

## CLUSTERING IN A NEUTRINO-DOMINATED UNIVERSE

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### ABSTRACT

We have simulated the nonlinear growth of structure in a universe dominated by massive neutrinos using initial conditions derived from detailed linear calculations of earlier evolution. Codes based on a direct  $N$ -body integrator and on a fast Fourier transform Poisson solver produce very similar results. The coherence length of the neutrino distribution at early times is directly related to the mass of the neutrino and thence to the present density of the universe. We find this length to be too large to be consistent with the observed clustering scale of galaxies if other cosmological parameters are to remain within their accepted ranges. The conventional neutrino-dominated picture appears to be ruled out.

*Subject headings:* cosmology — galaxies: clustering — neutrinos

### I. INTRODUCTION

In an earlier paper (Frenk, White, and Davis 1983, hereafter Paper I) we presented simulations of the growth of structure in a universe where the density distribution at early times possessed a large coherence length. In agreement with earlier work (Klypin and Shandarin 1981) we found that the nonlinear structure which grows from such conditions retains considerable large-scale coherence. It has a filamentary appearance reminiscent of the observed galaxy distribution (e.g., Einasto, Joeveer, and Saar 1980; Davis *et al.* 1982); this contrasts sharply with the structure in simulations started from white-noise initial conditions (Aarseth, Gott, and Turner 1979; Efstathiou and Eastwood 1981). A population of massive neutrinos could dominate the mass budget of the present universe and would possess just such a large initial coherence length (Doroshkevich *et al.* 1980; Bond, Efstathiou, and Silk 1980). Detailed linear calculations of the evolution of density fluctuations in a neutrino gas have been carried out by Peebles (1982) and Bond and Szalay (1983). Scaling our simulations to their results, we found a unique time at which the particle autocorrelation function matched that observed for galaxies. Three difficulties marred this apparent success. Our scaling required a Hubble parameter of  $80 \text{ km s}^{-1} \text{ Mpc}^{-1}$  which, in a flat universe, implies an uncomfortably low age. Our model had the right autocorrelation function after expansion by a factor of 2.5 from the first collapse of structure; this is too early to be consistent with observations of quasars at redshifts approaching 4. Finally, the model autocorrelation function should be

compared with that of neutrinos rather than galaxies in the real universe. Galaxies form only in dense, collapsed regions in the neutrino-dominated picture; as we shall see, this makes its large coherence length more difficult to reconcile with the relatively small scale of observed galaxy clustering.

This *Letter* presents extensions of our previous work. We consider open universe models, models with an initial fluctuation distribution derived from detailed linear calculations, and models which use an entirely different simulation method. These experiments allow us to check our numerical techniques and to investigate the three difficulties listed above.

### II. SIMULATION METHODS

Our earlier calculations followed the motion of 1000 representative particles with a direct  $N$ -body integrator. To obtain a large initial coherence length, we placed our particles on a uniformly expanding cubic lattice with a spherical outer boundary; we then perturbed their positions and velocities with a superposition of random waves. Following previous work (Klypin and Shandarin 1981) we required that the power density of the imposed waves should be flat between upper and lower wave-number cutoffs. We chose  $k_{\min} = 0.6 k_{\max}$ . This fluctuation spectrum is a poor representation of that expected after recombination in a neutrino-dominated universe. The latter is quite well fitted by

$$|\delta_k|^2 \propto k^n 10^{-2(k/k_\nu)^{1.5}}, \quad (1)$$

where

$$\begin{aligned} \lambda_\nu &= 2\pi/k_\nu \\ &= 41 m_{30}^{-1} \theta^{-1} \text{ Mpc} \\ &= 13 \Omega^{-1} h^{-2} \theta^2 \text{ Mpc}, \end{aligned} \quad (2)$$

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$m_{30}$  is the neutrino mass in units of 30 eV,  $\Omega$  is the cosmological density parameter,  $h$  is the Hubble parameter in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , and  $\theta$  is the microwave background temperature in units of 2.7 K (Bond and Szalay 1983). The wavelengths here are comoving and referred to the present day. The exponent  $n$  in equation (1) is set by whatever process originally produced the fluctuations. The value  $n = 1$  is often favored because the resulting curvature fluctuations all enter the particle horizon with equal amplitude; this is the spectrum predicted by inflationary models for the formation of irregularities (Barrow and Turner 1982). We have now performed simulations with the same direct  $N$ -body integrator using initial conditions in which the perturbing waves satisfy equation (1) between upper and lower cutoffs; these were chosen as  $k_{\nu} = 0.5 k_{\text{max}} = 4 k_{\text{min}} = 1.6 2\pi/R$ , where  $R$  is the radius of the region simulated. We have run models in which  $n = 0, 1$ , and 4. These new calculations can be used to check the scaling of our earlier models. As before, in order to estimate statistical uncertainties, we formed ensembles from four models run from different realizations of each set of initial conditions.

