

TESTING SOLAR MODELS WITH GLOBAL SOLAR OSCILLATIONS IN THE 5-MINUTE BAND

ROGER K. ULRICH

Department of Astronomy, University of California, Los Angeles

AND

EDWARD J. RHODES, JR.

Department of Astronomy and Space Sciences Center, University of Southern California

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ABSTRACT

We have computed frequencies of solar oscillation for normal modes described by spherical harmonics $Y_l^m(\theta, \phi)$ with values of l between 0 and 4. The frequencies for the standard solar model differ from the nearest observed frequencies by 1 to 15 μHz . Uncertainties in the interior physics, including nuclear cross sections, equation of state, and opacities, produce an uncertainty in the calculated frequencies of about 1 μHz . Because of uncertainties in the outer boundary condition, in the physics of the superadiabatic layer of the convection zone, and in the structure of the solar chromosphere and corona, we cannot rule out change in the frequencies by as much as 10 μHz ; however, modifications in the model producing such a change will also change the spacing between the frequencies of successive eigenmodes. Thus, an error in the outer layers could produce agreement for one mode only, and the remaining modes would disagree by more than the theoretical uncertainty of 1 μHz . We compare our frequencies to those of three independent groups and find substantial agreement, although the differences between our frequencies and theirs are larger than our estimated uncertainty. The additional range of the results of other investigators can be attributed to the use of a less accurate equation of state, less accurate opacities, or an inadequate number of mesh points. Even with these additional factors, no standard model yet computed is in agreement with the observations. We conclude that the discrepancy between the theoretical and observed frequencies represents a real failure of the standard solar model of significance comparable to the failure to predict the correct neutrino flux.

We have examined four nonstandard solar models: (1) lowered interior abundance of heavy elements; (2) increased interior abundance of heavy elements; (3) the interior hydrogen abundance distributions altered by mixing; and (4) a large relic magnetic field of the Cowling type. We find that the derived frequencies do not match the observed frequencies for any of the models.

Subject headings: stars: pulsation — Sun: atmospheric motions — Sun: interior

I. INTRODUCTION

The 5 minute oscillations have been observed and analyzed for the past 20 years following their discovery by Leighton, Noyes, and Simon (1962). The general line of investigation which has yielded results permitting the deduction of the solar interior structure began with the suggestion by Ulrich (1970a) that these oscillations are acoustic normal modes trapped below the solar surface. Wolff (1972) and Ando and Osaki (1975) subsequently suggested that the modes could be global p -modes.

The frequencies of solar oscillations can be used as diagnostic tools to probe the structure of the Sun. The precision of the frequencies determines the quality of the constraints which can be placed on the solar model. The oscillations can be regarded as standing waves trapped within the solar interior. The frequencies can be defined well only if the trapped wave is coherent and long-lived. In a quiescent medium with little dissipation the waves could be highly coherent, and the frequencies could be extremely well defined. The Sun has both a

turbulent convection zone and radiative damping. The damping process governs the width of the spectral lines which sets an ultimate limit on the precision to which the frequencies can be measured. The irregularities of the convection zone could cause shifts in the frequencies and increase the line width. Beyond some level of disorder, the modal character of the oscillations could be lost, and with the loss of the modal character would go the possibility of probing solar structure. Some preliminary theoretical discussions of these issues have been presented by Deubner, Ulrich, and Rhodes (1979), Gough (1980), and Gough and Toomre (1982). The complexity of the processes makes a definitive analysis difficult, and we rely on observations to indicate the degree of coherence of the oscillation modes.

The recent observations from the South Pole by Grec, Fossat, and Pomerantz (1980), from the Canary Islands by Claverie *et al.* (1981), and from the *Solar Maximum Mission* by Woodard and Hudson (1982) show that the frequencies of the largest scale solar oscillations are so

sharp that the motions must be globally coherent. The oscillations have been tentatively identified on the basis of their frequencies to be high overtones of the radial, dipole, quadrupole, and octopole modes. Note that the dipole mode is possible only in a compressible medium and involves the motion of the solar center. The overtone character means that there are nodes in the motion on spherical surfaces at various distances from the solar center. The number of nodal surfaces is commonly used along with the angular dependence of the motion to classify the modes of oscillation. Thus, a quadrupole mode with 17 nodal surfaces has $l = 2$ and $n = 17$. The solar motions must be well described by these eigenmodes in order for the frequencies to have the observed sharpness.

In contrast, work by Deubner (1975) and Rhodes, Ulrich, and Simon (1977) indicated the modal character of the oscillations of the modes with a highly nonradial character but did not establish that the modes are globally coherent. Indeed, such coherence has not yet been established for these modes. For incoherent oscillations which are limited to shallow layers, it is useful to think in terms of the horizontal wavelength λ and wave numbers k_x and k_y . Globally coherent modes, in contrast, should be thought of in terms of the spherical harmonics $Y_l^m(\theta, \phi)$ which describe the distribution of the vertical velocity over the spherical surfaces. Coherence in a vertical direction is all that is required to produce the ridge structure observed in the k - ω diagram. Each ridge is the locus of eigenfrequencies for a fixed value of n . Global coherence should further cause the power along each ridge to break up into a series of discrete points. Prior to 1979, observations of the oscillations were confined to wave numbers $k = (k_x^2 + k_y^2)^{1/2} \geq 0.2 \text{ Mm}^{-1}$ ($1 \text{ Mm} = 10^6 \text{ m}$). The wave-number and l are related by $k = [l(l+1)]^{1/2}/R_\odot$, so that the above limit on k corresponds to $l \gtrsim 140$. The resolution of the ridges into peaks corresponding to successive l values would require a nearly continuous observing run in excess of about 30 hr, but no spatially resolved observations of this duration have yet been published.

The analysis of the high l -modes by Rhodes, Ulrich, and Simon (1977) indicated that the power per mode is a function of frequency only. This result applied to the ridges with two or more radial eigenfunction nodes, while the fundamental mode (the one having no nodes) and first overtone (the one having one node) seemed to have somewhat less power per mode. The early work by Tanenbaum *et al.* (1969) in which the oscillatory motion was observed through fixed apertures of varying size showed that the amplitude was inversely proportional to the square root of the diameter of the aperture. An incoherent superposition of all modes having a wavelength larger than the radius of the observational aperture would lead to the observed behavior of the oscillatory amplitude. Thus, the amplitude per mode may be largely independent of l for all values of l . The observations by Grec, Fossat, and Pomerantz (1980) of modes with $l = 0$ to 4 and the

analysis by Christensen-Dalsgaard and Gough (1982) show that the power per mode is roughly uniform over l values less than 150.

The type of observing system developed by Isaak (1961), Fossat and Roddier (1971), and Grec, Fossat, and Vernin (1976), and used by both Grec, Fossat, and Pomerantz (1980) and Claverie *et al.* (1979, 1981), provides a stable and sensitive measurement of the velocity of the full solar disk. The frequencies for the detectable modes ($l = 0$ to 4) have been precisely measured ($\nu/\Delta\nu \approx 1000$). The observed frequencies are in the range of 2300 to 4000 μHz . The frequency spacing is not perfectly uniform, and the detailed set of frequencies has been used for a tentative identification with specific modes of pulsation. For example, a frequency given by Claverie *et al.* (1981) as 2496 μHz is identified as the $n = 17$ overtone of the $l = 0$ spherical harmonic. The choice of $n = 17$ for this identification is made strictly on the basis of comparison to theoretical calculations and could conceivably be incorrect. However, the discrepancy would be of order 140 μHz instead of being on the order of 10 μHz if the mode were reidentified as the $n = 16$ or 18 overtone of the $l = 0$ spherical harmonic. In principle, a purely observational determination could be made by tracing the ridges from the $l = 500$ part of the (l, ν) -plane down to $l = 0$. Observational data permitting the resolution of the modes in the range $4 \lesssim l \lesssim 200$ are not yet available.

