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THE CHEMICAL COMPOSITION AND EVOLUTIONARY STATE OF THE EARLY R STARS

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ABSTRACT

A sample of 11 stars classified R0–R5 has been analyzed to determine (i) iron-peak metal abundances, (ii) abundances of elements produced by the *s*-process, (iii) the light element abundances (C, N, and O), and (iv) the carbon isotopic ratio $^{12}C/^{13}C$. Abundances are reported relative to the Sun. With the exception of HD 100764, which has an iron abundance 0.6 dex lower than the Sun's, the iron abundance of 10 stars is near solar. The *s*-process elements have no demonstrable enhancement in the sample, unlike the Ba II stars of the same temperature and the cooler carbon stars.

The enhancement of carbon relative to the G–K giants found in the analysis of eight stars is 0.7 dex, with one star having $C \approx O$ (HD 113801), while another star (HD 95405) is probably not a carbon star ($C/O = 0.87$). Relative to the ordinary G–K giants, nitrogen is enhanced by a small amount (0.2 dex). In contrast, oxygen has nearly the value found in the Sun and in normal giants, except in the case of HD 100764 where O is probably as underabundant as Fe. The $^{12}C/^{13}C$ ratio ranges from near 4 to 15 for nine stars, indicating an enrichment of ^{13}C with respect to the majority of faint giants. For HD 156074 the oxygen isotopic ratios $^{16}O/^{17}O$ and $^{16}O/^{18}O$ do not have detectable departures from the solar system values.

The lack of oxygen depletion indicates that the CNO cycle operating near equilibrium is *not* responsible for the fact that $C/O > 1$ in these stars. Based on the early R stars' position in the H-R diagram, their space number densities, and their enhanced carbon abundances, it is here suggested that they result from mixing at the time of, or closely following, the helium core flash. The relatively low $^{12}C/^{13}C$ ratios and modest ^{14}N enhancements indicate additional processing of the pure ^{12}C produced by the core's 3α burning. It is suggested that the R stars undergo a core flash more dynamically extensive, although not necessarily at a higher temperature than the average red giant's. As a result, a mixing of envelope protons with upward-moving core material occurs on a time scale short enough and at a temperature low enough ($T \leq 70 \times 10^6$ K) to allow only the reactions $^{12}C(p,\gamma)^{13}N(\beta^+\nu)^{13}C$ and $^{13}C(p,\gamma)^{14}N$. In this way the production of neutrons via $^{13}C(\alpha,n)^{16}O$ is reduced and so is *s*-processing.

Subject headings: stars: abundances — stars: carbon — stars: evolution — nucleosynthesis

I. INTRODUCTION

Pickering (1896) recognized and later (Pickering 1908) delineated a small group of peculiar stars which were assigned class R to distinguish them from the redder N-type stars. The atomic-line spectra of the R0–R5 stars are equivalent to the G9–K2 normal giants, whereas the N stars are later than M3 under the same criteria (Gordon 1967, p. 95). Like the N stars, the spectra of the R stars contain strong C_2 Swan system bands in the visible, while red system CN bands dominate the near-infrared. The presence of such strong bands of carbon-containing molecules was recognized early as the signature of a carbon-rich ($C > O$) composition (Rufus 1916; Curtiss 1917; Shane 1920; Cambresier and Rosenfeld 1933; Russell 1934). In this paper attention is limited to the classical early R stars of Harvard classes R0–R5. Omitted are the H-deficient R stars which are supergiants (Richer 1975) and the metal-deficient giant CH stars (Wallerstein and Greenstein 1964; Luck and Bond 1982). Likewise, the later R stars are excluded since they are intrinsically at least 1.5 mag brighter (Vandervort 1958; Eggen 1971; Baumert 1974), have enhanced Ba II lines like the N stars (Gordon 1968), and possess a space distribution more similar to the N stars (Rybski 1972, p. 138).

The early R stars are spectroscopically similar to the N stars, but are physically quite distinctive. Whereas the early R stars are predominantly nonvariable, the cooler carbon stars are photometric variables. From a statistical parallax study (Vandervort 1958) it is known that the early R stars are faint giants, having $\langle M_v \rangle = +0.4$ ($L/L_\odot = 100$), a value similar to the K1 oxygen-rich giants. In contrast, the N-type carbon stars are brighter ($\langle M_v \rangle = -2.5$, $L/L_\odot = 3200$). The velocity ellipsoid solution of McLeod (1947) and the solar motion found by Dean (1972) are reminiscent of the dwarf G stars. The N-type stars, on the other hand, have a velocity ellipsoid similar to the dwarf F stars (Dahn 1964). The dispersion of the early R stars above the galactic plane (Stephenson 1973) also indicates that they are representative of a population which is older and less massive than is typical of the N stars. From this evidence, the early R stars must have masses in the range 0.7–1.3 solar masses, whereas the N stars probably

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TABLE 1
BASIC DATA FOR THE PROGRAM STARS

HD	DM	V[a]	$I(104)$[b]	R Type[c]	C Type[d]	v_r[d] (km s^{-1})
...	BD +21° 64	9.62	7.99	R2	C3,3	−1:
16115	SD −10° 513	8.07	6.33	R3–R5	C2,3J	+16
58337	BD +22° 1679	9.49	7.73	R5	C3,3J	+4
58364	BD +22° 1680	9.20	6.89	R5	C3,3	−3
90395	CD −49° 5234	9.3	...	R0	C[e]	...
95405	CD −25° 8383	8.29	6.75	Kp−R	C2,0	+12
100764	SD −13° 3407	8.80	7.26	R0	C1,1	+5
113801	SD −19° 3634	8.52	6.96	K5p−R	C2,1	−16
...[f]	BD +33° 2399	9.54	7.61	R2	C3,3	−30[g]
156074	BD +42° 2811	7.61	6.09	R0	C3,1	−16
...	BD +17° 3325[h]	8.72	7.06	R0	C2,0+	−43:

[a]From Mendoza and Johnson 1965, Vandervort 1958, or Stephenson 1973.
[b]From Baumert 1972.
[c]From the Henry Draper catalog or Vandervort 1958.
[d]From Yamashita 1972, 1975.
[e]From the Michigan catalog (Houk 1978).
[f]Identified as HD 122547 by some authors, this attribution is probably incorrect (Stephenson 1973).
[g]From Dean 1972.
[h]The position of this star on the Palomar Sky Survey confirms this DM assignment (see Yamashita 1974; Dominy 1983).

range from 2 to 10 (see also Eggen 1972*a*, *b*; Barbaro and Dallaporta 1974). Scalo (1976) has shown that the early R stars, like the Ba II stars, occupy the same region of the H-R diagram as the ordinary G–K giants.

An understanding of the N-type carbon stars has emerged. The observations indicate that the carbon in their atmospheres has been freshly synthesized by 3α He-burning (Kilston 1975; Thompson 1977). The theoretical investigations of the mixing driven by the He-shell flashes of a second giant branch star indicate that stars having a wide range of core masses can become carbon stars (Iben 1975, 1981; Renzini and Voli 1981). The classical early R stars are too faint ($M_{\rm bol} \approx -0.3$) and too warm ($T_{\rm eff} \approx 4600$ K) to be involved now in an episode of He-shell flashing (Scalo 1976). In addition, the space density of R stars is larger than that expected for stars in the He shell burning phase of evolution (Scalo and Miller 1979).

If an explanation for the existence of the early R stars lies in single-star stellar evolution (instead of, say, an event of mass transfer in an evolving binary system or accretion of interstellar dust), then their atmospheric composition should provide clues to their past evolution. This prospect is, in addition, promising if, as seems plausible from their masses and intrinsic faintness, the early R stars have undergone fewer stages of shell burning than the N-type carbon stars. Of particular interest is a comparison of the early R stars and the Ba II stars. Among the early R stars there exists apparently large enhancements of carbon (C > O) relative to the solar value of C/O = 0.6 (Goldman *et al.* 1983), while for the Ba II stars only modest carbon enrichment (C/O ≈ 0.7–0.8) has been demonstrated (Tomkin and Lambert 1979; Sneden, Lambert, and Pilachowski 1981; Smith 1983). The dichotomy between the early-R and Ba II stars is also made distinct by the observed enhancements of the elements formed by the *s*-process in the Ba II stars, while the warm carbon stars have no apparent *s*-process overabundances (e.g., Buscombe 1955; Warner 1962; Gordon 1968; Greene *et al.* 1973).

The general apparent faintness ($m_v > 8$) and blanketed spectra of the early R stars have conspired to limit progress in understanding their chemical composition. In a high dispersion study of HD 156074 by Greene *et al.* (1973) the finding that C/O ≈ 2.5 and N/C ≈ 8 could not be interpreted as consistent with a single, or set of sequential, nuclear burning episodes. To shed more light on the events producing the early R stars, several of the brightest stars were selected for observation in the interval R0–R5, as classified in the Henry Draper catalog, by Shane (1920), or by Vandervort (1958). The basic data for the program stars are listed in Table 1.

Using moderate to high resolution data from the visible, near-infrared, and infrared, abundance analyses were performed to determine (i) general metallicity (Fe and Ti), (ii) any enhancement of *s*-process elements, (iii) the isotopic ratio ^{12}C/^{13}C, and (iv) the abundances of C, N, and O. The results of standard LTE differential abundance analyses of these elemental groups are reported in §§ II–V. In § VI a discussion of the uncertainties in the analysis is presented. Based on these results, a discussion of the evolutionary state of the class of early R stars is presented in § VII.

II. IRON-GROUP METALS

The early R stars seem to be part of an old disk population (Eggen 1972*b*) with no evidence of metal depletion at classification dispersion (Keenan and Morgan 1941) or in detailed analysis (Greene *et al.* 1973). In order to test this conclusion quantitatively for a sample of stars, an analysis of nine early R stars was made. The stars observed are listed in Table 1.

a) Observations

The observations in the 4300–7680 Å region were obtained with the 4 m echelle spectrographs of Kitt Peak National Observatory and Cerro Tololo Inter-American Observatory. For each star a set of four long-focus camera plates was exposed. Spectra were widened to 0.8 mm and required an average of 95 minutes for exposure. The KPNO echellograms have a resolution of 180 mÅ (resolving power, $\lambda/\Delta\lambda = 3 \times 10^4$), while the CTIO data have a resolution of 140 mÅ. Simultaneous chemical processing of the IIIa-J echelle and calibration plates was done in the standard way. The plates were scanned with the *Skylab*/Department of Astronomy PDS microdensitometer of the University of Texas (Wray and Benedict 1974). The photographic densities were converted to spectral intensities and plotted using modified versions of the KPNO programs ECHORD and TRACE.

The echelle spectra of the early R stars are complicated by the blanketing of molecular lines and the narrow free spectral range of the instrument. In order to locate the continuum and derive equivalent widths, reference was made to an echelle spectrogram of β Gem (K0 IIIb) obtained during the same observing session. The Arcturus atlas (Griffin 1968) also served as a higher resolution comparison where atomic blending was severe.

