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COSMOLOGICAL CONSEQUENCES OF POPULATION III STARS

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ABSTRACT

We examine the possible cosmological consequences of Population III stars. Their dark remnants could provide the "missing mass." Their light could have produced either the entire 3 K background or observable distortions in its spectrum. Their heat or explosions could have reionized the universe and perhaps produced galaxies. Their helium yield could suffice to provide an alternative explanation for the observed "primordial" abundance, and their heavy-element yield could have produced a burst of pregalactic enrichment. We discuss which sort of stars could best fulfill these roles and conclude that the most plausible candidates would be "Very Massive Objects" in the mass range 10^2–10^5 $M_\odot$. Even if Population III stars played none of these roles, consideration of the associated effects places strong constraints on their mass spectrum and formation epoch.

Subject headings: cosmology — stars: interiors — stars: massive — stars: stellar statistics — stars: weak-line

I. INTRODUCTION

In this paper we will examine some of the cosmological consequences of Population III stars. The existence of such stars has been postulated for many different reasons: to explain the missing mass in clusters of galaxies and galactic halos (White and Rees 1978), to generate the 3 K background (Layzer and Hively 1973; Carr 1977*b*; Rees 1978) or the distortions in its spectrum (Rowan-Robinson, Negroponte, and Silk 1979; Puget and Heyvaerts 1980), to produce a burst of initial enrichment (Truran and Cameron 1971) or the cosmological helium abundance (Talbot and Arnett 1971), to reionize the universe (Doroshkevich, Zel'dovich, and Novikov 1967; Hartquist and Cameron 1977; Hogan 1979), or even to make galaxies themselves (Ostriker and Cowie 1981; Ikeuchi 1981; Hogan 1983; Carr and Rees 1983). In this paper we wish to put all these ideas together and examine how many of the cosmological roles envisaged for Population III stars are mutually compatible (cf. Tarbet and Rowan-Robinson 1982).

A question of particular interest concerns the epoch of Population III star formation. Many people have suggested that they could be pregalactic in origin, i.e., forming in the period 10^6–10^9 yr after the big bang. Not all the roles assigned to Population III stars require this, but certainly some of them do. One reason for expecting pregalactic stars to form is that the existence of galaxies implies that the early universe must have contained density fluctuations. Providing the fluctuations were isothermal, the form of the fluctuations required on a galactic scale and above, if extrapolated to smaller scales, would be of order unity on a scale 10^6–10^8 $M_\odot$ (Peebles 1974; Fall 1979). Thus bound regions could form well before galaxies, and one would expect these regions to fragment into stars. Calculating the characteristic mass of the fragments is complicated but the initial absence of metals, the influence of the background radiation, and the lack of initial substructure would all tend to make the stars considerably more massive than those which form in the present epoch and possibly bigger than 10^2 $M_\odot$ (Matsuda, Sato, and Takeda 1969; Yoneyama 1972; Silk 1977; Tohline 1980; Kashlinsky and Rees 1983; Silk 1983). If the initial density fluctuations were adiabatic, they would be erased by photon diffusion on subgalactic scales (Silk 1968) unless the mass of the universe were dominated by certain types of collisionless particles (Bond and Szalay 1983). Without such particles, one would get the usual "pancake" picture (Zel'dovich 1970). Even here, pregalactic stars could form in the initial pancake fragments, though in this case the stars would be forming at a relatively low redshift.

The possibility that Population III stars could be as massive as 10^2 $M_\odot$, independent of whether or not they are pregalactic, raises a problem in that the evolution of such stars is not well understood. Unlike stars smaller than 10^2 $M_\odot$, which burn stably until they reach their iron/nickel core phase (Arnett 1973; Weaver, Zimmerman, and Woosley 1978) or undergo thermally unstable electron degenerate ignition (Arnett 1969; Sugimoto and Nomoto 1980), and stars larger than 10^5 $M_\odot$, which go unstable to general relativistic instabilities even before igniting their nuclear fuel (Fowler 1966; Fricke 1973), stars in the intermediate-mass range go unstable to pair-production effects in their oxygen core phase (Fowler and Hoyle 1964). The study of such "Very Massive Objects," or VMOs as we term them, has been somewhat neglected. This is partly because there has been no obvious observational evidence that they exist, except perhaps in 30 Doradus (Cassinelli, Mathis, and Savage 1981) or η Carinae (Andriesse, Donn, and Viotti 1978), and partly because theoretical reasons have been voiced for why they could never form. However, in the pregalactic context at least, these reasons need no longer apply: the stars would have burnt out long ago (so that their present absence need cause no embarrassment), and various cosmological effects could inhibit fragmentation into smaller stars.

TABLE 1

PROPERTIES OF DIFFERENT TYPES OF STARS

Name	Mass Range	Defining Characteristics	Fate
SMO ..	$>10^5\ M_\odot$	relativistically unstable; never dynamically stable; fate decided at H ignition	black holes for Pop. III stars; complete disruption is possible for lower masses and Pop. I metallicity
VMO ..	10^2–$10^5\ M_\odot$	pulsationally unstable during H burning; proceed to pair-unstable O core	$M_O \approx 30\ M_\odot$: black hole?; $M_O \approx 30$–$10^2\ M_\odot$: complete disruption; $M_O > 10^2\ M_\odot$: complete collapse
MO	8–$10^2\ M_\odot$	stable burning until Fe core phase (12–10^2 $M_\odot$) or Ni core phase (8–12 $M_\odot$)	neutron star or low-mass black hole; SN II?
	4–8 $M_\odot$	degenerate C ignition	no remnant; SN I?
LMO ..	0.5–4 $M_\odot$	planetary nebula formation at 2 shell phase	carbon/oxygen white dwarf
	0.08–0.5 $M_\odot$	no instability; do not burn He	helium white dwarf
	$<0.08\ M_\odot$	no instability; never ignite H	hydrogen white dwarf or planet

These considerations motivate us in another paper (Bond, Arnett, and Carr 1984, hereafter BAC) to examine the evolution of VMOs in some detail. One of the most important questions we address concerns the end-state of VMOs. Numerical studies (Barkat, Rakavy, and Sack 1967; Fralay 1968; Arnett 1973; Wheeler 1977) indicate a general consensus that sufficiently large VMOs collapse to black holes, while smaller ones explode. By treating the VMO oxygen core as an isentropic $n = 3$ polytrope, we are able to explain this behavior by simple energetic-entropic arguments. We calculate the critical oxygen core mass which divides collapse from disruption, M_{OC}, to be about 100 $M_\odot$, a value which accords well with numerical results (Woosley and Weaver 1982; Öber, El Eid, and Fricke 1983). The critical mass associated with the initial hydrogen star, M_C, is very uncertain because VMOs are radiation dominated and therefore unstable to nuclear-energized pulsations (Schwarzschild and Harm 1959; Stothers and Simon 1970). While these pulsations are unlikely to be completely disruptive (Appenzeller 1970; Ziebarth 1970; Talbot 1971; Papaloizou 1973), they probably result in considerable mass loss during the hydrogen and helium burning phase, so M_C could be much larger than M_{OC}: it would have to be at least 200 $M_\odot$ even if there were no appreciable mass loss.

We have summarized elsewhere (Arnett, Bond, and Carr 1982; Carr, Arnett, and Bond 1982) a number of arguments for believing that the first stars may have been VMOs, and our present considerations will place considerable emphasis on this possibility, exploiting the results of BAC. Nevertheless, we do not commit ourselves in advance to the hypothesis that Population III stars were VMOs. After all, Population III stars could span a wide range of masses, and which range is most important may depend upon which effect is under consideration. We may loosely classify stars into four categories: Supermassive Objects (SMOs), Very Massive Objects (VMOs), Massive Objects (MOs), and Low-Mass Objects (LMOs). The defining characteristics and fates of these objects are indicated in Table 1. The fates should be regarded as merely indicative, since many uncertainties arise in all mass ranges. Some of the boundary masses are also uncertain: in particular, the 4–8 $M_\odot$ range may be much narrower.

The most unpredictable factor in determining the possible cosmological effects of Population III stars is their mass spectrum. For simplicity, we will often assume that the number of stars in the mass range M to $M + dM$ has the form

$$n(M)dM \propto M^{-\alpha}dM \qquad (M_{\min} < M < M_{\max}) , \tag{1.1}$$

where $M_{\min}$ and $M_{\max}$ prescribe the lower and upper cutoffs in the spectrum. This means that $\rho_*(M)$, the density of stars with mass around M, goes as $M^{2-\alpha}$, so most of the mass is in the largest or smallest stars according to whether $\alpha < 2$ or $\alpha > 2$. For present-epoch stars, $\alpha \approx 2.3$ (Salpeter 1955), at least in the solar neighborhood and over a limited mass range. However, there are observational and theoretical reasons (Silk 1977; Terlevich 1982) for believing that α may decrease and that $M_{\max}$ may increase with decreasing metallicity; α may even fall below 2 for $Z < 10^{-2}\ Z_\odot$ (Melnick, Terlevich, and Eggleton 1983). This suggests that the first stars (with zero metallicity) could be much more massive than the ones forming today. Of course, the assumption that the spectrum has the simple form given by equation (1.1) may be very misleading. Therefore, in many of our considerations, we will merely assume that the stars have a particular mass M, a density Ω_* (in units of the critical density), and a formation redshift z. The appropriate value for M is then the value which dominates the cosmological effect under consideration, a value which would be implicitly determined by the spectrum.

One of the most important cosmological questions is whether the black hole remnants of Population III stars could provide the "dark matter" inferred to exist in galactic halos and clusters of galaxies (Faber and Gallagher 1979). This is the issue which we consider in § II. The crucial point is that, if the dark matter in halos is contained in black holes, then the holes could not be larger than $10^6\ M_\odot$ else they would have dynamical effects inconsistent with observation (Carr 1978). Although there may be some mass range in which MOs leave black hole remnants, we will find that it is difficult to put most of the universe in such holes without overproducing heavy elements. This suggests that halo holes would have to derive from stars in the mass range M_C to $10^6\ M_\odot$, and we discuss some of the consequences of these holes, in particular, their generation of gravitational radiation.

In § III we consider the generation of light by Population III stars. We will show that, if the precursors of the objects which make up the missing mass were stars larger than 0.1 $M_\odot$, then they must have been pregalactic in order to avoid generating too much background light, and they must have been larger than 30 $M_\odot$ in order to burn out quickly enough. In this case, either pregalactic stars and their remnants generated a substantial part of the 3 K background (perhaps all of it) or there must be a large peak somewhere in the far-infrared background. This conclusion can be avoided only for SMOs larger than $10^6\ M_\odot$ since such objects could collapse before nuclear ignition. Of course, not all the energy released in the nuclear burning of pregalactic stars will go into background radiation. Some of it will go into heating up the background matter and perhaps reionizing the universe. Also, if the stars explode, the shocks thereby generated could produce the kind of fluctuations required to make galaxies or trigger further star formation. We examine these sorts of consequences in § IV.

Stars smaller than M_C should produce a lot of heavy elements, and one of the strongest constraints on the spectrum of Population III stars comes from the requirement that they do not generate an enrichment larger than that observed in Population I and II stars. One way to avoid overenrichment is to assume that all the stars collapse to black holes (i.e., $M_{\min} > M_C$), so that the heavy elements they produce are not returned to the background medium. However, from some points of view, an initial burst of enrichment would be desirable (Truran and Cameron 1971), and this motivates us in § V to consider various ways in which Population III stars could produce just a small amount of enrichment. In § VI we examine the more specific question of whether particular abundance problems (such as the oxygen and primary nitrogen anomalies) could be explained by Population III stars. We will also be interested in the circumstances under which such stars could generate the "primordial" helium abundance if the conventional cosmological nucleosynthesis scenario were to be discarded. We conclude that this could only be accomplished in a way which avoids overproduction of heavy elements if the stars responsible were VMOs larger than M_C.

In the final section, we will put all the various cosmological limits on (Ω_*, M, z) together. This will enable us to place important constraints on the spectrum and formation epoch of Population III stars and to assess which combination of cosmological roles alluded to above they could reasonably be expected to fulfill. Note that all our considerations are conditional on the standard Friedmann cosmology applying in the period between decoupling (10^6 yr) and galaxy formation (10^9 yr). In particular, we will assume that throughout this period redshift and time are related by $t \approx t_0 \Omega^{-1/2}(1+z)^{-3/2}$, where $t_0 \approx 10^{10}$ yr is the age of the universe, and Ω is the total matter density in units of the critical density. We leave open the question of whether the universe is hot or cold before decoupling.

II. POPULATION III STARS AND THE MISSING MASS

The considerations of BAC show that pregalactic VMOs with initial mass M exceeding $M_C = M_{OC}\phi_B^{-1}$ should leave black hole remnants of mass $M_B \approx M_O = M\phi_B$, where ϕ_B is the fraction of the initial mass which remains after the nuclear-energized pulsations of the hydrogen- and helium-burning phase. The value of ϕ_B, as well as its dependence on M, is very uncertain: it probably lies between 0.1 and 0.9. An equivalent quantity ϕ_B can also be defined for other types of star. SMOs are expected to collapse directly to black holes if they have no initial metallicity (Fricke 1973), in which case ϕ_B could be close to 1. On the other hand, ϕ_B is probably very small for MOs. In this section we will examine the effects of the black hole remnants. We will be particularly interested in whether they could constitute the dark matter in galactic halos.

a) *Black Hole Remnants*

Let us assume that the stars have a mass spectrum of the form described by equation (1.1). We suppose that $M_{\max}$ exceeds M_C, so that VMO or SMO holes do form, and for the moment we neglect the remnants from stars smaller than M_C. Then, if a fraction f_* of the universe's mass goes into pregalactic stars, the fraction of its mass destined to end up in black holes is

$$f_B = \begin{cases} f_* \phi_B & (M_{\min} > M_C) \\ f_* \phi_B[1-(M_C/M_{\max})^{2-\alpha}] & (M_{\min} < M_C, \quad \alpha < 2) \\ f_* \phi_B (M_C/M_{\min})^{2-\alpha} & (M_{\min} < M_C, \quad \alpha > 2)\,. \end{cases} \tag{2.1}$$

We are here assuming that ϕ_B is independent of M. The value of f_* is hard to predict *a priori*: it could be close to 1, although various feedback mechanisms might prevent this (Hartquist and Cameron 1977; Hogan 1979; § IV). If these black holes alone provide the missing mass, f_B certainly has to exceed 0.9 (Faber and Gallagher 1979). On the other hand, since observations of the cosmological deceleration parameter indicate that the black hole density Ω_B (in units of the critical density) cannot exceed around 1 (Sandage 1972), and since the material outside holes must have a density Ω_{U-B} of at least 0.01, the fraction $f_B = (1 + \Omega_{U-B}/\Omega_B)^{-1}$ cannot exceed 0.99. Thus the value of f_B is constrained to lie between 0.9 and 0.99. It might seem unlikely that the value specified by equation (2.1) could be this large, for this would require that nearly all the universe go into the stars ($f_* \approx 1$), that most of the stars be larger than M_C, and that they only lose a small fraction of their mass before collapsing ($\phi_B > 0.9$). We therefore need to consider ways in which the value of f_B can be increased above the value one would naively infer from equation (2.1).

An obvious way to boost f_B is to allow the holes to accrete. As discussed in earlier papers (Carr 1977*a*, 1981*b*), this is anyway necessary if one wants the holes to generate an appreciable part of the 3 K background radiation. If each hole of mass M_B accretes a mass $\Delta M \equiv \mu M_B$, then $f_B \to f_B(1+\mu)$. However, the value of μ may be constrained by background light limits. If accreted material generates radiation energy $E_{\rm rad}$ with an efficiency $\beta \equiv E_{\rm rad}/(\Delta M)c^2$, the radiation density generated must be

$$\Omega_R = \left(\frac{\beta\mu}{1+\mu}\right)\Omega_B(1+z_R)^{-1}\ , \tag{2.2}$$

where z_R is the redshift at which most of the accretion occurs, and Ω_B is the *present* (postaccretion) black hole density. If one believes that the holes cannot accrete faster than the Eddington limit, the accretion factor μ can exceed 1 (as required) providing

t_R, the age of the universe at z_R, exceeds the Eddington "mass-doubling" time scale $\sim 10^9\,\beta$ yr. However, in this case, equation (2.2) implies that Ω_R would exceed the observed background radiation density ($\Omega_R < 10^{-4}$ over all wave bands; see § III) unless $\beta < 0.04\Omega_B^{-3/5}$.