The experiments described above all model the evolution of an Einstein–de Sitter universe. In order to investigate how clustering depends on cosmological density, we multiplied the particle mass by a factor of 0.8 in the initial conditions of the eight pancake simulations of Paper I and then reran their evolution. This produces two ensembles of open universes which can be compared directly with the original models.

The results of these experiments may be uncertain because of the relatively small number of particles we used and the way we distributed them on a grid. In collaboration with G. Efstathiou we have now developed a number of simulation codes in which the interparticle force is calculated using fast Fourier transform methods. Details of these codes and of numerous tests of their performance will be published elsewhere. They allow us to study the sensitivity of our results to numerical technique. We have performed a number of simulations which follow the motion of 32,768 particles within a cubic cell of a periodic universe. Forces are calculated using a momentum-conserving cloud-in-cell scheme on a  $64^3$  density and potential grid (Hockney and Eastwood 1981). We set up initial conditions by placing the particles either on a grid or at random and then perturbing their positions and velocities with a superposition of growing mode fluctuations. For these experiments we chose  $\lambda_{\nu}$  equal to 20% of the side of the cube and set the amplitude in equation (1) to correspond to an rms density fluctuation of 22%. These experiments are very similar to that of Klypin and Shandarin (1981), but our force calculation has higher resolution than theirs, and our initial conditions are a much closer approximation to those of a neutrino-dominated universe. We have run

ensembles with  $n = 1$  and both grid and random initial conditions, and a grid ensemble with  $n = 4$ . Each ensemble contains four models. A model takes 1.5–3 hr to execute on a VAX 11/780.

### III. RESULTS

In an open universe, linear fluctuations grow more slowly than in a flat universe. We compare our ensembles of open models with their Einstein–de Sitter counterparts at times chosen to match the linear growth factor from the two sets of initial conditions; linear theory then predicts that the particle distributions should be identical. This expectation is remarkably well obeyed on large scales. Even after close inspection, equidensity plots for the open models are indistinguishable from those of Paper I. In Figure 1 we compare the correlation functions of an open ensemble and of the corresponding ensemble from Paper I. The agreement is excellent for values of  $\xi < 10$ ; as expected, clustering on smaller scales is stronger in the open ensemble. Velocity correlations in the open models have very similar form to those shown in Paper I, but their amplitude is smaller. When spatial correlations are matched as above, peculiar velocities scale by the expected factor of  $\sqrt{\Omega}$ .

Figure 1 also compares a correlation function from our earlier work with results from new Einstein–de Sitter models with initial conditions based on equation (1). Length scales were matched by equating the minimum wavelength present in the earlier calculations to  $1.19 \lambda_{\nu}$ , as is appropriate for  $n = 1$  (see Paper I and Peebles 1982). The correlation functions agree well in shape, but there is a slight scale discrepancy which reflects the uncertainty arising from the substantial difference between the initial fluctuation spectra. Our two integration schemes give results which agree very well; both methods seem entirely adequate for the problem at hand. In tests to be published elsewhere, we have found the correlation function of a Fourier model to be affected by potential softening only on scales somewhat smaller than the mesh size of the potential grid. The correlation functions of the two Fourier ensembles in Figure 1 are almost identical. This demonstrates the insensitivity of our results to the grid initial conditions of most of our models. It also shows that a long-wavelength feature with a peak power an order of magnitude above the white-noise level can imprint coherent large-scale structure on a universe which had white-noise initial conditions on small scales.

In a “standard” neutrino-dominated universe, galaxies, and presumably quasars, form once large-scale structures begin to collapse. We identify the onset of galaxy formation in our models as the time when 1% of the elemental volumes of our initial grid have undergone collapse; most galaxies are likely to form well after this. We parameterize later times by the expansion factor,

