

A SUPERCLUSTER CATALOG

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ABSTRACT

A catalog of superclusters, defined as clusters of rich clusters of galaxies, is constructed for $z < 0.1$ from a complete redshift sample of Abell clusters. The superclusters are identified by their density enhancement in space. The cataloged systems constitute the densest groups of the richest known clusters (to the given redshift limit). The dependence of the identified supercluster sample on the density enhancement is studied. Comparisons with random catalogs are used to assess the uncertainties. Large-scale structures are seen in the observed sample that have no counterparts in the random catalogs. Sixteen superclusters that contain $\sim 54\%$ of all rich clusters are cataloged at a density enhancement of $f = 20$. The number of clusters per supercluster varies from two to 15 clusters at $f = 20$. The larger superclusters appear to be elongated, with sizes that exceed $100 h^{-1}$ Mpc in some cases. While the superclusters are the largest known structures, they occupy only a tiny fraction of space: $\sim 3\%$ at $f = 20$ and 0.3% at $f = 100$. As the density enhancement increases, the larger superclusters break up into smaller subgroups and eventually into tight binaries and triplets. At $f = 400$, the identified superclusters, some of which are the cores of larger superclusters, consist of seven tight binaries and/or triplets with small separations ($r \lesssim 13 h^{-1}$ Mpc), containing 16% of all rich clusters. The multiplicity function of the superclusters is also determined. In addition, a larger sample of less rich galaxy clusters is analyzed; the results are consistent with the catalog constructed using the richer clusters. The sensitivities of the selected superclusters to uncertainties in the redshifts and to the selection algorithm are investigated.

Subject headings: galaxies: clustering

1. INTRODUCTION

Detailed studies of the spatial distribution of rich clusters of galaxies reveal strong correlations among clusters that extend to very large separations ($\sim 100 h^{-1}$ Mpc, $h \equiv H_0/100$); (Bahcall and Soneira 1983a and references therein). The observed correlation, which is stronger than the galaxy correlation function by a factor of ~ 18 (at a given separation), arises from the strong tendency of clusters to cluster themselves, i.e., to form superclusters. This tendency implies that rich clusters of galaxies provide a good tracer of the large-scale structure in the (luminous) universe. Although only a small fraction of all galaxies ($\sim 5\%-10\%$) are within the $1.5 h^{-1}$ Mpc radius defined by Abell (1958) for rich clusters, the clusters appear to highlight the galaxy distribution over a much larger volume (Seldner and Peebles 1977). A study of the clustering properties of rich clusters of galaxies yields information on the characteristic parameters of superclusters as well as clues to the formation and evolution of these large-scale systems (see, e.g., the review by Oort 1983 and references therein for a description of the various theoretical models).

In order to investigate the properties of the large-scale clustering of clusters, a complete, well-defined catalog of superclusters—defined as clusters of clusters of galaxies²—is required. We construct such a catalog in the present paper using a complete redshift sample and an objective selection

criterion. Previous lists of superclusters (e.g., Abell 1961; Rood 1976; Murray *et al.* 1980), used the projected distribution of rich Abell (1958) clusters plus estimated cluster redshifts (from Abell's magnitudes of the tenth brightest cluster galaxies), since very little redshift data was available. (The nearest 27 clusters with measured redshifts were used by Rood.) Thuan (1980) considered measured redshifts of the 77 nearest rich Abell clusters, but only a single selection parameter (with no comparison to random catalogs).

In the present paper we use the largest available complete redshift sample of rich clusters of galaxies (104 richness $R \geq 1$ Abell clusters with distance class $D \leq 4$) to construct a three-dimensional catalog of superclusters (Table 1), complete to $z \lesssim 0.08$. The criterion of membership in the catalog is quantitative: the density enhancement of clusters over the mean density. This method is equivalent to the percolation-analysis technique used in cluster analysis (see below). The supercluster catalog is presented for various values of the density enhancement parameter. The dependence of the identified sample of superclusters on the overdensity is studied, and the sensitivity of the identified superclusters to various quantities and selection procedures is investigated. The validity of the identification procedure is established by comparisons with random catalogs. A larger sample of 175 $R \geq 0$, $D \leq 4$ clusters with measured redshifts is also analyzed for supercluster membership.

The present catalog provides the best available list for systematic observational and theoretical studies of superclusters. Some global properties of the cataloged superclusters are investigated and are summarized in Table 2 and Figures 4 and 5. The multiplicity function of the superclusters is

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² Systems that contain a single rich cluster surrounded by a large halo of galaxies do not qualify according to this definition. We consider only systems that contain two or more rich clusters, and thus provide the densest groups of the richest known clusters.

TABLE 1
SUPERCLUSTER CATALOG

BS (1)	α, δ (1950) ($f = 20$) (2)	$\bar{z}$ ($f = 20$) (3)	ABELL CLUSTER MEMBERS ($R \geq 1$)				
			$f = 20$ (4)	$f = 40$ (5)	$f = 100$ (6)	$f = 200$ (7)	$f = 400$ (8)
1.....	0053-1238	0.0541	85,151	...	...	...	...
2.....	0112+0204 0103-0047 ^a	0.0433 0.0452 ^a	[119, 168 189, 193]	119, 168	119, 168	119, 168	...
3.....	0122+1759	0.0652	154, 225	...	...	...	...
4.....	0256+1307	0.0738	399, 401	399, 401	399, 401	399, 401	399, 401
5.....	0442-2122	0.0682	500, 514	500, 514	500, 514	500, 514	...
6.....	1049+4012	0.0795	1035, 1187	...	...	...	...
7.....	1113+3151	0.0347	1185, 1228	1185, 1228	1185, 1228	1185, 1228	...
8.....	1142+5547	0.0581	[1291, 1318 1377, 1383 1436,	[1291, 1318 1377, 1383 1436,	[1291, 1318 1377, 1383 1436	[1291, 1318 1383	[1291, 1318 1383
9.....	1145-0210	0.0992	1364, 1399	...	...	...	...
10.....	1120+2415	0.0218	1367, 1656	...	...	...	...
11.....	1345+0356	0.0782	1773, 1809	1773, 1809	1773, 1809	1773, 1809	...
12A.....	1348+2714 ^a	0.0699 ^a	[1775, 1793 1795, 1831 1927, 2022	[1775, 1795 1831	1775, 1831	...	...
12B.....	1527+3040 ^a	0.0710 ^a	2061, 2065 2067, 2079 2089, 2092 2124	[2061, 2065 2067, 2079 2089, 2092 2124	[2061, 2065 2067, 2079 2089, 2092 2124	[2061, 2065 2067, 2089	[2065, 2067 2089
12C.....	1607+2842 ^a	0.0945 ^a	2142, 2175	2142, 2175	...	...	...
13.....	1449+1506	0.0509	[1913, 1983 1991, 2040	...	...	...	...
14.....	1509+0605 1508+0650 ^a	0.0831 0.0774 ^a	[2028, 2029 2048	2028, 2029	2028, 2029	2028, 2029	2028, 2029
15.....	1549+1615 1602+1651 ^b	0.0388 0.0373 ^b	[2063, 2107 2147, 2151 2152	[2107, 2147 2151, 2152	[2147, 2151 2152 ^c	[2147, 2151 2152 ^c	[2147, 2151 2152 ^c
16.....	1627+4021	0.0308	2197, 2199	2197, 2199	2197, 2199	2197, 2199	2197, 2199

NOTE.—Supercluster catalog for $f = 20-400$ for the $D \leq 4$, $R \geq 1$ complete redshift sample; [using HGT redshifts, except $z(\text{A1318}) = 0.0566$; see text].

