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Abundances of elements in the atmospheres of sharp-lined Hg–Mn stars

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Summary. The abundances of 14 elements in the atmospheres of 20 sharp-lined Hg–Mn stars were derived from published equivalent widths and microdensitometer tracings of high-resolution spectra covering the wavelength range from 3900 to 4650 Å. The Hg–Mn stars were compared with the normal A0 IV star γ Gem by the differential curve-of-growth method. Numerous weak lines were used in the analysis. There are large excesses of P, Sc, Mn, Ga, Sr, Y, Zr, Pt and Hg in some stars, but Mg and Ni are generally deficient and Sc and V are deficient in the cooler stars. No correlation of the abundances with the periodic table was found. Although there are large variations in the relative abundances of elements even among stars with similar effective temperatures, overall trends with effective temperature may exist for P, Sc, Mn and Pt. The excesses of Pt in the seven coolest stars are large and remarkably similar, but the isotopic composition of Pt and Hg varies from star to star. Both diffusion and accretion mechanisms may be necessary to account for all the abundance patterns and anomalies.

1 Introduction

The Hg–Mn stars are early-type, main-sequence stars with distinctive, but rather puzzling, anomalies. Their optical spectra characteristically have strong lines of P, Mn, Ga, Y and Hg, and lines of Xe and Pt have been discovered in the spectra of a few stars. Most Hg–Mn stars have effective temperatures $T_{\rm eff}$ in the small range from 11000 to 14000 K, and yet stars with similar values of $T_{\rm eff}$ may have quite different spectra.

The spectroscopic anomalies are due to large abundance anomalies in the surface layers of the stars. All Hg–Mn stars have projected rotational velocities $v \sin i \lesssim 90$ km s^{-1}, while many normal, late B-type stars rotate more rapidly (Wolff & Preston 1978). Meridional circulation may prevent the occurrence of anomalies in rapidly rotating stars. Some Hg–Mn stars seem to have experienced severe rotational braking (Guthrie 1981). Van den Heuvel (1968) suggested that Hg–Mn stars may be remnants of binary or multiple systems in which mass transfer has occurred and that they may have accreted heavy elements from the explosions of former companion stars. The abundance anomalies have also been attributed to a separation of elements by diffusion in the outer layers of the stars (Michaud 1970). However, there are many unresolved problems with these ideas.

Abundances of elements in individual Hg–Mn stars have been found from high-dispersion spectra (2 Å mm^{-1}) by Aller and his colleagues (see the reviews by Dworetsky 1971 and Jaschek & Jaschek 1974). A survey of abundances was carried out by Heacox (1979) who analysed 6.8 Å mm^{-1} spectra of 21 Hg–Mn stars and derived abundances of 21 elements. These studies were based on model atmospheres for normal stars but, as Heacox pointed out, such models would not be valid for stars in which diffusion produces radial abundance gradients. Some alleviation of this problem may be obtained by selecting lines which are fairly insensitive to atmospheric parameters, but there is often a dearth of accurate *gf*-values for such lines.

For his survey of abundances, Heacox used only a few fairly strong lines of each element and assumed microturbulent velocities $\xi_t = 0$ or 3 km s^{-1} for all the stars. His logarithmic abundances for $\xi_t = 0$ differ from those for $\xi_t = 3$ km s^{-1} by amounts ranging up to 1.2 dex. The studies of individual Hg–Mn stars by Aller and others suggest that the mean value of ξ_t is about 3 km s^{-1} for stars with $v \sin i < 20$ km s^{-1}, but the value of ξ_t may vary from star to star. Weaker lines on the linear part of the curve-of-growth would be insensitive to changes in ξ_t, but weak lines are difficult to measure in stars with $v \sin i > 20$ km s^{-1} and their *gf*-values are often uncertain or unknown.

For some elements the effect of errors in *gf*-values may be reduced by comparing the Hg–Mn stars with normal stars. Kodaira & Takada (1978) used the differential curve-of-growth method in an analysis of 5.3 Å mm^{-1} spectra of 14 Hg–Mn stars and two normal comparison stars. The differential method may also be applied to lines with unknown *gf*-values, including those which are weak and fairly insensitive to changes in atmospheric parameters.

The aim of the present study is to derive information on abundances or relative abundances for 20 sharp-lined Hg–Mn stars with $v \sin i \leqslant 20$ km s^{-1}. Sharp-lined stars were chosen so that numerous weak lines could be used. The question of whether the chemical composition of such stars is representative of the Hg–Mn group as a whole remains open; Heacox plotted his abundances of several elements against $v \sin i$ but found no obvious correlations in the range from 0 to 80 km s^{-1}.

High-dispersion spectra of the 20 sharp-lined Hg–Mn stars are analysed by a method which minimizes the need for accurate *gf*-values. Curves-of-growth for a few elements are constructed from relative values of log *gf*, and these are adjusted to give curves-of-growth for other elements with comparable atomic weights. The Hg–Mn stars are compared with the normal A0 IV star γ Gem by a differential curve-of-growth analysis; this provides abundances of Mg, Zr and iron-peak elements without requiring additional *gf*-values. The relative abundances of Y and Zr are derived from the accurate *gf*-values which have recently become available (Grevesse *et al.* 1981; Hannaford *et al.* 1982). Star-to-star variations in the abundances of P, Ga and Pt are also studied. The results are discussed with regard to the diffusion and accretion hypotheses which have been proposed to account for the anomalies.

2 Determination of abundances

2.1 SELECTION OF STARS

The 20 Hg–Mn stars selected for study (Table 1) comprise nearly all the sharp-lined Hg–Mn stars in the *Bright Star Catalogue* (Hoffleit 1982) for which strong spectroscopic anomalies have been reported. They include the primary components of the double-lined spectroscopic binaries (SB2s) 112 Her, χ Lup and HR 4072 and both components of the SB2 46 Dra. Also included are four single-lined spectroscopic binaries (SB1s) with orbital periods $P < 25$ days (κ Cnc, HR 89, HR 7245 and 53 Tau). The 11 remaining stars are probably long-period binaries or single. The binary characteristics and the values of $v \sin i$ in Table 1 are quoted from a previous compilation (Guthrie 1981). All the stars have $v \sin i \leqslant 20$ km s^{-1}.

The A0 IV star γ Gem was selected as the comparison star for the differential curve-of-growth analysis, since its spectrum has many sharp lines ($v \sin i = 7$ km s^{-1}) in common with the Hg–Mn stars. The abundances of elements in γ Gem may be assumed to be essentially solar. The abundance of Fe in γ Gem is about three times higher than in the standard A0 V star α Lyr which is about three times deficient in Fe as compared with the Sun, and the relative abundances of Mg, Sc, Ti, V, Cr, Mn, Fe and Ni in γ Gem and α Lyr are very similar (Sadakane & Nishimura 1979; Cowley *et al.* 1982).

Table 1. Parameters for the Hg–Mn stars studied.

Star	HD	$v \sin i$ (km s^{-1})	$T_{\rm eff}$ (K)	$\log g$ (cgs)	θ	$\log P_{\rm e}$ (cgs)	Orbital period (days)	
κ Cnc	78316	6	13500	3.65	0.433	2.1	6.39	SB1
112 Her pr	174933	4	13500	3.8	0.433	2.15	6.36	SB2
HR 7361	182308	$\leqslant 5$	13500	3.5	0.433	2.0		
HR 8349	207857	14	13200	3.5	0.441	1.95		
HR 7664	190229	11	13100	3.4	0.444	1.9	61.7?	SB1
π^1 Boo	129174	16	12900	3.8	0.450	2.1		
μ Lep	33904	20	12700	3.7	0.456	2.05		
HR 89	1909	13	12300	3.8	0.469	2.1	<25	SB1
HR 7245	178065	5	12300	3.4	0.469	1.85	6.87	SB1
υ Her	144206	11	12000	3.7	0.480	2.0		
HR 7143	175640	<5	12000	3.8	0.480	2.05		
53 Tau	27295	$\leqslant 3$	11900	4.0	0.483	2.2	4.45	SB1
ϕ Her	145389	12	11700	3.8	0.490	2.05	560	SB1
46 Dra pr	173524	$\leqslant 5$	11700	3.7:	0 490	2.0:	9.81	SB2
46 Dra sec	173524	$\leqslant 5$	11400	3.9:	0.501	2.1:	9.81	SB2
HR 1800	35548	5	11100	3.7	0.513	2.0		
χ Lup pr	141556	$\lesssim 2$	11100	3.8	0.513	2.05	15.26	SB2
28 Her	149121	5	11000	3.75	0.517	2.0		
HR 4072 pr	89822	$\lesssim 2$	10900	3.7	0.521	2.0	11.58	SB2
HR 7775	193452	$\leqslant 2$	10900	4.0	0.521	2.15		

2.2 ATMOSPHERIC PARAMETERS

Effective temperatures $T_{\rm eff}$ and logarithmic surface gravities $\log g$ for the Hg–Mn stars were derived from the grid of line-blanketed models of solar composition calculated by Kurucz (1979). The dereddened indices c_0 and m_0 given by *uvby* photometry (Philip, Miller & Relyea 1976; Hauck & Mermilliod 1980) were used in conjunction with the $c_0 - m_0$ diagram for the Kurucz models (Relyea & Kurucz 1978) to derive values of $T_{\rm eff}$. The light ratios of the SB2s were used in estimating a value of $T_{\rm eff}$ for each of the components. The stars are arranged in Table 1 according to $T_{\rm eff}$. To derive values of $\log g$, equivalent widths of Hγ were deduced from Hβ photometry (Hauck & Mermilliod 1980) and compared with those calculated by Kurucz for his grid of models.

