

## A COMPARISON OF DISTANCE SCALES FOR EARLY-TYPE GALAXIES

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### ABSTRACT

The distance scales of elliptical and lenticular galaxies derived from several distance indicators have been intercompared and reduced to the scale (1) previously derived from the velocity dispersion indicator via the revised Faber-Jackson relation. This scale is found to be in near perfect agreement with the scale (5) derived from the luminosity index and from the 21 cm line width indicator via the revised Tully-Fisher relation for groups including a mixed population of early-type and spiral galaxies. It is also in excellent agreement with the distance scale (2) derived by Michard from his first magnitude-diameter-color relation (except for its arbitrary zero point), but not with that (3) derived from his second version of the relation. The latter agrees closely (except for zero point) with the distance scale (4) derived by Sandage and Visvanathan from the color-magnitude relation. Both (3) and (4) have scale errors of 15%–20% per magnitude compared to the other scales. The standard errors of distance moduli derived from the various scales after linear transformation to the standard scale (1) defined by the velocity dispersion indicator are as follows:  $\sigma(1) = 0.49$ ,  $\sigma(2) = 0.44$ ,  $\sigma(3) = 0.52$ ,  $\sigma(4) = 0.65$ ,  $\sigma(5) = 0.38$  mag. A general catalog of weighted mean distance moduli of 424 early-type galaxies based on distance indicators (1)–(4) is appended.

*Subject headings:* cosmology — galaxies: structure

### I. INTRODUCTION

We have previously derived the distance moduli of 157 elliptical and lenticular galaxies from a revised version of the Faber-Jackson relation (1976) between the velocity dispersion and the absolute luminosity of early-type galaxies (de Vaucouleurs and Olson 1982). This scale was shown to be in excellent agreement with several others, based on independent indicators (except for zero point, since all rest on the same system of primary and secondary distance indicators) (de Vaucouleurs 1978a–d, 1982a, b). In the present paper we report on a comparison of several other distance scales based on different indicators applicable to early-type galaxies. This comparison will allow us to determine the transformation equations and the external mean errors of the various indicators, and to obtain distance moduli for a large sample of early-type galaxies on the same scale and zero point as the moduli of several hundred spirals previously derived from tertiary indicators (luminosity index, de Vaucouleurs 1979a, b; superassociations, Wray and de Vaucouleurs 1980; 21 cm line widths, Bottinelli *et al.* 1980, 1983, 1984; and inner ring diameters, Buta and de Vaucouleurs 1983a, b).

### II. THE DATA

The material available for comparison is listed in columns (3)–(7) of Table 1.

*System (1).*—Distance moduli  $\mu_0(\sigma_v)$  derived by us from several revised versions of the Faber-Jackson relation, including possibly significant second-order terms depending on

surface brightness and color. The adopted values are the weighted mean distance moduli listed in column (9) of Table 4 given by de Vaucouleurs and Olson (1982). When questionable values {galaxies with uncertain or doubtful type and those with  $m.e.[\mu_0(\sigma_v)] > 1.00$  mag} are rejected, a sample of 126 “best-observed” galaxies remains to define our standard system.

*System (2).*—Distance moduli  $\mu_0(Ma)$  derived by Michard (1979a) from his first version of the magnitude-diameter-color (or magnitude-surface brightness–color) relation for early-type galaxies. A sample of 212 well-observed “main-sequence” galaxies remains after rejecting those classified by Michard as being in sequence II or sequence III, those marked as having anomalous color or surface brightness, and those with uncertain or doubtful type. We are indebted to Dr. R. Michard for permission to use a manuscript list of his catalog with detailed comments on individual objects.

*System (3).*—Distance moduli  $\mu_0(Mb)$  derived by Michard (1979b, c) from his second version of the magnitude-diameter-color relation. Similar rejection rules were applied, leaving a sample of 213 well-observed galaxies. Note that the data are the same as in system (2); only the equations used to derive absolute magnitudes are different.

The distance moduli in systems (2) and (3) are differential (i.e., with arbitrary zero points). Only their scale factors are of interest; the absolute values rest on an arbitrary choice of zero point corresponding roughly to 30.5 for the distance modulus of the Virgo E cluster.

TABLE I  
DISTANCE MODULI OF 424 EARLY-TYPE GALAXIES

Object	$T$	$\mu_0(\sigma_r)$	$\mu_0(\mathrm{Ma})$	$\mu_0(\mathrm{Mh})$	$\mu_0(\mathrm{SV})$	$\mu_0(\mathrm{EP})$	$\mu_0^1(1)$	$\mu_0^1(2)$	$\mu_0^1(3)$	$\langle \mu_0^1 \rangle$	$\delta\mu_0$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
N 16 -3	31.73	-	-	34.24	(32.62)	31.73	-	-	32.37	-25	N 499 -3
N 68 -3	1.00	-	34.80	.81	.80	1.00	-	.65	.35	.46	-
N 71 -3	-	.46	.63	-	-	.44	.44	.44	.46	.46	N 507 -2
N 80 -3	33.57	34.37	35.43R	.63	.63	-	34.36	34.37	34.63	34.63	N 508 -5*
N 83 -5	33.34	34.54	34.80	.63	.63	(33.97)	33.57	33.92	34.60	.63	N 524 -1
N 127 -2	-	34.90	34.74	-	(33.30)	-	34.43	34.03	.96	.46	N 533 -5*
N 128 -2	-	32.73	33.14	34.93	(33.30)	-	32.44	.52	.44	.48	N 541 -3*
N 147 -5	-	.46	.63	.81	.50	-	.44	.52	.44	.46	N 543 -5*
N 148 -1*	-	-	.81	-	-	-	.52	.65	.57	.58	N 545 -3
N 160 -1	-	34.16	34.38	-	-	-	31.32	31.32	-	-	N 547 -5
N 163 -5	-	33.91	34.13	-	-	-	33.72	33.73	-	-	N 560 -3
N 185 -5	-	.46	.63	-	26.38	-	.44	.52	.44	.44	N 564 -5*
N 193 -3	-	33.52	33.55	-	(33.30)	-	.52	.65	.57	.58	N 584 -5
N 194 -5	-	33.75	34.39	-	(33.30)	-	.44	.52	.44	.44	N 596 -5
N 227 -5*	-	.46	.63	.81	-	-	.44	.52	.44	.44	N 679 -3*
N 274 -3	-	-	-	.81	-	-	.44	.52	.44	.44	N 720 -5
N 315 -3	33.55	-	-	-	-	-	33.48	33.53	.50	.50	N 736 -3
N 379 -2	*.55	-	-	-	.81	(33.54)	33.55	.52	.44	.44	N 741 -5*
N 404 -3	-	33.48	33.60	-	(33.54)	-	.50	.55	.55	.55	N 750 -5
N 467 -2	32.96P	33.78	34.33	-	(33.54)	-	.44	.52	.44	.44	N 751 -5
N 474 -2	31.32	.46	.63	-	33.13	31.83	.50	.52	.44	.44	N 777 -5
92											
N 384 -5	-	33.99	33.73	-	(33.54)	-	33.55	33.20	-.14	-.14	N 890 -3
N 385 -3	-	.46	.63	-	(33.54)	-	.44	.52	.44	.44	N 936 -1
N 404 -3	-	.46	.63	-	-	-	.50	.44	.44	.44	N 1023 -3
N 467 -2	32.96P	33.78	34.33	-	(33.77)	-	32.96	33.35	-.65	-.51	N 1052 -5
N 474 -2	31.32	.46	.63	-	33.13	31.83	.50	.52	.44	.44	N 1052 -5

