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CNO ABUNDANCES AND THE STRENGTHS OF NOVA OUTBURSTS AND HYDROGEN FLASHES ON ACCRETING WHITE DWARFS

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ABSTRACT

Simple theories of the accretion phase and expansion phase of hydrogen shell flashes on accreting white dwarfs are used to find the conditions under which classical novae occur. Particular attention is paid to the possibility that the CNO abundances in the accreted envelope are greatly enhanced relative to the Sun. It is shown that such an enhancement is necessary for fast novae, yet slow novae, such as DQ Her, can still occur with large CNO enhancements. An analysis of the observed properties of nova ejecta indicates that the white dwarfs in nova systems have masses in the range 1.02–1.18 $M_\odot$. It is found that in some novae, including RR Pic, CP Lac, and V603 Aql, the accretion rate deduced from the optical luminosity in quiescence is too high to allow accretion of sufficient envelope material to get the strong hydrogen flash needed for the nova outburst. Some possible resolutions of this problem are discussed. We also find it unlikely that recurrent novae are thermonuclear runaways on accreting white dwarfs.

Subject headings: nucleosynthesis — stars: accretion — stars: novae — stars: white dwarfs

I. INTRODUCTION

Following pioneering work by Walker (1954) and Kraft (1964), it is now generally accepted that classical novae occur in close binary systems in which a Roche-lobe filling star is transferring mass via an accretion disk to a white dwarf companion (Gallagher and Starrfield 1978). Accretion of hydrogen-rich material by the white dwarf results in formation of a thermally unstable hydrogen shell source (Giannone and Weigert 1967). Detailed numerical hydrodynamic computations have shown that if the accreted material has approximately solar CNO abundances the ensuing thermonuclear runaway can give an outburst that resembles the slowest novae (Sparks, Starrfield, and Truran 1978; MacDonald 1979, 1980; Nariai, Nomoto, and Sugimoto 1980). Enhancement of the CNO abundances can give fast novae (Starrfield, Truran, and Sparks 1978). Hydrogen shell flashes on accreting white dwarfs may also be relevant to symbiotic stars (Paczyński and Zytkow 1978; Paczyński and Rudak 1980) and may play a role in the evolution of type I supernova (SN I) progenitors. Accumulation of helium by an accreting white dwarf as a consequence of hydrogen shell burning and its subsequent ignition has been proposed as a SN I mechanism (Fujimoto 1980; Woosley, Weaver, and Taam 1980). The accreted material may be ejected by nova explosions, preempting the supernova.

For these reasons alone, it is of interest to know how the hydrogen-flash strength depends on the binary system properties.

Also, in a recent paper, Shara, Prialnik, and Shaviv (1980) have proposed a quantitative picture relating the speed class of classical novae to the mass and CNO abundance of the accreted envelope. As pointed out by Fujimoto (1982*a*) their conclusions are partially based on computations by Prialnik, Shara, and Shaviv (1978, 1979) which have since been shown to be incorrect for a number of reasons (Sparks, Starrfield, and Truran 1978; MacDonald 1980; Fujimoto 1982*a*). That something is wrong with their scheme is immediately obvious from inspection of their Figure 1. The region of envelope mass–CNO abundance space in which the fastest novae are hypothesized to occur abuts onto a region in which "duds," i.e., no thermonuclear runaway, occur. This violates continuity.

In the present paper we use simple models of both the "accretion" stage, in which the hydrogen-rich envelope is accreted by the white dwarf, and the "expansion" stage, in which the envelope expands toward its maximum photospheric radius, to survey how the energetics of the hydrogen flash depend on the properties of the underlying binary system.

In the next section our model of the accretion stage is developed and used to find how much material can be accreted by the white dwarf before the onset of the thermonuclear runaway. Particular attention is paid to the possibility that the CNO abundances are greatly enhanced relative to the Sun. High CNO abundances have been observed in the ejecta of a number of novae (Collin-Souffrin 1977; Pacheco 1977; Ferland and

Shields 1978; Williams *et al.* 1978; Gallagher *et al.* 1980*a*; Stickland *et al.* 1981; Williams 1982). Possible mechanisms for this enhancement are discussed in § V.

In § III, an energy conservation argument is used to investigate the general behavior of the expansion stage of the hydrogen flash and to relate its strength to the mass and composition of accreted material. The observed correlation between principal absorption velocity and rate of decline for novae (McLaughlin 1960) is used to relate our results to novae of different speed classes and to give a quantitative picture of how envelope mass and composition determine speed class.

The validity and usefulness of the analysis in these last two sections is checked by comparison with detailed numerical calculations whenever possible.

In § IV we discuss the regions of parameter space in which the various classes of novae and related objects are expected to occur and check the conclusions, whenever possible, against observed properties of nova systems. Section V contains discussion and a brief summary of the main conclusions.

II. THE ACCRETION STAGE

Although the results of many detailed numerical calculations of the accretion stage have appeared in the literature (Taam and Faulkner 1975; Taam 1977; Paczyński and Zytkow 1978; Kutter and Sparks 1979; MacDonald 1980; Nariai, Nomoto, and Sugimoto 1980), the relevant parameter space has by no means been fully explored. Further, only a few of these calculations were carried through to the hydrodynamic stages of the hydrogen flash (Kutter and Sparks 1979; MacDonald 1980; Nariai, Nomoto, and Sugimoto 1980), and it is still not clear what it is that determines the speed class and energetics of nova outbursts. Hence, it is useful to have a simple but accurate model for the accretion stage that allows parameter-space exploration without the need for costly and complicated numerical calculations. In the following subsection we attempt to provide such a model. The results are presented and discussed in § II*b*.

a) *The Mass of Accreted Material*: *Simple Model*

Observations of old novae have shown that the transfer of hydrogen-rich material onto the white dwarf proceeds via an accretion disk. Although the accretion is not spherically symmetric, we adopt spherical symmetry for the sake of simplicity and leave discussion of this approximation to the final section.

The initial white dwarf mass, radius, and luminosity are denoted by $M_{\rm WD}$, $R_{\rm WD}$, and $L_{\rm WD}$, respectively. We use the approximation to the Chandrasekhar mass-radius relation derived by Nauenberg (1972) to relate $R_{\rm WD}$ and $M_{\rm WD}$. We assume the white dwarf is initially pure carbon and let accretion proceed at constant rate F.

If F is sufficiently small, the white dwarf envelope remains in thermal equilibrium in the sense that the luminosity is still supplied by the cooling of the nondegenerate ions in the core. An approximate condition for this is (MacDonald 1980)

$$F\frac{\mathfrak{R}T_c}{\mu} \ll L_{\rm WD}, \tag{1}$$

where T_c is the white dwarf central temperature and μ is the mean molecular weight of the accreted material. Condition (1) implies that the cooling time for the white dwarf is much less than the accretion time. When the converse is true, gravitational energy generation in the envelope becomes important, and the surface luminosity is increased above $L_{\rm WD}$. We consider first the case in which condition (1) is satisfied.

Suppose an amount M_a ($\ll M_{\rm WD}$) has been accreted by the white dwarf. Since the nondegenerate layers are thin and the bulk of the core is isothermal, it is a good approximation to take

$$\nabla \equiv \frac{d\ln T}{d\ln P} = \frac{3P}{4aT^4} \times \frac{\kappa L_{\rm WD}}{4\pi cGM_{\rm WD}} \tag{2}$$

in nonconvective regions of the star. In convective regions we assume complete efficiency and take

$$\nabla = \nabla_a \equiv \left(\frac{\partial \ln T}{\partial \ln P}\right)_S. \tag{3}$$

To find the structure of the accreted envelope, equations (2) and (3) are numerically integrated inward from the photosphere, taken to be at $r = R_{\rm WD}$, down to where the total mass of hydrogen-rich material is M_a. The composition is then changed to pure ^{12}C and the integration continued into the core until the core temperature T_c is found.

