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EFFECTS OF DRIFT ON THE TRANSPORT OF COSMIC RAYS. VI. A THREE-DIMENSIONAL MODEL INCLUDING DIFFUSION

J. KÓTA[1] AND J. R. JOKIPII
University of Arizona
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ABSTRACT

We present the first results from a series of computer simulations of the solar modulation of galactic cosmic rays using a full three-dimensional model which incorporates all known important effects on particle transport, particle drifts, convection with the solar wind, energy loss, and anisotropic diffusion. The model is time-independent in the coordinate frame rotating with the Sun, so corotating effects can be studied. We consider modulation in an interplanetary magnetic field model in which the current sheet separating the northern and southern solar hemispheres is warped and corotating with the Sun. The amplitude of the warp is varied to simulate possible solar cycle variation of the magnetic field. We find substantial effects due to the warp of the current sheet. Comparison of the model results with various data is presented. Among other things, we show that the intensity may decrease away from the current sheet for both signs of the magnetic field, as suggested by recent observations, and in contrast with inferences from earlier, more approximate calculations.

Subject headings: cosmic rays: general — interplanetary medium — particle acceleration — Sun: solar wind

I. INTRODUCTION

Previous papers in this series have explored the effects of particle drifts on the transport of cosmic rays, with particular emphasis on the effects on the solar modulation of the galactic cosmic-ray flux (Jokipii, Levy, and Hubbard 1977, hereafter Paper I; Isenberg and Jokipii 1978, hereafter Paper II; Jokipii and Kopriva 1979, hereafter Paper III; Jokipii and Thomas 1981, hereafter Paper IV; and Jokipii and Davila 1981, hereafter Paper V).[2] These papers established that for reasonable models of the large-scale structure of the interplanetary magnetic field and for plausible diffusion coefficients, the effects of drifts are of dominant importance in determining the modulation of galactic cosmic rays. Paper III presented the first detailed calculations using a two-dimensional code, and Paper V presented results using the same code for a more realistic set of parameters.

It is now widely believed that the large-scale interplanetary magnetic field has a basic structure in which the fields in the northern and southern hemispheres are Archimedean spirals in which the fields are of opposite sign. The regions of oppositely directed magnetic field are separated by a very thin current sheet which, near solar minimum, lies close to the equatorial plane (see, e.g., Smith, Tsurutani, and Rosenberg 1978, and Thomas and Smith 1981).

In the two-dimensional calculations discussed above, it was necessary to require that this current sheet be in the solar equatorial plane. Hence, although the two-dimensional models provided useful insights and gave quite reasonable results, it is clear that, especially at low latitudes near the ecliptic, the three-dimensional effects of "warping" or "flopping" of the current sheet could be important. This is potentially of great importance in comparing the calculations with observations, since all available observations come from this low-latitude region. Hence, it is imperative to carry out three-dimensional model calculations. An initial calculation, based on a somewhat crude approximation was reported in Paper IV, and some interesting results were obtained. More qualitative analyses have been published by Kóta (1979) and Tverskoi (1981). In Paper IV, it was assumed that the diffusion coefficients were very small, so that a simple calculation which neglected diffusion entirely provided a reasonable approximation. The results of Jokipii and Thomas suggested a large effect of the flopping current sheet and showed some agreement with observations, although they could not explain the more recent observations of Newkirk and Lockwood (1981) in which the intensity increased away from the current sheet for both signs of the interplanetary magnetic field.

The present paper presents the initial results of a program to extend the full two-dimensional calculations, including diffusion, to a three-dimensional computational code which permits a wavy or floppy current sheet to be incorporated. We have completely rewritten the code for the three-dimensional case. The results show a broad similarity to those of Jokipii and Thomas, lending support to the computational approach. However, these more accurate calculations now show general agreement with the observations of Newkirk and Lockwood.

[1] Permanent address: Central Research Institute for Physics, Budapest, Hungary.
[2] The fifth paper in this series, which appeared in *Ap. J.*, **248**, 1156, was incorrectly numbered. It should be considered Paper V.

II. THE MODEL

Inclusion of a wavy current sheet, for a rotating Sun, in general implies a time-dependent problem since the convection with the solar wind will cause crossings of the current sheet for an observer near the equator. However, an important class of models may be studied using a time-dependent code if one works in a coordinate frame rotating with the Sun. Phenomena which are static in this frame include a warped current sheet caused by a tilted magnetic equator at the Sun (this is the configuration studied in Paper IV for example). This is the magnetic field configuration to be reported on in this paper. In principle, phenomena involving corotating interaction regions, etc. could also be studied using our code, but these will be left to later papers. In the fixed frame this assumption is equivalent to requiring

$$\frac{\partial f}{\partial t} + (\boldsymbol{\Omega} \times \boldsymbol{r}) \cdot \nabla f = 0 \, , \tag{1}$$

where $f(p, \boldsymbol{r}, t)$ is the isotropic part of the cosmic-ray distribution in phase space as a function of momentum p, position $\boldsymbol{r}$, and time t. V is the (constant and radial) wind velocity, and Ω is the rotational velocity of the Sun, which will be assumed constant.

