

A CONVINCING M_v - $W(H\gamma)$ CALIBRATION FOR A AND B SUPERGIANTS

G. A. H. WALKER AND C. G. MILLWARD

Geophysics and Astronomy Department, University of British Columbia, Vancouver

Received 1984 July 3; accepted 1984 September 6

ABSTRACT

From Reticon spectra of 31 B0–A5 supergiants in the direction of η and χ Persei we find a very close correlation between the equivalent width of $H\gamma$ and V_0 , after correcting for a well-defined, linear dependence of $V_0/W(H\gamma)$ on spectral type (temperature) and of $V_0/W(H\gamma)$ on $W(H\gamma)$ within a spectral type (degree of ionization). We have established a *tentative*, absolute magnitude calibration by adopting a distance modulus of 11.8 and taking V_0 values from Wildey. Apart from the uncertainty in zero point, the probable error in M_v , from a single supergiant spectrum, is 0.18 mag which is remarkably small since many of the stars are probably not members of the double cluster. The photometry of such high-luminosity stars is less sensitive than main-sequence stars to multiplicity, and the small probable error means that early-type supergiants can be used with confidence as standard candles for galactic structure and extragalactic studies.

Our average values of M_v for luminosity class Ia stars agree well with the calibration of Schmidt-Kaler. Our luminosities for the supergiants in the double cluster agree well with the $W(H\gamma)$ luminosities of Balona and Crampton for the early B stars, but there are large differences at later spectral types. The Si II/He I ratio is not as useful as spectral type in defining the temperature dependence of $V_0/W(H\gamma)$ over the full range from B0 to A5. This supergiant calibration is quite independent of the main-sequence $W(H\gamma)$ - M_v calibration of Millward and Walker.

Subject headings: stars: luminosities — stars: supergiants

I. INTRODUCTION

The strength of the Balmer absorption lines in the spectra of early-type stars is effected by Stark broadening, electron pressure, degree of ionization, temperature, Doppler broadening, and the relative abundance of hydrogen in the photospheric layers of the star (see, for example, Aller 1963). Mass, radius, and temperature are uniquely related for main-sequence stars of normal hydrogen abundance, which means that a simple relation should exist between the equivalent width of a hydrogen absorption line, $W(H)$, and M_v for luminosity class V stars. Millward and Walker (1984, 1985) recently published such a calibration for class V and a few class III stars using values of $W(H\gamma)$ measured on Reticon spectra. There have been several other calibrations of which those by Balona and Crampton (1974) of $W(H\gamma)$ and by Crawford (1978) of the $H\beta$ index are among the more extensive.

The Balmer lines in the spectra of supergiants are much narrower and weaker than for main-sequence stars of the same spectral type. Not only is the surface gravity and hence electron pressure much lower, but the degree of ionization increases with increasing luminosity thereby depleting the population of neutral hydrogen in the $n = 2$ level. Petrie (1965) included supergiants in his comprehensive $W(H\gamma)$ - M_v calibration based on photographic spectra. He was forced to introduce so-called spectral type corrections to compensate both for the effect of luminosity on $W(H\gamma)$ as well as the effect of spectral type. The corrections for the class I stars were different from those for the class V to II stars later than spectral type B0 and became quite large, being up to -2.5 mag, for the early A stars. Balona and Crampton (1974) demonstrated that these corrections were not satisfactory and instead used corrections which varied smoothly with luminosity.

Hutchings (1966), using photographic spectra, pointed out that the strength of $W(H\gamma)$ in the supergiants reaches a

maximum at F0 as opposed to A0 for class V. He applied spectral type corrections to the equivalent widths and using a calibration based on observations of three galactic clusters, he determined distances to the Magellanic Clouds of 18.2 (LMC) and 18.3 (SMC) which agreed well with other determinations at that time.

Visually, the A and B supergiants are among the most luminous stars known. They have luminosities between -5.5 and -8 . In consequence, they can be detected at much greater distances than main-sequence objects, and, unless associated with another supergiant, they are largely unaffected, photometrically, by the presence of companions. This makes them very suitable standard candles for galactic structure studies in our own and in nearby galaxies.

Many supergiants exhibit variable emission at $H\alpha$, but the Balmer emission decrement is sufficiently steep that the associated emission at $H\gamma$ is usually weak or undetectable, and where it exists, allowance can be made for it in drawing the line profile. Sensitive detectors of high photometric fidelity such as the Reticon now make it feasible to reexamine the $W(H\gamma)$ - M_v calibration for supergiants using high signal-to-noise ratio spectra. Recently, Zhang (1983) has carried out such a study for B type supergiants on the *uvby* $H\beta$ system.