Based on the above identification, the comparison of the observed-to-theoretical eigenfrequencies provides a test of the theoretical model of unusual stringency. We will carry out this comparison in § III. Unfortunately, the $l = 0$ to 4 eigenfunctions penetrate throughout the solar interior, while the $l > 200$ eigenfunctions are confined to the outer part of the convective envelope. Consequently, our comparison between theoretical and observed frequencies will not permit us to localize the cause for any discrepancy beyond the statement that it is located either above or below the convective envelope. We will also show that intermediate l -modes could provide the necessary information to permit the development of a more definitive interpretation.

We discuss in this paper the method we use to compute eigenfrequencies and examine some of the uncertainties in the theory. We show that the standard solar model has eigenfrequencies which do not agree with the frequencies observed for the low l -modes to within the estimated accuracy of either the observed or theoretical frequencies. Four nonstandard models are examined: (1) the interior Z abundance is lower than the surface abundance; (2) the interior Z abundance is higher than the surface abundance; (3) the interior hydrogen abundance is altered by mixing; and (4) a large primordial magnetic field remains in the solar core. The effect of all these models on the solar neutrino flux is considered, with the result that the high- Z model is rejected. Our findings on the effect of a magnetic field differ from those of Bahcall and Ulrich (1971) wherein it was concluded that a primordial magnetic field

increases the neutrino flux. Differences between our approach and that of Bahcall and Ulrich (1971) will be discussed in § VI*d*. The magnetic field models take on added interest with the suggestions by Dicke (1981) and Isaak (1982) that the Sun has a rotating oblique magnetic field in its core.

II. SOLAR MODEL CALCULATION METHODS AND UNCERTAINTIES

Because of the high precision available for the observed solar frequencies, we have attempted to address any part of the theory which seems likely to cause a change in the calculated frequency in excess of 0.1%. This is a goal clearly beyond normal astrophysical accuracy and has required the modification of many well-entrenched habits. Many of the uncertainties in the solar models have been reviewed recently by Bahcall *et al.* (1982) for the purpose of determining the error in the prediction of the solar neutrino flux. Many of these uncertainties also apply to the calculation of the eigenfrequencies, although with differing importance.

The parameters for which we consider uncertainties are

- 1) The equation of state;
- 2) The cross section for the ${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma$ reaction;
- 3) The opacity;
- 4) The solar radius;
- 5) The solar luminosity.

The only uncertainty in the equation of state is due to the extra pressure of scattering states. We include a model without this term as a measure of the equation-of-state error. Although there are uncertainties in all the nuclear reaction cross sections, we have taken the case of the ${}^3\text{He}$, ${}^4\text{He}$ reaction as a representative example. We would expect the frequencies to be more sensitive to the p - p cross section, but the uncertainty is smaller than for the ${}^3\text{He}$, ${}^4\text{He}$ cross section. Since the estimated error in the opacity according to Huebner (1978) is at the 20% level, we have multiplied the opacity by 1.20 in order to estimate the uncertainty due to opacity errors. The solar radius is a more important parameter for this calculation than for the solar neutrino problem. Some care is needed here, since the measured solar diameter corresponds roughly to the distance between opposite $\tau(\text{tangential}) = 1$ points, whereas the point normally considered to define the solar surface is at $\tau(\text{normal}) = \frac{2}{3}$. We have carried out the tangential integration and find that it occurs at $\tau(\text{normal}) = 10^{-3}$. The distance between these two depths is 0.06% of a solar radius. Thus, it is just below the threshold of concern, but not enough below to be neglected entirely. The uncertainty in the measured radius according to Allen (1973) is 0.02%. We have not considered the uncertainty in the solar age, since we anticipate its effect to be very small in this context.

Beyond the uncertainties in defining the solar model, there are uncertainties in the eigenfrequencies which are present in the pulsation theory. Although not fundamental, one of the first concerns is that the eigen-

functions be adequately represented by the numerical method employed. This translates into a requirement on the number of zones in the model and is dependent on the integration techniques. We have used a fourth-order Runge-Kutta algorithm with 350 to 400 mass zones. Our first models included 200 mass zones, and frequencies proved sensitive to the number of zones. We chose 350 to 400 zones because the frequencies resulting from these models did not differ from those for a test case with 523 zones. More fundamental problems arise because of the radiative intereactions and the possibility that magnetic fields modify the outer boundary conditions. Although we do not believe that it represents a good approximation, we have used the Ulrich (1970*a*) formulation to provide an indication of the changes resulting from the radiative interactions. Somewhat improved formulations, such as those due to Unno (1965) or Ulrich (1970*b*), would considerably increase the cost of the computations, and these approaches are still less than exact. The radiative interaction process is one of the several processes which introduce uncertainties at the outer boundary. The boundary conditions discussed below in § IV are another source of uncertainty. Details of convection, such as turbulent viscosity and the exact distribution of the temperature gradient in the superadiabatic zone, are yet another area of uncertainty. As long as the uncertainties are confined to a thin surface layer, they have very similar effects on the frequencies. The pattern of frequency shifts possible because of these surface uncertainties will be discussed in detail in § IV. In principle, these effects could play an important role in the comparison of the theoretical and observed frequencies, but in practice the surface effects do not produce important uncertainties.

The method of calculating the solar model is that described by Ulrich (1982). We have used the equation of state described in that paper, nuclear cross sections from Bahcall *et al.* (1982), and opacities from the same source. Because our opacity table is configured to follow the ρ - T distribution within the sun, the problems of interpolation with a complicated algorithm do not arise for us. As indicated by Ulrich (1982), we are using the hydrogen mass fraction distribution scaled from the standard solar model of Bahcall *et al.* (1982) instead of following the evolution in detail. Our interpolation scheme utilizes the fact that the integral of the solar luminosity over the solar lifetime provides a strong constraint on the total consumption of hydrogen. The distribution of the fractional consumption over the solar interior should be governed by the ratio of the temperature at each mass shell to the central temperature. Based on this idea, we use the ratio of T to T_{central} as an interpolation parameter to locate the appropriate value of X in the standard model. The overall variations in X which are required to achieve a model with the correct luminosity are obtained by adding a constant change in X to all zones in the model. Although this method is not perfect, it is exact for the standard model and should be very good for models which differ only slightly from the standard model. This

method has the virtue of providing very accurate control of L and R , whereas numerical problems sometimes make it difficult to calculate an evolutionary sequence which ends up precisely on the correct value of L and R .

III. EIGENFREQUENCY CALCULATION

Although the equations of motion in their linearized adiabatic oscillation form are standard (e.g., Ledoux and Walraven 1958; Christensen-Dalsgaard, Dilke, and Gough 1974; Ando and Osaki 1975; Unno, Osaki, and Shibahashi 1979), we record here the form in which they are coded for the calculations reported in this series of papers (Ulrich 1970a; Ulrich and Rhodes 1977; Ulrich, Rhodes, and Deubner 1979; Lubow, Rhodes, and Ulrich 1980). The formulation is nonstandard because in 1968 Ulrich approached the problem as an extension of the theory by Lamb (1932) of long acoustic waves in an atmosphere rather than as an application of the theory of pulsating stars. Rather than change the basic code, it has been easier to correct the terms which break down for the largest scale modes. The important effects added since 1970 are the sphericity correction to the continuity equation (Ulrich and Rhodes 1977), the gravitational potential perturbation (this paper), and a modification to the Brunt-Vaisala frequency due to the chemical composition variation (this paper). The equations of motion are written in terms of the variables v_r , the radial velocity; P' , the Eulerian perturbation in the pressure (i.e., fixed in space rather than following the motion); and ρ' , the Eulerian perturbation in the density. The time dependence is assumed to have the form $e^{-i\omega t}$, where ω is related to the frequency ν by $\omega = 2\pi\nu$, and the velocity is replaced by a displacement-like variable J defined by

$$\rho v_r = i\omega J. \quad (1)$$

The radial variable, r , is replaced with a depth variable z , where

$$z = R - r, \quad (2)$$

and R is the radius of the outermost point of the model, which was equal to $1.05 R_\odot$ for these calculations. The Eulerian gravitational potential perturbation is Φ' , and we use its derivative, η , as one of the integration variables:

$$\frac{\partial \Phi'}{\partial r} = -\eta. \quad (3)$$

After separating the angular dependence and eliminating v_θ and v_ϕ in the usual way, the equations for the r or z dependence of P' , J , Φ' , and η can be written:

$$\frac{dP'}{dz} = g\rho' + \omega^2 J - \rho\eta, \quad (4)$$

$$\frac{dJ}{dz} = -\rho' + \frac{l(l+1)}{r^2} P' + \frac{\rho l(l+1)}{r^2} \Phi' + 2\frac{J}{r}, \quad (5)$$

$$\frac{d\Phi'}{dz} = -\eta, \quad (6)$$

$$\frac{d\eta}{dz} = \frac{2\eta}{r} + \frac{l(l+1)}{r^2} \Phi' + 4\pi G\rho', \quad (7)$$

$$\left(1 + \frac{i\omega_R}{\omega}\right)\rho' = -\frac{N^2 J}{g} + \left(1 + \frac{i\gamma\omega_R}{\omega}\right)\frac{P'}{c^2} \quad (8)$$