EXPLANATION OF COLUMNS.—(1) BS supercluster number. (2)–(3) Mean supercluster position and redshift for $f = 20$ (average of the $f = 20$ members, except where otherwise indicated). (4)–(8) A list of the Abell cluster members ($R \geq 1$) in the superclusters for each of the overdensities $f = 20-400$.

^a Mean position and redshifts of the $f \geq 40$ members.

^b Mean position and redshifts of the $f \geq 100$ members.

^c Included in the $f \geq 100$ superclusters if $z(\text{A2152}) = 0.0383$ is used (as measured from 22 galaxy redshifts; SRS).

presented in Table 3 and Figure 6. Additional studies of the superclusters, including percolation analysis, are presented in a following paper (Bahcall and Soneira 1983b).

II. THE SAMPLE

We analyze first Abell's (1958) statistical sample of rich clusters of galaxies with distance class $D \leq 4$ ($z \leq 0.1$); redshifts for all but one of these clusters were reported by Hoessel, Gunn, and Thuan (1980, hereafter HGT). This sample includes all Abell clusters at $D \leq 4$ that are of richness class $R \geq 1$ and are located at high galactic latitude (as specified in Abell's Table 1, plus the requirement of $|b| \geq 30^\circ$). A total of 104 clusters are included. (The cluster A415, for which no measured redshift is available, is assumed to have the redshift of 0.083 which is derived from the sample's $m_{10} - z$ relation of

Bahcall and Soneira 1983a, hereafter BS 1983a. This cluster is relatively isolated in the southern sky, and the uncertainty in its redshift estimate is not important for our analysis). The redshift of the cluster A1318 was recently reported to be $z = 0.0566$ (Fetisova 1981; see also BS 1983a for a similar suggestion) from measurements of five galaxy redshifts, as compared with the $z = 0.0189$ reported by HGT from a single (foreground) galaxy redshift. We use the redshift of $z = 0.0566$ for A1318 throughout this paper. The redshift of the cluster A2152, which is reported as 0.044 from the brightest two galaxies (HGT; Sarazin, Rood, and Struble 1982), is reduced to 0.0383 if the mean of 22 measured galaxies is used (Sarazin, Rood, and Struble 1982); the results are presented below for both of these values. All other redshifts are taken from HGT.

In order to test the sensitivity of the identified supercluster

TABLE 2
GLOBAL PROPERTIES OF SUPERCLUSTERS

Property	f			
	20	40	100	400
N_{sc}	16	12	11	7
$n_{cl/sc}$	2-15	2-7	2-7	2-3
ΔR_{max} (Mpc).....	145	36	36	13
$\langle \Delta R_x \rangle$ (Mpc).....	27.1 ^a	7.6	7.6	4.5 ^b
$\langle \Delta R_z \rangle$ (Mpc).....	28.6 ^a	14.5	13.5	4.5 ^b
$F_{cl}(sc)$	0.54	0.34	0.30	0.16
V_{sc}/V	0.03	0.008	0.003	0.0004

NOTE.—The quantities listed are (for the $R \geq 1$ sample): N_{sc} : total number of identified superclusters. $n_{cl/sc}$: number of $R \geq 1$ clusters per supercluster (i.e., supercluster richness). ΔR_{max} : linear size of the largest supercluster observed. $\langle \Delta R_x \rangle$ and $\langle \Delta R_z \rangle$: mean projected (1-D) and redshift separations (both in Mpc) of all cluster pairs in the superclusters [$\Delta R_x = (4/\pi 3^{1/2}) \Delta R_z$]. For a random distribution, $\Delta R_x \approx \Delta R_z$ is expected. $F_{cl}(sc)$: fraction of all $R \geq 1$ clusters that belong in superclusters. V_{sc}/V : fractional volume of space occupied by the superclusters.

^a The $f = 20$ mean is dominated by supercluster BS 12, which is elongated in projection, yielding a high $\langle \Delta R_x \rangle$.

^b At high f values $\langle \Delta R_x \rangle \approx \langle \Delta R_z \rangle$ due to the small radius of the selecting sphere.

sample to the input redshift data, we have also used the two recent compilations of cluster redshifts, i.e., those by Sarazin, Rood, and Struble (1982), and by Fetisova (1981). These two recent catalogs were combined to yield a catalog of cluster redshifts that uses the largest available number of measured galaxy redshifts for each cluster in the sample. We repeated the analysis of supercluster selection with the data from this combined catalog and compared the results with those obtained using the HGT redshifts. Only minor differences were found (see below).

The area subtended by the cluster sample is 4.26 sr: 2.64 in the north and 1.62 in the south. The space density of the clusters in the sample is approximately constant for redshifts $z \lesssim 0.08$ and decreases rapidly thereafter due to the imprecise sample redshift limit that results from estimating distances using m_{10} magnitudes (see Fig. 3 of BS 1983a). This observed variation of the Abell selection function with redshift is taken into account when we determine a density enhancement of clusters (see § III). Since the density of clusters in the sample is approximately constant to $z \lesssim 0.08$, the supercluster catalog can be regarded as complete to $z \lesssim 0.08$. At higher redshifts ($z \approx 0.08-0.1$), the catalog is incomplete (even though a correction for the decrease in density at these redshifts has been applied to the identification algorithm).

We have also analyzed an extended sample that includes, in addition to the complete $R \geq 1$ clusters, all richness $R = 0$, $D \leq 4$ clusters that have measured redshifts. Superclusters were selected from this larger sample (with 175 $R \geq 0$ clusters compared with 104 $R \geq 1$ clusters) using the same procedure. While the added $R = 0$ clusters do not comprise a complete sample, the density distribution with redshift of the $R \geq 0$ sample is similar to that of the complete sample, suggesting no appreciable selection bias for the $R = 0$ clusters. Even with some incompleteness, a comparison between the superclusters selected in the larger $R \geq 0$ sample and those obtained from the complete $R \geq 1$ sample provides important additional information regarding the clustering of clusters (§ V).

The distance to each cluster in the sample is calculated from the standard relation (e.g., Sandage 1961):

$$R = \frac{c}{H_0 q_0^2 (1+z)^2} \{q_0 z + (q_0 - 1)[(1 + 2q_0 z)^{1/2} - 1]\}. \quad (1)$$

A Hubble constant of $H_0 = 100 \text{ h km s}^{-1} \text{ Mpc}^{-1}$ and a deceleration parameter $q_0 = \frac{1}{2}$ are used throughout the paper. (The distances increase by $\lesssim 2\%$ for $q_0 = 0$). In equation (1) we assume that peculiar velocities are small compared to the Hubble velocity.

A summary of the sample properties, including the Hubble diagram, possible selection effects, the spatial density distribution, and the two-point correlation function, are all described in a preceding paper (BS 1983a).