In the corotating frame the transport equation may be written

$$\nabla(-K \cdot \nabla f) + V^* \cdot \nabla f = \frac{\mathrm{div}\, V^*}{3} \frac{\partial f}{\partial \ln p} \, , \tag{2}$$

with $V^* = V - \boldsymbol{\Omega} \times \boldsymbol{r}$, where K_{ij} is the full cosmic-ray diffusion tensor including the antisymmetric terms which give the gradient and curvature drifts, as discussed in Paper I.

We found it convenient to replace the azimuth ϕ by $\phi^* = \phi + \Omega r/V$, the azimuth where the local field line connects the Sun. Hence, the new azimuth ϕ^* and the polar angle θ, are constant along a field line. Expressing equation (2) explicitly in terms of r, θ, and ϕ, we arrive at

$$\begin{aligned}
-\frac{2V}{3r}\frac{\partial f}{\partial \ln p} &= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2(K_\parallel \cos^2\psi + K_\perp \sin^2\psi)\frac{\partial f}{\partial r}\right] + 2K_\perp \frac{\Omega}{V}\frac{\partial^2 f}{\partial r \partial \phi^*} \\
&+ \frac{K_\perp}{r^2 \sin^2\theta \cos^2\psi}\frac{\partial^2 f}{\partial \phi^{*2}} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(K_\perp \sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 K_\perp \frac{\Omega}{V}\right)\frac{\partial f}{\partial \phi^*} \\
&- V\frac{\partial f}{\partial r} - \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}(K^A r \sin\theta \sin\psi)\frac{\partial f}{\partial r} - \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\frac{K^A}{\cos\psi}\right)\frac{\partial f}{\partial \phi^*} \\
&+ \frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial r}(K^A r \sin\theta \sin\psi) + \frac{\partial}{\partial \phi^*}\left(\frac{K^A}{\cos\psi}\right)\right]\frac{\partial f}{\partial \theta} \, ,
\end{aligned} \tag{3}$$

with ψ denoting the spiral angle, $\tan\psi = r\Omega \sin\theta/V$. $K_\parallel$ and $K_\perp$ are the parallel and perpendicular diffusion coefficients; $K^A = pcw/3qB$ gives the antisymmetric component of the diffusion tensor for a particle of velocity w and charge q in the magnetic field B (c is the velocity of light) provided that the mean free path is sufficiently larger than the Larmor radius. The terms of equation (3) containing K^A are identical with the drift terms derived in Paper I.

We used the standard Parker spiral magnetic field configuration described in the Introduction which may be written

$$B = \frac{A}{r^2}\left(\hat{e}_r - \frac{r\Omega \sin}{V}\hat{e}_\phi\right)[1 - 2H(\theta - \theta^*)] \tag{4}$$

$$\cot\theta^* = -\tan\alpha \sin\phi^* \, , \tag{5}$$

where a positive A applies for the 1969–1980 period of the solar magnetic cycle. The tilt angle, α, giving the maximum latitudinal excursion of the neutral sheet serves as a measure of "waviness." There is evidence (Thomas and Smith 1981) showing that α is larger at near solar maximum and smaller at near solar minimum. See Paper IV, Fig. 2. Obviously, $\alpha = 0$ corresponds to a flat neutral sheet which was studied in previous papers.

When solving equation (3) we made use of the symmetry $f(\theta, \phi^*) = f(\pi - \theta, \phi^* + \pi)$. To reduce computing time and space requirements involved we considered a small value for the radius of the boundary. Such a model is sufficient to allow an insight into the nature of the effects introduced by the wavy neutral sheet. $f = 0$ was taken at the absorbing inner boundary at $r = 0.1$ AU and the condition $f = f_\infty \propto p^{-1}E^{-3.6}$ (E being the total particle energy) was imposed at the outer boundary placed at 10 AU. The parallel diffusion coefficient, $K_\parallel$, was taken as inversely proportional to the magnetic field strength and a constant ratio of $K_\perp/K_\parallel$ was maintained in the whole heliosphere

$$K_\parallel = K_0 P^{1/2} w/c \frac{|B_{\mathrm{earth}}|}{|B|} \quad \text{and} \quad K_\perp = 0.05 K_\parallel \, ,$$

where P represents particle rigidity in GV. In the calculations we used the value of $K_0 = 5 \times 10^{21}$ cm^2 s^{-1}.

The numerical code was substantially changed from that of Paper III. There, the δ-function singularity of the drift velocity at the neutral sheet was incorporated into a boundary condition. A similar jump condition would have led to several difficulties in the three-dimensional case. Thus, we chose to handle drift explicitly at the neutral sheet, as in the rest of the system. Instead of using an analytical formulation we determined the drift velocities by numerical differentiation, which automatically insures that drift velocity was divergence-free. The numerical differentiation gives finite drift at the neutral sheet; actually, it corresponded to smearing the field transition between the grid points adjacent to the neutral sheet.

