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THE POLARIZATION OF SUPERNOVA LIGHT: A MEASURE OF DEVIATION FROM SPHERICAL SYMMETRY

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ABSTRACT

The assumption of spherical symmetry, made in most theoretical models of supernova (SN) explosions, light curves, and spectra, and in attempts to use SN explosions as cosmological distance indicators, has never been justified observationally. An observational test of this assumption is suggested here, based upon the likelihood that SN atmospheres are scattering dominated and the fact that light from an unresolved, *asymmetric*, scattering atmosphere is linearly polarized.

We have calculated the linear polarization of SN light which a nonspherical, scattering-dominated, SN atmosphere would produce as a function of its asphericity. The effect of asphericity on the inferred total luminosity of the SN has been calculated as well. We have considered Thomson scattering, resonance scattering, and a mixture of scattering and absorption. The last of these cases can yield a significantly higher degree of polarization than that expected from the classical, pure scattering atmosphere.

The effects of interstellar matter on the observed SN polarization have been included. Interstellar dust not only changes the observed linear polarization but converts linearly polarized light into circularly polarized light as well, as a result of its linear birefringence.

Existing, preliminary observations of supernova polarization are reviewed in the light of these results. Suggestions are made for future observation.

Subject headings: interstellar: matter — polarization — radiative transfer — stars: supernovae

I. INTRODUCTION

Are supernova explosions spherically symmetric? As yet, there is no direct observational answer to this question. It is the purpose of this paper to propose such an observational test. The test is based upon the fact that an unresolved, nonspherical supernova atmosphere which is scattering-opacity dominated will emit linearly polarized light.

To date, most of the attempts to explain or model the supernova explosions and their accompanying light curves and spectra have assumed perfect spherical symmetry (for a recent review, see Wheeler 1981). Efforts to use supernovae as cosmological distance indicators have also depended critically upon the assumption that the supernova atmosphere is spherically symmetric (see Wagoner 1979). This is understandable; spherical symmetry is a reasonable first approximation, and departures from this symmetry make the already formidable computational problems of supernova modeling far more difficult.

Departures from spherical symmetry may strongly influence the occurrence and evolution of supernovae, however. Rotation, magnetic fields, and the growth of nonspherical instabilities are all potentially important sources of this departure. A dramatic example is provided by a numerical calculation by LeBlanc and Wilson (1970) of stellar core collapse in the presence of gravity, rotation, and magnetic fields. This calculation showed that differential rotation can amplify a seed magnetic field into a strong toroidal field, resulting under certain circumstances in the explosive expulsion of jets of matter along the rotation axis.

There has been other discussion of nonspherical effects on supernovae as well. Some of this discussion has been stimulated by the hope that the gravitational collapse of a stellar core can produce potentially detectable gravitational radiation and the knowledge that a spherically symmetric collapse precludes the emission of such radiation (see Saenz and Shapiro 1981; Smarr 1979). Some has been stimulated by the difficulty which the popular explanation of supernova explosions involving spherically symmetric core collapse and neutrino transport has had in producing a sufficiently strong explosion. The suggestion has been made, for example, that a Rayleigh-Taylor instability in the stellar core may develop which overturns the entire core during collapse, releasing enough neutrinos from the core that a sufficiently violent explosion *can* occur (e.g., Livio, Buchler,

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and Colgate 1980; Epstein 1979). There has even been some discussion of the possibility of off-center explosions and their connection to the formation of high velocity pulsars (e.g., Fryxell 1979; Shklovsky 1970). Recently, there has been a reconsideration of the effects of rotation and magnetic fields (Hillebrandt and Müller 1978, 1981; Müller and Hillebrandt 1979).

So far, this discussion of nonspherical effects has primarily concerned the supernova interior—the implosion of a dense stellar core and the generation of the supernova shock wave. One exception is the work of Chevalier and Klein (1978) which relates directly to the exploding supernova envelope. This work suggests that even an initially spherically symmetric stellar envelope, accelerated outward by the passage of the supernova shock wave, is susceptible to the growth of a Rayleigh-Taylor instability on scales which are an appreciable fraction of the envelope radius. In general, however, the possible effects of asphericity on the supernova *atmosphere* and the emergent *radiation* have not yet been examined. Most of the directly *observable* information about the supernova explosion, however, is contained in the light from this atmosphere. Since our goal here is to provide an observational *diagnostic* of departures from spherical symmetry, we shall limit our attention henceforth to the supernova atmosphere and the light which emerges from it.

As will be discussed in § II, supernova atmospheres are likely to be scattering-opacity dominated. It is well known that scattering atmospheres can transform initially unpolarized light into partially linearly polarized light. Chandrasekhar (1960, 1946) calculated this effect for a plane-parallel electron scattering atmosphere and derived the degree of polarization of the emergent light as a function of the angle between the normal to the plane and the direction of view. An observation of a spherically symmetric, electron scattering atmosphere which does not *resolve* the atmospheric disk, however, will always see zero net polarization. Only if the symmetry of the unresolved source with respect to rotation around the line of sight is *broken* can a net polarization be measured.