III. SUPERCLUSTER CATALOG

a) Method of Selection

The following selection procedure is adopted. For each cluster in the sample a sphere of maximum radius r_m , centered on the cluster, is determined. The radius is chosen so that the density of clusters within r_m , $n(r \leq r_m) = N(\leq r_m)/4(\pi/3)r_m^3$, satisfies

$$n(r \leq r_m) \geq f n_0(R). \quad (2)$$

Here $N(\leq r_m)$ is the number of sample clusters whose centers fall within the sphere of radius r_m , $n_0(R)$ is the mean space density of clusters at the cluster distance $R(z)$ from us, and f is the density enhancement factor. The radius r_m is the maximum radius around the given cluster at which the cluster density exceeds the mean level by a factor of f . The density of clusters in the sample, especially at high z , is observed to depend on the distance $R(z)$ from the Galaxy (BS 1983a), since the sample is magnitude limited (with some precision) rather than redshift limited (as defined in the Abell catalog); therefore, for any given value of f , equation (2) is calculated using the sample's actual value of $n_0(R)$ for each cluster at its appropriate distance R (as shown in Fig. 3 of BS 1983a). For each of the 104 clusters in the sample, a sphere of radius r_m is determined, resulting in a series of 104 spheres of various sizes. The spheres typically fall into two categories: (1) single, isolated spheres, and (2) distinct, nonoverlapping clumps of spheres, each clump containing two or more overlapping (i.e., connected) spheres. The exact division into these categories depends on the assumed value of f . The outside boundary of each clump of spheres approximates an isodensity enhancement contour with an average volume density of clusters within the boundary of $f n_0(R)$. Each of the distinct clumps of spheres at a given f is identified as a separate group of clusters, i.e., a supercluster. Any cluster lying outside the clump boundaries, i.e., all the isolated spheres, are considered single clusters for that value of f .

The sphere size for any value of f depends on the density of neighboring clusters. (A similar procedure, in a projected distribution, was used by Turner and Gott 1976 for selecting galaxy groups.) We have also carried out a somewhat different analysis using a constant sphere size; this analysis is discussed in § VI. Both of these methods are similar to the percolation-analysis technique (e.g., Zel'dovich, Einasto, and

Shandarin 1982), where the percolation-size parameter is r_m (or, approximately, $\propto f^{-1/3}$).

The groups and singles selected as described above, as well as the boundaries and richnesses of the groups (i.e., number of clusters per group) depend, by definition, on the value adopted for the density enhancement, f (or, equivalently, the size of the selecting sphere). Low values of f will tend to select groups of low density and high separation, some of which may not be real physical associations. Higher values of f will select tighter, more compact groups. Many of these tight groups constitute the dense cores of larger groups selected with lower f values (see below). The group boundaries clearly do *not* define strict physical limits to the superclusters but rather define volumes of various levels of overdensities.

In order to quantitatively study how the selection of superclusters is affected by the assumed value of f , we performed the analysis for various values of f , ranging from $f = 10$ to 400. The typical maximum radii r_m around single clusters are 17.5, 13.8, 10.9, 8.1, 6.4, and 5.1 h^{-1} Mpc, respectively, for $f = 10, 20, 40, 100, 200$, and 400 (at a distance of $R = 200 h^{-1}$ Mpc). Due to the procedure by which connected (i.e., “percolated”) spheres are selected, the maximum separation between any binary cluster is $2r_m$.

b) The Superclusters

The results for $f = 20, 40, 100, 200$, and 400 are summarized in Table 1 (for the complete sample of 104 $R \geq 1$ clusters). In this table we present a catalog of superclusters, complete to $z \leq 0.08$, for each of the various values of f . The mean position and redshift of the superclusters as determined from the members in the $f = 20$ catalog are also listed. These values change somewhat as f increases and the groups break-up into more compact subgroups (or vice versa).

Some global properties of the superclusters are summarized in Table 2. The total number of superclusters decreases from 16 at $f = 20$ to 7 at $f = 400$. This factor of 2 decrease is small considering the factor of 20 increase in the overdensity. (Random catalogs yield a factor of ~ 14 decrease in the number of random “superclusters”; see Table 3.) The multiplicity, or number of clusters per supercluster, $n_{cl/sc}$, varies from two to 15 for the $f = 20$ superclusters and reduces to a value of two to three clusters per $f = 400$ supercluster. The superclusters contain a large fraction of all clusters; this fraction, $F_{cl}(sc)$, is 54% at $f = 20$ and reduces to 16% at $f = 400$ (see also Fig. 4 below). The fractional volume of space occupied by the superclusters is, however, small; it varies from $\sim 3\%$ at $f = 20$ to 0.04% at $f = 400$ (see Fig. 5, below). The $f = 400$ superclusters have, as expected, the smallest cluster separations observed; all cluster pairs in these superclusters are separated by $\lesssim 13 h^{-1}$ Mpc (see also Table 4). These high-density superclusters are either identical to, or constitute the densest part (or parts) of, the larger superclusters selected with the lower f values. The effect of increasing f on the breakup of large groups into dense, compact subgroups, and eventually into cores of tight binaries and triplets, is apparent in Table 1.

The linear size of the largest observed supercluster at each f value is also listed in Table 2. This maximum “percolated” size, ΔR_{max} , increases rapidly from 13 h^{-1} Mpc for the highest density clumps ($f = 400$) to over 100 h^{-1} Mpc at $f = 20$. This increase is considerably steeper than observed in the random catalogs and indicates the stronger connections

among clusters in the real data (see Fig. 6 and Bahcall and Soneira 1983b). A comparison of the mean separation of all cluster pairs in superclusters as seen both projected on the sky and in the redshift direction is shown in Table 2. The results suggest a velocity broadening component superposed on the Hubble flow (see also BS 1983a). A more detailed analysis including comparisons with random catalogs, is presented in Bahcall and Soneira (1983b).

A map of all superclusters selected at $f = 20, 40, 100, 200$, and 400 is presented in Figure 1 for both the northern and southern galactic hemispheres. The density enhancements of the superclusters for $f = 20$ to 400 are indicated by the density contours. The supercluster numbers refer to those listed in Table 1. Figure 2 presents a similar map that includes only the higher density, $f \geq 40$, superclusters. The breakup into subsystems as f increases (or the coalescence into supersystems as f decreases) is apparent in these maps.

At low values of f ($\lesssim 20$), the connectedness (i.e., percolation) among the groups becomes greater due to the relatively large sphere sizes involved. The specific selection of those largest scale systems, especially in the somewhat “confused” region of highest cluster density (i.e., in the region of BS 12 to BS 16), may be more uncertain and therefore less meaningful for direct studies of individual supercluster properties. At $f = 10$, the entire volume containing BS 12+13+15+16 coalesces into a single large-scale system.

Some specific superclusters are noteworthy. Supercluster BS 8 is a rich Ursa Major Supercluster. BS 10 is the Coma supercluster (which contains A1656 and A1367 as its $R \geq 1$ members). BS 12B is a dense Corona Borealis Supercluster (that may extend to BS 12A and BS 12C, as shown, and possibly even farther; located in the region of highest density of rich clusters in the sample). BS 15 is the Hercules Supercluster, which, from galaxy redshift surveys (Chincarini, Rood, and Thompson 1981) as well as from our extended $R \geq 0$ cluster sample distribution (Fig. 3), appears to extend at least $\sim 50 h^{-1}$ Mpc north to connect with BS 16. It is apparent from the maps that a large concentration of superclusters (i.e., BS 12 through BS 16) is present in one quadrant of the northern hemisphere. The large galaxy void in Bootes (Kirshner *et al.* 1981) is located close by and is apparently related to this high concentration of rich galaxy clusters and superclusters (Bahcall and Soneira 1982).

c) Tests and Comparisons with Random Catalogs

In order to evaluate the statistical significance of the superclusters in Table 1, we used 100 random catalogs that were generated with the same selection functions as the real sample (see BS 1983a) and analyzed with an identical procedure to that used on the real clusters. The number of superclusters as a function of richness $n_{cl/sc}$ (i.e., number of cluster members per supercluster) is given in Table 3 for both the observed and the average of the 100 random catalogs. For all the values of f which were studied, the observed catalog yields a larger number of superclusters with high richnesses than do the random catalogs. In addition, for a density enhancement of $f \geq 100$, only a few binary superclusters (i.e., $n_{cl/sc} = 2$) are found in the random catalogs, while the observed catalog has a number of prominent binary and higher order superclusters. At $f = 20$, the five superclusters observed to have four to 15 members are not found in the random catalogs (see also

