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The lunar ephemeris ELP 2000

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Summary. A semi-analytical solution, for the motion of the Moon, has been elaborated by the authors, the most recent version being named ELP 2000-82. It contains all the sensible effects acting on the Moon (main problem, Earth and Moon figure perturbations including libration, planetary actions including secular variations of solar elements and motion of the ecliptic, relativistic perturbations and tidal effects). The main problem is solved using values of the lunar and solar parameters as listed in Table 1. The other effects are treated as perturbations to the main problem. Table 2 contains the constants used for building these perturbations. The secular variations of the motions of the perigee (w_2) and node (w_3) are gathered in Tables 5 and 7, using the preliminary constants mentioned above. An interesting feature of these tables is to separate the various effects.

An ephemeris has been built and compared, over one century, to JPL numerical integration, LE 200. From this analysis we derive a new set of lunar and solar elements: S_{200}. It is definitely this set (Table 9) which has been retained to elaborate the lunar ephemeris as it will appear in the *Connaissance des Temps* from 1984 onwards. The complete expressions of lunar and solar arguments are listed in Table 11.

ELP 2000-82 is referred to the dynamical ecliptic and dynamical equinox 2000, γ^I_{ELP} (I stands for inertial as opposed to R for rotational or observed equinox). Calling the equinox of LE 200, γ^R_{200}, the comparison brings the value of $\gamma^R_{200}\gamma^I_{\mathrm{ELP}}$ as well as the difference of the two obliquities: $\varepsilon^I_{\mathrm{ELP}}-\varepsilon_0$, ε_0 being the IAU value for $J2000$.

The set of constants S_{200} is compared with other sets resulting from previous fits of semi-analytical solutions ELP to numerical integration (LE 51). In particular one estimates the influence of the accuracy of the solution itself and of the time span of comparison, for determination of the constants.

Finally, the accuracy of the ephemeris ELP 2000 is discussed. It depends mainly on three components: the internal accuracy of the semi-analytical solution, the accuracy of the constants which receive assigned values (like Love number and phase, G', J_2 etc....), and the accuracy of the fitted constants. The semi-analytical solution can be numerically improved, by adding the difference ϱ between ELP and a given numerical integration, over one century. Hence, ELP$+\varrho$ can be compared to any other numerical integration, or observations, ϱ being not influenced by changes in the constants. It is a way to gain accuracy and reach the decimeter level to make comparison with laser observations, over a time span of 10 yr.

Key words: celestial mechanics – lunar theory

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Introduction

An ephemeris of the Moon, derived from the semi-analytical solution ELP 2000-82, will be introduced in the french ephemerides *Connaissance des Temps* from 1984 onwards. The construction of such an ephemeris can be divided in three fundamental major topics: The semi-analytical solution, the constants, the reference frame. This paper is intended to give a precise description of these contributions and a discussion of the accuracy of the ephemeris.

I. The semi-analytical solution ELP 2000-82

a) General characteristics

The general formulation in ELP 2000-82 for each polar coordinate, longitude, latitude and distance is:

$$w_1\delta_V+\sum_{n\geqq 0} t^n \sum_{i_1\ldots i_p} A^{(n)}_{i_1\ldots i_p}\sin(i_1\lambda_1+i_2\lambda_2+\ldots+i_p\lambda_p+\phi^{(n)}_{i_1\ldots i_p}), \quad (1)$$

where δ_V is 1 for longitude, 0 otherwise; t is the time, p is a fixed integer, $A^{(n)}_{i_1\ldots i_p}$ are numerical coefficients, $\phi^{(n)}_{i_1\ldots i_p}$ are numerical phases, λ_j $(1\leqq j\leqq p)$ are literal arguments standing for polynomial functions of the time

$$\lambda_j=\sum_{k\geqq 0}\lambda_j^{(k)}t^k. \quad (2)$$

In the expression (2), $\lambda_j^{(0)}$ is a literal coefficient and $\lambda_j^{(k)}$ $(k\geqq 1)$ are numerical. w_1 is the sidereal mean longitude of the Moon which has the same formulation as λ_j. ELP 2000-82 has been built with the following methods:

The main problem has been solved separately (Chapront-Touzé, 1980). It gives rise to series with the formulation (1) where $n=0$ for the three polar coordinates. The phase $\phi^{(n)}_{i_1\ldots i_p}$ is always zero for longitude and latitude and 90° for distance. The λ_j are reduced to the four Delaunay arguments D, l', l, F and they are linear functions of time [$k\leqq 1$ in (2)]. So does w_1. We have also obtained the first derivatives of the $A^{(0)}_{i_1\ldots i_4}$ and mean motions $\lambda_j^{(1)}$ with respect to the constants used (Chapront-Touzé, 1982). These constants are:

$$\begin{aligned}
&\nu=w_1^{(1)}, \text{ sidereal mean motion of the Moon}\\
&E, \text{ half coefficient of } \sin l \text{ in longitude}\\
&\Gamma, \text{ half coefficient of } \sin F \text{ in latitude}\\
&e', \text{ solar eccentricity}\\
&n', \text{ sidereal mean motion of the Sun}\\
&\sigma_1=m_L/(m_T+m_L)\\
&\sigma_2=(m_T+m_L)/(m_S+m_T+m_L)\\
&G'=Gm_T
\end{aligned} \quad (3)$$

m_T, m_L, m_S stand respectively for the masses of Earth, Moon,

Table 1. Initial values of the main problem constants

ν	1,732,559,343.″18 cy^{-1}
$2E$	22,639.″55
2Γ	18,461.″40
e'	0.01670924
n'	129,597,742.″34 cy^{-1}
σ_1	0.012150568
σ_2	3.040423956 10^{-6}
G'	3.986005 10^{14} m^3 s^{-2}

Table 2. Constants used for the computation of the perturbations in ELP 2000–82

J_2	0.00108263	k_2	0.30
J_3	−0.254 10^{-5}	δ	0.0407
$\varepsilon^{(0)}$	23°26′21.″448	β	0.632108 10^{-3}
$\varepsilon^{(1)}$	−46.″8150 cy^{-1}	γ	0.228443 10^{-3}
p	5029.″0966 cy^{-1}	J_{2L}	0.20215 10^{-3}
a_T	6,378,140 m	J_{3L}	0.12126 10^{-4}
$e'^{(1)}$	−0.42037954 10^{-4} cy^{-1}	C_{22L}	0.22304 10^{-4}
$e'^{(2)}$	−0.122196 10^{-6} cy^{-2}	C_{31L}	0.3071 10^{-4}
$\tilde{\omega}'^{(1)}$	1161.″2141 cy^{-1} [a]	S_{31L}	0.56107 10^{-5}
$\tilde{\omega}'^{(2)}$	0.″5453 cy^{-2} [a]	C_{32L}	0.48884 10^{-5}
$P'^{(1)}$	0.10186 10^{-4} cy^{-1}	S_{32L}	0.1687 10^{-5}
$P'^{(2)}$	0.46965 10^{-6} cy^{-2}	C_{33L}	0.1436 10^{-5}
$Q'^{(1)}$	−0.113467 10^{-3} cy^{-1}	S_{33L}	−0.33435 10^{-6}
$Q'^{(2)}$	0.12420 10^{-6} cy^{-2}	a_L	1,738,000 m

[a] These values from VSOP 80 are slightly different from those of table 11 coming from VSOP 82

and Sun. The nominal values of the constants used for the computation of the $A^{(0)}_{i_1 \ldots i_4}$ and $\lambda^{(1)}_j$ are given in Table 1.

The other effects are treated as perturbations to the main problem. The method used for their computation is similar to Brown's method (Chapront-Touzé and Chapront, 1980) as far as the forces result from a force function. An extent has been built to suit the general case (Lestrade and Chapront-Touzé, 1982). The effects due to secular terms in solar eccentricity and perigee have been treated separately (Chapront-Touzé, 1982) in the same manner as Brown. In any case, the methods used provide perturbations as additions under formulation (1) ($\delta_V = 0$) to the main problem constants: $m = n'/\nu$, Γ, E and arguments w_1, w_2, w_3. Variables w_2 and w_3 stand respectively for the sidereal mean longitudes of perigee and node. These additions must be substituted in the main problem series for the polar coordinates through the formal expressions of the coefficients

$$A^{(0)}_{i_1 \ldots i_4} = A^{(0)}_{i_1 \ldots i_4}(m, \Gamma, E, e', \sigma_1, \sigma_2) \quad (4)$$

and the arguments w_i ($i = 1, 2, 3$). We denote these additions as δm, $\delta\Gamma$, δE, and δw_i ($i = 1, 2, 3$). Practically the w_i appear through:

$$D = w_1 - T + 180°$$
$$F = w_1 - w_3 \quad (5)$$
$$l = w_1 - w_2 .$$

In (5), T is the sidereal mean longitude of the Earth:

$$T = n't + T^{(0)} .$$

For the latter substitution, the δw_i are separated in two parts: the secular parts and the remainders. The remainders are removed from the sine functions by means of derivatives. The secular parts are kept inside the sine functions so that D, F, l become polynomial functions of time with degree greater than 1. The secular terms $\bar{\tilde{\omega}}'$ and $\bar{e}'$ of the longitude of perigee $\tilde{\omega}'$ and eccentricity e' of the Earth must also be substituted in the main problem series respectively through the argument

$$l' = T - \tilde{\omega}'$$

and through the expressions (4). The additions δm, $\delta\Gamma$, δE contain unknown constant parts, arising from integration, which are computed in such a way that the value of the sidereal mean motion $w_1^{(1)}$, the coefficient of $\sin F$ in latitude, the coefficient of $\sin l$ in longitude remain the same as in the main problem. This effect is called constant adjustment and symbolically denoted as *C.A.* in the tables. These constant parts in δm, $\delta\Gamma$, δE contribute to the linear variations of δw_2 and δw_3, i.e., to the complete mean motions of perigee and node. As a rule, the additive quantities δm, $\delta\Gamma$, δE, δw_i must be computed by successive approximations.

b) Contents of ELP 2000-82

ELP 2000-82 contains the following limitations:

Only the first approximation has been performed in the computation of δm, $\delta\Gamma$, δE, δw_i. By first approximation we mean that, in the process to solve the system of differential equations, the coordinates of the Moon are simply those of the main problem. Nevertheless, lunar mean motions including approximate values of planetary and Earth figure perturbations have been introduced in the integration.