This suggests a direct observational test for the deviation of a supernova explosion from spherical symmetry. In the absence of some polarizing scattering opacity, the light emergent from the atmosphere should be unpolarized. Even *with* such a contribution to the opacity, an unresolved, *spherically symmetric* atmosphere will exhibit no *net* polarization. Hence, any detection of net polarization of the supernova light must be a direct measure of the asphericity of the atmosphere.

We discuss the net polarization which is to be expected as a function of the degree of this asphericity in the sections which follow. In § II, we summarize the basic assumptions about the supernova atmosphere which are relevant to the emergence of polarized light. In § III, we present the basic equations which must be solved in order to predict the net polarization, as well as describe their solution. Our results are presented in § IV. In § V, we summarize these results, discuss the important effects of intervening interstellar dust on the observed supernova polarization, and describe the preliminary observational results which already exist of supernova polarization.

II. THE SUPERNOVA ATMOSPHERE

Models of the atmospheres and of the radiation spectrum of both Type I and Type II supernovae generally assume that a thermal, blackbody continuum spectrum emerges from a well-defined photosphere (e.g., Branch 1980). The color temperature of this continuum emission is believed to decrease from a value $\gtrsim 15{,}000$ K near maximum luminosity to ~ 5000–6000 K during the first month. Spectral lines are assumed to be formed by resonance scattering of the continuum radiation in a homologously expanding atmosphere above the photosphere. This is responsible for the characteristic P Cygni-type profiles for the lines.

Wagoner (1981) has recently discussed the predominance of scattering over absorption in the supernova photosphere. The continuous opacity is dominated by electron scattering. In addition, as Karp *et al.* (1977) have discussed, the velocity gradient within the photosphere significantly enhances the bound-bound opacity as a result of the Doppler broadening of lines by an amount much greater than the thermal line width. The net effect of this Doppler broadening of many individual lines is the production of a quasi-continuous bound-bound opacity which, according to Karp *et al.* (1977), can be of a magnitude comparable to that of electron scattering. As Wagoner (1981) has pointed out, this bound-bound opacity is scattering dominated, since the ratio of radiative to collisional atomic transition rates is large in supernova photospheres. Although Wagoner's discussion is focused on the case of Type II supernovae, the statement that scattering dominates the photospheric opacity can be made more generally.

We shall assume in what follows, therefore, that the light emergent from the supernova atmosphere is produced by an optically thick, scattering atmosphere, with either electron scattering, resonance scattering, or both. Of course, *some* absorption *must* be present in this atmosphere in order to thermalize the continuum radiation. If the ratio of absorption to total opacity, λ, is much less than unity, however, it is well known that the optical depth of formation of the continuum is $\tau \approx \lambda^{-1/2}$, rather than $\tau \approx 1$, as it would be in an absorption-dominated case. Hence, the depth of continuum *formation* is much greater than that of the surface of last scattering. The neglect of absorption opacity in our calculation of emergent polarization would seem justified, therefore. Nevertheless, since *some* absorption is present throughout, and since our knowledge of the details of supernova atmospheric opacity is so uncertain, we shall also discuss cases in which both scattering and absorption are present. As we shall see, the presence of even a small amount of absorption can, under the right conditions, have the surprising effect of significantly enhancing the emergent polarization.

III. THE CALCULATION

In order to determine the net degree of polarization of light emergent from an optically thick, nonspherical, scattering atmosphere as a function of its degree of asphericity, the following three steps must be taken: (*a*) The vector equation of transfer for the full set of Stokes parameters must be solved. As long as the scattering is conservative and frequency independent, the effects of the expansion of the atmosphere can be ignored in solving the transfer equation. In that case, a plane-parallel, semi-infinite atmosphere approximation with constant net flux will be valid. As we shall discuss, the plane-parallel approximation is also valid, for the homologous expansion considered here, when the scattering is frequency dependent as well. (*b*) Some general characterization of the departure from spherical symmetry must be chosen which allows a range of degrees of asphericity to be explored. (*c*) The Stokes parameters for the net radiation emitted in a given direction by the entire atmosphere must be found by integrating the plane-parallel results over the face of the atmosphere. We outline each of these steps below.

a) *The Plane-Parallel Atmosphere*

i) *Pure Electron Scattering*

As mentioned in § I, the problem of the polarization of light emergent from a semi-infinite, plane-parallel, pure electron scattering atmosphere with constant net flux was solved by Chandrasekhar (1960, 1946). Chandrasekhar found that the polarization varied from a maximum of 11.7% for an atmosphere viewed edge-on to 0% for the face-on view. The polarization vector was found to lie in the plane of the atmosphere, perpendicular to the line of sight.

For the pure scattering case, we can use Chandrasekhar's results as the starting point for step (*c*) in our calculation, described above. In fact, the results for electron scattering apply equally well to classical resonance scattering, Rayleigh scattering, and any other process for which the angular distribution and the state of polarization of the scattered light can be described by the Rayleigh scattering phase matrix. We begin, therefore, by presenting those basic definitions and equations which are necessary in order to describe the application and extension of Chandrasekhar's results in the sections which follow. Our later discussion of the effect of combining absorption with scattering, as well as that of the effect of intervening interstellar dust on the observed polarization, will also utilize the quantities and notation introduced below.