II. THE OBSERVATIONS

The largest collection of bright, A and B supergiants in the northern hemisphere lies in the direction of η and χ Persei. We have observed 31 of them with a refrigerated 1872F/30 Reticon detector on the Cassegrain spectrograph of the DAO 1.88 m telescope at a dispersion of $60 \text{ \AA mm}^{-1}$ corresponding to $0.9 \text{ \AA}$ per diode. The detection system and data reduction procedures have been discussed by Walker, Johnson, and Yang (1984). Each star was observed only once and is listed in Table 1 together with an estimate of the average signal-to-noise ratio

TABLE 1
h AND χ PERSEI SUPERGIANTS

HD Number	Willey or Oosterhoff Number	Spectral Type	V_0	$W(H\gamma)$	$V_0/H\gamma$	h	ΔV	M_v	Si II/He I	N II/He II	S/N
12953	W3	A1 Ia	3.80	1.70	2.24	0.82	+0.55	-7.82	3.60	...	980
13267	W5	B5 Ia	5.04	1.72	2.93	1.31	-0.18	-6.83	0.945	...	880
13402 ^a	W6	B0.5 Ib	5.78	1.88	3.07	1.84	-1.51	-5.78	0.286	0.558	290
13476	W7	A3 Iab	4.97	3.68	1.35	1.43	-0.70	-6.60	...	...	500
13659	W8	B1 Ib	6.52	2.20	2.96	2.10	-1.80	-5.26	0.210	0.493	340
13744	W10	A0 Iab	5.38	2.69	2.00	1.42	-0.27	-6.61	3.60	...	380
13841	Oo 3/W11	B2 Ib	6.15	2.06	2.99	1.87	-1.26	-5.72	0.258	0.759	520
13854	Oo 16/W12	B1 Iab	5.12	1.66	3.08	1.58	-1.16	-6.29	0.264	1.098	840
13866 ^b	W13	B2 Ib	6.49	2.01	3.23	1.82	-0.74	-5.81	0.188	0.667	280
14052	Oo 662/W15	B1 Ib	6.71	2.35	2.86	2.24	-2.18	-4.98	0.266	0.522	500
14134 ^c	Oo 1057/W17	B3 Ib	4.75	1.57	3.03	1.35	-0.60	-6.76	0.526	0.564	880
14143	Oo 1162/W18	B2 Ia	4.77	1.34	3.56	1.21	-0.05	-7.03	0.348	1.272	880
14322	W22	B8 Ib	5.59	2.24	2.50	1.40	+0.05	-6.66	1.50	...	850
14433	Oo 2178/W25	A1 Ia	4.39	2.59	1.69	1.25	-0.57	-6.96	2.44	...	860
14443	Oo 2227/W26	B2 Ib	6.70	2.21	3.03	2.00	-1.25	-5.45	0.273	0.250	460
14489	W28	A2 Ia	4.42	2.74	1.61	1.19	-0.31	-7.07	3.00	...	1240
14535 ^d	Oo 2589/W31	A2 Ia	5.59	2.27	2.46	0.99	+1.67	-7.48	...	...	210
14542	Oo 2621/W32	B8 Ia	4.86	1.93	2.52	1.20	+0.08	-7.05	1.65	...	310
14818 ^e	W35	B2 Iab	4.82	1.54	3.13	1.39	-0.72	-6.67	0.300	1.132	740
14899	W37	B8 Ib	6.06	2.77	2.19	1.73	-0.80	-6.00	2.00	...	330
14956	W38	B2 Ia	4.63	1.27	3.65	1.15	+0.06	-7.16	0.289	1.355	320
15316	W40	A3 Iab	5.21	3.85	1.35	1.49	-0.72	-6.47	...	...	310
BD + 60°493	W41	B0.5 Ia	5.09	1.33	3.83	1.30	-0.07	-6.86	0.296	1.364	190
15497	W42	B6 Ia	4.62	1.61	2.87	1.16	+0.03	-7.15	1.063	...	280
15620	W43	B8 Iab	5.41	2.51	2.16	1.56	-0.80	-6.33	2.57	...	180
15690	W44	B1.5 Ib	5.35	1.64	3.26	1.52	-0.70	-6.41	0.313	0.922	250
16778	W47	A2 Ia	5.14	2.85	1.80	1.24	+0.22	-6.76	2.68	...	250
16779 ^f	W46	B2 Ib	6.01	1.49	4.03	1.35	+0.65	-6.98	0.257	0.653	180
17088	W48	B9 Ia	4.92	2.12	2.32	1.22	+0.07	-7.01	1.88	...	330
17145	W50	B8 Ia	5.81	2.33	2.49	1.45	+0.04	-6.55	1.77	...	150
17378	W51	A5 Ia	4.06	3.52	1.15	1.03	-0.04	-7.39	...	...	560

^a Willey does not give a luminosity subclass. Both Schild 1967 and Slettebak 1968 classify star as B0.5 Ib. From the luminosity-sensitivity line ratio N II 3995/He II 4144 we classify star as luminosity subclass Ib.