Equation (8) contains four quantities not previously defined. These are the radiative interaction rate ω_R which has been taken from Ulrich (1970a), the adiabatic exponent $\gamma = (\partial \ln P / \partial \ln \rho)_S$, the sound speed squared $c^2 = \gamma P / \rho$, and the Brunt-Vaisala gravitational frequency, N , which is evaluated from

$$\frac{N^2}{g} = \frac{1}{H_\rho} - \frac{1}{\gamma H_P}, \quad (9)$$

where H_ρ and H_P are the density and pressure scale heights. The form

$$\frac{N^2}{g} = \frac{Q}{H_P} (\nabla_{\text{ad}} - \nabla) \quad (10)$$

used in prior work in this series is thermodynamically identical to equation (9) as long as the composition is uniform. In the presence of a chemical gradient, equation (9) is correct. In equation (10), $Q = -(\partial \ln \rho / \partial \ln T)_P$. Equation (10) is still used in the convection zone where the composition is uniform and $\nabla - \nabla_{\text{ad}}$ is very small.

Equations (4)–(8) supplemented by boundary conditions define an eigenvalue problem for ω . For the outer boundary conditions we have set

$$J = 0, \quad (11)$$

and

$$\eta = (l+1)\frac{\Phi'}{r}, \quad (12)$$

at $r = 1.003 R_\odot$, which is the base of the corona. The outer boundary condition can alter the value of ω . This point will be discussed in more detail in § IV. The inner boundary condition on Φ' is more troublesome. Ledoux and Walraven (1958) give an equation which is ill defined. Christensen-Dalsgaard, Dilke, and Gough (1974) use an expansion which gives a relationship between η and Φ' . They have retained terms of lowest order in r and of next highest order in r . We have retained only the lowest order terms in r and extended the model analytically to $r = 4 \times 10^7$ cm where the higher order terms are certainly negligible. At this point Φ' and η are related by

$$\eta = -l\frac{\Phi'}{r}. \quad (13)$$

The amplitude of η and Φ' relative to J and P' is adjusted through iteration until the outer boundary condition is satisfied. A first guess is obtained by equating the next higher terms in the expansion of Φ' . Operationally, this

is done by writing the inner boundary conditions for $l > 0$ as

$$P' = - \left[\frac{r\omega^2}{l} + \frac{2\pi GR_{\odot}^2}{2l+3} \left(\frac{\omega^2}{lc^2} + \frac{N^2}{gr} \right) \rho r x \right] J, \quad (14)$$

$$\Phi' = \frac{2\pi GR_{\odot}^2}{2l+3} \left(\frac{\omega^2}{lc^2} + \frac{N^2}{gr} \right) r x J, \quad (15)$$

with η given by equation (13). For $l = 0$, the conditions are slightly different, viz.,

$$P' = \frac{3c^2}{r} J, \quad (16)$$

$$\Phi' = \frac{2\pi GR_{\odot}^2 x}{r}, \quad (17)$$

$$\eta = -4\pi GJ. \quad (18)$$

In Equations (14)–(17), x is a dimensionless scale factor which is adjusted iteratively. The typical value for an eigenfunction is 10^{-2} .

The above eigenvalue problem is solved with a shooting method. Values of ω and x are guessed, and a fourth-order Runge-Kutta integration is carried out from the center to the surface. The fourth-order accuracy makes it possible to use less than 400 points in the model and obtain results insensitive to the number of points. An error function based on the outer boundary condition is generated, and two additional integrations for small alterations in ω and x are carried out. The guesses of ω and x are then improved by a Newton-Raphson method, and a new sequence begun. The procedure typically converges to one part in 10^6 within five iterations. Because the code can handle only two variables, at present either ω_R is set to zero, or Φ' is set to zero. A third eigenfrequency is calculated with both ω_R and Φ' equal to zero, and the effects are assumed to be additive. Since the treatment of ω_R is highly approximate, this approach should be regarded as giving only an indication of the shift in ω due to the radiative interaction processes.

IV. THE INFLUENCE OF THE OUTER BOUNDARY CONDITION

The outer boundary condition has been applied near the photosphere (Iben and Mahaffy 1976; Christensen-Dalsgaard, Gough, and Morgan 1979), near the temperature minimum (Christensen-Dalsgaard and Gough 1980; Shibahashi and Osaki 1981; Ando and Osaki 1975), and at the top of the chromosphere (Scuflaire, Gabriels, and Noels 1981). Except for the first paper of this series (Ulrich 1970a), we have always applied the boundary condition near the base of the corona. Hill (1978) has argued that the boundary condition at the temperature minimum is uncertain and that an incoming wave condition should be applied. The physical uncertainties which lead to ambiguity about the boundary condition are the possible presence of a magnetic field and the existence of inhomogeneities. Both become potentially

important above the temperature minimum. The waves could also become nonlinear in the upper atmosphere, but there is no observational evidence that this is so (Athay and White 1979). Without any complications, the correct procedure is to apply the boundary condition so high that the frequencies are independent of the form of the condition as long as it requires there to be no divergence in the eigenfunction. Our standard outer boundary condition for this study is to set $J = 0$ at an altitude of 2500 km above the photosphere. This point is just below the temperature rise to the 20,000 K chromospheric plateau. We choose this point because it artificially shifts the chromospheric mode frequency out of the band of interest. The avoided crossing behavior discussed by Ulrich and Rhodes (1977) and Ando and Osaki (1977) can cause numerical difficulties when the interior and chromospheric modes accidentally have the same frequency. We have replaced our standard boundary condition with an outgoing wave boundary condition at $r = 1.05 R_{\odot}$ and find that the frequencies of interest change by less than $0.01 \mu\text{Hz}$. Thus, we can safely use the $J = 0$ condition as a standard.

Physical conditions in the chromosphere are incompletely understood. The region is inhomogeneous with spicules and plages, providing evidence of density and temperature variations. Magnetic field structures are rooted in the chromosphere, and the general field strength of a few gauss is large enough that magnetic forces can play a role in the hydrodynamics. The gas is not in local thermodynamic equilibrium, so that such quantities as the adiabatic sound speed should not be calculated from the usual thermodynamic formulae. We are not prepared to deal with these complications directly, but rather we will estimate the uncertainty in the eigenfrequencies due to the chromospheric complications by applying several outer boundary conditions at lower levels of the chromosphere. The complications which presumably are exterior to the outer boundary will then influence the eigenfrequencies by shifting the eigenfunction toward one boundary condition or another.

In accordance with the above philosophy, we have considered three outer boundary conditions: (1) an outgoing wave, (2) $J = 0$, and (3) $P' = 0$. The first is the best available if the unperturbed model is truncated prematurely and no exterior complications are present. The second represents a strong horizontal magnetic field which provides a stiff upper boundary to the motion, and the third condition represents the incursion of a low density medium into the low chromospheric regions. A vertical magnetic field has little impact, since it confines the motion to being parallel to the field, and the low-order modes already are almost precisely vertical. These boundary conditions are not intended to represent the physical models precisely.