If one interprets f_* in equation (2.1) as the fraction of the universe which goes into each generation of stars, then one can also increase f_B considerably by allowing many generations to form. Thus, if n generations form, the fraction of the universe destined to end up in black holes will be close to 1 providing $n > f_*^{-1}$. Since the nuclear-burning time of a massive star is of order 10^6 yr for $M > 10^2\,M_\odot$, and since the time of galaxy formation is around 10^9 yr, n could be as large as 10^3 even in the pregalactic context.

Boosting the value of f_B may, of course, be achieved by a combination of accretion and multiple-generation effects. In determining which of the two effects is likely to be most important, one should bear in mind that the second effect may increase f_B only at the expense of generating too many heavy elements or too much helium, whereas the first effect may do so only at the expense of generating too much radiation. It should also be stressed that the value of the combined factor $n(1+\mu)$ required depends sensitively on the spectral parameters of the stars. It would have to be much larger in the ($M_{\rm min} < M_C$, $\alpha > 2$) situation or if the holes derived from MOs (for which ϕ_B is much smaller).

b) Constraints on the Spectrum of Black Hole Remnants

The mass spectrum of the black hole descendants of Population III stars should just reflect the spectrum of the stars themselves, providing the factors ϕ_B and μ can be regarded as independent of M. On the assumption that the black holes *do* provide the missing mass in galactic halos, one can therefore specify how many of them there should be in each mass range. This is significant because we already have strong limits on the mass spectrum of any halo holes.

These limits have been summarized by Carr (1978), who concludes that the strongest one derives from considering the tidal disruption of loose star clusters by holes which are passing through the galactic disk. He claims that the fraction F_B of the halo's mass in holes of mass M_B is constrained to be less than $(M_B/10^5\,M_\odot)^{-1}$. However, a more careful calculation (J. P. Ostriker and M. Schmidt, private communication) shows that this limit is weakened for $M_B > 10^5\,M_\odot$. Instead, the most interesting limit in this mass range seems to be provided by the requirement that the traversing holes do not heat up the disk stars so much that their velocity dispersion exceeds the locally observed value of around 25 km s^{-1}. The associated limit has been calculated by Miller (1982), who gives the time scale for holes of mass M_B, number density n_B, and velocity V_B to heat the stars up to a velocity dispersion v as

$$t_{\rm heat} = \frac{v^2 V_B}{6\pi G^2 n_B M_B^2 \ln(v^2/n_B^{1/3} G M_B)} . \tag{2.3}$$

Lacey, Ostriker, and Schmidt (1983) have suggested that this effect could in fact be the mechanism which puffs up the disk, the relevant observation being that younger stars (for which the available $t_{\rm heat}$ is reduced) seem to have smaller scale heights. In any case, equation (2.3) with $t_{\rm heat} \approx t_0 \approx 10^{10}$ yr and $v \le 25$ km s^{-1} gives a limit $F_B \le (M_B/10^6\,M_\odot)^{-1}$; this is somewhat weaker than the original tidal disruption limit but of the same form. Assuming that the dark matter in galactic halos has about a tenth of the critical density (Faber and Gallagher 1979), and assuming that all pregalactic holes have in fact clustered inside galactic halos, we can express this limit as $\Omega_B(M) < (M/10^5\,M_\odot)^{-1}$. Although limits associated with the holes' accretion effects (Ipser and Price 1977; Carr 1979) or lensing effects (Canizares 1982) could be more stringent, these are less definitive in that they depend on extra assumptions.

The disk-heating limit immediately places an important constraint on the mass spectrum of any Population III VMOs and SMOs. For if their black hole remnants do provide the halo material, and if $\alpha < 2$ (so that most of the mass is in the largest holes), $M_{\rm max}$ certainly has to be less than $10^6\,\eta^{-1}\,M_\odot$, where $\eta \equiv \phi_B(1+\mu)$. If $\alpha > 2$, so that most of the halo is in holes of mass $\max(\eta M_{\rm min}, \eta M_C)$, one still requires

$$\alpha > 3 - \left\{\frac{\log[10^6\,M_\odot/\eta \max(M_{\rm min}, M_C)]}{\log[M_{\rm max}/\max(M_{\rm min}, M_C)]}\right\} \tag{2.4}$$

in order to ensure that the *largest* holes do not contravene the tidal limit; see Figure 1. This condition may be interpreted either as placing a lower limit on α for fixed $M_{\rm max}$ and $M_{\rm min}$ (e.g., for $M_{\rm min} = 10^3\,M_\odot > M_C$, $M_{\rm max} = 10^7\,M_\odot$, and $\eta = 1$, we get $\alpha_{\rm min} = 2.3$) or as placing an upper limit on $M_{\rm max}$ ($M_{\rm min}$) for fixed α and $M_{\rm min}$ ($M_{\rm max}$).

For $M > 10^7\,M_\odot$, the most stringent limit on F_B comes from dynamical friction effects rather than disk heating. This is because any holes at 10 kpc radius would have drifted into the center of the galaxy by now as a result of imparting their energy to smaller objects if they were this large (Carr 1978). Thus the appropriate limit in this regime becomes $\Omega_B(M) < 10^{-4}$ (the density associated with galactic nuclei). This is indicated in Figure 1.

We stress that these limits would not apply for holes which were *not* clustered inside halos: a uniform distribution of holes might even have $\Omega_B = 1$. However, it is at least clear that the hole mass which dominates the halo must lie in the range $M_{\rm OC}(1+\mu)$ to $10^6\,M_\odot$. Since $M_{\rm OC} \approx 10^2\,M_\odot$, this corresponds to an initial star mass between $10^2\phi_B^{-1}\,M_\odot$ and $10^6(1+\mu)^{-1}\phi_B^{-1}\,M_\odot$, which is a good indication that the precursors must have been VMOs or low-mass SMOs. However, if $\alpha > 2$, this would not preclude there also having been some *large* SMOs. Since it has been suggested that black holes which derive from large SMOs may themselves play an important cosmological role, such as acting as condensation nuclei for galaxies (Ryan 1972; Carr and Rees 1983) or generating the X-ray background (Carr 1980; Boldt and Leiter 1981), even if they do not provide the missing mass, this possibility should be borne in mind.

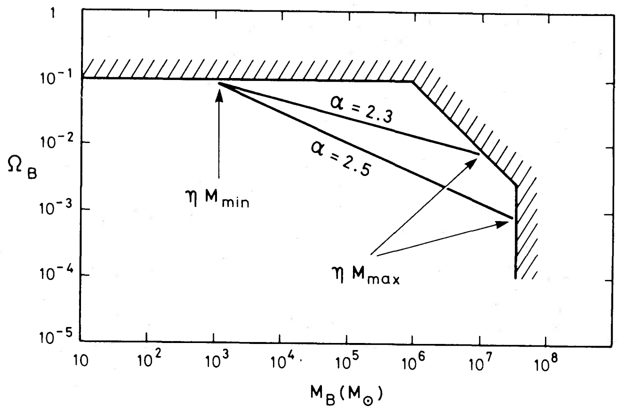

FIG. 1.—This shows the strongest observational limits on the density Ω_B of black holes with mass M_B which presently reside in the galactic halo. Disk-heating arguments imply that only black holes smaller than $10^6\ M_\odot$ could provide most of the halo mass, and they also limit the fraction of the halo in black holes larger than $10^6\ M_\odot$. Dynamical friction precludes there being halo holes larger than $10^7\ M_\odot$ if $\Omega_B > 10^{-4}$. These limits place a strong constraint on the mass spectrum of any Population III stars. For given values of $M_{\min}$ and η, defined in the text, the figure shows how the upper limit on $M_{\max}$ depends on α.

c) Gravitational Radiation from VMO Remnants

If there do exist black hole remnants of Population III stars, their formation would be accompanied by bursts of gravitational radiation. The gravitational wave energy released in each burst would be $\epsilon_g M_B c^2$, where ϵ_g, the gravitational radiation efficiency, could be as high as 0.1. Most of the energy would probably be emitted in a short initial broad-band burst, generated by the imploding matter, with a wavelength of order 10 times the Schwarzschild radius associated with the hole (Thorne 1978). Thus the present frequency of the radiation should be

$$\nu_0 \approx \left(\frac{10GM_B}{c^3}\right)^{-1}(1+z_B)^{-1}\ \text{Hz} \approx 10^2\left(\frac{M_B}{100\ M_\odot}\right)^{-1}(1+z_B)^{-1}\ \text{Hz}\,, \tag{2.5}$$

where z_B is the redshift at which the holes form. The duration of each burst, τ_0, should be of order ν_0^{-1}, and the ratio of the duration to the separation $(\Delta\tau)_0$ between bursts should be (Bertotti and Carr 1980)

$$\left(\frac{\tau}{\Delta\tau}\right)_0 \approx 10^2\Omega_B' z_B \Omega^{-2} \qquad (z_B \gg \Omega^{-1})\,, \tag{2.6}$$

where Ω_B' is the density of the holes *before* accretion, and Ω is the total matter density. The black holes therefore generate an overlapping background of gravitational waves rather than discrete bursts providing $\Omega_B' > 10^{-2} z_B^{-1}\Omega^2$.

The overlap condition, which is independent of M_B and ϵ_g, is likely to be satisfied for all interesting values of f_B, and it certainly is if the holes provide the missing mass. As shown in § III, the background light observations require $\Omega_* z_*^{-1} < 10^{-2}$, where z_* is the redshift at which light is generated through nuclear burning. If we write Ω_B' as $\phi_B\Omega_*$ and put $z_B \approx z_*$, then the background light limit and equation (2.6) imply $(\tau/\Delta\tau)_0 > 10^4\phi_B f_*^2$. Thus the light condition *guarantees* the overlap condition for holes with stellar precursors providing $f_* > 10^{-2}\phi_B^{-1/2}$. Having the bursts overlap is advantageous from the point of view of their detectability since the dimensionless amplitude associated with the gravitational wave background exceeds that of the individual burst by a factor $(\tau/\Delta\tau)_0^{1/2}$; the background amplitude is (Bertotti and Carr 1980)

$$h_0 \approx 10^{-19}\left(\frac{M_B}{10^2\ M_\odot}\right)^{1/2}\epsilon_g^{1/2}\Omega_B'^{1/2}(1+z_B)^{1/2}\,. \tag{2.7}$$

The density associated with the background (in units of the critical density) is

$$\Omega_g = \epsilon_g\,\Omega_B'(1+z_B)^{-1} \approx \epsilon_g\,\phi_B\,\Omega_*\,z_B^{-1}\,, \tag{2.8}$$

and this could be as high as 10^{-2} if the missing mass is contained in holes which formed at relatively low redshifts.

Population III stars could produce gravitational waves in another way if a substantial fraction of them were formed in binary systems (Bond and Carr 1983). In this case, continuous gravitational radiation will be produced as the components spiral inward as a result of the energy loss. If both components are larger than M_C and become black holes, this process will continue until the components merge to form a single black hole. The burst of radiation generated by the final coalescence will be similar to that produced by the collapse of a single hole, except that the time of the burst is postponed from the formation epoch of the stars to the lifetime of the binary (Peters and Matthews 1963):

$$\tau(a) = \frac{5c^5a^4}{256G^3M_A M_B(M_A+M_B)}\,, \tag{2.9}$$

where a is the initial separation, and M_A and M_B are the masses of the components. The density associated with the coalescence background is thus larger than indicated by equation (2.8) in that z_B is reduced, but smaller in that Ω_* is reduced by a factor $p(a)$ related to the fraction of stars in binaries with initial separation a. The optimal case occurs if the initial separation is such that the time scale τ corresponds to the present age of the universe. For $M_A = M_B = M$, this requires $a = a_0 \approx 114(M/10^2\ M_\odot)^{3/4}\ R_\odot$, and, in this case, the effective value of z_B in equations (2.5)–(2.8) is of order 1. In this situation, one could also hope to see the individual coalescence bursts: the amplitude and separation between coalescences in our halo, if dominated by holes, would typically be

$$h_{\rm burst} \approx 4 \times 10^{-17}\left(\frac{M}{10^2\ M_\odot}\right)\left(\frac{R_{\rm halo}}{60\ {\rm kpc}}\right)^{-1}, \qquad t_{\rm burst} \approx 10\left(\frac{M}{10^2\ M_\odot}\right)\left(\frac{M_{\rm halo}}{10^{12}\ M_\odot}\right)^{-1} p(a_0)^{-1}\ {\rm yr}\ . \tag{2.10}$$

The background associated with the continuous radiation generated during the preceding orbital decay phase could also be interesting. If the range of binary separations encompasses the critical value a_0 indicated above, then it can be shown that this continuous background will in general be dominated by the binaries with $a \approx a_0$. In this case, the background energy density at period P can be shown to be

$$\Omega_g(P) = \epsilon_g \phi_B \Omega_* p(a_0)\left(\frac{P}{P_{\rm min}}\right)^{-2/3} \quad {\rm for} \quad 10^{-2}\left(\frac{M}{10^2\ M_\odot}\right) {\rm s} < P < 4 \times 10^5\left(\frac{M}{10^2\ M_\odot}\right)^{5/8} {\rm s}\ . \tag{2.11}$$

The upper limit in the period range is associated with the orbital period of binaries with initial separation a_0, most of the radiation being generated at a frequency of twice the orbital frequency (for circular orbits). Note that the nearest such binary would be expected to be at a distance of order

$$d_{\rm min} \approx [n_B p(a_0)]^{-1/3} \approx 10 p(a_0)^{-1/3}\left(\frac{M}{10^2\ M_\odot}\right)^{1/3} {\rm pc}\ ; \tag{2.12}$$

the amplitude of the associated monochromatic waves is

$$h_{\rm ind} \approx 2 \times 10^{-18}\left(\frac{M}{10^2\ M_\odot}\right)^{11/12} p(a_0)^{1/3}\ , \tag{2.13}$$

although this is actually less than the amplitude of the background. These points are discussed in more detail by Bond and Carr (1983).

It is obviously interesting to ask whether the backgrounds of gravitational waves generated by individual or binary black holes could be observable. Laser interferometry detectors are most sensitive at periods of about 10^{-2} s, and the single hole burst background will peak in this range if the product $M_B(1 + z_B)$ is of order $10^2\ M_\odot$; in this case, $h_0 \approx 10^{-19}\epsilon_g^{1/2}\Omega_B^{1/2}$, which would be detectable (Weiss 1979). If $M_B(1 + z_B)$ is somewhat larger than this, the best method to detect the burst background would be the Doppler tracking of interplanetary spacecraft. Given the present stability of the H-maser clocks which regulate the frequency of the tracking beam, this technique could be used to detect backgrounds with $10^2\ {\rm s} < P < 10^7$ s; Bertotti and Carr (1980) argue that holes with $M_B > 3000 z_B\ M_\odot$ could be detectable. The prospect of detecting the binary background is even better since it covers a larger period range. Bond and Carr (1983) argue that binaries with $M < 400\ M_\odot$ might be detected by laser interferometers, whereas those with $M > 4 \times 10^4\ M_\odot$ might be detected by Doppler tracking.

d) Limits on Low-Mass Remnants

Since one cannot exclude the possibility that the missing mass may be in low-mass stars rather than black holes, we will also examine the consequences this would have for the Population III mass spectrum. The mass-to-light ratio of a star can only be as high as 100, as required for our own halo, if $M < 0.1\ M_\odot$. However, the largest contribution to the light and the mass may not come from the same stars. Since the main-sequence luminosity increases rapidly with mass, the largest stars may dominate the light even though the smallest ones dominate the mass. More specifically, for $L \propto M^\beta$, where $\beta \approx 4$ for $0.1\ M_\odot < M < 1\ M_\odot$ (Iben 1967), we have

$$\langle M/L \rangle \approx \begin{cases} \max\,[(M_{\rm max}/L_{\rm max}), 1] & (\alpha < 2) \\ (M_{\rm min}/L_{\rm min})[\min\,(M_{\rm max}, 1\ M_\odot)/M_{\rm min}]^{\alpha-1-\beta} & (2 < \alpha < 1+\beta)\ , \\ (M_{\rm min}/L_{\rm min}) & (\alpha > 1+\beta) \end{cases} \tag{2.14}$$

where $L_{\rm max} = L(M_{\rm max})$, $L_{\rm min} = L(M_{\rm min})$, and we assume stars larger than 1 $M_\odot$ would have burnt out by now. Thus, for $\alpha < 2$, we require $M_{\rm max} < 0.1\ M_\odot$; for $\alpha > 1 + \beta$, we require $M_{\rm min} < 0.1\ M_\odot$; and for $2 < \alpha < 1 + \beta$, we require $M_{\rm min} < 0.1\ M_\odot$ and

$$M_{\rm max} < 0.1\left(\frac{M_{\rm min}}{0.1\ M_\odot}\right)^{(2-\alpha)/(1+\beta-\alpha)} M_\odot\ . \tag{2.15}$$

Equation (2.15) immediately places an upper limit on how many pregalactic stars can have burnt out by now ($M > 1\ M_\odot$) or contributed significantly to pregalactic nucleosynthesis ($M > 4\ M_\odot$). Note that infrared observations of the halos of other galaxies suggest that the halo material must have a value for $\langle M/L \rangle$ of at least 38 in the K band (Boughn, Saulson, and Seldner 1981); this corresponds to a mass $M < 0.08\ M_\odot$, which may exclude any main-sequence stars.