Combinations $i_1\lambda_1 + i_2\lambda_2 + \ldots + i_p\lambda_p$ with periods longer than 5000 yr have been removed from δm, $\delta\Gamma$, δE, δw_i.

The substitutions of δm, $\delta\Gamma$, δE and non secular parts of δw_i, in the main problem series, have been performed while using first derivatives only.

In the polar coordinates, only the coefficients $A^{(n)}_{i_1 \ldots i_p}$ greater than 10^{-5} arc s for longitude and latitude, 2 cm for distance, have been retained.

The polar coordinates have the formulation (1) with $n \leqq 2$. Each one can be separated in two series: main problem and perturbations. In the main problem series, Delaunay arguments D, F, l, l', and w_1, for longitude, have the formulation (2) with $k \leqq 2$. In the perturbations series, all the λ_j are linear functions of time. They are: the four Delaunay arguments, the eight mean longitudes of planets, ζ, the mean longitude of the Moon referred to the mean equinox of the date. w_1 and the mean longitudes of planets are referred to a fixed origin.

ELP 2000-82 includes the following perturbations:

The Earth figure perturbations due to J_2 and J_3 (Chapront-Touzé, 1982). The motion of the true equator has been taken into account through the linear terms of precession included in the argument ζ, the four leading terms of the nutation in longitude and the three leading terms of the nutation in obliquity (coefficients, from Woolard, computed for J 2000), the linear term $\varepsilon^{(1)}$ of the ecliptic obliquity from (Lieske et al., 1977). The numerical values of J_2, J_3, $\varepsilon^{(0)}$, obliquity at J 2000, $\varepsilon^{(1)}$, p, precession constant at J 2000, a_T, Earth radius, are given in Table 2.

The effects of the secular terms of the eccentricity and perigee of the Earth. They are deduced from the secular parts of the variables:

$$K' = e' \cos\tilde{\omega}'$$
$$H' = e' \sin\tilde{\omega}'$$

Table 3. Tidal effect. Periodic and t-periodic terms in the coordinates

Longitude		Latitude		Distance	
sin	10^{-5} arc s	sin	10^{-5} arc s	cos	cm
$D+l'-F+193°$	82	$D+l'-2F+193°$	4	$D+l'-l-F+193°$	4
$D+l'-l-F+193°$	4	$D+l'+193°$	4	$D+l'+l-F+13°$	4
$D+l'+l-F+193°$	4				
Longitude/t		**Latitude/t**		**Distance/t**	
sin	10^{-5} ″cy^{-1}	sin	10^{-5} ″cy^{-1}	cos	cm cy^{-1}
l	58	F	−5	0	356
$2D-l$	21	$l-F$	3	l	− 72
$2D$	9	$l+F$	3	$2D-l$	− 19
$2l$	4	$2D-F$	1	$2D$	− 13
$2D-2l$	2			$2l$	− 3
$2D+l$	1				

Periodic and t-periodic terms are proportional to $k_2\delta$. Here $k_2\delta=0.01221$

in Bretagnon's theory VSOP 80 for the Earth-Moon barycenter (Bretagnon, 1980). For their main effect (direct substitution in the main problem coefficients and argument l') we have taken into account:

$$\bar{e}'=e'^{(1)}t+e'^{(2)}t^2,$$

$$\bar{\tilde{\omega}}'=\tilde{\omega}'^{(1)}t+\tilde{\omega}'^{(2)}t^2.$$

For the other effects, in δm, $\delta\Gamma$, δE, δw_i, only the linear parts of $\bar{e}'$ and $\bar{\tilde{\omega}}'$ have been taken into account (for numerical values, see Table 2).

The effects of the secular motion of the ecliptic. They are deduced from the secular parts $\bar{P}'$ and $\bar{Q}'$ of the variables:

$$Q'=\sin(i'/2)\cos\Omega',$$

$$P'=\sin(i'/2)\sin\Omega'$$

of VSOP 80 for the Earth-Moon barycenter. Their main effect is to tie the motion of the Moon to the mean ecliptic of the date instead of a fixed ecliptic. The other effects result from Coriolis forces. For their computation, we have taken into account:

$$\bar{P}'=P'^{(1)}t+P'^{(2)}t^2,$$

$$\bar{Q}'=Q'^{(1)}t+Q'^{(2)}t^2.$$

The other indirect planetary perturbations (Chapront and Chapront-Touzé, 1982) have been computed while using the solution VSOP 80 for the Earth-Moon barycenter up to the 3rd order in masses. In particular, the short periodic terms, due to the lunar effect on the motion of the Earth-Moon barycenter, have been included.

The direct planetary perturbations have been computed while using the planetary solution VSOP 80 up to the 1st order for all the planets and, up to the 3rd order (periodic terms only) for the major planets. In particular, the linear terms in the variables H', K', P', Q' for the planets have been taken into account.

The relativistic perturbations have been computed in the frame of isotropic coordinates and dynamical barycentric time. They include the main relativistic perturbations from (Lestrade and Chapront-Touzé, 1982) and the indirect relativistic perturbations resulting from the introduction of the relativistic terms of VSOP 80 while computing the indirect planetary perturbations of the Moon (Lestrade et al., 1982). The effects of the relativistic terms of the planetary theories on the direct planetary perturbations have been found negligible.

The tidal effects have been computed by using a model from (Williams et al., 1978) for the geocentric tidal acceleration of the Moon:

$$\begin{bmatrix}\Delta\ddot{x}\\ \Delta\ddot{y}\\ \Delta\ddot{z}\end{bmatrix}=-3k_2\frac{Gm_L}{r^3}\left(1+\frac{m_L}{m_T}\right)\left(\frac{a_T}{r}\right)^5\begin{bmatrix}x+y\delta\\ y-x\delta\\ z\end{bmatrix}$$

x, y, z are equatorial geocentric coordinates of the Moon, with z axis parallel to the Earth's spin axis; r is the Earth-Moon distance, k_2 is Love number, δ is phase. This gives rise to two kinds of effects: small contributions to the mean motions, proportional to k_2; quadratic terms on w_i ($i=1,2,3$), small periodic terms and t-periodic terms (mixed terms) on the polar coordinates, proportional to $k_2\delta$. Periodic and t-periodic terms are listed in Table 3. The quadratic term on w_1 is, of course, the most important contribution from tidal effects.

The Moon figure perturbations (Chapront-Touzé, 1983) include the effects of the harmonic coefficients up to the 3rd order in the development of the lunar potential and in the libration. Furthermore, a small t^2-term induced by the harmonics of order 4 has been retained in w_1. The solution used for the libration is the main problem of the physical libration from (Moons, 1982a, b). The numerical values adopted for the lunar parameters (Table 2) are those from (Ferrari et al., 1980) except for β and γ which have been corrected with a small tidal part.

Table 4 gives the number of coefficients $A^{(n)}_{i_1\ldots i_p}$ greater than 10^{-5} arc s for longitude and latitude, and greater than 2 cm for distance, for the various contributions to the polar coordinates in ELP 2000-82. Obviously, it is not possible to list the coefficients here, even truncated to a lower precision. The complete solution is available on magnetic tape. It is worth noticing that for the practical computation of the lunar ephemeris, we have limited ourselves to the truncation level of $5\,10^{-5}$ arc s in longitude and latitude and 10 cm in distance.

Table 5 gives the various contributions to the mean motions of perigee and node. The mean motions obtained in constructing solely the main problem are strongly dependent upon the numeri-

Table 4. Number of coefficients of the series for polar coordinates in ELP 2000–82 (coefficients greater than 10^{-5} arc s for longitude and latitude, 2 cm for distance)

		Longitude	Latitude	Distance
Main problem	P	1,023	918	704
Earth figure	P	347	316	237
	$t \cdot P$	14	11	8
Planetary perturbations	P	14,498	5,383	6,745
	$t \cdot P$	4,610	1,021	1,884
	$t^2 \cdot P$	28	13	19
Tidal effect	P	3	2	2
	$t \cdot P$	6	4	5
Relativity	P	11	4	10
Lunar figure	P	20	12	14
Total		20,560	7,684	9,628

P = periodic; $t \cdot P$ = t-periodic; $t^2 \cdot P$ = t^2-periodic

Table 5. Computed values of mean motions of perigee and node ("/cy) for constants of Table 1

		$w_2^{(1)}$	$w_3^{(1)}$
Main problem		14,642,537.9368	−6,967,167.2643
Earth figure	Without C.A.	615.8833	−588.2007
	C.A.	17.5201	−4.3350
Planetary perturbations	Indirect	−21.6127	6.2526
	Direct	267.9736	−142.8236
	Solar eccentricity	0.0339	0
	Moon on barycenter	3.3492	−8.2329
	C.A.	−2.6388	0.7306
Tidal effects	Without C.A.	0.0663	0.0002
	C.A.	0.0007	−0.0002
Lunar figure	Without C.A.	−2.2689	−16.8108
	C.A.	0.5217	−0.1335
Relativity	Main effect	4.4528	1.2534
	Indirect effect	0.6897	−0.1989
	C.A.	−3.3454	0.8474
Total		14,643,418.5623	−6,967,918.9157

C.A. means Constant Adjustment

cal values adopted for the constants (3). In Table 5, they have been computed with values of Table 1. Table 6 gives the derivatives of $w_2^{(1)}$ and $w_3^{(1)}$ with respect to those constants. In Table 5, we note the important contribution of the indirect planetary perturbations due to the short periodic terms of the Earth-Moon barycenter.