^b Broad-line spectrum. Slettebak 1968 classifies star as B2 II.

^c Willey classifies star as B3 Ia. Schild 1967 and Johnson and Morgan 1955 both classify star as B3 Ia. Slettebak 1968 classifies as B3 Iab. From the N II 3995/He II 4144 line ratio we classify star as luminosity subclass Ib.

^d Willey classifies star as A2 Ia. Johnson and Morgan 1955 classify star as A2 Iap?. Slettebak 1968 classifies as A2 Ia. Crawford, Glaspey, and Perry 1970 note the star is peculiar in their V_0 - β diagram. Star has a large residuals in the present $V_0/W(H\gamma)$ - $W(H\gamma)$ and ΔV - $h\gamma$ plots and has been omitted from the analysis.

^e Willey classifies star as B2 Ia. Johnson and Morgan 1955 and Slettebak 1968 classify star as B2 Ia. From N II 3995/He II 4144 we classify star as luminosity subclass Iab.

^f Star has large residuals in both $V_0/W(H\gamma)$ - $W(H\gamma)$ and ΔV - h plots. We measure an $H\gamma$ equivalent width of 1.49 Å. Crampton, Leir, and Younger 1973 give a value of 3.8 Å based on one photographic spectrum. The line strength is probably variable, and the star has been omitted from the analysis.

per diode in the last column. The W numbers in the second column are taken from Willey (1964) and the Oo numbers are taken from Oosterhoff (1937). The V_0 values, spectral types, and luminosity classes are taken from Willey (1964), with some additional spectral types from Schild (1967) and Slettebak (1968). The quantities in the other columns are discussed below.

Each spectrum was digitally filtered to 49% of the Nyquist frequency which gives a spectral resolution close to 2 Å. Two sample spectra are shown in Figure 1; one is a B2 Iab, the other is an A1 Ia. Although some 1800 Å of spectrum was registered, only the 4200–4500 Å region is shown. Our interpolated continuum and profile for $H\gamma$ are shown.

For such narrow $H\gamma$ lines, the continuum was fairly easy to place compared to the broader lines in main-sequence stars. The $H\gamma$ profiles were drawn in by hand making allowance for known blends with other absorption lines. The principal blends are Ti II 4337.9 Å, Ti II 4344.2 Å, Mn II 4348.5 Å, and O II + Fe II 4349 Å. Only the O II + Fe II blend is important at all spectral types. The other three are only detected in A and

later B stars. No emission was detected at $H\gamma$ in any of the spectra. The areas of the $H\gamma$ lines were measured in angstrom units with a planimeter and are listed under $W(H\gamma)$ in Table 1.

In Figure 2 the $W(H\gamma)$ values from Table 1 are plotted against $W(H\gamma)$ values taken from the catalog of Petrie's measures by Crampton, Leir, and Younger (1974). The line of unit gradient, shown by dashes, does fit the data, but the best fitting straight line, shown solid, is given by:

$$PW(H\gamma) = -0.038 + 1.081W(H\gamma) \quad (1)$$

where $PW(H\gamma)$ are Petrie's measures. This determination excludes the one deviant point for HD 16779 which is almost certainly variable. Although Petrie claimed to have taken account of the O II, blend this might have been incomplete and may account for his values being 8% larger than ours. In other respects the correction seems satisfactory.

The ratio N II(3995)/He II(4144) is sensitive to luminosity for B0–B3 stars. We have measured it on our Reticon spectra, and the values are listed in Table 1 and plotted against Willey's (1964) class I subclasses, Ia, Iab, and Ib in Figure 3. The correl-