TABLE 1—Continued

Object	$T$	$\mu_0(\sigma_r)$	$\mu_0(\text{Ma})$	$\mu_0(\text{Mb})$	$\mu_0(\text{SV})$	$\mu_0(\text{GP})$	$\mu_0^1(1)$	$\mu_0^1(2)$	$\mu_0^1(3)$	$\mu_0^1(4)$	$\langle \mu_0^1 \rangle$	$\delta\mu_0$	Object	$T$	$\mu_0(\sigma_r)$	$\mu_0(\text{Ma})$	$\mu_0(\text{Mb})$	$\mu_0(\text{SV})$	$\mu_0(\text{GP})$	$\mu_0^1(1)$	$\mu_0^1(2)$	$\mu_0^1(3)$	$\mu_0^1(4)$	$\langle \mu_0^1 \rangle$	$\delta\mu_0$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
N1172	-5*	30.57	—	—	33.66	30.78	30.57	—	—	32.17	.49	N1407	-5	31.50	30.81	30.46	33.40M	30.78	31.50	30.52	31.56	31.08	.30		
N1199	-5*	31.74	32.53	32.02	35.91	31.94	31.74	32.15	31.80	32.37	.58	N1411	-3	.48	.46	.63	.81	.21	.48	.44	.52	.65	.29		
N1201	-2	30.23	31.64	31.70	32.28M	30.78	30.23	31.30	31.54	30.65	.18	N1426	-5	31.48	32.27	31.62	31.29	30.78	31.48	31.90	31.47	30.25	31.36	.58	
N1209	-5*	32.56	32.20	31.64	33.81	31.94	32.56	31.83	31.49	32.10	.26	N1427	-5	.40	.46	.63	.81	.21	.40	.44	.52	.65	.27		
N1270	-5*	—	33.86	34.00	.63	—	33.65	33.43	33.42	—	.36	N1439	-5	31.65	—	—	31.93	30.78	31.65	—	—	30.77	31.31	.53	
N1272	-3	34.45	—	—	—	(33.65)	34.45	—	—	—	.80	N1440	-2	—	—	—	—	—	—	—	—	—	—		
N1273	-2	33.75	34.22	34.80	—	—	.50	.52	—	—	.34	N1453	-5	—	32.96	33.05	35.00	32.60	—	32.56	32.64	.33	.25	.65	
N1277	-1*	—	33.89	34.05	.63	—	(33.65)	—	.50	.55	.19	N1461	-2	—	.46	.63	.81	.25	—	.44	.52	.65	.37		
N1278	-5	32.94P	33.71	33.87	—	(33.65)	.50	.44	.52	—	.33	N1521	-5*	32.76	33.13	33.15	33.48	(33.06)	32.76	32.73	32.73	32.02	32.61	-.45	
N1316	-2	.64	.46	.63	—	31.22	—	.50	.64	.44	.47	N1527	-3	.43	.46	.63	.81	.50	.43	.44	.52	.52	.28		
N1326	-1	—	—	—	—	.81	—	—	—	.65	.19	N1533	-3	—	—	—	—	—	—	—	—	—	—		
N1332	-3	—	.46	.63	—	—	—	—	—	—	.34	N1543	-2	—	.46	.63	.81	—	—	.44	.52	.65	.37		
N1339	-5	—	.46	.63	—	—	.81	.21	—	.44	.52	N1546	-2*	—	.46	.63	.81	.32	—	.44	.52	.65	.37		
N1344	-5	—	31.21	30.67	31.26	—	.81	—	—	.37	.37	N1549	-5	—	—	.46	.63	—	.32	—	.44	.52	.65		
N1351	-3*	—	.46	.63	—	31.65	31.11	—	.44	.52	.65	N1553	-2	—	.36	.46	.63	.81	—	.44	.52	.65	.37		
N1374	-5	—	30.45	29.51R	—	—	30.15	29.74	—	—	.35	N1574	-3	—	31.54	31.29	32.39	—	—	31.20	31.14	31.18	—		
N1379	-5	—	.46	.63	—	—	.44	.52	—	—	.44	N1587	-5	—	.46	.63	.81	—	—	.44	.52	.65	.37		
N1380	-2	—	30.07	29.32R	31.40	31.11	—	—	.65	.65	.45	N1596	-2*	—	31.30	31.08	32.02	29.99	—	30.86	30.86	.30	.25	.86	
N1381	-2	—	.46	.63	—	31.22	31.11	—	.79	.58	.37	N1600	-5	33.07	32.47	32.45	32.51	29.99	—	30.97	31.03	.31	.24	.86	
N1387	-3	—	—	—	—	31.83	.81	—	—	.65	.65	N1638	-2*	—	30.19	.46	.63	.81	.32	.44	.52	.65	.37		
N1389	-5*	—	—	—	—	.81	.41	—	—	.65	.65	N1653	-5	31.72	—	—	32.69	—	—	—	—	—	.97		
N1395	-5	—	31.49	—	—	.81	.41	—	—	.46	.65	N1700	-5	.62	—	—	.35	.62	—	—	—	—	.97		
N1399	-5	—	.71	—	—	31.52	31.11	—	.43	.65	.68	N1726	-2*	.34	.46	.63	.81	.25	.34	.44	.52	.65	.25		
N1400	-3	32.10	31.67	31.43	31.12	30.78	32.10	31.33	31.31	32.54	.12	N2217	-1	31.57	31.26	31.20	33.25M	30.92	31.57	30.93	31.13	31.44	.38		
N1404	-5	—	.36	.46	.63	—	.81	.21	.36	.44	.52	N2300	-2	.48	.46	.63	.81	.25	.48	.44	.52	.65	.29		

TABLE 1—Continued

Object	$T$	$\mu_0(\sigma_t)$	$\mu_0(\text{Ma})$	$\mu_0(\text{Mb})$	$\mu_0(\text{SV})$	$\mu_0(\text{gp})$	$\mu_0^1(1)$	$\mu_0^1(2)$	$\mu_0^1(3)$	$\mu_0^1(4)$	$\langle \mu_0^1 \rangle$	$\delta\mu_0$	Object	$T$	$\mu_0(\sigma_t)$	$\mu_0(\text{Ma})$	$\mu_0(\text{Mb})$	$\mu_0(\text{SV})$	$\mu_0(\text{gp})$	$\mu_0^1(1)$	$\mu_0^1(2)$	$\mu_0^1(3)$	$\mu_0^1(4)$	$\langle \mu_0^1 \rangle$	$\delta\mu_0$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	
N2310	-2	-	-	-	-	30.57M	-	-	-	-	29.27	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
N2314	-5	33.58	32.81	32.54	.81	32.70	33.58	32.42	32.23	.65	33.05	.35	N2950	-2	31.32	-	-	-	-	32.52M	-	-	-	-	30.84	30.84
N2325	-5	.38	.46	.63	-	31.84M	-	.38	.44	.52	.29	.35	N2962	-1	.57	-	-	-	-	31.54M	31.55	.25	.57	-	.65	.65
N2434	-5	-	.46	.63	.81	-	-	31.37	31.24	30.29	31.00	-	N2974	-5	-	-	-	-	-	33.69M	-	-	-	-	31.79	-
N2549	-2	30.65	31.36	31.04	32.9M(30.65)	-	34.29	-	.51	.51	31.21	.65	N2983	-1	.46	.63	.81	.63	.46	32.28	31.87	33.45	.63	.65	.65	.65
N2563	-2*	-	.46	.63	.81	.71	.62	.44	.52	.65	.32	.28	N2986	-5	.46	.63	.81	.81	.46	31.87	31.62	33.42	.81	.65	.65	.65
N2629	-2	33.38	.46	.63	-	-	32.70	33.38	.44	.52	-	.38	N3032	-2	-	-	-	-	-	.46	.63	.81	.81	.44	.52	.52
N2646	-2	32.63	32.63	32.49	34.53	32.20	32.75	32.25	32.18	32.87	.59	.68	N3056	-1	-	-	-	-	-	.46	.63	.81	.81	.44	.52	.52
N2672	-5	33.24	33.27	33.45	34.96M(32.91)	-	33.24	32.86	32.97	33.22	33.10	.19	N3065	-2	.65	.65	.63	.63	.65	31.55	31.78	32.76	32.37	.65	.65	.65
N2685	-2	-	.46	.63	1.61	.50	.39	.44	.52	1.31	.29	N3078	-5	-	-	-	-	-	.65	.63	.63	.63	.65	.65	.65	
N2693	-5*	33.81	33.41	33.57	34.59	(33.53)	33.81	33.00	33.07	32.92	.21	N3091	-5*	-	-	-	-	-	.65	.62	.62	.62	.65	.65	.65	
N2732	-2	31.24	.66	-	-	32.75	32.37	31.24	-	31.43	1.03	N3115	-3	29.98	29.04	28.67	30.44	(28.50)	29.98	28.80	29.05	29.56	29.59	29.59	29.59	
N2749	-5	33.98	33.51	33.41	-	(32.91)	33.98	33.09	32.94	-	.64	N3136	-5*	.33	.46	.63	.63	.71	.33	.44	.52	.52	.52	.52	.52	
N2768	-5*	30.28	31.45	31.87	31.55	30.28	31.11	31.29	30.72	30.80	.75	N3156	-2*	.31	.05	-	-	-	.81	.81	.81	.81	.81	.81	.81	
N2781	-1	-	30.86	29.71R	.81	.25	-	.53	.44	.52	.65	-	N3158	-5*	.34	.05	34.03	34.59	35.36M(34.12)	34.05	33.59	33.91	33.15	33.83	33.83	33.83
N2783	-5	33.72	.56	.63	.63	-	-	.44	.52	-	.65	N3159	-5	-	-	.46	.63	.81	.32	.44	.52	.52	.52	.52	.52	
N2784	-2	-	30.42	30.36	33.96C	-	.56	.56	.56	.56	.56	N3163	-3	33.76	34.52	34.94	.46	.63	.50	.50	.50	.50	.50	.50	.50	
N2787	-1	30.96	.46	.63	1.61	-	34.66C(29.80)	30.96	.44	.52	1.31	.42	N3193	-5	30.83	32.04	31.68	.46	.63	.63	.63	.63	.63	.63	.63	.63
N2831	-5	-	34.73	34.55	-	(34.15)	.60	-	.60	.60	1.31	.55	N3203	-1	.39	.46	.63	.63	.81	.25	.39	.44	.52	.52	.52	.52
N2832	-4	33.36	33.89	34.63	-	(34.15)	33.36	33.46	33.94	-	.64	N3222	-2	.32	.96	-	-	-	.81	(33.69)	.32	.96	-	-	.73	
N2859	-1	30.47	31.03	30.13	33.08	31.40	30.47	30.71	30.25	31.70	.77	N3245	-2*	31.65	30.92	30.54	29.93	31.68	.46	.63	.63	.63	.65	.65	.65	
N2865	-5	.60	-	31.04	29.95R	30.91	.25	.60	.44	.52	.29	.31	N3250	-5	.54	.46	.63	.63	.81	.41	.41	.41	.41	.41	.41	.41
N2880	-3	30.61	.81	.46	.63	-	32.49	31.55	30.61	.44	.52	N3258	-5	.37	.57	.63	.63	.81	.46	.46	.46	.46	.46	.46	.46	
N2902	-2	-	-	-	33.81C	-	.81	.25	.81	.81	-	N3268	-5	-	-	.63	.63	.81	.25	.25	.25	.25	.25	.25	.25	
N2911	-2	-	-	-	1.61	-	-	-	-	-	-	N3271	-2	-	-	.46	.46	.63	.71	.71	.71	.71	.71	.71	.71	