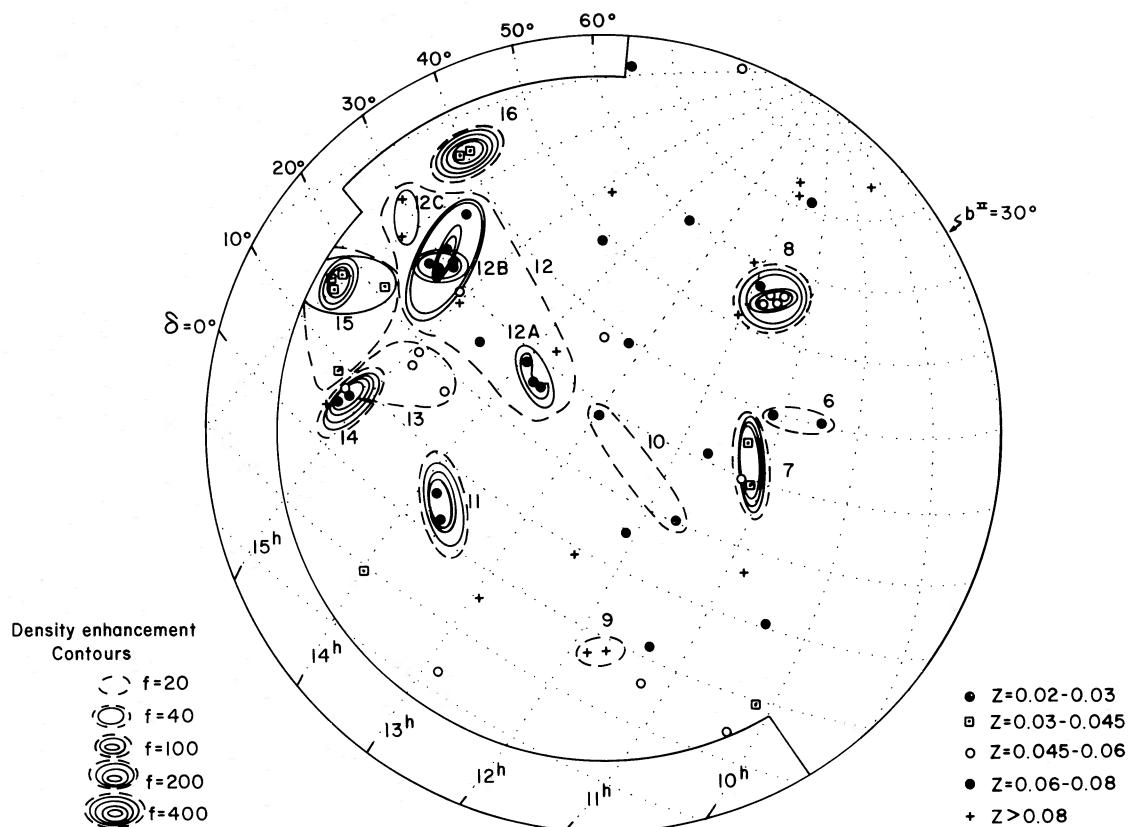


FIG. 1a

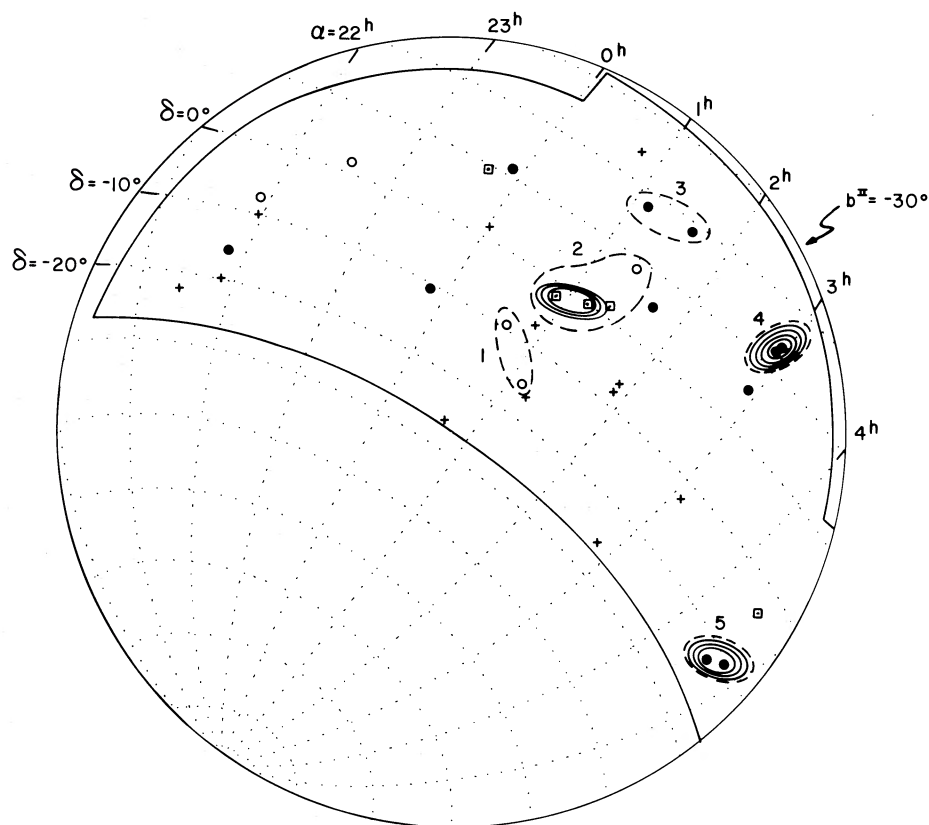


FIG. 1b

FIG. 1.—Projected equal area map of the northern (a) and southern (b) clusters and superclusters in the $R \geq 1$ sample. Outer contour is $|b| = 30^\circ$; inner contour is the completeness limit of the sample. The galactic poles are at the respective centers of these polar maps. Longitude l is 0° at the west and increases clockwise. Clusters at different redshift ranges are marked by different symbols. The density contours represent the three-dimensional density enhancement of the superclusters, from $f = 20$ to $f = 400$. The superclusters are numbered by their BS catalog numbers for $f = 20$ (Table 1). At $f = 10$, superclusters BS 12+13+15+16 coalesce into a single large-scale supercluster.