Table 6. Derivatives of the mean motions of perigee and node for the main problem

Derivatives	$w_2^{(1)}$	$w_3^{(1)}$
$\partial/\partial\nu$	−0.014817824	0.003745962
$\partial/\partial\Gamma$	−7,766,021″.5	1,157,847″.0
$\partial/\partial E$	−1,910,121″.9	−2,248,987″.8
$\partial/\partial n'$	0.31108026	−0.10383876
$\partial/\partial e'$	1,829,690″.9	−308,445″.2
$\partial/\partial\sigma_1$	−724″.2	381″.2
$\partial/\partial\sigma_2$	24,879,421″	−18,242,368″

N.B. Corrections to mean motions ν and n' have to be expressed in "/cy, other correction are dimensionless. The results are in "/cy

Table 7. Coefficients of t^2 ("/cy^2)

		$w_1^{(2)}$	$w_2^{(2)}$	$w_3^{(2)}$
Earth figure		0.1925	0.1003	−0.0958
Planetary perturbations	Indirect	0.0020	−0.0057	0.0016
	Direct	0.0005	−0.0008	0.0002
	Solar eccentricity	5.8664	−38.5481	6.5026
Tidal effects		−11.9473	0.1761	−0.0464
Lunar figure		−0.0151	0	0
Total		−5.9010	−38.2782	6.3622

Table 7 gives the various contributions to the quadratic terms in the mean longitude (w_1) and the longitudes of perigee (w_2) and node (w_3). The complete value of the quadratic term in w_1 is strongly dependent upon the tidal part which is proportional to $k_2\delta$. The values of k_2 and δ used in ELP 2000-82 are those of JPL numerical integration LE 200 (Williams, 1981). Their numerical values are listed in Table 2.

The argument of ELP 2000-82 is the dynamical barycentric time (TDB). The origin is J 2000 (Julian data 2 451 545.0)

II. The constants

A previous set of lunar and solar constants $S_{51}^{(1)}$ has been described in (Chapront and Chapront-Touzé, 1981). It was resulting from a comparison of ELP with JPL numerical integration LE 51 (Newhall et al., 1983). The constants S_{200}, which will be introduced in the ephemeris ELP 2000, have been obtained from a comparison to a more recent JPL numerical integration LE 200. In addition, the two comparisons differ on the following points:

The time span is 100 yr (1900–2000) for the latter instead of 20 yr for the former.

The semi-analytical solutions which are submitted to comparison are not exactly the same. We denominate the former one by ELP 2000-81. ELP 2000-81 suffers from some inconsistencies and errors which no longer exist in ELP 2000-82. In particular, some relativistic effects were added twice and the short periodic terms of the Earth-Moon barycenter had not been taken into account with enough completeness.

Table 8. Initial values of planetary longitudes and mean motions ("/cy) from VSOP 82

M_e	252°15′03″.25986	538,101,628.68898
V	181°58′47″.28305	210,664,136.43355
M_a	355°25′59″.78866	68,905,077.59284
J	34°21′05″.34212	10,925,660.42861
S	50°04′38″.89694	4,399,609.65932
U	314°03′18″.01841	1,542,481.19393
N	304°20′55″.19575	786,550.32074

Planetary longitudes are referred to the inertial dynamical equinox 2000 : $\gamma^I_{\rm VSOP}$

Table 9. Set S_{200} of lunar and solar constants fitted to LE 200

	Correction to Table 1	Complete value	RMS	Unit
ν	0.55604	1,732,559,343.73604	$49\ 10^{-5}$	"/cy
$2E$	0.03578	22,639.58578	$6\ 10^{-5}$	arc s
2Γ	−0.16132	18,461.23868	$6\ 10^{-5}$	arc s
e'	$-0.6244\ 10^{-6}$	0.0167086156	$15\ 10^{-10}$	
n'	−0.0642	129,597,742.2758	$37\ 10^{-4}$	"/cy
$w_1^{(0)}$		218°18′59″.95571	$18\ 10^{-5}$	arc s
$w_2^{(0)}$		83°21′11″.67475	$94\ 10^{-5}$	arc s
$w_3^{(0)}$		125°02′40″.39816	$129\ 10^{-5}$	arc s
$T^{(0)}$		100°27′59″.22059	$205\ 10^{-5}$	arc s
$\tilde{\omega}'^{(0)}$		102°56′14″.42753	$1780\ 10^{-5}$	arc s

Table 10. Computed values of mean motions of perigee and node ("/cy) for the constants S_{200}

$w_2^{(1)}$	14,643,420.26324
$w_3^{(1)}$	− 6,967,919.36222

Table 11. Lunar and solar arguments in the ephemeris ELP 2000

w_1	$218°18'59''.95571 + 1{,}732{,}559{,}343.73604\,t - 5.9010\,t^2$
D	$297°51'00''.73512 + 1{,}602{,}961{,}601.4603\,t - 5.8805\,t^2$
F	$93°16'19''.55755 + 1{,}739{,}527{,}263.0983\,t - 12.2632\,t^2$
l	$134°57'48''.28096 + 1{,}717{,}915{,}923.4728\,t + 32.3772\,t^2$
l'	$357°31'44''.79306 + 129{,}596{,}581.0474\,t - 0.5616\,t^2$
T	$100°27'59''.22059 + 129{,}597{,}742.2758\,t - 0.0205\,t^2$
$\tilde{\omega}'$	$102°56'14''.42753 + 1{,}161.2283\,t + 0.5411\,t^2$

The initial values and mean motions of planets, except for the Earth, which are substituted in the λ_j are those from VSOP 82 (Bretagnon, 1982) in the latter comparison, instead of VSOP 80 in the former. Let us note that VSOP 82 has been fitted to JPL numerical integration DE 200 (Standish, 1981b) which is consistent with LE 200. Initial values and mean motions of planets from VSOP 82 are reproduced in Table 8.

To take into account the values of G' and σ_1 used in LE 200:

$$G'_{200} = 3.98600448\ 10^{14}\ m^3\ s^{-2}$$

$$\sigma_{1,200} = 0.0121505816$$

which are different from the one introduced in ELP 2000-82 (Table 1), the Earth-Moon distance r computed from ELP 2000-82 has been multiplied by the quantity $a_{200}/a_{\rm ELP}$. a_{200} and $a_{\rm ELP}$ are two values of the keplerian semi-major axis a_0 derived from:

$$\nu^2 a_0^3 = G(m_T + m_L)$$

and computed respectively with the values of G' and σ_1 from LE 200 (a_{200}) and the one from Table 1 ($a_{\rm ELP}$). In both cases, ν has the value listed in Table 1. Hence:

$$a_{200} = 384{,}747{,}965.736\ \text{m},$$

$$a_{\rm ELP} = 384{,}747{,}980.645\ \text{m}.$$

The numerical substitution of the time in the series ELP 2000-82 has been performed using the method described in (Chapront, 1982).

The adjusted parameters are the same as in the previous comparison, i.e.,

Lunar parameters: $\delta w_1^{(0)}$, $\delta w_2^{(0)}$, $\delta w_3^{(0)}$, $\delta\nu$, $\delta\Gamma$, δE.

Solar parameters: $\delta T^{(0)}$, $\delta\tilde{\omega}'^{(0)}$, $\delta e'$, $\delta n'$.

Parameters of the reference frame: $\delta\phi$, $\delta\varepsilon$.

Bias parameters: $\delta w_2^{(1)}$, $\delta w_3^{(1)}$, $\delta w_1^{(2)}$.

The bias parameters are introduced to evaluate the accuracy of the ephemeris only. The parameters $\delta\phi$ and $\delta\varepsilon$ are analysed in the next section.

The new values of lunar and solar constants which are deduced from this comparison result in the set S_{200}. S_{200} is listed in Table 9. The initial values and mean motions of the planetary longitudes (except for the Earth) to be introduced in the ELP 2000 ephemeris are those of Table 8. The numerical values of the remaining parameters taking place in the ephemeris are still those of Table 1 and Table 2, especially for G' and σ_1. So the ephemeris is computed with the IAU system of constants except for small differences in the parameters of the lunar potential. The modifications in the values of ν, n', Γ, E, e' induce modifications in the total values of the mean motions of perigee and node given in Table 5, through the main problem part. The new mean motions to be substituted in ELP 2000-82 are listed in Table 10. All the angles referred to in Tables 7, 9, and 10 are reckoned from the dynamical equinox 2000, $\gamma^I_{\rm ELP}$ defined in Sect. III, the origin of time is J 2000. The complete expressions of lunar and solar arguments are listed in Table 11.

