

THE PLANETARY NEBULA IC 3568: A MODEL BASED ON *IUE* OBSERVATIONS

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ABSTRACT

The planetary nebula IC 3568 has been observed at both low and high dispersion with the *IUE* satellite. The spectrum of the star shows strong P Cygni profiles due to an 1840 km s^{-1} stellar wind. The nebular spectrum shows lines of He II, C III], C IV, N III], and O III]. The spatial distribution of the gas has been derived from a 6 cm VLA map. An ionization model is presented which fits the observed optical and UV line fluxes well. The abundances of C, N, O, Ne, and S are determined to be about half the solar values. The far-UV flux of the central star in the 912–228 Å range can be represented by a Hummer and Mihalas model atmosphere with an effective temperature of 50,000 K. The flux at wavelengths below 228 Å, however, is 35 times greater than the value given by the Hummer and Mihalas atmosphere.

Subject headings: nebulae: abundances — nebulae: individual — nebulae: planetary — stars: atmospheres — ultraviolet: spectra

I. INTRODUCTION

The planetary nebula IC 3568 is a fairly bright, very symmetric object of medium excitation with a bright central star. Its optical spectrum has been well studied and shows exceptionally few lines confined to a relatively small range in excitation (Lee *et al.* 1969; Aller and Walker 1970; Barker 1978). This simplicity makes it an attractive object for modeling.

In the next section we present new ultraviolet observations obtained with the *International Ultraviolet Explorer* (*IUE*) satellite. We discuss the interstellar extinction in § III, and the previous model by Buerger in § IV. In § V we examine all the available information on the morphology of this object and derive the spatial distribution of the emitting gas. In § VI we consider the relationship between the nebula's distance, mass, and filling factor. In § VII we show what can be determined about the stellar flux from the UV continuum and from the requirements of our nebular models. In § VIII we present a new detailed computer model for this nebula and compare the results with the observed line fluxes. In § IX we discuss the derived chemical abundances, and in § X we summarize the results of this study.

II. OBSERVATIONS

In Table 1 we list the observations of this nebula which have been made with the *IUE*. In Figure 1 we show two of the low-dispersion spectra, SWP 13875 and LWR

11446, merged together. There is a strong continuum due to the central star with P Cygni features at N v $\lambda 1240$, O v $\lambda 1371$, and C iv $\lambda 1549$.

Just as in the case of the optical spectra, the UV spectrum of the nebula is not very rich: C III] $\lambda 1908$ is the only line seen at low dispersion. The flux in this line is best determined from SWP 5432 ($7.11 \times 10^{-12} \text{ ergs cm}^{-2} \text{ s}^{-1}$) and SWP 13875 ($6.57 \times 10^{-12} \text{ ergs cm}^{-2} \text{ s}^{-1}$). SWP 5432 was reprocessed after the error in the intensity transfer function (ITF) had been corrected. We thus adopt for the C III] $\lambda 1908$ flux the mean value $6.84 \pm 0.27 \times 10^{-12} \text{ ergs cm}^{-2} \text{ s}^{-1}$.

The stellar continuum is fully exposed on the low-dispersion spectra so that fainter nebular lines can be observed only at high dispersion, where there is greater contrast between the lines and the continuum. The 1 hour high-dispersion exposure, SWP 5433, revealed only C III] $\lambda 1907$, $\lambda 1909$ lines, which were well exposed. The subsequent 6.45 hour exposure, SWP 14868, was able to record the stellar continuum and showed many interesting features. Tables 2, 3, and 4 list the nebular, stellar, and interstellar features, respectively. One important result was the separation of the narrow nebular C iv $\lambda 1548$, $\lambda 1551$ lines from the broad stellar P Cygni profile. This is shown in Figure 2. Figure 3 shows the He II $\lambda 1640$ and O III] $\lambda \lambda 1661$, 1666 lines, which cannot be seen on the low-dispersion spectra. The N III] lines are very weak, but there are emission features present at four of the five expected wavelengths. The N III] $\lambda 1749.7$ component seemed best suited for measurement. To obtain the total flux in the N III] quintet, we made use of the theoretical result that, at low densities, the 1749.7 component is 40% of the total flux (Nussbaumer and Storey 1979).

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TABLE 1
IUE OBSERVATIONS OF IC 3568

Date	IUE Image No.	Exposure (minutes)	Dispersion	Aperture
1979 Jun 2	LWR 4678	50	Lo	L
1979 Jun 2	SWP 5432	20	Lo	L
1979 Jun 2	SWP 5433	60	Hi	L
1979 Jun 2	SWP 5434	10	Lo	L
1981 May 3	LWR 10509	20	Lo	L
1981 May 3	SWP 13875	15	Lo	L
1981 Aug 30	LWR 11446	25	Lo	L
1981 Aug 30	SWP 14868	387	Hi	L

The equivalent widths of these lines are expressed in terms of the stellar continuum visible on the high dispersion spectrum. The last column of Table 2 gives the corresponding flux (corrected for reddening with $c = 0.25$) which is obtained from the absolute flux of the stellar continuum measured from SWP 13875.

Table 3 lists the broad features in the spectrum which are presumably of stellar origin. The $N\text{ v } \lambda 1240$ doublet is completely saturated, and the wavelength of the blue edge provides a good determination of the terminal velocity, which is 1840 km s^{-1} . The $O\text{ v } \lambda 1371$ line is not

saturated and gives an indication of the mass loss rate which is of the order of $4 \times 10^{-9} M_{\odot} \text{ yr}^{-1}$ (Harrington 1982).

Finally, in Table 4 we provide a list of the interstellar absorption features seen in this spectrum.

III. INTERSTELLAR EXTINCTION

From observations of the Balmer decrement, Barker (1978) obtained an extinction parameter of $c = 0.252 \pm 0.032$. This is in good agreement with the value of 0.23 ± 0.05 obtained by Sistla and Kaftan-Kassim (1976)

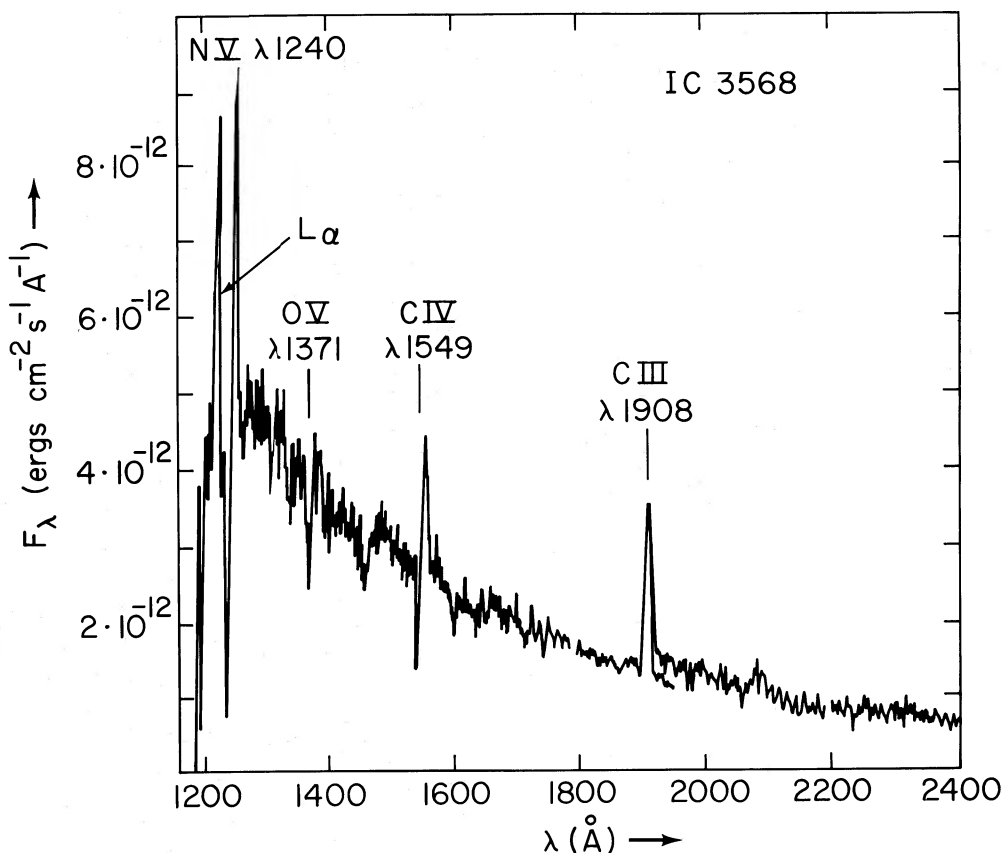


FIG. 1.—The UV spectrum of IC 3568, SWP 13875 and LWR 11446 have been merged. (Only part of the LWR spectrum is shown here.) These data have been corrected for extinction using $c = 0.25$ ($E_{B-V} = 0.17$).

