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THE LIQUID-MIRROR TELESCOPE AS A VIABLE ASTRONOMICAL TOOL

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ABSTRACT

The surface of a rotating liquid takes the shape of a paraboloid. This fact can be used to build very large transit telescopes having as primary mirror a reflecting film of mercury. The practical aspects of this type of telescope are discussed. It is shown that a great deal of research can be done with the instrument, notwithstanding the fact that it is strictly a transit telescope and can only observe the zenith.

1. *Introduction*. The last decade has seen a remarkable increase of the efficiency of astronomical detectors. Charge-coupled devices (CCD) can now reach quantum efficiencies as high as 80 per cent. Little gain can thus be achieved by increasing further the efficiency of the detectors. Any significant increase in the power of ground-based optical observations can come only from increases in the collecting area of the telescope. On the other hand, there has been no significant increase in the size of optical telescopes since the completion of the 5-meter Hale telescope, several decades ago. These facts have been clearly recognized by the participants at the conference on optical telescopes for the 1990s held in Tucson in January 1980 (Hewitt 1980). The participants (e.g. Faber 1980, Strittmatter 1980) make a strong case for the advances in astronomical knowledge that will come from a significant increase in telescope size, particularly from a telescope in the 25-meter class. However, the technology needed to build such an instrument is largely unexplored and both the cost and the time needed to build it are essentially unknown. A 10-meter telescope is probably more realistic, albeit very expensive, and might not be completed before the turn of the century.

In this paper, I will show that if one uses as primary mirror a rotating pool of mercury, a 30-meter telescope is within reach. Indeed much larger mirrors might be feasible. This type of telescope is strictly a transit instrument and can observe only the zenith. I shall argue in section 4 that, for a large class of observations, this is not a handicap. In section 2, I develop the basic concept and consider the practical aspects of the instrument in section 3.

2. *The Rotating Fluid Mirror*. Consider a cylindrical container partially filled with a liquid. The container rotates, with angular velocity ω, around its axis of symmetry which is perpendicular to the surface of the earth. The liquid assumes an equilibrium position such that its surface follows an equipotential surface. At every point, the surface of the liquid is thus perpendicular to the net force.

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Let us examine the shape of this surface of revolution by intersecting it with a plane that contains the axis of rotation. The liquid is subject to the acceleration of gravity **g** (parallel to the axis of rotation which is taken to be the y-axis) and to the centrifugal pseudo acceleration **a** which is parallel to the x-axis. At every point on the surface, the angle α between the normal to the surface and the parallel to the y axis passing through that point is given by

$$\tan \alpha = \frac{|\mathbf{a}|}{|\mathbf{g}|} = \frac{\omega^2 x}{g}. \tag{1}$$

This angle is equal to the angle between the tangent to the curve and the parallel to the x axis at that point, therefore

$$\frac{dy}{dx} = \frac{\omega^2 x}{g}. \tag{2}$$

Integration yields

$$y = \tfrac{1}{2}\omega^2 \frac{x^2}{g} \tag{3}$$

where the constant of integration has been set to zero by placing the origin of the axes at the vertex of the curve. Equation (3) describes a parabola and can be written in the parametric form (Eves 1976)

$$x^2 = 4ly \tag{4}$$

where l is the focal length of the parabola given by

$$l = \frac{g}{2\omega^2}. \tag{5}$$

We can readily see that, for a given value of the acceleration of gravity, the focal length of a liquid mirror is uniquely determined by the angular velocity. Table 1 gives some relevant parameters for a number of mirrors grouped by f-ratios. The acceleration of gravity used assumes a terrestrial latitude of 45° ($g = 980.6$ cm sec^{-2}). We can see that the angular and rim velocities are small and quite manageable. A man, walking, could keep up with the rim of an $f/2$ 5-meter mirror and a runner could follow the rim of an $f/1$ 30-meter mirror.

