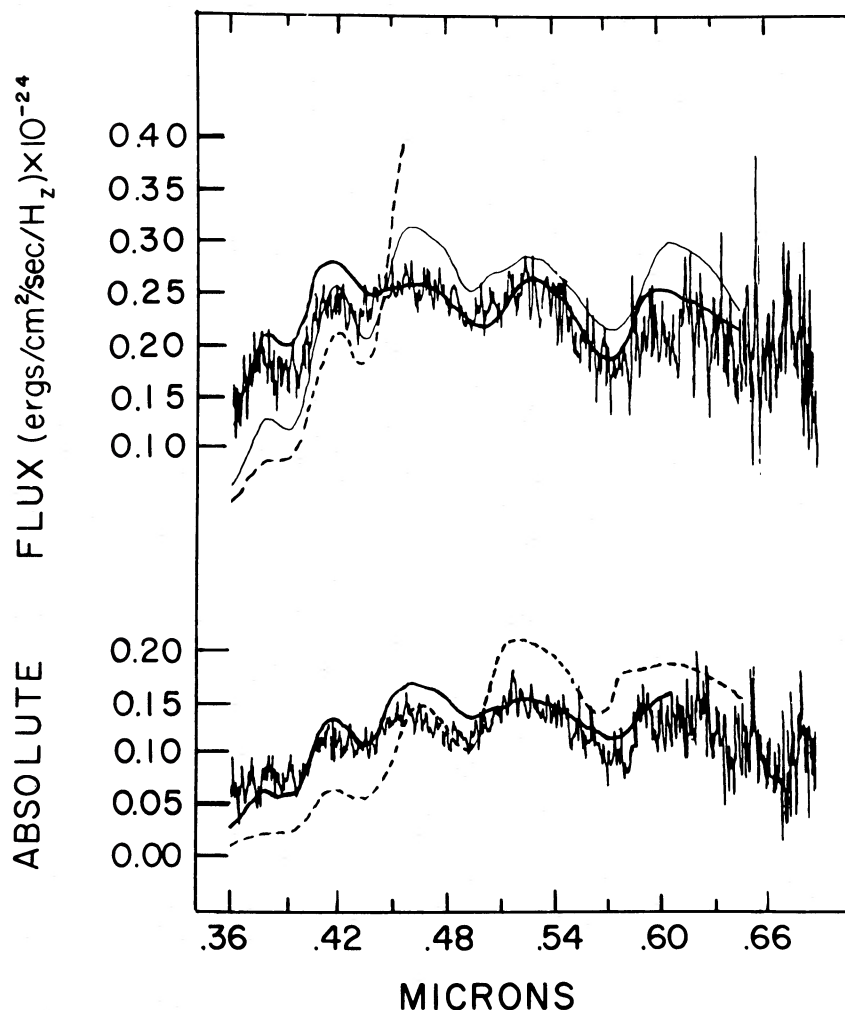


FIG. 8.—Fit to bright phase data of Visvanathan and Wickramasinghe (1979). Upper figure corresponds to their frame 2 and lower figure to their frame 9. For upper figure, our best-fit values ($B = 3.15 \times 10^7$ gauss, $kT = 8.7$ keV, $\theta = 90^\circ$) for $\Lambda = 3 \times 10^6$ (thick solid line) and $\Lambda = 10^6$ (thin solid line) are compared to best-fit values of WM ($B = 3.18 \times 10^7$ gauss, $kT = 10$ keV, $\theta = 90^\circ$, $\Lambda = 10^5$ (dashed line). For lower figure, our best-fit values (same as upper figure) for $\Lambda = 10^6$ (thick solid line) compared to WM (same as upper figure).

If the cooling mechanism in the postshock region is optically thin bremsstrahlung radiation, then the height h of the shock front above the surface of the white dwarf is given by (Aizu 1973)

$$h = 3.77 \times 10^6 (M/M_\odot)(R_9/R)(N_{16}/N). \quad (10)$$

Here N is the electron number density, $N_{16} = N/10^{16} \text{ cm}^{-3}$ and $R_9 = R/10^9 \text{ cm}$. The height of the shock above the stellar surface was found by SLB to be less than $0.04R$ or $\approx 4.4 \times 10^7 \text{ cm}$ from simultaneous photometry. Hence, by using equations (9) and (10), the electron number density in the postshock region during these observations may be deduced to be $N > 2.9 \times 10^{14} \text{ cm}^{-3}$. Therefore, an upper bound on the diameter of the accretion column $L = \Lambda B/4\pi eN$ is $< 3.4 \times 10^7 \text{ cm}$ for $\Lambda \approx 2 \times 10^6$ and $B = 3.15 \times 10^7$ gauss. Knowledge of h and L gives the apparent surface area, $S = hL$ of the postshock region. Therefore, an upper bound on the distance to VV Pup may be found:

$$D = (2kT_s S/\lambda^2 f_\lambda)^{1/2} < 350 \text{ pc}.$$

If the surface temperature of the white dwarf is $\sim 9000 \text{ K}$, as determined by Liebert *et al.* (1978), when VV Pup was very

faint with an observed flux $f_\lambda \approx 2 \times 10^{-27} \text{ ergs s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$, then the distance $D \approx 340$ parsecs.