TABLE 2
NEBULAR EMISSION LINES IN SWP 14868

λ (Å)	Ion	EW (Å)	$F(\lambda)$ (ergs cm ⁻² s ⁻¹)
1548.2	C IV	1.17	3.22×10^{-12}
1550.8	C IV	0.60	1.65×10^{-12}
1640.4	He II	0.78	1.77×10^{-12}
1660.8	O III]	0.25	0.54×10^{-12}
1666.2	O III]	0.63	1.35×10^{-12}
1746.8	N III]	^a	...
1748.6	N III]	^a	...
1749.7	N III]	0.22	0.41×10^{-12}
1754.0	N III]	^a	...
1816.0	?	0.3	0.4×10^{-12}

^a Line present, but not suitable for measurement.

from the ratio of H β and radio fluxes. Pottasch *et al.* (1977) derived $c = 0.29 \pm 0.04$ from broad-band UV observations of the $\lambda 2200$ interstellar feature. From a comparison of He II $\lambda 4686$ as given by Lee *et al.* (1969) with our observation of He II $\lambda 1640$, we obtain $c = 0.27 \pm 0.08$.

We adopt $c = 0.25$ and employ the reddening curve of Seaton (1976). This value is satisfactory in terms of removing the $\lambda 2200$ feature from the stellar continuum. (See Fig. 1.)

With a galactic latitude of $b = 34^\circ 55'$, IC 3568 is well above the galactic plane. The simple cosecant reddening model used, for example, by O'Dell (1962), $c = 0.141 \csc b$, also gives a value of precisely 0.25.

Phillips, Reay, and Worswick (1979) compared their H β isophotes to the 8 GHz radio map of Sistla and Kaftan-Kassim (1976) and concluded that there was a variable extinction across the nebula amounting to $A_V \approx 1$ mag near the center. Such an internal extinction would imply that radiation from the far side of the nebular shell would be less than half the intensity of that from the near side. The slit spectra of Bohuski, Smith, and Weedman (1970) show no such weakening of the redshifted side of the bowed nebular lines, which are in fact quite symmetric. The variable H β /radio ratio seen by Phillips *et al.* might be due to a greater broadening of their H β image by seeing and instrumental effects, which would appear to lower the central H β brightness by transferring the radiation outward.

TABLE 3
STELLAR FEATURES IN SWP 14868

λ (Å)	Ion	Comments
1238.8, 42.4	N V	Saturated, $V_\infty = 1840$ km s ⁻¹
1338.6	O IV	Broad, weak absorption feature
1343.0, 43.5	O IV	Unsaturated
1371.3	O V	Strong but unsaturated
1393.4	Si IV	Broad (> 1 Å) absorption feature, no emission
1548.2, 50.8	C IV	See Fig. 2
1718.6	N IV	Weak

TABLE 4
INTERSTELLAR ABSORPTION LINES IN SWP 14868

λ (Å)	Ion	EW (Å) ^a
1250.6	S II	0.2
1253.8	S II	0.3
1260.4	Si II	0.7 (<i>n</i>)
1302.2	O I	0.5 (<i>r, n</i>)
1304.4	S II	0.4
1334.5	C II	0.6 (<i>r</i>)
1526.7	Si II	0.5 (<i>r</i>)
1608.2	Fe II	0.5
1656.9	C I	0.15
1670.8	Al II	< 0.7 (<i>r, n</i>)
1808.0	S II	0.1

^a NOTE.—*n* indicates noisy; *r* indicates interference by resseau mark.

IV. PREVIOUS MODELS

A model of IC 3568 was constructed by Buerger (1973). This model fits most of the optical line fluxes well. It has one serious defect, however: the density distribution is irreconcilable with the nebula's appearance and surface brightness.

Buerger's model consists of a sphere at a density of 800 hydrogen atoms cm⁻³ surrounded by a concentric shell of higher density. The inner density of 800 cm⁻³ was presumably suggested by the value of Lee *et al.* (1969) from the H β surface brightness. Since this density is far too low to give the observed [O II] $\lambda 3729/\lambda 3726$ line ratio, the concentric shell was given the density implied by this ratio, i.e., 5000 cm⁻³. As a result, the emission from the shell completely dominates the flux from the model, which at the Earth (even after correction for interstellar extinction, which Buerger took to be zero) would be $\pi F(\text{H}\beta) = 2.6 \times 10^{-10}$ ergs cm⁻² s⁻¹, 17 times the observed flux. Furthermore, the model would have the appearance of a bright ring with the intensity maximum 13"0 in diameter, quite unlike the actual appearance of this object.

Thus, in addition to predicting the UV spectrum, we need to reexamine the problem of the density distribution of this object.

V. THE DENSITY DISTRIBUTION

IC 3568 is noted for its smooth appearance and high degree of circular symmetry. Also unusual is its expansion

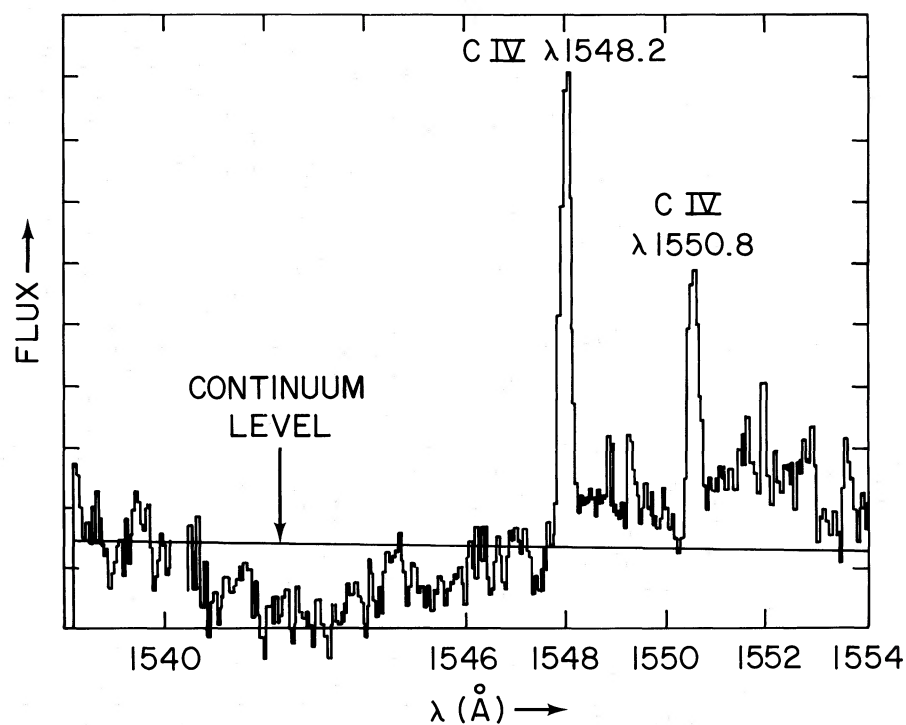


FIG. 2.—The nebular and stellar C IV $\lambda 1548$ and $\lambda 1551$ lines from the high-dispersion spectrum SWP 14686 (order 89)

