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AN APPLICATION OF DISCRIMINANT ANALYSIS TO VARIABLE AND NONVARIABLE STARS

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Discriminant analysis has been applied to nonvariable and δ Scuti variable stars in the lower part of the instability strip. The parameters used, $uvby\beta$ indices, M_v and $V \sin i$, are not sufficient for a satisfactory discrimination between the two groups; however, generally, variable stars tend slightly to be more luminous, to have higher m_1-index and lower $V \sin i$ than nonvariable stars. In clusters, variable stars have slightly higher m_1-index (and $V \sin i$) than nonvariable stars.

Key words: δ Scuti stars—multivariate analysis

I. Introduction

Several methods of multivariate statistical analysis have been applied to solve some problems of stellar classification (Deeming 1964), correlations between photometric indices and physical quantities (Heck 1976), comparison of different photometric systems (Janes 1974) and search for correlations between the parameters of variable stars (Antonello, Fracassini, and Pastori 1981; Fracassini et al. 1981). One of these methods is the discriminant analysis, which allows us to make a statistical separation (or discrimination) between two or more groups of cases (stars).

It is well known that, in the lower part of the instability strip, near the main sequence, there are some groups of stars with different characteristics: variable stars of δ Scuti type, nonvariable normal stars, Am, Am:, and δ Delphini stars. Some Am: and δ Del stars are pulsating as δ Scuti variables, while there is a well-known exclusion between pulsation and classical metallicity (Am stars). In particular, however, it is not clear what are the physical differences between variable and nonvariable stars with normal spectral type (Breger 1979; Antonello et al. 1981). As several parameters are observed for each star, an application of discriminant analysis to these two groups of stars could give more information about the physical nature of the variability. Our aim is to find some significant discriminant parameters among the observed ones.

II. Method

The objective of discriminant analysis is to weight and linearly combine the parameters (discriminating variables) in some fashion so that the groups are forced to be as statistically distinct as possible (Klecka 1975). The discriminant function is of the form

$$F = f_1 x_1 + f_2 x_2 + \ldots . f_n x_n \quad , \qquad (1)$$

where F is the score, the f's are the weighting coefficients and the x's are the values of the n-discriminating variables used in the analysis; the discriminant scores for the cases within a group will be very similar. When the discriminating variables are in standard form, the f's are standardized coefficients, and each case will have a standard discriminant score. This means that, over all cases in the analysis, the score will have a mean of zero and a standard deviation of one. Thus, any single score represents the number of standard deviations that case is away from the mean for all cases. The standardized coefficients are important because their absolute value represents the relative contribution of the respective associated variable to the discriminant function.

Discriminant analysis allows us to also make a classification of the cases, that is to identify the likely group membership of each case. By comparing predicted group membership with actual group membership, one can empirically measure the success in the discrimination by observing the proportion of correct classifications. The classification is achieved through the use of two functions, derived from the values of the discriminant scores; each case will have two scores, and it will be classified into the group with the highest score. Under the assumption of a multivariate normal distribution, the classification scores can be converted into probabilities of group membership.

As we do not know what are the discriminating variables necessary to achieve satisfactory discrimination, we will use a stepwise procedure to select the best of these variables. The stepwise procedure begins by selecting the single best-discriminating variable according to a criterion (see later); a second discriminating variable is selected as the variable best able to improve the value of the discrimination criterion in combination with the first variable, and similarly for the subsequent variables. At each step, variables already selected may be removed if they are found to reduce discrimination when combined with more recently selected variables (see Klecka 1975).

III. Data

We have considered δ Scuti stars (Breger 1979; Horan 1979) with known β, m_1, $(b-y)$, and c_1 indices (Hauck and Mermilliod 1980), and with known rotational velocity $V \sin i$ (Uesugi and Fukuda 1970); these are the mem-

TABLE I

Results of Discriminant Analysis

	n[a]	%[b]	DV[c]	SC[d]	UC[e]
Complete sample	160	62.5	δc_1	-0.900	-11.1
			m_1	-0.616	-32.7
			V sin i	0.355	0.006
			(const)		6.8
Cluster stars	63	58.7	m_1	-1.000	-59.3
			(const)		11.3
Hyades and Praesepe	41	63.4	m_1	-0.812	-58.5
			V sin i	-1.011	-0.021
			(const)		14.1
Stars not in clusters	93	62.9	M_V	0.853	1.31
			m_1	-0.608	-30.5
			V sin i	0.458	0.008
			(const)		2.73