If the missing mass *is* in low-luminosity stars, it will be very difficult to detect them directly, except perhaps as high-velocity infrared sources (Staller and de Jong 1981). However, Gott (1981) has pointed out that it may be possible to infer their presence indirectly by looking for their gravitational lens effects on distant quasars. He shows that, if a galaxy is massive enough and suitably positioned to image-double a quasar, then there is also a high probability that the lensing by an individual halo star will produce appreciable fluctuations in the quasar intensity. This effect could be detected for halo star masses larger than $10^{-4}\ M_\odot$. However, the time scale of the intensity fluctuations, being of order $40(M/M_\odot)^{1/2}$ yr, would only be noticeable over a period of 10 yr (say) if $M < 0.1\ M_\odot$.

III. RADIATIVE EFFECTS OF POPULATION III STARS

Any black hole remnants of Population III stars would tend to generate electromagnetic radiation, with consequent effects on the thermal history of the universe (Carr 1981*a*). However, the importance of this effect is very uncertain since it depends on the radiative efficiency of the accretion process as well as the wave band in which the radiation was emitted, both factors being model dependent. In this section we will consider the generation of radiation by the stars themselves; this effect is much less uncertain since the temperature of the radiation and the efficiency with which it is generated from nuclear burning can be predicted relatively unambiguously. We will find that the requirement that the stars not generate more background radiation than is observed enables one to place interesting limits on their formation redshift and mass spectrum.

a) *Integrated Background Light Limits*

Let us assume that a density $\Omega_*(M) = f_*(M)\Omega$ of stars with mass M burn their nuclear fuel at a redshift z_*, and that each star produces radiation energy $\epsilon_* Mc^2$. Since 7 MeV per baryon is released in the burning of hydrogen to helium, we may write ϵ_* as $0.007 f_b(M) X_0$, where $f_b(M)$ is the fraction of the star's mass burnt to helium, and X_0 is the primordial hydrogen abundance. We assume $1 \geq X_0 \geq 0.75$. The value of $f_b(M)$ is itself weakly dependent on X_0, but we neglect this dependence and use the approximation $f_b(M) \approx 0.8(M/10^2\ M_\odot)^{1/2}$ for $10\ M_\odot < M < 10^2\ M_\odot$. This expression is a fit to Figure 4 of Iben (1967); for the moment we neglect stars smaller than $10\ M_\odot$. Stars above $10^2\ M_\odot$ have $f_b(M) \approx (1 + X_0/2)/(1 + X_0) \approx 0.8$, independent of M (BAC), so f_b is continuous at $10^2\ M_\odot$. However, the value of f_b is much less for stars larger than $10^5\ M_\odot$ since such SMOs collapse before they can burn their nuclear fuel. Energetic arguments show that the effective value of ϵ_* for SMOs can never exceed $(M/10^2\ M_\odot)^{-1}$, which is small. In the interesting mass range, $10\ M_\odot < M < 10^5\ M_\odot$, we may therefore write

$$\epsilon_*(M) \approx 0.006 \min\left[\left(\frac{M}{10^2\ M_\odot}\right)^{1/2}, 1\right] X_0 . \tag{3.1}$$

The present radiation density generated by the stars should be (cf. Eichler and Solinger 1976)

$$\Omega_R = \epsilon_* f_R (1 + z_*)^{-1} \Omega_* , \tag{3.2}$$

where f_R is the fraction of the generated radiation which goes into the background light rather than into heating the matter content of the universe. One would expect f_R to be close to 1, even though a lot of the radiation may be reprocessed near the stars or in the background universe, since the thermal capacity of the matter should be much lower than that of the radiation.

If the observed background radiation density is $\Omega_R^{\rm obs}$ in units of the critical density, equations (3.1) and (3.2) imply

$$\Omega_*(1 + z_*)^{-1} < f_R^{-1} \epsilon_*^{-1} \Omega_R^{\rm obs} \approx 1.7 \times 10^{-2} \max\left[\left(\frac{M}{10^2\ M_\odot}\right)^{-1/2}, 1\right] X_0^{-1} f_R^{-1} \left(\frac{\Omega_R^{\rm obs}}{10^{-4}}\right) , \tag{3.3}$$

this limit pertaining for $10\ M_\odot < M < 10^5\ M_\odot$. Since the radiation density over all wave bands (with the possible exception of the far-infrared band, which is presently unobservable) cannot exceed 10^{-4} times the critical density, one immediately infers a lower bound on the redshift z_*. If the stars are VMOs, one requires $z_* > 60\Omega_* X_0$; thus if the stars had more than 0.2 of the critical density, they would certainly need to be pregalactic (the redshift of galaxy formation being assumed to be of order 10). If the stars were smaller than VMOs, the lower limit on z_* would be reduced by a factor $(M/10^2\ M_\odot)^{1/2}$. Of course, limit (3.3) would be much stronger if one knew that the radiation presently resided in a wave band where the background density was *less* than 10^{-4} times critical.

Equation (3.3) allows one to infer a limit on the spectrum of the stars. Since the stars produce their radiation in the period between t_f, their formation time, and $t_f + t_{\rm MS}$, where $t_{\rm MS}$ is their main-sequence time, we may take t_* to be max $(t_f, t_{\rm MS})$. Clearly, this is an approximation since radiation is being generated continuously between t_f and $t_f + t_{\rm MS}$. However, if $t_{\rm MS} \gg t_f$, the fractional contribution to Ω_R from an epoch t intermediate between t_f and $t_f + t_{\rm MS}$ is only $(t/t_{\rm MS})^{5/3}$ for $z > \Omega^{-1}$; and, if $t_f > t_{\rm MS}$, all the radiation comes from the redshift z_f anyway. It is therefore appropriate to assume that nearly all the radiation is generated at $z_* = \min(z_{\rm MS}, z_f)$, where $z_{\rm MS}$ is the redshift when the age of the universe is $t_{\rm MS}$. Now the main-sequence time of stars in the mass range above $10\ M_\odot$ (where electron scattering dominates the opacity) is given approximately as

$$t_{\rm MS} \approx 3 \times 10^6 \left(\frac{\epsilon_*}{0.006}\right) \max\left[1, \left(\frac{M}{10^2\ M_\odot}\right)^{-2}\right] \text{yr} , \tag{3.4}$$

where ϵ_* is given by equation (3.1). The relationship $t \approx t_0 \Omega^{-1/2}(1 + z)^{-3/2}$ therefore implies that

$$z_{\rm MS} \approx 300 \left(\frac{\epsilon_*}{0.006}\right)^{-2/3} \Omega^{-1/3} h^{-2/3} \min\left[1, \left(\frac{M}{10^2\ M_\odot}\right)^{4/3}\right] , \tag{3.5}$$

where $h \equiv H_0/(50 \text{ km s}^{-1} \text{ Mpc}^{-1})$, and $t_0 \approx 1.9 \times 10^{10}\, h^{-1}$ yr. Since z_* cannot exceed $z_{\rm MS}$, equation (3.3) immediately implies an upper limit on $\Omega_*(M)$:

$$\Omega_*(M) < 5 f_R^{-1} \left(\frac{\epsilon_*}{0.006}\right)^{-5/3} \left(\frac{\Omega_R^{\rm obs}}{10^{-4}}\right) \min\left[1, \left(\frac{M}{10^2\, M_\odot}\right)^{4/3}\right] \Omega^{-1/3} h^{-2/3} . \tag{3.6}$$

If one substitutes for $\epsilon_*(M)$ using equation (3.1), this limit becomes

$$\Omega_*(M) < 5 \left(\frac{\Omega_R^{\rm obs}}{10^{-4}}\right) \min\left[1, \left(\frac{M}{10^2\, M_\odot}\right)^{1/2}\right] (f_R^{-1} X_0^{-5/3} \Omega^{-1/3} h^{-2/3}) , \tag{3.7}$$

where the last bracketed term lies between 0.6 and 3.5 for reasonable cosmological parameters ($1 \geq X_0 \geq 0.75$, $1 \geq \Omega \geq 0.1$, $1 \leq h \leq 2$, $f_R = 1$).

Equations (3.4)–(3.7) do not apply for stars smaller than 10 $M_\odot$ since the opacity of such stars is no longer dominated by electron scattering, and also equation (3.1) fails. Assuming Kramers' opacity law for such stars, we get a main-sequence time which scales as M^{-4}, and this implies a steeper dependence on M than shown in equation (3.7). The limits on low-mass stars are therefore stronger than indicated above. When M becomes so small ($< 1\, M_\odot$) that the stars do not complete their nuclear burning in the age of the universe, the form of the limits changes again. The $L \propto M^\beta$ law used in equation (2.14) now implies that the radiation density produced over the age of the universe is

$$\Omega_R \approx \Omega_* \left(\frac{Lt_0}{Mc^2}\right) \approx 1.4 \times 10^{-3} \Omega_*(M) \left(\frac{M}{M_\odot}\right)^{\beta - 1} h^{-1} , \tag{3.8}$$

and so we deduce a limit

$$\Omega_*(M) < 7 \times 10^{-2} \left(\frac{\Omega_R^{\rm obs}}{10^{-4}}\right) \left(\frac{M}{M_\odot}\right)^{1-\beta} h \qquad (M < 1\, M_\odot) . \tag{3.9}$$

For $\beta \approx 4$, this implies that Ω_* could only be close to 1 if $M < 0.4\, M_\odot$. Peebles and Partridge (1967) have already used this sort of argument to exclude stars in the mass range 0.3–2.5 $M_\odot$ having a critical density. Note that this is slightly weaker than the constraint $M < 0.1\, M_\odot$ of § II*d*, which was associated with light limits for our own galactic halo.

Limits (3.7) and (3.9) are put together in Figure 2. This shows that Ω_* can exceed a specified value only if M is large enough or small enough; and if Ω_* is too large, M has to be in the low-mass range. Values much less than $10^{-1}(\Omega_R^{\rm obs}/10^{-4})$ are never excluded by the argument given above. However, even stronger limits would pertain if one knew that the stars were large enough to burn out on a time scale less than t_f. In this case, one should put $z_* = z_f$ in equation (3.3), and the dashed lines in Figure 2 indicate how this changes the form of the limits on $\Omega_*(M)$. There is no change for $z_f > 300$ since all stars must burn at least until the epoch corresponding to this redshift. However, the maximum value of Ω_* permitted for large stars progressively decreases as z_f decreases below 300.

b) Discrete Frequency Background Light Limits

The limits discussed above are rather simplistic in that the value for $\Omega_R^{\rm obs}$ is assumed to be fixed. However, $\Omega_R^{\rm obs}$ itself has an implicit dependence on M and z_*, since the wave band in which the radiation presently resides depends on both the surface

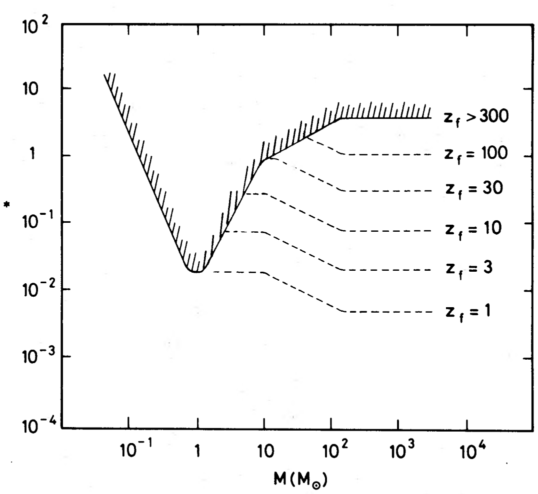

FIG. 2.—This shows the upper limits on the density Ω_* of stars of mass M which form at a redshift z_f, based on observations of the integrated background radiation density Ω_R. We assume $\Omega_R = 10^{-4}$, although it may be much smaller in particular frequency bands, and $X_0 = f_R = h = \Omega = 1$. The redshift at which the stars produce their radiation is the smaller of z_f and the redshift $z_{\rm MS}$ when the age of the universe is their nuclear-burning time. For $z_f > 300$, the nuclear-burning time necessarily exceeds t_f, and one gets the solid line; otherwise $t_{\rm MS} < t_f$ for sufficiently large M, and one gets the broken lines.

temperature of the stars and their burning epoch. It may therefore be possible to improve the radiative limits by using a more detailed model for the stars' light output. More detailed calculations have indeed been presented by Thorstensen and Partridge (1975). Their calculations neglect the effect of the background universe on the radiation and consider only the situation in which $\Omega_* = 1$, but the following discussion resembles theirs in several aspects.

Like Thorstensen and Partridge, we assume that the stars emit blackbody radiation at a temperature T_* with either total or partial absorption below the Lyman limit. As shown in BAC, $T_* \approx 10^5$ K independently of M for $M > 10^2\ M_\odot$; a fit to the results of Ezer and Cameron (1971) shows that it goes like $M^{0.3}$ for $10\ M_\odot < M < 10^2\ M_\odot$. Thus, as an approximation, we may write

$$T_* \approx 10^5 \min\left[\left(\frac{M}{10^2\ M_\odot}\right)^{0.3}, 1\right] \mathrm{K} \qquad (M > 10\ M_\odot)\,. \tag{3.10}$$

If $t_{\mathrm{MS}} < t_f$, the spectrum of the starlight today should just have its original form but with a redshifted temperature $T_*(1+z_*)^{-1}$. Thus the background intensity at present frequency ν_0 can be expressed (in ergs cm^{-2} s^{-1} Hz^{-1} sr^{-1}) as

$$i(\nu_0) = \left[\frac{c^3\rho_*(t_0)\epsilon_*}{4\sigma T_*^4}\right](1+z_*)^3 B'\left(\nu_0, \frac{T_*}{1+z}\right)\,, \tag{3.11}$$

where $B'(\nu, T)$ is the blackbody spectrum, modified for Lyman absorption, and the first term represents a volume diminution factor. If one defines an equivalent density parameter as

$$\Omega_R(\nu_0) \equiv 4\pi\nu_0\, i(\nu_0)/\rho_{\mathrm{crit}}\, c^3\,, \tag{3.12}$$

the predicted spectrum (uncorrected for absorption) is

$$\Omega_R(\nu_0) \approx 9 \times 10^{-4}\Omega_*(1+z_*)^{-1}\left(\frac{\epsilon_*}{0.006}\right)\left(\frac{x^4}{e^x - 1}\right)\,, \tag{3.13}$$

where $x \equiv h\nu_0(1+z_*)/kT_*$. This quantity peaks at a present frequency $\nu_{\max} \approx 8 \times 10^{10} T_*(1+z_*)^{-1}$ Hz, when the last term has a value of 5; at frequencies $\nu_0 \ll \nu_{\max}$, the value of $\Omega_R(\nu_0)$ is smaller than $\Omega_R(\nu_{\max})$ by a factor $13(\nu_0/\nu_{\max})^3$.