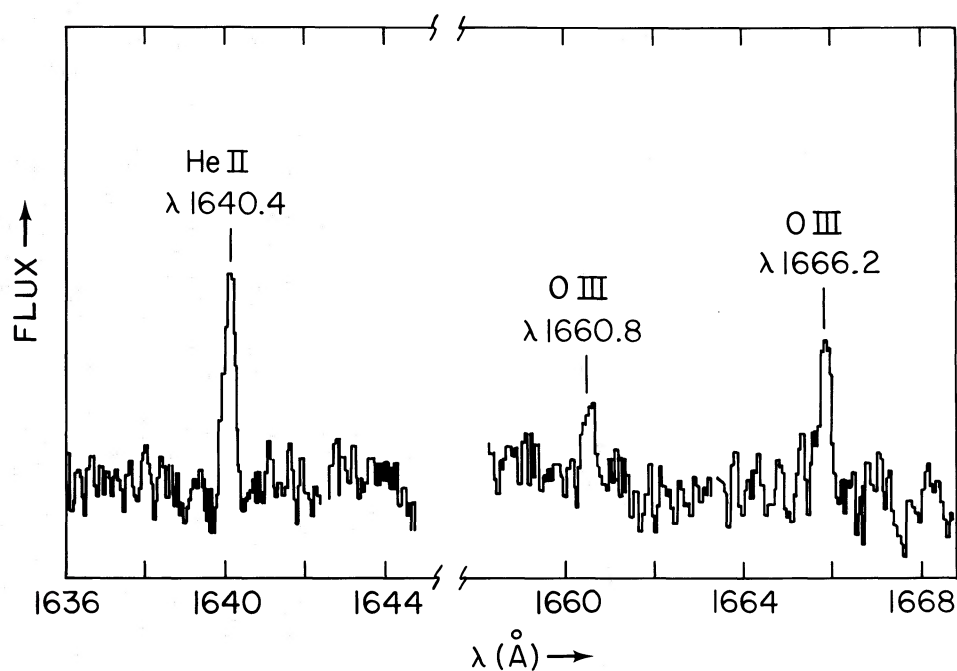


FIG. 3.—The nebular He II $\lambda 1640$ and O III $\lambda 1661$, 1666 lines from SWP 14866 (parts of orders 83 and 84 are shown)

velocity of 7.8 km s^{-1} , which is one of the smallest known for a planetary nebula (Bokuski, Smith, and Weedman 1970). Because of its regular structure, we have attempted, by introducing the approximation of spherical symmetry, to derive the spatial distribution of the gas from the observed projected brightness distribution.

Duncan (1937) found that this nebula has a faint outer envelope. He states that "IC 3568 *Camelopardalis* consists of a very bright circular disk $7''$ in diameter surrounded concentrically by a faint disk of diameter $16''$ " and he provides a photograph. The brightness as a function of angular radius for H I and [O III] emission has been given by Aller (1956, p. 265) and by Weedman (1968). Aller's results are based on 200 inch (5 m) photographs by Minkowski which are overexposed in the vicinity of the central star and which thus cannot supply information on the inner two arcsec of the nebula. Weedman found that his data could be well represented by the form $I(r) \propto \exp[-(r/2''.4)^2]$, which has a full width at half-maximum (FWHM) of $4''.0$. Phillips, Reay, and Worswick (1979) have given six isophotal maps of IC 3568. These isophotes show very little structure; they are characterized by a smooth, monotonic decrease in intensity from the center outward and have a slight ellipticity (10%–20%). We measured the Phillips *et al.* contours along eight radial directions and averaged the values to obtain a mean brightness curve for each spectral line map. The [O III], [O II], and [Ne III] brightness maps have a FWHM of $6''.5$, while their H α map (obtained with a different telescope) has a FWHM of $8''.5$. Thus

while the Phillips *et al.* maps show none of the inner ring–outer envelope structure seen by Duncan and by Aller, they cannot be said to agree with Weedman's data because they are much broader.

The nebula has also been observed at radio frequencies. Sistla and Kaftan-Kassim (1976) mapped it at 8 GHz, while Hjellming, Bignell, and Balick (1978) used the VLA to produce a 6 cm map. Both maps show a faint, patchy outer envelope surrounding a sharply defined mesa of emission.

The VLA map seems to have the best signal-to-noise ratio, and we have based the density distribution of our model on it. Since the published map (Hjellming, Bignell, and Balick 1978) has no scale, Dr. Hjellming provided a contour map which we have measured along various radial directions. The flux versus radius curves were then averaged to obtain a representative mean curve. The result is plotted as the series of squares in Figure 4.

Because of its small angular extent, all observations of IC 3568 are significantly broadened by instrumental effects and/or by atmospheric seeing. The cleaned VLA map we have used is presented as the convolution of the signal with a Gaussian beam with a FWHM of $2''.0$. To take this broadening into consideration, we found it convenient to represent the emission per unit volume by functions of the form

$$(\rho/H)^{2n} \exp[-(\rho/H)^2],$$

where $n = 0, 1, 2, \dots$, ρ is the radial distance from the center of the nebula and H is a scale factor. The

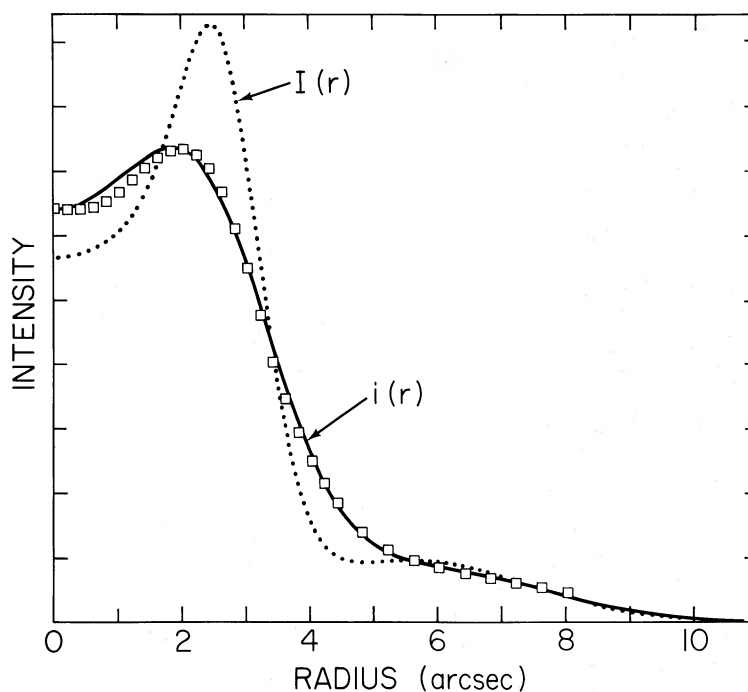


FIG. 4.—The averaged 6 cm brightness distribution from the VLA map of Hjellming, Bignell, and Balick (1978) is shown here as a series of squares. The dotted curve is projected brightness, $I(r)$, of the adopted density distribution, while the solid curve is the broadened distribution, $i(r)$, which results after applying a Gaussian beam of FWHM = $2''$ to $I(r)$. The vertical scale is linear but otherwise arbitrary.

maximum of this function occurs at $\rho_m = (n)^{1/2}H$. The projected brightness distribution which results from such emission is of the form

$$I(r) = \sum_{k=0}^n b_k (r/H)^{2k} \exp[-(r/H)^2], \quad (1)$$

where r is the impact parameter of the line of sight and the b_k are constants. It is remarkable that the convolution of this projected brightness with a Gaussian beam results in another expression of exactly the same form, with only the constants changed. Thus we have a convenient method of producing broadened brightness distributions for comparison with the mean curve from the VLA map. The equations are given in the Appendix.

Because of the sharp drop in the brightness of the nebula at about $3''$ from the center, it was necessary to use a function with a high value of n , which corresponds to a concentration of the emission in a relatively narrow shell. The distribution we have adopted is

$$e(\rho) = e_0 \{ (\rho/1''.15)^{12} \exp[-(\rho/1''.15)^2] + 0.25(\rho/2''.9)^{10} \exp[-(\rho/2''.9)^2] \}. \quad (2)$$

The second term accounts for the outer envelope. Figure 5 shows $e(\rho)$ and the resultant brightness distribution $I(r)$. It resembles the result for the $H\alpha$ radiation given by Aller (1956) rather closely.

Figure 4 shows the projected brightness $I(r)$ and the distribution $i(r)$ which results after convolution with a Gaussian beam of $\text{FWHM} = 2''.0$ (corresponding to $s = 1''.2$ in eq. [A6]). It can be seen that the broadened

distribution fits the mean VLA distribution very well. The real limitation to our adopted distribution is the assumption of spherical symmetry and the use of a mean intensity versus radius curve.

This adopted brightness distribution can to a large extent be reconciled with the Phillips *et al.* maps if we assume that the latter are sufficiently broadened by some mechanism. Figure 6 shows the broadened image $i(r)$ which results from the application of a Gaussian point-spread function with $\text{FWHM} = 3''.66$ ($s = 2''.2$). The squares and triangles are the mean curves from the Phillips *et al.* [O II] and [O III] maps, respectively. We cannot, however, reconcile our distribution with Weedman's (1968) data.

As discussed below, the temperature variations in this nebula are small, so that the radio emission will be proportional to N_H^2 . Our analysis therefore leads to a density distribution $N_H(\rho) \propto e(\rho)^{1/2}$, which is also shown in Figure 5. The absolute value of the density follows from equation (A10). With the observed $H\beta$ flux of 1.55×10^{-11} ergs $\text{cm}^{-2} \text{s}^{-1}$ (Collins, Daub, and O'Dell 1961) and the adopted reddening of $c = 0.25$, we obtain $N_0 = 500D^{-1/2}$, where D is the distance in kpc.