The incident and scattered light in a single scattering event are completely described by the set of four Stokes parameters I, Q, U, and V. We can represent the information contained in these Stokes parameters by a vector intensity $\boldsymbol{I}$ defined by

$$\boldsymbol{I}=(I_l, I_r, U, V), \tag{1}$$

where

$$I_l=\tfrac{1}{2}(I+Q), \tag{2a}$$

and

$$I_r=\tfrac{1}{2}(I-Q). \tag{2b}$$

Let us define the plane of scattering as the plane containing the incident and scattered beams. In that case, I_l and I_r represent the intensity of the light whose electric vector is parallel and perpendicular, respectively, to the plane of scattering. When a pencil of light described by this vector intensity $\boldsymbol{I}$ and confined to a solid angle $d\Omega$ is incident on a particle with scattering coefficient σ, the vector intensity scattered into a solid angle $d\Omega'$ around the direction making an angle Θ with the direction of incidence is given by

$$\sigma\frac{d\Omega'}{4\pi}R_{ij}\boldsymbol{I}_j\,d\Omega, \tag{3}$$

where R_{ij} is the 4×4 phase matrix for Rayleigh scattering given by

$$R_{11}=(3/2)\cos^2\Theta, \tag{4a}$$

$$R_{22}=3/2, \tag{4b}$$

$$R_{33}=R_{44}=(3/2)\cos\Theta, \tag{4c}$$

and

$$R_{ij}=0 \quad \text{for } i\neq j. \tag{4d}$$

As usual, the repeated indices in equation (3) imply summation.

Using the phase matrix defined above, it is possible to write a vector equation of transfer for $\boldsymbol{I}$. In order to do this, however, one must first refer all of the quantities written above to a coordinate system fixed to the meridian passing through the line of sight, rather than fixed to the scattering plane of each scattering event which contributes to the beam.

Let the line of sight through the point in question be along a direction defined by the spherical polar angles (θ, ϕ). Let a pencil of radiation of solid angle $d\Omega'$ directed along angles (θ', ϕ') be scattered into the direction (θ, ϕ). The vector equation of transfer must describe the contribution of this scattered radiation to the vector intensity along (θ, ϕ). The quantity $\boldsymbol{I}(\theta', \phi')$ as defined in equation (1) refers to the directions parallel and perpendicular to the plane of scattering, however. We must rotate these coordinates.

The procedure for doing this is discussed by Chandrasekhar (1960). The four-component vector equation of transfer derived in this way for a plane-parallel, electron scattering atmosphere ultimately reduces to a pair of coupled equations for the two components I_l and I_r. First, the equation for V is independent of the other equations. Second, the azimuthal symmetry of the radiation field requires that the plane of polarization, defined by the direction of propagation of the beam and the polarization vector perpendicular to it, must either be coincident with the plane of constant ϕ containing the beam (i.e., the meridian plane) or perpendicular to it. As a result, $U=V=0$, and the plane-parallel, vector equation of transfer reduces to the following pair of equations:

$$\mu \frac{dI_l(\tau,\mu)}{d\tau} = I_l - \frac{3}{8}\int_{-1}^{+1} d\mu' \{[2(1-\mu^2)(1-\mu'^2) + \mu^2\mu'^2] I_l + \mu^2 I_r\}, \tag{5a}$$

and

$$\mu \frac{dI_r(\tau,\mu)}{d\tau} = I_r - \frac{3}{8}\int_{-1}^{+1} d\mu' \{\mu'^2 I_l + I_r\}, \tag{5b}$$

where $\mu = \cos\theta$ and τ is the optical depth to scattering measured normal to the plane.

Equations (5a) and (5b), along with the condition of radiative equilibrium and the semi-infinite atmosphere boundary conditions, have been solved exactly by Chandrasekhar (1960). The solution is represented by the laws of darkening in the two states of polarization, with electric vector parallel and perpendicular to the meridian plane, respectively. These laws of darkening are just the values of $I_l(0,\mu)$ and $I_r(0,\mu)$ for $0 \lesssim \mu \lesssim 1$, expressed in units of F, where

$$F \equiv 2\int_{-1}^{+1} I(\tau,\mu)\mu\, d\mu. \tag{6}$$

The quantity πF is the net flux. Since we require the quantities $I(0,\mu)/F$ and $Q(0,\mu)/I(0,\mu)$ for our calculation in § III*c*, and since no closed-form analytical expression exists for these, we have tabulated these quantities for a set of values of μ in Table 1. We note that, since $U=0$, the degree of polarization is just

$$\Pi = \frac{I_r(0,\mu) - I_l(0,\mu)}{I_r(0,\mu) + I_l(0,\mu)} = Q(0,\mu)/I(0,\mu). \tag{7}$$

ii) *Resonance Scattering*

In the classical description of resonance scattering, the phase matrix is given by equation (4) just as it is for electron scattering. Hence, the electron scattering results summarized in Table 1 apply as well to the static atmosphere dominated by classical, resonance scattering. However, a correct quantum mechanical treatment indicates that the angular distribution and polarization properties of the scattered radiation are described by a *modified* phase matrix. This phase matrix is given by

$$R_{ij} = E_1(R_{ij})^A + E_2(R_{ij})^I, \tag{8}$$

where $(R_{ij})^A$, the anisotropic part, is the same as R_{ij} in equation (4), except that $(R_{44})^A = (E_3/E_1)\cos\theta$, while $(R_{ij})^I$, the isotropic part, is defined by

$$(R_{ij})^I = 1/2 \quad \text{for } (i,j) = (1,1),(1,2),(2,1),(2,2);$$
$$= 0 \qquad \text{for all other } (i,j) \tag{9}$$

(see Hamilton 1947). The $E_k (k=1,2,3)$ are constants which depend upon the total angular momentum quantum numbers of the upper and lower states of the resonance transition.