Equivalent widths from a master list of 60 well-defined Fe I and Ti I lines were measured by direct least-squares fitting of a Gaussian function. The measurements are found in Dominy (1983). The spectra allowed a maximum number of 38 Fe I and Ti I lines to be used (for HD 156074), with the least being 12 (HD 154124). The echellograms offered too few lines of other iron-peak elements for additional abundance analyses.

b) Model Atmospheres

Realistic model atmospheres for the warm carbon stars are necessary in the analysis of metal abundance. It can be expected that, owing to their similarity to the ordinary G–K giants, the R stars' atmospheres may be modeled using the classical assumptions. Compared with other carbon stars, their atmospheres are warmer and so less extended. The problems of non-LTE, sphericity, stratification, and pulsational or shock-induced dynamical effects can be no more serious than those existing for the ordinary giants.

The set of models used in this analysis was calculated by Johnson (Johnson 1979; O'Brien 1980) using a modified version of ATLAS5 (Kurucz 1970). Computing details are similar to those of an oxygen-rich set (Johnson, Bernat, and Krupp 1980) and a set of cool carbon-rich models (Johnson 1982) made in the same way. The input elemental abundances adopted for the R-star models were the solar values, with the exception of C, N, and O which were assumed to be altered by +0.46, +0.36, and −0.03 dex, respectively, relative to the Sun. The adopted model C/O ratio of 1.75, selected originally from the estimates obtained observationally by Gow (1975), is near the mean C/O found for the stars analyzed here. Yorka (1981, p. 105) has shown that the Johnson models are in excellent agreement with the models published by Olander (1981) using the code of Gustafsson *et al.* (1975) at the $T_{\rm eff}$, log g parameters of Johnson.

Line opacities were treated by the opacity sampling (OS) technique. No turbulent velocity was included in the solution for hydrostatic equilibrium; however, an isotropic turbulence of 2.0 km s^{-1} was used for the Doppler broadening of the line opacity absorption coefficients. For the CN radical a dissociation potential, $D_o^0 = 7.7$ eV, was assumed, as well as a vibrational band f-value, $f_{0,0} = 5.92 \times 10^{-3}$. This f-value is now recognized to be about a factor of 2.5 too large for this dissociation energy (Sneden and Lambert 1982).

Models were calculated at four temperatures ($T_{\rm eff} = 4200$, 4600, 5000, 5400 K) and two surface gravities, log $g = 2.0$ and 3.0. The line-blanketed emergent flux distributions from three of the models are shown in Figure 1. In this comparison of model and stellar flux, the model flux has been smoothed to match the instrumental resolution (30 Å) of the spectrophotometry of two early R stars (Gow 1975). It should be borne in mind that this is not a spectrum synthesis (all possible lines included), but is instead a smoothing of randomly sampled monochromatic fluxes. The agreement between observed and model spectra, both in general slope and degree of blanketing, provides encouraging evidence for the appropriateness of the models in representing the actual stars.

c) Effective Temperatures

The primary atmospheric parameters needed in the Fe and Ti analyses and $T_{\rm eff}$, log g, and the microturbulence, ξ_t. No fundamental determinations (from measured distances or angular diameters) of any of these quantities are currently available. Pilachowski *et al.* (1982) conclude that echelle spectra, owing to their inherent noise, do not provide equivalent widths sufficiently reliable for a determination of $T_{\rm eff}$. For these reasons, broad-band fluxes and, when these were not available, narrow-band fluxes were used to estimate $T_{\rm eff}$. Broad-band fluxes are taken from the Johnson photometry of Mendoza and Johnson (1965), Vandervort (1958), and Wing (1967). No reddening corrections are necessary (Richer 1971

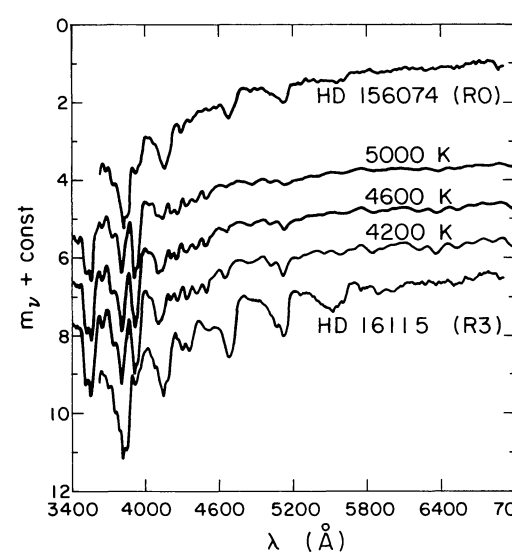

FIG. 1.—Observed and model fluxes of the early R stars. Effective temperatures of the log $g = 2$ Johnson models are indicated.

TABLE 2

DERIVED ATMOSPHERIC PARAMETERS OF THE PROGRAM STARS

Star	T_{eff}	$\sigma(T_{eff})$[a]	log g[b]	ξ_t[c]	$\sigma(\xi_t)$[d]
BD +21° 64	4400	280	2.0	3.1	0.5
HD 16115	4500	260	2.1	3.2	0.6
HD 58337	4600	250	2.1	3.1	0.6
HD 58364	4200	290	2.0	...[e]	...[e]
HD 90395	5000:	300	2.1	3.2	0.5
HD 95405	4600:	220	2.1	2.2	0.5
HD 100764	4850	190	2.2	2.6	0.4
HD 113801	4700	180	2.2	2.3	0.4
BD +33° 2399 ..	4460	200	2.0	3.0	0.4
HD 156074	4770	170	2.2	3.2	0.3
BD +17° 3325 ..	4520	210	2.1	1.0[f]	...[e]

[a]Error estimated from scatter among T_{eff}-determination methods, intrinsic errors in the methods themselves, and the applicability of each method to R stars.

[b]Derived assuming $M = 1.1\ M_\odot$.

[c]Determined from Fe I lines.

[d]Estimated from the error of the linear fit of line-to-line Fe abundance against lower potential.

[e]Not analyzed.

[f]From weak ^{12}CN lines only.

finds that $E_{B-V} \approx 0.0$) for most of the program stars which lie at distances of 200–350 pc and well above the galactic plane.

A fundamental photometric calibration of T_{eff} for carbon stars is not available. The early R stars lie close to the oxygen-rich giants in the $R - I$, $V - R$ two-color diagram of Mendoza (1968; see his Fig. 12). This close proximity of the warmest carbon stars to the oxygen-rich sequence (at least in these two colors) lends confidence to the idea that the T_{eff}-color relations established for the normal stars can yield reliable T_{eff} estimates for the early R stars.

The fundamental $V - K$ calibration of Ridgway *et al.* (1980), the model atmosphere calibration of $V - R$ by Bell and Gustafsson (1978), the $V - R$ calibration of the Barnes-Evans relation (Barnes, Evans, and Moffett 1978), and the K-magnitude result of the infrared method (Blackwell and Shallis 1977) calibrated with the Johnson R-star models were averaged to find T_{eff}. The $R - I$ index has little temperature sensitivity in the range encompassed by the early R stars (+0.25 to +0.45) and was not used. In the analysis of HD 100764 an apparent flux excess in the K band precludes its use as a T_{eff} discriminant. Only two methods, the Bell-Gustafsson and the Barnes-Evans calibration of $V - R$, were applied to this star. Only B, V photometry is available for HD 95405, and the recalibration of T_{eff} with $B - V$ by Böhm-Vitense (1981) was used.

The narrow-band ($\Delta\lambda = 60$ Å) photometry at 1.04 μm from Baumert (1972) was calibrated with T_{eff} using the theoretical fluxes of the Johnson models and the filter function provided by Wing (1981). The effective temperatures of two stars (HD 16115 and HD 58337) were found in this way. Whereas a total uncertainty of ~ 200 K or less is estimated for the T_{eff} values of stars with Johnson infrared photometry, an error of ~ 250–300 K is expected for the stars having only 1.04 μm data. Although not used in the determination of T_{eff}, the excitation temperatures derived from Fe I and ^{12}CN line data are consistent (within their larger errors) with the photometric temperatures. The adopted effective temperatures are given in Table 2.

d) Infrared Flux Excess

A powerful T_{eff} discriminant, the L band–to–total flux ratio, developed by Blackwell, Petford, and Shallis (1980) and used by Tsuji (1981*a*) for the M stars and the cool carbon stars (Tsuji 1981*b*), is thwarted by the early R stars. Similarly, the $I - L$ calibration of T_{eff} (e.g., Scalo 1974*a*) is made useless by this flux excess in the 3.5 μm region. The upturn in flux at the L band (relative to the G–K–M giants) is obvious in the magnitude differences plotted by Mendoza and Johnson (1965) and listed in Eggen's (1971) Table 13. Mendoza (1968) and Alksne and Ikaunieks (1981, p. 93) attribute its existence to the presence of circumstellar material reprocessing photospheric flux to longer wavelengths. The infrared flux excess is not a feature of the G–K giants but is seen in every early R star observed at infrared wavelengths. It thus seems that the excess may have a close connection with the process producing the R stars. With one exception, HD 100764, the additional emission does not appear shortward of the L band.

e) Surface Gravities

The heavily blanketed spectra of the program stars and the inherent errors of echelle equivalent widths together conspire in making few weak Fe II lines available for a spectroscopic evaluation of log g. Likewise, the damping wings of strong lines and other luminosity-sensitive line ratios are ruled out for this purpose.

To assign a value of log g to each program star, a mass of 1.1 $M_\odot$ for each star was assumed. This mass is near the mean mass of red giants in general (Scalo, Dominy, and Pumphrey 1978), and is the mass of the kinematically equivalent G0 V stars. A value of 1.1 $M_\odot$ is consistent with (but is not demanded by) the position of the early R stars on the clump giant branch in the H-R diagram (Scalo 1976). For each star in Table 1, log g is found from the adopted T_{eff}, the mean M_v of the class (+0.4), and the bolometric correction. Bolometric corrections are taken from Mendoza and Johnson (1965) for individual stars, or the mean B.C. of the class is adopted [−0.68, from Alksne and Ikaunieks 1981, p. 117, on the scale B.C. (☉) = −0.08].

f) Atomic Parameters

The oscillator strengths for the Ti I and Fe I lines used in the iron-peak element analysis were obtained from the solar spectrum. Solar equivalent widths were measured by Gaussian profile fitting of the full-disk flux data of Beckers, Bridges, and Gilliam (1976). The derived *gf*-values reflect the adopted solar abundances (Fe, 7.65; Ti, 5.05). The solar model of Holweger and Müller (1975), including its depth-dependent microturbulence structure, was used in the calculation. For Fe I lines the adopted abundance is the same as that derived by Blackwell and Shallis (1979) using the same model and solar center-of-disk equivalent widths. The Fe I *gf*-values may thus be expected to coincide with the Oxford scale.