The density thus obtained is too low to reproduce the observed density-sensitive line ratios, so we must consider small-scale density fluctuations. The simplest way to do this is to introduce a filling factor. We assume that within any small region a fraction $(1 - \xi)$ of the volume is empty while the complementary fraction ξ is filled at density $N_H(\rho)$. Then $N_H(\rho)$ is given by equation (A9) with $N_0 = 500(\xi D)^{-1/2}$, $a'_{1,6} = 1.0$, $a'_{2,5} = 0.25$,

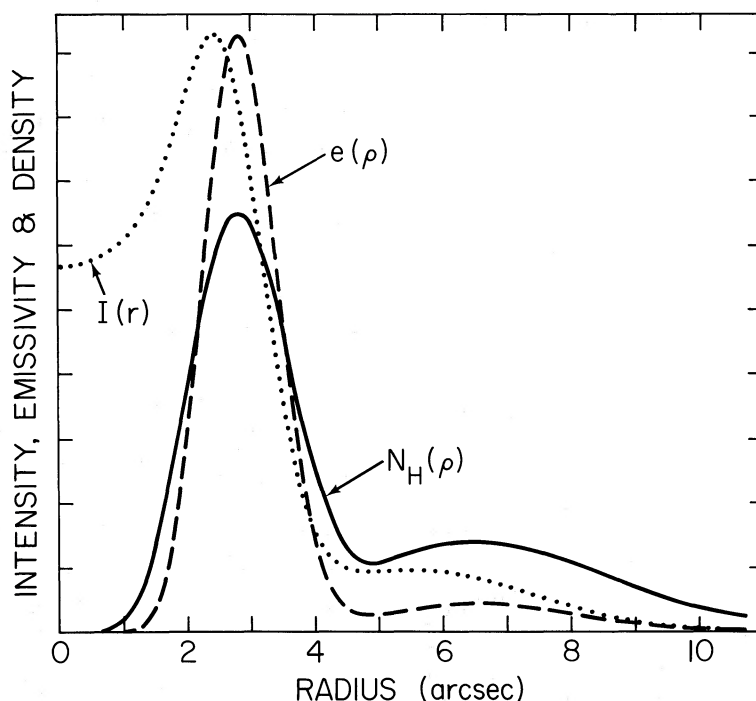


FIG. 5.—Solid curve, the adopted density distribution $N_H(\rho)$. Dashed curve, the corresponding emissivity, $e(\rho)$, as given by eq. (2). Dotted curve, $I(r)$, the projection of $e(\rho)$. The vertical scale is linear but otherwise arbitrary.

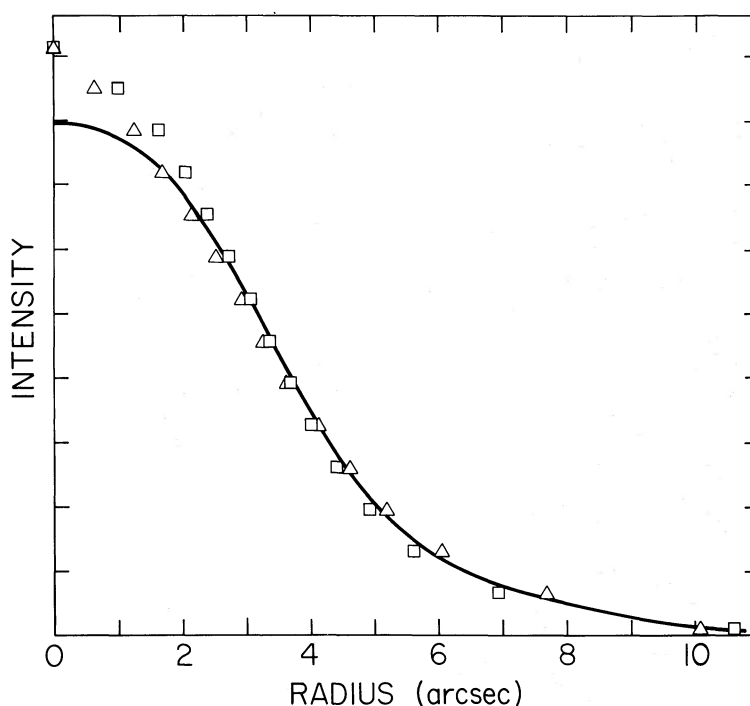


FIG. 6.—The triangles and squares are the averaged [O III] and [O II] contours of Phillips *et al.* (1979). These are compared to the broadened brightness distribution $i(r)$ which results from the application of a Gaussian of FWHM = $3''.66$ to $I(r)$.

$H_1 = 1.15(D/206.265)$ pc, and $H_2 = 2.9(D/206.265)$ pc. The maximum density is $5.38 \times 10^3 (\xi D)^{-1/2}$, which occurs at $\rho = 0.0137D$ pc. An important quantity is the mean value of $N_H(\rho)$ over the whole nebula which we define as

$$\langle N_H \rangle = \frac{\int N_e N_H dV}{\int N_e dV}. \quad (3)$$

Numerical integration over the distribution yields

$$\langle N_H \rangle = 1.71 \times 10^3 (\xi D)^{-1/2} \text{ cm}^{-3}. \quad (4)$$

We also obtain for the total mass of hydrogen in the object

$$M_H = 0.0117 \xi^{1/2} D^{5/2} (M_\odot). \quad (5)$$

VI. DISTANCE AND MASS

As for most planetary nebulae, distance estimates for IC 3568 rest upon the assumptions that the whole class of planetary nebulae has a well defined average mass and that this particular object can be approximated by that average value. A frequently quoted distance estimate for IC 3568 is that of O'Dell (1962), who gives 2.56 kpc. O'Dell's distance scale is based upon a mean hydrogen mass of $0.14 M_\odot$. It is also assumed that all planetary nebulae can be treated as spheres of uniform density which, however, are only 70% filled with radiating material. O'Dell adopted an angular radius of $9''.0$ for IC 3568. If we use instead our nonuniform density distribution, then from equation (5), with $M_H = 0.14$ and $\xi = 0.7$, we obtain $D = 2.90$ kpc.

Such a choice of M_H and ξ , however, implies that $\langle N_H \rangle = 1.2 \times 10^3 \text{ cm}^{-3}$, which is not acceptable because the model will be unable to match the observed density-sensitive [O II] $\lambda 3729/\lambda 3726$ or C III] $\lambda 1907/\lambda 1909$ line ratios. We find that to match the [O II] ratio the model must have $\langle N_H \rangle = 3.8 \times 10^3 \text{ cm}^{-3}$. From equation (4), this imposes the constraint that

$$\xi D = 0.2. \quad (6)$$

One could object to this constraint on the grounds that the [O II] ratio need not be representative of $\langle N_H \rangle$, but could be biased toward regions of higher density. For example, if there were regions of different densities at the same distance from the star, there would be more O^+ in the high-density regions because O^{+2} is always the dominant ion and the higher density increases the recombination rate (see eq. [9] and the discussion below). But this cannot be an important effect because the C III] line ratio implies an even higher density than the [O II] ratio. Since C^{+2} is the dominant carbon ion, C^{+2} will not be enhanced in regions of higher density. To fit both ratios would probably require a variable filling factor which would be smallest in the inner regions of the nebula, where there is less O^+ due to the more intense radiation field.

If we then accept equation (6) and attempt to retain the assumption that $M_H = 0.14 M_\odot$, we find that $\xi = 0.039$ and $D = 5.18$ kpc. This filling factor seems small for an object with such a smooth appearance. Furthermore, this distance would put the nebula 2.9 kpc above the

galactic plane. The scale height of planetary nebulae is about 0.12 kpc (Cahn and Wyatt 1976).

If, on the other hand, we take $\xi = 1.0$, we obtain the minimum distance of $D = 200$ pc. This would imply a very low luminosity for the central star, about $44 L_{\odot}$. Further, the nebular mass would then be only $M_H = 2 \times 10^{-4} M_{\odot}$. The wind from the central star would double the expansion velocity of such a nebular shell in 1000 years.

As a working value, we will adopt $D = 1.0$ kpc and $\xi = 0.2$. This corresponds to a nebular mass of $M_H = 0.0052 M_{\odot}$. It may be that this nebula with its unusually low expansion velocity also has an unusually small mass.

It should be noted that any combination of ξ and D which satisfies (6) will result in a model with exactly the same appearance and spectrum as the one we present here.