We now consider the effects of Lyman absorption. If absorption were complete above the Lyman limit of 13.6 eV (frequency ν_i), the present spectrum would be cut off above an energy $13.6(1+z_*)^{-1}$ eV. (This is below the energy at which the stellar spectrum peaks for $M > 10\ M_\odot$.) Thus observations of the background light at frequency ν_0 would permit no limits to be inferred on stars which produced radiation at redshifts exceeding $(\nu_i/\nu_0 - 1)$. Nevertheless, if one assumes that one Lyman-α photon (of frequency ν_α and energy 10 eV) is emitted for each UV photon absorbed, there will still be a contribution from these Lyman-α photons at frequency ν_0 providing the nuclear-burning period of the stars encompasses the redshift $(\nu_\alpha/\nu_0 - 1)$; i.e., providing $z_f > (\nu_\alpha/\nu_0 - 1) > z_*$. In this case the associated intensity can be shown to be (Thorstensen and Partridge 1975)

$$i_\alpha(\nu_0) \approx 2.9 i(\nu_{\max})\left(\frac{\nu_0}{\nu_\alpha}\right)^{3/2}\left(\frac{t_0}{t_{\mathrm{MS}}}\right)\left[\frac{\int_{\nu_i}^{\infty}\nu^{-1}B(\nu, T_*)d\nu}{\int_0^{\infty}\nu^{-1}B(\nu, T_*)d\nu}\right]\Omega^{-1/2}\,, \tag{3.14}$$

the factor $(\nu_0/\nu_\alpha)^{3/2}(t_0/t_{\mathrm{MS}})$ deriving from the fraction of its lifetime during which a star is producing Lyman-α photons at the required redshift. The factor in square brackets is about 0.6, with a weak dependence on T_*. If the actual line width is $(\Delta\nu)_\alpha$ and the bandwidth of the observations is $(\Delta\nu)_0$, then (for reasons which will become clear later) we define an equivalent density parameter as

$$\Omega_R(\nu_\alpha) \equiv \frac{4\pi\nu_0\, i_\alpha(\nu_0)}{c^3\rho_{\mathrm{crit}}}\frac{(\Delta\nu)_\alpha}{(\Delta\nu)_0}\,. \tag{3.15}$$

Determining $(\Delta\nu)_\alpha$ is complicated since it depends upon (1) the period for which the stars are burning ($\Delta\nu/\nu \sim \Delta z/z \sim t_{\mathrm{MS}}/t_f$), (2) processes intrinsic to the star and its surrounding H II region, and (3) scattering processes which may occur in our own Galaxy. It should certainly exceed 10 Å. For most of the observational limits invoked later, $(\Delta\nu)_0$ is about 10^3 Å, so we assume the last factor in equation (3.15) is at least 10^{-2}. There will also be a contribution to the background light from other recombination lines, but the Lyman-α component will generally be strongest.

Thorstensen and Partridge also consider the case of partial absorption. This is because they are only interested in the absorption which occurs locally, within the photosphere of the star. For massive stars, which are sufficiently hot to be highly ionized in their outer parts, this effect may be slight: for example, the discontinuity $\Delta_i \equiv i_\nu(\nu_i^-)/i_\nu(\nu_i^+)$ is only 1.6 for $M = 10^2\ M_\odot$ and 3.9 for $M = 10\ M_\odot$ (Mihalas 1965). However, photons above the Lyman limit may be absorbed in the background universe even if they are not absorbed in the photosphere of the star itself, so it is not clear that the value of Δ_i they use is relevant. A photon emitted at a redshift z will be absorbed within an expansion time if its energy lies between 13.6 eV and

$$E_p \approx 1.0(1+z)^{1/2}(1-x)^{1/3}\Omega_g^{1/3}\Omega^{-1/6}h^{1/3}\ \mathrm{keV} \tag{3.16}$$

(Carr 1981*a*); the coefficient is increased slightly for $E_p > 24.6$ eV if there is a primordial helium abundance. In this equation, x is the background ionization, and Ω_g is the density of the background gas in units of the critical density. Since the spectrum of radiation from Population III stars cannot extend much beyond 30 eV, equation (3.16) implies that absorption is negligible if the universe is sufficiently highly ionized that

$$(1-x) \ll 10^{-5}\Omega_g^{-1}\Omega^{1/2}h^{-1}(1+z_*)^{-3/2}\,. \tag{3.17}$$

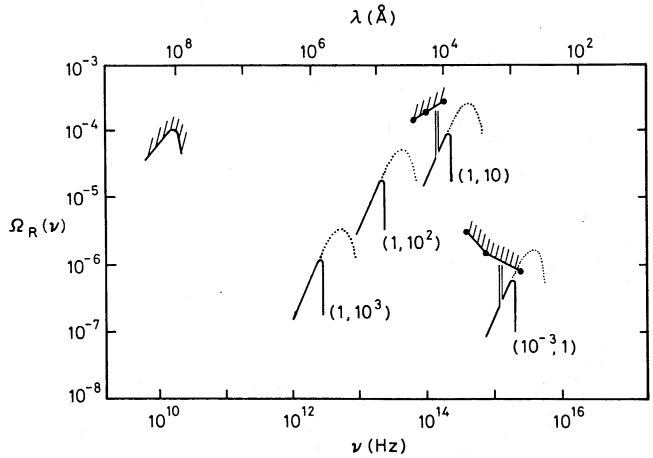

FIG. 3.—This shows the observational limits on the background radiation density at various frequencies and compares them with possible background spectra. The dotted curves apply if the clumpiness of the background gas is small enough that there is no cutoff beyond the Lyman limit. Otherwise, one expects a Lyman-α line. The height of this line is defined by eq. (3.15), where we assume $(\Delta\nu)_\alpha/(\Delta\nu)_0 = 10^{-2}$; it falls below the continuum for $z \gg 10$ under all circumstances. The spectra are labeled according to the density of the stars (Ω_*) and the redshift at which they burn most of their nuclear fuel (z_*). We assume the nuclear-burning time, t_{MS}, is less than the formation time, t_f. Otherwise, z_* is the redshift when the age of the universe is t_{MS}, in which case the radiation is generated over a range of redshifts ($z_f > z > z_{MS}$), and the spectra are more extended. We assume the stars are VMOs so that they produce radiation at 10^5 K; for smaller stars, the curves move to the left. For smaller values of Ω_*, they move down. We also assume that z_* is not large enough for the radiation to be absorbed at frequencies below the Lyman limit by grains.

This condition is necessarily satisfied for $z_* < 3$ (Gunn and Peterson 1965), and the ionization from the stars themselves could ensure it unless the clumpiness of the gas exceeds

$$\delta_{\rm crit} \approx 10^5 \Omega_* \Omega_g^{-2}(1 + z_*)^{-3/2}\Omega^{1/2}h^{-1} , \tag{3.18}$$

where we have used equation (4.18). However, such a large clumpiness is not necessarily implausible: if most of the gas goes into clouds when the first objects bind at redshift z_B, one would expect $\delta \sim (z_B/z_*)^3$, and this could be as large as 10^6. Thus Thorstensen and Partridge may be unjustified in neglecting the effects of the background medium, and total absorption below the Lyman limit may be a better approximation in some circumstances.

Let us first assume that there is indeed total absorption beyond the Lyman limit ($\delta > \delta_{\rm crit}$). Then, using equations (3.13)–(3.15), we may write the present density of radiation at frequency ν (or wavelength λ) as

$$\begin{aligned}\Omega_R(\nu) \approx\ & 5 \times 10^{-2}\Omega_*(1 + z_*)^{-1}X_0 \left(\frac{\nu}{\nu_{\rm max}}\right)^3 \min\left[\left(\frac{M}{10^2\, M_\odot}\right)^{0.5}, 1\right] \theta[\nu_i(1 + z_*)^{-1} - \nu] \\ & + 3 \times 10^{-1}\Omega_* h^{-1}\Omega^{-1/2}X_0 \left(\frac{\lambda}{5300\ \text{Å}}\right)^{-5/2} \min\left[\left(\frac{M}{10^2\, M_\odot}\right)^{1.7}, 1\right] \frac{(\Delta\nu)_\alpha}{(\Delta\nu)_0}\, \tilde{\theta}\left[\nu_\alpha(1 + z_*)^{-1} \pm \frac{1}{2}(\Delta\nu)_\alpha\right] . \end{aligned} \tag{3.19}$$

Here $\tilde{\theta}$ is a function which is zero outside the specified range, and we have substituted for ϵ_*, t_{MS}, and T_* using equations (3.1), (3.4), and (3.10). If there is no absorption beyond the Lyman limit ($\delta \ll \delta_{\rm crit}$), the Lyman-$\alpha$ contribution in equation (3.19) is absent, but the continuum contribution extends to higher frequencies. In the intermediate situation, the spectrum beyond the Lyman limit is multiplied by a factor $(1 - \delta/\delta_{\rm crit})$; the Lyman-$\alpha$ contribution is $\delta/\delta_{\rm crit}$ times the value given by equation (3.19), but it is spread out over a wider wave band since the recombinations occur on a long time scale.

The background light limits at 4100 Å, 5300 Å, 8700 Å, and 20,000 Å used by Thorstensen and Partridge were $i_\nu = 1.5 \times 10^{-20}$, 6.2×10^{-20}, 1.1×10^{-17}, and 1.5×10^{-17} ergs cm^{-2} s^{-1} sr^{-1} Hz^{-1}, respectively. The associated values for $\Omega_R(\nu)h^2$ are 1×10^{-6}, 3×10^{-6}, 4×10^{-4}, and 2×10^{-4}, using a definition for $\Omega_R(\nu)$ analogous to equation (3.12). More recent observations (Dube, Wicks, and Wilkinson 1979) have improved the 5300 Å limit: one now has $i_\nu = 4.3 \times 10^{-20}$, or $\Omega_R = 2 \times 10^{-6}$, at 5300 Å. One also now has a far-UV limit of $i_\nu = 1.4 \times 10^{-21}$, or $\Omega_R = 3 \times 10^{-7}$, at 1400 Å (Paresce and Jacobsen 1980) and an IR limit of $i_\nu = 0.9 \times 10^{-17}$, or $\Omega_R = 1 \times 10^{-4}$, at 24,000 Å (Hofmann and Lemke 1978). These limits are plotted and compared with the predicted spectra for various values of Ω_* and z_* in Figure 3. Note that the weakness of the IR limits is a result of instrument sensitivity; it does not arise from a positive detection of an IR background.

Using the 5300 Å limit, equation (3.19) implies that one can impose no limit if $z_* > (\nu_i/\nu_0 - 1) = 5$. If z_* is below this value, however, one gets a limit

$$\Omega_* < 1 \times 10^{-1} \min\left[\left(\frac{M}{10^2\, M_\odot}\right)^{0.4}, 1\right](1 + z_*)^{-2}X_0^{-1}h^{-2} \qquad (z_* < 5) , \tag{3.20}$$

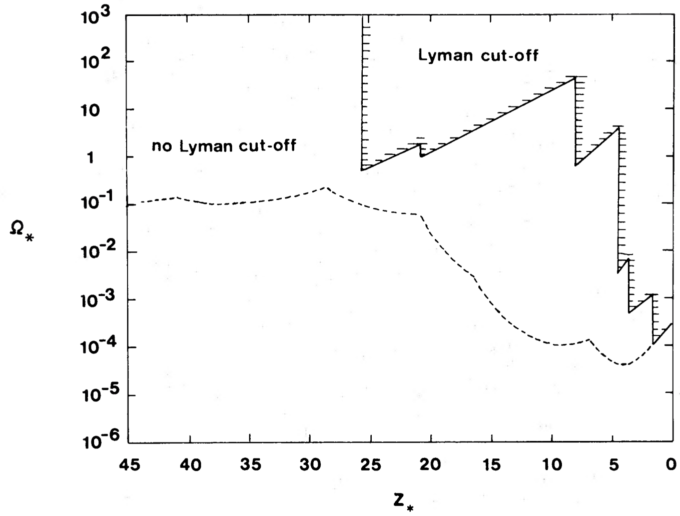

FIG. 4.—This shows the constraint on the function $\Omega_*(z_*)$ for $M \geq 10^2\ M_\odot$ implied by observations of the background light at 1400 Å, 4100 Å, 5300 Å, 8700 Å, 20000 Å, and 24000 Å. The region above the solid line is excluded even if there is complete absorption beyond the Lyman limit. In this case, there will be a stronger limit in narrow bands of z_* due to Lyman-α emission; these limits are not shown explicitly since they depend on astrophysical parameters which are very uncertain. If there is no Lyman cutoff, the region above the broken line is excluded. The limits assume $h = X_0 = 1$. They would be even stronger for $M < 10^2\ M_\odot$.

and this places a limit both on the spectrum of the stars and on their formation redshift. If z_* happens to be close to $(\nu_\alpha/\nu_0 - 1) = 3.4$, the Lyman-$\alpha$ component permits an even stronger limit if $\delta > \delta_{\text{crit}}$:

$$\Omega_* < 7 \times 10^{-6} \max\left[\left(\frac{M}{10^2\ M_\odot}\right)^{-1.7}, 1\right]\frac{(\Delta\nu)_0}{(\Delta\nu)_\alpha} X_0^{-1}\Omega^{1/2}h^{-1} . \tag{3.21}$$

The constraints imposed by the background light limits at other wavelengths have the same form as equations (3.20) and (3.21), but the numerical coefficients and critical values of z_* are different. For the observations at 1400 Å, 4100 Å, 8700 Å, 20000 Å, and 24000 Å, the coefficient and limiting value of z_* in equation (3.20) become $(3 \times 10^{-4}, 0.5)$, $(3 \times 10^{-2}, 3)$, $(100, 9)$, $(700, 21)$, and $(800, 25)$, respectively; the coefficient and value of z_* in equation (3.21) become $(3 \times 10^{-8}, 0.2)$, $(2 \times 10^{-6}, 2.4)$, $(4 \times 10^{-3}, 6.3)$, $(2 \times 10^{-2}, 16)$, and $(2 \times 10^{-2}, 19)$, respectively.

These limits imply constraints on the functions $\Omega_*(M)$, $\Omega_*(z_*)$, and $M(z_*)$. The $\Omega_*(z_*)$ constraints for $M > 10^2\ M_\odot$ are shown in Figure 4, the region above the solid line being excluded. If there were no absorption above the Lyman limit ($\delta < \delta_{\text{crit}}$), the region above the *broken* line would be excluded. The constraints in this case are much more interesting (although the Lyman-α limit no longer applies): for example, one could exclude the stars having a critical density at all redshifts below about 50. The Lyman-α limit is not shown explicitly since it depends upon the parameters $(\Delta\nu)_0/(\Delta\nu)_\alpha$ and $(\delta/\delta_{\text{crit}})$, both of which are very uncertain. We note that the constraint on the mass spectrum $\Omega_*(M)$ has a different form from the one shown in Figure 2 (which derived from integrated background light considerations); the discrete frequency constraint is usually stronger only for $z_* < 10$ and is therefore less interesting for pregalactic stars. We do not show the constraint of $M(z_*)$ explicitly since it has already been derived by Thorstensen and Partridge (1975) for the $\Omega_* = 1$ case.

The above analysis clearly fails if the burning time of the stars exceeds the cosmological time at their formation epoch ($t_{\text{MS}} > t_f$) or if the stars form over a range of redshifts rather than at a single redshift. In both cases, one expects the background light to have a considerably broader spectrum than indicated in Figure 3, and equations (3.19)–(3.21) must be modified. In particular, the Lyman-α line will no longer be narrow, and, if $\delta > \delta_{\text{crit}}$, it may well dominate the spectrum over most wave bands. These points are discussed in detail by McDowell (1983).

c) *Thermalized Background Light Limits*

The assumption that there is some wave band in which the directly emitted starlight still appears is crucial in deriving Figures 3 and 4. However, this assumption would fail if there were enough grains in the universe to absorb and perhaps thermalize the starlight even below 13.6 eV. Since the absorptivity of grains only falls off at wavelengths above about 1 μm, one might in principle be able to thermalize radiation up to a present wavelength of $10^4(1 + z_*)$ Å. This raises the question of whether the combination of grains and pregalactic stars could produce interesting distortions in the spectrum of the microwave background. Indeed, Rowan-Robinson, Negroponte, and Silk (1979) and Puget and Heyvaerts (1980) have suggested models in which this effect is invoked to explain the distortions reported just shortward of the peak by Woody and Richards (1979, 1981). The Rowan-Robinson *et al.* model requires that the grains have a density $\Omega_d \approx 10^{-5}$, that $\Omega_* \approx 0.1$, and that $z_* \approx 100$; also, implicitly, the stars must have $M > 30\ M_\odot$ in view of equation (3.5). The Puget and Heyvaerts model requires similar parameters except that

$z_* \approx 10$. In these models some 25%–30% of the energy density in the 3 K background has to be thermalized starlight. Rees (1978) has made the more radical suggestion that *all* of the 3 K background is grain-thermalized starlight. However, this scenario may be implausible since, besides the fact that it is not possible to explain the specific distortions reported by Woody and Richards, it is difficult to thermalize the starlight at long wavelengths unless one invokes a very exotic form of grain (Layzer and Hively 1973, Wickramasinghe *et al.* 1975; Alfvén and Mendis 1977; Rana 1981). A compromise scenario has been suggested (Carr 1981*b*) in which the initial 75% of the 3 K background is generated by black hole accretion and thermalized by free-free processes. In this scenario, energetic and thermalization criteria require that the holes produce their light at $z \approx 10^3$. This corresponds to a time of about 10^6 yr, which is somewhat less than the lifetime of a VMO but comparable to the collapse time of an SMO with $M \sim 10^6\, M_\odot$.