A model atmosphere was interpolated to each star's T_{eff}, log g from the Johnson grid for use in the abundance calcula-

TABLE 3

DERIVED ABUNDANCES[a]

Star	[⟨Ti⟩]		[⟨Fe⟩]		[⟨s⟩]	[⟨s⟩/⟨Fe⟩]		$^{12}C/^{13}C$		[C]		[N]		[O]		C/O	C/N	[C+N+O]
BD +21° 64	−0.53:	(0.42)	−0.09	(0.31)	+0.39	+0.48	(0.50)	7	(4)	+0.52*	(0.16)	+0.64*	(0.25)	[b]	…	1.9*	3.6*	+0.3*
HD 16115	−0.26	(0.48)	−0.21	(0.25)	+0.12	+0.33	(0.49)	5	(4)	+0.58*	(0.18)	+0.58*	(0.24)	[b]	…	2.1*	4.8*	+0.3*
HD 58337	−0.34	(0.51)	−0.40	(0.37)	+0.09	+0.49	(0.31)	5:	(5:)	[b]	…	…	…	[b]	…	…	…	…
HD 58364	[c]	…	[c]	…	[c]	[c]	…	6	(5)	+0.77*	(0.24)	+0.48*	(0.28)	[b]	…	3.3*	9.3*	+0.4*
HD 90395	+0.31	(0.22)	+0.19	(0.20)	+0.28	+0.09	(0.30)	[b]	[b]	+0.72*	(0.20)	[b]	…	[b]	…	2.9*	…	…
HD 95405	+0.06	(0.29)	+0.01	(0.20)	+0.07	+0.06	(0.24)	[b]	[b]	+0.23*	(0.13)	[b]	…	+0.04	(0.15)	0.9	…	…
HD 100764	−0.65	(0.32)	−0.59	(0.18)	−0.35	+0.24	(0.28)	4	(2)	+0.47	(0.20)	+0.44	(0.22)	−0.53:[d]	…	…	…	…
HD 113801	−0.25	(0.33)	−0.26	(0.21)	−0.25	+0.01	(0.27)	15	(2)	+0.12	(0.14)	+0.63	(0.25)	−0.14	(0.18)	1.0	1.5	+0.1
BD +33° 2399	+0.09	(0.40)	+0.03	(0.22)	+0.11	+0.08	(0.46)	5	(4)	+0.49	(0.28)	+0.82	(0.28)	−0.09	(0.19)	2.1	2.3	+0.3
HD 156074	−0.03	(0.34)	−0.10	(0.14)	+0.24	+0.34	(0.32)	8	(2)	+0.46	(0.11)	+0.64	(0.21)	−0.08	(0.15)	2.0	3.2	+0.3
BD +17° 3325	[b]	…	[b]	…	[b]	[b]	…	9	(3)	[b]	…	[b]	…	[b]	…	…	…	…

[a] Reported relative to the Sun; $[X] = \log X_* - \log X_\odot$; standard errors are shown in parentheses (calculated from line-to-line scatter); an asterisk indicates that the analysis was done *assuming* [O] = 0.0.

[b] No spectrum available.

[c] Spectrum not analyzed, very similar to HD 58337.

[d] Value obtained after correction for infrared flux excess.

tions. The results of the LTE analysis of Ti I and Fe I lines are given in Table 3. The depth-independent microturbulence parameters and their formal errors derived by minimizing any dependence of abundance on equivalent width are included in Table 2. The program LINES (Sneden 1973; modified by Luck 1977) was used to make all of the metal abundance calculations.

In most cases, the analysis of the program stars indicates that Ti and Fe have the solar value within the estimated errors of echelle spectroscopy. A remarkable exception is HD 100764. This star displays an iron-peak metal deficiency of 0.6 dex. The general weakness of the metal lines of this star is evident in the classification spectra of Keenan and McNeil (1976, Pl. [26]).

g) *Hydrogen*

An assumption of the model atmospheres is that hydrogen is the dominant constituent. Whereas this assumption is automatic for the majority of stars, in the case of stars possessing the products of extensive nuclear processing and mixing, this assumption may not be justified. It appears that the N-type carbon stars may be H-deficient (Yamashita 1972; Thompson 1972; Johnson *et al.* 1983).

The classical early R stars do not suffer the severe hydrogen deficiency seen in the supergiant R stars such as HD 182040 where the Balmer lines are absent. For modest deficiencies of H (for which H^- is still the dominant continuous opacity source), the Balmer line absorption (i.e., the ratio l_ν/κ_ν) will be nearly independent of both l_ν and κ_ν. Thompson (1972) and Böhm-Vitense (1979) show that CH lines will not display a strong sensitivity to the hydrogen content.

Using the fact that H^- remains the primary opacity source, Böhm-Vitense shows that the neutral atoms of elements having low ionization potentials (and so are mostly ionized in the atmosphere) have the greatest sensitivity to H deficiency (alternatively He enhancement). In the model atmosphere for HD 156074 (4770/2.2) the ratio Ca^+/Ca^0 is 240 in the uppermost parts and increases to 2300 at the lowest level. A search of the echellograms for the weakest and best defined Ca I lines produced only one: 5260.39 Å with $W_\lambda = 86 \pm 6$ mÅ measured on two plates. The predicted equivalent width is 95.8 mÅ (assuming the solar Ca abundance and the ξ_t from Fe I). If the He/H ratio were changed from 0.1 to 1, the Ca I equivalent width would be increased by nearly a factor of 3 according to Böhm-Vitense.

h) *Lithium and Magnesium*

Unlike the more luminous C-type stars, the early R stars do not have detectable Li lines at low dispersion (Yamashita 1972). In a spectrum synthesis of the Li region in HD 156074, all of the absorption can be accounted for by CN. A generous upper limit on the Li abundance is placed at $\log \epsilon(\mathrm{Li}) = 1.2$. This compares with a predicted value of +1.0 for a 2.0 solar mass star at the tip of the red giant branch (e.g., Lambert, Dominy, and Sivertsen 1980; Luck and Lambert 1982).

In the exposure of ^{14}N to α-particles, the sequence $^{14}N(\alpha, \gamma)^{18}F(\beta^+ \nu)^{18}O(\alpha, \gamma)^{22}Ne$ can be completed by $^{22}Ne(\alpha, \gamma)^{26}Mg$ at temperatures below 200×10^6 K and $^{22}Ne(\alpha, n)^{25}Mg$ above that temperature. The magnesium isotopic abundance ratios $^{24}Mg/^{25}Mg/^{26}Mg$ have the terrestrial pattern 79/10/11. Enhancement of ^{25}Mg and ^{26}Mg thus provides a probe of nucleosynthetic activity at temperatures associated with the helium core flash in low-mass stars. Unfortunately, the presence of C_2 hampers the analysis of MgH in even the most favorable region, 5101 Å, found at echelle resolution. There is, however, no evidence of a build-up of either ^{25}Mg or ^{26}Mg comparable to more than half the ^{24}Mg present.

III. HEAVY METALS

The elements having isotopes formed by the slow capture of neutrons have few useful lines in the visible spectra of the early R stars. This condition is brought about by the ever-present crowding of molecular lines among atomic lines. Contributing to the difficulty is the fact that, unlike the Ba II stars of the same equivalent spectral types, the early R stars do not have enhanced lines of the *s*-process elements. The early R

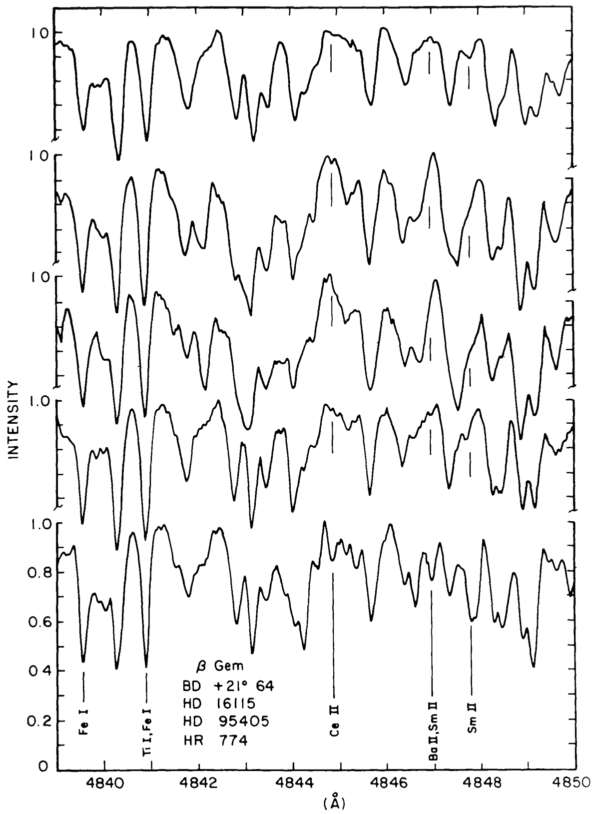

FIG. 2.—Comparison of Ce II line strengths in normal K giants, three early R stars, and the Ba II star HR 774.

stars are not cool enough to exhibit the oxides (e.g., LaO, ZrO, CeO, YO) and potential carbides accompanying the *s*-process enhancements found in the S-type stars.

A search of the spectra of the program stars in comparison with an extensive list of predicted lines yielded 45 lines representing neutral and singly ionized stages of eight elements (Y, Zr, Mo, Ba, La, Ce, Nd, Sm). Measured equivalent widths are found in Dominy (1983). The average number of lines analyzed per program star is 10 (minimum, eight for HD 58337; and, maximum, 29 for HD 95405). Figure 2 compares the strength of a Ce II line (4844.875 Å) found in four early R stars. To establish a wider comparison, the spectrum of the ordinary giant β Gem (K0 IIIb) and the Ba II star HR 774 (G8p Ba 3) are included. All spectra shown in Figure 2 were obtained in the same way with the echelle spectrographs. The line is strong in HR 774, reflecting this star's 1 dex abundance enhancement of Ce (Tomkin and Lambert 1983), but among the early R stars it is indiscernible in the echelle data, as it is in β Gem.

Solar *gf*-values for the elements formed by the *s*-process were derived from the Sacramento Peak atlas (Beckers, Bridges, and Gilliam 1976) equivalent widths. The microturbulence parameter derived from Fe I lines was adopted for the *s*-process elements. Since few lines are available, it is not practical to examine the distribution of abundance against atomic number or even the mean difference between the $A=90$ (Sr$-$Y$-$Zr) and the $A=138$ (Ba) magic number peaks. Instead, the mean differential abundances for the nine stars surveyed, $[\langle s\rangle]$ and $[\langle s\rangle/\langle \mathrm{Fe}\rangle]$, are found and presented in Table 3.

The *s*-process elements in the atmospheres of the early R stars do not show the sizable enhancements typical of the Ba II stars. The iron-peak metal deficiency of HD 100764 extends to the *s*-process nuclei also. HD 100764 is thus best described as an old-disk R star instead of being associated with the CH stars. Its [Fe] abundance (-0.6 dex) does not approach the severe Fe-peak deficiencies of the CH stars (-1 to -1.5 dex; Wallerstein and Greenstein 1964; Luck and Bond 1982). Its heavy element abundances ($[s/\mathrm{Fe}]\approx+0.2$) are far from that typical of the $+0.6$ to $+1.3$ dex found in CH stars. In addition, its radial velocity ($+5$ km s^{-1}; Sanford 1944) is not similar to the high velocities ($v_r>120$ km s^{-1}) of the giant CH stars.