The absolute flux distribution of γ Gem and an interferometric measurement of its apparent angular diameter yield $T_{\rm eff} = 9260$ K (Code *et al.* 1976). A comparison of the profile of Hγ for γ Gem (Wright *et al.* 1964) with the profiles computed by Kurucz gives $\log g = 3.4$ (cgs units).

For the curve-of-growth analysis the temperature T and logarithmic electron pressures $\log P_{\rm e}$ at the Rosseland mean optical depth $\tau_{\rm Ross} = 0.2$ in the Kurucz models were adopted. The values of $\theta = 5040/T$ and $\log P_{\rm e}$ for the Hg–Mn stars are given in Table 1. For γ Gem

$\theta = 0.596$ and $\log P_e = 1.85$ (cgs units). The corresponding values of the mass absorption coefficient at 4000 Å were found from the table given by Allen (1973). The choice of the representative value of τ_{Ross} is somewhat arbitrary, and some tests will be carried out later to see how sensitive the resulting abundances are to changes in T and $\log P_e$. Where possible, lines which are insensitive to the exact choice of atmospheric parameters will be selected, and in other cases useful information may still be obtained on the relative abundances of elements from lines with a similar dependence on atmospheric parameters.

2.3 CURVE-OF-GROWTH ANALYSIS

The curve-of-growth analysis will be based on high-dispersion spectra covering the wavelength range $\lambda = 3900$ to 4650 Å. Equivalent widths W have been measured on several 2 Å mm^{-1} Lick spectra of κ Cnc (Aller 1970), 112 Her (Seligman & Aller 1970), π^1 Boo (Montgomery & Aller 1969), 53 Tau (Auer *et al.* 1966), ϕ Her (Aller, Ross & Zimmermann 1970), χ Lup and HR 4072 (Dworetsky 1971) and on a 6.7 Å mm^{-1} spectrum of HR 89 (Buscombe, Chambliss & Kennedy 1968). A large number of lines were measured on tracings of single 2.4 Å mm^{-1} spectra of HR 7361, HR 7664, μ Lep, HR 7245, HR 7143, 46 Dra, HR 1800 and 28 Her taken at the Dominion Astrophysical Observatory (DAO). A magnetic tape listing average intensities from four DAO spectra of HR 7775 was analysed. The tracings and the tape were kindly lent by Dr C. R. Cowley. Equivalent widths for κ Cnc, HR 8349, and υ Her were measured on microdensitometer tracings of 6 Å mm^{-1} spectra taken with the Edinburgh 0.9-m telescope, and Edinburgh spectra of π^1 Boo, μ Lep and ϕ Her were also studied. For the comparison star γ Gem, equivalent widths have been published by Wright *et al.* (1964) and by Sadakane & Nishimura (1979); weak lines of some elements, especially Mn, were also measured on a 3 Å mm^{-1} Edinburgh spectrum (2.3 mm wide). Most of the equivalent widths derived from the DAO and Edinburgh spectra are listed in Table 2. The equivalent widths for each component of the SB2s were corrected for dilution by the other component using the appropriate light ratios (Seligman & Aller 1970; Conti 1970; Dworetsky 1971).

Heacox (1979) found that his equivalent widths were in good agreement with the Lick measures for ι CrB, 53 Tau and ϕ Her, but systematically lower for κ Cnc. The equivalent widths for κ CnC derived from the Edinburgh spectra are 12 per cent lower than Aller's measures. Aller's measures were therefore reduced by 12 per cent and averaged with the equivalent widths from the Edinburgh spectra.

Because of the possibility of small systematic errors in the equivalent widths, no explicit determinations of microturbulent velocities ξ_t will be attempted. The Lick studies of individual Hg–Mn stars indicate that ξ_t is typically a few km s^{-1}. Curves-of-growth were therefore derived for a few elements using *gf*-values, and these were adjusted to provide curves-of-growth for other elements with comparable atomic weights assuming $\xi_t = 3$ km s^{-1}. For γ Gem, ξ_t was taken to be 2 km s^{-1} (Sadakane & Nishimura 1979).

Phillips (1979) showed that the semi-empirical $\log gf$ values calculated by Kurucz & Peytremann (1975) from the known energy levels of Fe II were generally accurate for lines allowed in LS coupling. The same probably holds for LS lines of other elements studied by Kurucz & Peytremann (KP). For Fe II lines with excitation potentials χ in the narrow range from 2.5 to 2.9 eV, the $\log gf$ values derived from laboratory measurements (Warner 1967), the A2 supergiant α Cyg (Groth 1961), and the Sun (Phillips 1979) were adjusted to the KP scale using the relationships given by Phillips. Curves-of-growth for the Hg–Mn stars and γ Gem were determined by fitting the empirical solar curve-of-growth (Cowley & Cowley 1964) to plots of $\log W/\lambda$ against $\log gf\lambda - \theta\chi$, and these curves-of-growth were adjusted

Table 2. Equivalent widths W (mÅ).

λ (Å)	χ (eV)	HR 7361	HR 8349	HR 7664	μ Lep	HR 7245	υ Her	HR 7143	46 Dra pr	46 Dra sec	HR 1800	28 Her	HR 7775	γ Gem
Mg II														
4384.64	9.95	11	24	9	25	20	17	29	25	55	12	17	37	49
4390.58	9.96	31	22	11	41	31	31	41	46	70	25	34	57	71
P II														
4044.61	13.31	71		38		16		≤4						
4127.57	12.85	41		22		7								
4178.48	9.63	87	65			32								
4452.46	13.05	37		20										
4467.98	13.04	32	34	20		11								
4475.26	13.08	60	32	30	≤9	16		≤7						
4499.24	13.38	59	47	34		10		≤4						
4530.81	13.05	40	33	19										
4558.07	13.14	33	40	14										
4602.08	12.85	85	60		≤10	34		≤9						
Sc II														
4246.83	0.31		53	≤8	46		≤10	≤12	≤13		15	≤5	≤5	82
4314.08	0.62	13				9:								44:
4320.75	0.60		48			12:								41:
4374.46	0.62	12												46
Ti II														
4290.22	1.16			39	41	34	46	72	41	65	47	47	90	98
4294.10	1.08			45	32	35	55	58	44	62	31	39	73	
4300.05	1.18			48		55	86	83	54	88	45	58	98	111
4301.93	1.16			18		17	28	62	21	49	27	35	69	76
4314.98	1.16					32	40	54	32		31	42	83	
4367.66	2.59			18		13		35			17	18	54	46
4386.86	2.60			14		10	18	39	16	21	16		47	41
4394.06	1.22					10		29					46	43
4395.03	1.08	20	13	48	41	64	59	70	57	75	57	58	94	118
4395.85	1.24						18	28	15	18		12	41	34
4399.77	1.24	11		20		35	34	56	23	52	22	26	74	64
4411.08	3.09			21		13	18	34	13	29	12	17	49	39