TABLE 1—Continued

Object	$T$	$\mu_0(\sigma_r)$	$\mu_0(\text{Ma})$	$\mu_0(\text{Mb})$	$\mu_0(\text{Sv})$	$\mu_0(\text{gp})$	$\mu_0^1(1)$	$\mu_0^1(2)$	$\mu_0^1(3)$	$\mu_0^1(4)$	$\langle \mu_0^1 \rangle$	$\delta\mu_0$	$\mu_0^1(\sigma_r)$	$\mu_0(\text{Ma})$	$\mu_0(\text{Mb})$	$\mu_0(\text{Sv})$	$\mu_0(\text{gp})$	$\mu_0^1(1)$	$\mu_0^1(2)$	$\mu_0^1(3)$	$\mu_0^1(4)$	$\langle \mu_0^1 \rangle$	$\delta\mu_0$			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	
N3300	-2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
N3308	-3	-	-	.34	.07	.81	-	(32.75)	.50	-	.44	.33	.15	.33	.48	.30	.48	.30	.48	.30	.48	.32	.35	.33	.13	
N3309	-5	-	.46	.63	.46	.63	-	(34.21)	(32.75)	-	.44	.32	.95	.33	.07	.32	.61	.32	.88	.33	.83	.34	.47	.34	.10	
N3311	-2	-	.46	.63	.46	.63	-	(34.60R)	(32.75)	-	.44	.52	.65	.37	.27	.33	.91	.79	.54	.47	.31	.47	.32	.63	.30	.27
N3348	-5	.31	.81	.32	.30	.31	.87	33.76C	32.11	31.81	.50	.44	.52	.52	.52	.44	.52	.50	.44	.52	.50	.46	.52	.52	.37	
N3377	-5	.30	.50	.30	.04	.30	.40	29.92	30.50	30.61	.30	.18	.29	.53	.30	.28	.36	.30	.28	.36	.30	.28	.31	.05M	-	
N3379	-5	.29	.47	.46	.63	.46	.63	.30	.51	.30	.29	.31	.96	.29	.69	.30	.21	.30	.38	.30	.21	.30	.38	.31	.33	.02
N3384	-3	.29	.76	.30	.01	.29	.46	31.18M	29.92	29.76	.41	.44	.52	.52	.52	.44	.52	.44	.47	.52	.44	.46	.52	.52	.37	
N3412	-2	.29	.41	.46	.63	.46	.63	.30	.73	.29	.91	.30	.80M	29.32	.29	.06	.30	.42	.30	.07	.29	.45	.29	.74	.89	
N3414	-2	-.58	.46	.63	.46	.63	.46	30.66	-	-.58	.44	.52	.65	.31	.74	-	-.58	.44	.52	.65	.31	.74	.30	.89	.45	
N3415	-1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	.26		
N3489	-1	.29	.58	.29	.66	.28	.50	29.31	29.41	29.58	.52	.44	.52	.52	.52	.44	.52	.52	.47	.52	.44	.52	.52	.52	.53	
N3557	-5	.30	.75	.31	.60	.31	.52	.32	.56	.31	.88	.30	.75	.31	.26	.31	.39	.31	.28	.31	.25	.31	.25	.31	.29	
N3585	-5	-.75	.46	.63	.46	.63	.46	.40	.29	.60R	31.32	-	30.11	.29	.81	30.43	.30	.12	.30	.43	.30	.72	.31	.32	.30	
N3599	-2	-	.32	.35	.31	.57	-	-.46	.63	.63	-	-	31.98	.31	.43	.37	.37	.31	.75	.33	.27	.31	.27	.31	.27	
N3607	-2	.30	.98	.30	.68	.30	.34	32.09	31.28	30.98	.48	.44	.52	.52	.52	.48	.52	.52	.48	.52	.52	.48	.52	.52	.53	
N3608	-5	.31	.48	.31	.82	.31	.51	.32	.71	.31	.28	.31	.82	.31	.66	.31	.38	.31	.63	.35	.11	.1*	.10	.15	.04	
N3610	-5*	-	.43	.46	.63	.46	.63	-	30.64	-	.43	.44	.52	.52	.52	.43	.44	.52	.43	.44	.43	.44	.44	.44	-	
N3613	-5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	.46		
N3619	-1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	.46		
N3626	-1	-	30.77	29.83	30.18	31.28	-	30.46	30.00	29.35	.48	.44	.52	.52	.52	.48	.52	.52	.48	.52	.52	.52	.52	.52	-	
N3630	-2	-	.46	.63	.63	.81	.24	.44	.52	.65	.24	-	.33	.72	-	.32	.22	.32	.22	.32	.22	.30	.29	.30	.24	
N3640	-5	.30	.20	31.78	31.78	32.03	(31.04)	30.20	31.43	31.60	.54	.44	.52	.52	.52	.44	.52	.52	.44	.52	.52	.44	.52	.52	.46	
N3655	-2	.31	.27	-.54	.46	.63	.81	-	32.34C	(32.29)	31.27	-	.54	.52	.52	.52	.52	.52	.52	.52	.52	.52	.52	.52	.52	
N3801	-2*	-	.33	.79	34.05	34.05	-	-	-.61	.71	1.31	-	.33	.36	.33	.46	-	.44	.52	-	.44	.52	.52	.52	.09	

TABLE 1—Continued

Object	$T$	$\mu_0(\sigma_i)$	$\mu_0(\text{Ma})$	$\mu_0(\text{Mb})$	$\mu_0(\text{Sv})$	$\mu_0(\text{gp})$	$\mu_0^1(1)$	$\mu_0^1(2)$	$\mu_0^1(3)$	$\mu_0^1(4)$	$\langle \mu_0^1 \rangle$	$\delta\mu_0$	Object	$T$	$\mu_0(\sigma_i)$	$\mu_0(\text{Ma})$	$\mu_0(\text{Mb})$	$\mu_0(\text{Sv})$	$\mu_0(\text{gp})$	$\mu_0^1(1)$	$\mu_0^1(2)$	$\mu_0^1(3)$	$\mu_0^1(4)$	$\langle \mu_0^1 \rangle$	$\delta\mu_0$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
N4220	-1	-	-	-	-	31.78	-	-	-	30.65	.65	-	N4435	-2	30.75P	-	-	32.65	30.49	30.75	-	-	31.35	31.00	.51
N4251	-2	-	-	-	-	31.26	30.98	-	-	.65	.65	-	N4442	-2	30.83	30.59	30.49	.81	.25	.56	.25	.56	.25	.65	.43
N4261	-5	32.05	32.08	32.21	-	31.75	32.05	31.72	31.95	.27	.65	.65	N4459	-1	30.25	30.93	30.49	.81	.25	.39	.44	.52	.25	.39	.57
N4262	-3	31.18	31.91	31.50	32.33M	30.49	31.18	31.56	31.37	.29	.44	.25	N4461	-1*	30.81	31.07	30.49	.81	.25	.42	.44	.52	.25	.42	.57
N4267	-3	30.61	.62	.46	.63	.81	.25	.62	.44	.52	.65	.32	N4472	-5	30.17	30.98	30.27	.59	.46	.63	.25	.59	.44	.52	.35
N4270	-2	-	-	-	-	32.59M	30.49	30.61	-	.81	.40	.25	N4473	-5	30.47	31.25	31.01	31.78	.25	.45	.44	.45	.25	.45	.28
N4278	-5	31.06	30.98	30.37	32.55	30.98	31.06	30.66	30.45	.27	.65	.23	N4476	-3	.36	.46	.63	.81	.25	.31	.29	.30	.25	.31	.26
N4281	-1*	32.29	.46	.63	.63	.81	.27	.29	.44	.52	.65	.23	N4477	-2*	30.54	31.64	32.25	30.49	.25	.46	.46	.46	.25	.46	.65
N4283	-5	29.48	.46	.63	.63	.81	.29	.48	.44	.52	.65	.23	N4478	-5	29.74	31.40	30.47	31.25	.25	.47	.44	.47	.25	.47	.50
N4291	-5	32.88	-	-	-	32.77	(31.72)	32.88	-	.81	.29	.25	N4486	-4	30.72P	30.42	30.66	31.54C	.25	.53	.44	.44	.25	.53	.30
N4339	-5	30.17	-	-	-	30.59	30.17	-	-	.81	.51	-	N4494	-5	29.98	30.90	30.53	30.57	.25	.32	.46	.63	.16	.25	.31
N4340	-1	29.24	-	-	-	31.40C	30.49	29.24	-	.80	.37	.25	N4503	-3*	29.19	.40	.46	.63	.25	.36	.44	.44	.52	.52	.27
N4350	-2	31.45	-	-	-	31.80	30.49	31.45	-	.81	.25	.25	N4526	-2	1.03	-	-	-	.25	.1.03	.25	.1.03	-	.25	.1.30
N4365	-5	30.77	31.18	31.14	32.44C	30.49	30.77	30.86	31.08	.25	.65	.43	N4528	-3*	.37	.46	.63	.63	.25	.25	.47	.44	.52	.28	
N4371	-1	30.23	.45	.63	.63	.81	.25	.45	.44	.52	.61	.31	N4550	-1	1.24	-	-	-	.25	.1.24	-	-	-	.25	.14
N4373	-2*	-	31.62	31.45R	-	-	31.28	31.33	-	.80	.25	.25	N4551	-5*	30.53	-	-	-	.25	.31	.29	.31	.25	.25	.31
N4374	-5	30.55	30.75	30.72	31.54	30.49	30.55	30.73	30.45	.20	.65	.23	N4552	-5	30.73	31.18	31.11	32.50	.25	.45	.45	.45	.25	.45	.04
N4377	-3	30.89	.71	.71	-	31.22	30.49	30.89	-	.81	.25	.25	N4564	-5	30.76	31.91	31.56	32.37	.25	.35	.44	.44	.52	.65	.38
N4379	-3	-	.71	-	-	30.83	30.49	30.81	-	.81	.25	.25	N4570	-2	-	-	-	.81	.25	.54	.44	.52	.54	.25	
N4382	-1	29.30P	30.15	30.14	29.15	30.49	29.30	29.87	30.26	.81	.25	.25	N4578	-2	30.51	-	-	-	.25	.1.24	.25	.1.24	-	.25	.1.70
N4386	-3*	31.57	-	-	-	33.70	(31.72)	31.57	-	.81	.25	.25	N4589	-5	31.64	-	-	-	.25	.44	.52	.52	.1.31	.42	.47
N4387	-5	31.08	-	-	-	.81	.71	.58	-	.65	.43	.25	N4596	-1	29.39	-	-	-	.25	.49	.49	.49	-	.49	.38
N4406	-5	30.15	30.74	30.72	31.08	30.49	30.15	30.43	30.73	.25	.65	.25	N4608	-2	30.60	-	-	-	.25	.1.61	.25	.1.61	-	.25	.1.31
N4417	-2	-	31.52	31.37	31.48	30.49	30.49	31.18	31.27	.25	.65	.25	N4621	-5	30.61	31.01	30.79	31.19	.25	.46	.44	.44	.52	.65	.11
N4429	-1	30.69	31.37	31.63	.81	.81	.25	.81	.52	.46	.44	.25	N4623	-1	.47	.63	.81	.81	.25	.47	.44	.44	.52	.65	.33