Figure 1 gives the change in the frequency of the $l = 0$, $n = 17$ mode for the three boundary conditions as a function of the altitude of application. Clearly, the conditions cannot be applied near the photosphere without producing a frequency change which is larger than our

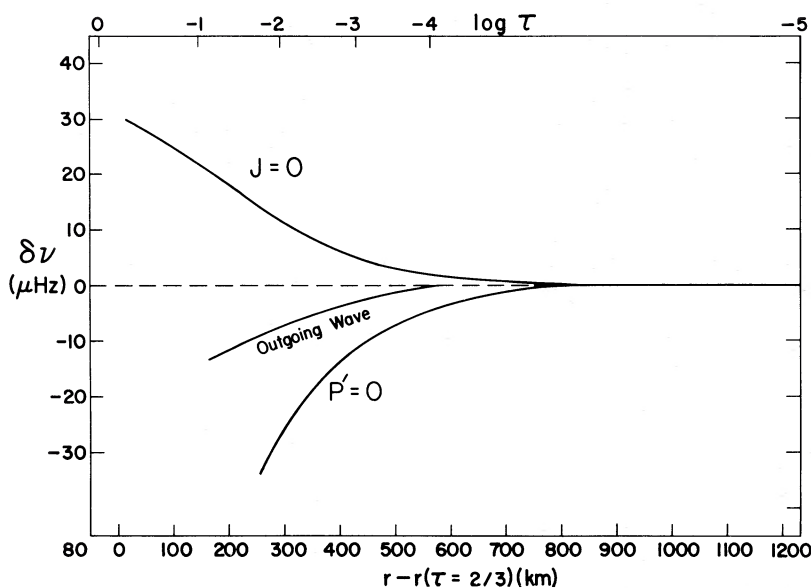


FIG. 1.—Variation in the frequency of the $n = 17$, $l = 0$ mode as a result of variations in the outer boundary condition. We normally applied our boundary condition at an altitude of 2330 km where the frequency change would be unplotable on the scale of this figure.

desired accuracy. This result is in good agreement with the findings of Christensen-Dalsgaard and Gough (1980). We believe that the frequencies calculated by Iben and Mahaffy (1976) were influenced by their outer boundary condition and cannot be readily compared to frequencies found by subsequent investigators. We believe that the application of the boundary condition at $\tau = 10^{-3}$ is only marginally acceptable. Variations in the frequencies at a level of $5 \mu\text{Hz}$ can be produced by the details of the outer boundary condition when it is applied at this point of the atmosphere. This difference would be noticeable but not significant. Unless there is some model requiring the equations of motion to be altered below $\tau = 10^{-3}$, the outer boundary condition should not be applied that low. We also feel that there is nothing in standard chromospheric models such as those of Vernazza, Avrett, and Loser (1981) to require such a modification of the equations of motion. Consequently, we feel that the outer boundary condition does not contribute to the uncertainty in the eigenfrequencies above the $5 \mu\text{Hz}$ level.

The spacing between the modes is also sensitive to the outer boundary condition. For frequencies in the range of interest, the spacing change is proportional to the frequency change over the range of optical depths shown in Figure 1. We write

$$\delta(v_{n+1} - v_n) = a_n \delta v_n. \quad (19)$$

The outgoing wave boundary condition yields $a_{17} = 0.1$, while the $J = 0$ and $P' = 0$ boundary conditions yield $a_{17} = 0.18$. A slope 0.18 is shown as a dashed line in Figure 2. The simultaneous use of the frequencies and their spacing permits us to distinguish between a boundary condition error and an error or defect in the interior model.

V. THE STANDARD MODEL AND ITS UNCERTAINTY

Our standard model is number 22 discussed by Ulrich (1982). This has the most complete equation of state and the best values of other parameters as discussed by Bahcall *et al.* (1982). It has a total of 363 mass zones

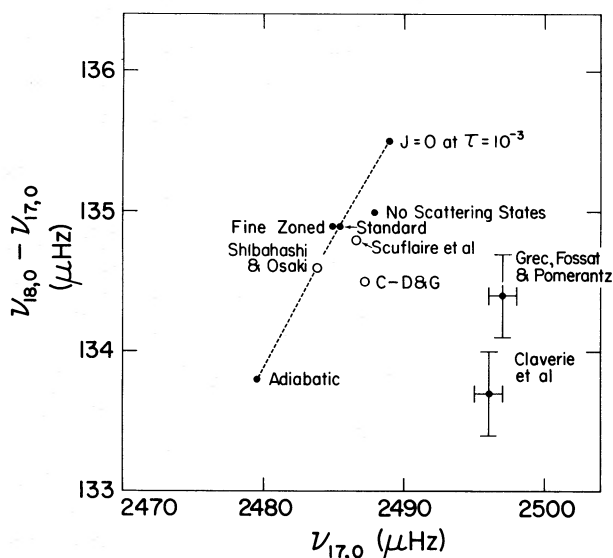


FIG. 2.—Comparison of frequencies and frequency spacing at $n = 17$. Observed points are shown with error bars; our points are shown as filled circles, and other investigators' results are shown as open circles. C-D & G stands for Christensen-Dalsgaard and Gough 1980. Error bars for the observed points are estimated from the half-width of the frequency spikes and from the variation in frequency spacing between $n = 15$ and $n = 20$. The points marked $J = 0$ at $\tau = 10^{-3}$ and adiabatic are identical to the standard model result, except for the treatment of the outer boundary condition or the treatment of the radiative terms.

TABLE 1
ERRORS IN EIGENFREQUENCIES^a

Source of Error	$\delta\nu_{17,0}$	$\delta(\nu_{18,0} - \nu_{17,0})$	$\delta\nu_{22,0}$	$\delta(\nu_{23,0} - \nu_{22,0})$
Radius error (0.01%)	0.20	0.01	0.24	0.01
State error (20% of scattering states)	0.52	0.02	0.50	0.06
Luminosity error (0.5%)	0.07	0.02	0.20	0.04
S_{34} (factor of 2 change)	0.21	0.00	0.36	0.01
Opacity error (20%)	0.90	0.34	0.50	0.28
Overall uncertainty	1.1	0.34	0.9	0.29

^a All values given in μHz .

which provides at least 12 mass zones for all radial wavelengths of the eigenmodes. The possible error due to inadequate zoning was evaluated by making a test calculation with a model having 523 zones, where all radial wavelengths have at least 20 mass zones. The frequencies for the fine-zoned model were 0.2 to 0.3 μHz higher than the standard model.

The impact of the other uncertainties discussed in § II is given in Table 1. The overall uncertainty is 0.9 to 1.1 μHz in the frequencies and one-third this uncertainty in the frequency spacing. These errors are well within our desired tolerance and are small enough to permit meaningful comparison of our theory with the observations. Christensen-Dalsgaard (1982) has given a larger estimate in the total error in the eigenfrequencies than we find because he used a less consistent set of opacities than we have, and he consequently overestimated the uncertainty in the opacity values. Table 2 gives our calculated frequencies for the standard model for selected

modes in the 5 minute band where the observed power is concentrated. These frequencies include the effects of radiative interaction and are, for the solar radius at $\tau = 10^{-3}$, equal to 6.960×10^{10} cm.