VII. STELLAR FLUX

The central star of IC 3568 is classified as O5f by Aller (1976). Although the V magnitude of this star is given as 11.39 mag by Kostjakova *et al.* (1968), they note that in this case "the correction for nebular radiation could not be done quite correctly." Extrapolation from our UV data suggests the star is about 1.8 mag fainter, $V \approx 13.2$ mag.

Observations of the UV continuum can be used to derive Zanstra temperatures. Following the notation used by Bohlin, Harrington, and Stecher (1982, BHS), we define

$$Q(\lambda_0, \lambda_*) = \int_{\lambda_0}^{\infty} \frac{F(v)}{v} dv / F(v_*) . \quad (7)$$

At a reference wavelength of $\lambda_* = 1640 \text{ \AA}$, the stellar continuum will dominate the contribution from the nebular continuum. After correction for interstellar extinction, our *IUE* data yield $F(\lambda_*) = 2.0 \times 10^{-24} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}$. We combine this with the corrected $H\beta$ flux of $F(H\beta) = 2.76 \times 10^{-11} \text{ ergs cm}^{-2} \text{ s}^{-1}$ (Collins, Daub, and O'Dell 1961) to obtain $Q(912, 1640) \gtrsim 0.18$. For a blackbody flux distribution, this implies a hydrogen Zanstra temperature of $T_Z(H \text{ I}) \gtrsim 32,000 \text{ K}$. This lower limit is not likely to be close to the true stellar temperature, however, because IC 3568 is almost surely optically thin in the H^0 continuum. This is indicated by the weakness of the singly ionized spectral lines in the nebular radiation (e.g., $[O \text{ II}]$, $[N \text{ II}]$, $[S \text{ II}]$) and by the observation that $[O \text{ III}] \lambda 5007$ and $[Ne \text{ III}] \lambda 3868$ emission extends to the outer edges of this object.

The He II Zanstra temperature can be obtained from the He II 1640 $\text{\AA}$ line. Since the equivalent width expresses a line flux in terms of the adjacent continuum, we can combine equation (7) with equation (3) of BHS to obtain

$$Q(228, 1640) = 1.15 \times 10^{-3} (10^{-4} T)^{0.1} W(\lambda 1640) , \quad (8)$$

where $W(\lambda 1640)$ is the equivalent width in angstroms of the nebular He II line referred to the stellar continuum. From SWP 14868 we find $W = 0.78 \text{ \AA}$ and hence that

$Q(228, 1640) = 0.00090$. The Zanstra temperature implied by this value of Q depends greatly upon the assumptions about the stellar atmosphere (see Fig. 4 of BHS). For a blackbody flux distribution, $T_Z(\text{He II}) = 53,000 \text{ K}$, while the fluxes given by the plane-parallel LTE model atmosphere calculations of Hummer and Mihalas (1970, HM) would imply $T_Z(\text{He II}) \approx 80,000 \text{ K}$.

Computed nebular models show that a 53,000 K blackbody spectrum gives far too little heating or ionization to the gas. The 80,000 K model atmospheres, on the other hand, supply far too much heating. Since less than 10% of the star's energy is radiated beyond the He^+ ionization threshold at 54.4 eV, it seemed reasonable to ignore $T_Z(\text{He II})$ and make use of a model atmosphere with an energy distribution between 13.6 and 54.4 eV that is able to provide the necessary heating. We found that HM model No. 209 ($T_{\text{eff}} = 50,000 \text{ K}$, $\log g = 4.0$) was satisfactory in this regard.

We may compare the flux distribution of this model with the UV continuum seen on our *IUE* spectra. Figure 7a shows the continuum fluxes from LWR 11446 and SWP 13875, corrected for extinction with $c = 0.25$. We also plot the *ANS* measurements by Pottasch *et al.* (1978). There is close agreement with the *IUE* data points. This data is compared with the flux from the HM model to which is added the nebular continuum emission as seen through the *IUE* large aperture, this later contribution computed from our nebular model. It is seen that the sum of the stellar and nebular continua is a good fit to the observed spectrum. While the nebular continuum is not very important for $\lambda < 2000 \text{ \AA}$, it is clearly present in the data at $\lambda > 2500 \text{ \AA}$.

We also plot the flux implied by the V magnitude of Kostjakova *et al.* (1968), corrected for extinction as is the other data. It is over a factor of 5 above the model flux. It seems likely that this magnitude measurement is contaminated not only by nebular continuous emission by nebular emission lines as well.

This fit of the observed UV continuum by the HM model atmosphere flux implies a stellar radius of $R_* = 0.28 R_{\odot}$ and a luminosity $L_* = 435 L_{\odot}$, assuming a distance $D = 1$ kpc. Models constructed with this stellar radius, however, were found to have too large a fraction of singly ionized species such as O^+ , N^+ , and S^+ . For example, the $[O \text{ II}] \lambda 3727$ doublet is 3 times too strong in such a model.

In all our models of IC 3568, over 90% of the oxygen is in the form of O^{+2} , and the fraction of H^0 is small enough that radiative recombination dominates charge transfer with hydrogen. Thus we can write the fraction of singly ionized oxygen at a distance ρ from the central star as

$$\frac{N(O^+)}{N(O)} = \left[\frac{\rho}{R_*} \right]^2 N_e \alpha(O^+) \left[\int_0^{\lambda_0} \frac{\pi F(\lambda) \lambda}{hc} a_{\lambda}(O^+) d\lambda \right]^{-1} , \quad (9)$$

where $a_{\lambda}(O^+)$ and $\alpha(O^+)$ are photoionization and recombination cross sections of O^+ , respectively, and the threshold $\lambda_0 = 353 \text{ \AA}$. The nebula is so thin—the absorp-