3. *Practical Considerations*. Although any high reflectivity fluid could be used, we will only consider mirrors using liquid mercury. A search of the scientific literature showed that an 0.5-meter diameter mirror was built by Wood (1909). He found that three main sources of ripples perturbed the surface of the liquid: a) jars of all sort (e.g. from the driving mechanism, the bearing and the floor), b)

TABLE I

PARAMETERS OF ROTATING LIQUID MIRRORS

Diameter (m)	ω	V_{rim} (km hr^{-1})	Scale (arcsec mm^{-1})	Resolution (arcsec $(30\mu)^{-1}$
		f/2		
2	1.11	4	53	1.60
5	0.70	6	21	0.63
10	0.50	9	10.5	0.32
15	0.40	11	7	0.21
30	0.286	15	3.5	0.11
		f/1		
2	1.57	6	105	3.05
5	0.99	9	42	1.26
10	0.70	13	21	0.63
15	0.57	15	14	0.42
30	0.40	22	7	0.21
		f/0.5		
2	2.21	8	210	6.05
5	1.40	13	84	2.50
10	0.99	18	42	1.25
15	0.81	22	28	0.84
30	0.57	31	14	0.41

imperfect levelling of the instrument that caused "tidal waves", c) variations in the speed of rotation. While the levelling problem could be easily overcome, the jars remained a minor problem as Wood's mirror could detect "a horse and carriage approaching" as well as "the footsteps of a person running fifty yards from the telescope house". Wood's final mirror proved satisfactory, with the exception of the periodic changes in focus that resulted from variations in rotational speed. Wood felt unsure however, that the instrument could ever be of any astronomical use because it is a transit instrument that can observe only the zenith.

I shall argue, in this section, and the next, that the problems encountered by Wood can be easily overcome with modern technology, and that a great deal of astronomy can be done with the transit telescope.

The jars and vibrations can be dampened by using an air bearing. These bearings are essentially frictionless, the technology is well-proven and they are routinely used to rotate large masses with a great precision. A manufacturer advertises turntables that have accuracies of 10^{-6} inches! Fortunately, these bearings are not expensive, a 144-inch diameter turntable that can carry 35 tons retails for U.S. \$17,000. This table can be used to rotate a mirror several times its diameter. Larger turntables are available on request.

The most severe problem encountered by Wood was caused by changes in rotational velocity which he attributed to the roughness of the bearing he used. The relative error in rotational velocity is related to the relative error in focal length by

$$\frac{dl}{l} = 2\frac{d\omega}{\omega}. \tag{6}$$

Let $d\theta$ be the maximum degradation in resolution that can be tolerated, it is related to the focal length variation by

$$\frac{dl}{l} = d\theta f.\ \text{constant} \tag{7}$$

where f is the f-ratio of the mirror. We can see that dl/l is a function of the f-ratio alone. If we take $d\theta = 0.2$ arc seconds as a reasonable value and consider an $f/1.5$ mirror, we see that $d\omega/\omega = 7 \times 10^{-7}$. The stability needed is thus very high and, probably, could not have been attained with the technology available to Wood. A simple solution to the high stability needed is to have a high inertia turntable rotating on a frictionless bearing. These conditions are clearly fulfilled for the large mirrors considered and the air-bearing turntable. After having attained the proper speed, the turntable should be driven by a loose belt with a weak motor. One should choose a synchronous electromotor driven by an oscillator stabilized a.c. power supply. Changes in speed can thus come only as slow drifts. The drifts can easily be checked, with encoders, against a stable clock. Temperature-controlled crystal oscillators are stable to 10^{-9}. Indeed, rubidium gas cells and hydrogen masers give clocks stable to 10^{-12}. We are thus comparing a poor clock (the turntable) to a good clock (the frequency standard). A correction can be applied to the a.c. power-supply frequency and to the motor. The stability of the turntable in the vertical direction is of equal concern. If we take a 5-meter $f/1.5$ mirror, a value of $d\theta = 0.2$ arcseconds corresponds to a focus change of 7 microns. A commercially available turntable running at 50 PSI on an air supply regulated at ±2PSI is stable at ±1 micron. Higher regulation and better stability are easily attainable.