Figures 2 and 3 show frequency versus frequency-spacing diagrams for the $l = 0$, $n = 17$ and 18 modes and for the $l = 0$, $n = 22$ and 23 modes. These diagrams compare our results to the results of Scuflaire *et al.* (1981), Shibahashi and Osaki (1981), and Christensen-Dalsgaard and Gough (1980). The last paper cited does not give a full set of frequencies. In order to compare their results to the others, we have assumed that their frequencies are distributed in a similar fashion to ours; i.e., the nonlinearity of ν as a function of n is assumed to be the same as what we find, while the zero point and average slope of ν versus n is taken from their paper. These figures also show the observed results of Grec, Fossat, and Pomerantz (1980), and of Claverie *et al.* (1981). We show points on these diagrams to which our

TABLE 2
EIGENFREQUENCIES FOR SELECTED MODES IN THE STANDARD MODEL

l														
n	0	1	2	3	4	6	8	10	14	20	26	34	40	
6	...	...	...	...	...	...	...	...	...	...	...	...	2129.0	
7	...	...	...	...	...	...	...	...	...	...	...	2203.0	2313.0	
8	...	...	...	...	...	...	...	...	...	...	2193.1	2374.2	2493.7	
9	...	...	...	...	...	...	...	...	...	2186.5	2348.8	2542.9	2671.1	
10	...	...	...	...	...	...	...	...	2146.7	2333.9	2503.5	2709.0	2845.1	
11	...	...	...	...	...	...	...	2148.8	2288.3	2480.9	2657.9	2872.5	3016.5	
12	...	...	...	...	...	2130.3	2212.3	2287.0	2429.8	2627.7	2810.9	3034.0	3184.0	
13	...	...	2073.2	2127.5	2127.3	2266.9	2348.6	2425.8	2571.1	2774.3	2962.4	3190.3	3348.7	
14	2083.8	2147.2	2206.0	2260.9	2312.7	2402.5	2485.5	2564.8	2712.8	2920.6	3112.9	3347.9	3510.1	
15	2217.0	2280.2	2339.9	2395.4	2446.6	2538.5	2623.8	2704.0	2854.6	3066.7	3262.5	3504.3	3668.6	
16	2350.7	2414.5	2474.9	2530.9	2582.2	2675.2	2762.2	2843.9	2997.0	3211.9	3412.0	3659.3	3823.7	
17	2485.3	2548.9	2609.7	2666.5	2718.7	2813.2	2901.2	2983.8	3139.5	3357.4	3559.6	3813.1	3975.9	
18	2620.2	2683.2	2745.6	2802.3	2855.4	2951.8	3040.3	3123.8	3281.5	3502.5	3706.6	3974.8	...	
19	2755.3	2818.8	2881.7	2939.1	2992.6	3089.7	3179.3	3263.7	3423.3	3648.1	3852.8	...	...	
20	2890.9	2955.5	3018.1	3076.0	3129.9	3227.9	3318.4	3404.1	3565.1	3791.8	...	...	...	
21	3027.3	3092.1	3154.8	3213.4	3267.0	3366.3	3457.6	3544.4	3706.8	3936.1	...	...	...	
22	3163.9	3229.3	3291.2	3349.9	3404.6	3504.8	3596.9	3684.5	3848.4	...	...	...	...	
23	3300.4	3364.7	3428.0	3487.1	3542.2	3643.4	3736.1	3824.4	...	...	...	...	...	
24	3436.6	3501.9	3564.9	3624.4	3679.9	3782.2	3875.7	...	...	...	...	...	...	
25	3573.3	3638.8	3702.0	3761.9	3817.8	...	...	...	...	...	...	...	...	
26	3710.1	3776.2	3839.1	...	...	...	...	...	...	...	...	...	...	
27	3848.5	...	...	...	...	...	...	...	...	...	...	...	...	

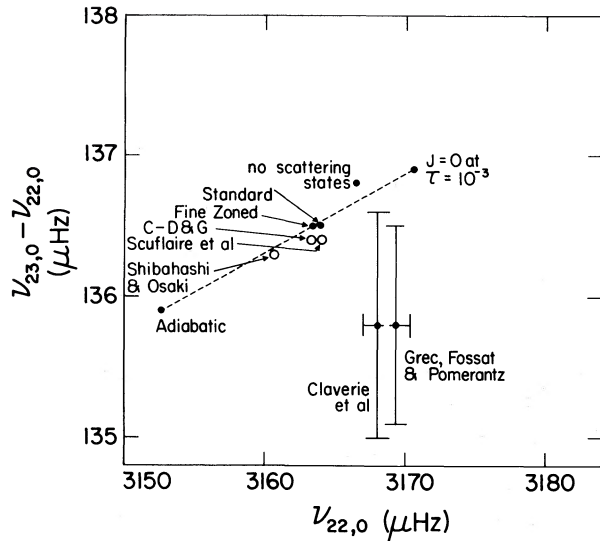


FIG. 3

FIG. 3.—Same as Fig. 2, except for $n = 22$. Here the frequency-spacing error bar was estimated from the variation in spacing between $n = 20$ and $n = 25$.

FIG. 4.—Comparison of frequencies from the standard model with $l = 0$ and $12 \leq n \leq 30$ to the observed frequencies and those of Shibahashi and Osaki 1981. Top axis shows the frequencies of the $n = 12$ mode. Successive values of the modal order, n , are shown with the zero point of frequency incremented by $135 \mu\text{Hz}$. Left axis shows the starting frequency for even values of n . Right axis shows the even values of n . Bottom axis gives the frequency offset from the starting frequency. Our frequencies are labeled with UR; those of Shibahashi and Osaki by SO; Claverie *et al.* 1981 is labeled by C; Grec, Fossat, and Pomerantz 1981 by GFP; and Woodard and Hudson 1982 by WH.

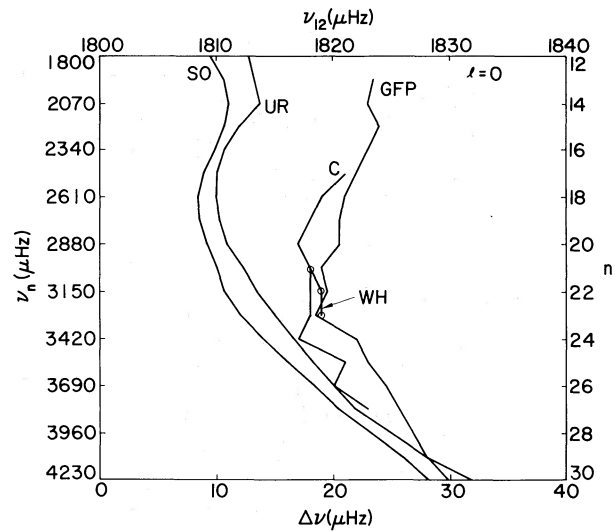
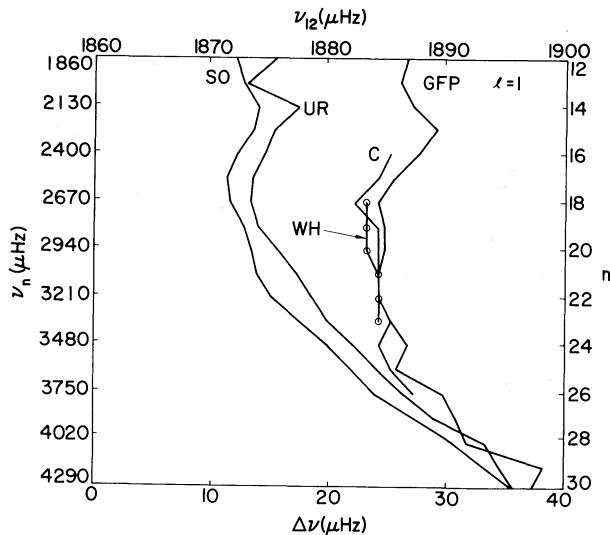
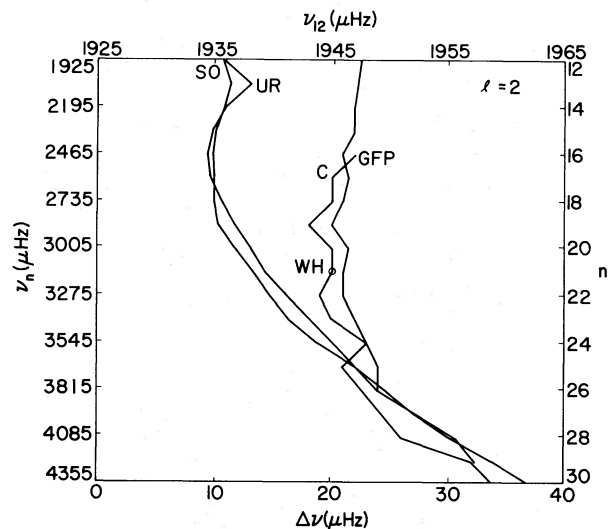


FIG. 4

standard model would move if we treated the oscillations as adiabatic or if we applied a $J = 0$ boundary condition at $\tau = 10^{-3}$. Figures 4–8 compare all of our frequencies for $l = 0$ –4 to the observed frequencies, including velocity observations of Scherrer and Wilcox (1981) at Stanford and the *SMM* irradiance results of Woodard and Hudson (1982), and to the calculations of Shibahashi and Osaki (1981). The discrepancy between the theory and observation is larger by a factor of 10 than we can account for by any known uncertainty in our model or

eigenfrequency calculation. We believe this discrepancy represents a failure of solar model theory which is comparable to the failure to account for the observed solar neutrino flux. The source of the discrepancy can potentially be found through further observation because of the large number of eigenmodes excited in the Sun. The frequency data provide information about the sound speed distribution throughout the Sun. The solar neutrino data provide information about the temperature in the innermost parts of the Sun. The two methods of probing structure are largely complementary.