TABLE 1—Continued

Object (1)	$T$ (2)	$\mu_0(\sigma_i)$ (3)	$\mu_0(\text{Ma})$ (4)	$\mu_0(\text{Mb})$ (5)	$\mu_0(\text{SV})$ (6)	$\mu_0(\text{gp})$ (7)	$\mu_0^1(1)$ (8)	$\mu_0^1(2)$ (9)	$\mu_0^1(3)$ (10)	$\mu_0^1(4)$ (11)	$\langle \mu_0^1 \rangle$ (12)	$\delta\mu_0$ (13)	Object (1)	$T$ (2)	$\mu_0(\sigma_i)$ (3)	$\mu_0(\text{Ma})$ (4)	$\mu_0(\text{Mb})$ (5)	$\mu_0(\text{SV})$ (6)	$\mu_0(\text{gp})$ (7)	$\mu_0^1(1)$ (8)	$\mu_0^1(2)$ (9)	$\mu_0^1(3)$ (10)	$\mu_0^1(4)$ (11)	$\langle \mu_0^1 \rangle$ (12)	$\delta\mu_0$ (13)
N4636	-5	30.15	31.26	31.31	31.51C	30.59	30.15	30.93	31.22	30.43	.13	N4889	-4	34.38	33.70	34.21	34.85	34.15	34.38	33.27	33.59	33.13	33.74	-.41	
N4638	-3	.32	.46	.63	1.61	.20	.32	.44	.52	1.31	.25	N4895	-2	.45	.46	.63	.81	.35	.45	.44	.52	.65	.28	.41	
N4645	-5	-.85	-	-	30.94	30.49	29.88	-	29.97	29.93	-.56	N4895	-2	-	34.82	35.73	-	34.15	34.84	-	31.35	34.84	-	34.56	
N4649	-5	-.44	.46	.63	1.61	.25	.25	.81	.81	.71	.25	N4896	-3	-	.46	.63	-	.35	.44	.52	.52	.52	.44	.44	
N4660	-5	31.26	31.40	31.40	31.90	30.49	31.26	31.45	31.29	30.74	31.21	.72	N4906	-5	.36	.46	.63	.35	.35	.44	.44	.52	.52	.44	.29
N4696	-4	-	32.45	32.73	33.41M	32.42	-	32.07	32.38	31.57	32.00	-.42	N4915	-5	-	-	-	-	-	-	-	-	-	-	.67
N4697	-5	29.46P	30.32	29.77R	30.21	30.81	29.46	30.03	29.95	29.37	29.62	-.19	N4923	-3	-	34.34	34.02	.81	.34	.35	.35	.38	.38	.38	.44
N4742	-5*	-.35	.46	.63	.81	.21	.35	.44	.52	.65	.25	N4923	-3	-	.46	.63	.35	.35	.44	.52	.52	.52	.52	.44	
N4754	-3	30.77	30.69	30.19	31.61M	30.49	30.77	30.39	30.30	30.11	30.49	.00	N4936	-5	-	32.05	31.52	33.81	(32.46)	-	31.69	31.39	32.29	31.79	-.67
N4762	-2	29.83	30.83	30.57	30.44	30.49	29.83	30.52	30.61	29.56	30.08	-.41	N4957	-5	.54	.46	.63	.81	.50	.44	.52	.52	.52	.37	.37
N4767	-5	-	-	-	29.90	-	-	-	-	29.12	-	N4958	-2	-	-	-	-	.34	.35	.34	.35	.35	.37	.37	
N4782	-5	33.95P	33.07	33.09	.81	.71	(33.11)	33.95	32.67	32.68	.65	N4976	-5*	-	.46	.63	.81	.81	.44	.52	.52	.52	.52	.44	
N4783	-5	.48	.46	.63	-.50	.50	.48	.44	.52	.52	.32	N5011	-5	-	33.26	.15	-	33.50	-	-	-	-	-	.45	
N4786	-5	.48	.46	.63	-.50	(33.11)	33.07	33.00	33.02	-	33.04	-.07	N5017	-5*	-	-	-	-	-	-	-	-	-	-	.45
N4825	-3	-	-	-	33.06	-	.48	.44	.52	.52	-	N5018	-5*	-	-	-	-	-	-	-	-	-	-	.45	
N4839	-3	.66	.46	.63	.81	.25	.46	.44	.52	.65	.29	N5044	-5	-	32.83	33.01	33.40	-	-	32.44	32.61	31.96	32.34	-	
N4853	-3	33.67	33.11R	-	-	.81	.71	-	-	-	.81	N5061	-5	-	.46	.63	.81	.81	.44	.52	.52	.52	.52	.45	
N4860	-5*	-.46	.46	.63	-.50	.50	.48	.44	.52	.52	.32	N5077	-5	-	32.51	32.19	34.35	-	-	32.13	31.94	32.73	32.26	-	
N4865	-5	34.55	34.54	34.54	34.15	34.75	34.15	34.37	34.04	-	34.23	.08	N5084	-2	.44	.63	.81	.81	.44	.52	.52	.52	.52	.51	.51
N4869	-5	-	35.60	36.03	-	34.15	34.15	34.15	35.09	-	35.65	.44	N5090	-5	-	31.54	-	.81	.81	-	32.88	34.88	-	.51	
N4873	-2	-	35.43	35.72	-	34.15	32.21	-	-	32.21	-1.94	N5102	-3	-	28.72	27.41	26.80	27.98	-	28.49	28.02	26.61	27.77	-.21	
N4874	-4	33.66	34.51	35.22	-	.35	.66	-	-	.66	.25	N5140	-2*	-	.46	.63	.81	.81	.44	.52	.52	.52	.52	.37	
N4881	-3	.47	.46	.63	-.35	.35	.47	.44	.52	.52	.32	N5193	-3	.20	.46	.63	.81	.81	.44	.52	.52	.52	.52	.44	
N4883	-2	-	35.07	34.74	-	34.15	34.09	34.09	33.87	-	34.44	.52	N5266	-3	-	-	-	-	-	-	-	-	-	-	.44
N4886	-5	-	35.10	34.65	-	34.15	34.15	34.15	33.96	-	34.62	.52	N5302	-1*	.19	31.74	33.45	33.74	-	(33.20)	31.74	33.03	33.21	-	.41