Since we are considering compositions other than solar for the accreted material, we require a general expression for the opacity as a function of density, temperature and composition. For the radiative opacity we adopt an approximation of the form suggested by Christy (1966)

$$\kappa = \kappa_{\rm es} + n_e(Xf_1 + Yf_2 + Zf_3) \tag{4}$$

Here $\kappa_{\rm es}$ is an approximation to the electron scattering contribution to the opacity (Iben 1975)

$$\kappa_{\rm es} = (1.0 + X)\left[0.2 - D - (D^2 + 0.0004)^{1/2}\right] \tag{5}$$

where

$$D = 0.05(\log T - 7.7), \tag{6}$$

n_e is the electron density, and f_1, f_2, and f_3 are functions of ρ and T found by fitting to Tables 19, 27, and 34 of Cox and Tabor (1976) and Table 19 of Cox and Stewart

(1970). The terms X, Y, and Z are the mass fractions of hydrogen, helium, and carbon, respectively (we treat all metals as carbon for opacity purposes). Conductive opacities are approximated by the formulae of Iben (1975). In the equation of state that relates ρ and n_e to P and T, we include all ionization states of H, He, and C, allow for nonrelativistic degeneracy of the electrons, and treat the ions as a perfect gas. Equilibrium nuclear energy generation rates are assumed and taken from Fowler, Caughlan, and Zimmerman (1975) with weak screening corrections from Cox and Giuli (1968).

Comparing our simple models with the detailed models of Lamb and Van Horn (1975) for a cooling 1 $M_\odot$ pure carbon white dwarf, we find our core temperatures are consistently high by ~ 25%.

The onset of nuclear burning (and the end of the accretion stage) is taken to be when the following three criteria are first simultaneously satisfied:

1. The total nuclear energy production in the accreted material, $L_{\rm nuc}$, is greater than $L_{\rm WD}$ (note $L_{\rm nuc}$ is not included in L in the envelope integration).

2. The thermonuclear time scale at the base of the accreted material is shorter than the time scale on which T_a, the temperature at the base of the accreted material, is changing due to accretion alone, i.e.,

$$C_P \dot{T}_a \leq \varepsilon_{\rm nuc} . \quad (7)$$

3. Conduction into the core is unimportant. We quantify this last condition by calculating conduction and nuclear time scales, $t_{\rm con}$ and $t_{\rm nuc}$, respectively, defined by

$$t_{\rm con} = 2\frac{l^2}{C} \quad (8)$$

and

$$t_{\rm nuc} = \frac{C_P T}{\eta \varepsilon_{\rm nuc}} , \quad (9)$$

where $2l$ is the thickness of the energy generation zone

$$2l = \frac{L_{\rm nuc}}{4\pi R^2 \rho \varepsilon_{\rm nuc}} , \quad (10)$$

C is the conduction coefficient (evaluated for core composition)

$$C = \frac{4acT^3}{3\kappa C_P \rho^2} , \quad (11)$$

and

$$\eta = \left(\frac{\partial \ln \varepsilon_{\rm nuc}}{\partial \ln T} \right)_P . \quad (12)$$

In the above all quantities are evaluated in the burning zone at the base of the accreted material. Conduction is taken to be negligible when

$$t_{\rm con} > t_{\rm nuc} . \quad (13)$$

The physical interpretation of criterion 1 is that, for the thermonuclear runaway to proceed, the total energy produced by thermonuclear reactions must be greater than the energy radiated away at the white dwarf surface.

For a range of accretion rates near $10^{-7}\ M_\odot\ {\rm yr}^{-1}$ stable nuclear burning can occur on white dwarfs with the radiative losses balancing the nuclear energy generation (Sienkiewicz 1980). The luminosity is close to the Eddington limiting luminosity and is given by the core mass-luminosity relation found by Paczyński (1971) for asymptotic giant branch stars. For higher rates the white dwarf is swamped by the accretion and the system develops into a common-envelope giant (Nomoto, Nariai, and Sugimoto 1979). For these high accretion rates our models, which have low initial luminosities, are applicable to the approach of the flash to these high luminosity configurations.

Stable nuclear burning can also occur for very low accretion rates, $F < F_{\rm max}$, where $F_{\rm max}$ depends on CNO abundance (Papaloizou, Pringle, and MacDonald 1982). For solar abundances $F_{\rm max} \approx 10^{-12}\ M_\odot\ {\rm yr}^{-1}$ and for $Z_{\rm CNO} \approx 0.5$, we estimate $F_{\rm max} \approx 10^{-14}\ M_\odot\ {\rm yr}^{-1}$. These accretion rates are, in general, too low to be relevant to cataclysmic variables.

For accretion rates outside these stable regimes, there are no equilibrium configurations, and once sufficient hydrogen-rich material has been accreted a thermonuclear runaway must occur (Giannone and Weigert 1967).

We now consider the case of high accretion rate for which inequality (1) is severely violated. As discussed in MacDonald (1980); also see Fujimoto 1982*b*) the accreted material attains a quasi-equilibrium with thermal energy generation rate

$$\varepsilon_{\rm th} \approx FT\frac{\partial S}{\partial m} \approx C_P T(\nabla_a - \nabla)\frac{F}{M-m} . \quad (14)$$

Here m is the mass coordinate, and M is the total mass $(M_{\rm WD} + M_a)$.

To obtain the total mass accreted before thermonuclear runaway we now integrate equation (2) or (3) together with

$$\frac{dL}{dm} = \varepsilon_{\rm th} , \quad (15)$$

where $\varepsilon_{\rm th}$ is given by equation (14).

The procedure used is to guess the photospheric luminosity and integrate from the surface inwards to the point where $L = 0$. (This is a justified approximation

TABLE 1

COMPARISON OF ACCRETION MODEL WITH DETAILED CALCULATIONS

$M_{WD}/M_{\odot}$	$L_{WD}/L_{\odot}$	$F(M_{\odot}\ yr^{-1})$	$M_{det}/M_{\odot}$	$M_{sim}/M_{\odot}$	Reference
0.4	1.7×10^{-4}	10^{-8}	8.44×10^{-4}	...	1
0.456	7×10^{-4}	10^{-13}	6.89×10^{-4}	4.00×10^{-4}	2
0.5	5.7×10^{-2}	10^{-10}	5.01×10^{-4}	4.16×10^{-4}	3
1.0	5×10^{-4}	10^{-10}	2×10^{-4}	1.37×10^{-4}	4
1.0	10^{-3}	10^{-13}	2.4×10^{-4}	1.64×10^{-4}	2
1.0	10^{-3}	10^{-10}	1.38×10^{-4}	1.35×10^{-4}	5
1.0	10^{-3}	10^{-8}	5.85×10^{-5}	6.11×10^{-5}	6
1.0	5×10^{-3}	10^{-10}	1×10^{-4}	9.47×10^{-5}	4
1.0	9.8×10^{-2}	10^{-10}	5.25×10^{-5}	6.49×10^{-5}	3
1.0	9.8×10^{-2}	10^{-7}	3.63×10^{-5}	2.91×10^{-5}	3
1.3	9.7×10^{-5}	10^{-10}	1.63×10^{-4}	...	1

REFERENCES—(1) Nariai *et al.* 1980. (2) Taam 1977. (3) MacDonald 1979. (4) Taam and Faulkner 1975. (5) MacDonald 1980. (6) Kutter and Sparks 1979.

because the accretion luminosity is much greater than L_{WD}.) L_{nuc} and $\dot{T}_a$ are then evaluated.

A second guess for the photospheric luminosity is then used and the procedure repeated. From these two guesses we estimate a third photospheric luminosity such that criteria 1 and 2 are approximately satisfied. (Conduction into the core has not been considered for this case). We then iterate until the minimum accreted mass for which criteria 1 and 2 are simultaneously satisfied is found.

For intermediate values of F we find the accreted mass by interpolating between the low and high F results. We take

$$M_a = \frac{M_{LO} + xM_{HI}}{1+x}, \tag{16}$$

where M_{LO} and M_{HI} are the low and high F accreted masses and

$$x = \left(\frac{\Re F T_c}{\mu L_{WD}}\right)^2. \tag{17}$$

As a test of this simple model we give in Table 1 a comparison with the results of a number of detailed numerical calculations. In all cases the composition of accreted material is essentially solar. The mean error in our approximate values is 24%, and, in general, our approximate value is an underestimate. We have not computed values corresponding to the parameters of the two models of Nariai, Nomoto, and Sugimoto (1980) which both have low luminosities ($\sim 10^{-4} L_{\odot}$) corresponding to cooling times of $\sim 5\times10^9$ yr. However, extrapolation of our results would give $M_a = 7.88\times10^{-4}$ $M_{\odot}$ and 2.35×10^{-5} $M_{\odot}$ for the 0.4 and 1.3 $M_{\odot}$ white dwarfs, respectively. The large discrepancy for the 1.3 $M_{\odot}$ white dwarf may be due to our neglecting the effect of conduction into the core for the high accretion rate regime. However, even if we ignore the gravitational energy release and apply the low accretion regime result, for which conduction is included, we would find $M_a \sim 6\times10^{-5}$ $M_{\odot}$, indicating that neglect of conduction is not the complete cause of the discrepancy.