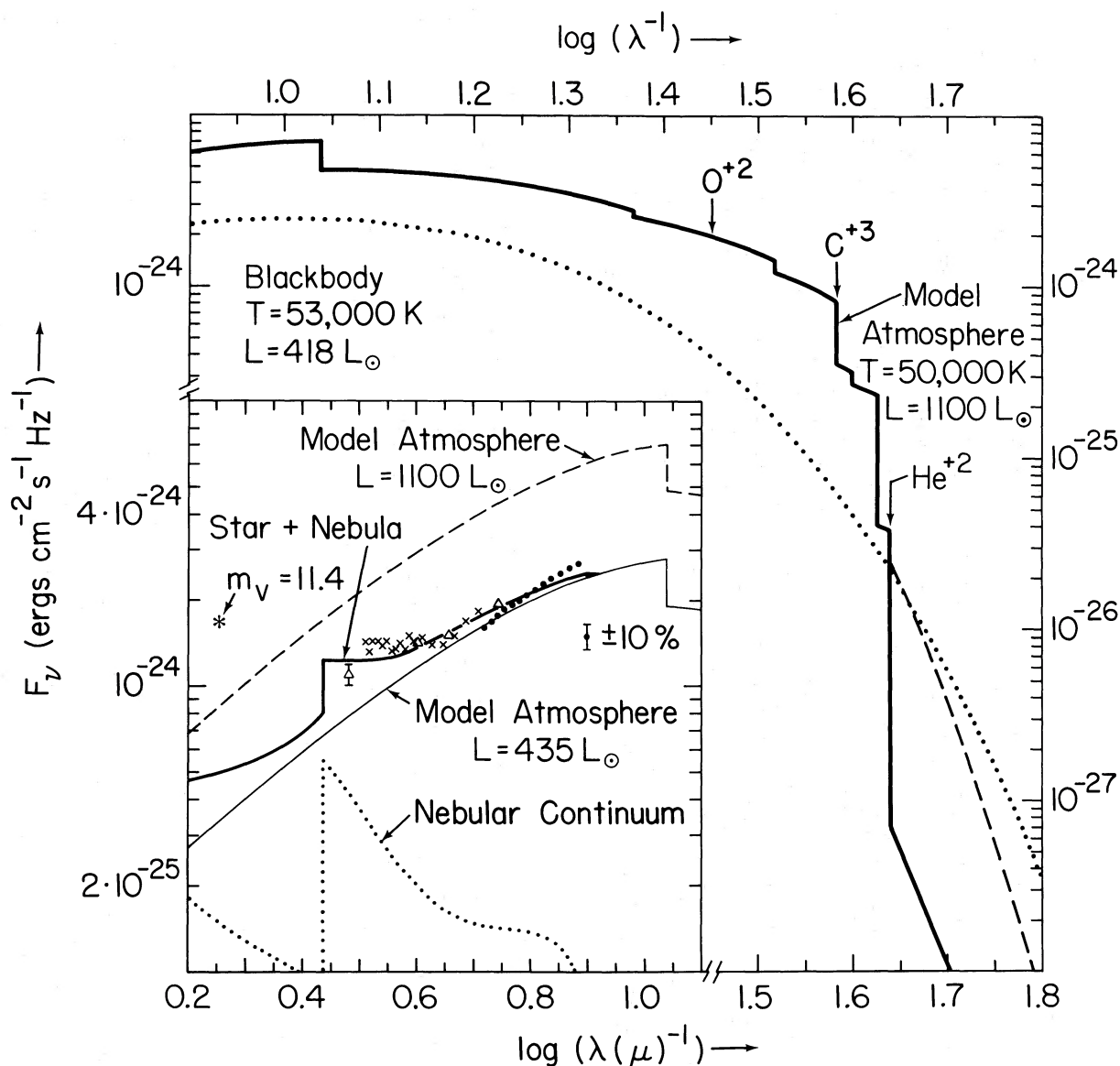


FIG. 7.—(a) (lower left inset) The ultraviolet flux distribution of the model atmosphere and the predicted nebular continuum. The sum (Star + Nebula) is compared with the measured continuum (corrected for extinction with $c = 0.25$) from SWP 13875 (●) and LWR 11446 (×). The triangles are the ANS values of Pottasch *et al.* (1978). (b) (upper right) The flux of Hummer and Mihalas (1970) model atmosphere No. 209 is shown as the heavy line. The dotted line is a blackbody at 53,000 K. The dashed segment shows how the model atmosphere was modified for $\lambda < 228 \text{ \AA}$. The ionization thresholds of several ions are indicated.

tion due to H^0 at $350 \text{ \AA}$ is less than 6%—that we can neglect the attenuation of the stellar radiation by the nebular gas.

We have already fixed N_e as a function of ρ from the VLA radio frequency map and the density-sensitive line ratios. If we try to lower the O^+ fraction by simply increasing the stellar flux $F(\lambda)$ for $\lambda < 353 \text{ \AA}$, the increased heating will result in a poor model. We therefore found it necessary to increase R_* to $0.444 R_\odot$, which corresponds to $L_* = 1100 L_\odot$.

It may be that the actual central star flux deviates from

the HM plane-parallel LTE model in such a way that it is fainter by a factor of 2.5 for $\lambda > 912 \text{ \AA}$. Alternatively, there could be a problem with the spatial distribution of the gas which we have derived—this would require that the material be moved 37% closer to the star. Finally, it might be that more complex variations of the gas density than those contemplated by our use of a constant filling factor ξ could produce the required $[\text{O II}]$ line ratio with a lower average value of $N(\text{O}^+)/N(\text{O})$. Thus, while the strength of the $[\text{O II}]$ lines indicates that the absolute value of the stellar flux corresponds to a luminosity of

TABLE 5
ADOPTED CHEMICAL ABUNDANCES

Ratio	Model	Sun	Reference ^a	Model/Solar
He/H	0.098	...	...	...
C/H	2.9 (-4)	4.7 (-4)	L	0.62
N/H	5.0 (-5)	9.8 (-5)	L	0.51
O/H	3.7 (-4)	8.3 (-4)	L	0.45
Ne/H	8.0 (-5)	1.5 (-4)	ACJ, BFR ^b	0.53
S/H	1.0 (-5)	1.7 (-5)	LL	0.59
Si/H	2.15 (-5)	4.3 (-5)	LL	0.50
Mg/H	2.1 (-5)	4.2 (-5)	LL	0.50

^a L, Lambert 1978. LL Lambert and Luck 1978. ACJ, Acton, Catura, and Joki 1975. BFR, Bertsch, Fichtel, and Reames 1972.

^b Mean of ACJ and BFR: Ne/O = 0.18.

1100 $L_{\odot}$ at 1 kpc, the uncertainties in the spatial distribution of the gas limits our confidence in this result.

Finally, the observed He II line fluxes require that we increase the model atmosphere flux $F(\lambda)$ by a factor of 35 for $\lambda < 228 \text{ \AA}$. The flux of the HM model and the required modification is shown in Figure 7b.

VIII. THE NEBULAR MODEL

An extensive description of the computer program used to construct the nebular models is given by Harrington *et al.* (1982), where references to the adopted atomic parameters are also given.

We present results for three models: B, A1, and A2. Model A2, computed with HM model atmosphere

No. 209 (modified for $\lambda < 228 \text{ \AA}$) at $L_{*} = 1100 L_{\odot}$, is intended to be a realistic representation of conditions in the nebula. Models A1 and B are presented solely to illustrate the discussion in the foregoing section concerning the stellar flux. Model A1 is identical to A2 except that $L_{*} = 435 L_{\odot}$, while Model B makes use of a black-body central star with $T_{*} = 53,000 \text{ K}$ and $L_{*} = 418 L_{\odot}$. In the discussion that follows, we will refer to model A2 unless otherwise specified.

Table 5 lists the chemical abundances adopted, along with the solar abundances for comparison. These abundances were adjusted for the best overall agreement with the observed emission line fluxes, with the exception of Si and Mg, which are not constrained by observed lines. They were simply set at half the solar value, in line with the trend of the other elements, although it is quite likely that these elements would be depleted by the formation of solid grains (e.g., Harrington and Marioni 1981).

To compare the models with observations, we have integrated the projected brightness of the ultraviolet lines over the $10'' \times 20''$ oval aperture which is appropriate for the IUE observations, and have integrated the optical lines over a $8'' \times 200''$ rectangular aperture to match the slit used by Barker (1978). We express the results on a scale of $H\beta = 100$. The brightness distribution of the model is such that 90% of the total $H\beta$ flux enters the $10'' \times 20''$ oval while 84% enters the $8'' \times 200''$ slit.

In Table 6 we compare these calculated values with the observed fluxes, which have been corrected for

TABLE 6
COMPARISON OF CALCULATED AND OBSERVED LINE FLUXES (corrected for extinction and relative to $H\beta = 100$)

Ion (1)	λ (Å) (2)	$f(\lambda)$ (3)	Model B (4)	Model A1 (5)	Model A2 (6)	Observed (7)	Ref. (8)
H I	4861	0.00	100.0	100.0	100.0	100.0	...
H I	6563	-0.323	286.0	286.0	286.0	274.0 ± 9.0	B
He I	4471	0.094	4.81	4.81	4.77	6.71 ± 1.6	B
He I	5876	-0.216	13.4	13.3	13.1	13.2 ± 0.7	B
He II	1640	1.136	7.80	2.69	6.93	7.1 ± 1.4	IUE
He II	4686	0.042	1.27	0.436	1.12	1.23	L
C II	4267	0.139	0.241	0.242	0.235	<0.25	L
C III]	1908	1.226	54.1	91.3	111.0	111.0 ± 4.4	IUE
C IV	1549	1.184	1.38	4.20	15.6	19.6 ± 4.0	IUE
[N II]	6584	-0.326	4.36	8.43	2.43	2.15 ± 0.41	B
N III]	1751	1.120	1.59	2.79	3.39	4.1 ± 3.0	IUE
[O II]	3727	0.256	36.6	20.8	6.78	6.88 ± 1.2	B
[O II]	7325	-0.409	3.40	2.12	0.743	1.17 ± 0.65	B
O III]	1663	1.128	3.34	6.43	8.53	7.6 ± 1.5	IUE
[O III]	4363	0.119	4.11	7.21	8.92	7.68 ± 1.0	B
[O III]	5007	-0.034	764.0	1015.0	1143.0	1033.0 ± 11.0	B
[Ne III]	3869	0.223	51.0	73.6	85.0	77.2 ± 2.9	B
[S II]	6725	-0.347	3.97	3.05	0.60	<1.1	L
[S III]	3722	0.256	1.19	1.16	0.63	0.65	D
[S III]	9531	-0.610	74.6	62.6	32.2	13.7 ± 3.4	D
[S IV]	10.5 μm	-0.996	20.7	38.3	62.6	80.0 ± 16.0	D
$\pi F(H\beta)$	...	...	2.89 (-11)	2.73 (-11)	2.68 (-11)	$2.76 (-11)$	CDO
Radio	2.7 GHz	...	0.0918	0.0896	0.0890	$0.089 \pm 0.005 \text{ Jy}$	SK

REFERENCES.—B, Barker 1978. CDO, Collins, Daub, and O'Dell 1961. D, Dinerstein 1980. IUE, this paper. L, Lee *et al.* 1969. SK, Sistla and Kaftan-Kassim 1976.