Equation (3) was solved by integrating with respect to ln p starting from a sufficiently high initial momentum value. We employed an unconditionally stable, implicit splitting method (Yanenko 1971). Schematically, equation (3) can be written in the form:

$$\frac{\partial f}{\partial \ln p} = \gamma_{rr}\frac{\partial}{\partial r}\left(\alpha_{rr}\frac{\partial f}{\partial r}\right) + 2\gamma_{r\phi^*}\frac{\partial^2 f}{\partial r\partial\phi^*} + \gamma_{\phi^*\phi^*}\frac{\partial^2 f}{\partial\phi^{*2}} + \gamma_{\theta\theta}\frac{\partial}{\partial\theta}\left(\alpha_{\theta\theta}\frac{\partial f}{\partial\theta}\right) + \beta_r\frac{\partial f}{\partial r} + \beta_{\phi^*}\frac{\partial f}{\partial\phi^*} + \beta_\theta\frac{\partial f}{\partial\theta}, \tag{6}$$

where the α, β, γ coefficients depend on momentum and spatial variables. The scheme we used solves this in three steps:

$$r\text{-inversion:}\quad \frac{f^{n+1/3} - f^n}{\Delta \ln p} = \gamma_{rr}\frac{\delta}{\delta r}\left(\alpha_{rr}\frac{\delta f^{n+1/3}}{\delta r}\right) + \beta_r\frac{\delta f^{n+1/3}}{\delta r} + \gamma_{r\phi^*}\frac{\delta^2 f^n}{\delta r\delta\phi^*}$$

$$\phi^*\text{-inversion:}\quad \frac{f^{n+2/3} - f^{n+1/3}}{\Delta \ln p} = \gamma_{\phi^*\phi^*}\frac{\delta^2 f^{n+2/3}}{\delta\phi^{*2}} + \beta_{\phi^*}\frac{\delta f^{n+2/3}}{\delta\phi^*} + \gamma_{r\phi^*}\frac{\delta^2 f^{n+1/3}}{\delta r\delta\phi^*}$$

$$\theta\text{-inversion:}\quad \frac{f^{n+1} - f^{n+2/3}}{\Delta \ln p} = \gamma_{\theta\theta}\frac{\delta}{\delta\theta}\left(\alpha_{\theta\theta}\frac{\delta f^{n+1}}{\delta\theta}\right) + \beta_\theta\frac{\delta f^{n+1}}{\delta\theta},$$

where superscripts refer to the momentum level, all the α, β, γ coefficients are taken at the momentum level $n + 1/2$. First derivatives are approximated by centered differences $\delta/\delta r$, $\delta/\delta\theta$, $\delta/\delta\phi^*$. The second derivative terms are approximated in a straightforward way (as shown, for instance, for the variable, r:

$$\frac{\delta}{\delta r}\left(\alpha\frac{\delta f}{\delta r}\right) = \frac{2}{r_{i+1} - r_{i-1}}\left(\alpha_{i+1/2}\frac{f_{i+1} - f_i}{r_{i+1} - r_i} - \alpha_{i-1/2}\frac{f_i - f_{i-1}}{r_i - r_{i-1}}\right),$$

while the cross derivative term can be written

$$\frac{\delta^2 f}{\delta r\delta\phi^*} = \frac{f_{i+1,k+1} - f_{i+1,k-1} - f_{i-1,k+1} + f_{i-1,k-1}}{(r_{i+1} - r_i)(\phi_{k+1}{}^* - \phi_{k-1}{}^*)},$$

where the subscripts i and k refer to the ith radial and kth azimuthal point. The coefficients $\alpha_{i+1/2}$ and $\alpha_{i-1/2}$ are taken at $r_{i+1/2} = (r_i + r_{i+1})/2$ and $r_{i-1/2} = (r_{i-1} + r_i)/2$, respectively.

III. RESULTS AND DISCUSSION

Numerical calculations were carried out for various values of the tilt angle, α, ranging from 0° to 30°. The results obtained, which will be discussed below, are in general agreement with those of Paper IV, although the inclusion of diffusion leads to quantitative modifications. In agreement with Paper IV, we found that "waviness" introduces much more pronounced effects for $qA < 0$ than it does for $qA > 0$. This charge asymmetry is a direct consequence of drift. In the IMF configuration $A > 0$, as pointed out in Paper IV, positively charged particles penetrate from the polar regions; thus, they are largely insensitive to the IMF structure at low latitude. For $A < 0$, on the other hand, protons propagate inward along the neutral sheet; a warped sheet obviously results in a longer time spent drifting inward, which, in turn, leads to larger deceleration and lower intensity.

a) *Azimuthal Variations at 1 AU*

To illustrate how waviness affects the cosmic ray distribution at a fixed heliocentric distance we plotted the intensity contours of 1.6 GeV and 53 MeV protons. The results are qualitatively the same for various radii from 0.5 to 5.0 AU; here we present (Figs. 1 and 2) typical contours obtained for $\alpha = 30°$ at $r = 1.06$ AU. Intensities are normalized to their interstellar values in each case. The charge asymmetry is obvious. For $qA > 0$ (Figs. 1*a* and 2*a*), intensity contours remain virtually unaffected at latitudes higher than the maximum excursion of the neutral sheet. Proton densities, like the case of a flat neutral sheet (see Papers III and V), increase toward the poles and have minima between the neutral sheet and the solar equatorial plane. This latter differs somewhat from the results of Paper IV, which obtained an intensity minimum right at the neutral sheet. The picture is more complex for the configuration $qA < 0$ (Figs. 1*b* and 2*b*). The longitudinal variation of cosmic ray density, in this case, is noticeable at high latitudes, too. Drift tends to produce a density gradient pointing toward the neutral sheet, while diffusion acts in the opposite way (the mean free path increases toward the poles). As a result, we obtain relatively little intensity variation in latitude, and minima are found at locations far from both the neutral sheet and the poles.