III. The reference frame

Originally, the semi-analytical solution ELP 2000-82 is referred to the mean dynamical ecliptic of the date and Brown's Departure Point $\gamma'(2000)$. By mean dynamical ecliptic of the date, we designate a plane deduced from the dynamical ecliptic 2000 (i.e., the reference plane of VSOP 82) by using the secular terms $\bar{P}'$ and $\bar{Q}'$ of variables P' and Q' of VSOP 82 for the Earth-Moon barycenter. The Departure Point $\gamma'(2000)$ is a point of the mean dynamical ecliptic of the date related to the dynamical equinox $\gamma(2000)$ by:

$$N\gamma'(2000) = N\gamma(2000)$$

N being the ascending node of the two planes.

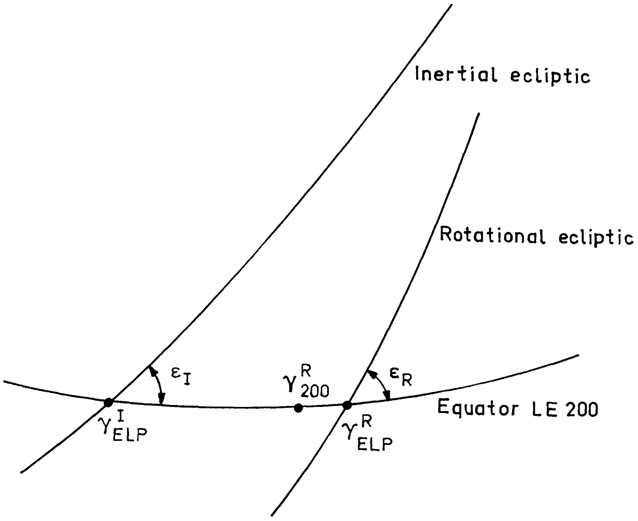

Fig. 1. Positions of the inertial and rotational dynamical equinoxes of the lunar Ephemeris ELP 2000 with respect to the equinox of LE 200

Afterwards, ELP 2000-82 is referred to the dynamical ecliptic and dynamical equinox 2000, by adding to the longitude V and latitude U the corrections:

$$\delta V = \sin\tilde{\pi}_A\{\sin\tilde{\Pi}_A \sin V + \cos\tilde{\Pi}_A \cos V\}$$
$$\cdot\{-tgU + \sin\tilde{\pi}_A(\tfrac{1}{2} + tg^2 U)(\sin\tilde{\Pi}_A \cos V - \cos\tilde{\Pi}_A \sin V)\}$$
$$\delta U = \sin\tilde{\pi}_A(\cos\tilde{\Pi}_A \sin V - \sin\tilde{\Pi}_A \cos V)$$
$$-\tfrac{1}{2} tgU \sin^2\tilde{\pi}_A(\sin\tilde{\Pi}_A \sin V + \cos\tilde{\Pi}_A \cos V)^2 .$$

The quantities $\sin\tilde{\pi}_A \sin\tilde{\Pi}_A$ and $\sin\tilde{\pi}_A \cos\tilde{\Pi}_A$ are derived from VSOP 82 expressions for $\bar{P}'$ and $\bar{Q}'$ in (Bretagnon and Chapront, 1981). Those are:

$$\sin\tilde{\pi}_A \sin\tilde{\Pi}_A = 4''.1997t + 0''.19396t^2 - 0''.000222t^3 ,$$
$$\sin\tilde{\pi}_A \cos\tilde{\Pi}_A = -46''.8093t + 0''.05105t^2 + 0''.000524t^3 ,$$

t in julian centuries from J 2000.

The position of the dynamical equinox $\gamma(2000)$ in the dynamical ecliptic 2000 is introduced in the semi-analytical solution through the initial values of the arguments w_1 and λ_j, i.e., in a literal way. So it has to be determined by comparison to observations or to an observational model. As a matter of fact, the dynamical equinox $\gamma(2000)$ introduced here, and denoted as γ^I_{ELP}, is the intersection of the dynamical ecliptic 2000, reference plane of VSOP 82, with the reference plane of LE 200. Later on, we shall assume that LE 200 reference plane is merged into FK 4 and FK 5 equators 2000, which are supposed to be the same. From now on, we shall refer to it as the Equator 2000. By FK 4 (or FK 5) equator 2000, we mean the plane derived from FK 4 (or FK 5) reference plane by means of precession formulae from 1950 to 2000.

We note that the dynamical ecliptic and equinox 2000, which are considered here, are inertial as defined in (Standish, 1981a). Besides, Standish defines a rotating ecliptic and a rotating equinox γ^R, which is the intersection of the rotating ecliptic with the equator. For J 2000, he gives:

$$\varepsilon^R(2000) - \varepsilon^I(2000) = 0''.00334$$
$$\gamma^I(2000)\gamma^R(2000) = 0''.09366 \qquad (6)$$

$\varepsilon^I(2000)$ and $\varepsilon^R(2000)$ being the obliquities of the two ecliptics 2000 on the equator 2000.

The origin of DE 200 and LE 200 is supposed to be the rotating equinox 2000. We shall denote it as γ^R_{200}. The comparison between ELP 2000-82 and LE 200, as described in Sect. II, yields numerical values of the angular distance:

$$\delta\phi = \gamma^R_{200}\gamma^I_{ELP}$$

along the Equator 2000 (Fig. 1) and of ε^I_{ELP} obliquity of the inertial dynamical ecliptic 2000 (reference plane of VSOP 82) on the Equator 2000. We denote:

$$\delta\varepsilon = \varepsilon^I_{ELP} - \varepsilon_0 .$$

ε_0 being the IAU value for J 2000:

$$\varepsilon_0 = 23°26'21''.448 .$$

We have determined:

$$\delta\phi = -0''.09245 \pm 0.00016$$
$$\delta\varepsilon = -0''.03917 \pm 0.00006 . \qquad (7)$$

Hence,

$$\varepsilon^I_{ELP} = 23°26'21''.40883 .$$

From (6) and (7), we derive the obliquity of the ELP rotational ecliptic 2000, ε^R_{ELP}, and the position of the ELP rotational equinox 2000, γ^R_{ELP}. So:

$$\varepsilon^R_{ELP} = 23°26'21''.41217$$

hence,

$$\varepsilon^R_{ELP} - \varepsilon_0 = -0''.03583$$

and,

$$\gamma^R_{200}\gamma^R_{ELP} = 0''.00121$$

along the Equator 2000. We note that γ^R_{200} and γ^R_{ELP} are two determinations of the same point $\gamma^R(2000)$. Standish (1982) gives a value ε^I_{200} of the obliquity of the inertial dynamical ecliptic deduced from analysis of DE 200. We have:

$$\varepsilon^I_{ELP} - \varepsilon^I_{200} = 0''.00027 .$$

Our determinations of $\gamma^I(2000)$ and ε^I can also be compared to Bretagnon's determinations denoted as γ^I_{VSOP} and ε^I_{VSOP} (Bretagnon, 1982). We have:

$$\gamma^I_{VSOP}\gamma^I_{ELP} = 0''.00055 ,$$
$$\varepsilon^I_{ELP} - \varepsilon^I_{VSOP} = -0''.00027 .$$

We note that our determinations of $\gamma^I(2000)$ and ε^I are close to Standish's determinations and to Bretagnon's and that the two sets of discrepancies are very likely below the expected accuracy.

As far as we admit, for J 2000, the relation:

$$\gamma^R_{200}\gamma_{FK\,5}(2000) = 0''.006 \qquad (8)$$

we derive from (7) and (8) the angular distance from the origin of the FK 5, referred to J 2000 by means of precession, to the origin γ^I_{ELP} of the ephemeris ELP 2000

$$\gamma_{FK\,5}(2000)\gamma^I_{ELP} = -0''.09845 .$$

Let us note that the relation (8) follows from Standish's results (1982) and Fricke's determination of the FK 5 equinox (1981) and that the former author considers it as fortuitous with respect to the expected accuracies of the two involved results.