If we use a cylindrical container having a flat bottom, the volume of liquid needed for a paraboloid is

$$V = \pi r^2 h - \frac{\pi g}{\omega^2} h^2 \tag{8}$$

where r is the radius of the mirror and h is the thickness of mercury at the rim, given by

$$h = \frac{\omega^2 r^2}{2g}. \tag{9}$$

A 5-meter $f/4$ mirror needs 10 metric tons of mercury at a cost of $300,000. A considerable saving in weight and cost can be achieved by using a container having the shape of a paraboloid. The mercury then serves as a liquid high-reflectivity coating, the thickness of which depends on the accuracy with which the parabolic container is manufactured. A 5-meter mirror needs 300 kilograms of mercury per millimeter thickness, costing $10,000. A 30-meter mirror needs 10 tons, costing $300,000 per millimeter thickness.

Of the many materials that could be used to manufacture the container, ferrocement is particularly attractive. It is cheap. It is a composite material and therefore its mechanical characteristics can be tailored to the particular application. Its technology is well-proven. It is routinely used in boat-building where it takes complicated shapes. Its application to astronomy is not new as it has been successfully used to build cheap telescope mounts (Labeyrie 1978, Vakili and Glentzlin 1980). The container could be built on site, upside down on a sand mold lying on the ground. Final figuring of the surface can be done by placing the container on the turntable which is then rotated at the appropriate speed. If a liquid coating (epoxy-like compound) is poured on the container and allowed to dry slowly, it will give the surface of the container a shape close enough to a paraboloid, for our purpose. The coating will also prevent the mercury from entering the pores and cracks of the cement.

Liquid mercury has a reflection coefficient varying between 79 per cent at 3100A and 78 per cent at 8700A (Hass and Hadley 1903) rising to 90 per cent at 13 microns (Schulz 1957), while evaporated aluminum has a reflectivity of 92 per cent at 3200A, rising to 98 per cent at 10 microns (Hass and Hadley 1963). At first sight, aluminum seems to have an advantage over mercury. Consider, however, that these values are for a fresh coating. Large telescope mirrors are never aluminized more often than once a year and a very large telescope is likely to be aluminized even less often. The aluminum coating deteriorates as the surface exposed to the air is coated with oil, dust, etc ... On the other hand, the mercury layer can be filtered every day if necessary and restored to perfect conditions for the night. In practice, aluminum and mercury should be equivalent.

Mercury is a toxic metal, it can however be handled safely with proper precautions. Because the telescope is exposed to the open air, the ambient density of mercury vapor should be small. The evaporation should probably be controlled in a large facility. This could be accomplished with suction pumps or by adding a transparent liquid on top of the mercury. This second solution has also the advantage that the liquid will dampen any remaining disturbance in the surface of the mirror (Wood 1909).

The transit telescope can view only the zenith and cannot track. However, electro-optical tracking can be accomplished by using a CCD detector and stepping the pixels, in the East-West direction, in a read-out mode, at a speed that matches the drifting of the image in the focal plane. This technique, to be used with a glass-mirror transit telescope, has been described in detail by McGraw, Angel and Sargent (1980). Particularly interesting is their description of the "average out" of the spatial and spectral responses of the CCD that comes as a byproduct of the technique. As they point out, this allows exceedingly high photometric accuracy. Notice also the performance expected (their Table 1) from a 76cm transit telescope. McGraw *et al.* (1980) also show that the quantity of data expected is large but manageable. Their project has recently been funded and the electro-optical tracking technique has successfully been tried with the Steward Observatory 90-inch telescope (McGraw, *et al.* 1982). The telescope was fixed at the zenith and diaphragmed to 70-inch diameter. One pass through the 512 × 320 30 microns pixel CCD (total exposure time one minute) detected objects fainter than $B = 21$.

For efficient operation, it is essential to keep the focal length of a very large telescope to a minimum. If the focal length is too large, a typical seeing of 1 to 2 arcseconds will give, at the focal plane, a point-spread function that is grossly oversampled by the typical size of CCD pixels (15 to 30 microns). Table 1 shows the plate scale for various mirrors. The liquid telescope has here the clear advantage that very fast mirrors can be made easily.