As with electron scattering, V transfers independently of the other Stokes parameters. Moreover, we can apply the same symmetry argument here as before to show that $U=V=0$ for the plane-parallel, semi-infinite atmosphere. In that case, equations (8) and (9) can be used to replace equations (5a) and (5b) for electron scattering by the following set for resonance scattering in a static atmosphere:

$$\mu \frac{dI_l(\tau,\mu)}{d\tau} = I_l - \int_{-1}^{+1} d\mu' (AI_l + BI_r), \tag{10a}$$

where

$$A \equiv \tfrac{3}{8}E_1[2(1-\mu^2)(1-\mu'^2) + \mu^2\mu'^2] + \tfrac{1}{4}E_2, \tag{10b}$$

$$B \equiv \tfrac{3}{8}E_1\mu^2 + \tfrac{1}{4}E_2, \tag{10c}$$

TABLE 1

LAWS OF DARKENING AND POLARIZATION: THE PLANE-PARALLEL ATMOSPHERE RESULTS

	PURE SCATTERING[a]		SCATTERING PLUS ABSORPTION[b]			
	$\lambda=0$		$\lambda=1/6$		$\lambda=\tau/(1+\tau)$	
μ	I/F	Q/I	I/F	Q/I	I/F	Q/I
0.00 ...	0.41441	0.117130	0.309	0.2290	0.354	0.2833
0.05 ...	0.47490	0.089790	0.350	0.2057	0.378	0.2624
0.10 ...	0.52397	0.074480	0.385	0.1910	0.391	0.2516
0.15 ...	0.57001	0.063110	0.419	0.1785	0.402	0.2421
0.20 ...	0.61439	0.054100	0.455	0.1669	0.415	0.2316
0.25 ...	0.65770	0.046670	0.492	0.1556	0.430	0.2192
0.30 ...	0.70029	0.040410	0.532	0.1444	0.451	0.2045
0.40 ...	0.78398	0.030330	0.621	0.1217	0.509	0.1691
0.50 ...	0.86637	0.022520	0.724	0.0989	0.602	0.1294
0.60 ...	0.94789	0.016270	0.847	0.0765	0.738	0.0912
0.70 ...	1.02882	0.011123	0.991	0.0551	0.927	0.0585
0.80 ...	1.10931	0.006818	1.161	0.0351	1.178	0.0329
0.90 ...	1.18947	0.003155	1.360	0.0167	1.501	0.0137
1.00 ...	1.26938	0.0	1.592	0.0	1.907	0.0

[a] Chandrasekhar 1960.
[b] Harrington 1969.

and

$$\mu\frac{dI_r(\tau,\mu)}{d\tau}=I_r-\int_{-1}^{+1}d\mu'(CI_l+DI_r), \quad (10d)$$

where

$$C\equiv\tfrac{3}{8}E_1\mu'^2+\tfrac{1}{4}E_2, \quad (10e)$$

$$D\equiv\tfrac{3}{8}E_1+\tfrac{1}{4}E_2. \quad (10f)$$

For the particular case of a transition from $J=0$ to $J=1$, the values of E_1, E_2 and E_3 are 1, 0, and 1, respectively. Hence, for this case, the resonance scattering phase matrix *is* just that for Rayleigh scattering, and the classical description is correct. For these transitions, equations (10a)–(10f) reduce to equations (5a) and (5b), and the electron scattering solution applies. Apparently, $J=0$ to $J=1$ transitions maximize the degree of anisotropy of the resonance scattering. Hence, they also produce the maximum polarization for the emergent radiation field. Since we are interested in determining the *range* of polarization expected from nonspherical atmospheres, we can indicate the *maximum* effect for the nonexpanding case by limiting our attention here to $J=0$ to $J=1$ resonance scattering.

Expansion, too, can be accounted for, even in the presence of resonance scattering. Consider the "expansion opacity" described by Karp *et al.* (1977). If, as Wagoner (1981) has suggested, this quasi-continuous expansion opacity represents the Doppler blending of many *scattering*, rather than absorption, lines, then equation (10) still applies if it is suitably modified. The plane-parallel approximation on which equation (10) is based remains valid as long as we make the standard assumption that the atmosphere is expanding radially and homologously, even if this expansion is not spherically symmetric. We can see this as follows. Homologous, radial expansion, even without spherical symmetry, means that the radial velocity v at time t at some point (r,θ,ϕ) in spherical coordinates is just r/t. As a result, the velocity *gradient* at that point is *isotropic*. This fact eliminates the dependence of the line opacities at any point on the direction of the beam at the point in question. This is sufficient to satisfy the plane-parallel atmosphere requirement of *axial* symmetry around the direction normal to the surface.