Table 12. Comparison between 4 sets of fitted parameters for Moon and Sun, related computed mean motions and t^2-terms of the mean longitude

	$S_{51}^{(1)}-S_{200}$	$S_{51}^{(2)}-S_{200}$	$S_{200}-S'_{200}$
v	-0.53304	-0.54032	-0.00871 ± 0.00088
$2E$	-0.00074	-0.00046	-0.00002 ± 0
2Γ	-0.00492	-0.00560	-0.00002 ± 0
e'	$-0.647\ 10^{-7}$	$-0.58\ 10^{-9}$	$-0.65\ 10^{-10}\pm0.41\ 10^{-10}$
n'	0.0442	0.0379	-0.00440 ± 0.00115
$w_1^{(0)}$	-0.07205	-0.07357	-0.00365 ± 0.00011
$w_2^{(0)}$	0.02343	0.02343	-0.00342 ± 0.00013
$w_3^{(0)}$	-0.00798	-0.01761	-0.00089 ± 0.00039
$T^{(0)}$	-0.00341	-0.00064	-0.00098 ± 0.00029
$\tilde{\omega}'^{(0)}$	0.04365	-0.03909	-0.00047 ± 0.00051
$w_2^{(1)}$	0.8727	0.1262	-0.0009
$w_3^{(1)}$	-0.2376	-0.1524	0.0005
$w_1^{(2)}$	-1.2560	-1.2560	0

The unit is ″/cy for mean motions; ″/cy² for t^2-terms; arc s for angles, E and Γ; e' is dimensionless. The angles are reckonned from the equinox γ_{ELP}^I

IV. Results from previous fits of semi-analytical solutions ELP to numerical integrations

ELP 2000-82 is presently the most accurate among several versions of the semi-analytical solution ELP 2000. Before fitting ELP 2000-82 to the numerical integration LE 200, we have performed two other fits which differ by the numerical integration and by the version of the semi-analytical solution involved. These comparisons provide us with informations which will be useful for discussing the accuracy of the ephemeris ELP 2000.

a) Fit of ELP *2000-81 to* LE *51*

We have mentioned in Sect. II a fit of the semi-analytical solution ELP 2000-81 to the numerical integration LE 51 which yielded a set of lunar and solar constants denoted as $S_{51}^{(1)}$. The time span was 20 yr. We give, in Table 12, Column 1, the differences between the set $S_{51}^{(1)}$ and the adopted set S_{200} and also the differences between the related computed mean motions of perigee and node, and t^2-terms of the mean longitude. Before computing the differences of Table 12, all the angles of $S_{51}^{(1)}$ have been referred to the adopted equinox γ_{ELP}^I by using correction (15) given below.

b) Fit of ELP *2000-82⁻ to* LE *51*

Later on another fit to LE 51 has been performed. It involved the semi-analytical solution denoted as ELP 2000-82⁻. ELP 2000-82⁻ was similar to ELP 2000-82, except that the parameters for the lunar potential and tidal effects were those from LE 51 and that a spurious t^2-term with coefficient $-0''.0505\ \mathrm{cy}^{-2}$ was introduced in the mean longitude. The small lack of accuracy of ELP 2000-82⁻ with respect to ELP 2000-82 arise from this spurious t^2-term only. It was induced by inaccurate computations of the perturbations due to short periodic terms of the Earth-Moon barycenter and to the shape of the Moon.

We have given in (Chapront-Touzé, 1983) the differences (ELP 2000-82⁻)-(ELP 2000-82) induced by using the set of coefficients for the lunar potential from LE 51 instead of the one from LE 200. The leading differences concern the mean motions:

$$\Delta_1 w_2^{(1)} = 0''.0025\ \mathrm{cy}^{-1}$$
$$\Delta_1 w_3^{(1)} = -0''.1334\ \mathrm{cy}^{-1}. \qquad (9)$$

The leading difference (ELP 2000-82⁻)-(ELP 2000-82) induced by using the tidal parameters from LE 51 instead of those from LE 200 is:

$$\Delta_2 w_1^{(2)} = -1''.2055\ \mathrm{cy}^{-2}. \qquad (10)$$

Let us note that the value of J_2 used in ELP 2000-82⁻ was the same as the one used in ELP 2000-82, denoted as J_2(IAU). The value of J_2 used in LE 51, denoted as $J_2(51)$, was slightly different:

$$J_2(51) - J_2(\mathrm{IAU}) = 0.7\ 10^{-8}. \qquad (11)$$

To introduce $J_2(51)$ instead of J_2(IAU) in the semi-analytical solution would be equivalent to add to ELP 2000-82⁻ the following correction:

$$\Delta_3 V = 0''.00005 \sin(\zeta - F)$$
$$\Delta_3 U = -0''.00005 \sin\zeta$$
$$\Delta_3 w_2^{(1)} = 0''.0041\ \mathrm{cy}^{-1}$$
$$\Delta_3 w_3^{(1)} = -0.0038\ \mathrm{cy}^{-1}. \qquad (12)$$

The initial values and mean motions of planetary longitudes (except for the Earth) which were substituted in the λ_j were yielded by a comparison of VSOP 82 with DE 102 consistent with LE 51. As it was done in the previous fit, the Earth-Moon distance from ELP has been corrected before comparison to take into account the LE 51 values of G' and σ_1.

We denote as $S_{51}^{(2)}$ the resulting set of lunar and solar constants. The differences between the set $S_{51}^{(2)}$ and the adopted set S_{200} are given in Table 12, Column 2, the angles of the set $S_{51}^{(2)}$ being referred to the equinox γ_{ELP}^I by means of correction (16) given below. Let us note that the differences $S_{51}^{(2)}-S_{200}$ for $w_2^{(1)}$ result from $\Delta_1 w_2^{(1)}$ and from the discrepancies between the constants of the two sets, the difference for $w_3^{(1)}$ result, for the most part, from $\Delta_1 w_3^{(1)}$, the difference for $w_1^{(2)}$ result from $\Delta_2 w_1^{(2)}$ and from the spurious t^2-term mentionned above.

c) The equinox

The positions of the inertial dynamical equinox 2000 yielded by the two fits to LE 51 were not the same for the following reasons. In the fit denoted as (1), the inertial dynamical equinox 2000, $\gamma_{(1)}^I$ (Fig. 2), was the intersection of the inertial dynamical ecliptic 2000 with the equator deduced from DE 102 equator by precession from 1950 to 2000. Before the fit denoted as (2), the LE 51 ephemeris has been rotated onto the equator and equinox of an other JPL numerical integration, DE 114. The rotation (Standish *in* Bretagnon and Chapront, 1981) is defined by:

$$r_{114} = \begin{bmatrix} 1 & \theta_z & -\theta_y \\ -\theta_z & 1 & \theta_x \\ \theta_y & -\theta_x & 1 \end{bmatrix} r_{102} \qquad (13)$$

with $\theta_x = -0''.0013$, $\theta_y = -0''.1163$, $\theta_z = 0''.6854$. r_{114} and r_{102} stand for the column matrix of rectangular equatorial coordinates in the two systems. We note here that DE 114 equator and equinox have been improperly denoted as FK 4 in (Bretagnon and Chapront,

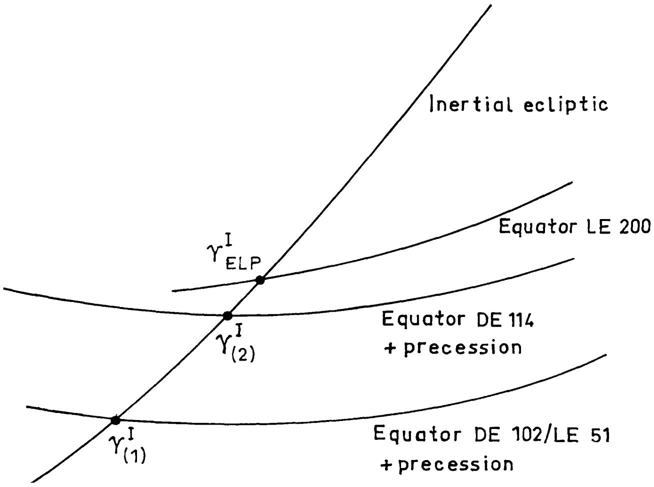

Fig. 2. Determinations of the inertial dynamical equinox 2000 yielded by different fits of semi-analytical solutions ELP 2000 to numerical integrations

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1981). Hence, in the comparison (2) the dynamical equinox $\gamma^I_{(2)}$ is the intersection of the inertial dynamical ecliptic 2000 with the equator deduced from DE 114 equator by precession between 1950 and 2000. We easily derive from (13):

$$\gamma^I_{(1)}\gamma^I_{(2)} = -\theta_y/\sin\varepsilon \quad (14)$$

along the dynamical ecliptic 2000. So:

$$\gamma^I_{(1)}\gamma^I_{(2)} = 0''.29237 .$$

On another hand, Standish (1982) gives the rotation transforming the ephemeris DE 102 into DE 102 $\sqrt{}$ referred to the same equator and equinox as DE 200. From Standish's value for θ_y, we deduce by means of (14):

$$\gamma^I_{(1)}\gamma^I_{\mathrm{ELP}} = 0''.31382 \quad (15)$$

along the dynamical ecliptic 2000. Hence:

$$\gamma^I_{(2)}\gamma^I_{\mathrm{ELP}} = 0''.02145 . \quad (16)$$

d) Comparison between the two sets of constants $S^{(1)}_{51}$ and $S^{(2)}_{51}$

A comparison of the first two columns of Table 12 shows the mixed influences of the accuracy of the semi-analytical solution and of the time span of the comparison on the fitted constants. The set $S^{(1)}_{51}$ has been determined with a less accurate solution than the set $S^{(2)}_{51}$. In particular, the difference between the two fitted values of e' is due to a missing $\sin l'$ term in the perturbations due to the shape of the Moon and an erroneous $\sin T$ term in the perturbations due to the short periodic terms of the Earth-Moon barycenter. The differences between the computed mean motions of perigee and node for the sets $S^{(1)}_{51}$ and $S^{(2)}_{51}$ are also due to the inaccuracies in the solution ELP 2000-81: parts of the relativistic perturbations had been added twice and the contributions due to the shape of the Moon were not computed with the most adequate values of the parameters.

V. The accuracy of the ephemeris ELP 2000

It depends on several components:

(i) The internal precision of the semi-analytical solution ELP 2000-82.