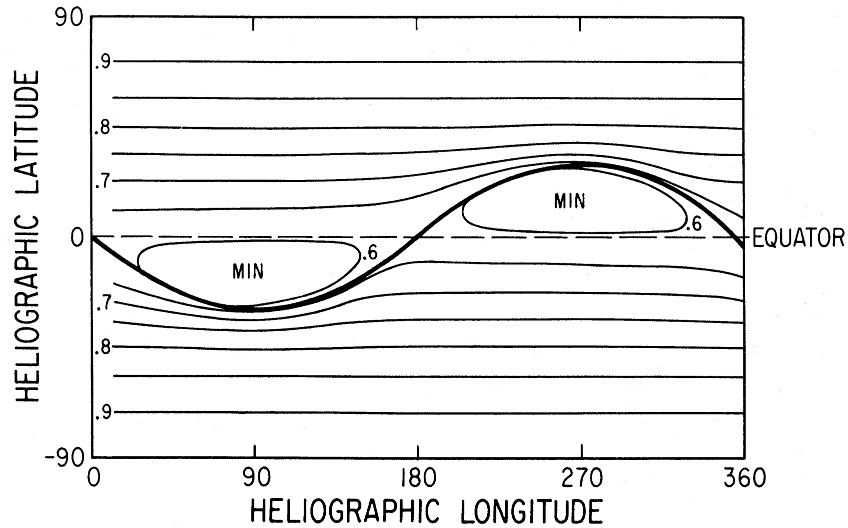

FIG. 1*a*

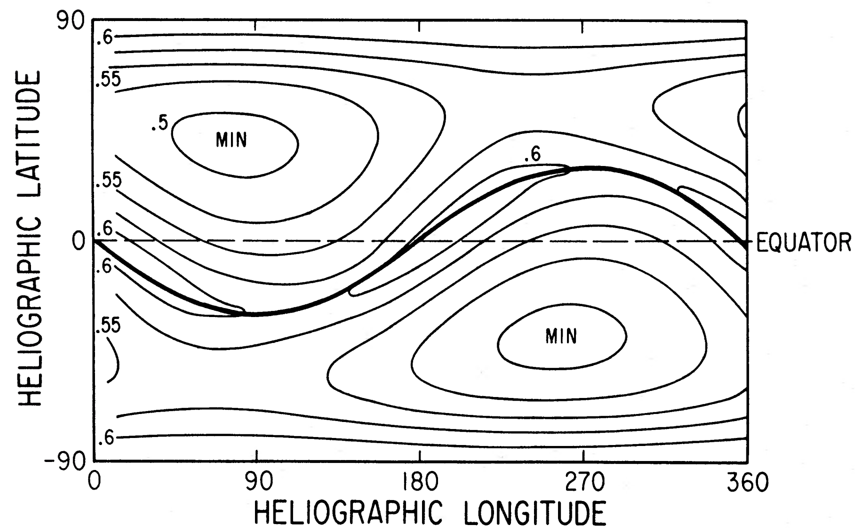

FIG. 1*b*

FIG. 1.—Intensity contours of 1.6 GeV protons at $r = 1.06$ AU as a function of heliographic latitude and longitude, for $qA > 0$ (Fig. 1*a*) and $qA < 0$ (Fig. 1*b*). Normalization is made to the interstellar intensity; heavy lines indicate the position of the neutral sheet. Note that, for $qA > 0$, the intensity maximum is found at the poles but the minima are not at the neutral sheet. For $qA < 0$, maxima are found both at the poles and the neutral sheet.

Recently Newkirk and Lockwood (1981) reported finding a negative correlation between cosmic-ray intensity and the Earth's heliomagnetic latitude (i.e., distance from the neutral sheet) for both halves of the 22 yr cycle. This result is seemingly in contradiction with the expectations of drift model which, for a flat neutral sheet at least, predict a fairly sharp increase of cosmic-ray intensity rising away from the neutral sheet for $qA > 0$. The present three-dimensional work is more appropriate to the situation studied by these authors and allows a direct comparison with their results. Inspection of Figure 1*a* shows that, for $qA > 0$, an observer, constrained to move at a constant latitude several degrees above or below the solar equatorial plane, should, on average, detect a lower cosmic-ray intensity as he moves in longitude away from the current sheet (or as the Sun rotates). This is what is found observationally, but is quite different from that found in our previous two-dimensional simulations or those which neglected diffusion. The negative correlation for $qA < 0$ is obvious from Figure 1*b*. A more detailed comparison and discussion are given in a separate paper (Jokipii and Kóta 1982). Here we merely stress the conclusion that the results of Newkirk and Lockwood (1981) are not inconsistent with a model in which drift is an important effect.