With the plane-parallel assumption justified, therefore, as long as we retain the standard assumption of radial, homologous expansion, the only modification of equation (10) which is necessary in order to account for atmospheric expansion is the replacement of the constants E_1 and E_2 by appropriate average values. Let $\kappa_i(\tau,\nu)$ represent the opacity contributed by resonance transition i to the total opacity at optical depth τ and frequency ν. Let $E^i_{1,2}$ be the constants for transition i. Then the appropriate average values with which to replace E_1 and E_2 in equation (10) are given by

$$\bar{E}_{1,2}(\tau,\nu)=\sum_i\kappa_i(\tau,\nu)E^i_{1,2}\Big/\sum_i\kappa_i(\tau,\nu). \quad (11)$$

A mixture of resonance scattering and electron scattering is easily accommodated in this way by the addition of the electron scattering opacity to the sums in equation (11), multiplied in the numerators by the values $E_1=1$ and $E_2=0$, respectively.

The detailed calculation of the opacities which contribute to the averages in equation (11) is an extremely complicated one which has not yet been achieved in efforts to model the supernova atmosphere. Even the detailed calculation of Karp *et al.* (1977), for example, does not distinguish the scattering from the absorption line opacities. Here again, therefore, we shall limit our attention to the illustrative case in which $\bar{E}_1(\tau,\nu)=1$ and $\bar{E}_2(\tau,\nu)=0$ for all values of τ and ν. This makes the electron scattering results applicable and serves to indicate the *maximum* polarization which can be produced by a radially and homologously expanding atmosphere containing both electron and resonance scattering.

iii) *Scattering and Absorption*

The problem of the generalization of the pure electron scattering atmosphere described in § III*a*(i) to include true absorption was first discussed by Code (1950). Assuming that the absorbing matter is in LTE, this generalization involves the addition of both an absorption contribution to the opacity and a corresponding contribution to the source function by isotropic thermal emission, according to Kirchoff's law. To describe this, we simply replace equations (5a) and (5b) by the following pair:

$$\mu\frac{dI_{l,\nu}(\tau,\nu)}{d\tau}=I_{l,\nu}-\tfrac{3}{8}(1-\lambda_\nu)\times\int_{-1}^{+1}d\mu'\{[2(1-\mu^2)(1-\mu'^2)+\mu^2\mu'^2]\times I_l+\mu^2I_r\}-\tfrac{1}{2}\lambda_\nu B_\nu(\tau),\tag{12a}$$

and

$$\mu\frac{dI_{r,\nu}(\tau,\mu)}{d\tau}=I_{r,\nu}-\tfrac{3}{8}(1-\lambda_\nu)\times\int_{-1}^{+1}d\mu'(\mu^2I_l+I_r)-\tfrac{1}{2}\lambda_\nu B_\nu(\tau),\tag{12b}$$

where τ is the total optical depth, λ_ν is the fraction of the total opacity at depth τ which is contributed by absorption, and $B_\nu(\tau)$ is the Planck function prevailing at the physical depth corresponding to optical depth τ.

Code solved these equations, along with the radiative equilibrium and semi-infinite boundary conditions, in a variety of approximations, including a gray atmosphere approximation and a monochromatic case. In the latter, λ_ν was assumed to be independent of the depth τ, while the Planck function was expanded to first order in τ, as follows:

$$B_\nu(\tau)=B^{(0)}+B^{(1)}\tau,\tag{13}$$

where $B^{(0)}$ and $B^{(1)}$ represent the first terms in a Taylor series expansion about $\tau=0$. The temperature distribution in this case was taken to be his approximate gray atmosphere result, written as

$$T^4=\tfrac{3}{4}T_{\rm eff}^4[\tau+q(\tau)],\tag{14}$$

where $q(\tau)$ is Code's version of the slowly varying Hopf function of order unity.

An important and somewhat surprising distinction between the polarizing efficiencies of the gray and monochromatic atmospheres emerges from the solutions presented by Code. For the gray atmosphere, adding true absorption to the total opacity results in a diminution of the net polarization emergent from the atmosphere, varying roughly linearly with λ from the pure scattering results for $\lambda=0$ to zero net polarization for $\lambda=1$. For example, for $\lambda=0.5$, the gray atmosphere seen edge-on will exhibit a polarization of only $\sim 5\%$, instead of the value of 11.7% exhibited by a pure electron scattering atmosphere. This is not very surprising, since the introduction of absorbing matter in LTE implies the addition of isotropic emission, as well. Hence, an overall reduction in the anisotropy of the radiation field and, with it, the polarization results.

For the monochromatic case considered by Code, however, the introduction of absorption has a novel consequence. The solutions indicate that, if the gradient of the Planck function is increased sufficiently (i.e., the ratio $B^{(1)}/B^{(0)}$ is increased), then the presence of absorption actually results in an *enhancement* of the polarization over the pure electron scattering result! This, too, can be understood, in retrospect, in terms of the effect of the absorption opacity on the degree of isotropy of the radiation as it flows toward the surface.