Table 2 – *continued*

λ (Å)	χ (eV)	HR 7361	HR 8349	HR 7664	μ Lep	HR 7245	υ Her	HR 7143	46 Dra pr	sec	HR 1800	28 Her	HR 7775	γ Gem
4417.72	1.16						37	58	28	34	25	22	73	74
4418.34	1.24							27				15	35	33
4443.80	1.08	18	27	46	33	52	50	73	62	75	39	52	82	103
4450.49	1.08			17			38	43				23	61	64
4464.46	1.17					14	25	30			15		49	44
4468.49	1.13		11	38	36	64	52	76	57	91	38	55	85	96
4488.32	3.12					16		42	18	26	15	16	56	50
4501.27	1.12		48	38	33	58		83	37	70	34	43	86	108
4563.76	1.22		23	38	27	45		73	41	81		46	81	107
4571.97	1.57	26	9		45	57		88	57	86		55	96	118
V II														
4005.71	1.82					≤6		≤6	≤8		≤9	≤11	3	48
4023.39	1.80					≤5		≤5	≤8		≤5	≤9	8	46
Cr II														
4224.85	5.33				14		14	28			13	21	30	26:
4252.63	3.86					12		30			30	16	35	35
4269.29	3.85							26			17	16	25	26
4275.58	3.86	14				24	20	45	16		35	32	43	54
4284.21	3.85					22		36	24		31	21	37	44
4558.66	4.07	53	51	21	36	74		94	52	99		63		111
4565.77	4.04					27		30				29	33	
4592.07	4.07	21	35			31		51	21	52		30		75
4616.64	4.07				17	29		43	20	34		34		56
4618.82	4.07	25			24	41		61	42	47		45		80
4634.10	4.07	22		20		53		70	32	52		36		68
Mn II														
4200.28	6.18			13	19		34	41			16			
4206.37	5.40		151	23	74		70	120	60	≤16	34	38	13	8
4239.19	5.37	56		10	25	20		61	16		22	18		
4240.39	6.18	47	44	14	25			43						
4244.24	5.37	53			31	15	47	44	16		24			

4251.74	6.18	67	54	18	35	49	44	65	26		30	20	12	
4259.20	5.40	104	98	20	48	54	62	98	46		42	30	15	13
4282.47	5.52	87				36	42	78						
4283.77	5.37	59			47	35	40	50			27	14		
4292.24	5.38	93	92	15	82	44	65	73	34		44		10	11
4325.03	5.40	45	72		40	28		38						
4326.63	5.40	108	116	25	84	74	90	92	67	≤29	69	35	16	12:
4356.62	5.42	43	65	19	27	21	22	44	15				4	
4363.25	5.57	51	44		29	23	26	39	24		15			
4365.22	6.57	51	52		32	24	30	48			15			
4377.74	5.44	42				11:	17	37						
4379.61	5.44	48	55			24	32	39			16			
4403.50	6.57	36				8		26						
4441.99	5.47	32			25	13		22						
4478.64	6.64	73	53	12	43	40	34	56	42		10	20	10	
4500.55	5.99					10		27						
4503.20	5.99	27				19		32						
4518.96	6.64	56	74	16		32		51	32		17			
Fe II														
3935.94	5.54	37		65	22	36		15	49	54	33	48	50	60
3938.97	5.89	27		52	16	19		16	48	44	21	24	41	29
4286.31	7.71	11		27		7			21		14	12	17	
4357.57	6.09	23		34	13	16	15		32			19	22	18
4402.88	6.14	19		17		14			16		12	12	15	16:
4451.55	6.14	29		38	18:	26	28	23:	39	29	16	31	33	29
4455.26	6.22	23		28		14	12	20	28	39	17	23	27	21
4596.02	6.22			49	15	25		11	26	29		23	31	
4635.33	5.93	32		76	10	33		27	62	49		32	46	
Ni II														
4015.50	4.03	9:		14	≤5	≤5		≤5			≤7	≤5		30
4067.05	4.03	18		30	≤5	≤16		≤14			≤6	≤7	≤11	
4362.10	4.03	≤8		≤6		≤6		≤5			≤5	≤5	≤5	20
Ga II														
4251.15	14.10	41			28	14		32	28				16	
4254.04	14.10	30				10		18	20				8	
4255.75	14.10	77	≤10	≤5	68	12	20	45	31	≤13	≤5	≤7	24	

Table 2 – *continued*

λ (Å)	χ (eV)	HR 7361	HR 8349	HR 7664	μ Lep	HR 7245	υ Her	HR 7143	46 Dra pr	46 Dra sec	HR 1800	28 Her	HR 7775	γ Gem
Sr II														
4077.71	0.00					12		32						87
4215.52	0.00							28	39	91	71	95	116	68
4161.80	2.94							⩽5			30	40	46	
4305.45	3.04							⩽14	15	42	27	42	55	
Y II														
3930.66	0.41							12			26	36	32	
3950.36	0.10	14			24	10		44			45	61	69	
3982.60	0.13	18			25			36			63	79	53	
4199.27	0.10										25		16	
4204.69	0.00							11			25	44	30	
4235.73	0.13							19			33	47	40	
4309.62	0.18	11						42			59	59	59	12:
4358.73	0.10							16			36	44	37	
4374.95	0.41	34	58	⩽18		16					76			
4398.01	0.13	15			15			21			53	49	56	
4422.59	0.10	10						18			49	45	47	
Zr II														
3958.22	0.52										12			24
3991.13	0.75										17			
4149.20	0.80	⩽10		⩽12	⩽8	⩽6		⩽5			9	⩽14	8	
Pt II														
4023.81	3.60											19	20	
4034.17	4.00											18:	29	
4046.45	4.52	⩽6		⩽10	⩽10	⩽10		⩽5	37	50	28	40	65	
4061.66	3.63								27	57	17	26	50	
4148.30	4.08											9	17	
4288.40	4.70											10:	26	
4514.17	3.63	⩽8		⩽10	⩽10	⩽12		⩽5	15	44	9	37	62	

according to atomic weight for other elements. The shape of the solar curve-of-growth in the linear-to-flat transition closely matches LTE calculations, and none of the lines used in the analysis lies on the damping portion.

The above procedure provided satisfactory curves-of-growth in most cases. For HR 7143 and 53 Tau, however, there is a lack of strong Fe II lines to define the 'flat' part of the curve-of-growth. The log *gf* values for Ti II lines with χ in the range from 1.08 to 1.24 eV (Warner 1967) were therefore used to derive curves-of-growth for these stars. Curves-of-growth for Sr, Y and Zr, which are considerably heavier than Fe, were derived from accurate log *gf* values for Y II lines (Hannaford *et al.* 1982). Lines of Mn II and Ga II are subject to hyperfine splitting; curves-of-growth for Mn II were therefore constructed from the log *gf* values given by KP for LS lines and laboratory values (Warner 1967) transformed to the KP scale, and these curves-of-growth were also used for Ga II. The special problem of the Pt II lines which are broadened by isotopic splitting will be discussed later.

The differential curve-of-growth analysis was based on values of log $W_{\rm corr}/\lambda$, i.e. the values of log W/λ which the lines would have had if the curves-of-growth were linear. The value of log $W_{\rm corr}/\lambda$ for each line was derived from the value of log W/λ by means of the appropriate curve-of-growth.

The lines used in the abundance analysis are those listed in Table 2, together with several additional lines of Sc II and Zr II which are present in some of the other Hg–Mn stars. The lines were selected as being unblended in most of the stars and singly ionized. Low-excitation lines of Cr II, Mn II and Fe II ($\chi < 3.8$ eV) were excluded as they are more sensitive to the choice of atmospheric parameters. Greater weight was given in the analysis to lines on the linear part of the curve-of-growth to minimize the effect of any errors in the construction of the curves-of-growth.

The resulting logarithmic abundances for the Hg–Mn stars are presented in Table 3 on the scale log N(H) = 12.0. The abundances of Mg, Sc, Ti, V, Cr, Mn, Fe, Ni and Zr are based on a differential curve-of-growth comparison with γ Gem, assuming that the abundances of these elements in γ Gem are solar. The adopted solar abundances are quoted from the compilation by Ross & Aller (1976), except for the values log N(Y) = 2.24 and log N(Zr) = 2.56 which are based on newly determined log *gf* values (Hannaford *et al.* 1982; Biémont *et al.* 1981). The recent work on log *gf* values was used to study the relative abundances of Sr, Y and Zr in Hg–Mn stars. No lines of P, Ga or Pt were found in γ Gem; the curve-of-growth results for P and Ga were scaled to fit Heacox's results for $\xi_t = 3$ km s^{-1}, and the abundances of Pt are based on an estimated abundance [log N(Pt) = 6.0] for the primary components of χ Lup and HR 4072 (*cf.* Dworetsky 1971; Cowley 1977a).

2.4 NOTES ON INDIVIDUAL ELEMENTS

Supplementary details on the abundance analysis for individual elements are now given. The elements are arranged according to their atomic numbers Z.