In both our simple accretion model and the detailed models, thermonuclear reactions are assumed to be in equilibrium. However, if the accreted hydrogen-rich material contains overabundances of nuclear fuels such as ^{3}He and ^{12}C which ignite at low temperatures compared to the equilibrium *p-p* chains or the CNO cycle, M_a can be significantly reduced. Moderate ^{3}He abundances, $Y_3 \sim 3\times10^{-4}$, are expected if the secondary is ascending the red giant branch for the first time (Iben and Truran 1978). Even higher ^{3}He abundances can occur in the surface layers of mass-losing main sequence stars, $Y_3 \approx 1.2\times10^{-3}(M_{\star}/M_{\odot})^{-2}$ for $M_{\star} \gtrsim 0.4$ $M_{\odot}$ (MacDonald 1979). The maximum nuclear luminosity from ^{3}He burning is (MacDonald 1979)

$$L_3 \sim 7\left(\frac{F}{10^{-10}\ M_{\odot}\ yr^{-1}}\right) Y_3\, L_{\odot}. \tag{18}$$

Comparing this with the expected white dwarf luminosity (see below), we see that the ^{3}He abundance can be an important factor in determining M_a, even at low accretion rates.

b) The Mass of Accreted Material: Results

Figures 1–6 are contour diagrams showing how M_a, the accreted mass, depends on M_{WD}, L_{WD}, and F for two accreted matter compositions: (1) "solar," $X = 0.7$, $Z = 0.02$, $Z_{CNO} = 0.014$; and (2) half-"solar" + half-carbon, $X = 0.35$, $Z = 0.51$, $Z_{CNO} = 0.507$. Representative values for the nonvaried quantities are taken to be $M_{WD} = 1$ $M_{\odot}$, $L_{WD} = 10^{-2}$ $L_{\odot}$, and $F = 10^{-9}$ $M_{\odot}$ yr^{-1}.

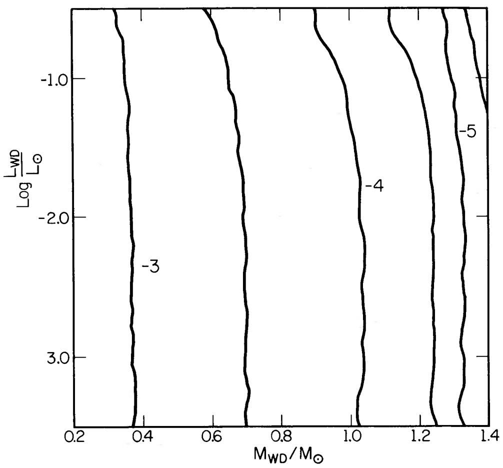

FIG. 1

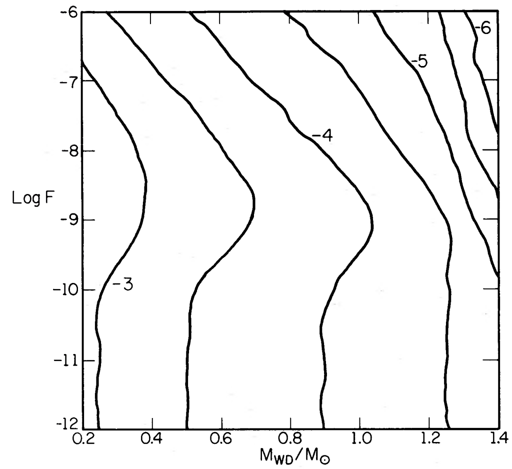

FIG. 2

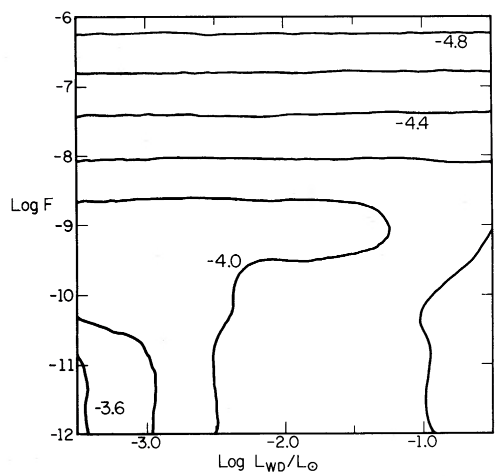

FIG. 3

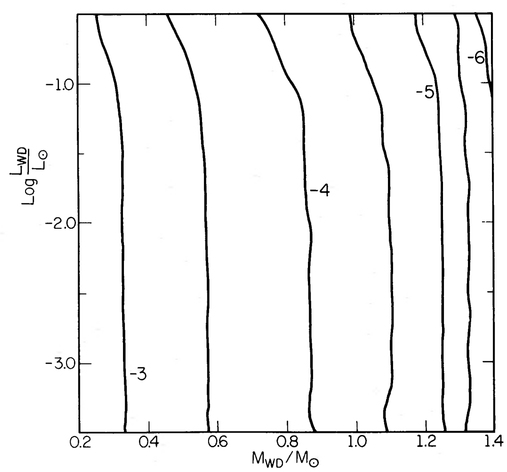

FIG. 4

FIG. 1.—Contours of accreted mass in white dwarf mass–white dwarf luminosity space for $X = 0.7$, $Z_{CNO} = 0.014$, and accretion rate, $F = 10^{-9}\ M_{\odot}\ yr^{-1}$. Contours are labeled with log $M_a/M_{\odot}$.

FIG. 2.—Contours of accreted mass in white dwarf mass-accretion rate space for $X = 0.7$, $Z_{CNO} = 0.014$, and white dwarf luminosity, $L_{WD} = 10^{-2}\ L_{\odot}$. Contours are labeled with log $M_a/M_{\odot}$.

FIG. 3.—Contours of accreted mass in white dwarf luminosity-accretion rate space for $X = 0.7$, $Z_{CNO} = 0.014$ and white dwarf mass, $M_{WD} = 1.0\ M_{\odot}$. Contours are labeled with log $M_a/M_{\odot}$.

FIG. 4.—As Fig. 1, but for $X = 0.35$, $Z_{CNO} = 0.507$

We immediately see that M_a is most sensitive to changes in M_{WD}. This is simply because surface gravity and hence pressure gradient increase steeply with M_{WD} so that less mass has to be accreted onto higher mass white dwarfs before thermonuclear temperatures and densities are reached. Also, from Figures 3 and 6, we see that, for given M_{WD} the higher mass envelopes occur for low F and low L_{WD}. In this case the onset of nuclear burning is due to the *p-p* chains rather than the CNO cycle, and hence there is only a weak dependence on composition via opacity and molecular weight. However, for high L_{WD} or high F the thermonuclear runaway is initiated by the CNO cycle and a higher CNO abundance leads to a significantly smaller accreted mass.

III. THE EXPANSION STAGE

In this section we consider the expansion stage, i.e. the expansion from the time of maximum shell source temperature, T_{ss}, to maximum photospheric radius and visual luminosity. Fujimoto (1982*a*) has also modeled this stage by approximating the accreted envelope by a polytrope. We use a different approach by making use of the fact that after maximum T_{ss}, the envelope is

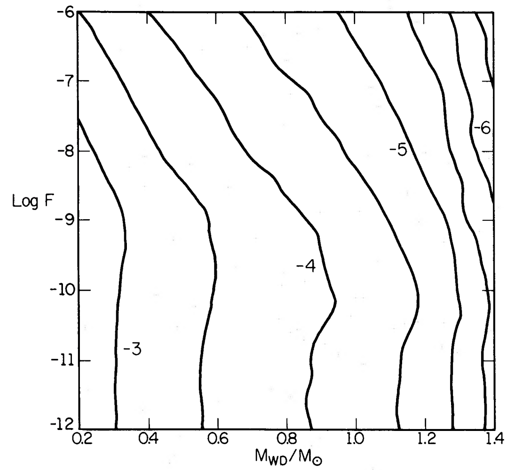

FIG. 5.—As Fig. 2, but for $X = 0.35$, $Z_{\rm CNO} = 0.507$

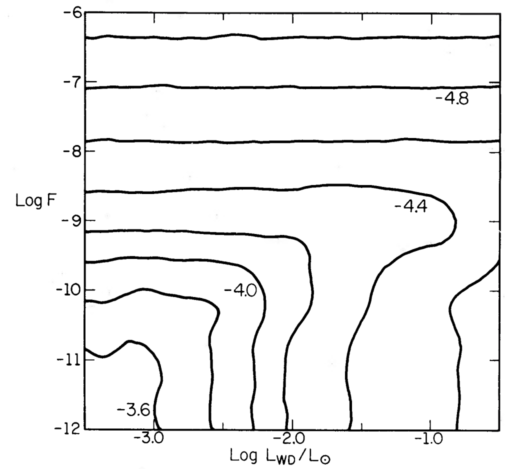

FIG. 6.—As Fig. 3, but for $X = 0.35$, $Z_{\rm CNO} = 0.507$

convective throughout. We also consider in detail the effects of enhanced CNO abundances, which Fujimoto considered only in a cursory way.