Deep within the atmosphere, the radiation field must be nearly isotropic and thermalized. In a diffusion approximation, a crude measure of the anisotropy of the radiation field is given by the quantity ρ defined by

$$\rho\equiv(\partial B/\partial\tau)/B(\tau).\tag{15}$$

It is roughly true that, in the gray atmosphere,

$$\rho_G\approx(2/3+\tau)^{-1},\tag{16}$$

while in the nongray case described above,

$$\rho_{\rm NG}\approx u_0/(1-e^{-u_0}),\tag{17}$$

where

$$u_0\equiv h\nu/kT_0,\tag{18}$$

where h is Planck's constant, k is Boltzmann's constant, and T_0 is the surface temperature. According to equations (17) and (18), if u_0 is large, so is $\rho_{\rm NG}$, while equation (16) indicates that ρ_G can never be very large. Hence, the ν dependence of the Planck function in the nongray case results in the possibility of a highly anisotropic radiation field. Of course, these arguments have simplified things by ignoring the effect of the scattering on the anisotropy. However, the qualitative fact remains; the nongray atmosphere approximation can yield a significantly higher anisotropy than in the gray atmosphere approximation and can even enhance the degree of anisotropy near the surface over that of the pure electron scattering atmosphere. Finally, the greater the part of the radiation field which is directed outward from the deep layers, the greater is the polarization emergent from the surface.

Harrington (1969) appealed to just this effect in order to explain the large intrinsic polarization of Mira variables. In this case the scattering mechanism was Rayleigh scattering by atomic and molecular hydrogen. Harrington obtained two illustrative solutions to the nongray problem first considered by Code (1950). One solution was for a constant ratio of absorption to total opacity, $\lambda = 1/6$, while the other, a more general solution than Code's, was for an increase in scattering toward the surface according to the relation $\lambda = \tau/(1+\tau)$. Both solutions were for a value of u_0 which was large enough (~ 20) to make the emergent polarization significantly exceed that from a pure electron scattering atmosphere. In the $\lambda = \tau/(1+\tau)$ solution, for example, the edge-on polarization was found to be 28%, a value more twice that of the pure electron scattering case.

It is quite possible that a similar effect will be present in the supernova atmosphere. After about a month, $T_0 \approx 5000$ K, making $u_0 \approx 6$ for $\lambda \approx 5000$ Å. Too little is known about the supernova atmosphere yet to make a meaningful quantitative statement about the relationship between u_0 and the actual gradient of B_ν in the atmosphere. However, it may still be true that the effect of the steep gradient described above will increase with time (as T_0 drops) and with decreasing wavelength. In the meantime, since we seek here to indicate the full range of possibilities for the polarization emergent from these atmospheres, we shall use Harrington's two solutions as well as the $\lambda = 0$ pure electron scattering case solved by Chandrasekhar in our calculation in § III*c*. We tabulate the $I(0,\mu)/F$ and $Q(0,\mu)/I(0,\mu)$ values for Harrington's solutions, as well, therefore, in Table 1.

b) *The Characterization of Asymmetry*

i) *Ellipsoids of Revolution*

We shall characterize the asphericity of the supernova atmosphere by considering the surface to be an ellipsoid of revolution, or spheroid, as shown in Figure 1 and described in spherical polar coordinates by

$$r(\theta) = a\left(1 - e_2 \cos^2\theta\right)^{-1/2}, \tag{19}$$

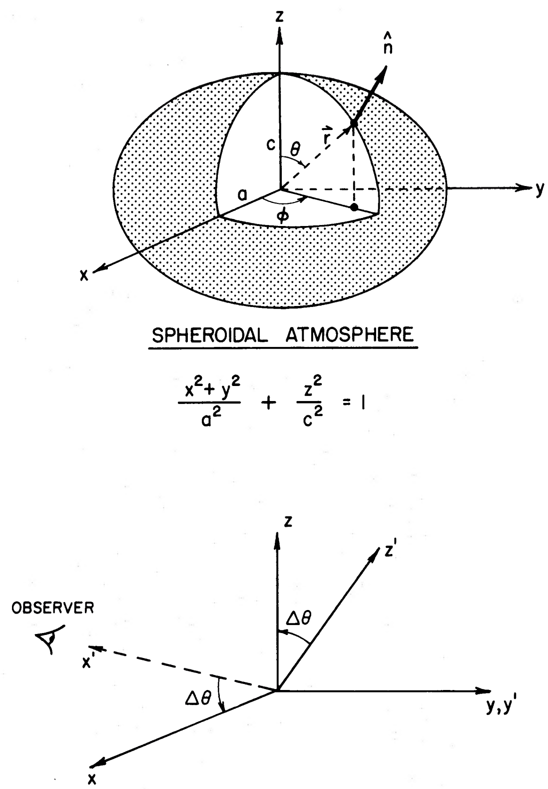

FIG. 1.—Spheroidal supernova atmosphere

where

$$e_2 \equiv 1 - (a/c)^2, \tag{20}$$

and a and c are the semiaxes perpendicular and parallel, respectively, to the symmetry axis. The unit vector $\hat{\boldsymbol{n}}$ normal to this surface is then given at each point (r, θ, ϕ) on the surface by the Cartesian components