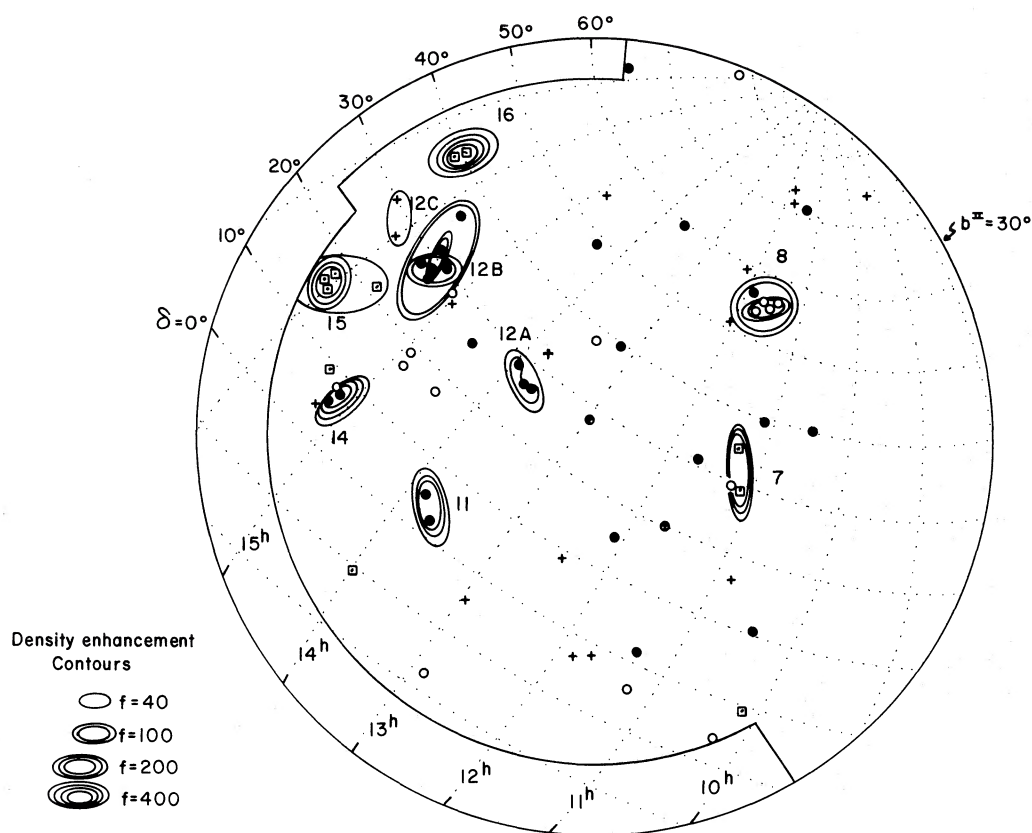


FIG. 2a

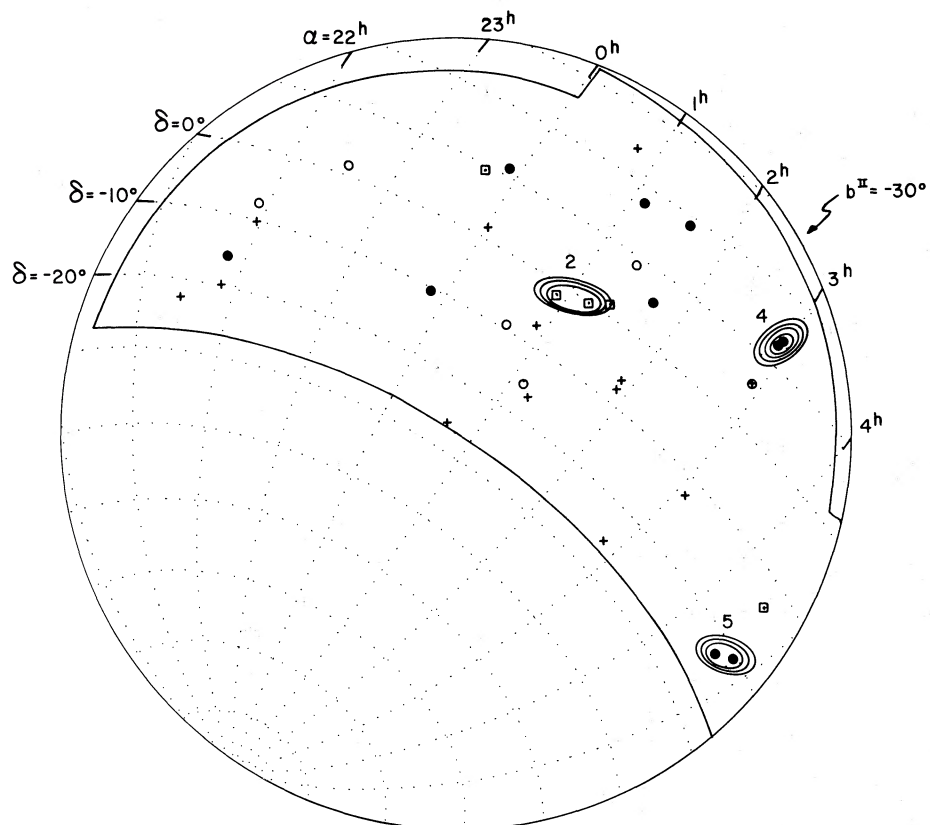


FIG. 2b

FIG. 2.—Same as Fig. 1 but only for the superclusters of highest overdensities ($f \geq 40$)