a) *The Expansion Velocity*: *Simple Model*

At and after maximum $T_{\rm ss}$, only gas and radiation pressure are important. Convective energy transport is efficient, and we assume the envelope is an adiabat (for a discussion of the effects of superadiabaticity, see Nariai, Nomoto, and Sugimoto 1980) so that

$$T\Delta S \equiv \Delta U - \frac{P}{\rho^2}\Delta\rho = 0. \quad (19)$$

This can be solved in terms of β, the ratio of gas to total pressure. We find

$$P = P_0 e^{32/3\beta}\frac{(1-\beta)^{5/3}}{\beta^{8/3}}, \quad (20)$$

$$\rho = \rho_0 e^{8/\beta}\frac{1-\beta}{\beta}, \quad (21)$$

and

$$T = T_0 e^{8/3\beta}\left(\frac{1-\beta}{\beta}\right)^{2/3}, \quad (22)$$

where

$$P_0 = \frac{\Re}{\mu}\rho_0 T_0 = \frac{1}{3}aT_0^4. \quad (23)$$

Further, at maximum $T_{\rm ss}$ and for some time after, the envelope is in hydrostatic equilibrium. If we take $P = 0$ at $r = R$, solving the hydrostatic support equation gives

$$r = \frac{R}{1 + qI(\beta)}, \quad (24)$$

where

$$I(\beta) = e^{8/3\beta}\frac{(1-\beta)^{2/3}}{\beta^{5/3}}\frac{8-3\beta}{2} \quad (25)$$

and

$$q = \frac{\Re T_0}{\mu}\frac{R}{GM_{\rm WD}}. \quad (26)$$

Our choice of boundary condition, $P = 0$ at $r = R$, is the only boundary condition consistent with the adiabat extending to the stellar surface and implies that $\beta = 1$, $T = 0$, and $\rho = 0$ there. Other boundary conditions would be appropriate if, for example, we had included in our model an optically thin photospheric region. Complications such as this defeat the object of our model, namely to provide a simple analytic means of surveying how the flash energetics depends on the physical parameters of the white dwarf system. The accuracy of our model will be determined by comparison with detailed computational studies.

The mass of the accreted envelope is given by

$$M_a/M_s \equiv z = q^4\int_{\beta_{\rm in}}^{1}\frac{J(\beta)}{[1+qI(\beta)]^4}d\beta, \quad (27)$$

where

$$M_s = 4\pi\left(\frac{\mu}{\Re}\right)^4\frac{a}{3}(GM_{\rm WD})^3, \quad (28)$$

$$J(\beta) = -\,e^{8/\beta}\frac{1-\beta}{\beta}I'(\beta), \quad (29)$$

and $\beta_{\rm in}$ is the value of β at the base of the accreted envelope. The value of $\beta_{\rm in}$ is found from

$$R = R_{\rm WD}\left[1 + qI(\beta_{\rm in})\right]. \quad (30)$$

For a given value of R, equations (27)–(30) give $\beta_{\rm in}$ and q. Equation (26) then gives T_0, and hence P_0 and ρ_0 are found from equation (23). We now know the envelope structure completely. The envelope evolution is then determined from the energy conservation equation,

$$\frac{dE}{dt} = L_{\rm nuc} - L_\star, \quad (31)$$

where

$$E = U + V, \quad (32)$$

and U and V, the internal and gravitational energies of the accreted envelope, respectively, are given by integrals over the envelope. The term $L_{\rm nuc}$ is the total nuclear energy generation in the envelope

$$L_{\rm nuc} = \int_{\beta_{\rm in}}^{1} 4\pi r^2 \rho \epsilon_{\rm nuc} \frac{dr}{d\beta} d\beta, \quad (33)$$

and $L_\star$ is the energy radiated at the photosphere and, in the frame of this model, is given by

$$L_\star = L_{\rm Ed}\left(1 - \beta_p\right)\frac{\left(32 - 24\beta_p\right)}{\left(32 - 24\beta_p - 3\beta_p^2\right)}, \quad (34)$$

where $L_{\rm Ed}$ is the Eddington luminosity, and β_p is the value of β at the photosphere, taken to be at optical depth $\tau = 2/3$. Electron scattering is the dominant opacity source, and hence

$$L_{\rm Ed} = \frac{4\pi c G M_{\rm WD}}{\kappa_{\rm es}} = \frac{6.5\times 10^4}{1+X}\frac{M_{\rm WD}}{M_\odot}L_\odot. \quad (35)$$

For given $M_{\rm WD}$, M_a, and envelope composition, we find the early evolution of the nova by integrating equation (31) from the time of maximum $T_{\rm ss}$ until either (1) the expansion velocity, $v_{\rm exp} = \dot{R}$, exceeds the escape velocity, $v_{\rm esc}$, so that some mass is ejected (also our assumption of hydrostatic equilibrium becomes invalid here), or (2) $v_{\rm exp} < 0$, i.e., the envelope has reached an equilibrium radius without any ejection of matter.

We quantify the strength of the nova outburst by the value of $v_{\rm exp}$ when escape velocity is reached. The results are presented and discussed in the next section. Here we discuss a few approximations and assumptions we have made.

First, we have neglected heat loss into the white dwarf core and mechanical work done at the core-envelope interface. The detailed models of MacDonald (1979) for very slow nova outbursts with solar abundances indicate that these terms are at most 10% of $L_{\rm nuc}$ and usually much less. For fast novae, these terms are $\sim 1\%$ (S. J. Kenyon and J. W. Truran 1982, private communication). Hence, within the accuracy of our model, our neglect of these terms is justified.

Second, we have assumed that μ remains constant with time. This is a reasonable approximation because very little hydrogen need be processed to helium to give energies typical of nova outbursts. The ratio of nuclear energy to binding energy of the accreted material is

$$0.007 X M_a c^2 \frac{R_{\rm WD}}{G M_{\rm WD} M_a} \approx 24 X \frac{R_{\rm WD}}{5\times 10^8\ {\rm cm}}\left(\frac{M_{\rm WD}}{M_\odot}\right)^{-1}. \quad (36)$$

Hence, typically, $\Delta X \sim 0.04$ and $\Delta\mu \sim 0.02$. Convection maintains uniformity of μ.

Third, we have assumed equilibrium CNO burning. This is a reasonable approximation for solar abundances for which convective time scales ($\gtrsim 10^3$ s) are reasonably long compared to the β-decay time scales ($\sim 10^2$ s) which govern the CNO cycle rate at high temperatures. However, for enhanced CNO, these time scales are comparable and fresh CNO nuclei can be mixed into the burning zone to undergo proton capture. In this case we may have underestimated the CNO energy generation rate and the strength of the hydrogen flash.

As a test of our expansion stage model we compare it to the results of detailed computations. Slow nova models with solar CNO have been calculated by Sparks, Starrfield, and Truran (1978) and Nariai, Nomoto, and Sugimoto (1980). In both these models, peak $T_{\rm ss}$ occurs away from the core-envelope interface, and hence we use the mass above the shell source for comparison with our simple models. Other models with solar CNO have been computed by MacDonald (1979, 1980) and Kutter and Sparks (1979). The results of this comparison are given in Table 2. We see that our simple model overestimates the velocity at $v_{\rm exp} = v_{\rm esc}$ and also the maximum value of $T_{\rm ss}$. However, the trend is correct. Comparison for models with enhanced CNO is made difficult by the lack of published velocity curves. The results of an unpublished calculation by S. J. Kenyon and J. W. Truran (1982, private communication) indicate that our simple model underestimates the nova outburst strength when CNO abundances are strongly enhanced. As discussed above, this may be because of our assumption of equilibrium CNO cycling.