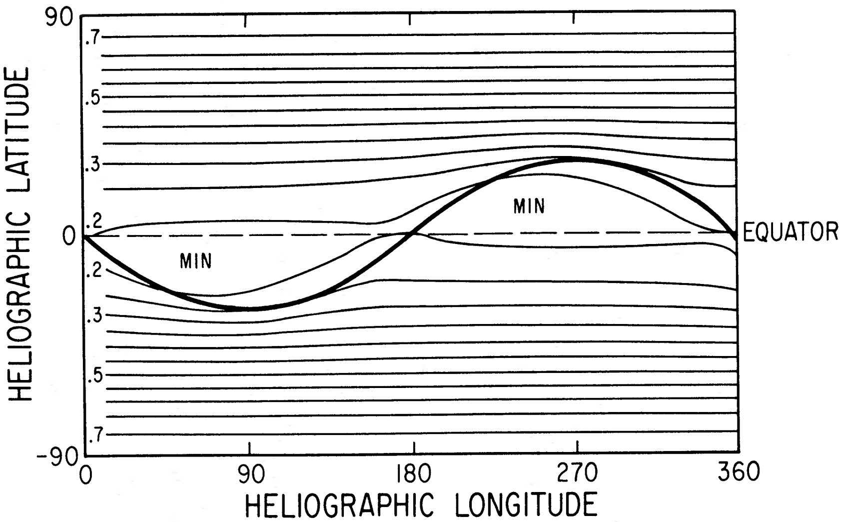

FIG. 2*a*

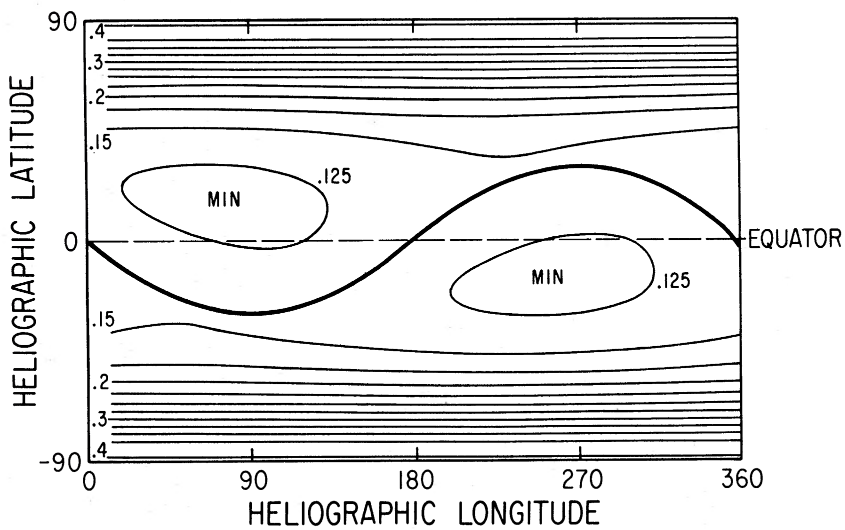

FIG. 2*b*

FIG. 2.—Same as in Fig. 1 for 53 MeV protons

b) *Radial Dependence and Energy Spectrum*

Now we turn to the discussion of the effect of waviness on the energy spectrum and radial dependence of cosmic ray intensity. In this section we consider only longitudinal averages of intensity, which correspond to observational averages obtained during a full solar rotation.

Figures 3*a* and 3*b* show the radial variation of the average proton intensities at various heliographic latitudes for tilt angles $\alpha = 30°$ and $\alpha = 0°$ (*dashed lines*), respectively. We find that waviness, as expected, has little effect for $qA > 0$ at either 1.6 GeV or 53 MeV. In both cases, a small but clear reduction of cosmic ray intensity is obtained in the latitude range 5°–40°. The immediate vicinity of the solar equator represents a somewhat specific area. The sharp transition of intensity taking place near the outer boundary for $\alpha = 0$ is replaced by a more gradual change for $\alpha = 30°$. As a result of this smoothing, waviness may even increase the cosmic-ray density at large radial distances. In the inner heliosphere, on the other hand, the proton density at the solar equator turns to be largely independent of α.

The picture is less transparent for $qA < 0$; curves belonging to various latitudes intersect each other. The effect of waviness is, however, far more significant and intense. At neutron monitor energies (e.g., 1.6 GeV; see Fig. 3*b*), the proton intensity is considerably reduced at all latitudes, implying that a large number of particles filling even the polar regions have still entered the heliosphere at low altitudes and propagated inward along the neutral sheet. At a