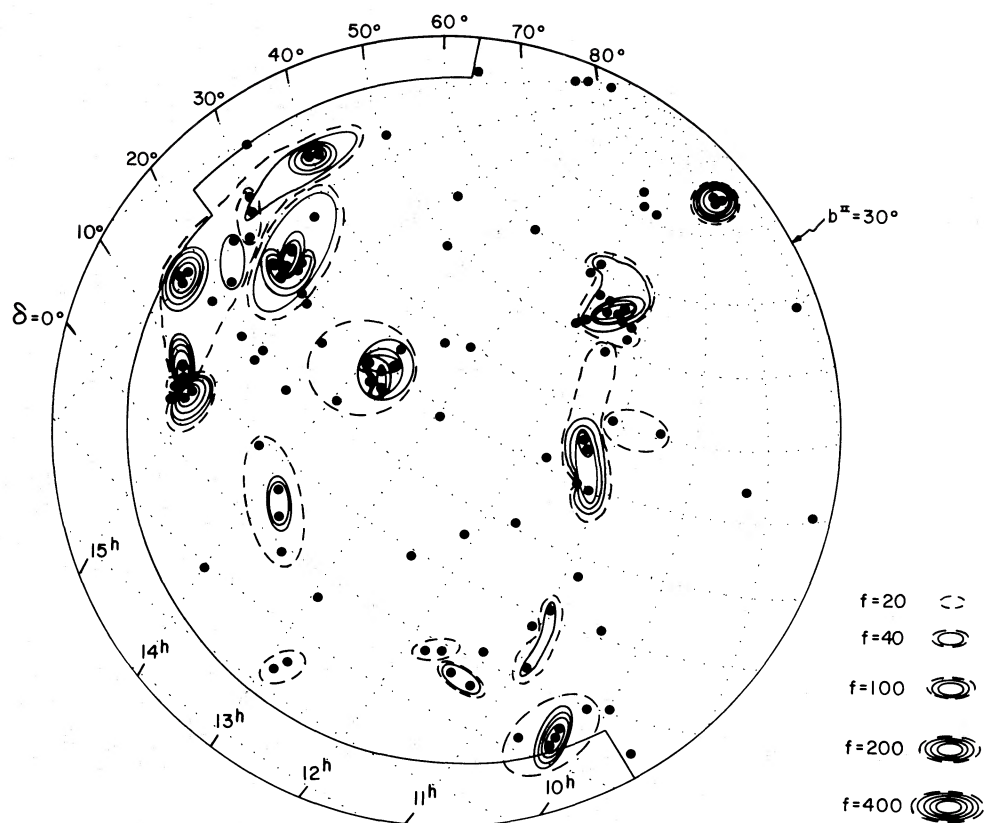


FIG. 3a

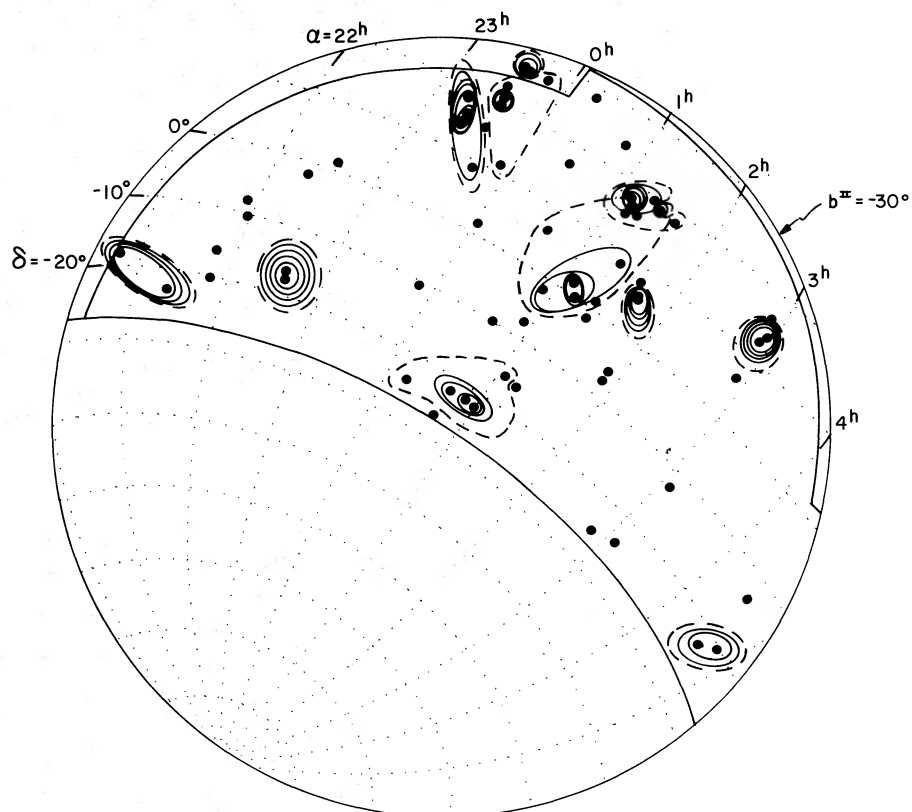


FIG. 3b

FIG. 3.—Same as Fig. 1 but for the extended sample of 175 $R \geq 0$ ($D \leq 4$) clusters with measured redshifts. (A single symbol is used for clusters of all redshifts). At $f = 10$, superclusters BS 12+13+15+16 (see Fig. 1) coalesce into a single large-scale supercluster.

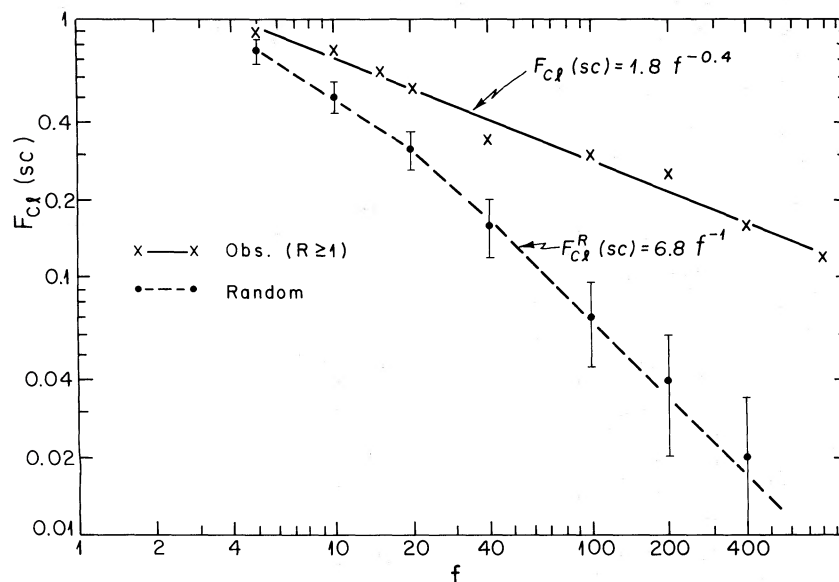


FIG. 4.—The fraction of all $R \geq 1$ clusters that are supercluster members as a function of the supercluster density enhancement f . Crosses represent the data; dots represent the ensemble average of 100 random catalogs. 1σ error bars are shown for the random points. The curves are power-law fits for $f \gg 1$.

§ IV and Fig. 6). Several of the low multiplicity systems are also real (e.g., BS 4, 5, 7, 11, 14, and 16), as indicated by their continuous existence as one varies f from 20 to ≥ 100 (see Table 1). At $f = 400$ the random catalogs yield only one binary supercluster while the data show seven superclusters: four binaries and three triplets. No special value of f describes all—and only—“physical” systems.

The fraction of rich clusters that belong in superclusters,

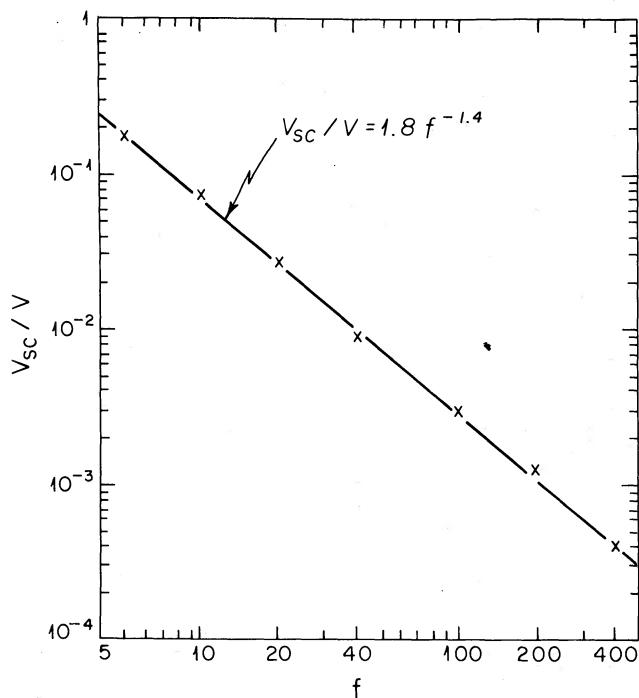


FIG. 5.—The fractional volume of space occupied by the superclusters as a function of the supercluster density enhancement f . The fractional volume at any f is estimated from $V_{sc}/V \approx F_{cl}(sc)/f$.

$F_{cl}(sc)$, is presented as a function of f in Figure 4 for both the observed and random catalogs. As expected, $F_{cl}(sc)$ decreases smoothly with f in both the observed and random catalogs. However, a larger fraction of clusters are supercluster members in the real data than in the random catalogs for $f \geq 10$. (The real physical number of supercluster members is at least the difference between the observed and random numbers, and may be larger.) In addition, the decrease in $F_{cl}(sc)$ with f is considerably less steep in the real data than in the random catalogs. The observed fraction of rich ($R \geq 1$) clusters that are in superclusters can be approximated by (Fig. 4)

$$F_{cl}(sc) \sim 1.8f^{-0.4} \quad (R \geq 1, f \gg 1). \quad (3)$$

The dependence on f of the fractional volume of space occupied by the superclusters is shown in Figure 5. This fractional volume is always small ($< 10\%$ for $f \geq 10$) and decreases rapidly with increasing density enhancement. The fractional volume can be approximated by the following relation:

$$V_{sc}/V \sim 1.8f^{-1.4} \quad (R \geq 1, f \gg 1). \quad (4)$$

In order to test the sensitivity of the catalog to uncertainties in the cluster redshift determination, we repeated the selection procedure using a somewhat different redshift catalog than that of HGT. We extracted cluster redshifts from two recent redshift compilations by Sarazin, Rood, and Struble (1982, hereafter SRS) and by Fetisova (1981, hereafter F). For each cluster we used the redshift corresponding to the largest number of measured galaxy redshifts per cluster. This “highest N_{gal} ” redshift catalog has 28 clusters with redshifts that differ from the HGT redshifts by $c\Delta z = 300\text{--}2300 \text{ km s}^{-1}$ (excluding the foreground galaxy measured in front of A1318, for which the correct redshift has been used throughout). The entire supercluster selection process was repeated, for various

TABLE 3
FREQUENCY DISTRIBUTION OF SUPERCLUSTER RICHNESSES

$n_{cl/sc}$	$f = 20$		$f = 40$		$f = 100$		$f = 400$	
	Obs.	Random	Obs.	Random	Obs.	Random	Obs.	Random
2	10	10.6	8	6.9	8	3.3	4	1.0
3	1	2.3	1	0.9	1	0.2	3	0.01
4	2	0.7	1	0.1	0	0	0	0
5	2	0.2	1	0	1	0	0	0
6	0	0.1	0	0	0	0	0	0
7	0	0	1	0	1	0	0	0
8	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0
15	1	0	0	0	0	0	0	0

NOTE.—Number of superclusters as a function of their richness $n_{cl/sc}$ ($n_{cl/sc}$ = number of $R \geq 1$ clusters per supercluster). Random corresponds to the ensemble average of 100 random catalogs.