2.4.1 Mg (Z = 12)

The analysis is based on the Mg II lines at 4384.64 and 4390.58 Å which are insensitive to moderate changes in the atmospheric parameters. The lines are not very strong in the Hg–Mn stars, and so the results do not depend heavily on the accuracy of the curves-of-growth, which were deduced from those for Fe II assuming that $\xi_t = 3$ km s^{-1}. In the case of γ Gem, an accurate curve-of-growth for Mg II may be derived from the curve-of-growth for Fe II, as ξ_t is known to be 2.0 ± 0.5 km s^{-1} (Sadakane & Nishimura 1979). Most of the Hg–Mn stars are somewhat deficient in Mg as compared with γ Gem and normal late B-type stars

Table 3. Abundances of elements in Hg–Mn stars [log N(H) = 12.0].

Star	T_{eff} (K)	Mg	P	Sc	Ti	V	Cr	Mn	Fe	Ni	Ga	Sr	Y	Zr	Pt
κ Cnc	13500	6.9	7.4		5.7		6.2	7.2	7.8		7.0		3.8:	≤3.9	
112 Her pr	13500	≤6.4	7.1	3.9:	5.8		5.7	6.5	8.1	≤5.3	6.4		≤3.8	≤3.8	
HR 7361	13500	6.9	7.4	4.4	5.4		6.0	7.4	7.7	5.7	6.6		4.4	≤3.9	≤4.9
HR 8349	13200	6.9	7.1	4.8	5.2:		6.2	7.4	7.5		≤5.7		4.7:		
HR 7664	13100	6.5	7.0	≤3.5	5.9		5.6	6.3	8.0	6.0	≤5.4		≤3.7	≤3.8	≤5.0
π^1 Boo	12900	7.2	≤6.1	4.7	5.4		6.2	7.2	7.1	≤5.5	7.0		5.4	≤3.5	
μ Lep	12700	7.3	≤6.0	4.5:	5.6		5.9	6.9	7.2	≤5.0	6.8		4.3	≤3.5	≤5.1
HR 89	12300	6.9	≤6.2		6.0		5.6	7.0	7.6		6.7		4.6:	5.0:	
HR 7245	12300	6.9	6.8	3.8	5.5	≤3.9	6.1	6.6	7.4	≤5.5	6.4		3.4	≤3.2	≤5.1
υ Her	12000	6.9		≤3.1	5.7		5.7	6.7	7.3		6.7				
HR 7143	12000	7.1	≤6.3	≤3.1	6.0	≤3.7	6.2	7.0	7.2	≤5.5	7.1	3.1:	4.6	≤3.0	≤4.8
53 Tau	11900	6.9		≤3.1	6.0		5.9	6.7	6.7	5.4	≤6.5	3.1:	3.4	4.0	
φ Her	11700	7.3		4.7	5.7		6.4	6.6	7.5	≤5.7	≤6.8	3.2:	5.0	4.3	≤5.1
46 Dra pr	11700	7.1		≤3.0	5.4	≤3.8	5.7	6.5	7.7		7.1				5.6
46 Dra sec	11400	7.6			5.5		6.0	≤5.9	7.8		≤6.8				6.1:
HR 1800	11100	6.7		2.8	5.1	≤3.4	5.8	6.4	7.4	≤5.2	≤6.5	4.5	4.8	3.1	5.6
χ Lup pr	11100	7.4		≤2.1	5.4	3.6	5.9	5.6	7.7	5.8	≤6.5	4.5	3.8	3.1	6.0
28 Her	11000	6.9		≤2.3	5.1	≤3.7	5.7	6.2	7.5	≤5.3	≤6.7	4.8	4.9	≤3.0	6.0
HR 4072 pr	10900	7.2		≤2.1	5.6	3.3	6.0	6.2	7.6	≤5.5	≤6.6	5.0	4.9	3.4	6.0
HR 7775	10900	7.5		≤2.2	5.7	3.4	6.1	5.8	7.8	≤5.6	7.4	5.1	4.7	2.6	6.3
Sun	5800	7.60	5.50	3.04	5.05	4.02	5.71	5.42	7.50	6.28	2.80	2.90	2.24	2.56	1.75

(e.g. ν Cap, Adelman 1973). The deficiencies are > 1.0 dex for the primary component of 112 Her and HR 7664.

2.4.2 P (Z = 15)

The P II lines which appear in some of the hotter Hg–Mn stars are sensitive to changes in temperature. Curves-of-growth were derived by adjustment of the curves-of-growth for Fe II. The differential curve-of-growth method was used to compare the abundances of P in several stars, and the results were scaled to fit the abundances found by Heacox from model atmospheres with $\xi_t = 3$ km s^{-1}. The five Hg–Mn stars with $T_{\rm eff} > 13\,000$ K have large excesses of P.

2.4.3 Sc (Z = 21)

Curves-of-growth for Sc II were derived from those for Fe II. Although the results are sensitive to changes in the adopted atmospheric parameters, there are clearly large star-to-star variations in the abundance of Sc among Hg–Mn stars.

2.4.4 Ti (Z = 22)

There are numerous lines of Ti II, Cr II, Mn II and Fe II in the spectra of Hg–Mn stars, and the analysis is based on selected lines in the wavelength range from 4200 to 4650 Å which is covered by the V portions of the DAO plates. The star-to-star variations in the abundance of Ti are much smaller than for Sc.

2.4.5 V (Z = 23)

Weak lines of V II have been found only in three of the coolest Hg–Mn stars (χ Lup, HR 4072 and HR 7775). It is very likely that V is deficient in at least some of the Hg–Mn stars.

2.4.6 Cr (Z = 24)

The 11 selected Cr II lines have values of χ in the range from 3.85 to 5.33 eV and are moderately sensitive to changes in the atmospheric parameters. The differential curve-of-growth comparisons with γ Gem indicate that the abundances of Cr in the Hg–Mn stars are normal or somewhat above normal.

2.4.7 Mn (Z = 25)

Since Mn II lines may have hyperfine structure, special curves-of-growth were constructed using log *gf* values for 30 Mn II lines with χ between 5.3 and 7.0 eV. The log *gf* values calculated by KP for LS lines were averaged with laboratory values (Warner 1967) transformed to the KP scale. The differential curve-of-growth comparisons of the Hg–Mn stars were based on Mn II lines with values of χ from 5.37 to 6.64 eV, and comparisons with γ Gem were made from four lines with $\chi = 5.4$ eV. These four lines are rather weak in γ Gem; their equivalent widths were measured on the well-widened Edinburgh spectrum and by Wright *et al.* (1964). The results, which are fairly insensitive to changes in atmospheric parameters, confirm that Mn is overabundant in the Hg–Mn stars, especially those with high effective temperatures.

2.4.8 Fe (Z = 26)

High-excitation Fe II lines (χ from 5.5 to 7.7 eV) were chosen for the differential curve-of-growth analysis, since they are insensitive to atmospheric parameters and are suitable for deriving fairly reliable Mn–Fe abundance ratios. (The measurements of high-excitation lines in HR 8349 and 89 were inadequate, and the abundances of Fe in these stars were deduced by comparison with other Hg–Mn stars having similar values of $T_{\rm eff}$ using Fe II lines with values of χ around 2.7 eV.) Both excesses and deficiencies of Fe were found among the Hg–Mn stars. The Mn–Fe abundance ratios are highest in HR 8349, π^1 Boo and 53 Tau where they are about unity.

2.4.9 Ni (Z = 28)

Lines of Ni II are weak or absent in the Hg–Mn stars but present in γ Gem. Values or upper limits for log N(Ni) were deduced from measurements of three lines which are not very sensitive to changes in atmospheric parameters. The strongest line at 4067.05 Å is blended with an Fe I line in γ Gem and some of the cooler Hg–Mn stars. The results indicate a general deficiency of Ni in Hg–Mn stars.

2.4.10 Ga (Z = 31)

The three Ga II lines used in the analysis have $\chi = 14.1$ eV and are sensitive to changes in temperature. Their identification is confirmed by the presence of very strong Ga II lines in the ultraviolet spectra of several Hg–Mn stars (Jacobs & Dworetsky 1981), other Ga II lines in the Lick spectra of κ Cnc and π^1 Boo, and the Ga I line at 4172.04 Å in HR 7775. Bidelman & Corliss (1962) noted various hyperfine structures in the three Ga II lines and approximate allowance for this line splitting was made by adopting the curves-of-growth for Mn II. The abundances of Ga in the Hg–Mn stars were compared by a differential curve-of-growth analysis and the results were scaled to fit the abundances derived by Heacox for $\xi_t = 3$ km s^{-1}. There are large excesses of Ga in at least half of the stars, and in the hotter stars the abundances of Ga and Mn are often comparable.