A very fast paraboloid has a very small usable field, because of coma. It is therefore crucial to find a corrector capable of enlarging the field to several minutes of arc. Baker (1969) has discussed a family of two-mirror correctors similar to those described by Paul (1935). These correctors, designed for parabolic primary mirrors can achieve a surprisingly large corrected field. Angel, Woolf and Epps (1982) have studied the Paul corrector for fast primaries. They find that an $f/1$ paraboloid can be corrected to yield images smaller than 0.2 arcseconds over a one-degree diameter field.

To be useful for astronomical observations, the surface of a liquid mirror should not be perturbed by more than a fraction of a wave. This seems a stringent limit if one considers the ease with which ripples are set-up on the surface of a liquid. We, therefore, want to use a precise bearing that will not jar the container. The stability of gas-lubricated bearings is thus of major concern. Gas-lubricated bearings are known to suffer from instabilities (Constantinescu 1969); but these instabilities occur either at very high rotational speeds or because of unbalanced loads or varying dynamical loads. None of these conditions is of concern to us. Instabilities can also arise from poorly regulated air supplies (Constantinescu 1969). This is, however, not a serious problem as a commercial turntable stable to 10^{-6} inches (250Å) runs from a 50 ± 2 psi air supply. If needed, better regulation can be attained and the use of a pressure tank, acting as a buffer, can further isolate the turntable from the compressor. Air bearings can be manufactured to a very high accuracy, depending on the roughness and flatness of the mating surfaces. In this respect, Wunsch and Nimmo (1972) list the reduced significance of the surface texture and small geometric deviations in the bearing components as one of the advantages of air bearings. As astronomers well know, it is well within our ability to polish large glass or metal surfaces within a fraction of a wave (flat mirrors) that could give ideal mating surfaces for very stable bearings. An air bearing 150-inch in diameter could carry 35 tons and could probably support a 30-meter liquid mirror and container. Such an exquisitely precise bearing might, however, not be needed as molecular forces are expected to dampen ripples. The fact that Wood (1909) could produce decent images with a relatively crude metal bearing is very encouraging. One of the disadvantages of gas-lubricated bearings, in many applications, is their flexibility under varying loads. This is actually an advantage in our case as it provides some isolation from vibrations transmitted through the ground.

Prototype mirrors, from one-meter diameter up, are clearly needed to evaluate the feasibility of the concept and of large facilities. Should surface ripples be a major problem, it will still be possible to dampen them by pouring a layer of viscous transparent liquid on the mirror. Wood (1909) experimented successfully with water and glycerin.

4. *The Scientific Case*. The transit telescope is a survey instrument and the scientific information it gives is the same as that given by a Schmidt telescope. A very large number of projects can be carried out with medium (5-meter) to large (30-meter) liquid-mirror telescopes. Searches for (and study of) primordial galaxies, studies of the colours of distant faint galaxies, activity of galactic nuclei, galactic structure (star counts), counts and distribution of faint galaxies for cosmological studies, discovery (and estimation rates of) distant supernovae, search for faint quasars, are only a few. Two projects (search for supernovae and studies of optical variability have also been described in detail by McGraw *et al.* (1980)).

Let us examine the characteristics of two telescopes, their expected performance and some of the research that would derive from them. For optimum performance, telescope and detector must be matched. In what follows I shall only consider CCD detectors that are presently available (512 × 320, 30-micron pixels).

Let us consider first a 5-meter telescope in a typical Canadian dark site, having average seeing of 2 to 3 arcseconds; at *f*/1 the plate scale is such that a 30-micron pixel subtends 1.25 arcseconds, giving a moderate oversampling in average seeing. One CCD detector covers 640 arcseconds in the East-West direction and 400 arcseconds in the North-South direction. This configuration gives (at a latitude of 45°) a 60 seconds exposure per object every night. The strip of sky, 6.7 arcminutes wide, covered through the whole celestial sphere (at a declination of 45°) has a total area of 28 square degrees.