FIG. 5.—Same as Fig. 4, but for $l = 1$ FIG. 6.—Same as Fig. 4, but for $l = 2$

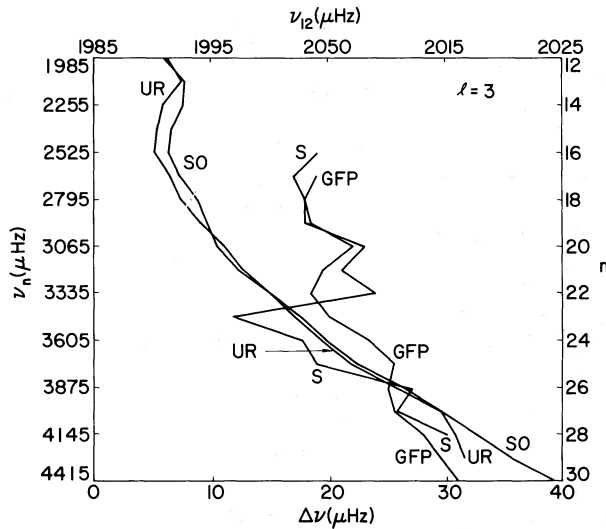


FIG. 7.—Same as Fig. 4, but for $l = 3$. This figure also shows the results from Stanford of Scherrer and Wilcox 1981 labeled with S.

In order to illustrate the sensitivity of the eigenfrequencies to the details of solar structure, we show in Figures 9 and 10 eigenfunctions for those modes nearest $\nu = 2500 \mu\text{Hz}$. Figure 9 shows the complete eigenfunction for the $l = 2, n = 16$ mode. The parameter giving the amplitude is ρv^2 , the kinetic energy density. Figure 10 shows the maxima connected by straight lines for modes with l values between 2 and 40. The important characteristic of these modes is the similarity of the eigenfunctions for layers exterior to a critical radius which increases with l . Interior to this critical radius the eigenfunction quickly approaches zero, and the eigenfrequency of that mode cannot be influenced by the structure. Modes with successive values of l can be used to selectively determine the characteristics of the solar

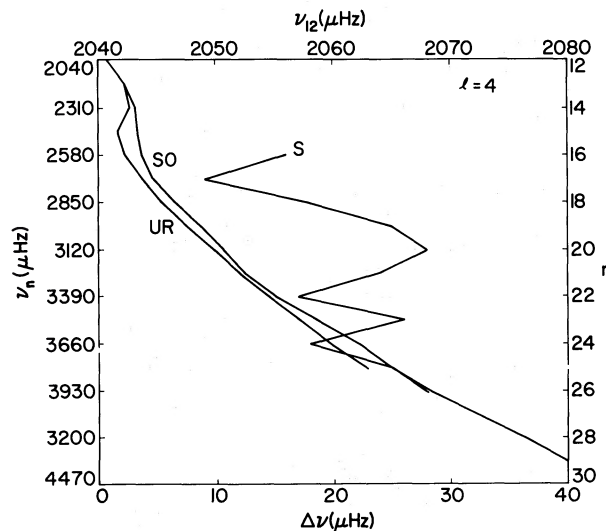


FIG. 8.—Same as Fig. 7, but for $l = 4$

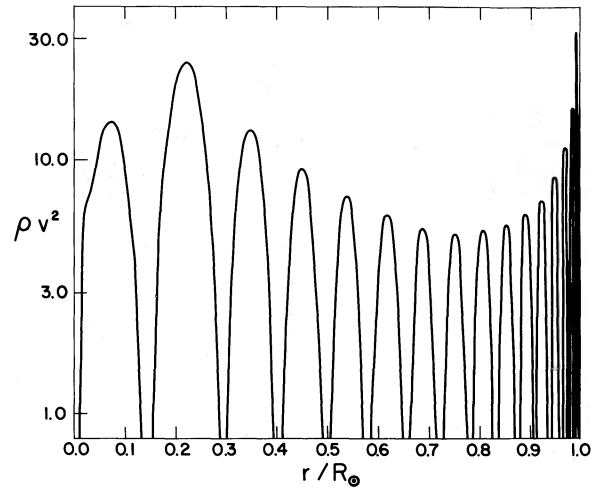


FIG. 9.—Amplitude of the eigenfunction for $l = 2, n = 16$ as a function of $r/R_\odot$.

structure near the critical radius. If eigenfrequencies of precision comparable those for $l = 0-3$ were available for modes with $4 < l \lesssim 40$, we would be able to determine the layer where the error in the solar model begins by finding the critical radius for the mode with the smallest value of l which has a frequency agreeing with the model frequency. Figure 11 shows an expansion of the key section of the (l, ν) -plane with the eigenfrequencies for our standard model. Since highly precise observed frequencies are not yet available even for large l modes, we may, of course, ultimately find that no part of the solar model predicts correct eigenfrequencies.

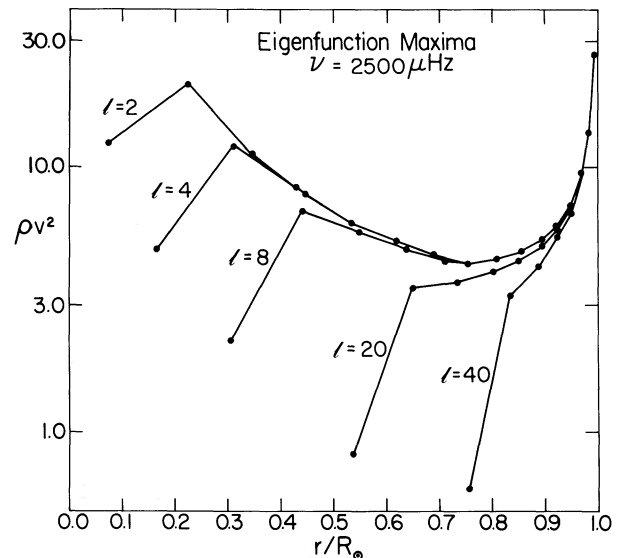


FIG. 10.—Eigenfunction maxima for l values between 2 and 40 vs. $r/R_\odot$. The values of n were selected so that the frequency was closest to $2500 \mu\text{Hz}$.

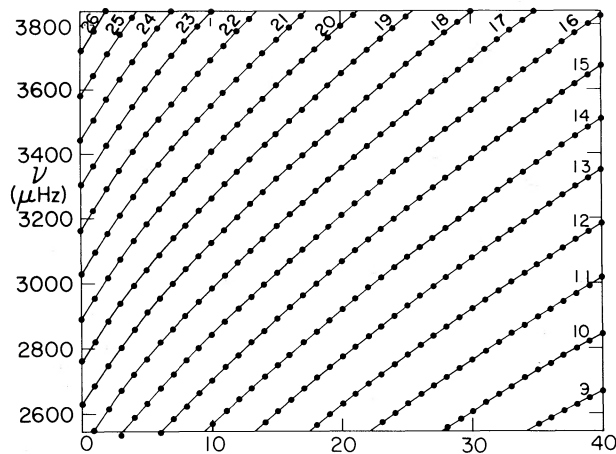


FIG. 11

FIG. 11.—An expansion of the l, ν diagram in the region where the amplitude of the oscillation is likely to be large. The inability to select a single value of l due to the inability to observe more than a limited fraction of the solar surface will cause a band in l to be combined in any power spectrum.