TABLE 7
DENSITY-SENSITIVE LINE RATIOS

Ratio	Model A2	Observed	Reference
C III] $\lambda 1907/\lambda 1909$	1.25	1.16	Feibelman <i>et al.</i> 1981
[O II] $\lambda 3728/\lambda 3726$	0.50	0.51	Lee <i>et al.</i> 1969

extinction using the reddening function $f(\lambda)$ given in column (3) and taking $c = 0.25$. Where the observers give an estimate of the errors, these are included. The agreement of model A2 with observation is good. Table 7 shows the comparison with observations of the density sensitive line ratios of model A2.

The electron temperature and ionization structure of the model are quite simple. The optical depth at the hydrogen ionization threshold of 912 Å is $\tau(H^0) = 0.41$. As a result, the absorption of the radiation by the nebular gas does not greatly distort the stellar continuum and conditions are rather uniform throughout the nebular volume. The highest temperature, which occurs at the inner edge, is 13,500 K, but this drops to 10,600 K outside the very small He^{+2} zone. The minimum temperature is 9940 K, and the mean value is 10,170 K. Define the average fractional abundance of the ij ion as

$$f(N_{ij}) = \langle N_{ij} \rangle / \sum_k \langle N_{ik} \rangle, \quad (10)$$

where the average abundance $\langle N_{ij} \rangle$ is defined by equation (3) and the summation is over all ionization stages of the element i . The fraction of doubly ionized helium is small, $f(He^{+2}) = 0.0084$. The fractional abundances of C^{+2} , N^{+2} , O^{+2} , and Ne^{+2} are 0.86, 0.88, 0.98, and 0.98, respectively. The abundances of C^{+3} and N^{+3} are both 0.11. Only S and Si show much diversity of ionization, with the major stages being $f(S^{+3}) = 0.71$ and $f(Si^{+3}) = 0.57$. Because of this simple ionization structure, the abundances of C, O, and Ne are well determined on the basis of the C III], [O III] and [Ne III] line fluxes, respectively.

The fractional abundances of the singly ionized species N^+ , O^+ , S^+ , and Mg^+ are sensitive to the details of the model density distribution and to the intensity of the stellar radiation as discussed above. Since the average fraction of neutral hydrogen is only $f(H^0) = 0.0007$, charge transfer is of secondary importance. A model constructed without charge transfer showed only a 25% reduction in the [O II] and [N II] line intensities.

For completeness, Table 8 lists the strongest lines in the computed UV spectrum of model A2, regardless of whether they were observed. The last column gives the flux expected at the Earth including the effects of extinction. (These values are for the entire nebula, not just the flux through the projected area of the IUE large aperture as in Table 6.)

Our failure to detect the Si III] lines is probably an indication that the Si abundance is lower than that assumed in the model. Because of interstellar absorption, the C II $\lambda 1334.5$, 1335.7 feature is not measurable.

TABLE 8
PREDICTED UV SPECTRUM OF IC 3568

Ion	λ (Å)	$f(\lambda)$	$100 \cdot \frac{F(\lambda)}{F(H\beta)}$	$10^{-d(1+f(\lambda))} \cdot F(\lambda)$
C II	1335	1.416	13.5	0.926
Si IV	1397, 1403	1.312	4.19	0.305
S IV]	1398-1424	1.291	0.363	0.027
N IV	1487	1.229	0.319	0.024
C IV	1549	1.184	14.9	1.17
He II	1640	1.136	6.25	0.504
O III]	1663	1.128	8.42	0.682
N III]	1751	1.120	3.36	0.273
Si III]	1883, 1892	1.196	12.6	0.981
C III]	1908	1.226	110.0	8.42
C III]	2297	1.431	0.875	0.060
[O III]	2321	1.367	2.12	0.150
C II]	2326	1.354	6.75	0.480
[O II]	2470	1.025	0.540	0.046
Mg II	2800	0.646	10.4	1.11

NOTE.—Values in the last column in units of 10^{-12} ergs cm^{-2} s^{-1} .

IX. THE CHEMICAL ABUNDANCES

The oxygen and neon abundances should be well determined. With regard to carbon, there is the possibility, not considered in these models, that the observed C IV $\lambda 1549$ line is attenuated by internal dust (Harrington, Lutz, and Seaton 1981). However, even if there were 3 times as much C^{+3} as we have inferred from C IV $\lambda 1549$, this would increase the total carbon abundance by only 20%. We cannot be very sure of the nitrogen abundance because, on one hand, the [N II] $\lambda 6548$ line is not strong and represents only a minor stage of ionization [$f(N^+) = 0.0103$], while on the other, the measured N III] $\lambda 1751$ flux is uncertain.

Comparison of the observed [S III] $\lambda 9531$ and [S IV] $10.5 \mu m$ fluxes with the model predictions indicates that our model overestimates the S^{+2}/S^{+3} ratio. This problem has little effect on the abundance because the value adopted is a compromise taking both lines into account.

Our abundance analysis of IC 3568 shows little evidence for the enhancement of particular elements such as carbon or nitrogen. All the heavy elements seem to be systematically below solar values by a factor of 2. This may be compared with the situation in a number of other planetary nebulae where the oxygen and neon abundances are about half solar, but the carbon and nitrogen abundances are enhanced (e.g., Harrington *et al.* 1980; Harrington, Lutz, and Seaton 1981; Marionni and Harrington 1981; Peimbert 1981).

X. CONCLUSIONS

Although the UV nebular lines in IC 3568 are few, they are important for the analysis of this object. The O III $\lambda\lambda 1661, 1666$ lines provide confirmation of the electron temperature obtained from the optical [O III] $\lambda 5007/\lambda 4363$ ratio and the He II $\lambda 1640$ line provides an independent measure of the interstellar extinction. The most important lines, however, are C III $\lambda 1908$ and C IV $\lambda 1549$, which enable us to determine the carbon abundance. We find no enhancement of carbon relative to other elements.

We have used the nebular model for the determination of the chemical abundances, but it should be clear that for IC 3568 a model is not actually needed. Using the electron temperature from the [O III] line ratio, one could obtain essentially the same result by simply computing ionic abundance from the observed [O II], [O III], [Ne III], C III, and C IV lines. If we were to use only the [N II] line flux and make the usual assumption that $f(\text{N}^+) = f(\text{O}^+)$, we would infer only 30% more nitrogen than results from the computed $f(\text{N}^+)$. Even with the model, uncertainties in the nitrogen abundance of this magnitude remain. The real importance of the model is in the information we are able to extract regarding the far-UV flux of the central star.

With the *IUE* we directly observe the stellar flux in the 1250 Å–3000 Å range. The slope of this continuum (after correction for the nebular continuous emission) can be fitted well enough by a stellar atmosphere—or a blackbody—with an effective temperature of about 50,000 K.

The nebular model complements this by probing the stellar radiation field at wavelengths below 912 Å. This occurs in several ways. First, the total flux which the nebula radiates in collisionally excited lines is determined by the slope of the stellar continuum between 912 Å and 228 Å. This is equivalent to measuring the temperature of the star by Stoy's method. In addition, the ionic fractions of O^+ , C^{+3} , and He^{+2} are determined by the number of photons below 350 Å, 260 Å, and 228 Å, respectively. The $f(\text{O}^+)$ and $f(\text{C}^{+3})$ are also sensitive to the density and spatial distribution of the nebular gas, since the nebula is optically thin for $\lambda > 228$ Å. The stellar flux for $\lambda < 228$ Å which we infer from the He II lines is model independent.

We find, not surprisingly, that blackbody distributions are not suitable. Furthermore, no available model atmosphere seems to have the necessary properties. The required slope in the 912 Å–228 Å range can be provided by a plane-parallel model of Hummer and Mihalas with an effective temperature of 50,000 K, but the absolute flux has to be raised a factor of 2.5 above the level implied by extrapolation of the observed *IUE* continuum in order to give the needed ionization level. This result is not certain because of the dependence on our adopted density distribution.

What is certain is that such a model atmosphere fails by a factor of 35 to provide the required flux below 228 Å responsible for the He^{+2} ionization. IC 3568 does not seem to be unusual in this respect: Kaler (1976)

evaluated the Stoy temperature for a large number of lower excitation nebulae assuming model atmosphere flux distributions. He found that those nebulae with faint nebular He II $\lambda 4686$ (i.e., those with stars just hot enough to have a few percent of helium doubly ionized) had Stoy temperatures of about 46,000 K, much cooler than the He II Zanstra temperatures of around 85,000 K inferred from the He II line flux. Our detailed study of this object supports his conclusion on this point. Kaler, however, found blackbody flux distributions to give consistent temperatures, while we find them unsatisfactory.