The best observational upper limit on the amount of dust grains in the universe at present is provided by quasar-reddening measurements (Wright 1982). These suggest that $\Omega_d < 1.2 \times 10^{-4} h^{-1}$, at least for dust which is uniformly distributed. Wright calculates that this would be sufficient to explain the Woody-Richards distortions, the best fit parameters being $\Omega_* = 0.3$, $z_* = 240$, and (implicitly) $M > 80\, M_\odot$, but that it would *not* suffice to thermalize 100% of the background. In any case, it is clearly possible that much of the original Population III starlight could now be part of the microwave background. In this case, the limits implicit in Figure 3 need no longer apply for large z. However, Figure 2, which depends only on the integrated background density being less than 10^{-4} times critical, would still apply; and limits (3.20) and (3.21) would still pertain since the optical depth of the universe to dust would be low for the small values of z involved. Of course, the upper limits on the distortion in the 3 K background could themselves impose constraints on (Ω_*, M, z_*). However, calculating these limits is complicated since it depends on the type of grains and their thermal history. Since these features are very uncertain, we do not discuss the distortion limits here, although they are implicitly contained in Negroponte, Rowan-Robinson, and Silk (1981).

One can anyway make the following qualitative conclusion: if pregalactic stars did exist with an appreciable density, then either there should exist an excess somewhere in the IR background or they must have contributed to the 3 K background. For example, if the remnants of the stars provide the dark matter, then equation (3.2) implies that Ω_R must be in the range 10^{-5} to 10^{-3}, and there is no other wave band where such a high density could reside. If the stars were SMOs with $M > 10^6\, M_\odot$, one might avoid this conclusion because such stars may not release much energy before undergoing collapse. However, we have seen that the dark matter cannot be in holes this large unless they can avoid clustering inside halos. We note that Matsumoto, Akiba, and Murakami (1983) have recently claimed to detect an IR background with $\Omega_R \sim 10^{-4}$ in the wave band 2–5 μm. Carr, McDowell, and Sato (1983) have argued that the form of the data could be explained by pregalactic stars forming at a redshift exceeding 40, in which case it would be less likely that pregalactic stars also generated the 3 K background.

IV. THE EFFECTS OF POPULATION III STARS ON THE BACKGROUND MATTER

In this section we will discuss the effects of the fraction of the radiative energy generated by Population III stars which goes into heating the background matter. By modifying the thermal history of the universe, this can have an important feedback effect on the formation of further Population III stars, especially if the universe is reionized. Most of the radiation energy will be released during the stars' main-sequence phase, and, during this period, the steady input of heat into the universe allows its consequences to be calculated quite simply. For those stars which are in a mass range such that they end up exploding, energy will also be released in the explosive phase. This energy will generally be less than that emitted in the preceding phase. However, because of the impulsive nature of its release, it can generate shock waves and thus produce extra qualitative effects on the background universe.

a) *The Main-Sequence Phase*

During their main-sequence phase, VMOs will be associated with large H II regions. Our main purpose in this section is to determine the structure and evolution of these regions; in particular, we wish to determine when the H II regions overlap, since this will specify when the background universe is reionized. Much of what follows will also apply if the stars are SMOs, or if the sources of the H II regions are *clusters* of VMOs, because all these objects radiate at the Eddington limit. We will discuss at the end the modifications necessary if the stars are MOs.

The H II region surrounding a pregalactic VMO differs from the usual sort of H II region associated with present-epoch O stars in several respects. First, VMOs are hotter: their surface temperature is $T_* \approx 10^5$ K $= 8.6$ eV, so the photon number flux and energy flux peak at $1.6T_* = 13.8$ eV and $2.8T_* = 24.1$ eV, respectively. This implies that there is a high flux of both hydrogen- and helium-ionizing photons (the ionization energies being $E_0 = 13.6$ eV and $E_1 = 24.6$ eV, respectively). Second, we will find that VMOs do not live long enough for their H II regions to expand sufficiently to reach pressure equilibrium with the cosmological background. We can therefore assume that the particle density within the H II regions just reflects that of the cloud from which the stars formed. Third, various cosmological effects which do not pertain at low redshifts, such as the inverse Compton cooling of the 3 K background radiation, may have an important influence on the H II region.

Since each star radiates as a blackbody of temperature T_*, the photon flux at a distance R from the star can be expressed as

$$F(E)dE = \frac{0.15L_*}{4\pi R^2 kT_*}\left[\frac{(E/kT_*)^2 e^{-\tau(E,R)}}{(e^{E/kT_*}-1)}\right]\frac{dE}{kT_*}, \tag{4.1}$$

where $L_* \approx L_{ED} = 1.2 \times 10^{38}(M/M_\odot)$ ergs s^{-1}, and the factor of 0.15 arises from normalizing the number flux to the luminosity. The factor $\tau(E, R)$ is the optical depth at radius R for photons of energy E, given by

$$\tau(E, R) = \int_0^R n_B[Y_H(1-x)\sigma_H(E) + Y_{He}\sigma_{He}(E)]dR\,, \tag{4.2}$$

where n_B is the baryon density; Y_H and Y_{He} are the number of hydrogen and helium atoms per baryon. The ionization, x, is here defined as the ratio of the number of protons to the total number of hydrogen atoms (ionized or neutral); we neglect the small number of electrons which arise from single helium ionizations (i.e., we assume $Y_e = xY_H$). The ionization within the H II region is determined by

$$Y_H \frac{dx}{dt} = -Y_H^2 x^2 n_B \alpha + (1-x)Y_H \zeta_H\,, \tag{4.3}$$

where $\alpha = 2.6 \times 10^{-13}(T/10^4\ \mathrm{K})^{-0.8}$ cm^3 s^{-1} is the recombination rate in the appropriate temperature range (Kaplan and Pikel'ner 1970), recombinations to the ground state being neglected since these produce secondary ionizing photons. The quantity ζ_H is the ionization rate per hydrogen atom:

$$\zeta_H = \int_{E_0}^\infty \sigma_H(E)F(E)dE\,. \tag{4.4}$$

Since $E_0/kT_* = 1.6$ for VMOs, we may simplify the analysis by neglecting the (-1) term in the denominator of equation (4.1). At values of R sufficiently small that $\tau(E) \ll 1$, equation (4.4) and $\sigma_H(E) = \sigma_0(E_0/E)^3$, with $\sigma_0 \approx 7 \times 10^{-18}$ cm^2, then give

$$\zeta_H = \frac{0.6L_*}{4\pi R^2 kT_*}\sigma_0\left(\frac{E_0}{kT_*}\right)^3 \epsilon_1\left(\frac{E_0}{kT_*}\right) = 3 \times 10^{-6}\left(\frac{R}{\mathrm{pc}}\right)^{-2}\left(\frac{M}{10^2\ M_\odot}\right)\mathrm{s}^{-1}\,, \tag{4.5}$$

where the exponential integral term $\epsilon_1(E_0/kT_*)$ equals 0.09 for 10^5 K sources. The recombination rate per proton is

$$t_{rec}^{-1} = Y_H \alpha x n_B = (7 \times 10^5\ \mathrm{yr})^{-1}\left(\frac{T}{10^4\ \mathrm{K}}\right)^{-0.8}\left(\frac{z}{10^2}\right)^3 x\eta\,, \qquad \text{where} \qquad \eta \equiv \left(\frac{\Omega_g}{0.1}\right)h^2(1+\delta)Y_H\,, \tag{4.6}$$

and $1+\delta$ specifies the ratio of the density in the H II region to the density in the background Friedmann universe.

Providing the time scale t_{rec} is less than the lifetime of the star, which from equation (3.4) requires that z exceed

$$z_S = 60\left(\frac{T}{10^4\ \mathrm{K}}\right)^{0.3}\eta^{-1/3}\,, \tag{4.7}$$

the ionization in the H II region is determined by the balance of the two terms on the right-hand side of equation (4.3), and one gets a fully developed H II region with the usual Strömgren radius (Spitzer 1978):

$$R_S = \left(\frac{\dot N}{4\pi n_B^2 Y_H^2 \alpha}\right)^{1/3} = 450 T_4^{0.3} M_2^{1/3} z_2^{-2} \eta^{-2/3}\ \mathrm{pc}\,, \tag{4.8}$$

where we have normalized T to 10^4 K, M to $10^2\ M_\odot$, and z to 10^2; $\dot N$ is the rate of production of ionizing photons. In terms of R_S, the ionization and optical depth are given simply by

$$\frac{1-x}{x^2} = \frac{13.5}{n_B Y_H \sigma_0 R_S}\left(\frac{R}{R_S}\right)^2\,, \qquad \tau = 4.5\left(\frac{R}{R_S}\right)^3\left(\frac{E}{E_0}\right)^{-3}\,. \tag{4.9}$$

Within R_S, τ remains small and x is close to 1; beyond R_S, τ increases suddenly and x falls off exponentially in the usual way. There will also be an He II region within the H II region, where the structure will be modified.

If z is less than the value z_S specified by equation (4.7), the recombination time exceeds the lifetime of the star, and so the H II region just expands to contain the number of hydrogen atoms which can be ionized by the total number of photons with $E > E_0$ coming from the star, $N = 1.5 \times 10^{64}$. This gives a size

$$R_{max} = \left(\frac{3N}{4\pi n_B Y_H}\right)^{1/3} = 750 z_2^{-1}\eta^{-1/3}M_2^{1/3}\ \mathrm{pc}\,, \tag{4.10}$$

which is less than R_S for $z < z_S$ and becomes equal to R_S at z_S. After the star has burnt out, the H II region will survive for a time t_{rec}, so the average H II region will have a size somewhat smaller than R_{max}.

The universe will be fully ionized once the H II regions overlap. The distance between H II region centers,

$$d_{II} = [\Omega_{II}(z)\rho_{crit}M^{-1}]^{-1/3} = 23 z_2^{-1} M_2^{1/3}\left(\frac{\Omega_{II}h^2}{0.1}\right)^{-1/3}\ \mathrm{pc}\,, \tag{4.11}$$

is related to their instantaneous number density, which we have multiplied by the star mass to make a density $\Omega_{\rm II}\rho_{\rm crit}$. When $\Omega_{\rm II}$ has increased to the M-independent value

$$\Omega_i(z) = \begin{cases} 1.4 \times 10^{-5} z_2^3 T_4^{-0.8} \eta^2 h^{-2} & (z > z_{\rm S}) \\ 2.9 \times 10^{-6} \eta h^{-2} & (z < z_{\rm S}) \end{cases} \tag{4.12}$$

$d_{\rm II}$ will just equal min ($R_{\rm S}$, $R_{\rm max}$). We take this to be the criterion for reionization, although the evolution of the background just prior to this may be complicated since it may be compressed into cool clouds (cf. Mészáros 1975). There are three possible cases, each having a particular interpretation of Ω_i: (1) for $z > z_{\rm MS}$, $\Omega_i(z)$ is the density of stars generated by redshift z, the Ω_* of § II; (2) for $z_{\rm S} < z < z_{\rm MS}$, it is the star density generated within a main-sequence lifetime of the epoch z; (3) for $z < z_{\rm S}$, it is the star density created within a recombination time, since the H II regions survive long after the stars have ceased shining. The last is essentially the Hartquist and Cameron (1977) case.

Equation (4.12) only determines Ω_i implicitly in the $z > z_{\rm S}$ situation since it still remains to calculate the temperature in the H II region. In general this is determined by

$$\frac{d}{dt}\left[\frac{3}{2}kT(1+x) - E_0(1-x)\right] + 3\frac{\dot{a}}{a}kT(1+x) = \zeta_{\rm H}(1-x)\langle E\rangle_{\rm inj} - x^2 n_B \alpha \langle E\rangle_{\rm rec} + \dot{q}Y_{\rm H}^{-1}\ ; \tag{4.13}$$

$3kT(1+x)/2$ and $\dot{q}Y_{\rm H}^{-1}$ are the internal energy and cooling rate per hydrogen atom. The $\dot{a}/a$ term arises from adiabatic cooling and can be neglected so long as the other rates exceed the rate with which the H II region expands; $\langle E\rangle_{\rm rec}$ is the mean energy emitted during a recombination, which can be shown to be $E_0 + 0.66kT$ (the coefficient 0.66 being accurate at 10^4 K but slightly erroneous at higher or lower temperatures); $\langle E\rangle_{\rm inj}$ is the mean energy injected per photoionization—for 10^5 K sources, this is $1.44E_0$. At high redshifts ($z > 10$), the dominant cooling mechanism is inverse Compton cooling by electrons Thomson scattering off the 3 K background photons. The associated cooling rate is

$$\dot{q}_c = (8 \times 10^3\ {\rm yr})^{-1} Y_{\rm H} z_2^4 k(T_r - T)\ , \tag{4.14}$$

where $T_r = 2.7(1+z)$ K is the background radiation temperature. In the absence of this effect, the balance of the first two terms on the right-hand side of equation (4.13) would ensure a temperature $T_{\rm rec} = 9.1$ eV, providing both terms exceed the cosmological expansion rate. In its presence, however, equation (4.13) implies

$$T = \frac{T_r + \psi T_{\rm rec}}{1+\psi}\ , \qquad \text{where} \qquad \psi = 0.23\left(\frac{T}{T_r}\right)^{-0.8} z_2^{-1.8}\eta\ , \tag{4.15}$$

which determines T implicitly. Providing $T_{\rm rec} > T > T_r$, the approximate solution is

$$T \approx \psi T_{\rm rec} = 3.2 \times 10^3 z_2^{-0.6}\eta^{0.6}\ {\rm K}\ . \tag{4.16}$$

Thus T is close to T_r for $\psi \ll 1$, close to $T_{\rm rec}$ for $\psi \gg 1$, and given by equation (4.16) in between. Putting these expressions into equation (4.12) determines $\Omega_i(z)$ for $z > z_{\rm S}$.

The parameter η in the above discussion depends on the quantity δ which measures how underdense or overdense the H II region is relative to the background. Two effects are relevant here, one tending to make δ negative and the other tending to make it positive. The first effect arises because the H II region will try to expand in order to attain pressure balance with the Friedmann background, i.e., until its density has fallen to a value $\langle nT\rangle/T$. Roughly speaking, this expansion will occur on the dynamical time scale

$$t_{\rm dyn} = (3.4 \times 10^7\ {\rm yr}) M_2^{1/3} z_2^{-2} \eta^{-2/3} T_4^{-0.2}\ . \tag{4.17}$$

However, since this appreciably exceeds the lifetime of the star unless $z > 300M_2^{1/6}$, this effect is not very important. A more detailed calculation shows that δ evolves as $-(3t/t_{\rm dyn})^2$.

The second effect arises because one might expect stars to form in overdense clumps ($\delta > 0$), thus reducing the size of the H II region [$R_{\rm S} \propto (1+\delta)^{-2/3}$]. On the other hand, one would expect *many* VMOs to form in such a clump, with the interstar separation being reduced by a factor $(1+\delta)^{1/3}$. One might therefore expect the individual H II regions to overlap and form a super H II region around the whole clump before the sources turn off. Since the luminosity scales with M, the previous equations should still apply except that M must be replaced with the value appropriate for the whole clump of stars. Thus, for example, a $10^5\ M_\odot$ clump would produce a 5 kpc H II region at $z = 10^2$. This would usually exceed the size of the clump itself, so throughout most of the super H II region the density would just be the background density ($\delta = 0$).

Once the whole universe is ionized, the temperature is still determined by equation (4.15) except that the parameter δ, on which η and hence ψ depend, now reflects the volume-averaged *clumpiness* of the background gas. In terms of this clumpiness factor, the background ionization has the value

$$(1-x) \approx 10^{-10}\Omega_*^{-1}\Omega_g T_4^{-0.8}(1+\delta) \tag{4.18}$$

determined by the balance of photoionizations and recombinations. This ionization is maintained so long as the stars are burning.