VI. NONSTANDARD MODELS

In view of the failure of the standard model to account for the observed solar frequencies, we have also considered four nonstandard models. The results of the calculations are shown in Figures 12 and 13. An important discriminant for choosing between these various models is the detailed spacing of the eigenfrequencies for the values of l between 0 and 3. The presently available data provide an indication of this spacing which can be compared to the theory. The results for the models below are given in Table 3.

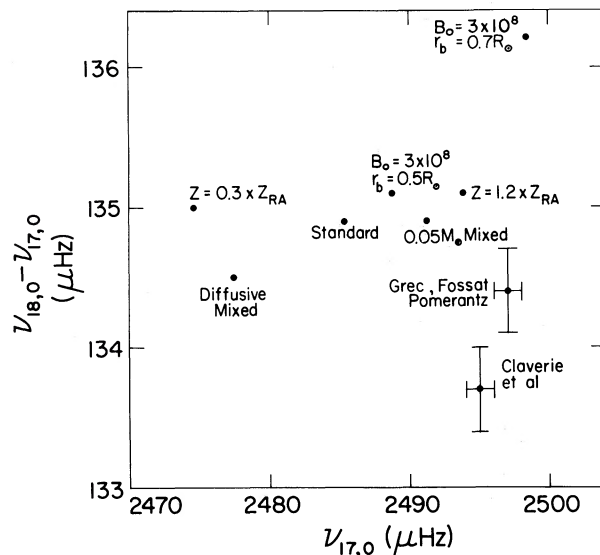


FIG. 12

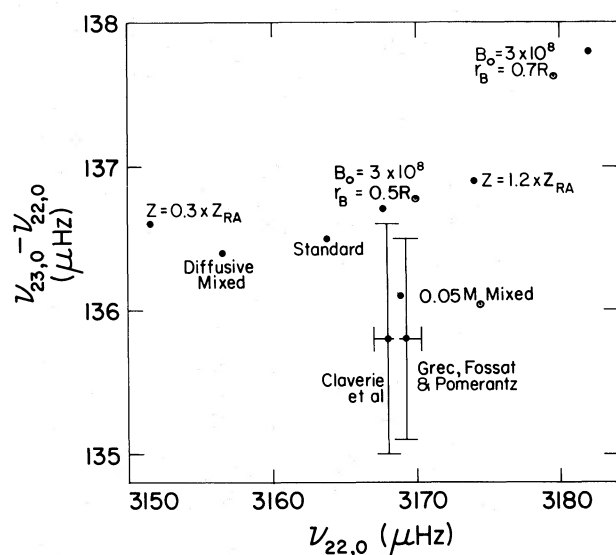


FIG. 13

FIG. 12.—Like Fig. 2, except that nonstandard solar models are shown.

FIG. 13.—Like Fig. 3, except that nonstandard solar models are shown.

a) Low Z Model

This model uses opacities appropriate to a heavy element abundance 0.3 times the normal as defined by the Ross and Aller (1976) mix. We have used the astrophysical library of opacities prepared by the Los Alamos group (Huebner *et al.* 1977) to compute opacities for all the opacities used in this study. We have changed the composition at the base of the convective envelope. To avoid the possibility of doubly diffusive mixing, we have assumed that a helium abundance change is associated with the Z boundary so that μ remains constant. Since no theory leading to an inhomogeneous Sun is known, we may impose this helium discontinuity as an additional constraint. Contrary to the results of other investigators (Iben and Mahaffy 1977; Christensen-Dalsgaard and Gough 1980), we find that the eigenfrequency spacing is not changed while the zero-point frequency is decreased. This may be a result of the relatively modest underabundance we assume, viz., 30% of normal, as opposed to the 10% and 20% values used by the others. Our zero-point frequency decreases, as does that of the other investigators, so there is general agreement as to the sense of the change in this type of model. The different result for the frequency spacing could be due to the treatment of the atmospheric zones. When we modified the initial helium abundance in order to match the solar luminosity, we also modified the atmospheric helium abundance and recalculated the atmospheric opacities. The neutrino flux for this model is 3.5 SNU. The neutrino flux and the oscillations thus give contradictory indications for the low- Z model.

b) High Z Model

The second nonstandard model is similar to the low- Z model, except that we have used a 20% increase in

heavy element abundance. This model shifts the value of the frequency in the correct direction but does not change the frequency spacing. Presumably, a combination of modified boundary conditions and an enhanced Z could produce a satisfactory result for the solar oscillations. The neutrino flux for this model is a totally unacceptable 15 SNU, so we feel that this type of model is not promising.

This model is unusual in that it contains a naturally convective core of mass $0.05 M_{\odot}$. We have homogenized the composition in this core and then abruptly changed to the scaled solar abundance outside the core boundary. Because of the hydrogen abundance dependence of the opacity, this nonhomogenized region has a superadiabatic temperature gradient but would be stable if given the core composition. This is the classic condition of semiconvection. Presumably, if the convective core were to shrink during the evolution as is typical of low-mass stars, the deposited composition gradient would be stable. The eigenfrequencies are sensitive to the details of the composition distribution at a level of $5 \mu\text{Hz}$ —a level which is potentially verifiable by observational means.

c) Mixed Interior

We tried two types of mixed interior model: (1) we homogenized the hydrogen and helium abundances for $M_r \leq 0.05 M_{\odot}$, and (2) we used the hydrogen distribution given by Schatzman *et al.* (1981) for diffusive mixing. The ^3He was assumed to be in local steady state, so our models correspond to an assumption of slow mixing.

The first model is primarily a test of the sensitivity of the eigenfrequencies to a modification restricted to the innermost regions of the Sun. Because of the small mass which is mixed and our assumption of local steady state ^3He abundance, the neutrino flux for this model is the same as for the standard model: 9.7 SNU. The frequencies and their spacings look attractive, especially for the

$n = 22$ and 23 modes. A major deficiency with this model is the frequency difference between the $n, l = 0$ mode and the $n - 1, l = 2$ mode. As Table 2 shows, other models have a value of this spacing close to the observed value. This spacing is sensitive to the central temperature gradient and the central mean molecular weight gradient since the eigenmodes are virtually identical exterior to $r = 0.22 R_{\odot}$. In spite of the attractiveness of the frequencies and their spacing in n , we feel that the difficulty with the $\nu_{n,0} - \nu_{n-1,2}$ spacing makes this model unlikely. It is encouraging that this modification to the inner 5% of the mass produces a potentially observable change in the frequency spacing.

One second mixed model based on the work by Schatzman *et al.* (1981) involves a broad change in the structure, with the reduction of the hydrogen abundance gradient to the point where the central value of X is only 0.11 smaller than the surface value of X . The same quantity in the standard model is 0.38. The central temperature is dropped to 14.8×10^6 K as opposed to the standard value of 15.7×10^6 K. As found by Schatzman *et al.*, the neutrino flux is substantially reduced to 3.5 SNU. Unlike the model in the preceding paragraph, this model has no abrupt change in mean molecular weight. As is shown in Figures 2 and 3, the frequencies found for this model are very similar to those for the low- Z model. This model also suffers from the same problem as the preceding one in that the splitting between $\nu_{n,0}$ and $\nu_{n-1,2}$ is too large.

d) Large Magnetic Field

The possibility that the Sun could have a magnetic field large enough to alter its structure has been considered by Iben (1969), Bahcall and Ulrich (1981), Ulrich (1974), Chitre, Ezer, and Stothers (1973), and Bartenwerfer (1973). These earlier studies were primarily concerned with the effect such a field could have on the solar neutrino fluxes. Except for the work by Bahcall

TABLE 3
SPACING OF EIGENFREQUENCIES FOR $l = 0, 1, 2$, AND 3
($\nu_{n,l}$ in μHz)