2.4.11 Sr (Z = 38)

Lines of Sr II are present in some of the Hg–Mn stars, but it is difficult to obtain reliable abundances of Sr as the ionization corrections are large, especially in the hotter stars. An attempt was made to determine Sr–Zr abundance ratios in some of the cooler stars by a curve-of-growth analysis. The curves-of-growth for Y II were generally adopted, and log *gf* values for Sr II and Zr II lines (Grevesse *et al.* 1981) were used. The weaker lines of Sr II at 4161.80 and 4305.45 Å were accorded greater weight than the resonance lines at 4077.71 and 4215.52 Å. The scale for log N(Sr) was approximately fixed by the logarithmic Sr–Zr abundance ratios and the values of log N(Zr) for six Hg–Mn stars. Values of log N(Sr) on this scale are given for eight of the cooler Hg–Mn stars.

2.4.12 Y (Z = 39)

Log *gf* values for 11 Y II lines with $\chi \leqslant 0.41$ eV were derived from the *f*-values given by Hannaford *et al.* (1982) and used to construct curves-of-growth. For some stars there were too few Y II lines to define the curves-of-growth, and adjusted curves-of-growth for Fe II and Ti II were used instead. Only one of the 11 Y II lines was measured by Auer *et al.*

(1966) on their Lick spectra of 53 Tau, and so the equivalent widths of eight of the Y II lines measured by Allen (1977) on a DAO spectrum of 53 Tau were adjusted to the Lick scale and used in the analysis. As the Y II lines are very weak or blended in the spectrum of γ Gem, the log *gf* values for Y II and those for Zr II (Grevesse *et al.* 1981) were used to determine logarithmic Y–Zr abundance ratios in seven of the Hg–Mn stars and hence to fix the scale for log N(Y). Values of log N(Y) on this scale are presented for most of the Hg–Mn stars.

2.4.13 Zr (Z = 40)

The curves-of-growth used for Y II were also adopted for Zr II. HR 89 and ϕ Her have the strongest lines of Zr II, and 14 lines with values of χ from 0.52 to 1.53 eV were selected for study. The differential curve-of-growth comparison of the Hg–Mn stars with γ Gem was based on the lines at 3958.22, 3998.97, 4211.88 and 4379.78 Å. A solar abundance [log N(Zr) = 2.56] was assumed for γ Gem (Biémont *et al.* 1981). Since Sr, Y and Zr are mainly in a doubly ionized state in the Hg–Mn stars, there may be large systematic errors in the derived abundances. However, the Sr II, Y II and Zr II lines have a similar dependence on atmospheric parameters, and the relative abundances of Sr, Y and Zr should be fairly reliable for the cooler stars. There are apparently large variations from star to star in the relative abundances of the three elements.

2.4.14 Pt (Z = 78)

Several Pt II lines with values of χ from 3.6 to 4.7 eV have been found in the spectra of the coolest Hg–Mn stars. Their exact wavelengths differ significantly from the laboratory wavelengths and vary from star to star. These shifts are probably due to variations in the isotopic composition of Pt. This is confirmed by studies of line profiles and the fact that the transitions involving two *s*-electrons have larger shifts than those involving one *s*-electron (Guthrie 1969; Dworetsky 1971; Dworetsky & Vaughan 1973), although no laboratory study of the isotope shifts appears to have been carried out yet. There are similar shifts and variations in profile for the Hg II line at 3984 Å (Cowley & Aikman 1975; White *et al.* 1976). Isotopic splitting would tend to desaturate the stronger lines, and this must be taken into account in a curve-of-growth analysis.

Isotope shifts for the Pt II and Hg II lines which involve two *s*-electrons are listed in Table 4 where the laboratory wavelengths λ_{lab} (Shenstone 1938; Cowley & Aikman 1975) are noted. The stellar wavelengths λ_{star} were taken from the papers by Dworetsky & Vaughan (1973) and White *et al.* (1976) or, in the case of 28 Her and HR 7775, derived from nearby lines of other elements using the DAO data. Since the wavelengths of the Hg II line are

Table 4. Isotope shifts (mA) for cool Hg–Mn stars.

Star		$\lambda_{star}-\lambda_{lab}$	
	Pt II 4046.45	Pt II 4288.40	Hg II 3983.93
46 Dra pr	67:		81 ± 6
46 Dra sec	67 ± 4		93 ± 6
HR 1800	82 ± 7		70 ± 4
χ Lup pr	85 ± 6	70	122 ± 5
28 Her	70:		130:
HR 4072 pr	46 ± 4	50	81 ± 2
HR 7775	50	40	100

3983.91, 3983.99 and 3984.07 Å for ^{200}Hg, ^{202}Hg and ^{204}Hg respectively, it is evident that nearly all the Hg in the primary component of χ Lup is in the form of the heaviest stable isotope ^{204}Hg. The Pt II lines in this star also have large isotope shifts and narrow profiles, suggesting a dominance of the heaviest isotope ^{198}Pt and little isotopic splitting. Since $\xi_t = 1.9$ km s^{-1} (Dworetsky 1971), the curve-of-growth for Pt II may be derived from the curves-of-growth for Fe II and Y II. The Pt II and Hg II lines in the primary component of HR 4072, however, have smaller isotope shifts and are subject to isotopic splitting (Dworetsky 1971). There is no significant difference between the abundances of Pt in HR 4072 pr and χ Lup pr according to the weak Pt II lines, and so a curve-of-growth for HR 4072 pr was constructed such that the stronger lines also yielded no difference in abundance. Curves-of-growth for the other cool stars were derived from those for χ Lup pr and HR 4072 pr according to whether the isotope shifts suggested a dominance of ^{198}Pt or a mixture of isotopes.

No log *gf* values have been measured for the Pt II lines, and an estimated abundance [log N(Pt) = 6.0] was assumed for χ Lup pr and HR 4072 pr (*cf.* Dworetsky 1971; Cowley 1977a). The abundances of Pt in the other cool Hg–Mn stars were derived by comparison with χ Lup pr or HR 4072 pr, giving greater weight to the weaker lines. The available tracings of spectra of the hotter stars were searched for Pt II lines, and upper limits for the abundance of Pt in six stars were derived. The Pt II lines are fairly insensitive to changes in the atmospheric parameters. Although the abundances of Pt in the seven cool stars are greatly enhanced, the variation from star to star is remarkably small. The cool stars have significantly higher abundances of Pt than the six hotter stars which have been studied.

2.4.15 Au (Z = 79)

Dworetsky (1971) suggested that the weak lines at 3804.01, 4016.10 and 4052.80 Å in the Lick spectra of χ Lup pr and HR 4072 pr may be due to Au II, and Cowley & Aikman (1979) reported the presence of at least 10 Au I and Au II lines in HR 7775. The equivalent widths W of the Au II lines at 4016.10 and 4052.80 Å in HR 7775 are 12 and 14 mÅ respectively, indicating that the abundance of Au is several orders of magnitude higher than in the Sun.

2.4.16 Hg (Z = 80)

A detailed analysis of the Hg II line at 3984 Å was not attempted because of the problem of isotopic splitting (see White *et al.* 1976). However, it was noted that the Hg II 3984 line in 28 Her is weak (W = 37 mÅ) as compared with other cool Hg–Mn stars with similar Pt II line strengths (46 Dra, HR 1800, χ Lup pr, HR 4072 pr and HR 7775). The Hg II and Pt II lines have small and similar dependences on temperature, and the Pt–Hg abundance ratio was found to be about eight times higher in 28 Her than in χ Lup pr. No lines of Os (Z = 76) were found in 28 Her or the other cool stars.

2.4.17 Bi (Z = 83)

Strong Bi II lines were recently discovered in the ultraviolet spectrum of HR 7775 by Jacobs & Dworetsky (1982). A fairly strong, broad line (W = 36 mÅ) occurs at 4259.41 Å in the DAO spectra of HR 7775 but not in the spectra of the other cool Hg–Mn stars. This line is probably due to the strong, broad Bi II line at 4259.4 Å (Crawford & McLay 1934). Lines of Pb (Z = 82) are very weak or absent in the DAO spectra of HR 7775.