(ii) The accuracy of the constants introduced in the solution. Two kinds of constants appear: On one side, the constants with an assigned value such as the geocentric constant of gravitation, the masses, Love number and phase, the coefficients of the potentials of Earth and Moon. On the other side, the constants with a fitted value such as the constants in the set S_{200}. The initial values and mean motions of planetary longitudes can be also considered as fitted constants since the adopted values derive from a fit of the planetary solution to a numerical integration.

(iii) The accuracy of the determination of the reference frame.

The last point has been analysed in Sect. III. We shall consider here the two first points.

a) The accuracy of the semi-analytical solution ELP *2000-82*

It both depends upon the inaccuracies on the actual contributions and upon the neglected contributions. For instance, the accuracy of the ephemeris over a long time span is certainly lowered because of the missing t^3-terms in the lunar arguments and the missing t^2-periodic terms in the series. The amplitude of the former likely amounts to several 10^{-3} arc s cy^{-3}.

A test of the accuracy of ELP 2000-82 over a time span of one century is provided by the fit to LE 200 through the bias parameters $\delta w^{(1)}_2$, $\delta w^{(1)}_3$, $\delta w^{(2)}_1$. and through the post fit residuals. $\delta w^{(1)}_2$, $\delta w^{(1)}_3$, $\delta w^{(2)}_1$ are unknown corrections to the computed mean motions for perigee and node and t^2-term in the mean longitude respectively. They are determined by the least square fit in order to perform a better adjustment of the semi-analytical solution to the numerical integration. The results obtained during the comparison to LE 200 are listed in Table 13, Column 1. We note that they are very small comparatively to the RMS values. The post fit residuals are illustrated by Fig. 3. They do not include the effects due to the different values of G' and σ_1 which are used in the ephemeris ELP 2000 and LE 200 since ELP 2000 is corrected from those effects before comparison. The values of the other assigned constants either are the same in ELP 2000 and LE 200 (potential coefficients, tidal coefficients) or provide negligible discrepancies

Table 13. Values of bias parameters determined by least square fits

	LE 200 − (ELP 2000–82)	LE 51 − (ELP 2000–82⁻)	LE 200 − (ELP 2000–82 + ϱ) over 1 cy	LE 200 − (ELP 2000–82 + ϱ) over 10 yr
$\delta w^{(1)}_2$	−0.00111 ± 0.00346	0.00604 ± 0.00337	−0.00704 ± 0.00120	−0.01936 ± 0.00059
$\delta w^{(1)}_3$	0.00470 ± 0.00241	−0.00590 ± 0.00235	0.01053 ± 0.00084	0.00628 ± 0.00160
$\delta w^{(2)}_1$	−0.00664 ± 0.00051	0.07247 ± 0.00050	−0.07910 ± 0.00018	−0.07750 ± 0.00176

The unit is ″/cy for $\delta w^{(1)}_2$ and $\delta w^{(1)}_3$, ″/cy^2 for $\delta w^{(2)}_1$

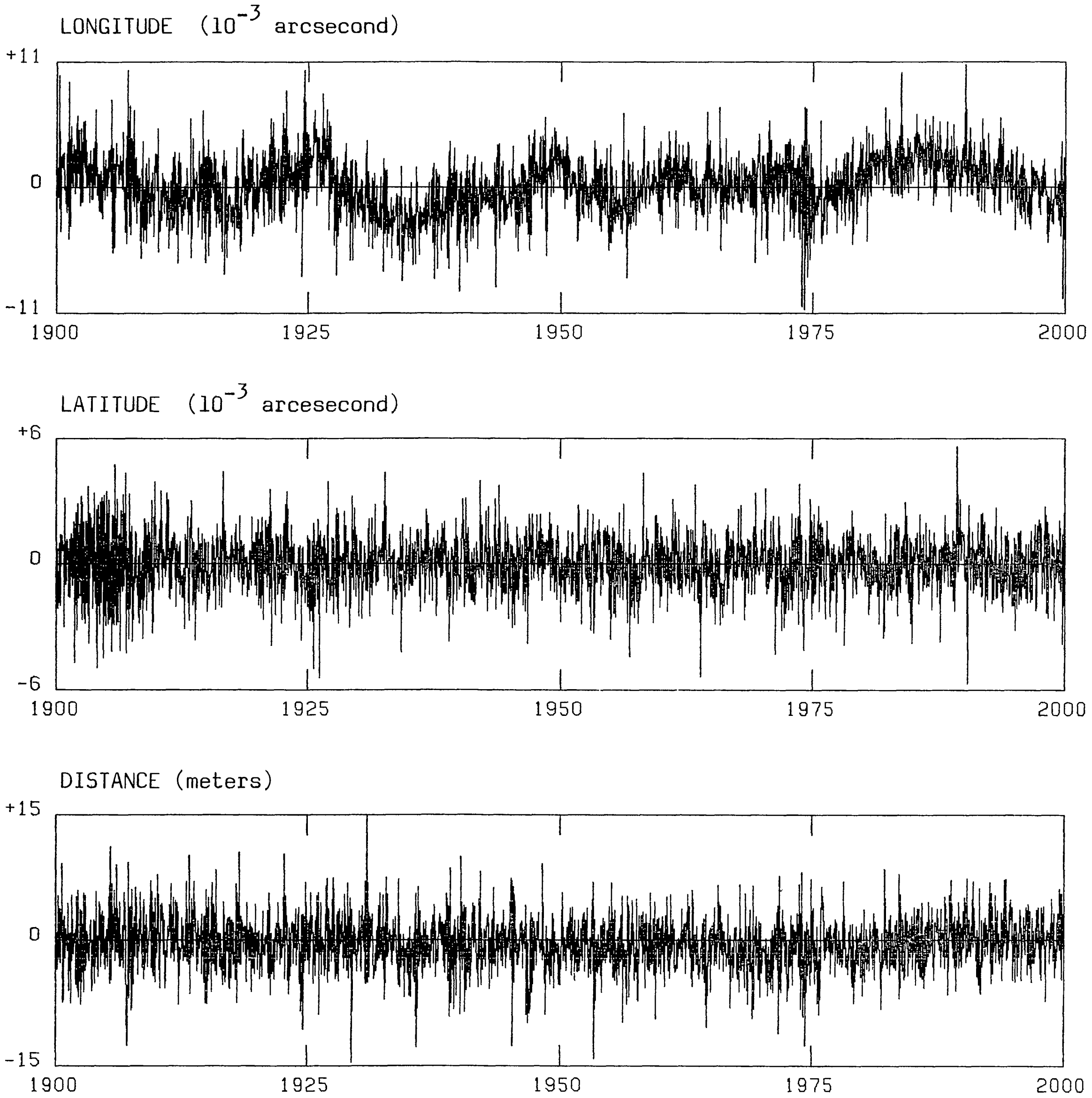

Fig. 3. Post fit residuals from comparison ELP 2000-82–LE 200 over one century

(planetary masses). As far as the other constants are concerned, they are fitted either on LE 200 or on DE 200. So if we assume that LE 200 is perfect, the values of the bias parameters and the post fit residuals of Fig. 3 result from the inaccuracies in the semi-analytical solution ELP 2000-82 and neglected contributions.

The values of the bias parameters, resulting from the fit of ELP 2000-82$^-$ to LE 51, as described in Sect. IVb, are listed in Table 13, Column 2. In the latter case, let us note that the quantities $\Delta_3 w_2^{(1)}$ and $\Delta_3 w_3^{(1)}$, as quoted in (12) should be subtracted from the values of $\delta w_2^{(1)}$ and $\delta w_3^{(1)}$ respectively to represent properly the accuracy of ELP 2000-82$^-$ independently from the values of constants. Let us note also that the value of $\delta w_1^{(2)}$ results for a great part from the spurious t^2-term included in ELP 2000-82$^-$.

b) The accuracy of constants which receive assigned values.

The most inaccurate are Love number and phase. It is more convenient to speak about the accuracy of the tidal t^2-term in longitude since it is the observed quantity. Love number receives an assigned value and the phase results from those two quantities and from the model described in Sect. I for the tidal forces. A recent determination (Dickey et al., 1982) provides a tidal $w_1^{(2)}$ value: $-11''\!.9 \pm 0.75\,\mathrm{cy}^{-2}$. We see that, past 20 yr, the uncertainty on the lunar longitude yielded by the tidal acceleration is greater than the one yielded by the inaccuracy of the analytical solution.

It follows from Sect. II that the leading effect of the differences between the IAU values of G' and σ_1 and those from LE 200 is a constant discrepancy of 14.91 m on the distance. The part due to σ_1 is -1.79 m; the part due to G' is 16.70 m. The LE 200 values of G' and σ_1 are certainly more accurate than the IAU values since they are in good agreement with the most recent determinations. Hence, we can consider the above discrepancies as estimations of the error made when using IAU values. As a matter of fact, it is very easy to introduce more modern values of G' and σ_1 in the ephemeris ELP 2000 by multiplying the distance by the ratio of the semi-major axes as explained in Sect. II. The other changes due to σ_1 are negligible (below 10^{-5} arc s) and G' has no other contribution.