TABLE 1—Continued

Object (1)	$T$ (2)	$\mu_0(\sigma_i)$ (3)	$\mu_0(\text{Ma})$ (4)	$\mu_0(\text{Mb})$ (5)	$\mu_0(\text{SV})$ (6)	$\mu_0(\text{gp})$ (7)	$\mu_0^1(1)$ (8)	$\mu_0^1(2)$ (9)	$\mu_0^1(3)$ (10)	$\mu_0^1(4)$ (11)	$\langle \mu_0^1 \rangle$ (12)	$\delta h_0$ (13)	Object (1)	$T$ (2)	$\mu_0(\sigma_i)$ (3)	$\mu_0(\text{Ma})$ (4)	$\mu_0(\text{Mb})$ (5)	$\mu_0(\text{SV})$ (6)	$\mu_0(\text{gp})$ (7)	$\mu_0^1(1)$ (8)	$\mu_0^1(2)$ (9)	$\mu_0^1(3)$ (10)	$\mu_0^1(4)$ (11)	$\langle \mu_0^1 \rangle$ (12)	$\delta h_0$ (13)		
N5322	-5	30.94	31.38	31.22	30.85	31.87	30.94	31.14	29.89	30.76	-1.11	-	N5898	-5	-	31.98	31.16	34.54M	-	-	31.62	31.09	32.48	31.74	-		
N5328	-5*	.71	.46	.63	.81	.29	.71	.44	.52	.65	.33	-	N5903	-5	-	.46	.63	.81	-	-	.44	.52	.65	.37	-		
N5338	-	-	-	-	33.36	-	-	-	-	31.93	-	-	N5922	-5	-	32.09	31.99	31.78R	32.16	32.20	32.09	31.63	31.60	30.95	31.72	-48	
N5353	-2	-	32.13	32.40	-	.81	-	-	-	31.77	32.11	-	N6020	-5	-	.44	.63	.81	-	-	.40	.44	.52	.65	-		
N5354	-2	-	.46	.63	-	.25	-	.44	.52	-	32.22	32.39	-	N6028	-1	-	.44	.63	-	-	-	.29	.37	.50	.65	-72	
N5360	-3	31.25	-	.46	.63	-	32.81	.25	.44	.52	-	31.25	-1.56	-	.44	.63	-	-	-	.70	-	.70	-	-			
N5362	-	.90	-	-	-	.27	.90	-	-	.90	-	-	N6041A	-3	-	.46	.63	-	-	-	.44	.52	.52	.44	-		
N5422	-2	-	-	-	-	32.97	31.87	-	-	31.61	-2.26	-	N6047	-3	-	.46	.63	-	-	-	.46	.52	.52	.44	-		
N5473	-3*	-	-	-	-	.81	.29	-	-	.65	.65	-	N6051	-5	-	.46	.63	-	-	-	.46	.52	.52	.44	-		
N5485	-2	-	-	-	-	32.00	31.87	-	-	.82	.82	-1.05	N6055	-5	-	.46	.63	-	-	-	.46	.52	.52	.44	-		
N5493	-2	-	-	-	-	.81	.29	-	-	.65	.65	-	N6086	-5	-	.46	.63	-	-	-	.46	.52	.52	.44	-		
N5532	-2	33.39	33.25	33.53R	-	.81	(34.30)	33.39	32.84	33.04	.65	-	N6166	-4	-	.46	.63	-	-	-	.46	.52	.52	.44	-81		
N5557	-5	.46	.63	-	.90	.41	.44	.52	-	.30	.30	-	N6482	-5*	-	.46	.63	-	-	-	.46	.52	.52	.44	-		
N5576	-5	31.06	31.02	30.00R	30.18	30.95	31.06	30.70	30.14	29.35	30.38	-.57	N6587	-3	-	.46	.63	-	-	-	.46	.52	.52	.44	-		
N5631	-2	.58	.46	.63	.81	.21	.58	.44	.52	.65	.31	-	N6684	-2	-	.46	.63	-	-	-	.46	.52	.52	.44	-		
N5638	-5	31.40	32.30	31.91	-	30.46	-	31.40	31.93	31.71	31.18	30.68	N6702	-5*	-	.46	.63	-	-	-	.46	.52	.52	.44	-		
N5791	-5*	31.73	-	.46	.63	.21	34.54	32.08	31.73	-	.58	.44	.52	N6703	-3	-	.46	.63	-	-	-	.46	.52	.52	.44	-	
N5796	-5	33.50	-	-	-	35.54	31.84R	32.98	-	31.81	31.62	31.70	N6721	-5	-	.46	.63	-	-	-	.46	.52	.52	.44	-		
N5812	-5	.51	-	-	-	35.18	.81	.35	.51	-	.44	.52	N6758	-5	-	.46	.63	-	-	-	.46	.52	.52	.44	-		
N5813	-5	30.83	32.54	32.91	33.00	31.06	30.83	32.16	32.53	31.63	31.71	.65	N6776	-5	-	.46	.63	-	-	-	.46	.52	.52	.44	-		
N5820	-2	.55	.46	.63	.81	.29	-	.55	.44	.52	.65	.30	N6851	-5*	-	.46	.63	-	-	-	.46	.52	.52	.44	-		
N5831	-5	31.64	-	.46	.63	.21	33.23	31.08	31.64	-	.58	.44	.52	N6854	-5	-	.46	.63	-	-	-	.46	.52	.52	.44	-	
N5838	-3	-	.49	-	-	.81	.35	.69	-	.65	.65	.39	N6861	-3	-	.46	.63	.63	.71	.39	.44	.52	.52	.44	-		
N5839	-2	31.41	-	.46	.63	.81	.35	.51	-	.44	.52	.65	N6866	-5	-	.46	.63	.63	.71	-	.44	.52	.52	.44	-		
N5846	-5	30.71	32.02	-	32.32	33.90M	31.08	30.71	31.66	32.04	31.96	.65	N6875	-5*	-	.46	.63	-	-	-	.46	.52	.52	.44	-		
N5854	-1	-	31.48	30.50R	-	.63	.81	.25	.44	.52	.65	.35	N6893	-2	-	.46	.63	.63	.71	-	.46	.52	.52	.44	-		
N5866	-1	-	29.84	29.12	31.18	-	29.57	29.42	30.16	29.71	-	.44	.52	.65	N6893	-2	-	.46	.63	.63	.71	.81	.42	.42	.42	.42	.65

TABLE 1—Continued

Object	$T$	$\mu_0(\sigma_r)$	$\mu_0(\text{Ma})$	$\mu_0(\text{Mb})$	$\mu_0(\text{SV})$	$\mu_0(\text{gp})$	$\mu_0^1(1)$	$\mu_0^1(2)$	$\mu_0^1(3)$	$\mu_0^1(4)$	$\langle \mu_0^1 \rangle$	$\delta\mu_0$	Object	$T$	$\mu_0(\sigma_r)$	$\mu_0(\text{Ma})$	$\mu_0(\text{Mb})$	$\mu_0(\text{SV})$	$\mu_0(\text{gp})$	$\mu_0^1(1)$	$\mu_0^1(2)$	$\mu_0^1(3)$	$\mu_0^1(4)$	$\langle \mu_0^1 \rangle$	$\delta\mu_0$				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)				
N6909	-5	-	-	-	-	30.580 (32.18)	-	-	-	29.67	-2.51	-	-	N7240	-3*	-	34.37	34.25	-	-	-	33.92	33.63	-	33.80	-			
N6942	-3	-	-	-	-	1.61	.71	-	-	1.31	.31	-.46	.63	-	-	-	33.90	34.94	-	-	-	33.47	34.19	-.52	33.77	-			
N6944	-3*	-	-	-	-	33.68	33.88	-	(33.25)	-	.65	.65	-.46	.63	-	-	33.00	32.91	33.85	-	-	32.60	32.53	32.32	32.49	-			
N6958	-5	-	.46	.63	-	.50	-	.44	.52	-	-	33.26	33.32	.03	-	N7302	-3	-	.46	.63	.81	-	-	.44	.52	.65	.37	-	
N6963	-5*	-	-	-	-	32.19	-	-	-	-	-	30.98	-	-	-	N7317	-5	-	35.15	35.16	-	-	-	34.67	34.57	-	34.54	-	
N6964	-5	-	.46	.63	-	.50	-	.44	.52	-	-	34.15	33.24	-	-	N7318A	-5	-	.46	.63	-	-	-	34.44	.52	-	34.44	-	
N7007	-3*	-	.46	.63	-	.50	-	.44	.52	-	-	33.86	33.13	.30	-	N7332	-2	31.86P	30.09	28.85R	30.14	30.56	31.86	29.81	29.20	29.32	30.12	-.44	.31
N7014	-5	-	.46	.63	-	.50	-	.44	.52	-	-	31.54	.44	.60	.46	-	31.34	.60	.44	.52	.65	.52	.65	.31	31.34	-1.32			
N7029	-5*	-	.46	.63	-	.50	-	.44	.52	-	-	33.47	33.49	33.81	.19	N7383	-2	.76	-	.50	.76	-	-	-	35.29	.52	-	35.29	-
N7041	-3	-	.46	.63	-	.50	-	.44	.52	-	-	30.56	.37	.65	-	N7385	-5	-	34.18	34.35	37.25C	-	-	33.74	33.71	35.08	33.86	-	
N7049	-2	-	.46	.63	-	.50	-	.44	.52	.37	-	31.24	31.23	30.98	.31	N7386	-2	-	.46	.66	1.61	-	-	33.79	33.96	.13	33.87	-	
N7079	-2	-	.46	.63	-	.50	-	.44	.52	.37	-	31.00	31.17	32.92	.10	N7389	-2	-	.46	.63	.63	-	-	-	33.87	.52	-	33.87	-
N7097	-5	-	-	-	-	.81	-	-	-	-	-	30.52	.52	.65	-	N7413	-3*	-	34.60	34.17	.46	-	-	34.14	33.56	-	33.90	-	
N7135	-3	-	-	-	-	.81	-	-	-	-	-	31.12	.65	.65	-	N7454	-5	30.72	.63	.63	.63	-	-	.44	.52	.52	.44	-	
N7144	-5	-	.46	.63	-	.50	-	.44	.52	.65	-	31.40	.40	.65	-	N7457	-3	.61	-	31.27	30.89	.29	.61	-	30.94	30.87	29.57	30.49	-.07
N7145	-5	-	-	-	-	.81	-	-	-	-	-	31.40	.40	.65	-	N7503	-5*	-	.46	.45	.63	-	-	34.95	34.61	.65	34.81	-	
N7155	-2	-	-	-	-	.81	-	-	-	-	-	31.40	.40	.65	-	N7507	-5	30.67	31.42	31.20	33.27	31.34	30.57	31.09	31.13	31.85	31.04	-.30	
N7166	-3	-	.46	.63	-	.50	-	.44	.52	.65	-	31.65	.55	.65	-	N7562	-5	.41	.46	.63	.63	.81	.41	.44	.52	.52	.27		
N7168	-5	-	.46	.63	-	.50	-	.44	.52	.65	-	31.65	.55	.65	-	N7619	-5	32.57	33.57	32.90	32.90	.81	.41	.44	.52	.52	.65		
N7173	-5*	-	.46	.63	-	.50	-	.44	.52	.65	-	31.65	.55	.65	-	N7623	-1	.39	.46	.63	.63	.81	.50	.44	.52	.52	.33		
N7176	-5	-	.46	.63	-	.50	-	.44	.52	.65	-	31.54	.54	.65	-	N7626	-5	32.56P	33.25	33.61	34.16	(33.11)	32.56	32.84	33.10	32.57	32.69	-.42	
N7192	-5	-	.46	.63	-	.50	-	.44	.52	.65	-	31.66	.55	.65	-	N7660	-5	32.84	.34	.63	.63	.81	.50	.44	.52	.52	.25		
N7196	-5	-	.46	.63	-	.50	-	.44	.52	.65	-	32.03	.53	.65	-	N7702	-1	.85	.86	.86	.86	.50	.85	.47	.47	.47	.85		
N7236	-3	-	.46	.63	-	.50	-	.44	.52	.65	-	31.77	.53	.65	-	N7720	-3*	35.36	34.31	35.00	35.00	(34.91)	.50	.44	.52	.52	.44		
N7237	-3	-	.46	.63	-	.50	-	.44	.52	.65	-	34.79	.57	.65	-	N7744	-3*	-	33.74	.74	.81	-	-	-	-	.34.58	-.33		