IV. $^{12}C/^{13}C$ RATIOS

The spectra of carbon stars provide a fertile source of molecular lines for the determination of the isotopic ratio $^{12}C/^{13}C$. With the exception of the pioneering study by McKellar (1948) and subsequent reanalysis of the brightest early R star (HD 156074), the carbon isotopic ratio of the warmest carbon stars has not received attention. The discrepancies in the derived $^{12}C/^{13}C$ ratio among various observers of HD 156074 attest to the difficulty of obtaining a reliable determination. Early estimates generally placed the ratio below 5 (McKellar 1948; Climenhaga 1960; Wyller 1960, 1966). Later work favored a value exceeding 10 (Fujita and Tsuji 1976; Dominy 1978). In order to obtain reliable estimates of the $^{12}C/^{13}C$ ratios of the program stars, observations of two molecular systems were made.

a) *The* CN *Red System*

The open structure of the red system ($A\ ^2\Pi_i - X\ ^2\Sigma^+$) of CN offers the only source of single rotational lines which are resolvable with present instrumentation. The advantages inherent in a line-by-line analysis, in contrast to the synthesis of a head-head region (e.g., an independent determination of ξ_t), make CN lines a valuable source of isotopic information.

b) *Observations*

The spectra of ^{12}CN and ^{13}CN lines in nine stars were obtained with the McDonald Observatory's 2.7 m telescope and the 1024 element Reticon detector (Vogt, Tull, and Kelton 1978) of the coudé spectrograph. Two integrations per star were made. These overlapping observations, centered at 7872 and 7965 Å, each included ~96 Å with a spectral resolution of 0.2 Å. The mean integration time for stars fainter than HD 156074 was 2.2 hr, resulting in continuum signal-to-noise ratios of 60–380. For HD 156074 several additional spectral regions were examined. Nearly every observation of a program star was divided by an observation of a rapidly rotating A- or B-type star obtained at a similar air mass. This procedure successfully removes diode-to-diode sensitivity variations and contaminating telluric lines.

Continuum positions available in the spectra of all the program stars were found at 7910.46 and 8000.97 Å. A portion of the spectrum from 7970 to 7995 Å for five of the

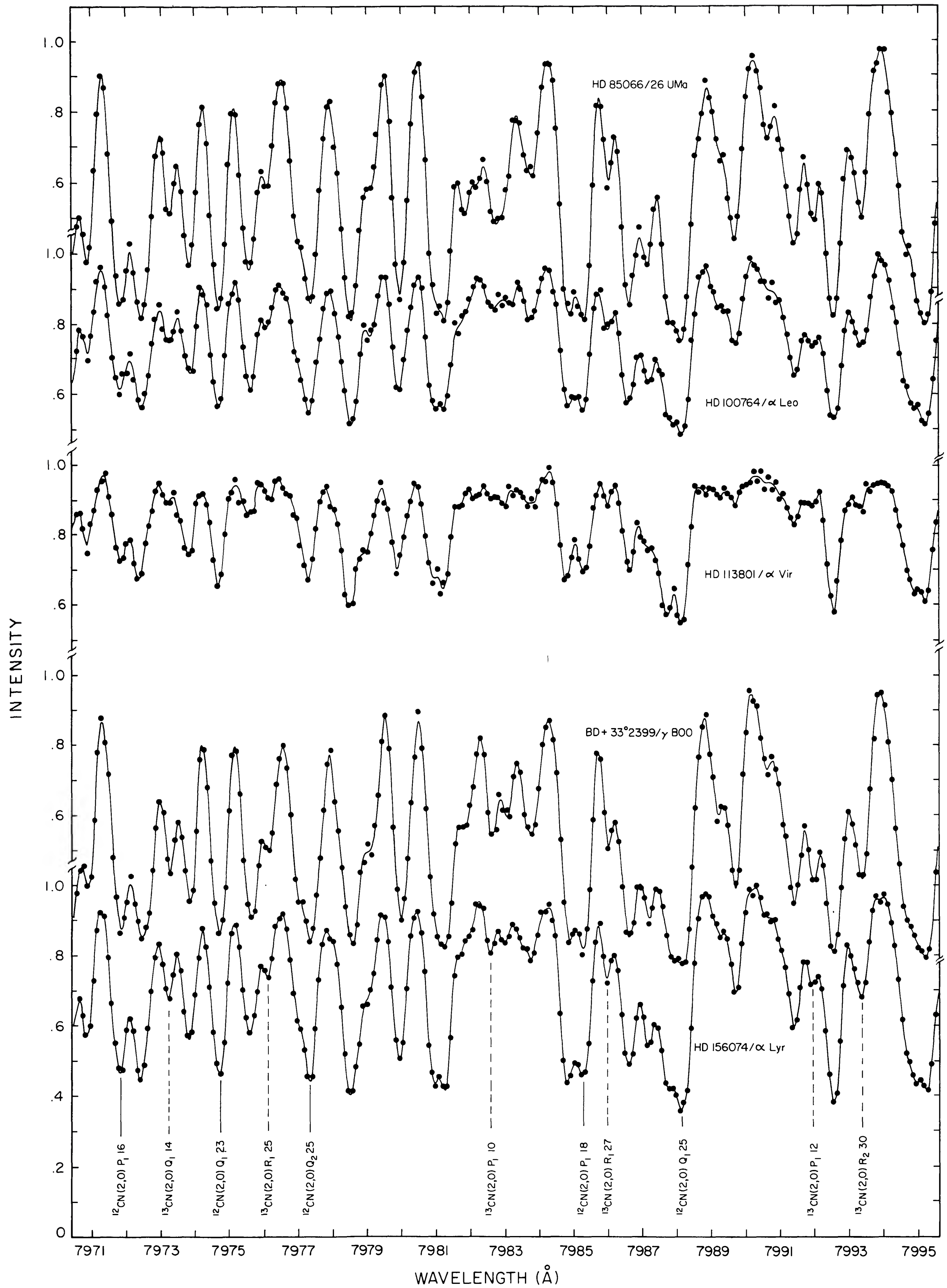

FIG. 3.—Near-infrared Reticon spectra of five early R stars in the 7971–7995 Å wavelength interval. The division of each program star's spectrum by an early-type star eliminates telluric contamination. The observed flux ratio (*dots*) has been smoothed by eliminating high frequency Fourier components in the spectrum (*interpolated curve*). Features of ^{12}CN are identified by solid lines; single ^{13}CN features are indicated by dashed lines.

program stars is shown in Figure 3. Equivalent widths were found by a Gaussian approximation using measures of central depth and line breadth. Several high quality lines from the echellograms are also included in the analysis. The equivalent widths are listed in Dominy (1983).

The *gf*-values for CN lines ($gf = f_{v'v''} S_{N'N''} \lambda_{v'v''} / \lambda_{v'J'v''J''}$) were calculated from the Hönl-London factor ($S_{N'N''}$) formulae of Kovács (1969) and the solar band oscillator strengths ($f_{v'v''}$) of Sneden and Lambert (1982). Line ($\lambda_{v'J'v''J''}$) and band origin ($\lambda_{v'v''}$) wavelengths were taken from Davis and Phillips (1963) or calculated using the program CALC written by Kotlar (1978, p. 448). Excitation potentials for ^{12}CN lines were calculated using Kotlar's program and the molecular constants of Kotlar *et al.* (1980), while the ^{13}CN line potentials were calculated from the constants of Swensson *et al.* (1970).

c) *Analysis*

The observations rely heavily upon lines of the $\Delta v = +2$ sequence. Shortward of the ^{12}CN (2,0) *R*-branch band head (7872.65 Å) lie weak satellite lines which have equivalent widths similar to the ^{13}CN lines of the region. These lines, first used to advantage by Fujita and Tsuji (1976), offer an opportunity to use ^{12}CN and ^{13}CN lines having closely matching parameters ($\chi, W_\lambda, gf\varepsilon$), and so $^{12}C/^{13}C$ may be found without precise knowledge of $T_{\rm eff}$ and ξ_t.

A curve-of-growth approach to finding $^{12}C/^{13}C$ was used for the majority of the program stars. The analyses for two stars, HD 113801 and HD 156074, are shown in Figures 4 and 5, respectively. In these figures, the observational abscissa is designed to reference a ^{12}CN line's excitation potential, χ, to the mean of the ensemble of all ^{12}CN lines used. The excitation temperature is composed of a constant appropriate for weak lines (20 mÅ) and a theoretically calculated correction term, $\delta\theta(W_\lambda)$, which accounts for the cooling of $\theta_{\rm exc}$ experienced by stronger lines formed higher in the photosphere. A theoretical curve of growth for a fictitious line having the mean wavelength and excitation of the ^{12}CN line set is also shown. The model atmosphere from the Johnson grid denoted by $T_{\rm eff}/\log g/\xi_t$ in Figures 4 and 5 was used in the theoretical curve calculation. The same $T_{\rm eff}$ and gravity was assumed for all stars. This assumption recognizes the fact that the shape of the curve of growth does not depend critically on the choice of model parameters. Line-by-line analysis was used for those program stars having too few weak lines to define the curve of growth. The $^{12}C/^{13}C$ ratios from the CN analysis are contained in Table 3.

d) CO

The vibration-rotation bands of the ground state ($X^1\Sigma^+$) of CO provide an additional opportunity to determine $^{12}C/^{13}C$ in the atmospheres of the warm carbon stars. The second overtone ($\Delta v = +3$) bands in the 1.6 μm region are generally too weak and blended by CN to provide a useful source of isotopic information at low resolution. The first overtone ($\Delta v = +2$) bands near 2.3 μm, however, represent a valuable confirmation of the infrared CN results. These harmonic transitions may safely be assumed to be in LTE (Thompson 1973; Hinkle and Lambert 1975; Carbon, Milkey, and Heasley 1976).

e) *Observations*

The CO observations were obtained with the 1.8 m Fourier transform spectrometer (FTS) at the coudé focus of the KPNO 4 m Mayall telescope (Hall, Ridgway, and Yarborough 1978). The spectra were obtained during two observing sessions: 1980 April 24–27 and 1981 February 21–23. In each case a low-noise observation of an early-type star was used to eliminate telluric lines, the instrumental function, and filter transmission ripple by division. The observations have apodized spectral resolutions of 0.25–1.0 cm^{-1}. Signal-to-noise ratios are typically 60.

f) *Analysis*

The spectral resolution, except for the observation of HD 156074, is insufficient to resolve individual lines. Consequently, a spectrum synthesis attack is made. The region of

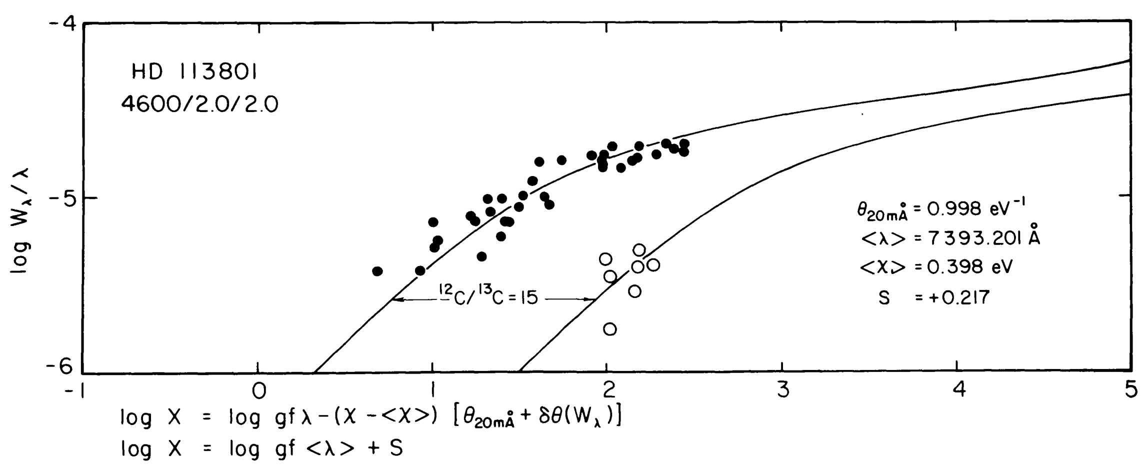

FIG. 4.—Analysis of the carbon isotopic ratio ($^{12}C/^{13}C = 15$) for HD 113801. Filled circles are ^{12}CN lines; open circles, ^{13}CN. Both theoretical and observational abscissae are indicated.