Finally we discuss how the above analysis is modified for MOs. If one assumes that the luminosity of such stars is $L_{\rm ED} M_2^2$ and that their temperature is $10^5 M_2^{0.3}$ K (cf. eqs. [3.4] and [3.10]), then one can show that the number of ionizing photons is reduced by a factor

$$f(M) \approx 1.6 M_2^{1.8} e^{-1.6 M_2^{-0.3}} (0.8 M_2^{0.6} + 1.3 M_2 + 1) . \tag{4.19}$$

From equations (4.8) and (4.10), $R_{\max}$ and R_S just scale as $f(M)^{1/3}$, and so the value of Ω_i specified by equation (4.12) is increased by a factor $f(M)^{-1}$. Clearly, M cannot be too small if the stars are to reionize the universe without producing too much enrichment. The limits discussed in § V require $M > 30\, M_\odot$. The value of $(1 - x)$ given by equation (4.18) is also scaled by a factor f^{-1}.

b) The Explosive Phase

In BAC, we showed that any VMOs smaller than the critical mass M_C should end up exploding, and we now consider the consequences of this. The energy released per unit rest mass in each explosion is approximately (BAC, § III*h*)

$$\epsilon = \frac{(472 \text{ keV}) f_c B_{\rm O}}{Mc^2} \approx 5 \times 10^{-4} f_c \left(\frac{M_{\rm O}}{M}\right) , \tag{4.20}$$

where 472 keV is the energy released per nucleon in burning oxygen to silicon, f_c is the fraction of the oxygen core mass burnt, $B_{\rm O}$ is the number of nucleons in the oxygen core, and M is the mass of the initial hydrogen star. A rough analytic expression for f_c as a function of M was shown to be

$$f_c = f_0 \left(\frac{M_{\rm O}}{10^2\, M_\odot}\right)^b \qquad \text{with} \qquad f_0 \approx 0.7, b \approx 1.8 . \tag{4.21}$$

Providing $\phi_L < \frac{1}{2}$, $M_{\rm O}/M \approx 0.5$, ϕ_L being the fraction of the initial mass lost during hydrogen burning; $M_{\rm O}/M \approx (1 - \phi_L)$ for $\phi_L > \frac{1}{2}$. Thus we get

$$\epsilon(M) \approx 3.5 \times 10^{-4} \left(\frac{M}{10^2\, M_\odot}\right)^{1.8} \min\, [0.13, (1 - \phi_L)^{2.8}] . \tag{4.22}$$

If the stars are MOs rather than VMOs, they may still explode in some mass range below $10^2\, M_\odot$, and, in this case, the value of ϵ lies in the range 10^{-4} to 10^{-5} (Bookbinder *et al.* 1980). MOs may therefore generate explosive energy with efficiency comparable to VMOs. SMOs can explode in some mass range above $10^5\, M_\odot$ only if they contain metals (Fricke 1973), so we assume that at least the first Population III SMOs collapse. Stars with $M_C < M < 10^5\, M_\odot$ can produce explosive energy only if they eject their envelopes during hydrogen shell burning (BAC); in this case, $\epsilon \approx 3 \times 10^{-5}$.

Exploding stars or clusters would generate shock fronts, and, as discussed by Ostriker and Cowie (1981) and Ikeuchi (1981), the shells swept up by these shocks could fragment into new bound objects. In some circumstances the new objects would be bigger than the first, thus initiating a bootstrap process in which ever larger objects form; Ostriker and Cowie were particularly interested in whether this process could generate galaxies from smaller pregalactic objects. In our context, the redshift may be so high that the shell fragments may be stars of comparable mass to the original ones. In this situation, the formation of a few MOs or VMOs might trigger the production of many more; one could thus amplify the value of f_*, the fraction of the universe's mass in stars. Our aim in what follows is to determine under what circumstances this can happen.

We start off by reviewing the Ostriker-Cowie argument. Let us first assume that all the relevant time scales are much longer than the burning time of the stars. In this case each cluster of stars of total mass M will, upon exploding, produce a single spherical shockfront which expands adiabatically according to the Sedov solution until its cooling time becomes comparable to the cosmological time. For $z > 5$, the dominant cooling process is inverse Compton cooling off the 3 K background, for which the cooling time is (cf. eq. [4.14])

$$t_{\rm cool} \approx 2.4 \times 10^{12} (1 + z)^{-4} \text{ yr} . \tag{4.23}$$

This is less than the VMO burning time for $z > 30$ and less than the cosmological expansion time for $z > 10$. When the shock front begins to cool, its radius is

$$R_{\rm cool} \approx 4(1 + z)^{-2.2} E_{56}^{0.2} \Omega_g^{-0.2} h^{-0.4} \text{ Mpc} . \tag{4.24}$$

Here E_{56} is the explosive energy released in units of 10^{56} ergs; this corresponds to $M \sim 10^6\, M_\odot$ if $\epsilon = 10^{-4}$. The mass swept up at $t_{\rm cool}$ is

$$M_{\rm cool} \approx 2 \times 10^{13} (1 + z)^{-3.6} E_{56}^{0.6} \Omega_g^{0.4} h^{0.8}\, M_\odot , \tag{4.25}$$

and the "amplification" factor is therefore

$$\xi_{\rm cool} \equiv \frac{M_{\rm cool}}{M} \approx 2 \times 10^7 (1 + z)^{-3.6} \epsilon_{-4}^{0.6} M_6^{-0.4} \Omega_g^{0.4} h^{0.8} , \tag{4.26}$$

where $M_6 = M/10^6\, M_\odot$ and $\epsilon_{-4} = 10^4 \epsilon$. This exceeds 1 providing

$$M < 3 \times 10^{24} (1 + z)^{-9} \epsilon_{-4}^{1.5} \Omega_g h^2\, M_\odot . \tag{4.27}$$

We also need to know the velocity and Mach number of the shell at $t_{\rm cool}$; these are, respectively,

$$V_{\rm cool} \approx 0.7(1+z)^{1.8}\Omega_g^{-0.2}h^{-0.4}E_{56}^{0.2} \ \ {\rm km\ s^{-1}} \tag{4.28}$$

and

$$\mathscr{M}_{\rm cool} \approx 3(1+z)^{1.3}E_{56}^{0.2}\Omega_g^{-0.2}\,h^{-0.4}\left(\frac{T_c}{T_r}\right)^{-1/2}, \tag{4.29}$$

where T_c is the temperature of the cooled shell, and T_r is the microwave background temperature. These are Ostriker and Cowie's equations, renormalized to different values of E, M, and h. For simplicity, we henceforth assume $\Omega_g = \Omega$.

The shell can fragment at $t_{\rm cool}$ providing there exists a range of scales, a, within the shell over which pancakes are unstable to gravitational collapse. Ostriker and Cowie show that this requires that the shell expansion time exceed

$$t_{\rm crit} \approx 1.3\mathscr{M}^{-1/2}t_E\,, \tag{4.30}$$

where t_E is the cosmological time. From equations (4.23) and (4.29), fragmentation can only occur at $t_{\rm cool}$ if

$$M > 1\times 10^{-16}(1+z)^{18.5}\,\epsilon_{-4}^{-1}h^{-8}\Omega_g^{-4}\left(\frac{T_c}{T_r}\right)^{5/2} M_\odot\,; \tag{4.31}$$

otherwise it cannot occur until later. The typical fragment mass is that associated with the fastest growing instability at $t_{\rm cool}(>t_{\rm crit})$; it has a scale

$$\frac{a_m}{R} = \sqrt{6}\left(\frac{t_{\rm cool}}{t_{\rm crit}}\right)^{-2}\mathscr{M}_{\rm cool}^{-1}\,, \tag{4.32}$$

and hence a mass

$$M_{\rm frag} \approx 3\times 10^3(1+z)^{1.2}h^{-1.6}E_{56}^{-0.2}\Omega_g^{-0.8}\left(\frac{T_c}{T_r}\right)^2 M_\odot\,. \tag{4.33}$$

It is interesting that this mass is in the VMO range for reasonable values of z and E.

In order for the Ostriker-Cowie mechanism to initiate a bootstrap process in the fraction f_* at $t_{\rm cool}$, one must satisfy both the amplification condition (4.27) and the fragmentation condition (4.31). The intersect of the two lines specified by these equations in the (M, z) plane occurs at

$$z \approx 30h^{0.4}\Omega_g^{0.2}\epsilon_{-4}^{0.1}\left(\frac{T_c}{T_r}\right)^{-0.1}, \qquad M \approx 2\times 10^{11}\epsilon_{-4}^{0.7}\Omega_g^{-0.6}h^{-1.3}\left(\frac{T_c}{T_r}\right)^{0.8} M_\odot\,. \tag{4.34}$$

As indicated by the broken lines in Figure 5, the bootstrap mechanism could then occur only to the left of this point. This means that it could only be initiated well after decoupling and only if the seed mass M were itself of galactic scale. Both these conditions would fail if, for example, one considered the sort of cluster of stars which might form at decoupling. However, equations (4.27) and (4.31) are the conditions for amplification and fragmentation to occur at $t_{\rm cool}$. We will now show that the conditions may still be satisfied *after* $t_{\rm cool}$, thus extending the (M, z) domain in which a bootstrap process can be initiated.

During the period after $t_{\rm cool}$ (not considered by Ostriker and Cowie), the shell expands according to the usual "radiative" solution, in which

$$R = R_{\rm cool}\left(\frac{t}{t_{\rm cool}}\right)^{1/4}, \qquad V = V_{\rm cool}\left(\frac{t}{t_{\rm cool}}\right)^{-3/4}, \qquad \mathscr{M} = \mathscr{M}_{\rm cool}\left(\frac{t}{t_{\rm cool}}\right)^{-3/4}. \tag{4.35}$$

This solution will apply until V becomes comparable to the Hubble velocity, which occurs at

$$t_{\rm max} = \frac{3}{2}\left(\frac{V_{\rm cool}\,t_{\rm cool}}{R_{\rm cool}}\right)t_E = \frac{3}{5}\,t_E\,. \tag{4.36}$$

The amplification factor at this time is

$$\xi_{\rm max} = \xi_{\rm cool}\left(\frac{t_{\rm max}}{t_{\rm cool}}\right)^{3/4} \approx 4\times 10^5\epsilon_{-4}^{0.6}M_6^{-0.4}(1+z)^{-1.7}\Omega_g^{0.02}h^{0.04}\,, \tag{4.37}$$

and, using equations (4.23) and (4.26), this will exceed 1 providing

$$M < 1\times 10^{20}(1+z)^{-4.3}\epsilon_{-4}^{1.5}h^{0.1}\Omega_g^{0.05}\,M_\odot\,. \tag{4.38}$$

This condition is much easier to satisfy than equation (4.27). Since amplification ceases after $t_{\rm max}$, because the shell stops expanding relative to the cosmic flow then, amplification can never occur if equation (4.38) is not satisfied. The fragmentation condition at $t_{\rm max}$ corresponds to

$$\mathscr{M}_{\rm cool} > 3.2\left(\frac{t_E}{t_{\rm cool}}\right)^{3/4}, \tag{4.39}$$

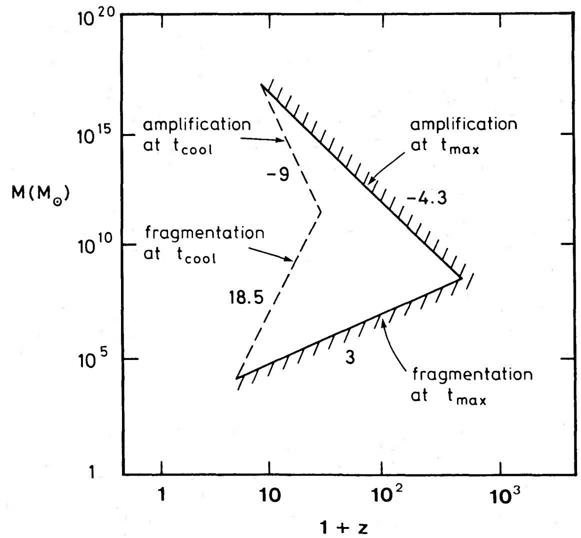

FIG. 5.—This shows the situations in which a cluster of stars with total mass M, exploding at a redshift z, can initiate a bootstrap process in which more stars form via fragmentation in the shock front generated by the explosions. The broken lines specify the conditions for amplification and fragmentation to occur when the shock front first begins to cool at t_{cool}; this is derived from the analysis of Ostriker and Cowie. The solid lines specify the conditions for amplification and fragmentation to occur at some time after this but before the shock front stops expanding at t_{max}. The bootstrap process can only be initiated within these two lines (i.e., not in the shaded region). It can therefore only occur after decoupling and, even then, only within the range of values of M indicated. The numbers by the lines indicate their z-dependence. We have assumed that explosive energy is released from the stars with an efficiency of order 10^{-4}, that the shock temperature is the background radiation temperature, and that $\Omega_g = h = 1$.

and, using equations (4.23) and (4.29), this implies

$$M > 3 \times 10^{-2}(1+z)^3 \epsilon_{-4}^{-1} h^{-1.8} \Omega_g^{-0.9} \left(\frac{T_c}{T_r}\right)^{2.5} M_{\odot} . \tag{4.40}$$

The lines in the (M, z) plane given by equations (4.38) and (4.40) intersect at

$$z \approx 9 \times 10^2 \epsilon_{-4}^{0.3} h^{0.2} \Omega_g^{0.1} \left(\frac{T_c}{T_r}\right)^{-0.3} , \qquad M \approx 2 \times 10^7 \epsilon_{-4}^{-0.1} h^{-1.0} \Omega_g^{-0.5} \left(\frac{T_c}{T_r}\right)^{1.4} M_{\odot} , \tag{4.41}$$

as indicated in Figure 5. Thus, one can initiate a bootstrap process shortly after decoupling ($z \sim 10^3$) providing the initial seed mass is about $10^7 \, M_{\odot}$; the seed mass can be even smaller at later times.

From equation (4.32) with $t_{cool} \to t_{crit}$, the typical fragment mass at t_{crit} is

$$M_{frag} \approx 1.5 M(\xi \mathscr{M}^{-2})_{crit} \approx 8 \times 10^3 z^{0.5} E_{56}^{-0.2} \Omega_g^{-0.6} h^{-1.2} \left(\frac{T_c}{T_r}\right)^{1.9} M_{\odot} . \tag{4.42}$$

This would appear to be in the VMO rather than SMO mass range. However, equation (4.42) should obviously be regarded with a certain amount of scepticism, since many extra processes could complicate the simple fragmentation criterion invoked in deriving it (Vishniac 1983). In particular, Ikeuchi, Tomisaka, and Ostriker (1983) and Bertschinger (1983) have shown that the preceding analysis must be modified once the shell has been expanding for a time comparable to t_E since the lowering of the background density due to cosmological expansion must then be taken into account. This apparently results in a density enhancement within the shock, which would reduce the fragment mass. It also means that the factor of $\frac{3}{5}$ in equation (4.36) is inaccurate; indeed, the shell may continue to sweep up material long after t_E if fragmentation is inefficient. Nevertheless, Carr and Ikeuchi (1983) show that equation (4.37) is still approximately correct.

The qualitative implication of this discussion is clear. Providing the initial bound regions lie within the shaded lines in Figure 5, the formation of exploding fragments within these regions will inevitably generate a shell of new stars which collectively contain more mass than the original bound region. In the standard big bang model with isothermal fluctuations, the mass of the original region might be of order $10^6 \Omega^{-1/2} \, M_{\odot}$, the Jeans mass at decoupling, and this is interestingly close to the sort of mass required by equations (4.41). If the new stars also explode, they could generate a new spherical shock front, roughly concentric with the original one, but with a larger value of R_{cool} since, in equation (4.24), z is decreased and E is increased. The process can then repeat itself either until M and z fall into the shaded region of Figure 5 or until M_{frag} falls outside the exploding range. How this picture links up with the Ostriker-Cowie-Ikeuchi picture of galaxy formation is discussed in more detail by Carr and Rees (1983).

Note that the preceding discussion assumes that all the stars at each stage can be regarded as exploding simultaneously. However, equation (4.24) applies only if the cooling time scale t_{cool} is much greater than the main-sequence time t_{MS} associated with the stars. The latter is given by equation (3.4), and, even for stars with $M > 10^2 \, M_{\odot}$, equation (4.23) implies that one needs $z < 30$; for smaller stars, z has to be even less. Thus at sufficiently large redshifts one cannot regard each cluster of exploding stars as producing a single expanding shell. Nevertheless, once the first star has exploded and produced a shock front, the density behind this front will be very low. This means that, when the second star explodes, the velocity at which the new shock propagates outward will be very large, so it will soon reach the first shock and merge with it. Similarly, as further stars explode, their shocks will just catch up with and enhance the already existing one. After the time t_{MS}, the scenario should revert to the previous one.

The previous analysis must also be modified if the nuclear burning time of the stars exceeds their formation time. In this case, the values of z and t_E are themselves determined by the individual star mass. For example, if the stars are VMOs, equations (4.38) and (4.40) become

$$3 \times 10^9 \epsilon_{-4}^{1.5} h^3 \Omega_g^{1.5} > \frac{M}{M_\odot} > 7 \times 10^5 \epsilon_{-4}^{-1} h^{-3.8} \Omega_g^{-1.9} \left(\frac{T_c}{T_r}\right)^{2.5} . \tag{4.43}$$

This range of values corresponds to where the line $z = z_{MS}$ cuts the boundary lines in Figure 5. Having $t_{MS} > t_E$ thus cuts off the corner of the permitted (M, z) region.