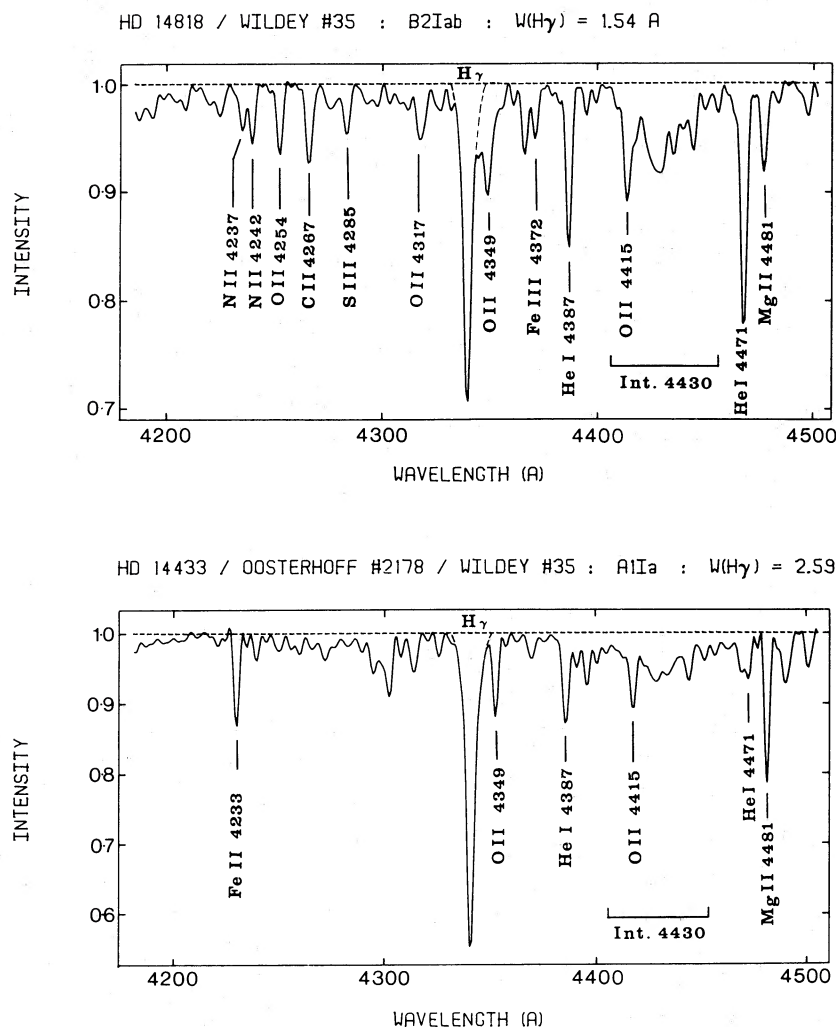


FIG. 1.—Samples from Reticon spectra showing the 4200–4500 Å region. The spectra have been normalized to a continuum and filtered to a resolution of 2 Å. The interpolated continuum and the profile adopted in the measurement of $W(H\gamma)$ are shown. The upper spectrum is a B2 Iab ($S/N = 740$); the lower one is an A1 Ia ($S/N = 860$).

ation is good except for two stars which Wildey gives as Ia: HD 14818 which, from Figure 3, we classify as Iab; and HD 14134 which we classify as Ib. Wildey quotes HD 13402 simply as I. We agree with Slettebak (1968) in classifying it as Ib. These reclassifications also agree better with the values of $W(H\gamma)$. We have changed the classifications accordingly in Table 1.

Among later spectral types, the star HD 14322 is given as B8 Ib by Wildey, but in the analysis which follows in the next section its position in Figures 4 and 5 indicate that it is almost certainly a Ia. However, we have not modified Wildey's classification.

III. THE CALIBRATION

The relationship $V_0/W(H\gamma)$ is plotted against spectral type in Figure 4 for all of the stars in Table 1. Points for luminosity class Ia stars are shown as solid points, Iab as X, and Ib as open circles. Two stars known to have peculiar spectra have a p beside them. Of the latter, HD 16779, as mentioned in the previous section, is probably variable and is, in any case, unlikely to be a member as it lies 6° from the double cluster. The other peculiar star, HD 14535, lies within 14' of the center

of χ Per. Johnson and Morgan (1955) classify the latter as A2 Iap?, and Crawford, Glaspey, and Perry (1970) found that the star deviated from their mean V_0 - $H\beta$ calibration for h and χ Persei.

If the stars all lie at the same distance and $W(H\gamma)$ varies only as some power of the luminosity, $V_0/W(H\gamma)$ should have the same value for all of the stars, within the errors of measurement (since V_0 varies as the log of the luminosity). However, there is a clear, linear trend with spectral type and, within a spectral type, residuals from the trend are luminosity dependent.

The trend with spectral type reflects the variation of photospheric temperature, while the dependence of the residuals on luminosity reflects increasing ionization with luminosity. It is quite straightforward to correct for both of these effects.

The best fitting straight line to the points for 13 of the 14 Ia stars is shown in Figure 4. The point for the peculiar Ia star HD 14535 was omitted from the calculation. The equation of this line is:

$$V_0/W(H\gamma) = 3.971 - 0.187s, \quad (2)$$

where s is the number of spectral subclasses measured from B0, positively toward later spectral types, where each spectral sub-

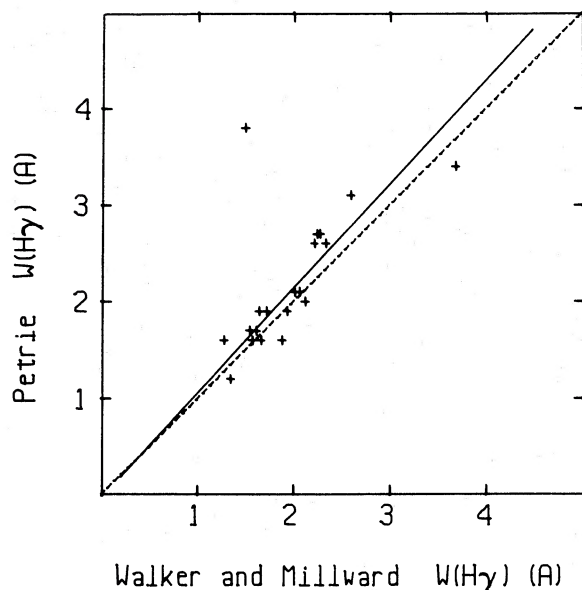


FIG. 2.—A plot of Petrie's $W(H\gamma)$ values from Crampton, Leir, and Younger (1974) against ours from Table 1 for the supergiants in common in η and χ Persei. The best fitting straight line is shown by the solid line, while the dashed line is of unit gradient.