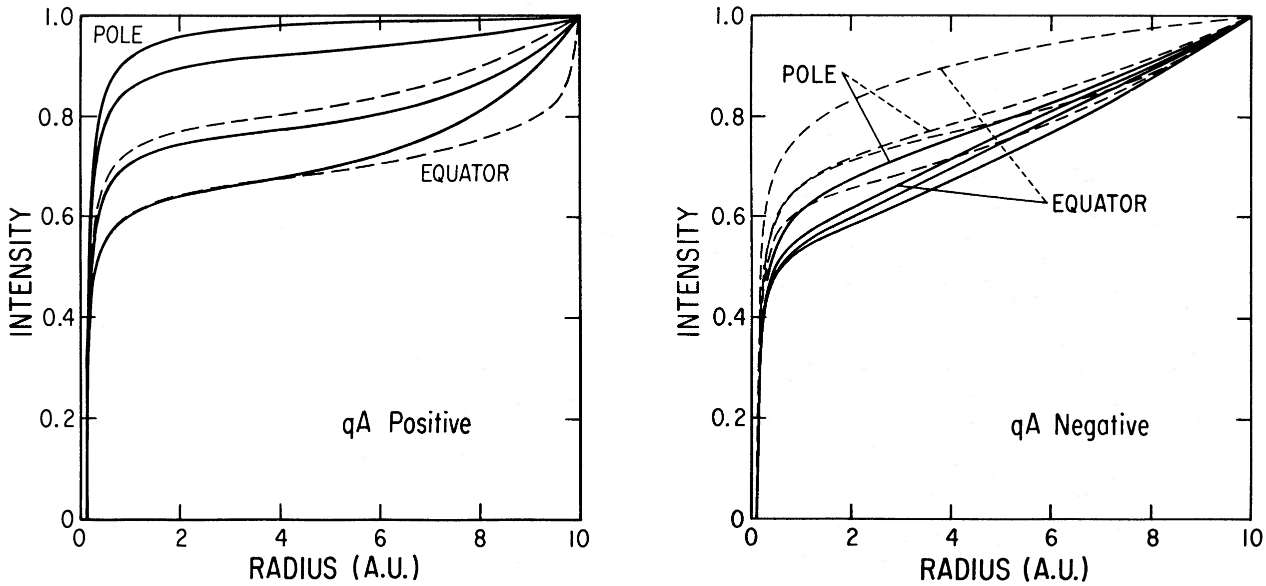

FIG. 3*a* FIG. 3*b*

FIG. 3.—Radial dependence of the computed 27-day averages of 1.6 GeV proton intensities at latitudes 0°, 30°, 60°, and 90°, for $qA > 0$ (Fig. 3*a*) and $qA < 0$ (Fig. 3*b*). Solid lines refer to a wavy sheet with $\alpha = 30°$; dashed lines refer to a flat sheet. Note that for $qA > 0$ the curves for a flat and wavy sheet essentially overlap at the pole and at 60°. For $qA < 0$, the wavy sheet results in an intensity reduction which is largest at the equator and which is still noticeable at the pole.

lower energy of 53 MeV, where drift is less dominant, the effect of waviness is negligible at the poles, but it still remains important at low latitudes.

In Figure 4*a* and 4*b* we present the computed intensity averages of 1.6 GeV protons near the Earth (latitudes 0° and 7°, $r = 1.06$ AU) as a function of tilt angle. Inspection of Figure 5*b* shows that, for $qA < 0$, the cosmic-ray intensity rapidly decreases as the amount of warp is increased. For $qA > 0$, we find a weaker dependence at the latitude 7° (Fig. 4*a*); the decreasing tendency is obvious in this case, too. These results are in good agreement with those of Paper IV and with the predictions of Kóta (1979). The lack of a clear correlation between the tilt angle and cosmic-ray intensity at the solar equator (Fig. 4*a*) confirms the conclusion of Paper IV that "for $+qA$, [virtually] the only effect on increasing the waviness is to change the percentage of times one observes particles which have drifted in from the north or south poles."

The energy spectra were calculated for tilt angles 0° and 30°, respectively. Figure 5 presents the resulting spectra obtained from an observer located at 1.06 AU at the solar equator, averaged over a solar rotation, for $qA < 0$ ($qA > 0$ would give rise to much smaller variation in the spectrum). Waviness is found to significantly reduce cosmic ray flux even at low energies; the ratio of intensities ($\alpha = 30°$ with respect to $\alpha = 0°$) changes from 73% to 63% as the energy decreases from 1.6 GeV to 53 MeV. It is noted that the variation of the spectrum is not as large as that in Paper IV. The difference, most probably, may primarily be attributed to the inclusion of diffusion, but, at the same time, the different parameters used in the two calculations (in particular the smaller distance to the outer boundary in the present work) may play a role, too.

To illustrate the possible effect of waviness on the long-term variation of the cosmic ray flux, as in Paper IV, we constructed a hypothetical model of a 22 yr solar cycle. The tilt angle was assumed to vary from 10° to 30° according to the expression

$$\alpha = a + b \cos(2\pi t/11 \text{ yr}) , \tag{7}$$

with $a = 20°$, $b = 10°$, $t = 0$ at sunspot maximum, and an alternate sign of A was applied for the two halves of the 22 yr cycle. The result shown in Figure 6 is, again, in good qualitative agreement with that of Paper IV; the flat maximum for $qA > 0$ and a larger variation for $qA < 0$ also reflect a most prominent feature of observations (see, e.g., Fig. 7 of Paper IV).