V. THE ENRICHMENT PROBLEM

In this section we will discuss one of the most stringent constraints on the spectrum of Population III stars: the requirement that they do not generate an excessive amount of enrichment. This constraint derives from the fact that there exist Population I stars with metallicity as low as 10^{-1} times solar, or $Z \sim 10^{-3}$. The Population III enrichment, therefore, cannot exceed this value. There also exist Population II stars with Z as low as 10^{-5} (Bond 1981). While conventional wisdom would say that Population II stars form after Population III, in which case the Population III enrichment would need to be below 10^{-5}, one cannot exclude the possibility that both populations form at the same epoch. This is because one would expect the first bound regions in the universe to form stars with a spectrum of masses, and the exploding stars might not enrich the background until after the smaller stars have begun their main-sequence phase. For the same reason, one would only expect the amount of pregalactic enrichment to be reflected in a lower cutoff in the metallicity distribution of present-epoch stars if all Population III stars were large enough to complete their evolution by the present epoch. Thus the maximum enrichment Z_{max} cannot be specified precisely. The problem is compounded by the possibility that mixing of enriched material may be incomplete: the amount of pregalactic enrichment might only be defined in some sort of average way, and there could be stars with Z much less than this average value. We will assume $10^{-3} > Z_{max} > 10^{-5}$.

a) Limits on the Mass Spectrum from Nucleosynthesis

The pregalactic nucleosynthesis constraint could be potentially embarrassing to the Population III star hypothesis. This is certainly true if the stars have a mass in the range 15–10^2 $M_\odot$. For, in this range, numerical calculations of Weaver and Woosley (1980) show that the fraction of mass ejected as heavy elements can be expressed as

$$Z_{ej} \approx 0.5 - \left(\frac{M}{6.3\, M_\odot}\right)^{-1} \qquad \left(15 < \frac{M}{M_\odot} < 10^2\right) ; \tag{5.1}$$

i.e., the yield lies between 20% and 50% in this mass range. This is related to the mass of the carbon-oxygen core: $Z_{ej} = (M_O - 1.5\, M_\odot)/M$ if it is assumed that such stars leave 1.5 $M_\odot$ remnants. The ejected metallicities of Arnett (1978) agree with this. It can be inferred that the fraction of the universe going into stars in the mass range 15–10^2 $M_\odot$ over the whole history of the universe cannot exceed

$$f_{max} = \frac{Z_{max}}{Z_{ej} + Z_{max} Z_{ej}} < 5 \times 10^{-3} \left(\frac{Z_{max}}{10^{-3}}\right) . \tag{5.2}$$

Similar limits apply for other exploding mass ranges. Stars smaller than 4 $M_\odot$ are assumed to produce no enrichment. However, stars in the mass range 4–8 $M_\odot$ end up with degenerate carbon cores and eject a large fraction of their mass as iron ($Z_{ej} \approx 0.2$). Stars in the range 8–15 $M_\odot$ may have either a large yield ($Z_{ej} \sim 0.1$) or a small yield ($Z_{ej} \sim 0.01$), depending on whether or not they leave condensed remnants. Stars in the range 10^2 $M_\odot$ to M_C will form exploding VMOs and therefore also contribute to the Population III enrichment. The precise yield of these stars is uncertain because of the uncertainty in the fraction of mass lost during the hydrogen- and helium-burning phase, but, for a given value of ϕ_L, the considerations of BAC show that the yield in elements heavier than helium should be

$$Z_{ej} = \min\,[(1 - \phi_L), 0.5] . \tag{5.3}$$

For plausible values of ϕ_L, Z_{ej} could lie anywhere between 0.5 and 0.1.

These limits on f_* are shown in Figure 6, where we assume $\Omega_g = 0.1$, and one can use them to put restrictions on the mass spectrum of Population III stars. For example, if $\alpha < 2$ (so that most of the density is in high-mass stars), we require

$$M_{max} > M_C \left(\frac{Z_{ej} f_*}{Z_{max}}\right)^{1/(2-\alpha)} . \tag{5.4}$$

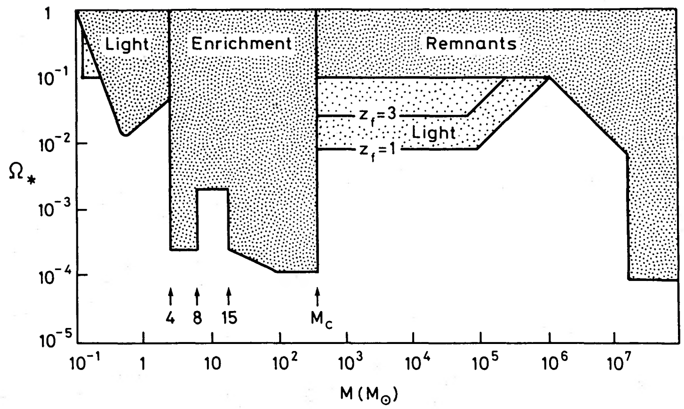

FIG. 6.—This summarizes the various restrictions on the function $\Omega_*(M)$. The background light limit depends on the formation epoch of the stars, z_f, but can provide the most stringent constraint for $M < 4\ M_\odot$ and for $M > M_C$ if $z_f < 10$. The black hole limit is independent of z_f and provides the most interesting constraint for $M > 10^6\ M_\odot$. The limit applies only if the holes reside in galactic halos; unclustered holes could in principle have the critical density. The enrichment limit is also independent of z_f, and it provides the strongest constraint over the mass range $4\ M_\odot < M < M_C$. We have assumed that the maximum enrichment $Z_{\rm max}$ is 10^{-3}, although it could conceivably be as small as 10^{-5}. The form of the enrichment limit for $M > 10^2\ M_\odot$ depends sensitively on ϕ_L, the fraction of mass lost by the stars during their hydrogen- and helium-burning phase. We assume $\phi_L \ll 1$; if it were close to 1, the limit would be much weaker in this mass range. The form of the helium limit for $M > 10^2\ M_\odot$ (not shown) is also very sensitive to ϕ_L. The amount of helium generated is very small, giving only weak constraints, if ϕ_L is either very small or very close to 1.

On the other hand, if $\alpha > 2$ (so that most of the density is in low-mass stars), we require

$$M_{\rm min} < 4\ M_\odot \left(\frac{Z_{\rm ej} f_*}{Z_{\rm max}}\right)^{1/(2-\alpha)} . \tag{5.5}$$

In these equations f_* is the fraction of the universe's mass which goes into stars over *every* mass range. If $f_* = 0.5$, $Z_{\rm max} = 10^{-5}$, and $Z_{\rm ej} = 0.2$, the factor $Z_{\rm ej} f_*/Z_{\rm max}$ is 10^4, and the restriction on the mass spectrum can be expressed very simply. For example, $M_{\rm max}$ is hardly likely to exceed $10^6\ M_\odot$ if $\alpha < 2$ because of the halo dynamical limit; thus equation (5.4) requires $\alpha < 1$. Similarly, $M_{\rm min}$ is hardly likely to be less than $0.004\ M_\odot$ (say) in any fragmentation scenario; thus equation (5.5) requires $\alpha > 3$.

b) Ways of Producing a Small Enrichment

As indicated in § III, some of the explanations for spectral distortions in the microwave background *require* a pregalactic enrichment of 10^{-5} in the form of dust. People have also tried to explain the G-dwarf problem in terms of a prompt initial enrichment of $Z \sim 10^{-5}$ (Truran and Cameron 1971). In this case the enrichment would not necessarily need to be pregalactic, but it could be. If one demands $Z \sim 10^{-5}$, the inequalities in equations (5.4) and (5.5) must be treated as equalities. Clearly, providing $M_{\rm max} > 4\ M_\odot$ or $M_{\rm min} < M_C$, it is always possible to satisfy these equalities if the values of α and $M_{\rm max}$ or $M_{\rm min}$ are suitably chosen. However, these situations are somewhat contrived. In view of the missing mass problem, it might seem more natural to assume either $M_{\rm max} < 0.1\ M_\odot$ or $M_{\rm min} > M_C$ since, in the first case, all of the dark matter would be in low-mass stars, and, in the second case, it would be in massive black holes. Naively, one would infer that there could be no enrichment in either situation. However, we will now show that this need not be the case.

First, the assertion that all stars larger than M_C collapse is clearly too simplistic since parameters other than mass may affect a VMO's evolution. For example, the VMO could have some angular momentum or a magnetic field, and, if it is part of a binary system, its evolution could be affected by its companion. These complications presumably mean that some fraction of stars larger than the critical mass M_C may be able to explode after all. In particular, it is shown in BAC that the effect of rotation is to increase the critical mass by a factor $[1 + 3.5(J/J_o)^2]$, where J/J_o is the angular momentum in units of the breakup value. This will have two effects: it will increase M_C to some "effective" value $\langle M_C \rangle$, associated with the average value $\langle J \rangle$; and, for any particular J distribution, there will always be a small fraction of VMOs larger than $\langle M_C \rangle$ which have sufficient angular momentum to explode. Another possibility is that rotation could drive convective dredge-up of oxygen through the hydrogen- and helium-burning shells before core collapse.

A second way of generating a large density of dark remnants as well as a small amount of enrichment through Population III stars is to postulate that there were *two* generations of such stars. One of these generations could contain a large fraction of the mass of the universe and provide the dark matter; the other could contain only a small fraction but comprise stars whose mass is such that they produce a large individual heavy-element yield. One fairly natural way in which this could come about is as follows: One would expect the density fluctuations from which the first stars derive to have a Gaussian distribution on any particular scale. Thus, if the first regions to bind have a mass M_1, then the distribution in the overdensity at decoupling, $\delta_{\rm dec}$, over regions of mass M_1 should have the form

$$P(\delta_{\rm dec}, M_1) d\delta_{\rm dec} = (2\pi)^{-1/2} \delta_*(M_1)^{-1} \exp\left[-\frac{\delta_{\rm dec}^2}{2\delta_*(M_1)^2}\right] d\delta_{\rm dec} , \tag{5.6}$$

where $\delta_*(M_1)$ is the root-mean-square density fluctuation on the scale M_1. Since the time at which an overdense region binds is $t_B \approx t_{\rm dec}\,\delta_{\rm dec}^{-3/2}$, the fraction of regions which bind between t_B and $t_B + dt_B$ should be

$$P(t_B)dt_B = (2\pi)^{-1/2}\left(\frac{t_{\rm dec}}{\langle t_B\rangle}\right)^{-2/3} \exp\left(-\frac{\langle t_B\rangle^{4/3}}{2t_B^{4/3}}\right) dt_B\,. \tag{5.7}$$

Here $\langle t_B\rangle$ is the time at which most of the overdense regions would bind, i.e., the value of t_B associated with δ_*. This is just $\langle t_B\rangle \approx 10^{-5}(M_1/M_\odot)\delta_*^{-3/2}$ s. Integrating equation (5.7) over t_B implies that the fraction of the universe which has already gone into stars at a time $t \ll \langle t_B\rangle$ is

$$f_*(t) \approx \exp\left(-\frac{\langle t_B\rangle^{4/3}}{2t^{4/3}}\right). \tag{5.8}$$

As this fraction rises, the heat generated by the stars will also increase, and eventually one would expect the whole universe to be reionized, in the manner described in § IV. Thereafter, the binding of further regions will be inhibited by Compton drag either until a redshift $z_{\rm drag} \approx 140(\Omega h^2)^{1/5}$, or until the stars burn out, whichever comes first (Hogan 1979).

Eventually, more regions will bind, with further associated star formation. If these later stars are not themselves to produce heavy elements, we require that they be either much smaller than the first stars (e.g., because the small amount of metals present decreases the effective fragment mass [Silk 1977]) or much larger (e.g., because the Jeans mass has been boosted above the critical value M_C). Providing that one of these alternatives applies, one would expect the initial enrichment to be generated only by the fraction of the universe, $f_{\rm max}$, which goes into stars prior to reionization. This possibility, which was first suggested by Hartquist and Cameron (1977), is essentially a way of producing a *bimodal* mass spectrum for Population III stars, rather than a continuous spectrum.

In order to determine the expected value of $f_{\rm max}$ in this scenario, we must calculate the value of Ω_* when the universe is reionized (Ω_i). Hartquist and Cameron use simple energetic arguments to determine Ω_i and obtain the $z < z_S$ result of equation (4.12). However, if z exceeds z_S, the considerations of § IV show that the appropriate value is somewhat larger. In any case, this scenario would clearly limit the initial enrichment to a value $Z_{\rm max} = f_{\rm max} Z_{\rm ej}$, where $Z_{\rm ej}$ is the yield of the first stars. In fact, equation (5.8) permits one to predict the value of $Z_{\rm max}$ and the epoch of reionization z_i more precisely. For it requires that z_i be the solution of equation (4.12) and the equation

$$\log\left[\frac{\Omega_i(z)}{\Omega_g}\right] \approx -\frac{z^2}{2\langle z_B\rangle^2}\,. \tag{5.9}$$

Thus z_i and hence Ω_i are determined by $\langle z_B\rangle$. The quantity Ω_i/Ω_g is just 3×10^{-5} for $z_i < z_S$, and, in this case, $z_i \approx 5\langle z_B\rangle$.

A somewhat different way of producing a bimodal mass spectrum of stars would be to invoke the Ostriker-Cowie scenario discussed in § IV. In this case, the first generation of exploding stars may only produce a small enrichment if f_* is small. However, since the amplification factor $\xi_{\rm max}$ given by equation (4.37) can be large, one may automatically generate a large density of second-generation stars. These later stars will collapse, without producing further enrichment, providing $M_{\rm frag}$ is large enough.

So far we have assumed the exploding stars form first. However, it is also possible that the nonenriching stars could form *before* the enriching ones. The self-limiting scenario discussed above might suggest that one could never have a large value for Ω_* before $z_{\rm drag}$. However, if one believes that the 3 K background is itself generated by Population III stars or their remnants, as suggested in § III, then Compton drag cannot operate while the first stars are forming since no radiation is present. In this situation, self-limiting star formation does not begin until after the dark matter has formed, and, in contrast to the first situation, we now require the first stars to be *more* massive than the ones which form later. Lacking any reliable theory of fragmentation, it is difficult to judge which of these two scenarios is more plausible. However, it should be stressed that, if one wants to explain the distortions in the spectrum of the 3 K background as deriving from grains, then the grains definitely have to be produced *after* the bulk of the 3 K background.

VI. OTHER NUCLEOSYNTHETIC CONSEQUENCES OF POPULATION III STARS

In this section we examine some specific abundance features which may throw light on nucleosynthetic processes involving Population III stars. We will suggest that some of these features may *require* particular types of stars, perhaps VMOs, while others may impose important constraints on how many of them ever formed. Most of our considerations will be qualitative since, for the most part, we lack detailed quantitative information about the nucleosynthetic products of the stars.

a) Elements Heavier than Oxygen

Abundance ratios of heavy elements to oxygen have been found to vary in astronomical objects with metallicity (e.g., French 1980). In particular, Sneden, Lambert, and Whitaker (1979) argue that stars with [Fe/H] < -1 have about 3 times the solar oxygen-to-iron ratio, with the ratio in stars with [Fe/H] > -1 decreasing with increasing metallicity until it attains the solar value at [Fe/H] $= 0$. This high [O/Fe] is found in both field metal-poor stars and globular clusters. Twarog (1980) models this behavior by assuming that the galactic disk was born with [O/H] $= -0.5$ and [Fe/H] $= -1$, the remaining 90% of iron and 66% of oxygen being produced during the lifetime of the disk. The implication is that predisk stars had different nucleosynthetic yields, and presumably therefore a different mass spectrum, from the postdisk ones. Since more massive stars tend to have large oxygen cores, which yield higher [O/Fe] ratios than lower mass exploding ones, the predisk stars are presumably exploding MOs or exploding VMOs.

Another relevant point is that observations of extragalactic giant H II regions (French 1980) show that the sulfur-to-oxygen ratio decreases with increasing oxygen abundance. This contrasts with the situation in the disk of our own Galaxy, where sulfur and oxygen are not clearly correlated. Since sulfur is a direct product of oxygen burning, this ratio may be a good indicator of what sort of exploding oxygen cores are required. Field metal-poor stars also have enhancements of oxygen-burning products: in particular, calcium, silicon, and titanium.