TABLE 1—Continued

Object	$T$	$\mu_0(q)$	$\mu_0(\text{Ma})$	$\mu_0(\text{Mb})$	$\mu_0(\text{SV})$	$\mu_0(\text{SP})$	$\mu_0^1(1)$	$\mu_0^1(2)$	$\mu_0^1(3)$	$\mu_0^1(4)$	$\langle \mu_0^1 \rangle$	$\delta\mu_0$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
N7795	-5	32.15	33.16	33.24	35.55	(33.47)	32.15	32.76	32.80	33.70	32.65	-.82
N7796	-5	.40	.46	.63	.81	.50	.40	.44	.52	.65	.27	-
I 708	-5	34.11	-	-	.81	-	-	-	-	32.52	32.52	-
11459	-5	.55	-	30.98	30.65	32.42	31.19	.55	-	.65	.65	-
12035	-2*	-	.46	.63	.81	.28	-	.44	.52	.65	.37	-
13370	-5	-	-	-	28.84C	-	-	-	-	28.26	28.26	-
13896	-5	-	.46	.63	.81	-	-	-	-	1.31	1.31	-
13998	-2	-	35.25	34.80	-	-	-	34.76	34.08	-	34.47	-
14012	-5	-	35.24	34.45	-	34.15	-	.44	.52	-	.44	.20
14021	-5	-	35.32	34.44	-	34.15	-	.44	.52	-	.44	.24
14026	-2	-	35.23	34.79	-	34.15	-	.44	.52	-	.44	.35
14042	-2	-	34.98	34.75	-	34.15	-	.44	.52	-	.44	.31
14051	-5	-	35.39	35.74	-	34.15	-	.44	.52	-	.44	.16
14296	-5	-	32.48	32.88	32.91	-	.35	.44	.52	-	.44	.73
14329	-3*	33.16	32.95	33.34	-	(33.20)	33.16	.44	.52	-	.44	-
14797	-5	.55	.46	.63	-	.50	.55	.44	.52	-	.34	.31
14842	-5*	-	.46	.63	.81	-	-	.44	.52	-	.44	.37
14889	-5	-	33.50	33.18	-	-	-	.33.08	32.75	-	32.94	-
15063	-3*	-	.46	.63	-	32.06	-	.44	.52	-	.44	-
15105	-5	-	.46	.63	.81	-	-	-	-	30.87	30.87	-
15181	-2	-	-	-	32.49	-	-	-	-	.65	.65	-
15267	-2	-	31.00	30.33	-	31.19	-	.30.68	30.41	-	.65	.65
15269	-2	-	.46	.63	-	.28	-	.44	.52	-	.44	.62
15328	-5	-	-	-	.81	-	-	-	-	30.24	30.24	-
										31.69	31.69	-
										.65	.65	

## NOTES

Cols. (1), (2).—Designation and morphological type  $T$ .Col. (3).—Modulus  $\mu_0(\sigma_v)$  from velocity dispersion and mean error (line 2).Col. (4).—Modulus  $\mu_0(\text{Ma})$  from Michard 1979a and mean error (line 2).Col. (5).—Modulus  $\mu_0(\text{Mb})$  from Michard 1979b, c and mean error (line 2).Col. (6).—Modulus  $\mu_0(\text{SV})$  from Sandage and Visvanathan 1978 and mean error (line 2).Col. (7).—Modulus  $\mu_0(\text{SP})$  from group membership (in parenthesis if from redshift only).Col. (8).— $\mu_0^1(\sigma_v)$  (standard system); same as in col. (3).Col. (9).— $\mu_0^1(\text{Ma})$  reduced to standard system with eq. (6a).Col. (10).— $\mu_0^1(\text{Mb})$  reduced to standard system with eq. (6b).Col. (11).— $\mu_0^1(\text{SV})$  reduced to standard system with eq. (6c).Col. (12).— $\langle \mu_0^1 \rangle$  = weighted mean of 8, 9, 10, 11.

Col. (13).—Residual 12 — 7.

## DISTANCE SCALES FOR EARLY-TYPE GALAXIES

101

There is one class of objects which is retained here but which was rejected by Michard, even if the color, surface brightness, and type of the galaxy were all well observed. The 29 galaxies with distance moduli marked "R" in column (5) of our Table 1 were rejected by Michard because their distance moduli showed large residuals relative to their Hubble (redshift) distance. Such galaxies are included here, since they may, eventually, be of use in mapping the local velocity field.

**System (4).**—Distance moduli  $\mu_0(\text{SV})$  derived by Visvanathan and Sandage (1977) and Sandage and Visvanathan (1978) from their version of the color-luminosity relation. The zero point rests on the assumptions of the "long" distance scale, including the postulate of zero absorption in galactic polar windows (for a discussion, see de Vaucouleurs 1982a, b). The adopted values are taken from columns (6) and (7) in Table 1 given by Visvanathan (1979). A sample of 197 well-observed galaxies remains after rejecting those marked by Visvanathan as having uncertain magnitude or uncertain color and those with uncertain or doubtful type.

**System (5).**—In addition, for early-type galaxies in groups, a fifth estimate of distance modulus,  $\mu_0(\text{gp})$ , is available. As described in de Vaucouleurs and Olson (1982), for a given elliptical or lenticular in a group, we compute  $\mu_0(\text{gp})$  as the weighted mean of all the available distance moduli  $\mu_0(\Lambda_c)$

from luminosity index and  $\mu_0(V_M)$  from 21 cm line width for spirals in the group, and of  $\mu_0(H^*)$ , computed from the mean group redshift and the Hubble ratio  $H^*$  if the group redshift is  $V_0 < 3000 \text{ km s}^{-1}$  (or from  $H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$  if  $V_0 > 3000 \text{ km s}^{-1}$ ).

These various systems of distance moduli are compared two-by-two in Figures 1 and 2.

## III. METHOD OF ANALYSIS

In order to reduce all of the distance moduli from sources (1)–(5) to a common scale, taken to be that of source (1), least-squares solutions of the form

$$\mu_0(1) - \langle \mu_0(1) \rangle = a(i)[\mu_0(i) - \langle \mu_0(i) \rangle] \quad (1)$$

must be made for the galaxies in common. The systems differ in scale [ $a(i) \neq 1$ ] and in zero point [ $\langle \mu_0(1) \rangle \neq \langle \mu_0(i) \rangle$ ]; indeed, the zero points of systems (2) and (3) were chosen arbitrarily. After zero-point shifts and scale corrections have been determined and used to remove systematic differences as far as possible, intercomparisons of the transformed data can be used to evaluate the mean accidental errors of each data set.

