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*Letter to the Editor*

# Anti-gravity and galaxy rotation curves

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Summary.

A modification of Newtonian gravitational attraction which arises in the context of modern attempts to unify gravity with the other forces in nature can produce rotation curves for spiral galaxies which are nearly flat from 10 to 100 kpc, bind clusters of galaxies, and close the universe with the density of baryonic matter consistent with primordial nucleosynthesis. This is possible if a very low mass vector boson carries an effective antigravity force which on scales smaller than that of galaxies almost balances the normal attractive gravity force.

Keywords: Gravitation, Galactic dynamics, cluster of galaxies, cosmology

Observations of non-decreasing rotation curves in the outer parts of spiral galaxies and of the large virial masses of clusters of galaxies have led naturally to the suggestion that much of the matter in the universe is non-luminous (Ostriker, Peebles, and Yahil 1974). This suggestion is consistent with current trends in cosmological theory which require the universe to be asymptotically closed ($\Omega_0 \simeq 1$) (Guth 1983) essentially by non-baryonic matter (neutrinos, photinos, gravitinos); although there may be some difficulty in forming dark halos of galaxies, binding clusters of galaxies, and closing the universe all with the same flavor of dark matter (Primack and Blumenthal 1984). A second possible solution to these large-scale dynamical problems is more radical: modifications of Newtonian gravity or dynamics on the large scale. For example, addition of a $r^{-1}$ term in the gravitational force law operative on length scales greater than galactic dimensions (Tohline 1983) or amending Newton's second law ($F = ma^2/a_0$) in the limit of small accelerations (Milgrom 1983a) both produce flat rotation curves for spiral galaxies and reasonable mass-to-light ratios for clusters of galaxies. But these suggestions are rather ad hoc, and the cosmological implications are likely to be extreme; the expansion rate of the universe depends upon length scale and is thus inconsistent with the cosmological principle.

Modifications of Newton's inverse square law do arise naturally in modern theories which attempt to unify gravity with the other fundamental forces of nature (Gibbons and Whiting 1981). This is due to the possibility that, in addition to the presumably massless graviton, particles with mass may carry the gravitational force, and this introduces a length scale ($r_0$) dependent upon the mass ($m_0$) of such a particle; i.e.

$$r_0 = \frac{h}{m_0 c} = 2 \times 10^{-5} \left(\frac{1\ \mathrm{ev}}{m_0}\right)\ \mathrm{cm} \qquad (1)$$

Therefore, the gravitational potential contains a component of the well-known Yukawa form:

$$U(r) = \frac{G_\infty M}{r}\left(1 + \alpha\, e^{-r/r_0}\right) \qquad (2)$$

where $\alpha$ is the coupling constant for this additional component of gravity and $G_\infty$ is the gravitational constant as measured at infinity. The local value of G is

$$G_0 = G_\infty\,(1 + \alpha) \qquad (3)$$

There have been several attempts to constrain $\alpha$ and $r_0$ (and hence $m_0$) by looking for deviations from the inverse square law on scales of 1 cm to 1000 km (Hut 1981, Gibbons and Whiting 1981) corresponding to $10^{-13}$ ev $< m_0 < 10^{-5}$ ev. But suppose the mass of the particle carrying this additional gravitational interaction is much smaller than has previously been imagined (i.e. $m_0 \sim 10^{-28} - 10^{-27}$ ev). Then deviations from the inverse square law would be present on length scales somewhat larger than that of galaxies (20 kpc to 40 kpc). A potential of the form of eq. 2 could then account for flat rotation curves of galaxies if $\alpha \sim -1$. This would imply that gravity on small scales is a mixture of the normal attractive force and a repulsive or anti-gravity force. Anti-gravity also arises naturally in supergravity theories and results from the exchange of vector bosons such as the 'graviphoton' (Sherk 1979 a, b). On scales larger than $r_0$ the anti-gravity component cuts off and pure attraction remains. Obviously, $G_\infty$ could be much larger than $G_0$.