3 Discussion

3.1 ACCURACY OF THE ABUNDANCES

Errors in the abundances could arise through errors in the equivalent widths W or in the course of the analysis. The values of W depend on the correct identification and measurement of the lines. On the tracings of the DAO and Edinburgh spectra, the lines were identified by the traditional method which uses the laboratory wavelengths and intensities of lines in multiplets. As the line density in the spectra of Hg–Mn stars is not high, misidentifications should be rare and unblended lines may be easily selected. The equivalent widths were measured by drawing the continuum on the tracings and integrating line profiles. Star-to-star comparisons indicate that the random errors are about ±6 mÅ for the values of W derived from single DAO spectra and about ±5 mÅ for the Lick measures based on several spectra. The effect of such random errors is probably small, as about 20 Fe II or Ti II lines were used to construct the curves-of-growth and several lines of each element to derive the abundances.

The values of W may also be subject to systematic errors due to errors in plate calibration and in locating the continuum. Mention has already been made of the case of κ Cnc where Aller's (1970) values of W had to be reduced by 12 per cent to give better agreement with other measures, and systematic errors may exist in the values of W for other stars. The effect of systematic errors on the derived abundances has been reduced by using lines on the linear part of the curve-of-growth where possible; for such lines the errors in the logarithmic abundances resulting from systematic errors in W should be $\lesssim 0.1$ dex. For stronger lines the values of $\log W_{\rm corr}/\lambda$ are more sensitive to systematic errors in W, but the effect on the derived abundances is to some extent cancelled as the lines used in constructing the curves-of-growth are subject to the same systematic errors as the lines of the element in question. The curves-of-growth for Fe II and Ti II were constructed using lines in the wavelength range from 4120 to 4650 Å, and most of the stronger lines used in the abundance analysis are also in this range.

Comparisons by the differential curve-of-growth method are most accurate when the strengths of the lines of the element in question are similar in the stars being compared. When there is a large variation in line strength from star to star, the sharp-lined stars with intermediate line strengths provide valuable linkages between the stars with the strongest lines of the element and those with the weakest lines. The curve-of-growth for a sharp-lined star is usually well defined over a wide range of $\log W/\lambda$, so that all the stars may have some suitable lines in common with those measured in the sharp-lined stars with intermediate line strengths.

A rough indication of the degree to which the derived abundances for each element are sensitive to changes in the atmospheric parameters was given in Section 2.4. The values of $T_{\rm eff}$ for the 20 Hg–Mn stars studied range from 10 900 to 13 500 K and the values of $\log P_{\rm e}$ at $\tau_{\rm Ross} = 0.2$ from 1.85 to 2.2 (cgs units). If $\log P_{\rm e}$ is taken as 2.0 and $T_{\rm eff}$ is allowed to vary over the whole range from 10 900 to 13 500 K, the ranges of $\log N$ which would be found for lines with typical excitation potentials χ and constant values of $\log W_{\rm corr}/\lambda$ are listed in Table 5. These ranges are all $\leqslant 1.2$ dex, and so the errors in $\log N$ due to the errors in $T_{\rm eff}$ (±400 K) are small (generally $\lesssim 0.2$ dex). The errors in the values of $\log P_{\rm e}$ at $\tau_{\rm Ross} = 0.2$ due to errors in $\log g$ are about ±0.1 dex, and the resulting errors in $\log N$ are again small ($\lesssim 0.1$ dex). Some of the authors who studied Lick spectra of Hg–Mn stars listed the changes α and β in their derived values of $\log N$ which would result from changes of 1000 K and 1.0 dex in their adopted values of $T_{\rm eff}$ and $\log g$ (Seligman & Aller 1970; Zimmermann, Aller & Ross 1970; Dworetsky 1971). These values of α and β confirm that the overall errors

Table 5. Ranges of $\log N$ ($\log W_{corr}/\lambda$ constant, $\log P_e = 2.0$, T_{eff} varying from 10 900 to 13 500 K).

Ion	Ionization potential (eV)	Chosen χ (eV)	Range of $\log N$
Mg II	15.0	10.0	0.1
P II	19.7	13.0	1.1
Sc II	12.8	0.3	1.2
Ti II	13.6	1.2	1.0
V II	14.6	1.8	0.9
Cr II	16.5	4.1	0.6
Mn II	15.6	6.1	0.2
Fe II	16.2	6.1	0.2
Ni II	18.2	4.0	0.1
Ga II	20.5	14.1	1.2
Sr II	11.0	3.0	0.9
Y II	12.2	0.2	1.2
Zr II	13.1	0.7	1.2
Pt II	18.6	4.1	0.1

in $\log N$ due to errors in T_{eff} and $\log g$ in the present study are quite small, and the comparisons of Hg–Mn stars with similar effective temperatures are fairly accurate.

A source of concern is the possibility of systematic errors in the values of $\log N$ due to the somewhat arbitrary choice of the representative value of τ_{Ross} and the fact that the effective temperature of γ Gem (9260 K) lies outside the range covered by the Hg–Mn stars. These errors would be most serious for the elements with the largest ranges of $\log N$ in Table 5 and for the stars which are much hotter than γ Gem. Normal stars with effective temperatures in the range covered by the Hg–Mn stars unfortunately do not have many spectrum lines in common with the Hg–Mn stars. The adoption of a representative value of τ_{Ross} does not allow for radial abundance gradients which might be produced by diffusion.

The overall effect of moderate changes in the representative value of τ_{Ross} is fairly small. Thus if τ_{Ross} were, say, reduced from 0.2 to 0.1, the derived logarithmic abundance of the sensitive element Sc in the hot Hg–Mn star HR 7361 would be formally reduced by 0.2 dex. Nevertheless, it is worth noting that the singly ionized lines of Sc and other elements with low second ionization potentials (Table 5) are formed high in the atmospheres of Hg–Mn stars where LTE may not strictly hold. Further information on the abundances of such elements may eventually be obtained by analysing doubly ionized lines in high-resolution ultraviolet spectra.

The present values of $\log N$ may be compared with those derived by Heacox (1979) for the eight Hg–Mn stars in common. The agreement with Heacox's values for $\xi_t = 3$ km s^{-1} is generally good for Mg and the iron-peak elements Sc, Ti, Cr, Fe and Ni, the absolute differences in $\log N$ being nearly all $\leqslant 0.4$ dex. However, the present values of $\log N$ for Mn are on average 1.0 dex lower than those derived by Heacox; this may be partly due to the neglect by Heacox of the probable hyperfine splitting of the Mn II lines. The present values of $\log N$ for Fe are in excellent agreement with those found by Allen (1977) for the six Hg–Mn stars in common; his values were based on a model-atmosphere analysis of both Fe I and Fe II lines. The 5.3 Å mm^{-1} spectra used by Kodaira & Takada (1978) for their differential curve-of-growth analysis had lower resolution than the DAO and Lick spectra, and some of their equivalent width measurements may be inaccurate due to line blending. For the eight Hg–Mn stars in common with the present study the agreement in the values of $\log N$ is fairly good for Ti, Cr, Mn and Fe but poor for other elements for which Kodaira & Takada measured only a few unblended lines.

Dworetsky (1971) reviewed published analyses of individual Hg–Mn stars and adjusted the values of $\log N$ to conform with the *gf*-value scales and normal abundances which he adopted for his own study of HR 4072 and χ Lup. He tabulated the logarithmic abundance anomalies $\Delta \log N = \log N_{\rm star} - \log N_{\rm normal}$ for eleven Hg–Mn stars, 10 of which are in the present sample. Most of the results compiled by Dworetsky were based on equivalent widths measured on high-resolution Lick spectra, and many of these equivalent widths have also been used for the present work. There is good agreement between the results of the present differential curve-of-growth analysis and Dworetsky's tabulation for individual elements, and the overall rms difference between the values of $\Delta \log N$ for Sc, Ti, V, Cr, Mn, Fe, Ni and Zr is 0.3 dex. The present study provides stronger evidence for a general deficiency of Mg in Hg–Mn stars.

Although the results of the present and previous studies of Hg–Mn stars are generally in satisfactory agreement, they should be treated with caution as significant radial abundance gradients may exist.