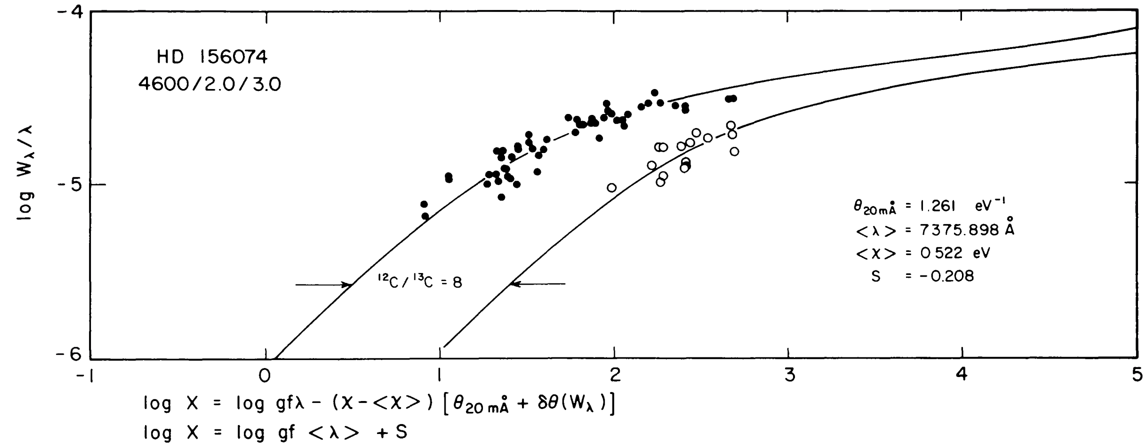

FIG. 5.—Analysis of the carbon isotopic ratio ($^{12}C/^{13}C = 8$) for HD 156074

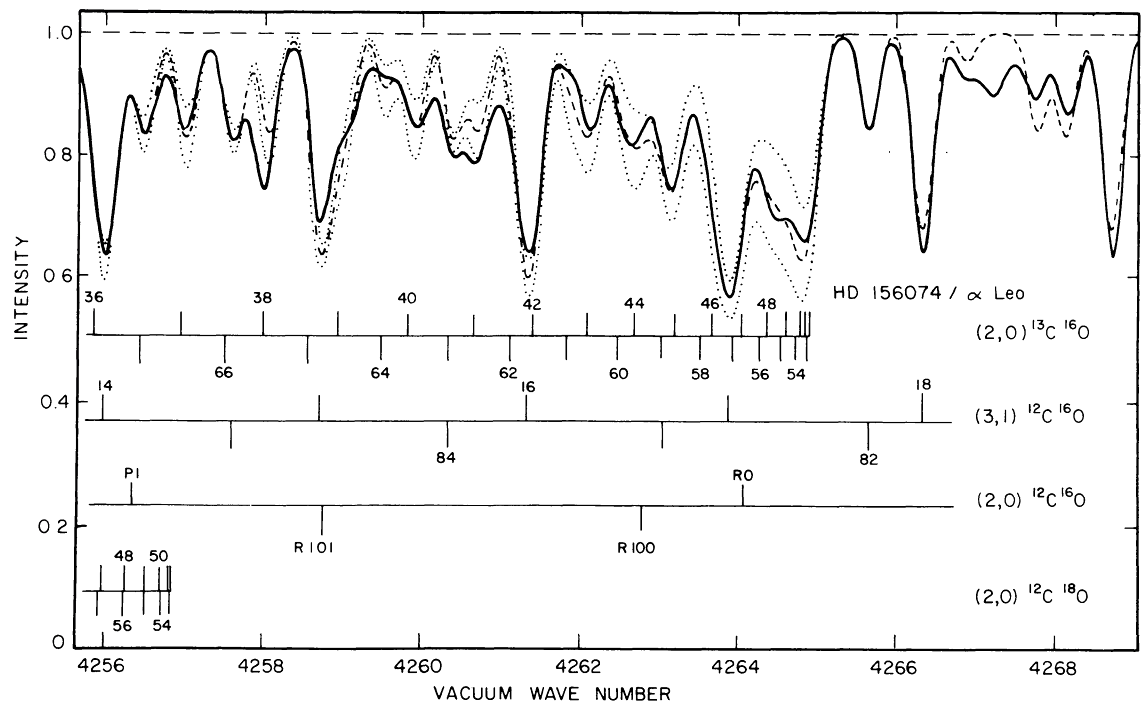

FIG. 6.—Determination of $^{12}C/^{13}C$ from CO lines in the spectrum of HD 156074. Ratios of 4 and 16 (*dotted curves*), as well as the adopted ratio of 8 (*dashed curve*), are shown on the observations (*solid curve*). Observed and calculated spectra are matched at the ^{12}CO (3,1) *R* 82 line.

synthesis is 4273–4254 cm^{-1}, encompassing the $^{13}C^{16}O$ (2,0) band head at 4264.7 cm^{-1}. The spectrum synthesis program MOOG (Sneden 1973; Luck 1977) was used. Transition wavenumbers and excitation potentials for the theoretical spectrum calculation were derived from the constants of Mantz *et al.* (1975) by Hinkle (1978). The *gf*-values for these CO lines were taken from Tipping (1980).

In the calculation of the synthetic spectrum the red system CN lines of the $\Delta v = -2$ sequence were omitted from the line list. The terrestrial oxygen isotopic ratios $^{16}O/^{17}O = 2696$ and $^{16}O/^{18}O = 500$ (Holden 1977) were assumed in the calculation. Using the appropriate interpolated model from the Johnson grid and the ξ_t found in the Fe I analysis (Table 2), a set of instrumentally broadened spectra of varying $^{12}C/^{13}C$ ratio was calculated for each star. The technique, as applied to HD 156074, is shown in Figure 6. The derived ratios for five stars were found this way, and are averaged with the CN results of Table 3. The agreement between the two methods is good.

g) Oxygen Isotopes

The low-resolution and signal-to-noise characteristics of the data do not support a detailed analysis of the oxygen isotope ratios. The $^{12}C^{17}O$ (2,0) band head at 4305.6 cm^{-1} coincides with the (3,1) band head of $^{12}C^{16}O$ at 4305.4 cm^{-1}. The (3,1) band head (4252.3 cm^{-1}) lies to the short wavelength side of the $^{12}C^{16}O$ (4,2) band head at 4250.9 cm^{-1} and is at 4204.6 cm^{-1}, both of which are relatively clear of strong lines of other isotopic forms of CO. Figure 6 includes the $^{12}C^{18}O$ (2,0) head. The spectrum synthesis of these regions indicates that the $^{17}O/^{16}O$ and $^{18}O/^{16}O$ abundance ratios can be no greater than a factor of 5 and 2, respectively, above the terrestrial values.

V. CNO ABUNDANCES

The analysis of the C, N, and O composition of carbon stars depends initially on a firm knowledge of the carbon abundance. In this study several spectral features sensitive to the carbon abundance were examined. The only oxygen abundance indicator available from the present data is CO. CN is the only usable carrier of nitrogen abundance information.

The carbon abundances are derived from a group of C_2 and CH blends. The chosen features are well isolated on 4 m echelle spectra and, in most cases, have continuum regions nearby (≤ 2 Å). The C_2 blends are found at 4936.7, 4963.0, 4992.3, 5344.3, and 5611.5 Å, and the CH blends occur at 4835.3, 4869.2, 4877.3, and 4878.7 Å. The blends contain lines of both ^{12}C- and ^{13}C-carrying molecules. The total equivalent widths of the features are listed by Dominy (1983). The molecular lines customarily used in the Sun and G–K giants (e.g., Greene 1969; Lambert 1978; Sinha 1979; Lambert and Ries 1981) are too strong for reliable analysis in the R stars' spectra. No feature having over 200 mÅ in equivalent width was used.

A possible secondary source of carbon abundance information is the C I line at 5380.32 Å. Owing to its high excitation potential (7.68 eV), the abundance inferred from this line is sensitive to errors in the adopted effective temperature and possible non-LTE excitation.

a) Analysis

The C_2 and CH blends were synthesized by program MOOG using the adopted model parameters for each star (T_{eff}, $\log g$, ξ_t, $^{12}C/^{13}C$) determined earlier. For a predetermined series of assumed O abundances the corresponding C abundance which reproduced the observed equivalent width was found.

The spectral feature used to fix the oxygen abundance was the $^{12}C^{16}O$ band head (4360.1 cm^{-1}). The entire span 4362–4305 cm^{-1} was synthesized using MOOG. The instrumentally broadened theoretical spectrum was compared directly with the FTS observations. A series of O abundances which match the observations were derived from a preselected set of C abundances. The simultaneous solution for both C and O are shown for three program stars in Figures 7–9 (where the plus [+] indicates the weighted mean abundances).