The basic character of the solutions and the quantitative conclusions drawn should remain valid for any reasonably chosen set of input parameters ($K_{\perp}$, $K_{\parallel}$, etc.). When making direct comparison with experimental data, however, one should bear in mind that alteration of these parameters may lead to quantitative changes (in the present work we did not attempt to achieve a good fit to the data, but instead attempted to illustrate the basic phenomena). The directions of the anticipated modifications are obvious or easily predictable in most cases. Solutions for most $qA > 0$ seem largely

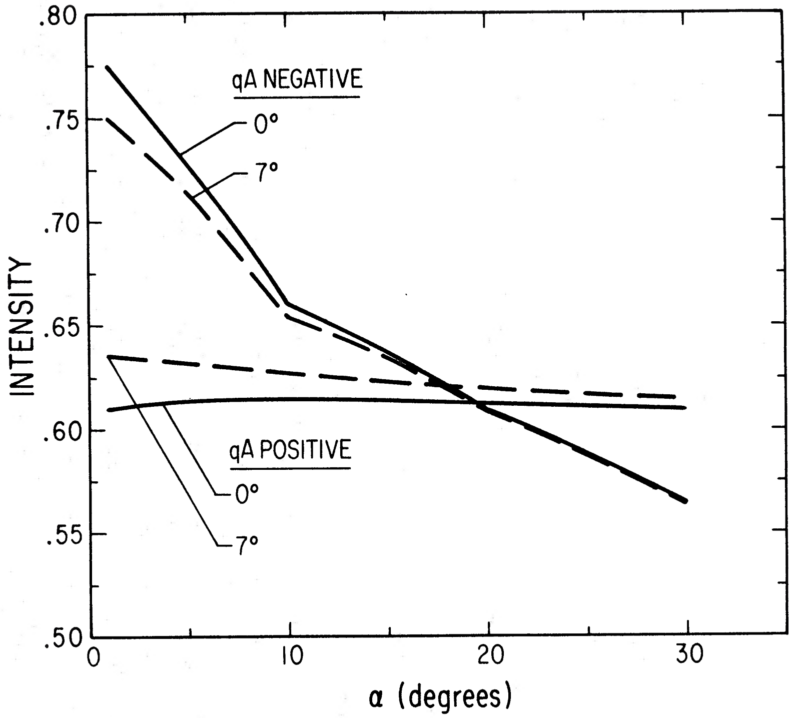

FIG. 4.—Dependence of 1.6 GeV proton intensities on the tilt angle, α, at $r = 1.06$ AU for observers at latitude 0° (*solid lines*) and 7° (*dashed lines*), respectively. For $qA < 0$, the intensity rapidly decreases as the amount of warp is increased. For $qA > 0$, the intensity also decreases, but by a smaller amount, at latitude 7°, while there is little variation at the equator.

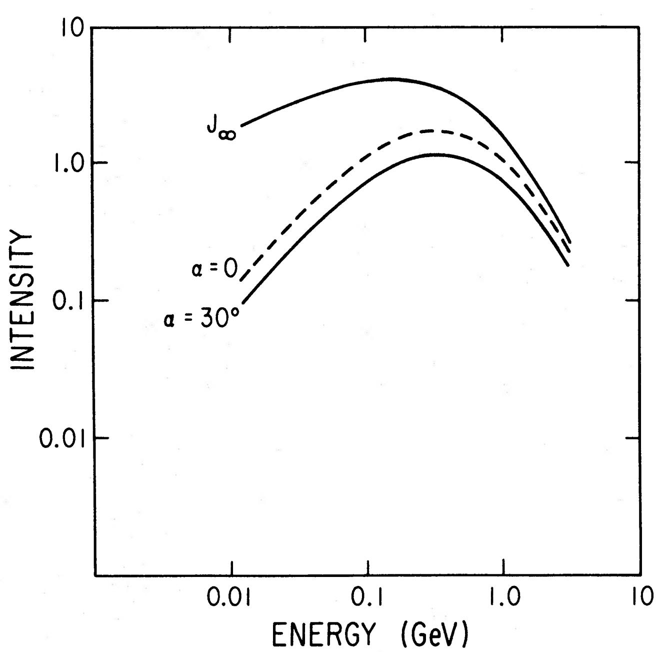

FIG. 5.—Computed energy spectra of protons near the earth for $qA < 0$. Inclusion of a wavy sheet ($\alpha = 30°$, *solid line*) results in lower intensity than does a flat sheet (*dashed line*).