Fujimoto (1982*a*) has shown that, for solar CNO abundances, the outburst strength depends mainly on the "proper" pressure in the shell source.

$$P_\star = \frac{G M_{\rm WD}}{4\pi R_{\rm WD}^4} M_a. \quad (37)$$

TABLE 2

COMPARISON OF EXPANSION MODEL WITH DETAILED CALCULATIONS

$M_{WD}/M_{\odot}$	$M_p/M_{\odot}$	Z_{CNO}	v_{det} (km s^{-1})	v_{sim}(km s^{-1})	T_{det} (10^8 K)	T_{sim}(10^8 K)	Reference
1.00	5.85×10^{-5}	0.013	0	0	1.67	1.76	1
1.00	1.38×10^{-4}	0.014	0	0	2.11	2.10	2
1.25	3.3×10^{-5}	0.3	1000	490	2.34	2.47	3
1.25	6.5×10^{-5}	0.015	50	81	2.9	3.19	4
1.30	5.7×10^{-5}	0.02	150	220	3.45	3.63	5

REFERENCES—(1) Kutter and Sparks 1979. (2) MacDonald 1979. (3) S. J. Kenyon and J. W. Truran 1982, private communication. (4) Sparks *et al.* 1978. (5) Nariai *et al.* 1980.

The present calculations are in exact agreement with this conclusion. We find, as did Fujimoto, that $P_\star \gtrsim 10^{20}$ dyn cm^{-2} is required for ejection with solar CNO abundances. With $Z = 0.51$, the critical value of $P_\star$ is reduced to $\sim 2\times10^{19}$ dyn cm^{-2}, indicating that substantially smaller envelope masses are sufficient for nova outbursts to occur.

b) Results of Expansion Model

Figures 7 and 8 are contour diagrams of v_{exp} at the time when $v_{exp} = v_{esc}$ for the two compositions considered above. Also shown are the regions of (M_{WD}-M_a) space which our simple accretion model predict to be realizable. From Figure 7 we see that no ejection of material is expected for solar CNO unless $M_{WD} \gtrsim 1.1$ $M_{\odot}$ and the ejection velocity will be low ($\lesssim 250$ km s^{-1}) even for optimal conditions. From Figure 8 we see that $M_{WD} \gtrsim 0.8$ $M_{\odot}$ is required for ejection when $Z_{CNO} = 0.51$. Now, however, the highest ejection velocity is ~ 1600 km s^{-1} comparable to the principal absorption velocities observed in the fastest novae, e.g., V1500 Cyg (Duerbeck and Wolf 1977).

This clearly shows that strongly enhanced CNO abundances are necessary for fast novae. However slow novae can also occur with CNO greatly enhanced. In Table 3 we have collected together abundance estimates for the ejecta of a number of recent novae and translated them into mass fractions. Also given are decline time, t_3, principal absorption velocity, v_{princ}, and ejecta mass estimate, M_{ej}, adjusted to the given distance, D. The most uncertain of these quantities are the ejecta masses which are usually determined from emission-line strengths with the assumption of a uniform nebula. Clumping in the ejecta will lead to an overestimate of M_{ej}. Conversely only the mass of ionized hydrogen is measured, and M_{ej} may be underestimated if neutral hydrogen is present. Taking this data at face value we

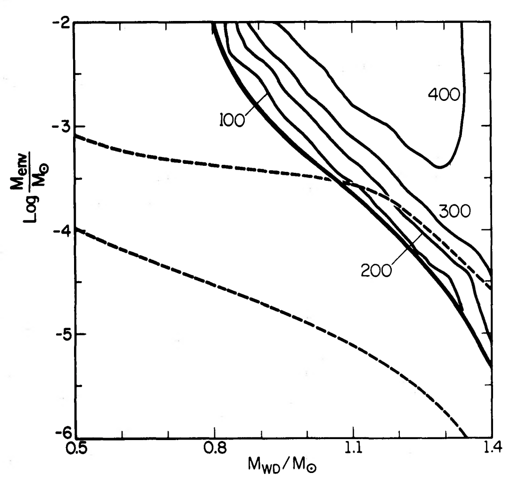

FIG. 7

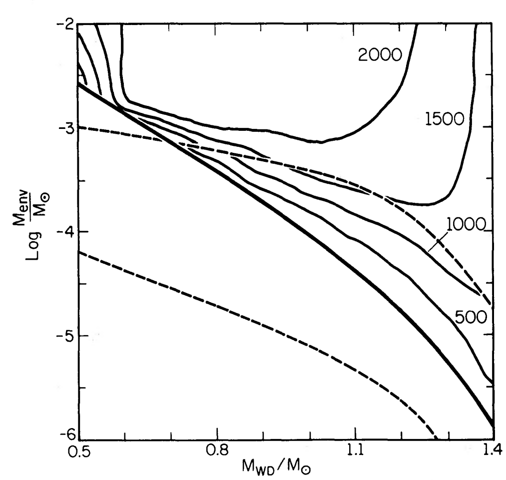

FIG. 8

FIG. 7.—Contours of expansion velocity, v_{exp}, when it equals escape velocity in white dwarf mass-accreted mass space for $X = 0.7$, $Z_{CNO} = 0.014$. Contours are labeled with v_{exp} in km s^{-1}. Also shown (*dashed lines*) is the domain of M_{WD}-M_a space accessible by accreting white dwarfs.

FIG. 8.—As Fig. 7, but for $X = 0.35$, $Z_{CNO} = 0.507$

TABLE 3

ABUNDANCES AND EJECTA MASSES

Object	D(kpc)	t_3(d)	$v_{\rm princ}$(km s^{-1})	$M_{\rm ej}/M_\odot$	X	Y	$Z_{\rm CNO}$	Reference
V1500 Cyg 1975 ...	1.6	4	1600	3×10^{-4}	0.49	0.21	0.28	1, 2
CP Lac 1936	1.3	10	1300	3×10^{-5}	0.60	0.26	0.13	3, 4
V1668 Cyg 1978 ...	3.9	24	740	2×10^{-4}	0.47	0.22	0.31	5
DK Lac 1950	2.1	29	870	...	0.47	0.47	0.06	6
IV Cep 1971	3.6	42	1000	8×10^{-5}	0.40	0.21	0.39	4, 7
T Aur 1891	0.83	85	400	...	0.46	0.35	0.19	8
DQ Her 1934	0.42	100	310	5×10^{-5}	0.30	0.17	0.53	9
RR Pic 1925	0.48	150	300	3×10^{-4}	0.49	0.45	0.04	10
HR Del 1967.......	0.86	220	350	1×10^{-4}	0.43	0.47	0.073	11

REFERENCES.—(1) Ferland 1978. (2) Ferland and Shields 1978. (3) Pottasch 1959. (4) Ferland 1979. (5) Stickland *et al.* 1981. (6) Collin-Souffrin 1977. (7) Pacheco 1977. (8) Gallagher *et al.* 1980*a*. (9) Williams *et al.* 1978. (10) Williams and Gallagher 1979. (11) Tylenda 1979.

can, in principle, find the masses of the white dwarfs on which these particular novae occurred. We need to make one further assumption: that all the material above the shell source is ejected so that we can identify M_a and $M_{\rm ej}$. Starrfield (1979) has considered the time scale on which novae turn off and concluded that the bulk of the envelope must be ejected by some means, either by the hydrogen flash or interaction with the secondary (MacDonald 1980) or possibly an OB stellar wind, on a short time scale and is not consumed by steady nuclear burning. With this assumption we find, with one exception, that the white dwarf mass falls in the range 1.02–1.18 $M_\odot$. The exception, CP Lac, is a very fast nova but has a small ejecta mass and only a moderate CNO enhancement. A high white dwarf mass, $M_{\rm WD} \approx 1.37\ M_\odot$, is then required to attain the shell source temperatures necessary for a strong outburst. In deriving the white dwarf masses, we have applied a correction factor that allows for our neglect of the effect of non-equilibrium CNO cycling. In view of the above discussion we assume that this factor depends only on the ratio of the convective turnover timescale, τ_c, to the β-decay timescale, τ_β, and is such that

$$\log v_{\rm exp}/v_{\rm sim} = a + b \log \frac{\tau_c}{\tau_\beta},$$

where a and b are found by fitting to the results given in Table 2. Here $v_{\rm sim}$ is the expansion velocity given by our simple expansion phase model.

DQ Her, a slow nova with a very high CNO abundance (Williams *et al.* 1978) has always been a problematic object for thermonuclear runaway theories of novae. We see, however, that a low accreted mass consistent with the observed ejecta mass can give a slow nova outburst even when the CNO abundances is as high as it is in DQ Her. The deduced white dwarf mass of 1.1 $M_\odot$ is also in good agreement with the observed value (Robinson 1976). We suggest that the differences in rates of light curve development for DQ Her and V1500 Cyg are primarily due to differences in accreted mass rather than a difference in white dwarf mass, as had been suggested by Truran (1982).