To see how such a potential could affect the rotation curves of galaxies, note that circular or Keplerian velocity V is given by

$$\frac{V^2}{r} = \frac{dU}{dr} = \frac{G_\infty M}{r^2}\left[1 + \alpha\left(1 + \frac{r}{r_0}\right) e^{-r/r_0}\right] \qquad (4)$$

Fig. 1 shows this velocity law with various values of $\alpha$ for a test particle orbiting a point mass. For $r < r_0$ the circular velocity is Keplerian with $V \propto r^{-1/2}$ and for $r > r_0$ it is also Keplerian but with a larger effective constant of gravity. If $-0.95 < \alpha < -0.90$ there is a substantial region ($0.4\ r_0 < r < 2.5\ r_0$) where the circular velocity is nearly constant. Choosing $\alpha = -0.92$ and $r_0 = 40$ kpc, can result in the galactic rotation curves shown in Fig. 2.

Here the dashed curve is the usual inverse square gravity ($\alpha = 0$) rotation curve in the plane of a thin disk with a truncated exponential surface density density distribution ($\sigma = \sigma_0 \exp(-r/a)$, $r < r_g$). This corres-

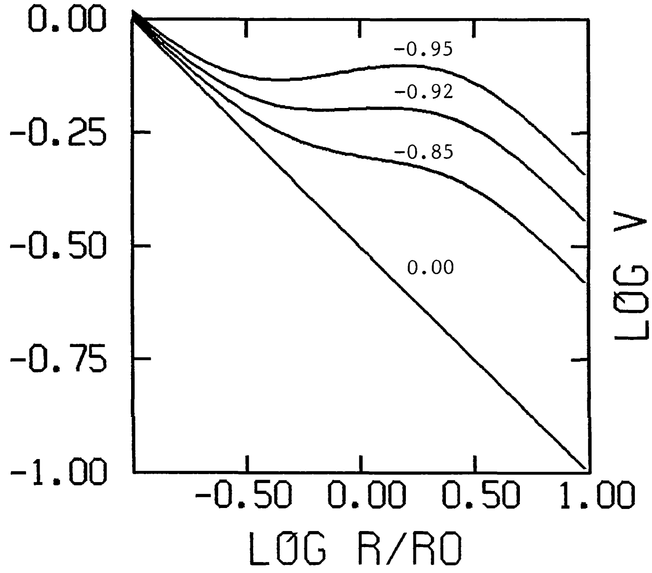


Figure 1: The circular orbit velocity of a test particle with negligible mass orbiting a point mass M for various values of the coupling constant $\alpha$. The velocity is in units of $\left[10\,\frac{G_0 M}{r_0}\right]^{\frac{1}{2}}$.

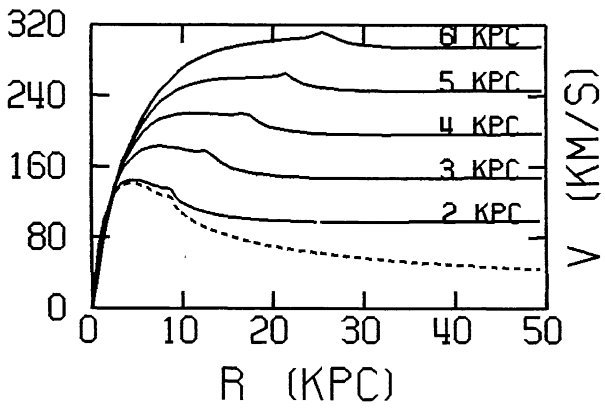


Figure 2: The dashed curve is the pure inverse square gravity rotation curve for a galaxy consisting of a truncated exponential disk ($\sigma = \sigma_0$ exp $(-r/a)$ where $r < r_g = 4.3$ a) with a = 2 kpc. The solid curves are rotation curves resulting from similar mass distributions but with a modified gravitational potential (eg. 2) integrated over the mass distribution. Here $\alpha = -0.92$ and $r_0 = 40$ kpc. The central surface density ($\sigma_0$) is assummed constant and the disk length scale (a) is indicated for each curve.