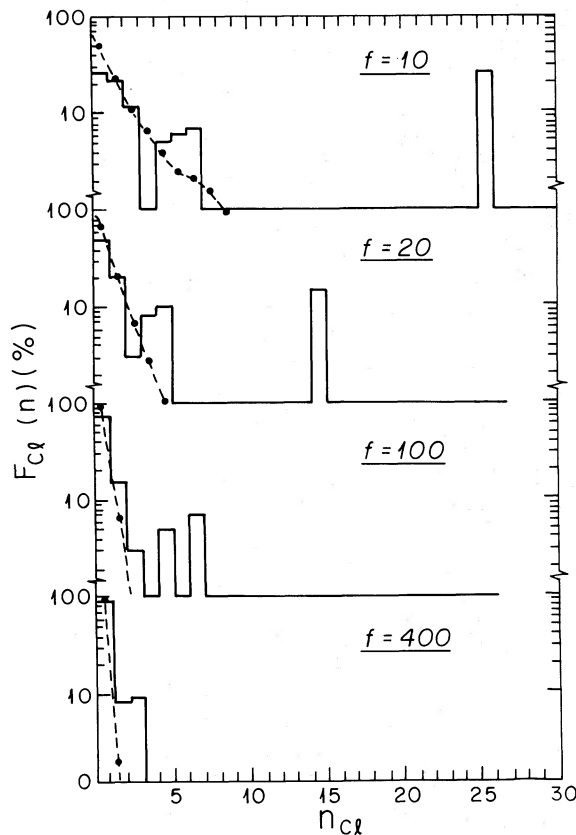


FIG. 6.—The frequency distribution of $R \geq 1$ clusters that are members of superclusters of various richnesses $n_{cl/sc}$. (The first bin, $n_{cl/sc} = 1$, refers to single clusters; $n_{cl/sc} = 2, 3$, etc., represent superclusters with two, three, etc., cluster members). The distributions are given for four different density enhancements f . The histogram represents the observed catalog; the dashed line is the average of 100 random catalogs.

overdensities f , using these somewhat revised redshifts. The identified superclusters were found to differ only slightly from those listed in Table 1 (which were constructed using the HGT redshifts). The minor differences consisted of an occasional loss or gain of a single member in a supercluster or a slight change in the value of f when a binary cluster is first identified. The selection of the main superclusters remained essentially unchanged. This insensitivity was expected since the typical redshift uncertainties are smaller than the size of the selection radius r_m for all the values of f studied.

We conclude that the main features of the superclusters identified in this paper are insensitive to the precise determination of the cluster redshifts within the typical expected uncertainties.

IV. THE MULTIPLICITY FUNCTION

The number of superclusters with different richnesses, $n_{cl/sc}$ (where $n_{cl/sc}$ is the number of clusters in a supercluster) is shown in Table 3. The different—but related—frequency distribution of clusters among superclusters of different richnesses ($n_{cl/sc}$) is plotted in Figure 6. This figure shows the fraction of clusters that are members of superclusters at any given richness; the distribution, $F_{cl}(n_{cl/sc})$, is the multiplicity function of the clusters.

The frequency distributions are plotted in Figure 6 for different values of f , from $f = 10$ to 400, for both the observed and the average of 100 random catalogs. Figure 6 shows that the observed and random catalogs yield different distributions. The fraction F_{cl} for the random catalogs falls off smoothly and steeply with increasing $n_{cl/sc}$ (for $f \geq 10$); thus the random catalogs have essentially no power at large $n_{cl/sc}$. The observed superclusters have systems with more members than seen in the random catalogs for all $f \geq 10$ (see also Table 3). The observed high richness superclusters appear to grow rapidly (in richness and size) as f decreases. As these richest, largest scale structures grow, a gap of medium richness superclusters appears to be rapidly forming (see Fig. 6). Neither the gap nor the related largest scale structure exist in the random catalogs.

V. THE EXTENDED $R \geq 0$ SAMPLE

The extended sample of 175 clusters with $R \geq 0$, $D \leq 4$, and $|b| \geq 30^\circ$ for which measured redshifts are available (see § II) has also been used for supercluster identification. Measured redshifts for the added $R = 0$ clusters were taken from the compilations of HGT, SRS, F, and Schneider, Gunn, and Hoessel (1983). The supercluster selection analysis as described above was repeated using this larger sample. The appropriate $n(R)$ distribution for this sample was used in the selection procedure.

The superclusters identified in the $R \geq 0$ sample at various density enhancement levels are presented in a map in Figure 3. A comparison of Figure 3 with Figures 1 and 2 reveals that the superclusters identified in the $R \geq 0$ sample are generally consistent with those discussed previously for the smaller $R \geq 1$ sample. A few new superclusters appear in the $R \geq 0$ sample, and most of the $R \geq 1$ superclusters become more prominent in the $R \geq 0$ case. Some structures, such as the Hercules and A2197/2199 superclusters (BS 15 and BS 16) unite, at a given f value, into a single large and elongated system. This coalescence is also suggested by redshift observations of the galaxies in this region (Chincarini, Rood, and Thompson 1981). The length of the entire structure exceeds $60 h^{-1}$ Mpc, but it is much narrower ($\lesssim 20 h^{-1}$ Mpc) along its other two axes.

High values of density enhancement tend to select small groups or the inner cores of larger, lower density systems. These high density cores are usually small systems with no apparent structure. As the value of f decreases, the connectedness among the clusters increases, the selected superclusters become larger, and it is possible to distinguish between spherical or elongated distributions. The maps of Figures 2 and 3 show that some of the large, rich superclusters at low f values exhibit, in projection, elongated structures. While the structure in the redshift dimension is more difficult to interpret because of possible peculiar velocities, it appears that in some cases the redshift separation is considerably smaller than the major axis (see, e.g., the BS 15+16 supercluster in Fig. 3). As f decreases even further, the statistical significance decreases and the groups coalesce.

VI. SELECTION USING SPHERES OF CONSTANT RADII (i.e., PERCOLATION METHOD)

The analysis was also carried out using a similar method, called percolation analysis, in which a sphere of constant size is used around each cluster in the sample, independent of the local density. We considered spheres of radii ranging from $5 h^{-1}$ to $30 h^{-1}$ Mpc. (The sphere size correlates with the density enhancement parameter f , such that smaller radii correspond to higher values of f). As before (§ III), all connecting spheres form superclusters; the separation of any binary cluster can thus reach a maximum of two sphere radii.