In Figure 9 we show the relationship between speed class, CNO abundance, and envelope mass. A value of $M_{\rm WD} = 1.1\ M_\odot$ has been adopted. The values $v_{\rm princ} = 500$ km s^{-1} and $v_{\rm princ} = 1000$ km s^{-1} have been taken to be the dividing lines between slow, moderate, and fast novae. Duds are defined to be objects that do not give ejection. EUV objects are a subset of the duds which expand only slightly but brighten to near Eddington luminosity so that the bulk of their radiation is emitted in the EUV. Both novae and duds will also pass through

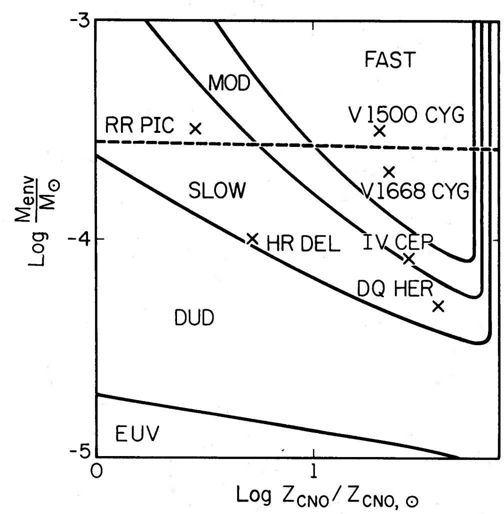

FIG. 9.—Nova speed class against CNO abundance and envelope mass. $M_{\rm WD} = 1.1\ M_\odot$ has been assumed. The crosses mark the positions of classical novae. Also shown is the maximum envelope mass that can be accreted by a 1.1 $M_\odot$ white dwarf before thermonuclear runaway. See text for definition of speed classes.

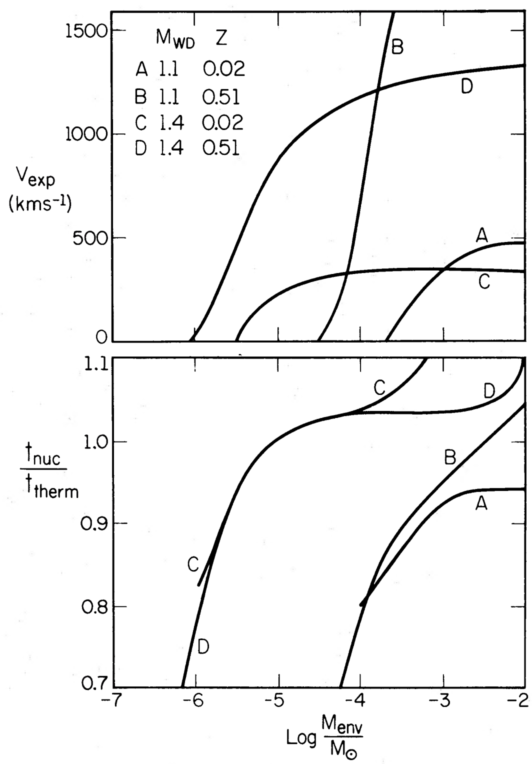

FIG. 10.—Expansion velocity, v_{exp}, and ratio of nuclear decay time to thermal time against envelope mass for $M_{WD} = 1.1\ M_{\odot}$ and $1.4\ M_{\odot}$ and $Z_{CNO} = 0.014$ and 0.507. v_{exp} is in km s^{-1}.

an EUV stage as their photospheres shrink due to mass loss or consumption of nuclear fuel.

Our expansion model also gives a clue to why the deduced kinetic energy of the ejecta (see Table 3) is so small when compared to the binding and nuclear energies of the accreted envelope. There are two time scales of interest: t_{nuc}, the time for the total nuclear luminosity to decrease from its maximum value to e^{-1} its maximum value, and t_{therm}, the time scale on which nuclear energy is being used to unbind the accreted envelope. More exactly,

$$t_{therm} = \frac{|E_{bin}|}{L_{nuc}}, \quad (38)$$

where E_{bin} is the binding energy of the accreted envelope at the time of maximum L_{nuc}. If $t_{nuc} \gg t_{therm}$ the envelope would become unbound before L_{nuc} decreased significantly and a large amount of kinetic energy would be generated. If $t_{nuc} \ll t_{therm}$, L_{nuc} decreases too rapidly to unbind the envelope, and no ejection occurs. From our expansion model we find, in general, $t_{nuc} \approx t_{therm}$, and hence a small kinetic energy results. The reason why $t_{nuc} \approx t_{therm}$, is because it is the envelope expansion, which occurs on time scale $\sim t_{therm}$, that shuts off the nuclear burning. To illustrate this, we show in Figure 10, v_{exp} and t_{nuc}/t_{therm} plotted against envelope mass for $M_{WD} = 1.1$ and $1.4\ M_{\odot}$.

IV. ACCRETION RATES AND WHITE DWARF LUMINOSITIES

In this section we put together the results of the previous two sections to find the conditions under which the hydrogen flash gives envelope ejection and to relate these conditions to the properties of the binary. We consider, in turn, classical novae, recurrent novae, and type I supernovae.

a) Classical Novae

The regions of F-L_{WD} space that give rise to envelope ejection can be best seen from Figures 11 and 12 which, for our two envelope compositions, are contour diagrams of F_{max}, the maximum value of accretion rate that gives ejection, against M_{WD} and L_{WD}. We see that, for given M_{WD} and composition, critical values of F and L_{WD} can be defined, F_c and L_c say, such that if $F \le F_c$ and $L_{WD} \le L_c$ then envelope ejection occurs and results in a nova. Simple analytic fits to F_c and L_c as functions of M_{WD} are

$$\log F_c = -8.775 - 15.088\left(\frac{M_{WD}}{M_{\odot}} - 1.459\right)^2 \quad (39)$$

and

$$\log L_c = -0.629 - 5.923\left(\frac{M_{WD}}{M_{\odot}} - 1.766\right)^2 \quad (40)$$

for $Z = 0.02$ and

$$\log F_c = -8.632 - 4.596\left(\frac{M_{WD}}{M_{\odot}} - 1.334\right)^2 \quad (41)$$

$$\log L_c = -1.375 - 7.027\left(\frac{M_{WD}}{M_{\odot}} - 1.308\right)^2 \quad (42)$$

for $Z = 0.51$.

Similar results can be derived for each nova speed class. We content ourselves with discussion of some of the better observed novae. The most energetic classical nova observed to date is V1500 Cygni which has $v_{princ} \approx 1600$ km s^{-1} and $t_3 = 4$ days. For the ejecta mass and composition given in Table 3, we find $M_{WD} = 1.04\ M_{\odot}$ is required. The large ejecta mass, $M_{ej} \approx 3 \times 10^{-4}\ M_{\odot}$, can be accreted if the accretion rate is low, $F \lesssim 10^{-11}\ M_{\odot}$ yr^{-1} and the underlying white dwarf was initially cool, $L_{WD} \lesssim 10^{-3}\ L_{\odot}$. The absolute visual magnitude of the accretion disk can then be estimated to be $M_v \approx 9.8$ which, for distance 1.6 kpc and $E(B-V) = 0.5$

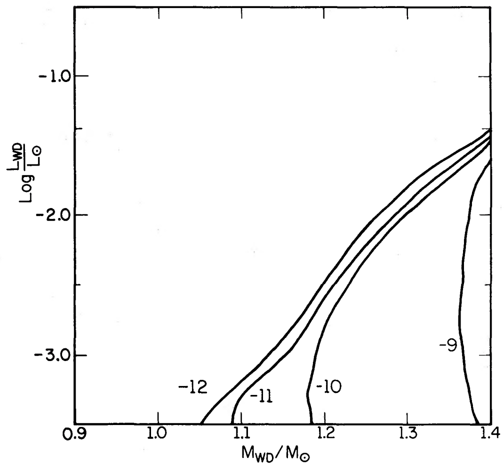

FIG. 11

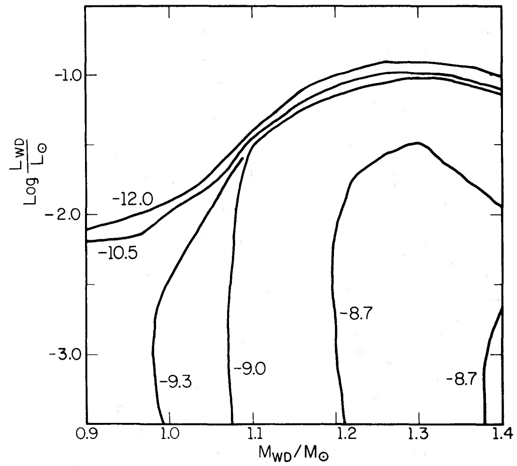

FIG. 12

FIG. 11.—Contours of maximum accretion rate that permits a nova outburst in white dwarf mass–white dwarf luminosity space for $X = 0.7$, $Z_{CNO} = 0.014$. Contours are labeled with log F_{max}, where F_{max} is in $M_\odot$ yr^{-1}.