$$\hat{n}_x = \sin\tilde{\theta}\cos\phi, \tag{21a}$$

$$\hat{n}_y = \sin\tilde{\theta}\sin\phi, \tag{21b}$$

$$\hat{n}_z = \cos\tilde{\theta}, \tag{21c}$$

where

$$\tilde{\theta} = \tan^{-1}\left[(1-e_2)^{-1}\tan\theta\right], \tag{21d}$$

and $\tilde{\theta}$ is the angle between $\hat{\boldsymbol{n}}$ and the symmetry axis (the z-axis). The differential surface area element dA at each point is given by

$$dA = r^2 \sin\theta\, d\theta\, d\phi / |\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{r}}|, \tag{22a}$$

$$= a^2 \sin\theta\left[1 - e_2(2-e_2)\cos^2\theta\right]^{1/2} \times \left[1 - e_2\cos^2\theta\right]^{-2} d\theta\, d\phi, \tag{22b}$$

where $\hat{r}$ is the unit radial vector. Hence, if $\hat{k}$ is a unit vector pointing along the line of sight, then the *projected* surface area element dA_p, the projection of dA onto a plane perpendicular to the line of sight, is given by

$$dA_p = |\hat{n} \cdot \hat{k}|\, dA = |\mu|\, dA, \tag{23}$$

where μ is the cosine of the angle between $\hat{n}$ and $\hat{k}$. Lastly, we shall henceforth parameterize the degree of asymmetry of the surface in terms of the asphericity ξ, defined by

$$\xi \equiv 1-(a/c), \quad a<c \text{ (prolate)}, \tag{24a}$$

$$\equiv 1-(c/a), \quad a>c \text{ (oblate)}, \tag{24b}$$

a quantity which varies from zero (sphere) to unity (limiting departure from symmetry).

ii) *Nonuniform Surface Flux Distribution*

In addition to the surface *shape* asymmetry described above, we must also consider the effects of variation of brightness across the surface. We take account of this possibility by considering a smooth, axisymmetric angular variation of the quantity F (defined in eq. [6]) from pole to equator of the form

$$F(\theta) = F_{\text{pole}}(1-\alpha \sin^2\theta)^{\beta}, \tag{25}$$

where α and β are constants which are allowed to vary from $(-\infty)$ to $(+1)$ and from 0 to $(+\infty)$, respectively.

c) *The Net Stokes Parameters*

In this section, we solve for the net polarization of the light emitted in a particular direction $\hat{k}$ by the full disk of the supernova atmosphere. The additivity of the Stokes parameters allows us to determine the net Stokes parameters emergent from the atmosphere by adding the contributions from the Stokes parameters emergent from each point on the disk. The results of the plane-parallel atmosphere calculations described in § IIIa, however, refer the emergent Stokes parameters at each point to the coordinate system in which I_l and I_r represent the intensity of the light whose electric vector is parallel and perpendicular, respectively, to the plane containing the normal $\hat{n}$ and the direction of view $\hat{k}$. In order to add the Stokes parameters for light emergent in the same direction $\hat{k}$ but from different points on the surface (i.e., different $\hat{n}$ vectors), we must refer all of the Stokes parameters to the same coordinate system, as follows (see Fig. 1).

Choose a new coordinate system (x', y', z') fixed to the observer, with $\hat{x}'$ pointing along $\hat{k}$ and with the "L" and "R" directions referring to the directions parallel and perpendicular, respectively, to the (x', z')-plane. In order to express the Stokes parameters emergent from some point on the surface of the atmosphere in the new coordinate system, we must rotate the original coordinate system about the $\hat{x}'$-direction by the amount necessary to bring the plane containing $\hat{n}$ and $\hat{x}'$ parallel with the (x', z')-plane. Let $\mathbf{L}(\psi)$ be the rotation matrix which describes the linear transformation of the vector intensity which results from the clockwise rotation of the original coordinate system through an angle ψ about the propagation direction. Then $\mathbf{L}(\psi)$, it can be shown, is given by

$$\mathbf{L}(\psi) = \begin{pmatrix} \cos^2\psi & \sin^2\psi & \frac{1}{2}\sin 2\psi & 0 \\ \sin^2\psi & \cos^2\psi & -\frac{1}{2}\sin 2\psi & 0 \\ -\sin 2\psi & \sin 2\psi & \cos 2\psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{26}$$

In this case the rotation angle ψ is given by

$$\psi = \cos^{-1}\{\hat{z}' \cdot [\hat{x}' \times (\hat{n} \times \hat{x}')]\}, \tag{27}$$

since $\hat{x}' \times (\hat{n} \times \hat{x}')$ is the projection of $\hat{n}$ on the (y', z')-plane and μ is the cosine of the angle between $\hat{n}$ and $\hat{x}'$. If we let $\boldsymbol{I}'(0, \mu)$ be the transformed vector intensity, then (recalling that $U = V = 0$) equation (26) implies

$$I_l' = I_l \cos^2\psi + I_r \sin^2\psi, \tag{28a}$$

$$I_r' = I_l \sin^2\psi + I_r \cos^2\psi, \tag{28b}$$

$$U' = (I_l - I_r) \sin 2\psi, \tag{28c}$$

$$V' = 0. \tag{28d}$$

We must relate this (x', y', z') coordinate system fixed to the observer to the (x, y, z) coordinate system fixed to the supernova atmosphere as described in § IIIb. As shown in Figure 1, we shall choose the y'-axis to coincide with the y-axis. The inclination angle of the spheroid with respect to the line of sight is then just $\Delta\theta$, the angle between the x- and x'-axes. It is clear that the direction $\Delta\theta = 0$ (the equator view) is the direction of maximum polarization, while the direction $\Delta\theta = \pi/2$ (the pole view) exhibits no polarization at all, by symmetry. We specialize hereafter to these two limiting cases, therefore, in order to show the range of polarization which is to be expected from observations of randomly oriented spheroids.