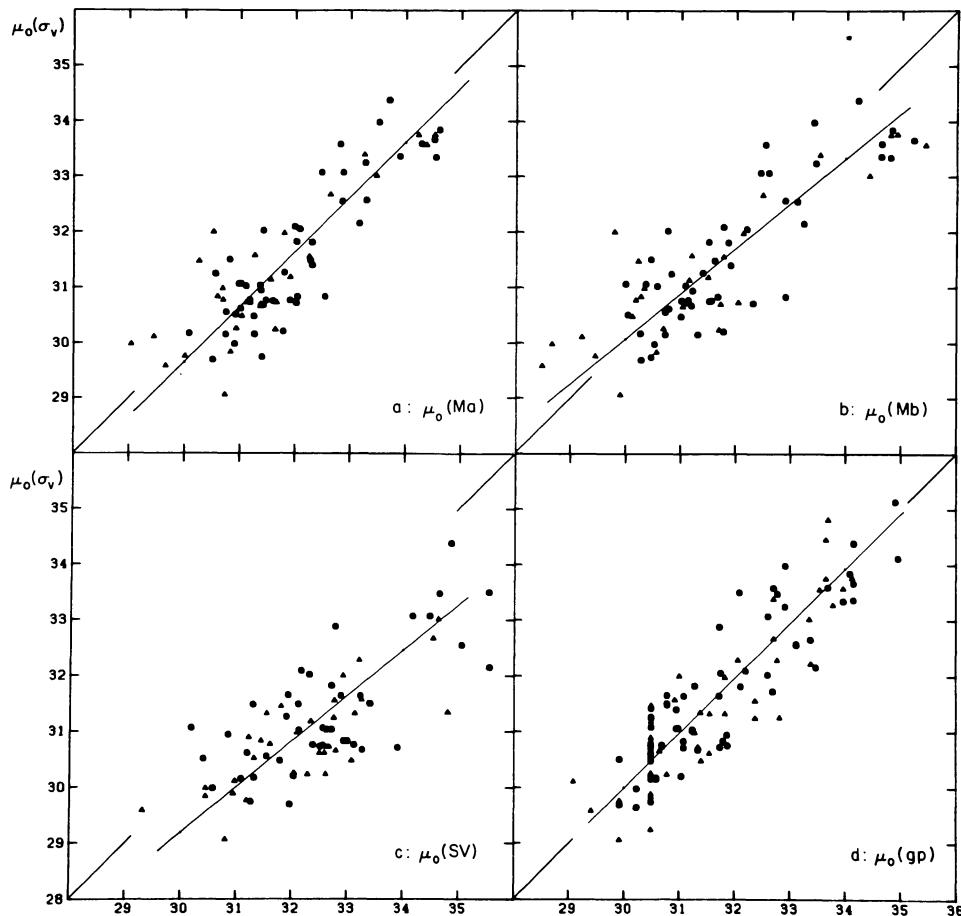


FIG. 1.—Comparison of distance moduli derived from velocity dispersion,  $\mu_0(\sigma_v)$ , and Michard-a (Ma), Michard-b (Mb), Sandage and Visvanathan (SV), and group (gp) scales. Note scale agreement with (a) and (d), disagreement with (b) and (c).

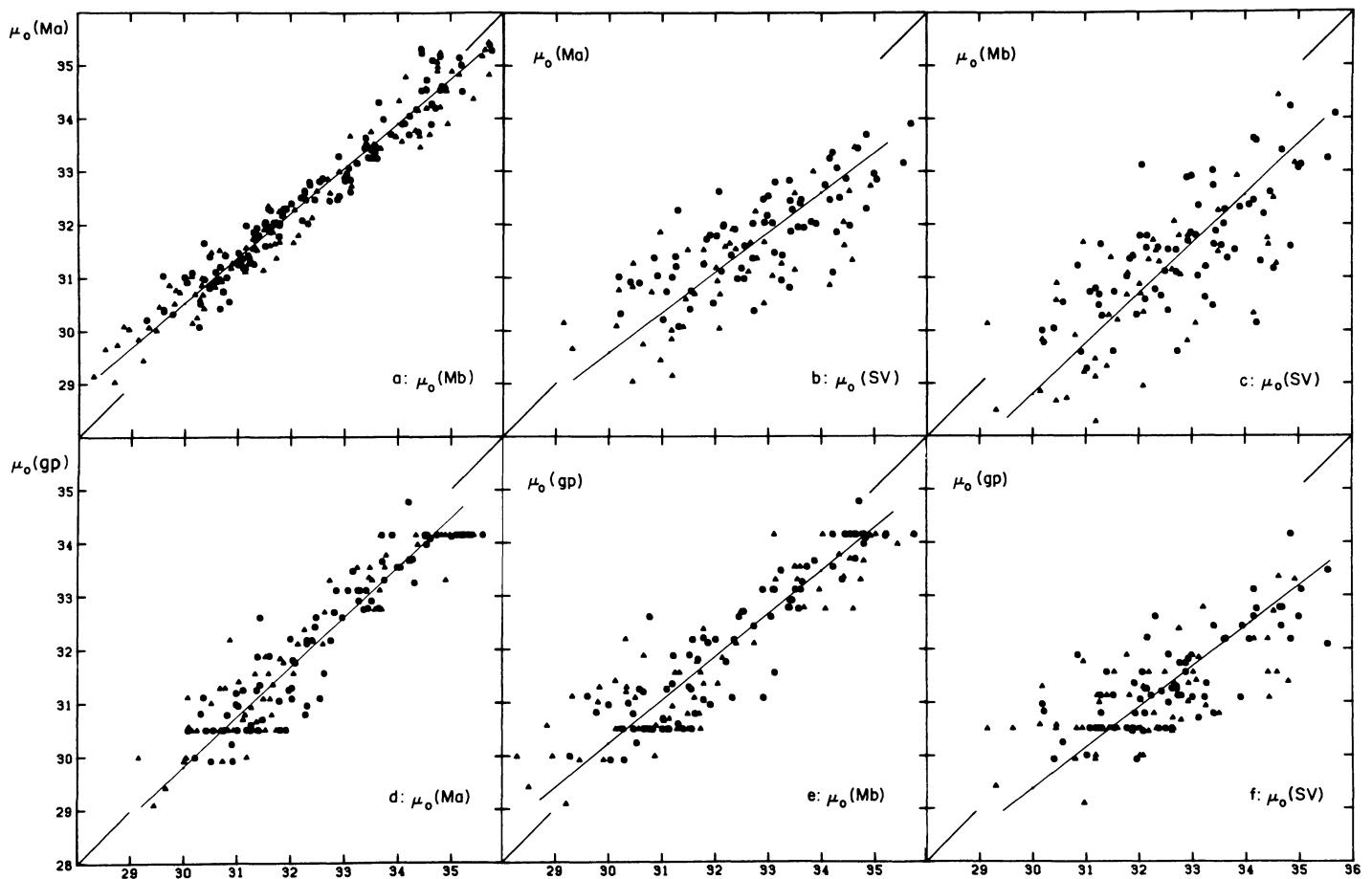


FIG. 2.—Comparison of distance moduli: (a) Ma vs. Mb; (b) Ma vs. SV; (c) Mb vs. SV; (d) groups vs. Ma; (e) groups vs. Mb; (f) groups vs. SV.

The least-squares solutions of equation (1) are obtained by a method developed by Jefferys (1980, 1981), which correctly takes into account errors in each variable. Therefore, in order to get an accurate regression line, it is necessary to know beforehand the mean random error in each variable. On the other hand, we must know the slopes and zero point shifts of the regression lines so that the rather large systematic differences between the distance scales can be taken out and so that the mean squared differences in distance moduli can be used to evaluate accurately the actual accidental errors in each scale.

We adopt an iterative approach: (i) we assume first that all the distance scales are equally accurate, with a starting guess of  $\sigma_i = 0.50$  mag for all  $i$ . (ii) We then compute the straight-line fits of equation (1), using these values for the  $\sigma_i$ . (iii) With the resulting slopes and zero points, we compute reduced moduli  $\mu^1(i)$  for each galaxy and compute the mean-squared differences  $\sigma_{ij}^2$ . (iv) From the 10 values for  $\sigma_{ij}$  (five sets of data taken two-by-two), we derive provisional values for the five individual  $\sigma_i$ . This overdetermined system is solved in two ways, a triplet method and an overall least-squares method, as described in the following section. With estimates for the five  $\sigma_i$ , we return to step (ii) and repeat until convergence is obtained.

In practice, only four iterations were required to give convergence to the values given in Table 2. These external error estimates are derived, therefore, entirely from the intercomparisons between distance moduli for galaxies in common between the scales.

#### IV. EXTERNAL ERRORS AND WEIGHTS

Consider several different independent sets of measurements  $\mu_0^1(i)$  reduced to the same system (i.e., no systematic differences). If the accidental errors are uncorrelated, then the covariances can be expressed as

$$\sigma_{ij}^2 = \sum [\mu_0(i) - \mu_0(j)]^2 / (N_{ij} - 2) = \sigma_i^2 + \sigma_j^2, \quad (2)$$

where  $N_{ij}$  is the number of objects in common between data sets  $i$  and  $j$ , and where  $\sigma_i^2$  and  $\sigma_j^2$  are the unknown individual variances for each data set.

If only three data sets ( $i, j, k$ ) are compared, then the system of equations

$$\sigma_{ij}^2 = \sigma_i^2 + \sigma_j^2, \quad \sigma_{jk}^2 = \sigma_j^2 + \sigma_k^2, \quad \sigma_{ki}^2 = \sigma_k^2 + \sigma_i^2, \quad (3)$$

TABLE 2  
COVARIANCES AFTER REDUCTION TO SYSTEM (1)

	$\mu_0(\sigma_v)$	$\mu_0(\text{Ma})$	$\mu_0(\text{Mb})$	$\mu_0(\text{SV})$	$\mu_0(\text{gp})$	
$\mu_0(\sigma_v)$	...	0.67 79	0.72 79 (0.34)	0.75 76 0.79	0.64 114 0.55	$\sigma_{ij}$ $N_{ij}$
$\mu_0(\text{Ma})$		...	210	117 0.84	153 0.61	$\sigma_{ij}$ $N_{ij}$
$\mu_0(\text{Mb})$			...	116 ...	153 0.77	$\sigma_{ij}$ $N_{ij}$
$\mu_0(\text{SV})$				...	134 ...	$\sigma_{ij}$ $N_{ij}$
$\mu_0(\text{gp})$						
Estimated mean errors $\sigma_i = \text{m.e.}[\mu_0^1(i)]$ after reduction to system (1):						
Least squares ....	0.49	0.43	0.51	0.65	0.37	
Triplets.....	0.48	0.44	0.52	0.65	0.38	
Estimated mean errors $\text{m.e.}[\mu_0(i)]$ before reduction to system (1):						
Least squares ....	0.49	0.45	0.62	0.81	0.38	
Triplets.....	0.48	0.46	0.63	0.80	0.39	

can be solved exactly, in principle, to give

$$\sigma_i^2 = (\sigma_{ij}^2 + \sigma_{ki}^2 - \sigma_{jk}^2)/2, \quad (4)$$

and so on.