Let us consider the performance of the instrument equipped for slitless wide-field spectroscopy. We shall not examine the details of the design of the disperser but, as Baker (1969) points out, the Paul corrector is well-suited for wide-field spectroscopy as the beam from the tertiary to the secondary is collimated. Let us choose a dispersion of 1700Amm^{-1}. In typical seeing (2 to 3 arcsec.) the spectral resolution is 80A to 120A. To reduce the contribution from the red part of the spectrum of the night sky, let us block, with a filter, the response of the CCD redwards of 7,000A (such a filter is commercially available). Using the formula given by Strittmatter *et al.* (1980) with the adequate values of the various parameters, we find that in one-hour total exposure (corresponding to 60 clear moonless nights) we can determine the continuum distribution (assuming a flat spectrum and CCD response) to a 3 per cent accuracy per spectral element down to visual magnitude 20 and to a 10 per cent accuracy down to magnitude 21.2. Emission-line objects, such as quasars, can obviously be detected to fainter magnitudes. Objects brighter than magnitude 20 are measured to a precision better than 3 per cent considering the "averaging out" effect discussed by McGraw *et al.* (1980). Because the data are gathered as a sum of short exposures, the dynamic range is greater than would be allowed by a single long exposure. Selected bright interesting objects (such as cataclysmic variables or OVV Quasars and BL Lac objects) can be monitored from clear night to the other. Assuming nights of eight hours duration, the instrument obtains slitless spectra having 80A to 120A resolution for every object down to at least $21^{\mathrm{m}}.5$, in a total area of 28 square degrees, in 180 moonless nights. This number of clear moonless nights can be obtained in less than 4 years. Notice that this performance is obtained in a typical Canadian site. In a very good site, having better seeing, a telescope of longer focal length would be used, yielding fainter limiting magnitudes. This, coupled with the larger number of clear nights would give more impressive performance figures. Table 2 summarizes the performance of the telescope.

The quantity, quality and scientific value of these data can best be appreciated if one considers that the spectral information obtained has a resolution comparable to the data obtained with the enormously successful multichannel spectrometer used with the Hale 5-meter telescope (e.g. Yee and Oke 1978), and this to very faint magnitudes and over an area comparable to the area of a 48-inch Schmidt plate. The amount of data is large but manageable (McGraw *et al.* 1980). Indeed, it is not so much the data reduction that is the problem, but rather obtaining all the scientific information from the data.

Among all the possible uses of the data, let us see how they can increase our knowledge of quasars. We have a statistically unbiased, magnitude-limited sample of quasars, including magnitudes, spectral distributions and redshifts to a limit two to three magnitudes fainter than previously available. Taking into account only the strip of sky having $b^{\mathrm{II}} \geq 30°$, the total area covered at a terrestrial latitude of 45° is

TABLE II

CASE STUDY OF TWO TELESCOPES
(*f*/1 mirrors)

Diameter of mirror (m)	Resolution arcsec $(30\mu)^{-1}$	Area Covered[1] (square degrees)	Dispersion (bandpass)	*V* Limit[2] ($s/n > 10$)
6	1.25	28*	1700Amm^{-1}	21.5
30	0.21	6.7†	*V* band	28

[1] Area of the complete strip of sky covered in one year by using a single 520 × 312 micron CCD detector.

[2] One hour total exposure per object.

*At a latitude of 45°.

†At the equator.

12 square degrees. The surface density of faint quasars is not very well known, but from Koo and Kron (1982), we can see that there are about 150 quasars per square degrees to magnitude 22. Our survey should discover over 2,000 quasars. This number is larger than the number of quasars listed in the Hewitt and Burbidge (1980) catalogue. This catalogue includes all objects found in 15 years and is essentially useless for statistical purposes as it is plagued by selection effects. By comparison, also, the best deep homogeneous survey, the CTIO 4-meter survey, covers 5 square degrees, is complete to magnitude 19.5 and lists 71 quasars (Osmer 1981). The large and unbiased sample that would be given by the 5-meter survey could therefore supply us with the data necessary to increase enormously our knowledge of the space density, luminosity function, evolution and clustering properties of quasars. We will also obtain an unprecedented view of the universe at high redshift.