3.2 ANALYSIS OF THE ABUNDANCES

One of the difficulties in interpreting the abundances of elements in Hg–Mn stars is that the principal mechanism responsible for the anomalies has not been definitely identified. The diffusion hypothesis has many attractive features (Michaud 1970, 1980), but the separation of elements by diffusion under the influence of gravitation and radiation pressure could occur only if the outer layers of the stars are fairly quiescent. Although meridion circulation may be slight in the Hg–Mn stars because of their slow rotation, there are indications that at least some Hg–Mn stars have disturbed atmospheres. Some Hg–Mn stars rotate non-synchronously as components of SB2s with orbital periods of only a few days (Guthrie 1981), and the mutual irradiation of the components would induce surface streaming which might hinder any separation of elements by diffusion (Guthrie & Napier 1980; Tassoul & Tassoul 1982). The Hg–Mn star α And may have an unstable atmosphere and a circumstellar envelope, since N V, Si IV and S IV lines have been found in its ultraviolet spectrum and there are blueshifted components of some Si II lines and variations on time-scales of a few hours (Aydin & Hack 1978; Rakos, Jenkner & Wood 1981; Derman 1982). Two Hg–Mn stars, β Scl and the SB2 46 Dra, are candidate identifications for X-ray sources; the atmospheres of these stars may be unstable if the X-ray emission is due to chromospheric or coronal activity rather than to companion or unrelated stars (Cash & Snow 1982).

There are also some problems with the accretion hypothesis. Selective magnetic accretion of ionized gas or charged grains from the interstellar medium has been proposed for magnetic Ap stars (Havnes & Conti 1971; Havnes 1979), but this idea does not seem to be immediately applicable to the Hg–Mn stars for which no surface magnetic fields have been detected. The lack of large magnetospheres around Hg–Mn stars may also limit accretion of heavy elements from supernova explosions, although accretion of ejecta from former companion stars in binary systems or clusters might have occurred. Cowley (1977b) discussed the interesting idea that the infall of planetesimals might contaminate the atmospheres of early-type stars, but only small objects or fragments would be substantially vaporized in the photospheres.

The great variety of abundance anomalies found among Hg–Mn stars suggests that more than one mechanism may be responsible, and it may well be that both diffusion and accretion processes are involved. The abundances presented in Table 3 will now be discussed in an attempt to elucidate the situation.

An examination of Table 3 shows that both excesses and deficiencies of elements occur among the Hg–Mn stars. There are no large excesses of the elements which already have high abundances in normal stars (e.g. Mg, Cr, Fe and Ni), but large excesses are found for some elements with lower abundances in normal stars (e.g. P, Sc, Mn, Ga, Sr, Y, Zr, Pt and Hg). The highest value of $\log N$ in Table 3 is 8.1, and this is due to a modest excess of Fe in 112 Her pr. On the diffusion hypothesis there is, for each element, a maximum abundance which might result from radiation pressure acting through atomic transitions. This upper limit depends on the degree of atmospheric turbulence and the value of T_{eff} for the star as well as on the atomic weight of the element and details of its atomic structure. In the absence of horizontal magnetic fields, some elements would be expelled from the star by radiation pressure; an excess accumulation of an element in the atmosphere would occur only if the radiative acceleration decreases steeply above the line-forming region. Calculations of the expected abundances are complicated and have been carried out for only a few elements (Michaud 1981). The values of $\log N(\text{Mn})$ in Table 3 may be plotted against T_{eff}, and they have an upper envelope with a slope similar to that predicted by Alecian & Michaud (1981) for models with $\log g = 4$. However, the observed upper envelope lies about 1.2 dex below the predicted envelope, and this may indicate that there is some turbulence or instability in the atmospheres of Hg–Mn stars.

Slight deficiencies of elements may possibly reflect variations in the composition of the interstellar material out of which the stars formed. Thus, for example, the rms scatter in the logarithmic abundances of Fe in normal stars is about 0.3 dex (Cowley *et al.* 1982). However, it would be difficult to account for the large deficiency of Fe in 53 Tau (about 0.8 dex), the large deficiencies of Sc and V in cool Hg–Mn stars, and the general deficiency of Mg and Ni in Hg–Mn stars in this way. Such deficiencies cannot be attributed to a simple accretion process, unless the stellar atmospheres were essentially replaced by material which was deficient in the appropriate elements relative to hydrogen. The existence of deficiencies does not necessarily imply, however, that there has been no contamination of the atmospheres of Hg–Mn stars by accretion. According to atomic weight and structure, some of the accreted elements might not be supported by radiation pressure and sink below the photosphere, while radiation pressure might prevent other elements from reaching the photosphere.

Fairly reliable abundances of Mg, Ti, Cr, Mn and Fe are given in Table 3 for all the 19 selected Hg–Mn stars (excluding the secondary component of 46 Dra). To search for correlations between the abundances of these elements and trends with effective temperature, a matrix of 15 correlation coefficients r was constructed from the values of $\log N$ and T_{eff} (*cf.* Cowley 1981). With one exception the absolute values of r are $\leqslant 0.51$ and not significant; the exception is the high value $r = +0.73$ found from the values of $\log N(\text{Mn})$ and T_{eff}. This apparent dependence of the abundance of Mn on effective temperature may be partly due to the fact that the stars were selected as having strong spectroscopic anomalies. Thus hot stars with mild enhancements of Mn would be excluded unless other spectroscopic anomalies were present. Nevertheless, the absence of cool stars with large excesses of Mn is worth noting, since it is consistent with the diffusion hypothesis (Alecian & Michaud 1981). The possible anticorrelation between $\log N(\text{Cr})$ and T_{eff} (Heacox 1979) is not confirmed by the present data ($r = +0.08$). The data for the other elements are less extensive and possibly subject to selection effects and systematic errors. Examples of other likely trends are the correlations of $\log N(\text{P})$ and $\log N(\text{Sc})$ with T_{eff} and the anticorrelation between $\log N(\text{Pt})$ and T_{eff}.

Although trends with effective temperature are expected on the diffusion hypothesis, Hg–Mn stars with similar effective temperature often differ considerably in composition. The most reliable information on star-to-star variations in the relative abundances of elements may be obtained by using Table 5 to select elements with lines that are fairly insensitive to changes in atmospheric parameters (e.g. Mg, Mn, Fe, Ni and Pt) or groups of elements with lines having similar excitation and ionization potentials (e.g. P–Ga and Sc–Ti–Y–Zr). Examples of large star-to-star variations in relative abundances may be easily found in Table 3. Thus there are large ranges in the P–Ga, Mn–Fe and Y–Zr abundance ratios. The P–Ga and Mn–Fe ratios are respectively at least 300 times higher and 60 times lower in HR 7664 ($T_{\rm eff} = 13100$ K) than in π^1 Boo ($T_{\rm eff} = 12900$ K). The Y–Zr ratio in HR 7143 ($T_{\rm eff} = 12000$ K) is at least 160 times higher than in 53 Tau ($T_{\rm eff} = 11900$ K). It is clear that the composition of Hg–Mn stars does not depend only on effective temperature.

The Hg–Mn stars in the present sample were plotted on an $H\beta - c_0$ age diagram. As expected, they lie above the zero-age main sequence for normal, slowly rotating stars and cover the whole of the main-sequence band (*cf.* Wolff & Preston 1978). No dependence of the values of log N(Mn) or the Mn–Fe abundance ratios on age was detected.

The primaries of the cool SB2s 46 Dra ($P = 9.81$ days), χ Lup ($P = 15.26$ days), and HR 4072 ($P = 11.58$ days) may be compared with the single stars and long-period SB1s having similar effective temperatures (ϕ Her, HR 1800, 28 Her and HR 7775). As already mentioned, the abundances in Table 3 for the primaries of the SB2s are based on equivalent widths which have been corrected for dilution by the light from the secondaries. For the elements which have been adequately studied, there are no significant differences between the mean values of log N for the SB2 primaries and those for the long-period SB1 and single stars. This is true even for Pt which is enhanced by about 4 dex. The only obvious exceptions are that large excesses of Sc and Bi have been found in ϕ Her and HR 7775 respectively, but not in the SB2 primaries. The general similarity between the SB2 primaries and the single stars suggests that the radiation from secondaries in SB2s with $P \gtrsim 10$ days does not disturb the atmospheres of the primaries significantly, although it has to be pointed out that SB2s with milder anomalies would not be easily detected as Hg–Mn stars. The effect of secondaries on the atmospheres of primaries becomes significant for shorter periods; binary periods less than three days are conspicuously absent among Hg–Mn stars but quite common among normal, late B-type stars (Aikman 1976).

The present analysis of the SB2 46 Dra and the study by Conti (1970) both indicate that the abundance of Mn is significantly higher in the primary than in the secondary. The same may also hold for 74 Aqr (Wolff 1974) and HR 7694 (Dworetsky 1974); Y II and Pt II lines are visible in the secondary of HR 7694 but not in the primary. The components of 66 Eri are nearly identical in luminosity, but Cr II, Y II and Hg II lines are found in only one component (Young 1976). Such differences in composition would be difficult to explain on a simple accretion hypothesis.