ponds to the typically observed light distribution in spiral galaxies (Freeman 1970, van der Kruit and Searle 1981); the radial scale length is taken to be a = 2 kpc and the truncation radius is $r_g = 4.3a = 8.6$ kpc which implies a total mass of $M_g = 2.2 \times 10^{10}$ $M_\odot$. Apart from the truncation bump (Casertano 1983, Bachall 1983) the rotation curve monotonically decreases beyond the disk scale length in contrast to observed rotation curves. The modified gravity rotation curves for truncated exponential disks are shown by the solid lines where the force has been calculated by numerical integration of the potential (eq. 2) over the mass distribution. The different curves correspond to different values for the disk length scale (a), and, because the central surface density ($\sigma_0$) is assumed to be constant (Freeman 1970), this represents a mass sequence with $M_g \propto a^2$. Here we see that the rotation curves are quite flat both inside and outside the disk; although, there is a decrease at the truncation radius which is also consistent with observations in the few cases where gas is seen to extend beyond the optical disk (Sancisi 1976, Sancisi and Allen 1979).

The general properties of extended galactic rotation curves are reproduced with $-0.95 < \alpha < -0.92$ and $r_0 \simeq 25 - 50$ kpc which requires that $m_0 \simeq 2.5 - 5 \times 10^{-28}$ ev. This range of $\alpha$ implies that $G_0 \simeq 10 - 20$ $G_\infty$; that is to say, the gravitational constant measured locally is only 5 - 10% of its value at infinity due to the local effect of anti-gravity. Since the length scale of clusters of galaxies is typically larger than $r_0$, the virial mass

$$M \simeq \frac{V^2 r}{G_\infty}$$

is reduced by a factor of 10 - 20 and the mass-to-light ratio becomes comparable to that of elliptical galaxies (Faber and Gallagher 1979). Moreover, as the force law is $1/r^2$ on large scale, the uniform expansion of the universe is retained. The density parameter of the universe

$$\Omega_0 = \frac{8\pi G_\infty \rho}{3H_0^{\ 2}}$$

constrained by considerations of primordial nucleosynthesis to be < 0.1 for baryonic matter is increased to about one without the addition of a substantial non-baryonic contribution. The outstanding success of standard big-bang cosmology --the synthesis of the light elements-- is retained because nucleosynthesis takes place at an epoch (<10 minutes) well before the horizon has expanded to $r_0$ (~$10^5$ years), so the usual, locally determined value of G governs the expansion during that era. When the horizon has expanded beyond $r_0$, the expansion rate of the universe remains independent of length scale and is set by $G_\infty$. This is not obvious but is due to the fact that for any spherical region of a uniform medium the repulsive gravitational force due to the interior mass is exactly canceled by the repulsive gravitational force from the external mass (the Birchoff theorem applies only to the attractive component of gravity).

It would seem fortuitous that $r_0$ corresponds to the maximum length scale of galaxies, but this may not be accidental. Perhaps the scale of galaxies is in some sense determined by $r_0$. The growth of density fluctuation on a scale smaller than $r_0$ is inhibited relative to larger scale fluctuations due to the smaller effective constant of gravity.

It is well-known that the luminosity of spiral galaxies is proportional to some power of the gas rotational velocity; i.e.

$$L \propto V^b$$

where V is usually determined from global 21 cm line profiles and $2 < b < 4$ depending upon the color band in which luminosity is measured (Tully and Fisher 1977, Aaronson, Huchra, and Mould 1979, Bottinelli, et. al. 1980). Such a relation could provide an observational test of hypotheses on modified gravity or dynamics (Milgrom 1983b). In the context of the present hypothesis the mass-velocity law for spiral galaxies defined by relating mass to the "plateau" rotational velocity (at $r_0$) should be $M_g \propto V_{r_0}^{\ 2}$. The observed luminosity-velocity law, however, may differ due to systematic variations of mass-to-light ratio with galaxy mass (Burstein 1982) or to the fact that, except for the larger more massive spiral galaxies, it is not the plateau velocity that is typically measured. Because of these uncertainties a more sensitive test of a modified gravity law of the form proposed here would be to determine whether or not the same values of $r_0$ and $\alpha$ can reproduce galaxy rotation curves over a wide range of mass and size. This requires further observations, primarily of the extended rotation curves of low mass galaxies.

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