It is not surprising that plane-parallel LTE model atmospheres cannot reproduce the extreme UV flux from a star with mass outflow via an 1800 km s^{-1} stellar wind. A “warm” region in the wind just above the photosphere might produce enough radiation at $\lambda < 228$ Å to cause the observed He^+ ionization. Thus the modeling of planetary nebulae surrounding nuclei with mass loss may be important to stellar wind theory, for it provides information about the far-UV stellar flux not available in the case of early type main-sequence stars.

Finally, we find that this nebula probably has a rather low mass and, in spite of its smooth appearance, a filling factor substantially less than unity. The C III line ratio indicates that the filling factor may be smallest in the inner part of the nebula. This could result from the observed stellar wind, which upon encountering the nebular shell produces a rarefied gas with a temperature of 5×10^7 K. Pikel'ner (1968, 1973) has shown how such hot gas might produce and contain filaments via a mechanism similar to the Rayleigh-Taylor instability. There are, however, significant unanswered questions regarding the interaction of this hot component with the nebular gas (Harrington 1982).

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Note added 1982 July 12.—After this paper was accepted for publication, it was discovered that the software used in the reduction of the low-resolution *IUE* spectra (e.g., Fig. 1) did not incorporate the final absolute calibration. The necessary corrections can be found in “Comprehensive Revision to the *IUE* Absolute Calibration in Low Dispersion,” by R. Bohlin and A. Holm, *IUE Newsletter* No. 10, 1980 June. Use of the new calibration constants would result in small (8–12%) increases in our observed SWP fluxes [e.g., $F(\lambda)$ in Table 2], which in turn imply small adjustments in the model parameters, none of which affect the conclusions of this paper.

Note added in manuscript 1982 September 9.—Recent calculations of collisional excitation rate parameters for S IV (P. L. Dufton, A. Hibbert, A. E. Kingston, and G. A. Doschek 1982, *Ap. J.*, **257**, 338) and for S III (C. Mendoza, *J. Phys. B*, to be published) are significantly

larger than those used in this paper. With these new rates, a reduced sulfur abundance of $S/H = 4.0 \times 10^{-6}$ will give exact agreement with the observed [S IV] 10.5 μm line and improve agreement with the [S III] $\lambda 9531$ line.

APPENDIX

Let the emission per unit volume from a spherically symmetric object have the form

$$e(\rho) = \sum_{k=1}^K \sum_{n=0}^N a_{k,n} \left(\frac{\rho}{H_k} \right)^{2n} \exp \left[- \left(\frac{\rho}{H_k} \right)^2 \right], \quad (\text{A1})$$

where ρ is the radial distance and the H_k and $a_{k,n}$ are constants. Then the projected brightness along a line of sight whose closest approach to the center is r is given by

$$I(r) = \frac{1}{2\pi} \int_r^\infty \frac{e(\rho) \rho d\rho}{(\rho^2 - r^2)^{1/2}} = \sum_{k=1}^K \sum_{n=0}^N b_{k,n} \left(\frac{r}{H_k} \right)^{2n} \exp \left[- \left(\frac{r}{H_k} \right)^2 \right], \quad (\text{A2})$$

where

$$b_{k,n} = \frac{H_k}{4(\pi)^{1/2}} \sum_{l=n}^N \frac{l! (l-n)! 2^{n-l}}{(l-n)! n!} a_{k,l}. \quad (\text{A3})$$

Here, $(0)!! = 1$ and $m!! = 1 \cdot 3 \cdots (2m-1)$ for $m \geq 1$.

If the effects of seeing and instrumental broadening can be represented by a Gaussian point spread function

$$S(x, y) = \frac{1}{\pi s^2} \exp \left(- \frac{x^2 + y^2}{s^2} \right),$$

then the observed intensity distribution is given by the convolution integral

$$i(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(x - x', y - y') I(x', y') dx' dy',$$

or, in polar coordinates,

$$i(r, \theta) = \int_0^\infty \int_0^{2\pi} S(r, r', \theta - \theta') I(r', \theta') d\theta' r' dr',$$

where $S(r, r', \theta - \theta') = (\pi s^2)^{-1} \exp(-d^2/s^2)$ and the distance d is given by $d^2 = r^2 + (r')^2 - 2rr' \cos(\theta - \theta')$. Then, if $I(r)$ is independent of θ , we can integrate over θ' to obtain

$$i(r) = \frac{2}{s^2} \exp \left[- \left(\frac{r}{s} \right)^2 \right] \int_0^\infty \exp \left[- \left(\frac{r'}{s} \right)^2 \right] J_0 \left(\frac{2rr'}{s} \right) I(r') r' dr', \quad (\text{A4})$$

where, under the integral sign, $i = (-1)^{1/2}$ and J_0 is the Bessel function.

If $I(r)$ is given by (A2), the integration over r' yields degenerate hypergeometric functions ${}_1F_1$ (Gradshteyn and Ryzhik 1965, pp. 716 and 1058) which are of the form

$${}_1F_1(n+1, 1; z) = e^z \sum_{l=0}^n \frac{n! z^l}{(n-l)! (l!)^2}.$$

The resultant $i(r)$ can be written

$$i(r) = \sum_{k=1}^K \sum_{n=0}^N c_{k,n} \left(\frac{r}{Q_k} \right)^{2n} \exp \left[- \left(\frac{r}{Q_k} \right)^2 \right], \quad (\text{A5})$$

where

$$Q_k^2 = H_k^2 + s^2 \quad (\text{A6})$$

and the constants $c_{k,n}$ are

$$c_{k,n} = (\zeta_k)^{n+1} \sum_{l=n}^N \frac{(l!)^2 (1 - \zeta_k)^{l-n}}{(n!)^2 (l-n)!} b_{k,l}, \quad (\text{A7})$$

with $\zeta_k = (H_k/Q_k)^2$ and $(1 - \zeta_k) = (s/Q_k)^2$.

If coefficients of (A5) are chosen to represent the observed brightness, then for a given s we can infer the H_k and $b_{k,n}$ and thus find the $a_{k,n}$ which define the emission $e(\rho)$. The $a_{k,n}$, however, are very sensitive to the highest terms $c_{k,N}$. It is better to vary the $a_{k,n}$ and compare the resultant $i(r)$ with the observed distribution.

The total emission from the object is given by

$$\mathcal{L} = 4\pi \int_0^\infty e(\rho) \rho^2 d\rho = \pi^{3/2} \sum_{k=1}^K H_k^3 \sum_{n=0}^N \frac{(n+1)!!}{2^n} a_{k,n}. \quad (\text{A8})$$

This result is useful in setting the density scale. We write

$$N_H(\rho) = N_0 \left\{ \sum_{k=1}^K \sum_{n=0}^N a'_{k,n} \left(\frac{\rho}{H_k} \right)^{2n} \exp \left[- \left(\frac{\rho}{H_k} \right)^2 \right] \right\}^{1/2}. \quad (\text{A9})$$

Let the total $H\beta$ flux observed at the Earth be $\pi F(H\beta)$ so that

$$\mathcal{L}(H\beta) = 4\pi D^2 \pi F(H\beta) 10^c,$$

where D is the distance of the nebula and c the extinction at $H\beta$. Assume that helium is singly ionized so that $N_e = [1 + N(\text{He})/N(\text{H})]N_H$ and that the electron temperature is uniform. The volume emissivity in $H\beta$ which corresponds to the density distribution (A9) is then just (A1) with $a_{k,n} = e_0 a'_{k,n}$ where

$$e_0 = h\nu\alpha_{\text{eff}}(H\beta)[1 + N(\text{He})/N(\text{H})]N_0^2.$$

Making use of $h\nu\alpha_{\text{eff}}(H\beta) = 1.24 \times 10^{-25} t^{-0.87}$ ergs $\text{cm}^3 \text{s}^{-1}$, where $t = 10^{-4} T_e$, we finally obtain from equation (A8)

$$N_0^2 = \frac{5.176 \times 10^{19} \pi F(H\beta) 10^{c 0.87}}{[1 + N(\text{He})/N(\text{H})] D_{\text{kpc}}} \left[\sum_{k=1}^K h_k^3 \sum_{n=0}^N \frac{(n+1)!!}{2^n} a'_{k,n} \right]^{-1}, \quad (\text{A10})$$

where D_{kpc} is in kiloparsecs and the h_k are angular radii in arcsec corresponding to the radial scale factors H_k .

For the models presented in this paper, we take $K = 2$, $N = 6$, $h_1 = 1.15$, $h_2 = 2.9$, and all $a'_{k,n} = 0$ except $a'_{1,6} = 1$ and $a'_{2,5} = 0.25$.

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