In the present case with five data sets, the number of pairs and covariances  $\sigma_{ij}^2$  is 10. The number of triplets, i.e., of systems of equation (4), is also 10, leading to five different estimates from triplets for each of the five individual variances  $\sigma_i^2$ . This overdetermined system is solved in two different ways (see de Vaucouleurs and Head 1978).

The first solution was made by solving exactly the 10 triplets of the form of equation (4) and by taking weighted means of the five estimates for each  $\sigma_i$ . The weights were taken to be the smallest of the  $N_{ij}$  entering in each triplet, except that the weight was taken to be zero for any triplet including both of the Michard data sets, scales (2) and (3). The dispersion  $\sigma_{23} = 0.34$  mag is not an accurate measure of the accidental errors  $\sigma_2$  and  $\sigma_3$ , since the data and errors for the two Michard scales are certainly not uncorrelated. Eliminating such triplets still left three different estimates for  $\sigma_2$  and  $\sigma_3$ , and five different estimates for the other  $\sigma_i$ , with the results given in Table 2.

The second solution was made by a straightforward least-squares solution of the 10 equations in five unknowns. The least-squares values given for the  $\sigma_i$  in Table 2 are those values which minimize the statistic

$$S = \sum_{i < j} W_{ij} (\sigma_{ij}^2 - \sigma_i^2 - \sigma_j^2), \quad (5)$$

where  $W_{ij} = N_{ij}$ , except that  $W_{23} = 0$  for the two Michard scales.

Estimates of the mean errors  $\sigma_i$  from the two methods, triplets and least-squares, agree closely. Table 2 gives the values for  $\sigma_i = \text{m.e.}[\mu_0^1(i)]$ , after reduction to system (1), and for  $\text{m.e.}[\mu_0(i)] = \text{m.e.}[\mu_0^1(i)]/\alpha(i)$ , for the original data sets before reduction. As it happens, the scale factors are such that

the  $\mu_0^1(i)$  have smaller mean errors than the  $\mu_0(i)$ . For example, the  $\mu_0(\text{Mb})$  distance moduli have a mean error 0.63 mag, but the scale factor  $\alpha(3) = 0.82$  gives  $\mu_0^1(\text{Mb})$  with a mean error of only 0.52 mag.

Although they do not give individual errors for the galaxies in their samples, Michard-a (1979a), Michard-b (1979b, c), and Visvanathan (1979) do give general mean error estimates of 0.40, 0.50, and 0.69 mag, respectively. These can be compared with the values 0.46, 0.63, and 0.81 mag, respectively, derived from the present analysis using intercomparisons among the distance scales.

The present analysis also gives  $\text{m.e.}[\mu_0(\sigma_v)] = 0.49$  mag in set (1) which we use as our standard of reference. de Vaucouleurs and Olson (1982) assign individual errors to the  $\mu_0(\sigma_v)$  of each galaxy. The average of the quoted mean errors for the same galaxies used here in the intercomparisons is 0.52 mag. This good agreement suggests that the individual errors we calculated by propagation of errors were reasonably accurate.

##### V. REDUCTION EQUATIONS AND EXTERNAL ERRORS

After iterating 4 times, following the scheme of § III, the equations required to reduce the other distance scales to the scale of system (1),  $\mu_0(1) \equiv \mu_0(\sigma_v)$ , converged to the forms

$$\mu_0(1) - 31.47 = (0.96 \pm 0.06)[\mu_0(\text{Ma}) - 31.82], \\ (\text{ZPC} = -0.35), \quad (6a)$$

$$\mu_0(1) - 31.52 = (0.82 \pm 0.06)[\mu_0(\text{Mb}) - 31.68], \\ (\text{ZPC} = -0.16), \quad (6b)$$

$$\mu_0(1) - 31.18 = (0.81 \pm 0.07)[\mu_0(\text{SV}) - 32.44], \\ (\text{ZPC} = -1.26), \quad (6c)$$

$$\mu_0(1) - 31.61 = (0.99 \pm 0.05)[\mu_0(\text{gp}) - 31.64], \\ (\text{ZPC} = -0.03), \quad (6d)$$

where the zero-point corrections (ZPC) are evaluated at the mean distance moduli of the samples (2)–(5).

#### VI. WEIGHTED MEAN DISTANCE MODULI

Table 1 lists the weighted mean distance moduli of 336 “well-observed” early-type galaxies ( $-5 \leq T \leq -1$ ) from four sources, 126 with  $\mu_0(\sigma_v)$ , 212 with  $\mu_0(\text{Ma})$ , 213 with  $\mu_0(\text{Mb})$ , and 197 with  $\mu_0(\text{SV})$ . For each galaxy, distance moduli  $\mu_0^1(i)$ , reduced to the  $\mu_0(\sigma_v)$  system, and a weighted  $\langle \mu_0^1 \rangle$  are given. Since  $\mu_0(\text{Ma})$  and  $\mu_0(\text{Mb})$  are not independent, they were averaged before combining them with the others. The smaller of the two errors,  $\sigma(\text{Ma})$  or  $\sigma(\text{Mb})$ , was assigned to the mean. Individual errors, taken from de Vaucouleurs and Olson (1982), are given for  $\mu_0(\sigma_v)$ , as was justified in § IV. For the others, the average mean errors calculated in § IV were used.

In addition to the “well-observed” sample, Table 1 also includes distance modulus estimates for galaxies which were rejected in the intercomparisons because of uncertain (\*) or doubtful (\$) type, or for various other *a priori* reasons, as described in § II. None of these galaxies were used in deriving the reduction equations or external error estimates, but it is of interest to apply the standard reduction equations to these galaxies.

Distance moduli  $\mu_0(\text{SV})$  marked “M” or “C” are those described by Visvanathan (1979) as having been derived from uncertain magnitudes or colors. Since a bad color seriously affects  $\mu_0(\text{SV})$ , such values were used with weight 1/4 in computing the average  $\langle \mu_0^1 \rangle$ .

Distance moduli  $\mu_0(\sigma_v)$  marked “P” indicate galaxies which have measured values of  $\sigma_v$  but were not included in the catalog of de Vaucouleurs and Olson (1982) because, in each case, the galaxy had been classified as peculiar or as a member of an interacting or colliding pair. These galaxies are included in Table 1 since most of them were included in the samples of Michard (1979a) and of Sandage and Visvanathan (1978), but none of these peculiar galaxies were used to derive the reduction equations to the  $\mu_0(\sigma_v)$  system.

With these additions, there are 170 galaxies with  $\mu_0(\sigma_v)$ , 253 with  $\mu_0(\text{Ma})$ , 254 with  $\mu_0(\text{Mb})$ , and 284 with  $\mu_0(\text{SV})$ , and Table 1 lists distance moduli reduced to a common system for a total of 424 galaxies.

The weighted mean moduli  $\langle \mu_0^1 \rangle$  are compared in Figure 3 with the group distances  $\mu_0(\text{gp})$ , after excluding galaxies with uncertain type, those with  $m.e.[\langle \mu_0^1 \rangle] > 1.0$  mag, and those for

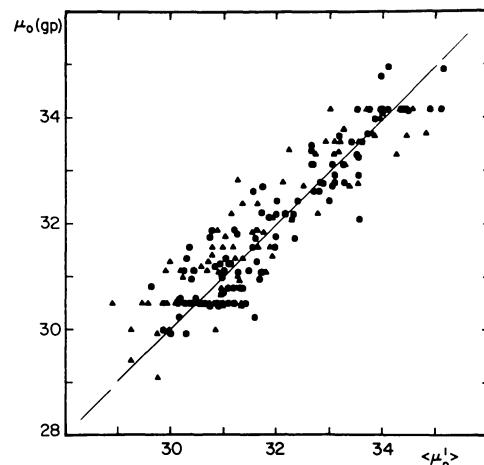


FIG. 3.—Comparison of mean distance moduli  $\langle \mu_0^1 \rangle$  of early-type galaxies from Table 1 with distance moduli  $\mu_0(\text{gp})$  of the groups of which they are probable members derived from independent distance indicators for spirals in the same groups, from mean redshifts.

which  $\mu_0(\text{gp})$  is based on the on the redshift of a single galaxy. For this sample of 216 galaxies the weighted least-squares solution is

$$\mu_0(\text{gp}) - 31.87 = (0.99 \pm 0.03)[\langle \mu_0^1 \rangle - 31.84]. \quad (7a)$$

If the sample is further reduced to 151 galaxies by excluding all group distances derived from redshifts, the solution becomes

$$\mu_0(\text{gp}) - 31.89 = (0.98 \pm 0.03)[\langle \mu_0^1 \rangle - 31.87]. \quad (7b)$$

The zero-point difference is only 0.02 or 0.03 mag, and the scale factor is not significantly different from unity. The mean distance moduli of early-type galaxies in Table 1 are, therefore, strictly homogeneous with the distances of spirals previously derived from the luminosity index, the 21 cm line width, and other independent distance indicators (except for zero point).

This report is part of a general comparison of tertiary distance indicators and distance scales. A companion study dealing with spirals ( $T \geq 0$ ) will be reported later.

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