If the diffusion mechanism is operating in the outer layers of Hg–Mn stars, the abundances of elements would depend on atomic properties and some correlation with the periodic table might be expected. Simple correlations between the abundances of elements in a group in the periodic table may not always occur, however, as the various elements in a group have different atomic weights and some of the lighter elements already have high abundances in normal stars and so might not be greatly enhanced in the Hg–Mn stars. Thus the apparently normal abundances of Ca are not inconsistent with the excesses of Sr in Hg–Mn stars on the diffusion hypothesis, since the 'normal' abundance of Ca is high and the radiation pressure is affected by line saturation (Borsenberger, Michaud & Praderie 1981). Nevertheless, the behaviour of elements in groups in the periodic table is potentially a severe test of the diffusion hypothesis. Elements in the same group often have lines with similar excitation and ionization potentials, and so the study of their relative abundances is easier. Four groups in the periodic table are represented by more than one element in Table 3 (Mg and Sr; Sc and Y; Ti and Zr; Ni and Pt); there are no obvious correlations between the elements in each group, but abundances are not available for all the stars. Despite the apparent excesses of Sc and Y in some Hg–Mn stars and the occurrence of strong rare-earth lines in some magnetic stars with similar effective temperatures (e.g. α^2 CVn), rare-earth lines are very weak or absent in Hg–Mn stars. In view of the large excesses of Hg, the spectra of nine Hg–Mn stars were searched for the strong Cd II at 4415.63 Å, but it was not found ($W \leqslant 10$ mÅ in HR 7361, HR 7664, HR 7143, 46 Dra pr, HR 1800 and 28 Her; $W \leqslant 5$ mÅ in μ Lep, HR 7245 and HR 7775).

The elements in both normal and Hg–Mn stars were originally formed by nuclear processes. Abundance patterns in normal stars may be related to the nuclear properties of elements; for example the iron-peak reflects nuclear stability, and the s-process peak at Sr–Y–Zr is due to low neutron-capture cross-sections. The anomalies in Hg–Mn stars have to be attributed to unusual nuclear processes or to subsequent modification of abundance patterns by non-nuclear processes (e.g. diffusion). Hg–Mn stars lie in the main-sequence band, but some of them may have accreted products of nuclear processes in former companion stars.

A characteristic of many nuclear processes is that the resulting abundances of even-Z elements are usually higher than those of their odd-Z neighbours which are less stable. This preference for even-Z elements is readily discerned in the abundances of elements in normal stars. Both violations and enhancements of the odd–even rule are found among Hg–Mn stars. Thus the abundance of P ($Z = 15$) is higher than that of S ($Z = 16$) in HR 7361 (Heacox 1979). The abundances of Mn ($Z = 25$) are as high as those of Fe ($Z = 26$) in HR 8349, π^1 Boo and 53 Tau, and the abundances of Y ($Z = 39$) are apparently greater than those of Sr ($Z = 38$) and Zr ($Z = 40$) in HR 7143, ϕ Her and HR 1800 (Table 3). It is also likely that the abundance of Ga ($Z = 31$) in some Hg–Mn stars is higher than the abundances of its even-Z neighbours. The relative abundances of Sc ($Z = 21$), Ti ($Z = 22$), V ($Z = 23$) and Cr ($Z = 24$) in the five coolest Hg–Mn stars show enhancements of the odd–even effect.

It may be difficult to account for all the violations and enhancements of the odd–even rule by nuclear processes alone. The relative abundances of Sr, Y and Zr are not well established, since these elements have low second ionization potentials and the Sr II, Y II and Zr II lines may not be formed in LTE. The possible dominance of Y over both Sr and Zr in HR 7143, ϕ Her and HR 1800 would be inconsistent with any abundance pattern predicted for the s- or the r-process of neutron capture, as well as being difficult to understand on the diffusion hypothesis (Allen 1977). However, if some Hg–Mn stars had former companion stars in binary systems or clusters, the products of nuclear processes in the companion stars may have been violently ejected and exposed to protons accelerated in shock waves. This interaction with energetic protons might result in an increase of P–S, Sc–Ti, Mn–Fe and Y–Sr abundance ratios, and the atmospheres of the Hg–Mn stars might become contaminated with material which was enriched in odd-Z elements (Guthrie 1971). Nevertheless, the violations of the odd–even rule in some Hg–Mn stars may be too severe to be accounted for in this way, and there would still be the problem of the enhancements of the odd–even effect from Sc to Cr in the cool Hg–Mn stars.

The isotopic composition of Pt and Hg in Hg–Mn stars seems to vary from star to star and is often heavier than the terrestrial composition in the cooler stars (Dworetsky & Vaughan 1973; White *et al.* 1976). Proposals that the isotopes of Pt and Hg have been separated according to mass have to face the difficulty that the differences in mass between the various isotopes are small in comparison with the atomic masses. The separation of Hg isotopes by diffusion, as proposed by Michaud, Reeves & Charland (1974), would require very stable atmospheres and may be ruled out for short-period SB2s. A predominance of the heavier isotopes of Hg has been observed in several SB2s (e.g. 41 Eri, 46 Dra, χ Lup and

HR 4072). Alternatively, one might seek an explanation in terms of nuclear processes. The Pt and Hg may have been formed in the cyclic part of the r-process and accreted by the Hg–Mn stars. If the neutron flux was removed gradually so that the final β-decays took place in a neutron-rich environment, the Pt and Hg produced would be dominated by the heavier isotopes (Guthrie 1972). One difficulty with this proposal is that it does not provide an obvious explanation for the probable dependence of the isotopic composition of Hg on effective temperature (White *et al.* 1976). Another problem is that the Pt and Hg in χ Lup pr seem to be concentrated in the heaviest stable isotopes, instead of being distributed as expected among several of the heavier isotopes of each element. Cowley & Aikman (1975) and White *et al.* (1976) presented some arguments against the possibility that the shifts in the prominent Hg II line at 3984 Å are due to blending with lines of other elements, and the consistent shifts of both Pt II, and Hg II lines suggest real, though unexplained, variations in isotopic composition. A laboratory study of isotopes shifts for Pt II lines would be very useful.

4 Concluding remarks

The foregoing discussion may suggest some key tests for the diffusion and accretion hypotheses for the origin of the abundance anomalies. The diffusion mechanism depends on the atmospheres of Hg–Mn stars being fairly quiescent and would be hindered by surface streaming or turbulence. A comparison of single stars with SB2s having periods less than 10 days may therefore prove valuable. Since meridional circulation would inhibit the formation of abundance anomalies by diffusion or accretion, a decrease of the anomalies with increasing rotation would be expected. Such a trend does appear on a plot of the combined indices of the strengths of Mn II and Hg II (Aikman 1976) against $v \sin i$ for Hg–Mn stars, excluding SBs with $P < 25$ days, in the range $12\,000 \leq T_{\rm eff} \leq 13\,500$ K. However, Aikman's indices were based on eye estimates of line strengths, and the trend should be checked by measuring equivalent widths for a complete sample of Hg–Mn stars.

Some more specific tests may be suggested for the diffusion hypothesis. Further study of the relative abundances of elements in each group in the periodic table would be useful. Since deficiencies of elements and trends with effective temperature are hard to explain by accretion, it would be worth checking whether all the observed deficiencies and temperature trends are consistent with diffusion. The large variations in the composition of stars with similar effective temperatures may pose difficulties for the diffusion hypothesis. Curve-of-growth results are useful for exploring the diffusion hypothesis, but detailed tests would have to take radial abundance gradients into account.

The behaviour of Pt deserves further investigation. The excesses of Pt in the seven coolest Hg–Mn stars are large and remarkably similar, despite variations in isotopic composition from star to star. If the excesses were solely due to accretion in various circumstances, the abundance of Pt would vary greatly from star to star. The high and fairly constant abundance of Pt in the cool stars may therefore represent the upper limit which can be supported by radiation pressure. When accurate *gf*-values of Pt II lines become available, the actual abundances of Pt could be determined and compared with those predicted by diffusion for atmospheric parameters which are consistent with the observed abundances of other elements.

Accretion is probably excluded as the sole cause of the anomalies in Hg–Mn stars by the observed deficiencies and temperature trends of some elements and by the constancy of the large excesses of Pt in the cool stars. However, an accretion mechanism may be necessary to explain the isotopic anomalies of Pt and Hg, especially in SB2s. As the astronomical context of possible accretion mechanisms is very uncertain, further assessment of the role of accretion may have to await the outcome of the tests of the diffusion hypothesis.

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