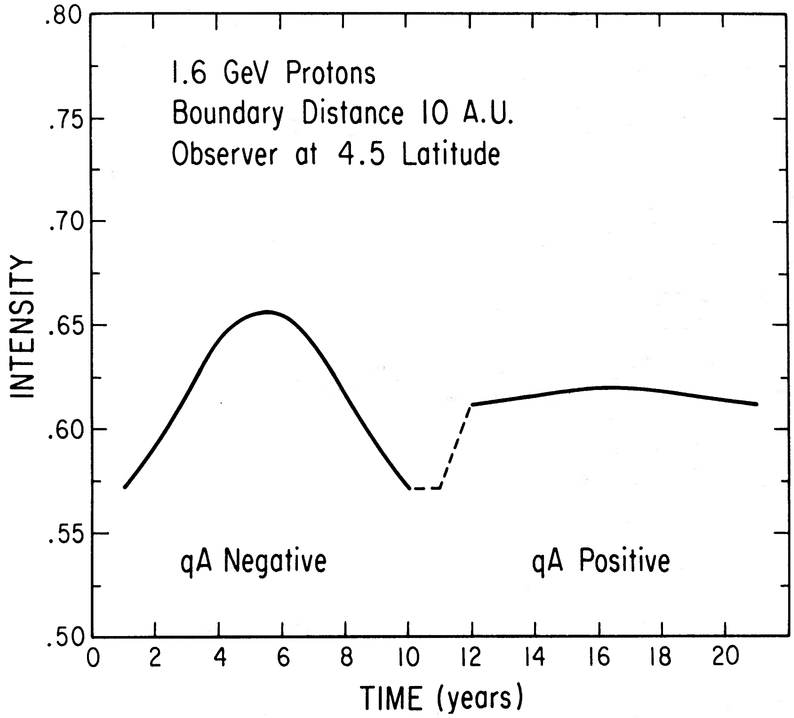

FIG. 6.—Illustration of the variation of 1.6 GeV protons during a hypothetical 22 year solar magnetic cycle

insensitive to the input data. A smaller value of $K_\perp$ and/or $K_\parallel$ or a larger distance of the outer boundary would certainly give a somewhat lower cosmic ray intensity at the Earth (see Paper V); the effect of waviness, however, should remain little. The solutions for $qA < 0$ are more sensitive to the parameters. Placing the outer boundary at a larger distance would obviously result in a lower cosmic-ray flux at 1 AU and a larger effect of waviness, too. Allowing the tilt angle to vary in a wider range would enhance the 11-yr variation of cosmic ray flux. Numerical calculations carried out for a four-sector IMF model show that, for $qA < 0$, cosmic ray flux at the Earth is lower for this configuration that it is for a dipole field with the same latitudinal excursion of the neutral sheet. Similarly, it seems plausible that any additional kind of warp would further lower the near-Earth cosmic-ray flux for $qA < 0$.

IV. CONCLUSIONS

The calculations reported here represent the first study of the steady state modulation of cosmic rays by the solar wind in which all major transport effects—drift, diffusion, convection, and energy loss—are included in a fully three-dimensional model. In general, the results support earlier, less comprehensive calculations which included drifts and, in addition, they show that including diffusion as well as drift produces qualitatively new behavior near the "wavy" current sheet. In particular, we find that the intensity decreases away from the current sheet for both signs of the solar magnetic field, in contrast with the earlier results reported in Paper IV which did not include diffusion.

The observations of Newkirk and Lockwood (1981) which had been suggested as major evidence against drift, are now shown to be a natural consequence of the model if diffusion is included.

Our major conclusion, therefore, is that a model of modulation which incorporates drift together with diffusion in a plausible three-dimensional model heliosphere is in at least qualitative agreement with available observations. In these models, drift plays a dominant role in determining the large scale motions of the cosmic-ray particles, as inferred from earlier calculations in this series.

Future work will extend the range of parameters considered to allow a more quantitative assessment of the degree of agreement with observations, and to further our understanding of the fundamental processes underlying modulation.

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REFERENCES

Isenberg, P. A., and Jokipii, J. R. 1978, *Ap. J.*, **219**, 740 (Paper II).
Jokipii, J. R., and Davila, J. 1981, *Ap. J.*, **248**, 1156 (Paper V).
Jokipii, J. R., and Kopriva, D. A. 1979, *Ap. J.*, **234**, 384 (Paper III).
Jokipii, J. R., Levy, E. H., and Hubbard, W. B. 1977, *Ap. J.*, **213**, 861 (Paper I).
Jokipii, J. R., and Thomas, B. 1981, *Ap. J.*, **243**, 1115 (Paper IV).
Kóta, J., 1979, *Proc. 16th Int. Cosmic Ray Conf.* (Kyoto)., **3**, 23.
Kóta, J., and Jokipii, J. R. 1982, *Geophys. Res. Letters*, **9**, 656.
Newkirk, G., and Lockwood, J. A. 1981, *Geophys. Res. Lett.*, **8**, 619.
Smith, E. J., Tsurutani, B. T., and Rosenberg, R. L., 1978, *J. Geophys. Res.*, **83**, 717.
Thomas, B. T., and Smith, E. J. 1981, *J. Geophys. Res.*, **86**, 11,105.
Tverskoi, B. A. 1981, in *Cosmic Rays in the Heliosphere*, ed. A. J. Somogyi, J. Kóta, and K. Kecskemety (New York: Pergamon), p. 5 (*Adv. Space Res.*, **1**, No. 3, 5).
Yanenko, N. N. 1971, *The Method of Fractional Steps* (Berlin: Springer Verlag), p. 23.

J. R. Jokipii and J. Kóta: Department of Planetary Sciences, University of Arizona, Tucson, AZ 85718