FIG. 12.—As Fig. 11, but for $X = 0.35$, $Z_{CNO} = 0.507$

(Ferland 1977), corresponds to apparent magnitude $m_v \approx 22.3$. This is consistent with V1500 Cygni being too faint to appear on the Palomar Sky Survey (Beardsley *et al.* 1975).

In quiescence, DQ Her has $m_v = 14.6$ which corresponds to $M_v = 6.1$ for distance 420 pc and 0.38 mag of reddening (Ferland 1980). For our mass estimate, $M_{WD} = 1.10\ M_\odot$, we find $F \approx 8 \times 10^{-10}\ M_\odot$ yr^{-1} from the accretion disk luminosity, in close agreement with $F = 1.1 \times 10^{-9}\ M_\odot$ yr^{-1} found from analysis of period changes (Nelson 1976). The luminosity is then $L_{WD} \lesssim 5 \times 10^{-3}\ L_\odot$. We concluded earlier that V1500 Cygni and DQ Her essentially differed only in accreted mass. We now see that this difference is probably mainly due to a two orders of magnitude difference in accretion rate.

For a number of novae, including RR Pic, CP Lac, and probably V603 Aql, we find that the accretion rates deduced from visual light in quiescence are too high, typically $\sim 10^{-8}\ M_\odot$ yr^{-1}, for the estimated ejecta mass to be accreted. It has also been pointed out that the soft X-ray emission from classical novae is two to four orders of magnitude less than predicted by boundary layer theory (Ferland *et al.* 1982). One possible resolution to both these problems is that part of the optical luminosity is due to reprocessing by the accretion disk of UV radiation from a white dwarf that is still hot from the outburst. This is supported by observations of V841 Oph = Nova Oph 1848 which has declined by ~ 1 mag over the past century (R. F. Webbink and S. J. Kenyon 1982, private communication) indicating that novae decline long after outburst. Alternatively, the accretion rate deduced from the optical flux is correct, but the secondary is in a perturbed thermal state due to the recent nova outburst and is presently transferring matter at a rate much greater than in equilibrium.

Our model for the accretion phase of novae is strictly valid only if the thermal structure of the white dwarf is not significantly perturbed by previous hydrogen flashes so that we can legitimately assume a uniform radiative flux in the envelope. Sequences of hydrostatic hydrogen flashes on accreting hot white dwarfs have been calculated by Paczyński and Zytkow (1978), Sion, Acierno and Tomczyk (1979), and Iben (1982). The initial conditions and accretion rates for all of these sequences are such as to exclude hydrodynamic evolution and hence are not relevant to classical nova outbursts. Until a sequence of nova outbursts has been calculated we can only estimate the heating and subsequent cooling of the white dwarf core due a nova outburst. From Table 2 we see that peak temperature, $T_p \sim 3 \times 10^8$ K for flashes that eject material. Let t_{hot} be the time in days for which high temperatures (i.e., significantly greater than the white dwarf core temperature) in the shell source are maintained. Heat will diffuse inwards into a mass, M_{hot}, of the core, where

$$\frac{M_{hot}}{M_\odot} \approx 2 \times 10^{-6} \left(\frac{R_{WD}}{5 \times 10^8 \text{ cm}} \right)^2 \left(\frac{T_p}{10^8 \text{ K}} \right)^{3/2} t_{hot}^{1/2}. \quad (43)$$

The value of t_{hot} is not accurately known but can be estimated from observations of novae. V1500 Cygni was a strong UV source 100 days after visual maximum (Wu and Kester 1977), whereas DQ Her had returned to white dwarf dimensions after ~ 8000 days (Walker

1956). Hence, we take $10^2 < t_{\rm hot} < 10^4$ so that $M_{\rm hot} \approx 10^{-4}$ to $10^{-3}\ M_\odot$. Since the bulk of the hydrogen-rich envelope must be ejected in a nova outburst (Starrfield 1979; Truran 1982), the late evolution will be determined by the cooling of the reheated core edge. After a time

$$\tau_{\rm rem} \sim \frac{\kappa_{\rm es} M_{\rm hot}}{4\pi c R_{\rm WD}} \approx 7\left(\frac{M_{\rm hot}}{10^{-4}\ M_\odot}\right)\frac{5\times 10^8\ {\rm cm}}{R_{\rm WD}}\,{\rm yr} \quad (44)$$

(Nariai 1974), the remnant will have contracted to white dwarf size and be cooling at roughly constant radius. The value of $\tau_{\rm rem}$ is typically 7–70 yr and is much shorter than the typical accretion time for a nova envelope, $\tau_{\rm acc} \gtrsim 10^4$ yr. Hence our assumption of uniform radiative flux in the white dwarf envelope is completely justified for classical novae.

b) Recurrent Novae

Similarities in the spectra and light curves of classical and recurrent novae have naturally led to the suggestion that recurrent novae are also thermonuclear runaways on white dwarfs. The smallest accretion times occur for white dwarf masses close to the Chandrasekhar limit, $M_{\rm Ch} = 1.454\ M_\odot$ in our model. For these high masses, conversion of hydrogen to helium must be taken into account. When this is included in our expansion model, we find the shortest accretion times for models that give ejection are 400 yr for $Z = 0.51$ and 450 yr for $Z = 0.02$ and occur for accretion onto 1.44 and 1.45 $M_\odot$ white dwarfs, respectively. In each case the change in hydrogen mass fraction is $\Delta X \approx 0.3$–0.4. These minimum accretion times are much greater than the interval, typically ~ 35 yr, between recurrent nova outbursts. They are also greater than $\tau_{\rm rem}$, the time for the nova remnant to evolve to a state of cooling at constant radius. This allows us to apply our accretion model to recurrent novae, and so we can conclude that recurrent novae are not thermonuclear runaways on accreting white dwarfs.

Shara (1979) has suggested that an enhancement of ^{3}He in the accreted material will decrease the envelope mass required to trigger the thermonuclear runaway and hence reduce the accretion time scale. Reducing the accreted envelope mass, however, decreases the strength of the thermonuclear runaway. Since our limits on the envelope mass required for ejection are strict lower bounds, increasing the ^{3}He abundance exacerbates the recurrent nova problem. The minimum accretion time scale is set by the maximum accretion rate consistent with achieving the minimum envelope mass sufficient to give a nova outburst.

In addition to this theoretical argument against a thermonuclear runaway model for recurrent novae, there are a number of observations which seem to rule out a thermonuclear model. The large mass estimate for the blue component of T CrB (Paczyński 1965) is inconsistent with it being a white dwarf but is consistent with the accretion model proposed by Webbink (1976). Also the luminosity of the unusual recurrent nova, WZ Sge, at maximum is much less than the Eddington limit (Fabian *et al.* 1980) in contrast to the classical novae which always attain or exceed the Eddington limit in agreement with the thermonuclear models. Abundance analyses of recurrent nova ejecta indicate essentially solar abundances and no significant CNO enhancement (Williams *et al.* 1981; Williams 1982). One of these novae is U Sco which has the fastest decline rate of all novae (Payne-Gaposchkin 1957) and a very high ejection velocity $v_{\rm ej} \sim 5000$ km s^{-1} (Barlow *et al.* 1981). If U Sco were a thermonuclear runaway, it should have a large CNO enhancement. Further, the ejecta mass estimate for U Sco (Williams *et al.* 1981) is roughly three orders of magnitude less than is typical of classical novae.

c) Type I Supernovae

Recently, theoretical interest in exploding white dwarf models for type I supernovae has revived (Fujimoto and Sugimoto 1979; Fujimoto 1980; Nomoto 1980; Taam 1980*a*, *b*; Woosley, Weaver, and Taam 1980). Such models are attractive on observational grounds because this is the only type of supernova known to occur in elliptical galaxies (Tammann 1977) which are systems of old stars and also because, by definition, SN I show no hydrogen in their spectra (Oke and Searle 1974).