[a] number of cases
[b] percent of cases correctly classified
[c] discriminant variables
[d] standardized coefficients
[e] unstandardized coefficients

bers (or cases) of the variables group. The "constant" stars were taken from the surveys of Millis (1967), Jorgensen, Johansen, and Olsen (1971), Breger (1969, 1970, 1972), Slovak (1978) and Horan (1979). These stars lie in the instability strip, but they do not show any appreciable variation of luminosity. However, we cannot exclude that they actually pulsate with a very small amplitude, which is less than the usual limit of 0.01 mag; for example, on the main sequence, all normal stars could be considered as variables, but 70% of them have an amplitude less than 0.01 mag (Baglin et al. 1980). Moreover, it is possible that, owing to amplitude variations, some of these stars were apparently constant during the observing time. We must keep these considerations in mind when discussing the results of the analysis; in any case, one may think that the discrimination is made between more- and less-excited variable stars, rather than between variable and nonvariable stars.

As there is a well-known exclusion between pulsation and classical metallicity, we have taken into account only constant normal stars; and since constant stars are more numerous than variable stars, we have selected those stars observed for a longer time. The two samples have a like number of cases: 80 stars.

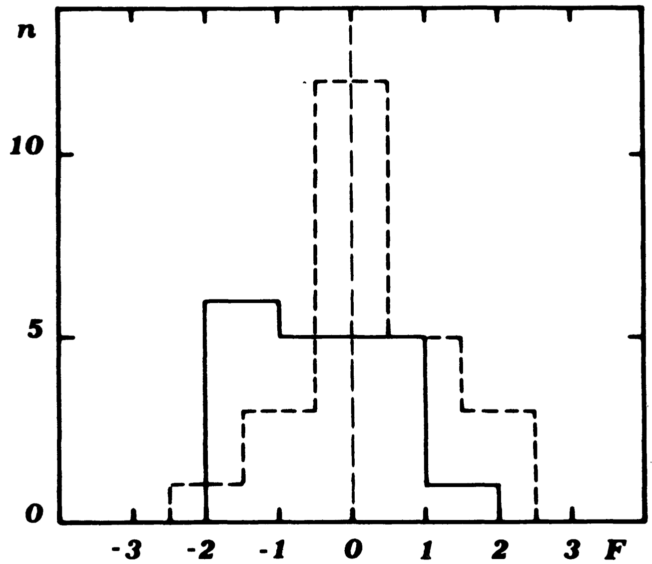

FIG. 1—Discrimination between variable stars (continuous line) and nonvariable stars (dashed line) of the complete sample. The F values are derived by means of unstandardized coefficients.

From $uvby\beta$ indices, we have obtained δm_1, δc_1 and the absolute magnitude M_v (e.g., Antonello and Pastori 1981), both for variable and constant stars; hence the complete set of dependent parameters is: β, $(b-y)$, m_1, c_1, δm_1, δc_1, M_v, and $V \sin i$.

IV. Discriminant Analysis

We have used the SPSS code, subprogram DISCRIMINANT (see Klecka 1975), on the computer UNIVAC/80 of the University of Milan. We have used several stepwise selection criteria, but the analysis has yielded always the same results.

The analysis was made for all the stars (complete sample), and for stars in clusters (cluster stars). Clearly, the number of cases is not high, and the analysis of each single cluster is poorly significant; however, the results are interesting, as can be seen from the data in Table I and Figures 1, 2 and 3. Here, F values of variable stars tend to be smaller than F values of constant stars.

In the case of the complete sample, the best discriminating variables are δc_1, m_1, and $V \sin i$, whereas the best discriminating variables for the clusters are m_1 and $V \sin i$. In any case, the canonical correlation coefficient is not high, about 0.3, and this shows that the separation between variable and constant stars is never good. The negative result is confirmed by the low percent of cases correctly classified.