Let us now consider a very large telescope at a very good site near the equator, having average seeing of the order of 1 arcsecond. An *f*/1 30-meter telescope gives a resolution of 0.2 arcseconds per 30-micron pixel and an adequate oversampling of the point-spread function. A512 × 320 CCD gives an integration of 7.2 seconds per object per night. The total area of sky covered is 6.7 square degrees in a strip 67 arcseconds wide. Using the equation given by Strittmatter *et al.* (1980), I find that, in the *V*-band, a one-hour integration yields a precision of 8 per cent at $V = 28$. One nightly observation (integration of 7.2 seconds) yields a 12 per cent precision at $V = 25$. Table 2 summarizes this information. For a more detailed analysis (of a 10-meter telescope) see Faber (1980), especially her table 1. As in sky-background-limited observations the limiting magnitude increases linearly with telescope diameter, a 30-meter will reach 1 magnitude fainter than a 10-meter for an equal integration time. To obtain a one-hour total exposure per object across the whole 67-arcsecond-strip of sky one needs, assuming 8-hour dark nights, a total of 1500 dark nights. Taking into account moonlit nights and a 20 per cent loss to weather, the survey lasts 10 years if one CCD only is used. Because a Paul corrector can correct a full degree, the survey can be extended, and the time needed to complete the survey shortened, by using more than one CCD detector. Moonlit hours can be used to extend the survey to the infrared with the soon to be available infrared imaging devices.

The 30-meter telescope will be a valuable supplement to the Space telescope much as the 48-inch Schmidt has been a valuable complement to the 5-meter Hale telescope. Notice, however, that at some wavelength, the 30-meter can go as faint as the space telescope for point sources and substantially fainter for extended sources (>1 arcsecond) (Strittmatter *et al.* 1980).

5. *Conclusion.* The main advantage of a liquid-mirror telescope is clearly its cost, so that it is feasible to build very large (and fast) mirrors. The liquid mirror and frame of a 6-meter *f*/1 telescope can be built for a few tens of thousands of dollars. A further reduction in cost results from the fact that a rotating dome is not needed. A sufficiently strong telescope frame might not need a dome at all. The frame might have to be enclosed if wind gusts shake it beyond tolerance; however, a silo-like enclosure will still be cheaper than a rotating dome.

The main limitation of the instrument is that it is strictly a transit telescope and cannot track mechanically. This is presumably the reason why the liquid mirror concept was abandoned by Wood and was never considered seriously by astronomers. This limitation is now overcome by the advent of CCD detectors and the electro-optical tracking technique pioneered by McGraw *et al.* As shown in section 4, the liquid-mirror transit telescope promises to be a valuable survey instrument that can be used to carry out the type of research usually done with Schmidt reflectors. In particular, a very large liquid-mirror telescope, built to the limits imposed by funds, technology and detectors, promises to be the ultimate instrument to use in cosmological studies and might yet give us our deepest glimpse into the early universe.

The remaining problems encountered by Wood (1909) can be easily overcome with modern technology and the time has now come to take a fresh look at the liquid-mirror concept. As shown in the preceding section, the case in favor of this type of telescope is strong enough for this writer to take the challenge and he will experiment with mercury paraboloids of increasing sizes leading, eventually, to a 5-meter *f*/1 mirror. This should, he hopes, show the way to building much larger telescopes.

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NOTE ADDED IN PROOF

The analysis in section 2 neglects the effects caused by the Coriolis pseudo-force due to the Earth rotation. An analytical analysis, including the Coriolis force, neglecting viscosity and assuming that the liquid reacts instantly to the force, shows a practical limit of diameters of about 15 meters. The surface of the liquid sees a force that tries to set up a wave traveling with the angular velocity of the turntable and having amplitude proportional to $\omega^3 x^3$. Because mercury is an incompressible fluid, this wave can only be maintained by mass motions (in the time scale of half a period of rotation) inside the liquid itself. On the other hand, the viscosity of the liquid will slow down the material flow. This makes the forementioned practical limit a worse-case limit. A high viscosity transparent liquid (such as glycerin) will also greatly impede the mass motions, further extending the limit in size due to the Coriolis force.

Because the effects of the Coriolis force are functions of ωx, it is possible to study what will happen to large mirrors by spinning rapidly smaller mirrors. Experiments are now being undertaken to study the exact effects of the Coriolis force and ways to decrease them or correct them. Masking the worse parts of the mirror (depending on seeing) and using Multiple Mirror Transit Telescopes are also viable means to increase further the total collecting area. A detailed discussion will be submitted at a later date.

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