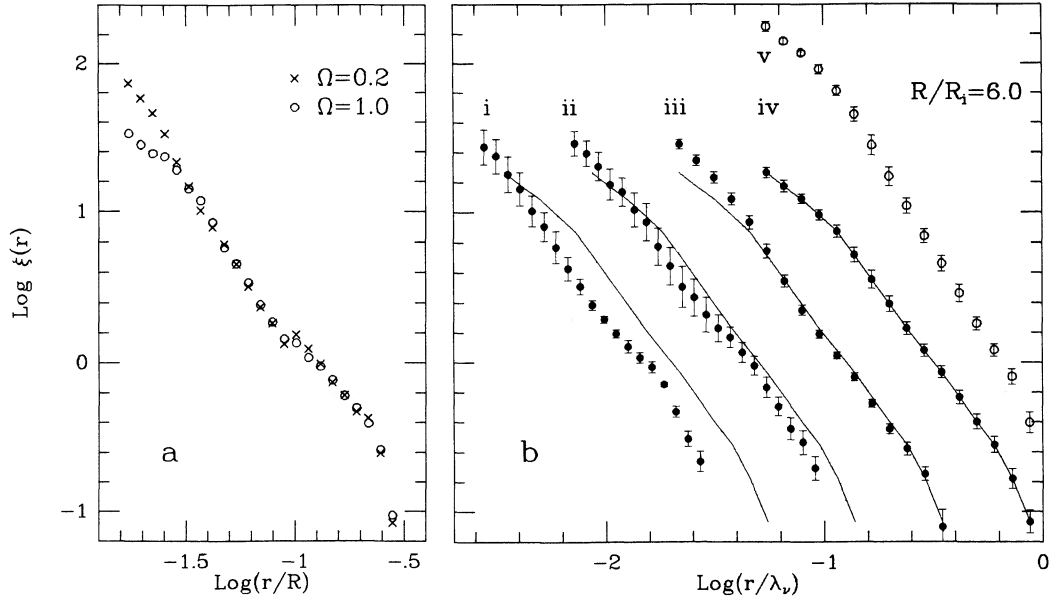


FIG. 1.—(a) The correlation function of ensemble A of Paper I (*open circles*) is compared with the correlation function of an open ensemble which initially had the same particle positions and velocities but smaller particle masses (*crosses*). The data for ensemble A are shown at an expansion factor of 6, while the data for the open ensemble are shown at an expansion factor of 15.6 when  $\Omega = 0.2$  and the linear growth factor from the initial conditions is 6. (b) Correlation functions for four Einstein-de Sitter ensembles are compared at an expansion factor of 6. Plot (i) is ensemble B of Paper I; (ii) is an ensemble of 1000 particle direct  $N$ -body simulations with initial conditions derived from eq. (1) with  $n = 1$ ; (iii) and (iv) are ensembles of 32,768 particle Fourier simulations with this same perturbation spectrum imposed on random and grid underlying particle distributions respectively; (v) is the “galaxy” function for the Fourier grid ensemble. The abscissa is correct for (iv) and (v), but for clarity the other three functions have been shifted to the left by multiples of 0.4. The solid lines show similarly shifted versions of (iv) superposed on each of the “neutrino” correlation functions. Error bars give the standard deviation in the plotted points derived using the four simulations in each ensemble.

$1 + z_{\text{GF}}$ , since this first collapse, and we measure the scale,  $r_c$ , of nonlinear clustering using the integral relation

$$r_c^{-3} \int_0^{r_c} \xi(r) r^2 dr = 1. \quad (3)$$

In our models, this length can be expressed in units of the initial coherence length,  $\lambda_\nu$ . The observed value of  $r_c$  for the galaxy distribution is  $5.0 \pm 0.7 h^{-1}$  Mpc (Davis and Peebles 1983a), and the value of  $\lambda_\nu$  for a neutrino-dominated universe is related to the observable parameters  $\Omega$ ,  $h$ , and  $\theta$  through equation (2). Equating the model value of  $r_c/\lambda_\nu$  to the observational prediction thus gives a relation between the cosmological parameters

$$(r_c/\lambda_\nu)_{\text{model}} = 0.39 \pm 0.06 \Omega h \theta^{-2}. \quad (4)$$

For any ensemble of models,  $\Omega$  and  $r_c/\lambda_\nu$  are functions of time and thus of  $z_{\text{GF}}$ . For any particular value of  $z_{\text{GF}}$ , equation (4) then gives the value of  $h\theta^{-2}$  required for consistency with observation.

In Figure 2 we plot this relation for three of our ensembles of simulations. For  $\Omega$  significantly smaller

than 1, the length scale of a neutrino-dominated universe is much too large to be compatible with the relatively small scale of galaxy clustering; this is reflected in the very large values of  $h\theta^{-2}$  required for “consistency” with observation. In a flat universe the conservative requirement that  $z_{\text{GF}} > 3$  requires  $h\theta^{-2} > 1$  unless  $n > 3$ . This forces a choice between an uncomfortably young universe and an initial power spectrum with substantially more power on small scales than the constant curvature spectrum which is now generally favored; if such a spectrum were indeed present at early times, the universe would have been chaotic on the horizon scale well after the epoch of nucleosynthesis.