The nitrogen abundance of each star is found from the set of CN lines used in the $^{12}C/^{13}C$ analysis. Adopting the $^{12}C/^{13}C$ ratio, together with the derived C and O abundances, the mean nitrogen abundance was found by program LINES.

b) Results

The [C], [N], and [O] abundances (relative to the solar abundances of Lambert 1978) for five early R stars are given in Table 3. The oxygen abundance is, within the estimated errors, equal to the Sun's for four stars having CO data

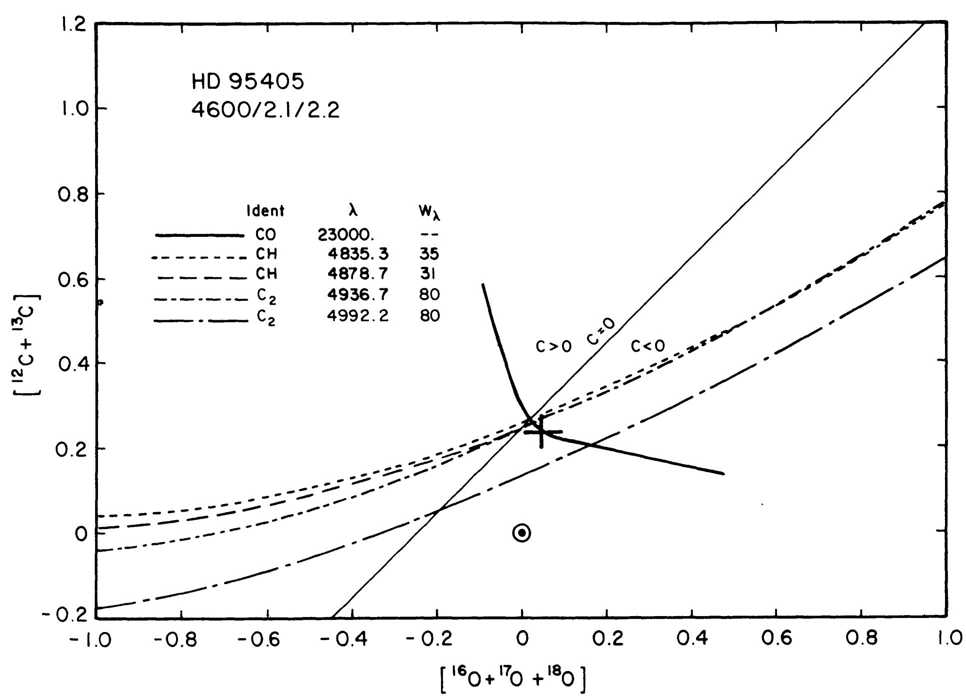

FIG. 7.—Simultaneous solution for [C] and [O] for HD 95405 (*plus*). Loci of [C], [O] abundance pairs which reproduce the observed equivalent widths (or CO band head depths) in the indicated model are shown for five spectral features. The minor oxygen isotopes, ^{17}O and ^{18}O, are considered to be solar.

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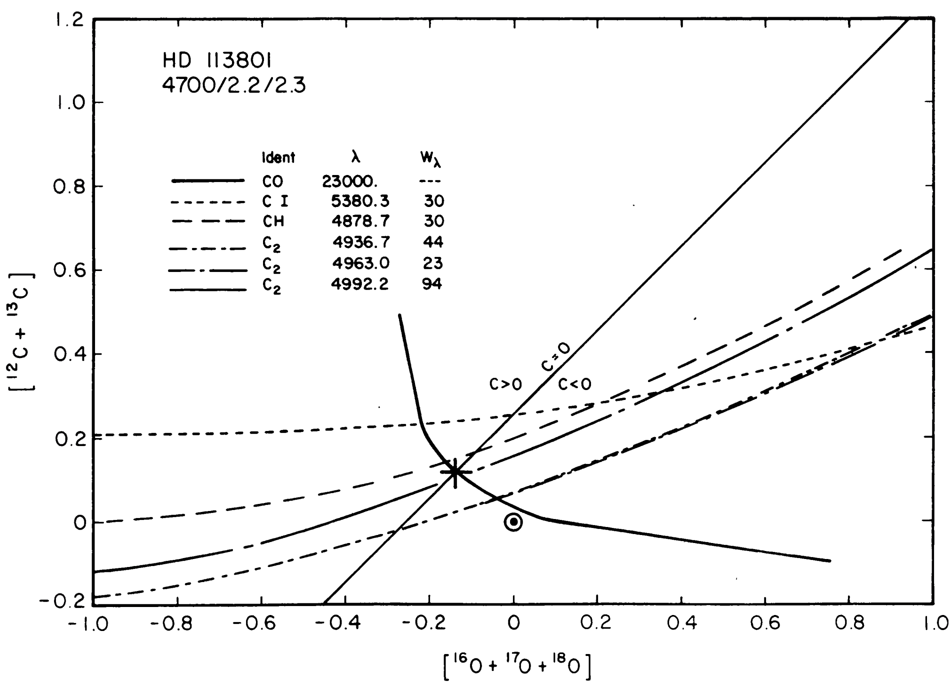

FIG. 8.—Simultaneous solution for [C] and [O] for HD 113801

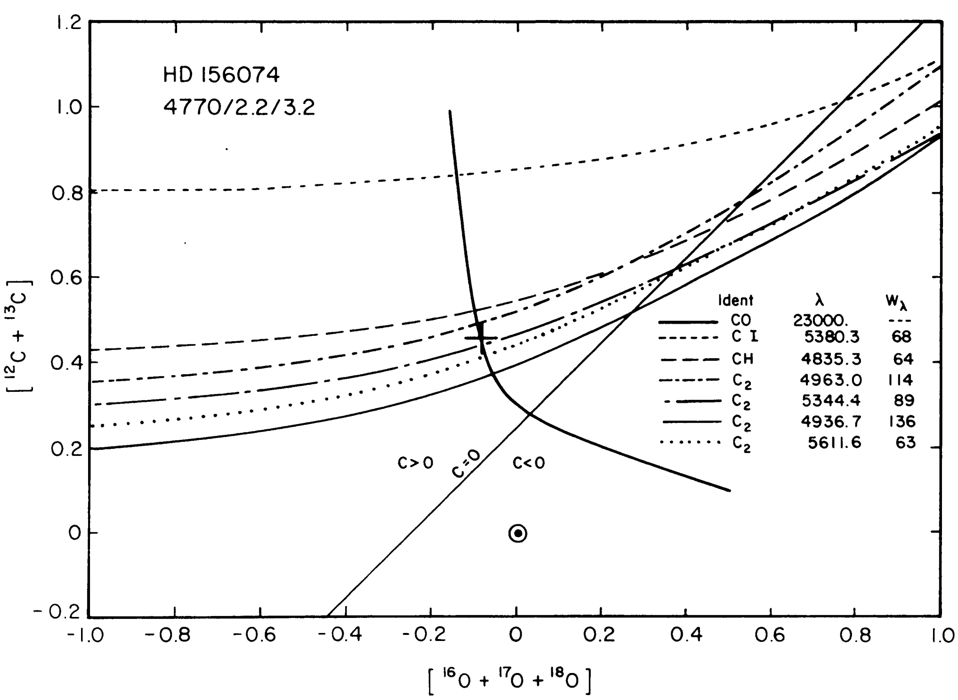

FIG. 9.—Simultaneous solution for [C] and [O] for HD 156074

available (HD 100764 is discussed separately below). In all stars carbon has been enhanced relative to the Sun and normal G–K giants. The average enhancement of nitrogen, [⟨N⟩], is +0.6 dex, i.e., only 0.2 dex greater than the average G–K giant.

One star, HD 95405, is probably not a carbon star (Fig. 7). The classification of this star as a CN-strong G8 IIIp giant by Schmitt (1971) is well justified. Yamashita's (1972) assignment of C2,0 for this star indicates the lowest carbon strength available in the classification system. At the resolution of 4 m echelle plates the C_2 (0,0) band head is 54% deep in the spectrum of HD 95405. Other program stars have observed depths as follows: HD 16115, 92%; HD 58337, 90%; HD 90395, 91%; HD 100764, 71%; HD 113801, 61%; and HD 156074, 74%; in contrast with β Gem, 31%. The C enhancement of HD 95405 of 0.2 dex indicates that it may be a precursor or weakly representative of the early R star phenomenon.

The analysis of HD 113801 (Fig. 8) indicates that C/O is near unity. The molecule CO is not fully associated in stars having the effective temperature of HD 113801. The model for HD 113801 (4700/2.2/2.8 C = O = 8.79, N = 8.62) indicates that carbon is 63% in C^0, 37% in CO, with the remainder distributed as 0.02% in CN, 0.006% in CH, and 0.003% in C_2, while O is 62% in O^0 and 38% in CO (at $\tau_{5000} = 0.11$, where CO reaches its maximum number density). For this reason, the dramatic changes in spectral appearance with small changes in C/O ratio experienced by the cooler, lower gravity S stars (Scalo 1974*b*; Scalo and Ross 1976; Wyckoff and Clegg 1978; Piccirillo 1980) are not seen in the early R stars. The weakness of CN in the spectrum of HD 113801 is obvious in Figure 3.

Another noteworthy star is HD 100764. The (2,0) band head of ^{12}CO has a central depth of only 14%, compared with 49% for HD 95405 and 60% for the C2,3 star HD 85066 obtained at the same resolution (0.5 cm^{-1}). This weakness is a manifestation of the infrared flux excess; i.e., a continuous contribution dilutes the photospheric line spectrum. The synthesis of this band head region implies a differential abundance, [O] ≈ −1.3 dex (C/O ≈ 0.7). Lacking detailed spectrophotometry, an estimate of the amount of continuum flux may nevertheless be made by finding the excess of $V-K$ over the $V-K$ of an oxygen-rich red giant of the same temperature. This procedure rests on the hope that neither the V nor K bands are blanketed differently in the C- and O-rich stars. A normal red giant with $T_{\rm eff} = 4850$ K has $V-K = 2.26$ (Ridgway *et al.* 1980). Thus, HD 100764, is 0.92 mag (2.3 times) brighter at K than predicted. The adopted level of zero photospheric flux in the CO spectrum of HD 100764 was altered to reflect the continuous contribution, and the band head was resynthesized. The new estimate of [O] is −0.5 (C/O = 5.6), which is near the −0.6 dex value found for [Fe], and so iron and oxygen are similarly underabundant relative to the Sun in this crude estimate.

The finding that [O] ≈ 0 for the early R stars makes available an opportunity to find the carbon and nitrogen abundances when CO data are not available. Previous analyses of carbon stars have adopted this assumption or estimated [O] from the sensitivity to the C/O ratio displayed by different carbon-containing molecules (e.g., Kilston 1975; Greene *et al.* 1973). The direct observation of [O] for a sample of classical R stars places the assumption on firmer ground here. The [C] and [N] abundances indicated by asterisks in Table 3 have been derived under this assumption of solar oxygen and are included for comparison.

A comparison of the values listed in Table 3 indicates that no apparent correlation exists among the abundances. For example, the relatively high $^{12}C/^{13}C$ ratio of HD 113801, together with its small C enhancement, do not demonstrate an extremum of a general trend of decreasing $^{12}C/^{13}C$ with increasing [C]. It should be noted, however, that the range of [N] (or N/Fe) is about half that found for [C] (or C/Fe).

VI. UNCERTAINTIES, PREVIOUS WORK

a) Uncertainties

The statistical errors in this analysis are estimated from two sources: (i) data quality (including signal-to-noise ratio, continuum setting uncertainty, and equivalent width measurement and calibration); (ii) errors in the adopted model parameters. The errors that have been quoted in Table 3 are larger than those usually stated for G–K giants. This tendency to derive larger error estimates is primarily a direct result of data quality. With increasing blanketing, the error caused by continuum placement dominates. With the exception of the coolest stars (e.g., HD 16115) where the carbon-rich and oxygen-rich model $T(\tau)$ structures differ the most, the C/O ratio assumed in the model atmosphere has little influence on the inferred abundances. In this respect, had the program stars been analyzed with standard oxygen-rich models, the results would not be very different. The large uncertainties associated with the Ti and *s*-process abundances, in comparison with those of Fe, indicate the effect of fewer numbers of lines used for these species.