class from 0 to 9 is counted for the B stars and includes classes A0–A5 for the A stars. For example, $s = 12$ for A2.

Using equation (2), modified values of $W(H\gamma)$, h , were calculated for each star, where:

$$h = (1 - 0.0471s)W(H\gamma). \quad (3)$$

This has the effect of removing the dependence of $W(H\gamma)$ on spectral type and, for the class Ia stars, sets $V_0/h = 3.971$, the value at B0. The values of h are listed in Table 1.

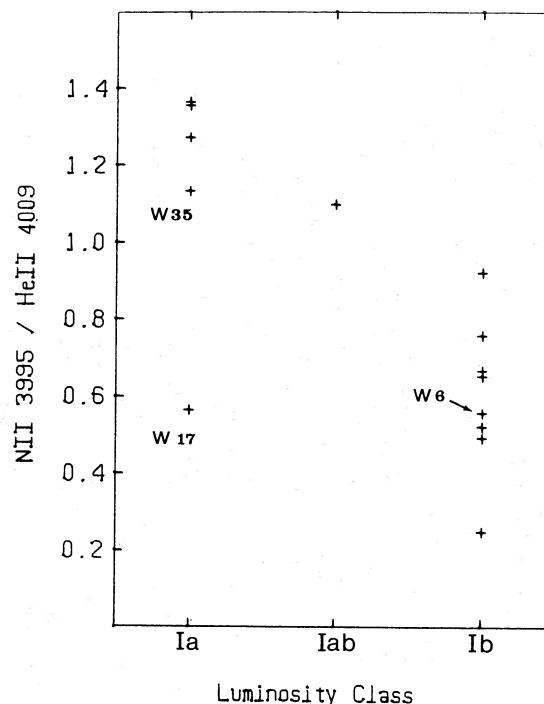


FIG. 3.—The luminosity-sensitive ratio $N\text{ II}(3995)/\text{He II}(4144)$ measured from Reticon spectra plotted against the luminosity subclasses given by Wildey (1964) for the B0 to B3 supergiants in Table 1. Stars for which we have reclassified the luminosity subclass are indicated by their Wildey number.

The residual $\Delta V = (V_0 - V_0')$ was then formed, where V_0' was calculated from equation (2), and V_0 is the observed value. ΔV is plotted against h in Figure 5. This plot isolates the luminosity dependent component of $W(H\gamma)$. Apart from the two peculiar stars, there is a well-defined relation between ΔV and h and a clear separation between the luminosity subclasses. The best fitting straight line is given by:

$$\Delta V = 2.344 - 1.971h. \quad (4)$$

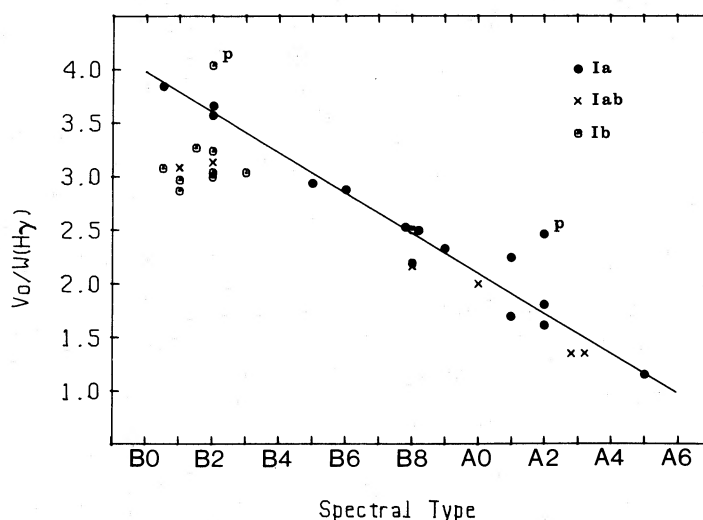


FIG. 4.—A plot of $V_0/W(H\gamma)$ against spectral type for all 31 stars observed in the direction of η and χ Persei, where V_0 is the apparent magnitude corrected for interstellar extinction and $W(H\gamma)$ is the equivalent width of the $H\gamma$ line measured from spectra such as those in Fig. 1. Luminosity class Ia stars are shown as solid circles, Iab as crosses, and Ib as open circles. The best fitting straight line to the Ia points is shown. The peculiar A2 Ia star HD 14533, indicated by "p," was not included in the determination of the line.