The superclusters selected using the method of constant size spheres are similar to those selected with the constant density method. At large values for the sphere radius (i.e., low f values), connections among groups in the highest density region proceed in slightly different ways before coalescence. For example, with $r_m = 15 h^{-1}$ Mpc, BS 12B+13+15 unite to

TABLE 4
CLOSEST CLUSTER PAIRS ($\Delta R \leq 15 h^{-1}$ Mpc)

Cluster Pair (1)	$\bar{z}$ (2)	$c\Delta z$ (km s $^{-1}$) (3)	$\Delta\theta^\circ$ (4)	ΔR (Mpc) (5)
A2147, 2152 ^{a,b}	0.0380	180	0.9	2.3
A2197, 2199	0.0308	270	1.4	3.2
A2147, 2151 ^a	0.0369	510	1.9	5.6
A1291, 1318 ^a	0.0576	600	1.1	5.8
A2151, 2152 ^{a,b}	0.0372	690	1.3	6.5
A2028, 2029	0.0774	150	1.8	6.6
A399, 401	0.0738	810	0.6	6.6
A1291, 1383 ^a	0.0592	360	2.6	7.8
A2079, 2092 ^a	0.0663	360	2.5	8.3
A1318, 1383 ^a	0.0582	960	1.8	9.3
A2065, 2089 ^a	0.0733	630	2.3	9.3
A1185, 1228	0.0347	150	6.0	10.4
A2065, 2067 ^a	0.0724	120	3.2	10.7
A119, 168	0.0452	330	5.0	11.2
A500, 514	0.0682	930	2.8	11.6
A2067, 2089 ^a	0.0734	510	3.5	12.7
A2061, 2067 ^a	0.0754	1680	0.4	13.1
A1383, 1436 ^a	0.0622	1440	2.4	13.5
A1773, 1809	0.0782	360	4.0	14.6
A1318, 1377 ^a	0.0538	1710	1.8	15.0

EXPLANATION OF COLUMNS.—(1) Cluster pair (Abell no., $R \geq 1$). (2) Mean redshift of pair. (3) Pair radial velocity separation. (4) Pair angular separation. (5) Pair spatial separation (assuming redshift distances).

^a Part of a richer supercluster at $f = 40$.

^b If $z(\text{A2152}) = 0.0383$ is used (from the mean of 22 galaxy redshifts [SRS], rather than 0.0444 from HGT).

form a single supercluster, while at $f = 20$ superclusters BS 12A+12B unite into a single system. At $r_m = 20 h^{-1}$ Mpc, as well as at $f \approx 10$, the entire region containing BS 12A+12B+13+15+16 becomes a single supercluster. (BS 12C, at $z > 0.08$, is picked up as part of the larger supercluster BS 12 only if a correction to the sphere radius due to the reduced density of clusters at $z > 0.08$ is applied [see $n_0(R)$ in eq. (2)].)

The supercluster catalog at $f \approx 20$ (Table 1) is generally consistent with that reported by Thuan (1980), who used a smaller sample (77 clusters to $z \leq 0.08$) and a constant size sphere (radius = $17.9 h^{-1}$ Mpc in the present notation, where the maximum separation is two radii). However, Thuan's choice of a single large sphere size for the supercluster selection (corresponding to a low f value), results in uncertain large-scale superclusters. Using an identical selection procedure to that described by Thuan (with $r_m = 17.9 h^{-1}$ Mpc), we find also some results that differ from his conclusions. For example, superclusters BS 12B+13+15 are found, in our test, to be connected into a single system. In addition, due to the revised redshift of A1318, this cluster is *not*, as claimed by Thuan, a "new" member of the Coma Supercluster, but is rather a member of the more distant Ursa Major Supercluster (BS 8). Two clusters used by Thuan, A1793 and A2142, with $z > 0.08$, do not belong in his $z \leq 0.08$ sample.

VII. BINARY CLUSTERS

A list of the closest cluster pairs in the sample is presented in Table 4, in increasing order of linear spatial separation.

Twenty pairs with separations $\leq 15 h^{-1}$ Mpc are listed; only three pairs with this small a separation would be expected for a random distribution. The number of observed pairs increases to 34 for separations $\leq 20 h^{-1}$ Mpc (the extended number of random pairs increases to 7). No pairs with separations $\leq 2 h^{-1}$ Mpc are observed, mostly due to the selection definition of Abell cluster radii of $1.5 h^{-1}$ Mpc. Possible peculiar velocities among the clusters would also tend to widen their redshift separation. Comparison with Table 1 indicates that while some of the close pairs remain isolated to relatively low values of f (e.g., A399/401, A500/514, A1185/1228, A2028/2029, A2197/2199), other tight pairs appear to belong to the dense cores of larger groups with somewhat smaller overall density enhancement (e.g., A1291/1318/1383, A2079/2092, A2065/2067/2089, A2147/2151/2152).

The average observed radial velocity difference of the binary systems with $\Delta R \leq 15 h^{-1}$ Mpc is $\langle \Delta v_r \rangle \approx 560 \text{ km s}^{-1}$, comparable to the mean one-dimensional separation $\langle \Delta R \rangle / 3^{1/2} \approx 5 h^{-1}$ Mpc. An analysis of the velocity dispersion in superclusters and its interpretation in terms of the possible role of gravitational interactions within superclusters will be presented in another paper.

VIII. CONCLUSIONS

A complete catalog of superclusters that are selected on the basis of density enhancement is presented in Table 1 and Figures 1 and 2. The catalog extends to a redshift of $z \lesssim 0.1$ and is complete to $z \leq 0.08$. The catalog is presented for different values of the parameter f specifying the minimum density enhancement. The manner in which the catalog evolves with f (see the last five columns of Table 1) is equivalent to the percolation analysis; it shows the breakup of large-scale systems into smaller, denser substructures as the density enhancement increases. Comparisons with random catalogs establish that the data exhibit more power on large scales than do the random distributions. The fraction of clusters that are supercluster members is higher in the data than in the random catalogs for any $f \gtrsim 10$. The $f = 20$ superclusters contain more than half of all rich clusters and their maximum size exceeds $100 h^{-1}$ Mpc. An elongated structure is suggested in some cases. As the density enhance-

ment increases, only the densest cores of superclusters are selected. At $f = 400$, for example, seven high density clumps are identified, all binaries or triplets, that contain $\sim 16\%$ of all rich clusters.

Some global properties of the superclusters are summarized in Table 2. A large fraction of all rich clusters appear to belong to superclusters; $\sim 54\%$ of all $R \geq 1$ clusters are members of $f = 20$ superclusters, and $\sim 30\%$ of the clusters are members of $f = 100$ systems. The fractional volume of space that is occupied by the superclusters is small. The superclusters occupy only $\sim 3\%$ of all space at $f \approx 20$ and $\sim 0.3\%$ at $f \approx 100$ (Figs. 4 and 5).

The multiplicity function of the superclusters is determined (Table 3 and Fig. 6). A larger number of large-scale, rich superclusters is observed in the real data than in the random catalogs for all $f \geq 10$. Very large scale structures of high richness, accompanied by an absence of intermediate richness superclusters, are seen in the observed catalog at low f 's that have no counterparts in the random catalogs.

Superclusters identified from the larger (but partially incomplete) sample of $R \geq 0$ clusters are also presented (Fig. 3). The characteristics of these superclusters agree well with, and add to the significance of, the complete supercluster catalog of Table 1. The larger, low density superclusters appear to be elongated (see Figs. 2 and 3).

The superclusters selected using a sphere of constant size around each cluster (i.e., percolation analysis) are similar to those identified using a minimum density enhancement.

The sensitivity of the supercluster sample to uncertainties in the cluster redshifts and to the precise method of supercluster selection is discussed. The main features of the supercluster sample are found to be insensitive to plausible variations in these quantities.

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