Widely varying values are given for the area of the polar cap $A = 4\pi R^2 f$. Here, $R \approx 10^9 \text{ cm}$ is the radius of the white dwarf and f is the polar cap fraction. These values range from $f \approx 0.1$ (King and Shaviv 1984), to 10^{-3} (Raymond *et al.* 1979), and to $f \approx 10^{-4}$ – 10^{-5} (Patterson *et al.* 1984; Imamura 1984). Using our diameter of the postshock region, we find $f = 6 \times 10^{-5}$. Our results, then, favor the smaller polar cap size.

WM placed constraints on the orbital inclination i and the angle δ between the rotation axis and the magnetic dipole field axis. Their conclusion listed the angle between the line of sight and the direction of the magnetic field $\theta > 70^\circ$ because the cyclotron features were present throughout the entire bright phase. This conclusion and the fact that the bright phase was present for $\sim 40\%$ of the orbital period implied geometries of $0^\circ < i < 30^\circ$, $90^\circ < \delta < 100^\circ$, or $80^\circ < i < 90^\circ$, $150^\circ < \delta < 180^\circ$. Preference was given to the former set of values because of the absence of the eclipse of the accretion column by the secondary. Brainerd and Lamb (1984) obtained the geometry for VV Pup by analyzing linear polarization position angle data. They found $i = 75^\circ \pm 5^\circ$ and

$\delta = 148^\circ \pm 5^\circ$, which agrees with the second set of values due to WM and our results.

Finally, WM concluded from their analysis that another component of radiation either unpolarized or weakly polarized dominates in the blue and UV regions of the spectrum. They claim this is necessary in order to fit the cyclotron spectrum and reduce the calculated polarization. Our analysis does not support this conclusion. We see no need for the presence of a weakly polarized or unpolarized component of radiation in the blue and UV spectral regions.

V. CONCLUSIONS

We have made calculations of cyclotron line spectra for homogeneous plasmas and applied them to VV Pup. Our main conclusions are as follows:

1. The visibility of cyclotron lines in the optical spectrum is limited to a small parameter space in magnetic field strength, plasma temperature, and system orientation. The magnetic field strength probably needs to be between $2\text{--}10 \times 10^7$ gauss in order to observe optical cyclotron lines. Hence, cyclotron lines would be difficult to observe in most DQ Her binaries, since their magnetic fields are on the average $\sim 10^6$ gauss. The plasma temperature needs to be $\lesssim 15$ keV, and the accretion column perpendicular to the line of sight for extended periods of time (~ 10 minutes, depending on the brightness).

2. Equation (6) generalizes the equation of Hirshfield *et al.*

(1961) for the frequency of the cyclotron line to a three-dimensional Maxwellian electron distribution and propagation at an angle θ to the magnetic field. This equation is shown to be useful in determining the magnetic field strength of the postshock plasma and in placing limits on the system orientation from observational spectra.

3. VV Puppis has a polar magnetic field strength of 3.15×10^7 gauss, and a postshock plasma temperature of 8.7 keV.

4. The orientation of the accretion column varies between $\theta = 70^\circ\text{--}90^\circ$, during the bright phase, confirming the orientation determined by Brainerd and Lamb (1984).

5. The size of the polar cap ($f = 6 \times 10^{-5}$) for VV Pup is consistent with estimates of the polar cap fraction proposed by Patterson *et al.* (1984) and Imamura (1984).

6. The suggestion by WM for a weakly polarized or unpolarized component of radiation in the blue and UV spectral region is not required by our analysis.

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APPENDIX

Petrosian (1981) gives the equation

$$\omega/\omega_B = n\gamma/(1 + t_0^2), \quad (\text{A1})$$

where $t_0^2 = \gamma_0^2 \beta_0^2 \sin^2 \theta$. For the nonrelativistic case ($\omega/\omega_B \mu \ll 1$), $\beta_0 \gamma_0 \approx 2\omega/\omega_B \mu$ and $\gamma_0^2 \approx 1$. This gives

$$\omega/\omega_B = n/[1 + 2\omega \sin^2 \theta/\omega_B \mu]. \quad (\text{A2})$$

Some additional algebra gives

$$\omega/\omega_B = [-1 + (1 + 8n \sin^2 \theta/\mu)^{1/2}]/(4n \sin^2 \theta/\mu). \quad (\text{A3})$$

If $8n \sin^2 \theta/\mu \ll 1$, then we obtain

$$\omega/\omega_B = n(1 - 2n \sin^2 \theta/\mu). \quad (\text{A4})$$

For $\theta = 90^\circ$, equation (A4) is similar to that obtained by Hirshfield *et al.* (1961) (see eq. [5]).

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