The relatively small error estimates (±2 to 3) for the $^{12}C/^{13}C$ ratios in Table 3 reflect the use of appreciable numbers of CN lines. When these estimates are larger (~5), as for the more blanketed stars, the use of a few strong lines makes the ratio determination more uncertain. Any component of error contributed by model parameters (except ξ_t) has no serious influence on the derived isotopic ratio.

Among the C, N, and O abundances, the C abundance is the most secure. It is derived from the strengths of several low-excitation (and often weak) features of carbon-containing molecules. The oxygen abundance rests on the modeling of a single band head, and so its precision is influenced by data quality (in this case, spectral resolution and incomplete division of telluric lines). The nitrogen abundance is made uncertain, not only by data quality, but also by the propagation of the errors in [C] and [O] and the error committed by assuming $\log g$. The luminosity effect in derived [N], $\partial[\mathrm{N}]/\partial \log g$, is estimated to be 0.2 dex dex^{-1} at $T_{\rm eff} = 4700$ K. The mean [N] enhancement of the normal G–K giants, i.e., +0.26 dex (Ries 1981; Lambert and Ries 1981), and the +0.6 dex average enhancement found here for the early R stars should be judged as a "one sigma" difference.

One star, HD 100764, may have larger real errors than the other program stars. The value of $\log g = 2.2$ found from the assumption $M = 1.1\ M_\odot$ may be too large. A crude analysis of [Fe] based on six Fe II lines indicates that $\log g = 0.2$ (± 1.3 s.e.) to establish Fe^0/Fe^+ ionization equilibrium in LTE. The [Fe] and [Ti] analyses for all program stars are based on neutral lines, and so have low sensitivity to errors in log *g*. The *s*-process analysis is more strongly tied to the analysis of lines from the singly ionized species, however.

b) *Earlier Work*

The results of the metal abundance analyses are in agreement with earlier published work. It has been known from classification studies that the early R stars are not, on average, underabundant in iron-peak elements or overabundant in those elements with isotopes formed by the *s*-process. The [Fe] abundance reported for HD 156074 by Greene *et al.* (1973), -0.10, is the same as the value found here.

With the exception of HD 156074, $^{12}C/^{13}C$ ratios have not been previously reported for the program stars. For this star, the low ratios (e.g., 2.4–3.2, McKellar 1948; 5.3, Wyller 1966) are ruled out by the higher resolution study undertaken here. Likewise, the large ratio ($^{12}C/^{13}C = 16$) found by Fujita and Tsuji (1976) would require an excitation temperature precluded by the adopted effective temperature. The ^{13}C index of Gordon (1967, p. 85) shows good correlation with the $^{12}C/^{13}C$ ratios found here. For HD 113801 ($^{12}C/^{13}C = 15$) she assigns the lowest ^{13}C index, i.e. 1, and for BD +33° 2399 ($^{12}C/^{13}C \approx 5$), she assigns 4. The *J*-type (^{13}C-enhanced) nature of HD 16115 and HD 58337 found in classification studies (Gordon 1967, Yamashita 1972) is confirmed here.

The only program star for which C, N, and O abundances have been previously derived is HD 156074. In comparing the [C], [O] abundance solution plane used here (Fig. 9) with that of Greene *et al.* (their Fig. 1), it is seen that the loci of CH and C_2 features measured here follow closely their solution for the 5000/2.6 model (after adjustment for differences in assumed solar C and O abundances). Both analyses attempted to use the C I 5380.32 Å line (with equivalent widths of 75 and 68 mÅ). In Figure 9 it is seen that this high excitation line does not intersect the molecular line loci anywhere in the C > O region of the [C], [O] abundance plane for the adopted model. A demonstration of the temperature sensitivity of the C I line is the fact that an increase in T_{eff} to 4950 K (about one standard deviation from the adopted T_{eff}) is needed to bring the C abundance implied by C I into agreement with the molecular lines. The possible non-LTE excitation of this line rules out its use in an LTE analysis as a reliable carbon (or, as in the method of Greene *et al.*, an oxygen) abundance indicator.

Yorka (1981, p. 146), using a narrow-band scanner photometric system, concluded that ⟨N⟩, based on the NH index, is enhanced in six of the early R stars relative to main-sequence stars, but not to the extent found in the normal giants. This conclusion is the result of her inclusion of the O-rich star HD 95405 and the H-deficient supergiant HD 182040 in the sample. Omitting these stars, as well as the metal-deficient star HD 100764 which has the lowest derived N abundance reported here, the remaining three R stars in Yorka's sample (HD 16115, HD 113801, and HD 154124) have NH indices lying well above the average G–K giant's.

VII. EVOLUTIONARY CONSIDERATIONS

Since the early R stars are not rare, amounting to ~1% of the G–K giants, they therefore represent a stage of evolution that is available to an appreciable fraction of stars, and are not the result of anomalous initial conditions or statistically unlikely events. It has already been demonstrated that the early R stars (along with the Ba II stars) are not currently in an episode of He shell pulses (Scalo 1976; Scalo and Miller 1979). It may then be supposed that the early R stars have undergone less evolution (fewer stages of shell burning and nucleosynthesis) than the more luminous peculiar red giants of types M, N, and S. These two starting points to understanding the early R stars presuppose that the R-star phenomenon is not a transitory phase of every red giant's evolution (e.g., Wallerstein 1973). For comparison, Table 4 collects the C, N, and O abundances found in the G–K giants, the Ba II stars, and the early R stars. The abundances found in this study offer constraints to certain scenarios that have been offered to account for the existence of the warm, faint carbon stars. Four of these evolutionary suggestions are discussed here.

a) CNO *Cycle*

The low $^{12}C/^{13}C$ ratios found by the earliest investigators (e.g., McKellar 1948; Climenhaga 1960) indicated that the carbon-rich nature of the early R stars must be a result of the CNO cycle operating at equilibrium. Later Greene *et al.* (1973) found a depletion of oxygen of nearly 1 dex in HD 156074, seemingly pointing to destruction of O by CNO burning. Greene *et al.*, however, note that the N/O ratio they derived (20) is much smaller than the predicted ratio (47 when the CNO cycle has made C/O = 2; Irgens-Jensen 1976). Continuous deep mixing, possibly by meridional currents, offered a plausible explanation for the early R stars. The understanding that the early R stars cannot now be in a state of shell helium burning and the fact that the earliest theoretical results on the core flash (e.g., Schwarzschild and Härm 1962; Thomas 1967) indicated that the flash did not promote mixing added further hope that the CNO hypothesis would explain the early R stars.

The finding here that the oxygen abundance in several early R stars is near the solar value casts considerable doubt on the idea that the mixing of CNO-equilibrium material is responsible for the early R stars. At CNO equilibrium both C and O are depleted, but the oxygen found in the early R stars has not been significantly altered from the solar system value by evolution. At the temperatures prevailing in the H-burning shells of red giants with inert He cores (30 to 60×10^6 K), there is no plausible method to recreate ^{16}O after proton capture has destroyed it in the CNO cycle. Only at temperature above 100×10^6 K can α-captures by ^{12}C generate ^{16}O. The fact that C exceeds the solar value (and that found in the mixed O-rich K giants) is also indicative of the failure of the CNO hypothesis in accounting for the classical warm carbon stars.

In the CNO cycle the production of ^{17}O is a result of both equilibrium and nonequilibrium H-burning. The destructive reaction, $^{17}O(p,\alpha)^{14}N$, is slow, and so the $^{17}O/^{16}O$ ratio may become detectable. The lack of a detectable $^{12}C^{17}O$ band head

TABLE 4

C, N, O ABUNDANCE COMPARISON

Group	[C/H][a]	[N/H]	[O/H]	[C/Fe]	[N/Fe]	[O/Fe]	$^{12}C/^{13}C$
26 G–K Giants[b]	−0.35	+0.26	−0.03	−0.24	+0.38	+0.08	16
Three Ba II Stars[c]	−0.05	+0.21	−0.15	+0.04	+0.30	+0.06	21
Three Early R Stars[d] ...	+0.36	+0.70	−0.10	+0.47	+0.80	+0.01	9
HD 156074	+0.46	+0.64	−0.08	+0.56	+0.74	+0.02	8

[a] [C/H] is [^{12}C/H] for the G–K giants and Ba II stars, but [(^{12}C+^{13}C)/H] for the R stars.

[b] Values normalized to H are taken from Ries 1981, values relative to Fe from Lambert and Ries 1981.

[c] Omitting the bright giant ζ Cap; values from Sneden, Lambert, and Pilachowski 1981 based on the lower temperature scale and [M] = [Fe].

[d] HD 113801, BD +33° 2399, HD 156074.

also speaks strongly against a purely equilibrium CNO interpretation.

b) *Mass Transfer*

There is a current suggestion that the Ba II stars are the result of the transfer of processed envelope material from a now invisible (or white dwarf) companion (McClure, Fletcher, and Nemec 1980). Can the early R-type stars be the result of mass transfer from a former N-type star? The suggestion that the N stars can donate a portion of their envelopes to an available companion seems plausible when the relative numbers of stars are considered. Blanco (1965) estimates that R/N ≈ 10. If the more massive N stars evolve through the N-star stage (from M giant, to N giant, to planetary nebula and carbon white dwarf) 10 times more rapidly than their companions, the observed R/N ratio can be established. The circumstellar material generating the infrared flux excesses of the early R stars thus finds an explanation as the portion of the planetary nebula remnant of the old N star not captured by the present R star. The fact that planetaries are carbon-rich (Aller and Czyzak 1983) adds additional support to the idea.

The near-planar galactic distribution of the N stars is indicative of a younger population than that of the early R stars. In the framework of a mass-transfer explanation, the N stars responsible for the current early R stars must be older, and, perhaps, more carbon-rich than the presently observable N stars. In this respect, the N stars making the R stars must be more like the rarer N-type long-period variables which are not confined to the plane.

If the transport of N-star material into a K giant's envelope[2] is to account for the early R stars, that material must be extremely carbon-rich. There is little available information on the C/O ratios of the N stars. The C/O estimates of Kilston (1975) for eight stars, of Gow (1977) for 61 stars, and of Johnson *et al.* (1983) for two stars indicate that the ratio is probably less than 2.5 and rarely exceeds 5.