These comparisons are somewhat misleading because they identify the model correlation function with the observed galaxy function. In fact, galaxies form only in dense “pancake” shocks in the neutrino picture, and the low-density regions between pancakes should be devoid of observable systems. In such a situation the correlation length of the galaxy distribution is expected to be greater than that of the neutrinos, thus exacerbating scaling problems. To illustrate this we have calculated correlation functions for our models assuming that particles are “visible” only after their cell of the initial particle grid has collapsed. In Figure 1 we plot such a “galaxy”

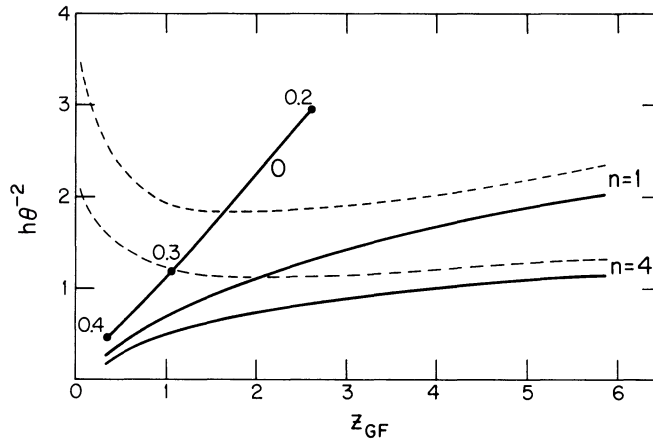


FIG. 2.—The value of  $h\theta^{-2}$  required for the coherence length of a neutrino-dominated universe to be consistent with the observed scale of galaxy clustering is plotted as a function of redshift since the onset of galaxy formation for three kinds of initial condition. The line marked O refers to the open version of ensemble A of Paper I scaled assuming  $n = 1$ . The value of  $\Omega$  is shown at three points of its evolution. The solid lines refer to two Fourier ensembles with initial conditions given by eq. (1), while the dashed lines show the required parameters if the “galaxy” correlation function of these same ensembles (rather than their “neutrino” function) is matched to the observations.

correlation function for comparison with the corresponding total correlation function. The “galaxy” function has a higher amplitude than the “neutrino” function and has a large comoving clustering length which actually decreases until the time when half the particles have become visible. Using equation (4) we can derive the scaling to observation implied by this length; its evolution is plotted in Figure 2 for Fourier ensembles with  $n = 1$  and 4. If  $\Omega \leq 1$ , agreement with observation is not possible for any acceptable values of  $h$ ,  $\theta$ , and  $n$ .

#### IV. CONCLUSIONS

We have used two quite different numerical techniques to carry out simulations of the nonlinear growth of structure in a neutrino-dominated universe. Our initial conditions reproduce the perturbation spectra obtained from detailed linear calculations. We find that the predicted neutrino clustering scale in such a universe substantially exceeds the observed scale of galaxy clustering. Agreement can be obtained only by going well outside the range of currently fashionable cosmological parameters. If  $\Omega > 2$ , acceptable values of the Hubble constant, the redshift of galaxy formation, and the age of the universe are obtained for a constant curvature initial power spectrum ( $n = 1$ ). If  $\Omega \approx 1$ , acceptable values are obtained only for  $n > 3$ , implying a highly inhomogeneous universe at quite recent epochs. If the universe is open by a significant margin, no conventional neutrino-dominated cosmology is possible. This scale discrepancy is exacerbated by the requirement that galaxies form in “pancakes” and thus avoid low-density regions; it could be alleviated only if galaxy formation were strongly suppressed in regions of high neutrino density. Figure 2 shows that scaling difficulties cannot be avoided by assuming galaxy formation to switch off after the collapse of the first pancakes.

Unconventional cosmological assumptions might rescue the neutrino-dominated picture. Increasing the number of massive neutrino species makes things worse, since the neutrino coherence length is then larger for given values of  $\Omega$  and  $h$ . Allowing a highly inhomogeneous universe at relatively recent epochs might help by providing extra entropy generation and so increasing the photon-to-neutrino ratio; in addition it might produce a steep fluctuation spectrum with  $n \approx 4$ . (Other ways to increase the photon-to-neutrino ratio or otherwise salvage the neutrino picture are discussed by Davis *et al.* 1981). A positive cosmological constant might make a high-density universe consistent with observational constraints on the deceleration parameter and the age of globular clusters; however, it does not affect upper bounds on  $\Omega$  from the dynamics of the Local Supercluster (cf. Davis and Peebles 1983*b*). Finally, large-amplitude isothermal perturbations in the baryon component of the universe might allow galaxy formation to precede pancake formation; the latter might then occur recently enough to be consistent with the observed clustering. None of these alternatives will seem attractive unless direct measurements of a neutrino mass force us to consider them. Until then, the discrepancy between the large coherence length of neutrino-dominated universes and the small scale of observed galaxy clustering makes it appear unlikely that neutrinos provide the missing mass.

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