In the model of Woosley, Weaver, and Taam (1980), ignition of $\sim 0.6\ M_\odot$ of helium accreted at a rate $\sim 10^{-8}\ M_\odot$ yr^{-1} by a low-mass (0.5 $M_\odot$) carbon-oxygen white dwarf is violent enough to cause detonation waves to propagate both outward through the accreted helium and inward into the carbon-oxygen core. Essentially all the nuclear fuel is processed into iron peak elements producing a supernova explosion of total energy $\sim 2\times 10^{51}$ ergs and a light curve similar to that of "fast" type I supernovae (Weaver, Axelrod and Woosley 1980). The envisaged site for the accretion to occur is in a cataclysmic binary. Woosley, Weaver, and Taam (1980) argue that, for their choice of parameters, the hydrogen shell flashes are too weak to reeject the accreted material in nova outbursts. From equation (39) we see that this is indeed true if the accreted material has solar abundances. However, some ejection of material is necessary to emove accreted angular momentum since a white dwarf accreting from a viscous disk is spun up to break up when its mass has increased by $\lesssim 20\%$ (MacDonald 1979). The amount of ejection required depends on the coupling between the inner and outer parts of the accreted envelope. We also note that for the parameters chosen by Woosley, Weaver, and Taam (1980), the red dwarf component of the cataclysmic binary must be initially more massive than the white dwarf, and hence mass transfer would initially occur on a dynamical timescale (Webbink 1977*a*, *b*). The mass transfer rate, $F \sim 10^{-4}$ to 10^{-3} $M_\odot$ yr^{-1} is supercritical, and the white dwarf component expands to a red giant (Paczyński and Zytkow 1978; Nomoto, Nariai, and Sugimoto 1979), overflows its Roche lobe, and forms a common envelope binary. The ultimate fate of such a system is not clear. Dynamical friction between the secondary and the common envelope might result in the ejection of a planetary nebula (Paczyński 1976; Webbink 1979) and/or cause the secondary to spiral inward to crash into the white dwarf.

Choosing a higher white dwarf mass may avoid this possibility. The accretion rate is then determined by the nuclear time scale of the secondary or by the rate at which gravitational radiation removes angular momentum from the system (Faulkner 1971) and is typically 10^{-10} to 10^{-9} $M_\odot$ yr^{-1}. Nova outbursts are then expected to occur before the white dwarf reaches 1.1 $M_\odot$. These outbursts can be avoided if the cool main-sequence secondary is magnetically coupled to a stellar wind, leading to loss of orbital angular momentum from the system and an enhanced mass transfer rate, typically 10^{-8} $M_\odot$ yr^{-1} (Verbunt and Zwaan 1981).

A further problem with the Woosley, Weaver, and Taam (1980) model is that a 0.5 $M_\odot$ white dwarf in a cataclysmic variable is likely to be helium rather than carbon-oxygen (Webbink 1979), although this may depend on the history of the progenitor system (cf. van der Linden 1980). The difficulties encountered by accreting He white dwarf models for SN I are discussed by Wheeler (1981).

From the above discussion it is clear that the occurrence of SN I depends critically on the properties and evolution of the underlying binary system.

V. DISCUSSION AND CONCLUSIONS

We have shown how the energetics of nova outbursts depend on white dwarf mass, accreted envelope mass, and envelope composition. In particular, it has been shown that enhancement of CNO is necessary for fast novae. The source of this enhancement is still unclear, but there are two promising enhancement mechanisms. Durisen (1977) and Kippenhahn and Thomas (1978) have suggested that shear instabilities mix chemical composition and angular momentum between the accretion disk and the surface layers of the white dwarf. If the white dwarf has no outer helium layer, significant carbon and/or oxygen enrichment can occur. A second possibility for CNO enrichment is a helium shell flash, initiated after a series of weak hydrogen flashes that eject little or no material. Since the nuclear energy content of helium is significantly less than hydrogen, a large helium fraction, $\Delta Y \gtrsim 0.2$, will be converted into carbon and/or oxygen and convectively mixed throughout the envelope. Shear instabilities can then mix freshly accreted solar composition material with this CO-enriched material. Alternatively, since the white dwarf envelope expands to engulf the secondary during the helium flash, the red dwarf can accrete CO-rich material and later transfer it back to the white dwarf. Abundance analyses of the accretion disks in old novae would show whether this is a viable possibility.

We note an interesting property of novae in which shear instabilities play a role. The shear mixing causes more matter than accreted to be involved in the outburst (MacDonald 1982). As discussed earlier, the bulk of the envelope must be ejected to shut off the nuclear burning shell, and so $M_{\rm WD}$ should decrease with time! This would effectively rule out accretion models for SN I in classical nova systems.

The narrow range for $M_{\rm WD}$, 1.02–1.18 $M_\odot$, deduced from the observations also removes some of the uncertainty in using novae as standard candles (Truran 1982). If we adopt this range for all novae, L_P, the luminosity in the "plateau" phase of the bolometric light curve is (Paczyński 1971)

$$L_P = (0.77\text{–}0.88)\, L_{\rm Ed,\odot}$$

where $L_{\rm Ed,\odot}$ is the Eddington limiting luminosity of a 1 $M_\odot$ object,

$$L_{\rm Ed,\odot} = \frac{6.41 \times 10^4}{1+X} L_\odot .$$

The hydrogen mass fraction for the nova ejecta range from 0.3 to 0.6 (see Table 3). The value of X is unlikely to be known for extragalactic novae, and hence the total uncertainty in L_P amounts to about 0.5 mag. As an example of the power of this method, consider V1668 Cygni. The plateau luminosity is (Stickland *et al.* 1981)

$$L_P = 3.5 \times 10^3 D^2 L_\odot ,$$

where D is distance in kpc. We estimate $D = 3.3\text{–}3.5$ kpc, in excellent agreement with the distance found from the maximum magnitude-rate of decline relation of Schmidt (1957), $D = 3.9 \pm 0.9$ kpc. Stickland *et al.* (1981) and Gallagher *et al.* (1980*b*) argue that because no acceleration is seen in the lines at maximum light, the maximum bolometric luminosity, $L_{\rm max} = 10^4 D^2 L_\odot$, should be equated with the Eddington luminosity. Adopting $D = 3.4$ kpc, we find $L_{\rm max} = 1.2 \times 10^5 L_\odot$, roughly a factor of 3 greater than $L_{\rm Ed}$. The spectral type of V1668 Cygni at maximum was F5 Ib (Ortalani *et al.* 1978), and we can estimate $T_e \sim 6300$ K and $R \sim 2 \times 10^{13}$ cm. The acceleration due to radiation pressure in the photospheric regions is then only ~ 1 km s^{-1} per day which is certainly undetectable.

Under certain conditions, the hydrogen shell flash leads to formation of a common envelope binary without itself causing ejection of the envelope. Dynamical friction between the secondary and the common envelope can then produce sufficient energy quickly enough to give ejection and an outburst with the characteristics of a slow nova (MacDonald 1980). This leads to the possibility that our white dwarf masses for the slower novae are slight overestimates. Binary interaction may also be effective at reducing the amount of helium accreted after weak hydrogen flashes and hence can have important consequences for type I SN models. The conditions under which the secondary influences the development of the hydrogen flash will be investigated in a subsequent paper.

To summarize, our main conclusions are:

1. Enhanced CNO abundances are necessary for fast novae. Slow novae, e.g. DQ Her, can still occur if $Z_{\rm CNO}$ is large.

2. The mean white dwarf mass in classical nova systems is predicted to be $\sim 1.1\ M_\odot$.

3. Ejection of accreted material occurs only if the proper pressure at the base of the accreted envelope is greater than a critical value, P_c, which depends strongly on composition but only weakly on white dwarf mass. We find, as did Fujimoto (1982*a*), $P_c \approx 10^{20}$ dyn cm^{-2} for solar abundances and further that $P_c \approx 2 \times 10^{19}$ dyn cm^{-2} for $Z_{\rm CNO} = 0.5$.

4. There exist critical accretion rates and white dwarf luminosities. The critical values F_c and L_c, which both depend strongly on $M_{\rm WD}$ and $Z_{\rm CNO}$ such that both $F < F_c$ and $L_{\rm WD} < L_c$, are necessary for a nova outburst. Accretion rates, deduced from the optical luminosity of the old nova, are found to be appreciably higher than F_c for some novae including RR Pic, CP Lac, and V603 Aql. Ferland *et al.* (1982) have suggested that some of the optical luminosity of postnovae is reprocessed UV radiation from a hot white dwarf. Alternatively, the postoutburst accretion rate may be significantly higher than average.

5. Recurrent novae are not thermonuclear runaways.

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