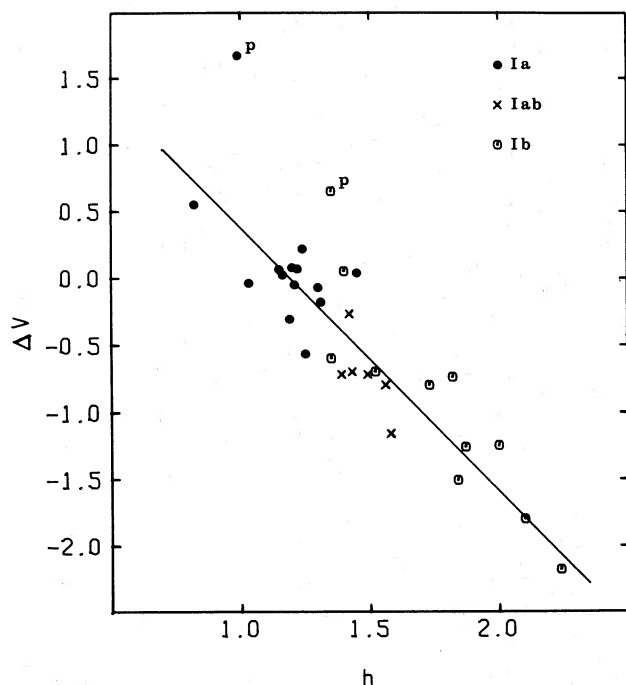


FIG. 5.—A plot of the residual $\Delta V = V_0 - V_0'$ against h , where h is the value of $W(H\gamma)$ corrected to its equivalent at B0 using the straight line in Fig. 4. V_0' is the calculated value of V_0 using h . The symbols have the same meaning as those in Fig. 4.

Substituting for V_0' and ΔV in the relation $V_0 = V_0' + \Delta V$ gives:

$$V_0 = 2.344 + (2.00 - 0.0942s)W(H\gamma). \quad (5)$$

To establish a zero point in absolute magnitude, we must either adopt a distance modulus for the double cluster or apply equation (5) to supergiants in another cluster with an independently, or better, determined distance. Unfortunately we do not have such extensive data for any other cluster or association, and we do not yet have spectra of main-sequence stars in the double cluster with which to tie this to the Millward and Walker (1985) calibration.

There is a range of distance estimates for the double cluster. Seven made since 1956 are listed in Table 2 from which we adopt a value of 11.8. (Where authors list separate moduli for h and χ , we have taken a mean.) While this is the most frequent value, it is not clear that all of the values are independent, and the two distances based on Balmer line luminosities are some 0.3 mag lower. Using the relation $M_v = V_0 - 11.8$ gives the following, tentative, luminosity calibration:

$$M_v = -9.456 + (2 - 0.0942s)W(H\gamma). \quad (6)$$

If we ignore the uncertainty in the zero point reflected by the range of values in Table 2, the probable error in M_v from a single spectrogram is 0.18 mag based on the range of calculated values of $V_0 - M_v$, when the two peculiar stars are neglected. The calculated values of M_v are listed in Table 1.

IV. DISCUSSION

While the high quality of the Reticon data has improved the accuracy with which $W(H\gamma)$ can be measured and allows the well-known temperature dependence to be better defined, the key to this calibration, in our view, is the precision with which

TABLE 2

h AND χ PERSEI DISTANCE MODULI FROM THE LITERATURE

Source	Value
Johnson and Hiltner 1956	11.8
Becker 1963	11.82
Borgman and Blaauw 1964	11.65
Schild 1967	11.8
Willey 1964	11.9
Crawford, Glaspey, and Perry 1970	11.5
Balona and Crampton 1974	11.45
Adopted	11.8

we can correct for the effect of ionization. This is demonstrated by the small scatter about the correlation in Figure 5. The small dispersion in the values of $V_0 - M_v$ in Table 1 implies that the stars must be largely at the distance of the double cluster, and that their photometry is little affected by the presence of companions. The latter confirms that supergiants can be used with confidence as standard candles. This seems to be particularly true for stars in luminosity class Ia which form a closely defined group in luminosity as can be seen in Figures 4 and 5.

Crawford, Glaspey, and Perry (1970) have pointed out that the intrinsic colors and reddening assumed by Willey (1964) in forming the V_0 values in his paper are probably incorrect. We have recalibrated relations (2)–(5) using more recent intrinsic supergiant colors and find that the gradients are unchanged in all cases. Only the errors are marginally increased. For this reason we have retained Willey's V_0 values. It should be stressed that the transformation of V_0 to M_v in equation (6) depends on the distance adopted to the double cluster from Table 2 and is independent of any small systematic error in the scale of V_0 .