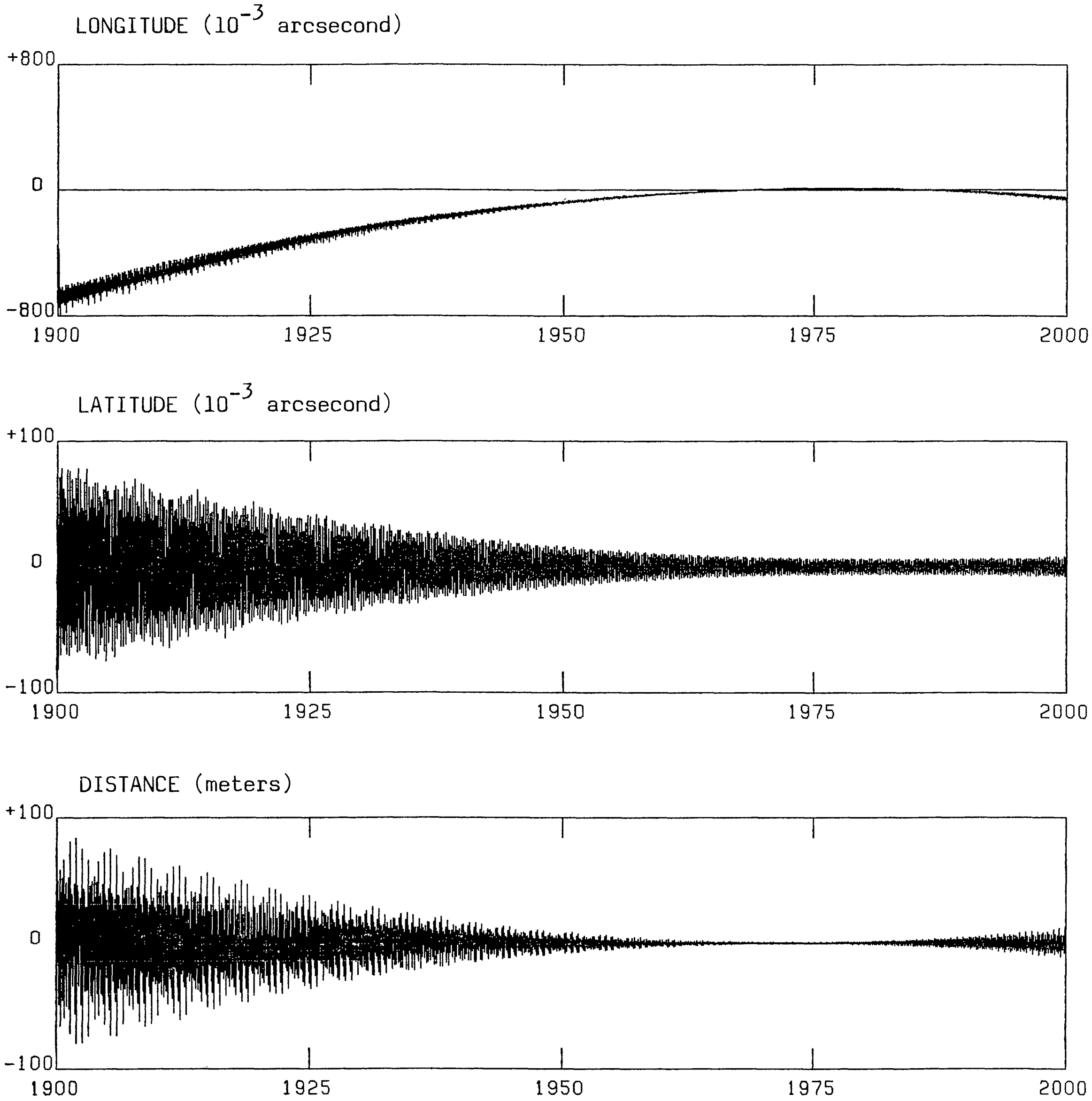

Fig. 4. Differences LE 51–LE 200. The two ephemerides are referred to the same ecliptic and equinox. LE 51 distance is corrected from discrepancies due to G' and σ_1. The differences arise mainly from the discrepancy between the values adopted for the tidal t^2-term in the two numerical integrations

A comparison, between the IAU values for planetary masses (including the Earth) and the LE 200/DE 200 values, makes fairly probable that planetary masses are well known enough and introduce but negligible errors in the lunar theory.

As far as coefficients of the potential of the Earth are concerned, we can consider the difference between LE 51 value and LE 200/IAU value, as quoted in (11) as a maximum of the uncertainty on J_2. This difference induces the small effects quoted in (12) upon the semi-analytical solution.

The coefficients of the lunar potential seem to be less accurately known. The leading effects induced by the differences between the two sets, LE 51 set and LE 200 set, are quoted in (9).

c) The accuracy of the fitted constants

It follows from Sect. IVb that the differences between the two sets of fitted constants S_{200} and $S_{51}^{(2)}$ are not due to the inaccuracy of ELP 2000-82$^-$ with respect to ELP 2000-82. So the values of the fitted constants depend on the numerical integration involved in the fit. For instance the values of E and Γ are slightly modified but the value of ν is strongly modified between the two numerical integrations. The large differences in the values of ν and $w_1^{(0)}$ are related to the large difference between the tidal t^2-terms of the mean longitudes in the two numerical integrations as quoted in (10).

To illustrate the influence of the assigned and fitted constants, Fig. 4 shows the behaviour of the two numerical integrations LE 51 and LE 200, when we remove the discrepancies due to the reference frame and the one due to G' and σ_1 values in the distances. The two longitudes and distances fit very well in the time span of laser observations but the two latitudes do not fit so nicely. The large discrepancies over 75 yr are due to the discrepancies between the tidal t^2-terms in longitude and also between the values of ν.

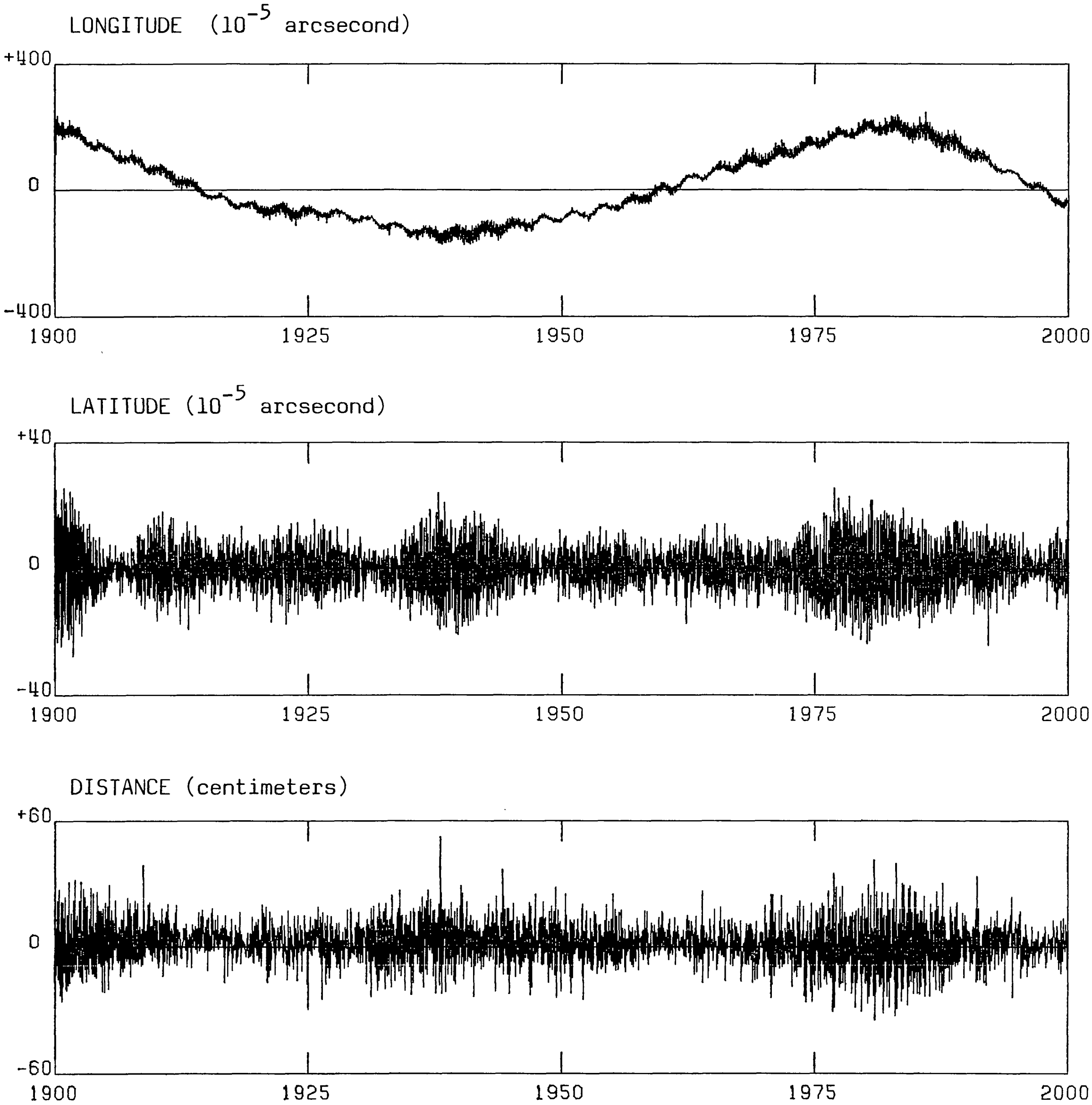

Fig. 5. Post fit residuals from the comparison of ELP 2000-82 + $\{\varrho_\sigma\}$ with LE 200 over one century

d) A numerical improvement of the solution

The post fit residuals obtained in the comparison of ELP 2000-82$^-$ to LE 51 are very close to those obtained during the comparison to LE 200 and given in Fig. 3. As we mentioned above, both post fit residuals and bias parameters describe the discrepancies between the numerical integration and the semi-analytical solution when the set of fitted parameters and the same values of assigned constants have been substituted in the latter. For their larger part, they are due to inaccuracies of the semi-analytical solution but they can also contain bias of the numerical integration. As far as they are small, we can expect that they are independent of small variations in the values of the constants used. They are also independent of the determination of the equinox. It is the reason why we have attempted to give them a numerical representation. The representation, we mention here, results from the comparison of ELP 2000-82$^-$ to LE 51 as described in Sect. IVb.