However, in spite of this result, we can derive some information. In the complete sample, the variables tend to be only slightly more luminous, to have higher m_1 and lower rotational velocity than constant stars. If we interpret the difference in the m_1 index mostly as a difference in the metallicity of the stars, our result, and the percent

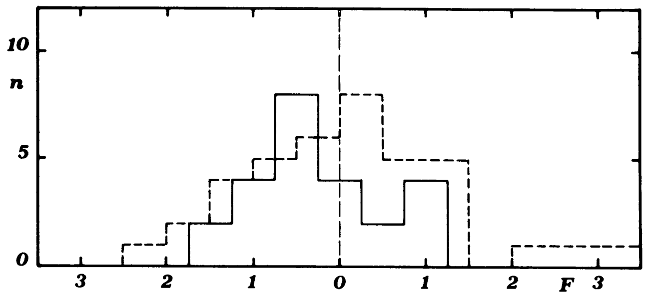

FIG. 2—The same as Figure 1, cluster stars.

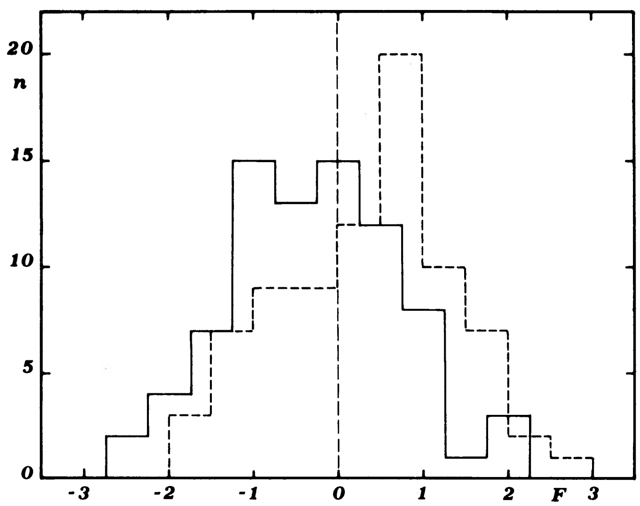

FIG. 3—The same as Figure 1, Hyades and Praesepe clusters.

of cases correctly classified, confirm Breger's suggestion (1979) on the difference between variable and nonvariable stars, and his estimation of the probability a star is variable or not. The m_1 index effect could be ascribed to the presence of several δ Del stars among variable stars.

In the clusters, the variable stars seem to have slightly higher m_1 (and rotational velocity) than constant stars. We must point out that our cluster stars have normal spectra; excluded are HR 1368 (variable star in Hyades, Am) and HR 1414 (constant star in Hyades, Am:). Recently, we have noted the different incidence of binary stars in the groups of variable and nonvariable normal stars (in the lower part of the instability strip) of the Hyades cluster (Antonello 1981); about 80% of variable stars are spectroscopic binaries, while only about 30% of nonvariable stars are (see Table II). If this incidence were substantiated in other clusters, the m_1 index effect could be ascribed to the presence of later type, less-luminous companions (e.g., Strömgren 1966); this would confirm our hypothesis (Antonello 1981) on the importance of duplicity (and/or high rotational velocity) for pulsation in dwarf stars.

TABLE II

Spectroscopic Binaries in Hyades Cluster

	HR[a]	H[b]	K[c]	CP[d]
VAR.	1351	SB	SB1	SB1
STARS	1356	-	SB1	SB1
	1392	-	SB1	SB1
	1394	SB		
	1412	SB	SB1	SB1
	1444	SB2	SB?	SB2
	1547	-	-	
	1368	SB	SB1	SB1
CONST.	1331	-	-	-
STARS	1380	-	-	SB
	1388	-		-
	1408	-	-	SB2
	1422	SB	-	SB
	1427	-		-
	1430	-	-	-
	1472	-	-	-
	1479	SB2	-	-
	1507	-	-	-
	1620	-	-	-
	1414	-		-

[a] HR number
[b] Hoffleit (1964)
[c] Kraft (1965)
[d] Crawford and Perry (1966)

V. Conclusion

The application of discriminant analysis to variable and nonvariable stars in the lower part of the instability strip, made by means of the known *uvbyβ* indices, M_v, and $V \sin i$, is not sufficient to give a satisfactory separation between variable and nonvariable stars. It confirms Breger's estimation (1979) of the probability a star is variable or not, but our results have been obtained without considering Am stars.

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