In Figure 6 we have plotted the mean or individual values of M_v against spectral type, separately, for the luminosity class Ia and Ib stars. A range of ± 0.18 mag is indicated. The solid lines are the luminosity calibrations of Ia and Ib stars given by Schmidt-Kaler (1982). The agreement for the class Ia stars is remarkably good. There are fewer stars classified as Ib, and they range in luminosity between the limits set for Ia and Ib by Schmidt-Kaler. This suggests that class Ib is less well defined in luminosity than class Ia.

In Figure 7 the residuals, $\Delta M_v = (BC - WM)$, are plotted against spectral type for the stars observed in the double cluster. BC corresponds to our values of $W(H\gamma)$ from Table 1 converted to M_v using the Balona and Crampton (1974) calibration which involves $W(H\gamma)$ and spectral type only since their calibration involving $(B - V)_0$ is only valid for spectral types earlier than B4. WM refers to the M_v values based on our calibration listed in Table 1. Agreement is good for the early B stars, but the residuals become large and there are systematic differences between the Ia and Ib for the later type stars. These residuals are strongly dependent on spectral type and presumably reflect the different ways in which corrections have been applied in the two calibrations.

Given the close, linear correlation between $W(H\gamma)$ and spectral type for the class Ia stars in Figure 2, we attempted to generate a more continuous variable than spectral type by using the Si II/He I ratio. We measured the central depths of the Si II 4128 and 4130 Å lines and the He I 4121 Å line on most of

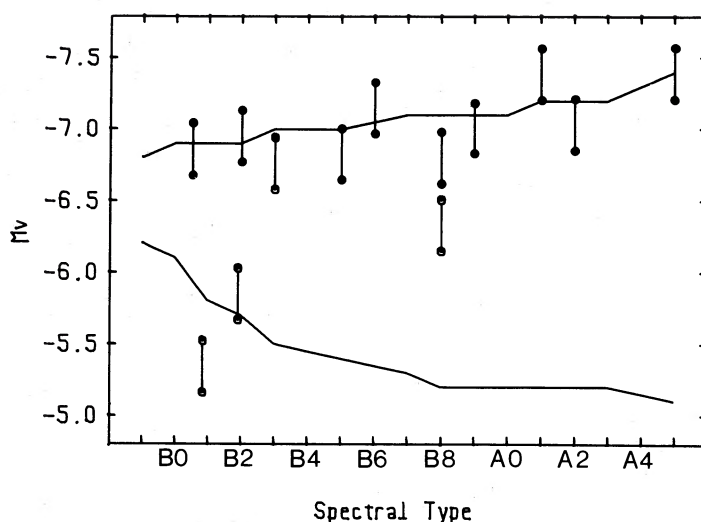


FIG. 6.—Mean or individual values of M_v estimated from our calibration for class Ia and Ib supergiants plotted against spectral type. The length of the bar is 0.36 mag corresponding to twice the probable error estimated for M_v based on a single spectrum. The solid lines are the luminosity calibrations for class Ia and Ib stars by Schmidt-Kaler (1982). The symbols have the same meaning as those in Fig. 4.

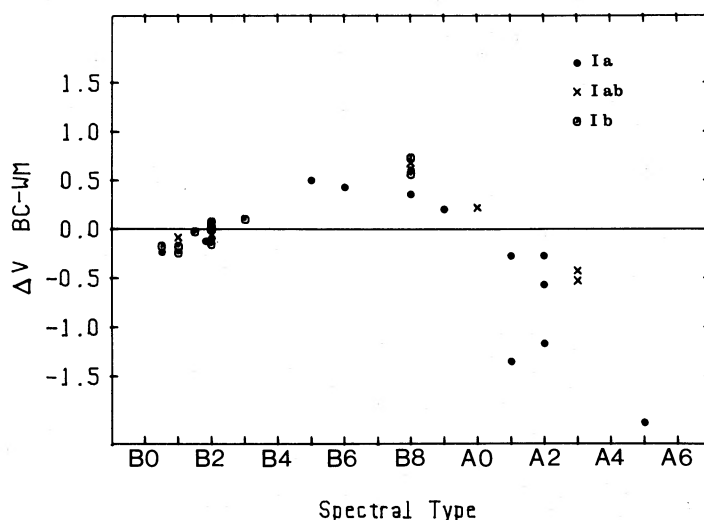


FIG. 7.—Residuals $\Delta M_v = BC - WM$ plotted against spectral type for the h and χ supergiants, where BC are absolute magnitudes estimated from our $W(H\gamma)$ and the calibration of Balona and Crampton (1974). WM are the M_v values based on our calibration and listed in Table 1.