For each coordinate σ (longitude, latitude, and distance) we define ϱ_σ as:

$$\varrho_\sigma = \{\text{LE 51-ELP 2000}(S_{51}^{(2)})\}_\sigma . \qquad (17)$$

To compute ϱ_σ, LE 51 ephemeris has been rotated onto the inertial dynamical ecliptic and equinox $\gamma_{(2)}^I$ as defined in Sect. IVc. By ELP 2000($S_{51}^{(2)}$), we mean that we have substituted the set of constants $S_{51}^{(2)}$, and the related computed mean motions, in ELP 2000-82$^-$. So the (ϱ_σ) contain periodic variations similar to those of Fig. 3 and the effects of secular terms $\delta w_2^{(1)} t$ and $\delta w_3^{(1)} t$ in Moon's perigee and node and $\delta w_1^{(2)} t^2$ in Moon's mean longitude. Here $\delta w_2^{(1)}$, $\delta w_3^{(1)}$, $\delta w_1^{(2)}$ stand for the bias parameters obtained in comparison 2 (Table 13, Column 2). Each ϱ_σ is represented over one century (1900–2000) by a set of Chebyschev polynomials with 15 coefficients. Each polynomial represents the function over 8 d in such a way that $\{\text{LE 51-ELP 2000}(S_{51}^{(2)})\}_\sigma - \varrho_\sigma$ remains smaller than $5\,10^{-5}$ arc s.

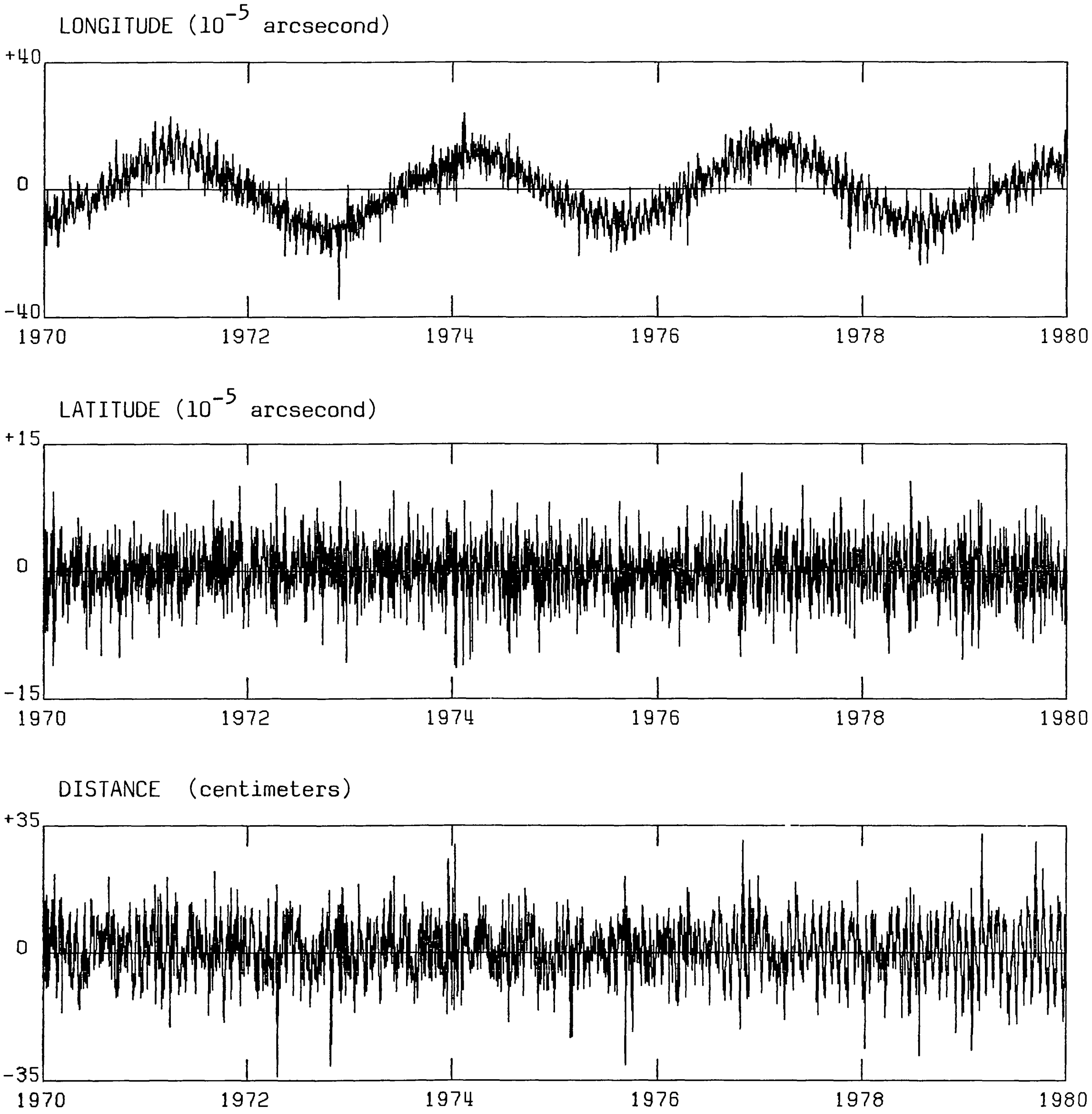

Fig. 6. Post fit residuals from the comparison of ELP 2000-82 + $\{\varrho_\sigma\}$ with LE 200 over 10 yr

We have compared ELP 2000-82 + (ϱ_σ) to LE 200 in the same way as we compared ELP 2000-82 to LE 200. We mean that the version of the semi-analytical solution, the values of the assigned parameters, the initial values and mean motions of planetary longitudes, the time span are the same in the two comparisons. The least square fit does not provide any significant modification to the set of lunar and solar constants S_{200} and to the determination of the equinox γ^I_{ELP}. It means that those quantities had been accurately determined. The bias parameters are listed in Table 13, Column 3. As we expected, they represent the difference between the bias parameters obtained from the comparison of ELP 2000-82 to LE 200 (Column 1) and those obtained during the comparison of ELP 2000-82$^-$ to LE 51 (Column 2) because the latter are included in the (ϱ_σ). So in the comparison of ELP + ϱ to LE 200, we are charged with the errors made during the computation of (ϱ_σ) by (17). They consist in a spurious t^2-term in the mean longitude included in ELP 2000-82$^-$ and a discrepancy between the J_2 values adopted in LE 51 and ELP 2000-82$^-$. The resulting effects are included in the bias parameters LE 51-(ELP 2000-82$^-$). The post fit residuals are given in Fig. 5. They are larger than we expected. We do not believe they are related to the constants or to the error made in the evaluation of (ϱ_σ), as described above, or to the reference frame. The problem lies more likely in the consistency of the two numerical integrations, concerning for instance the modelisation of forces or the integration of physical libration. Nevertheless, we can consider that the (ϱ_σ) increase the accuracy of the semi-analytical solution by a factor 5.

Besides, we have performed a similar least square fit over a time span of 10 yr (1970–1980) so as to simulate a comparison of ELP 2000-82 + (ϱ_σ) with laser observations. The results are different. The post fit residuals (Fig. 6) become smaller, but the lunar and solar parameters and the bias parameters are submitted to significant modifications. We denote S'_{200} the so-obtained set of lunar and solar parameters. The bias parameters are given in Table 13, Column 4. The differences between S_{200} and S'_{200} and the RMS are given in Table 12. Let us note here that a comparison of ELP 2000-82 without (ϱ_σ) to LE 200, over a time span of 10 yr, did not yielded any significant results concerning lunar and solar parameters since the RMS were too large. As a rule, the smaller is the time span of the comparison, the more accurate must be the solution submitted to comparison.

Conclusion

The ephemeris ELP 2000 has been derived from the semi-analytical solution ELP 2000-82, the internal precision of which is estimated to 0."01. The computation of an ephemeris requires the choice of values for the assigned constants and the evaluation of fitted constants. The values chosen for the assigned constants are those of IAU system 1976, as far as they are included in it, except for the parameters of the lunar potential. The parameters of the lunar potential and of the tidal effects are those of LE 200. With respect to the time span of observations, the internal precision of ELP 2000-82 is not large enough to allow a determination of the fitted constants by comparison to laser observations covering 10 yr, unless it is completed by some numerical approximation like (ϱ_σ) as it is described in Sect. V. The values of the fitted constants to be introduced in ELP 2000-82 have been determined from a fit to LE 200 over one century. We have observed large discrepancies between these values and values determined from a fit to LE 51.

The ephemeris ELP 2000 is referred to the inertial dynamical ecliptic 2000 and dynamical equinox γ^I_{ELP}, where γ^I_{ELP} is the intersection of the inertial dynamical ecliptic with the equator of LE 200.

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