It is interesting to inquire whether VMOs exploding in low-metallicity regions could account for these features. In BAC, we calculated the fraction of a VMO's oxygen core which undergoes burning. VMOs with $M_{\rm O} < M_{\rm OC}$ were found to completely disrupt, except perhaps when $M_{\rm O} \approx M_{{\rm O}m} \approx 30\ M_\odot$, the minimum mass required for oxygen cores to experience the pair-instability. We will not make a large error in assuming that all burned products are ejected for $M_{{\rm O}m} < M_{\rm O} < M_{\rm OC}$, and that none are if $M_{\rm O} > M_{\rm OC}$. The fraction of the oxygen core mass which is burned and ejected can be approximated by equation (4.21). Comparison with the numerical results of Arnett (1973) and Woosley and Weaver (1982) suggests f_0 may be somewhat smaller; also, b may be smaller for $M_{\rm O} \sim M_{\rm OC}$ and larger for $M_{\rm O} \sim M_{{\rm O}m}$. We take our results as indicative; more detailed numerical computations over a wide mass range would be needed to improve the yield formula.

If the minimum and maximum masses in our assumed power-law spectrum span the range $M(M_{{\rm O}m})$ to M_C, and if $M/M_{\rm O}$ is mass independent, then a simple relation can be obtained for the ratio of oxygen-burning products to oxygen. Using equation (4.21), we obtain

$$\left(\frac{X(>0)}{X(0)}\right)_{\rm VMO} = \frac{A}{1-A}, \tag{6.1}$$

where

$$A = \left|\left(\frac{2-\alpha}{b+2-\alpha}\right)\left[\frac{(M_{\rm OC}/M_{{\rm O}m})^{2+b-\alpha}-1}{1-(M_{\rm OC}/M_{{\rm O}m})^{2-\alpha}}\right]\right| f_0 \left(\frac{M_{{\rm O}m}}{10^2\ M_\odot}\right)^b. \tag{6.2}$$

The yield of oxygen-burning products is sensitive to the stellar mass spectrum since more massive VMOs burn more oxygen before exploding. For example, using $b = 1.8$ and $f_0 = 0.7$, the ratio given by equation (6.1) is 0.36 if $\alpha = 2.35$, and it decreases from 0.51 to 0.09 as α increases from 0 to ∞. The solar ratio is 0.48 (if we disregard the carbon-burning product neon). BAC also calculate the iron yield of VMOs. While their result is certainly an overestimate, the results of Woosley and Weaver (1982) confirm that one expects low values of Fe/O, as required.

b) Primary Nitrogen

It is usually assumed that nitrogen is produced by CNO processing in stars after a prior generation has produced CO. If nitrogen is "secondary" like this, one expects (N/O) $\propto$ (O/H)^2. However, Pagel and Edmunds (1981) have reviewed arguments which suggest that some nitrogen may be primary (i.e., produced at the same time as the oxygen), so that N/O is constant. For example, though there is a great deal of scatter in the data, irregular and compact galaxies with a range of low metallicities (such as the SMC and LMC), as well as very metal-poor stars (Barbuy 1983), seem to have N/O approximately constant. A possible solution is that the CNO processing of later-generated metals superposes secondary nitrogen upon primary nitrogen produced by Population III stars.

A mechanism for this may be found in VMOs. If convective dredge-up can pass carbon and oxygen from the helium-burning core through the hydrogen-burning shell, in such a way that it is CNO processed to nitrogen before entering the hydrogen envelope, then nitrogen emission may occur in a wind driven by nuclear pulsations. Since the adiabatic index is near 4/3 in radiation-dominated stars, it does not cost much energy to pass metal-rich material into metal-poor zones, so this scenario is not implausible. Either meridional circulation or convective penetration could be the mechanism for an upwelling of nitrogen. Woosley and Weaver (1982) already have indications from their 500 $M_\odot$ Population III star evolution that this dredging may occur. If such a mechanism allowed metals to be ejected from stars with $M > M_C$, then a strong limit could be placed upon the number of black holes resulting from Population III VMOs (cf. Klapp 1982; Tarbet and Rowan-Robinson 1982).

c) Helium Production

One of the chief successes of the standard big bang picture is the prediction of the helium abundance generated by cosmological nucleosynthesis. An abundance $Y \approx 0.23$ would be generated if the photon entropy per baryon, $S_\gamma \equiv 5 \times 10^8 (\Omega_N h^2)^{-1}$, were 7×10^{10} (Yang *et al.* 1979); here Ω_N specifies the nucleon density at the epoch of primordial nucleosynthesis. In the standard model, helium is overproduced if S_γ is smaller than this unless neutrinos are partially degenerate. On the other hand, we saw in § III that VMOs could, in principle, have generated part of the background radiation; this would necessitate a smaller primordial entropy per baryon ratio than today. In addition, BAC show that VMOs could have generated a lot of helium through winds. Thus the formation of Population III stars may involve an overproduction of helium on account of both an increased primordial abundance and a nonprimordial contribution.

If Population III remnants are to provide the missing mass, one may need to give up the conventional cosmological nucleosynthesis picture anyway. For, in the standard picture, the implication of Y not exceeding 0.23 in some regions of the universe is that $\Omega_N h^2 < 0.007$; in particular, the density of remnants which arise after cosmological nucleosynthesis must satisfy this. Thus, the dark matter problem cannot be solved by black holes or low-mass stars of nonprimordial origin unless one modifies the conventional cosmological nucleosynthesis scenario. This is the well-known argument that the bulk of the mass in the universe was in nonbaryonic form at the epoch of primordial nucleosynthesis (Bond, Efstathiou, and Silk 1980; Schramm and Steigman 1981).

These problems are not, in fact, insurmountable. For example, if the baryon asymmetry in the universe is generated as a consequence of baryon-nonconserving processes occurring at very early times, then a lepton asymmetry would presumably also be generated. In standard grand unified models (Georgi and Glashow 1974; Harvey *et al.* 1982), one would expect the lepton asymmetry to be of order the baryon asymmetry. This leads to a degeneracy parameter (chemical potential divided by temperature) for neutrinos which is proportional to S_γ^{-1} and thus exceedingly small unless S_γ is itself small. However, scenarios have been constructed in which a much larger lepton number could arise from the grand unified era (Kolb 1981), leading to large neutrino degeneracy parameters even for large S_γ. In these models, the reaction $e^- + p \rightarrow n + \nu_e$ is inhibited by Fermi statistics, and so the neutron-to-proton ratio is smaller than in models without degeneracy; this implies that a smaller helium abundance is generated during primordial nucleosynthesis. The extreme example of this is a cold universe ($S_\gamma < 1$) with a lepton-to-baryon ratio in excess of 1.5: no neutrons and hence no alphas form (Kaufman 1970; Carr 1977*b*). In such a universe, pregalactic stars would be needed to generate both the 3 K background and the observed helium. On the other hand, if pregalactic stars could be shown to be incapable of significantly affecting either S_γ or Y, then universes with $S_\gamma < 7 \times 10^{10}$ would become untenable.

It seems clear that stars smaller than $\sim 10^2\ M_\odot$ cannot produce a value $Y \approx 0.23$ without overproducing heavy elements at the same time. VMOs with $M > M_C$, however, could avoid ejecting *any* metals, even though they could eject a lot of helium as a result of pulsations (Talbot and Arnett 1971). If the fraction of mass lost ϕ_L is less than $(1 - Y_i)/(2 - Y_i)$, where Y_i prescribes the *initial* helium abundance, then BAC find that the fraction of mass returned as extra helium is

$$\Delta Y = (1 - \tfrac{1}{2} Y_i)\phi_L^2 \; . \tag{6.3}$$

Larger mass loss fractions lead to smaller helium yields, so the maximum yield occurs when ϕ_L has the critical value. Thus

$$(\Delta Y)_{\max} = 0.25(1 - Y_i)^2/(1 - Y_i/2) \; . \tag{6.4}$$

This corresponds to a situation in which the mass loss just keeps up with the shrinkage of the convective core. If $Y_i = 0.23$, as would apply in the conventional cosmological nucleosynthesis scenario, $(\Delta Y)_{\max} = 0.17$, a value also obtained in the numerical calculations of Talbot and Arnett (1971). If $Y_i = 0$, as would apply in a cold universe with no cosmological nucleosynthesis, $(\Delta Y)_{\max} = 0.25$. It is, of course, intriguing that this is so close to the observed abundance.

We stress that helium generation would be reduced if ϕ_L was smaller than the critical value. However, in BAC we discuss a mechanism whereby, for Population I stars, the helium-rich envelope above the final helium core and hydrogen-burning shell could be ejected via a super-Eddington luminosity. During this catastrophic mass loss, $\dot{M}$ becomes so large that all the synthesized helium above the hydrogen-burning shell could be ejected. This would produce exactly the optimum yield $(\Delta Y)_{\max}$ derived above. Woosley and Weaver (1982) find a similar mechanism operative in their 500 $M_\odot$ Population III stars. While it drives the helium-rich envelope to very low densities ($\sim 10^{-14}$ g cm^{-3}), it does not eject it. However, winds would presumably be enhanced in such low-density envelopes, and this could lead to helium ejection.

We have described elsewhere (Bond, Carr, and Arnett 1983) a scenario in which VMOs with $M > M_C$ can generate the entire primordial helium in this way. However, we caution that the problem cannot be solved without further study of nucleosynthesis, convection, and mass loss in VMOs. For example, it may turn out that too many metals are dredged up in either the hydrogen- or helium-burning phase (cf. the discussion of nitrogen production), in which case one could *not* invoke VMOs to generate the helium. On a more positive note, we emphasize that, even at low metallicities, the helium abundance appears to vary widely with location (Peimbert 1980). It is not clear how much of this spatial variation is due to observational uncertainties and how much is intrinsic, but VMOs could certainly impose fluctuations of say $\Delta Y \sim 0.05$ upon a primordial background value of $Y \approx 0.23$. In a hot universe one could clearly use these sorts of considerations to constrain the amount of matter going into VMOs. However, we do not discuss these constraints explicitly since they are obviously very dependent on the unknown parameter ϕ_L.

d) *Deuterium*

Primordial deuterium provides another strong nucleosynthesis argument favoring a low baryon density, high entropy, big bang model. Standard models with $S_\gamma < 5 \times 10^9$ produce $X_{\rm D} < 10^{-5}$ (Yang *et al.* 1979), and having partially or completely degenerate neutrinos does not aid in deuterium production. VMOs are like other stars in that they destroy rather than create deuterium. Indeed, since VMOs are initially almost completely convective, one would expect *all* deuterium processed through them to be destroyed. The deuterium problem may therefore impose another strong constraint on how much of the universe can have gone into Population III VMOs: the present deuterium abundance would have to be smaller than its initial value by a factor $(1 - f_*)$ in a hot universe. In a cold universe, of course, the problem is to produce any deuterium at all. We note, however, that Population III stars or their remnants may well be associated with particle acceleration mechanisms, generating cosmic rays and γ-rays. Epstein, Lattimer, and Schramm (1976) show that spallation of helium by cosmic rays could generate deuterium providing it occurs at a large redshift (to avoid overproduction of γ-rays when the universe is optically thin to them), but the cosmic rays have to be of high energy (>30 GeV) to avoid overproduction of lithium. Ozernoi and Chernomordick (1975) and Bond, Carr, and Hogan (1983) have discussed the possibility of generating deuterium by photodissociating helium with γ-rays. It is not inconceivable that these sorts of mechanisms could produce the tiny amount of deuterium required.

TABLE 2

COSMOLOGICAL EXPLANATORY POWER OF DIFFERENT TYPES OF STARS

Cosmological Feature	LMO	MO	VMO ($M < M_C$)	VMO ($M > M_C$)	SMO	Comments
Dark matter	✓	?		✓	✓	LMOs need $M < 0.1\ M_\odot$. Ω_B too small for MOs unless much accretion. SMOs excluded by dynamical effects if $M > 10^6\ M_\odot$.
3 K background				✓	✓	Only black hole accretion can produce radiation early enough ($z_* \sim 10^3$) for it to be thermalized by free-free processes. The holes need $\Omega_B \sim 0.1$, excluding large SMOs.
3 K distortion.........		?	✓	✓	✓	Black holes not necessarily required; nuclear burning suffices if $\Omega_* \sim 0.1$ and $z_* \sim 10^2$. Need some grains and hence some exploding stars, but MOs may overenrich.
Helium				✓		Only VMOs with $M > M_C$ can avoid overenrichment. They produce helium via winds in their hydrogen- and helium-burning phase.
Enrichment		✓	✓			Ω_* must be small, viz., self-limiting star formation. SMOs probably collapse completely if $Z = 0$ initially.
Oxygen anomaly		✓	✓			Either VMOs or MOs could explain this but very uncertain.
Primary nitrogen			✓	✓		Conceivably produced by upwelling in VMOs but uncertain. Could severely constrain number of $M > M_C$ remnants.
Galaxies...............		✓	✓	?		Need exploding stars. SMOs probably do not explode if $Z = 0$ initially. $M > M_C$ VMOs may eject envelopes.

VII. DISCUSSION

The considerations of this paper may be seen as placing various limits on $\Omega_*(M, z)$, the density of Population III stars of mass M which formed at a redshift z. Which is strongest depends on the particular values of M and z: roughly speaking, the background light limit is best for low M, the black hole limit is best for high M, and the nucleosynthesis limit is best for intermediate M. The combined limits are represented in Figure 6. Only the light limit is sensitive to z; and only if $z < 10$ does it become the most interesting one in the $M > M_C$ regime. Clearly, Population III stars can provide the missing mass in halos and clusters only for $M < 0.1\ M_\odot$ or for $M > M_C$ and $z > 10$. However, stars in the intermediate range could still be used to reionize the universe or provide a burst of pregalactic enrichment or generate galaxies through explosions. The form of the $\Omega_*(M)$ limits in Figure 6 in turn constrains the mass spectrum of Population III stars. Thus, if we know any one of the parameters α, $M_{\max}$, $M_{\min}$ in equation (1.1), we can infer limits on the other two. Some of these spectral constraints have already been given explicitly in §§ II and V. For example, we have seen that, for a large total star density, the spectrum can intersect the mass range $4\ M_\odot < M < M_C$ only if $\alpha > 3$ or $\alpha < 1$.

The limits on $\Omega_*(M, z)$ are obviously of interest in their own right, but it is of particular importance to examine what light they throw on the plausibility of the various cosmological roles attributed to Population III stars. In Table 2, we assess which types of star could explain the particular cosmological features discussed in this paper. In terms of the number of check marks, collapsing VMOs fare best, while LMOs fare worst. Of course, this mode of assessment may be misleading. If there is more than one generation of Population III stars, each in a different mass range, there would be no need for the same type of star to explain more than one cosmological conundrum. On the other hand, there is clearly an aesthetic appeal in having a single type of star explain as much as possible.

An immediate implication of Table 2 is that no type of Population III star can perform *all* of the cosmological roles. However, since the mass spectrum of Population III stars could span several types of star, even if it is continuous, one would not necessarily require this anyway. One might expect a continuous spectrum to span at least two neighboring columns of Table 2, and it is therefore interesting to consider the combined explanatory power of pairs of neighboring columns. On this criterion only the two VMO columns can explain everything. One should interpret this result with caution in view of the aesthetic character of Table 2, but it obviously lends support to the notion that Population III stars were VMOs.

Recently Öber, El Eid, and Fricke (1983), Woosley and Weaver (1982), and Tarbet and Rowan-Robinson (1982) have also considered the cosmological roles of Population III stars. Öber *et al.* and Weaver and Woosley have focused on their nucleosynthetic yields. In particular, they have argued from the theoretical estimates of the relatively low yield of elements around calcium from MOs that a pregalactic generation of VMOs may be required. Tarbet and Rowan-Robinson have considered the circumstances under which Population III stars could explain some combination of the first five cosmological conundrums in

Table 2. While questioning some of their stellar evolution assumptions, we are in broad agreement with their conclusion that, in order to solve all these conundrums, $M_{\max}$ must be large and α must be small.

We should point out that only the second, third, and last of the cosmological roles listed in Table (2) definitely require that the Population III stars form at a high redshift ($z > 10^2$). The other roles could equally be played by the sort of Population III stars which might form in pancake fragments at relatively low redshifts ($z < 10$). Even in the latter context, VMOs would appear to be the most plausible candidates as judged by the check marks in Table 2. This also applies if the Population III stars form *after* the protogalaxies have bound, though before Population I and II stars.

Finally, we stress that the following observations would help one to determine the possible existence and characteristics of Population III stars: (1) the confirmation of spectral distortions in the 3 K background; (2) the determination of the far-infrared background spectrum; (3) ascertaining that there is a definite lower cutoff in the metallicities of Population II stars; (4) the confirmation of various abundance anomalies; and (5) the detection of a gravitational wave background.

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