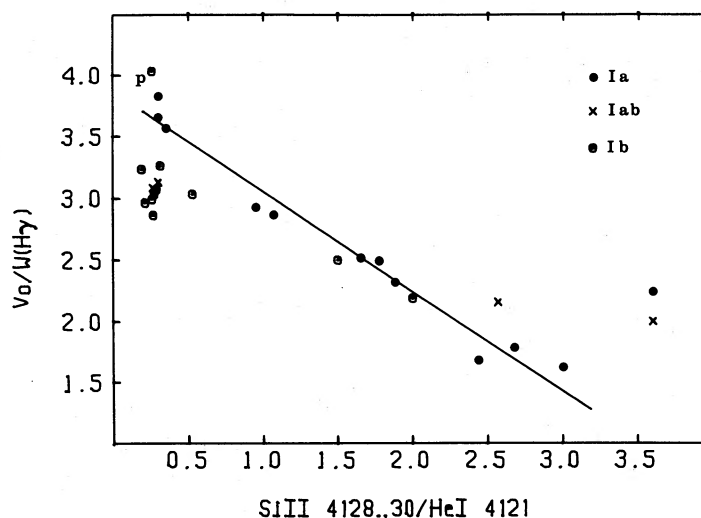


FIG. 8.— $V_0/W(H\gamma)$ plotted against the ratio $\text{Si II}(4128 + 4130)/\text{He I}(4121)$ based on central depths measured on the Reticon spectra. The latter ratio is a possible substitute for spectral type.

our spectra. The ratio is listed in Table 1. Unfortunately the ratio does not differentiate well between the early B stars and it is not very useful for the A supergiants where He I becomes very weak; indeed, we were unable to measure it for the later A stars. Figure 8 is a plot of $W(H\gamma)$ against $(Si\ II/He\ I)$ for those stars with measured values of the ratio. The correlation for the class Ia stars is as good as that in Figure 2 over the mid-B star range but suggests that a combination of ratios which more closely mimics the full range of spectral types is necessary. For example, we find that the ratio $Si\ III(4552)/Si\ IV(4089)$ is an

excellent discriminant between the early B stars in the limited range B0–B2.

We thank the Director of the Dominion Astrophysical Observatory, Victoria for observing time on the 1.88 m telescope. One of us (G. A. H. W.) acknowledges a helpful conversation with Dr. D. Mihalas about the interpretation of these results. This research was supported with funds from the Natural Sciences and Engineering Research Council of Canada.

REFERENCES

- Aller, L. H. 1963, *The Atmosphere of the Sun and Stars* (New York: Roland Press).
 Balona, L. A., and Crampton, D. 1974, *M.N.R.A.S.*, **166**, 203.
 Becker, W. 1963, *Zs. Ap.*, **30**, 164.
 Borgman, J., and Blaauw, A. 1964, *Bull. Astr. Inst. Netherlands*, **17**, 358.
 Crampton, D., Leir, A., and Younger, F. 1974, *Pub. Dominion Ap. Obs.*, **14**, 151.
 Crawford, D. L. 1978, *A.J.*, **83**, 48.
 Crawford, D. L., Glaspey, J. W., and Perry, C. L. 1970, *A.J.*, **75**, 822.
 Hutchings, J. B. 1966, *M.N.R.A.S.*, **132**, 433.
 Johnson, H. L., and Hiltner, W. A. 1956, *Ap. J.*, **123**, 267.
 Johnson, H. L., and Morgan, W. W. 1955, *Ap. J.*, **122**, 429.
 Millward, C. G., and Walker, G. A. H. 1984, in *Proceedings of MK Workshop: Criteria and Applications*, ed. R. F. Garrison (Dordrecht: Reidel), in press.
 ———. 1985, *Ap. J. Suppl.*, **57**, in press.
 Oosterhoff, P. T. 1937, *Leiden Ann.*, **17**, Part 1.
 Petrie, R. M. 1965, *Pub. Dominion Ap. Obs.*, **12**, 317.
 Schild, R. E. 1967, *Ap. J.*, **148**, 449.
 Schmidt-Kaler, Th. 1982, *Landolt-Bornstein*, ed. K. H. Hellwege (Berlin: Springer), New Ser., Group 6, Vol. 2, p. 17.
 Slettebak, A. 1968, *Ap. J.*, **154**, 933.
 Walker, G. A. H., Johnson, R., and Yang, S. 1983, in *Proceedings of the 8th Symposium on Photoelectric Image Devices*, ed. B. L. Morgan (Dordrecht: Reidel), in press.
 Wildey, R. L. 1964, *Ap. J. Suppl.*, **8**, 439.
 Zhang, E. 1983, *A.J.*, **88**, 825.

CHRISTOPHER G. MILLWARD and GORDON A. H. WALKER: Department of Geophysics and Astronomy, University of British Columbia, 2219 Main Mall, Vancouver, B.C. V6T 1W5, Canada