INVESTIGATOR OR CASE	$\nu_{n,0} - \nu_{n-1,2}$		$\nu_{n,1} - \nu_{n,0}$		$\nu_{n,1} - \nu_{n-1,3}$	
	$n = 17$	22	17	22	17	22
Observed:						
Grec, Fossat, and Pomerantz	11.0	9.5	63.4	65.5	15.0	15.0
Claverie <i>et al.</i>	9.0	8.0	63.0	66.0	...	...
Our Models:						
Standard	10.38	9.18	63.59	65.41	18.04	15.99
No scattering states	9.99	9.63	63.93	65.70	17.68	15.50
Low Z	11.14	9.68	63.85	65.82	19.64	17.50
High Z	9.52	8.27	64.81	66.80	18.161	16.28
$0.05 M_{\odot}$ mixed	17.35	15.44	58.86	63.92	19.69	20.30
$B_0 = 3 \times 10^8, r_B = 0.7 R_{\odot}$	10.53	9.09	64.75	66.49	18.92	16.41
Other Investigators:						
Scuflaire <i>et al.</i>	15.9	15.8	61.5	62.6	24.9	12.5
Shibahashi and Osaki	9.0	7.4	62.3	64.3	14.8	12.6

and Ulrich (1971), the treatment of the magnetic field has been qualitative. Iben appealed to the concept of equipartition and assumed the magnetic pressure scales with the gas pressure. He found little influence on the neutrino fluxes. Chitre, Ezer, and Stothers (1973) and Bartenwerfer (1973) assumed the field is centrally concentrated and tangled. Parker (1974, 1979) has objected to such models on the grounds that magnetic buoyancy will cause the field to leave the Sun on a time scale of no less than 10^8 yr. Bahcall and Ulrich (1971) based their magnetic field configuration on the work by Cowling (1945) which showed that a dipole-like field decays very slowly due to ohmic dissipation. If such a large-scale field were present in the Sun, Parker's buoyancy arguments would not apply because there is no nonmagnetized fluid to replace the rising magnetized fluid. Parker (1979) argues that large-scale symmetry is unlikely ever to be present because the fluid out of which the Sun formed would necessarily have been turbulent before settling into a spherical configuration. We find this to be a plausible but not compelling argument. We also point out that the time scale of 3×10^8 yr found by Parker (1979, p. 150) is long enough that nuclear reactions can increase the core mean molecular weight and anchor the magnetic field.

Bahcall and Ulrich (1971) found that the field of a Cowling type raised the solar neutrino flux. However, they included only the magnetic pressure $|B|^2/8\pi$ in the equation of hydrostatic equilibrium. They based this treatment on the concept that the pressure terms are relatively insensitive to the details of the field configuration, whereas the tension terms $\mathbf{B} \cdot (\nabla \mathbf{B})$ are more subject to errors in the model of the field geometry. We feel that the longevity of the field requires that it have the specific Cowling geometry. Consequently, we feel the tension terms should be included, and we have found the magnetic force directly from $\mathbf{j} \times \mathbf{B}$. This force is added to the gravitational force in the equation of hydrostatic equilibrium. The inclusion of the magnetic tension terms by this procedure changes the result concerning the neutrino flux because these terms redistribute the effective pressure. The central region receives an extra outward force, while the regions near the surface are pulled inward. This distribution of force has the effect of reducing the neutrino flux. For the case we have calculated with $B_0 = 3 \times 10^8$ gauss, the neutrino flux is reduced by 30%. Consequently, we feel that the large magnetic field model is attractive. An objection in principle to this model is the field strength of 10^7 gauss near the base of the convective envelope. We would expect to see occasional flux tubes of intensity 10^5 gauss pop out of the solar surface if the field were really this strong below the convective envelope. A more rapid decrease of the field with radius would avoid this problem. A deep convective envelope early in the solar life could have expelled the field inward to some critical radius, and the subsequent ohmic decay would not yet have allowed the field to reach the present base of the convection zone. Clearly, the initial field cannot be too small in extent or Parker's buoyancy arguments will

come into force. We have not explored such a model in detail and offer these ideas here as a conceptual framework for future investigation. As representative examples of such possible field configurations, we have considered Cowling-type modes which have zeros in B at $r = 0.7 R_\odot$ and $r = 0.5 R_\odot$.

The full treatment of the equations of hydromagnetic oscillation would require an extension of the eigenfrequency code beyond its present capability. The Cowling field will break the m degeneracy of the modes and require additional equations and variables. We feel such an investigation is justified only if a preliminary analysis shows the magnetic field model to be promising. In order to carry out a preliminary analysis, we assume that the wave phase velocity is increased from c to $(c^2 + V_A^2 \sin^2 2\Psi)^{1/2}$. Alfvén and Fälthammer (1965) show this to be a good approximation when $c \gg V_A$. In this equation, V_A is the Alfvén velocity $|B|^2/8\pi\rho$, and ψ is the angle between $\mathbf{B}$ and the wave vector $\mathbf{k}$. This angle depends on both the field geometry and the oscillation eigenfunction. As a simplifying assumption, we take $\mathbf{k}$ to be parallel to $\mathbf{r}$. Our assumption is exact for $l = 0$ and should be quite good for $l = 1-3$. As a further approximation, we average $\sin^2 \psi$ over the spherical surface and take the phase velocity to be a function of r only. We then replace the sound velocity with the phase velocity and carry out the eigenfrequency analysis as before.

The results of the above investigation given in Figures 12 and 13 in Table 3 show that the magnetic field moves the frequencies in the right direction. The spacing is also increased, so this model has a result similar to that of changing the outer boundary condition. It is clear that the Cowling mode geometry for the magnetic field is not successful. The general trend is encouraging, and perhaps a more centrally concentrated field model would have a better result. We reiterate here the importance of the $l = 4-40$ modes because of their ability to probe the solar interior layer by layer.

VI. CONCLUSIONS

We have found that a model of the Sun with standard assumptions does not produce frequencies which agree with observation. The disagreement is larger than can be caused by any known errors in the model. Our standard model results agree with those of three other groups at a level which is about 20% of the discrepancy with observations. This strengthens our belief that the discrepancy is real and not a result of an inadequacy in the input physics or numerical techniques. An important factor in our conclusion is the study of the effect of boundary conditions and radiative interactions on the eigenfrequencies. Both these processes are uncertain and could conceivably shift the frequencies or their spacing to agree with observation. For example, we could apply a condition of $J = 0$ at $\tau = 10^{-2}$ and have $\nu_{17,0}$ agree with the observed value. However, the spacing of $\nu_{18,0} - \nu_{17,0}$ would then be 136.8 μHz instead of the observed 133.7 μHz . Thus, only $\nu_{17,0}$ would agree with observation. Alternatively, we could use an adiabatic treatment of the oscillations to match the observed

spacing, but then our frequencies are 16 μHz too low. Because both the boundary conditions and the radiative interactions apply at the outermost part of the model, they provide only one parameter, whereas the observations are providing two constraints. The one parameter available does not permit the simultaneous satisfaction of both constraints. Hence, we conclude that some parameter describing the interior is incorrect. As a cautionary note, we point out that the modal identification could be incorrect. In this case, major changes in the model will be required.

In an effort to find a model permitting the frequencies to match the observations, we considered four non-standard models: low Z , high Z , mixed interior, and strong magnetic field. None of these models is successful. The high- Z and mixed interior models move the frequencies and their spacings in the right direction. The high- Z model is unacceptable on the basis of its neutrino flux. One model of mixed interior failed because it missed badly in accounting for the $\nu_{n,0} - \nu_{n-1,2}$ spacing. The other—the diffusive mixed case—moved the fre-

quencies the wrong way. It is possible that some other distribution of helium abundance would be more successful, but we have been unable to define such a model. The preliminary version of a strong magnetic field model is also unsuccessful in matching the observations. In a general way, such a model seems attractive because it now alters the neutrino flux in the right direction and because there are a number of degrees of freedom left to explore. The magnetic field model and the mixing model offer the best possibility for future success in explaining the solar oscillation frequencies.

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EDWARD J. RHODES, JR.: Department of Astronomy and Space Sciences Center, University of Southern California, Los Angeles, CA 90007

ROGER K. ULRICH: Department of Astronomy, University of California, Los Angeles, Los Angeles, CA 90024