Suppose 0.5 $M_{\odot}$ of an N star's envelope, with C/O = 5 (and solar system O), is mixed with 0.7 $M_{\odot}$ of a "typical" red giant's envelope (with C/O = 0.27; Lambert and Ries 1981). The adoption of 0.7 $M_{\odot}$ for the receiving star is based on the fact that the convective envelope extends down to a mass fraction of 0.3 in a 1 $M_{\odot}$ red giant (Iben 1967), and we assume total envelope mixing. Introducing the notation $Z_p(\mathrm{C})$ for the mass fraction of carbon contributed by the mass-losing N star (mnemonic, p = "processed") and $Z_e(\mathrm{C})$ for the mass fraction of carbon in the receiving star's envelope, we have

$$Z(\mathrm{C}) = f_e Z_e(\mathrm{C}) + f_p Z_p(\mathrm{C}). \quad (1)$$

Here $Z(\mathrm{C})$ is the (observable) mass fraction of carbon resulting in the fully mixed envelope and the factors f_p and f_e ($=1-f_p$) are the relative mixing fractions of each type of material. In this example, with $f_e = 0.58$, $f_p = 0.42$, $Z_e(\mathrm{C}) = 0.0025$, and $Z_p(\mathrm{C}) = 0.0208$, we have that $Z(\mathrm{C}) = 0.01$, or [C/H] = +0.38, close to the average seen in the early R stars (Table 4).

All of the N stars appear to have enhancements of Tc and other *s*-process elements by as much as 1 dex (Utsumi 1970; Perry 1971; Perry, Keenan, and Marenin 1971; Kilston 1975; Yamashita 1972, 1975). In the contaminated 1.5 $M_{\odot}$ example here, the Tc would have time to decay, but the enhancements of the stable *s*-process nuclei would be 0.6 dex. This enhancement is larger than the expected errors of the determination of [⟨s⟩] found in § III. In a similar argument, the large Li abundances attributed to many N stars (Torres-Peimbert and Wallerstein 1966; Warner and Dean 1970) may be expected to be transported to the R star. Only if the N-star Li is confined to the stellar surface, or is destroyed in the transfer process, could the mass-transfer hypothesis explain the lack of R-star Li enhancement.

This need for an amount of new C-rich mass equal to that of a red giant's envelope places severe constraints on the mass-transfer hypothesis. First, the N stars responsible for the R stars must have carbon abundances in excess of the average field N-type stars observed. An increase in carbon abundance just prior to the planetary nebula stage of an N star could produce this needed observational constraint. Second, in order for efficient mass transfer to occur, the components must be close so that transfer takes place through the inner Lagrangian point of the system, and not as the capture of a more isotropic stellar wind available in a much wider system.

A radial velocity survey of a sample of early R stars would be helpful in further resolving the duplicity question for close binaries. An attempt to detect a white dwarf companion to HD 156074 did not meet with success (Dominy and Lambert 1983). An examination of the infrared flux excess of the early

[2] Or, into the envelope of a main-sequence star thus hastening its motion up the giant branch and so eliminating the potential for dwarf R stars.

R stars to determine if signatures of a carbon- or oxygen-rich composition are present would be fruitful. Such observations might answer the question of whether the circumstellar material comes from a former N-star companion or, instead, from a single R star at an earlier, O-rich, epoch.

c) *Single-Star Mass Loss and Post Red Giant Evolution*

The observation that the early R stars are not *currently* in a stage of double-shell source evolution does not rule out the possibility that they *once* were. If the present early R stars have evolved to high luminosities (as, say, M giants), and then, during violent shell flashes lost much of their envelope mass, they could appear now as carbon-rich faint giants with circumstellar shells. This premature creation of a pulsing, thin envelope star should produce observable consequences, including, most likely, an observable planetary nebula.

It may be expected that if substantial envelope loss occurs, then the remnant star will be hydrogen-deficient. This is the case for the R CrB–type stars (e.g., Feast 1975). The exhaustion of fuel for additional burning would ensure rapid evolution of such an object to the white dwarf stage. The numbers of early R stars are greater than can be provided by such rapid evolution.

d) *The* He *Core Flash*

With the CNO, mass-loss, and mass-transfer scenarios encountering difficulties in explaining the existence of the early R stars, we turn to an examination of mixing at the He core flash. At the outset the supposition of mixing at the time of the flash is promising: (i) carbon is produced via 3α He-burning; (ii) recent theoretical work indicates that the flash is an energetic event, with sufficient energy available to engender strong global mixing; (iii) it may be energetic enough to initiate an episode of mass loss and so explain the observed circumstellar emission.

The degenerate He core flash was originally invoked to explain the Ba II stars and Population II CH stars (Burbidge *et al.* 1957; Wallerstein and Greenstein 1964; Caughlan and Fowler 1964). Chemical evolution of stars after the flash is imagined to follow three paths. The first, followed by most stars, is one in which little or no mixing occurs and core He burning produces an ordinary G–K clump giant. A second path yields the Ba II stars of Population I and CH stars of Population II. In this event an injection of protons from the hydrogen shell into carbon-rich material from the core occurs. The abundance of ^{13}C is increased by the reaction sequence $^{12}C(p,\gamma)^{13}N(\beta^+\nu)^{13}C$. The ^{13}C, in turn, produces neutrons for *s*-process nucleosynthesis by way of the α-capture $^{13}C(\alpha,n)^{16}O$. Depending on the amount of ^{12}C and ^{13}C destroyed, the result of mixing these products to the surface may yield either a C-rich (CH star) or O-rich (Ba II star) atmosphere.[3] A third result of the He core flash could be the early R stars. In this case more extensive and more rapid mixing occurs. The introduction of protons into core material (^{12}C and ^{4}He) yields only enhanced ^{12}C, and ^{13}C, with perhaps a small enhancement of ^{14}N in the photosphere. This is the suggestion favored here.

In contrast with previous theoretical work, Cole and Deupree (1981) and Deupree and Cole (1981, 1983) find that convection fails to contain the flash-generated heating, resulting in the formation of hydrodynamic flow. The flash is not spherically symmetric but instead starts as a set of point-source explosions and continues as a propagating burning front. They find that the core contained interior to the hydrogen shell has been processed via α-captures, $^{12}C(\alpha,\gamma)^{16}O(\alpha,\gamma)^{20}Ne(\alpha,\gamma)^{24}Mg(\alpha,\gamma)^{28}Si$. The distribution (by mass) in the core ranges from 50% to 97% He and from 40% to less than 1% Si, depending on the location of the core boundary. The mass fraction of ^{12}C in the outer core is always ~ 3%. The heaviest α-rich nuclei are located near core center, while the He and C remain in the outer core.

The published hydrodynamic treatments of the core flash assume a Population II composition and do not include the hydrogen-burning shell. In one off-center calculation (Deupree and Cole 1983) a bubble of low density and high temperature ($T \approx 450\times10^6$) formed and rose to the vicinity of the H-burning shell. The velocity of the bubble seems large enough to effect penetration and mixing of core and shell matter.

These recent theoretical calculations of the core flash offer a strong promise of understanding the early R stars. If, for ~1% of normal giants, the core flash does present C-rich material into the H-burning shell, there is every reason to believe that the results of 3α and CN-processing can reach the envelope's convective zone and ultimately the surface. This scenario of core material leaving the confines of the core and undergoing proton capture on its way to the surface provides a more realistic explanation than the early suggestions of the mixing *downward* of envelope hydrogen, followed by the mixing *upward* of processed material.

The core composition predicted by the violent core-centered flash, if mixed to the surface, is at variance with the observations. In HD 156074, for example, the series of α-captures seen in the theoretical results is not confirmed by the near-solar abundance of ^{16}O found here and the near-normal values found for Mg (factor of 3) and Si (factor of 2) by Greene *et al.* (1973). For consistency between the observations and the theory of the violent core flash, one would have to insist that the α-capture chain ended at the observationally inaccessible ^{20}Ne or that mixing of only the outermost core occurs.

Disappointingly, the observations do not conform with the results of the off-center, lower temperature, flash calculations. Too little carbon is produced. Using equation (1), now with Z_p(C) indicating the mass fraction of proton-exposed core material finding its way into the envelope (mnemonic, p = "proton") and ignoring any contribution of the H-burning shell to the final composition, we can write,

$$f_p = \frac{Z(C) - Z_e(C)}{Z_p(C) - Z_e(C)}. \qquad (2)$$

Upon inserting $Z(C) = 0.01$ (typical of the early R stars), $Z_e(C) = 0.0025$ (typical of the envelopes of G–K giants), and $Z_p = 0.03$ from Cole and Deupree's (1983) case NC1, we

[3]It has been suggested (Smith and Demarque 1980; Luck and Bond 1982) that even more extensive mixing produces the subgiant CH stars in which core H-burning resumes.

obtain $f_p = 0.3$, a large mixing fraction of core material. This demand that 30% of the resulting core + envelope mixture be contributed by the core is equivalent to the requirement for very high C/O ratios in the N-star companion in the mass-transfer hypothesis. With $Z_p(\text{He}) = 0.97$, $Z_e(\text{He}) = 0.09$, and $f_p = 0.3$, the predicted fully mixed He content is $Z(\text{He}) = 0.35$. This three-fold He enhancement is inconsistent with the lack of a detectable departure of the Ca abundance from the solar value (§ II*g*).

VIII. CONCLUSIONS

In light of recent core flash calculations, the abundance pattern found here for a sample of early R stars leads to two related conclusions. First, if internal mixing is responsible for their existence, the temperatures at which the processing occurs must be low. In order to avoid the destruction of the ^{12}C produced by 3α burning, the flash temperature must be less than 200×10^6 K in the core material eventually reaching the base of the convective envelope. After core ^{12}C has been exposed to envelope protons, and much of it converted to ^{13}C, to prevent the formation of neutrons via $^{13}\text{C}(\alpha, n)$ and so the creation of *s*-process nuclei, the temperature in the mixing zone must not exceed 100×10^6 K. During the exposure to envelope hydrogen in the processing zone, in order that $^{12}\text{C}/^{13}\text{C}$ be reduced to near 9 and yet ^{13}C and ^{14}N not be destroyed, a temperature not exceeding 70×10^6 K is necessary.

The second conclusion is that more carbon must be produced by the core flash than hitherto found in the hydrodynamic treatments of the event. (Alternatively, a less extensive convective envelope than assumed here can resolve the inconsistency). The carbon abundances inferred from the C-containing molecules in these giants, coupled with a lack of a detectable He enhancement, indicates that most of the He (at least near the core surface) must be burned to ^{12}C. This indicates that (in the absence of *severe* mass loss) the core flash occurs in the outer (cooler) parts of the core with substantial ^{12}C production and that an appreciable fraction of this new ^{12}C is injected into the envelope where it is exposed to warm protons.

The conditions under which the core flash occurs in the 1% of red giants becoming R stars may be generated by rapid core rotation. As investigated by Mengel and Gross (1976), the noncentral flash within a rapidly rotating core will occur farther from core center, and at lower temperatures than in the nonrotating case. The rotating core is not spherically symmetric, and so neither is the He-burning zone; a fact important in the hydrodynamic development of the flash. If the flash occurs near the core boundary, the flash may provide enough energy to relieve electron degeneracy and initiate mixing only at the boundary, leaving the bulk of the core unaffected. In this event, as the core mass is reestablished from additional H-burning, another flash is possible. A series of flashes could then build up the envelope C abundance through successive flash and mixing events. Providing the best potential for the explanation of the early R-star phenomenon, the dynamics of the degenerate He core flash, including core rotation, deserve further investigation.

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