For the equator view, equations (21) and (27) can be used to show that

$$\psi = \tan^{-1}\left[(1-e_2)^{-1}\tan\theta\sin\phi\right]. \tag{29}$$

The cosine μ in this case is $\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{x}}$. From equation (21), this gives

$$\mu = \frac{\sin\theta\sin\phi}{\left[1-e_2(2-e_2)\cos^2\theta\right]^{1/2}}. \tag{30}$$

In general, whenever $V=0$, the degree of polarization is Π, defined by

$$\Pi = \left[(I_r - I_l)^2 + U^2\right]^{1/2}/I. \tag{31}$$

However, for the case at hand, symmetry ensures that the value of U' averaged over the face of the spheroid is zero. Hence, the integrated emergent polarization is

$$\Pi_{\text{net}} = \left[(I_r')_{\text{net}} - (I_l')_{\text{net}}\right]/I_{\text{net}} \tag{32}$$

($I' = I$ under a rotation transformation), or

$$\Pi_{\text{net}} = Q'_{\text{net}}/I_{\text{net}}. \tag{33}$$

The integrated quantities Q'_{net} and I_{net} are found by the following surface integrals over the visible disk:

$$Q'_{\text{net}} = (A_p)^{-1} \times \int_{\substack{\text{projected}\\ \text{surface}}} dA_p \cos 2\psi\left[\frac{Q(0,\mu)}{I(0,\mu)}\right]\left[\frac{I(0,\mu)}{F}\right]F, \tag{34}$$

and

$$I_{\text{net}} = (A_p)^{-1}\int_{\substack{\text{projected}\\ \text{surface}}} dA_p\left[\frac{I(0,\mu)}{F}\right]F, \tag{35}$$

where $A_p = \pi ac = \pi a^2(1-\xi)$ (oblate case) or $\pi a^2/(1-\xi)$ (prolate case), dA_p is given by equations (22), (23), and (30), ψ is given by equation (29), Q/I and I/F are the solutions of the plane-parallel atmosphere calculations described in § III*a* and summarized in Table 1, and F can vary across the surface as prescribed by equation (25). There is a fourfold symmetry which allows us to evaluate the integrals in equations (34) and (35) only over the first quadrant (i.e., $\cos\theta$ ranges from 0 to 1, while ϕ ranges from 0 to $\pi/2$) and to multiply this result by 4. We evaluate these two-dimensional surface integrals numerically by quadrature, using a third-order spline fitted to the tabulated values of Q/I and I/F as functions of μ shown in Table 1, with a higher density of discrete points used than is shown in the table.

The quantity Π_{net} calculated in this way is independent of the total observed flux from the supernova atmosphere. That is, the factors in equations (34) and (35) which reflect the overall scale of the integrated vector intensity divide out of the ratio in equation (33). This allows us to determine Π_{net} as a function of asphericity only, independent of the size, total luminosity, or surface brightness of the supernova.

The same departure from spherical symmetry which is responsible for producing a net polarization also affects the observed net flux emergent from the atmosphere. In principle, this information is already contained in the net intensity I_{net} given by equation (35). Unlike Π_{net}, however, I_{net} depends upon *both* the asphericity *and* the overall scale of the supernova atmosphere (i.e., the size and the surface brightness). There is no unique way to present the variation of I_{net} with asphericity, therefore. We could, for example, hold the projected surface area of the spheroid fixed, along with the surface flux F at some point (e.g., the pole) and calculate how I_{net} varies with asphericity. Alternatively, we could fix the total luminosity instead of the projected surface area. We choose, instead, to report this information in the following way.

Suppose an observer measures the net flux from the nonspherical supernova but assumes that the supernova is spherically symmetric. What error will the observer make as a result of his assumption of symmetry when he solves for the supernova luminosity? To answer this, we must compare the results of our calculations for the spheroid to those for a sphere at the same distance and with the same net observed flux. Let F_0 be the uniform brightness, or surface flux, assumed by the observer for the sphere. If L is the luminosity of the spheroid and L_{ss} is the luminosity inferred by the observer who assumes spherical symmetry, then

$$L = \int_{\substack{\text{entire}\\ \text{surface}}} dA(\pi F), \tag{36}$$

while

$$L_{\text{ss}} = 4\pi r_{\text{ss}}^2 \pi F_0, \tag{37}$$

where r_{ss} is the radius of the sphere. In order for the sphere and spheroid to produce the same observed net flux, it must be true that

$$\pi r_{\text{ss}}^2 F_0 = A_p I_{\text{net}}. \tag{38}$$

Therefore,

$$(L/L_{\text{ss}})_{\text{eq}} = (4A_p I_{\text{net}})^{-1}\int_{\substack{\text{entire}\\ \text{surface}}} dA\cdot F, \tag{39}$$

where the subscript "eq" refers to the equator view. The

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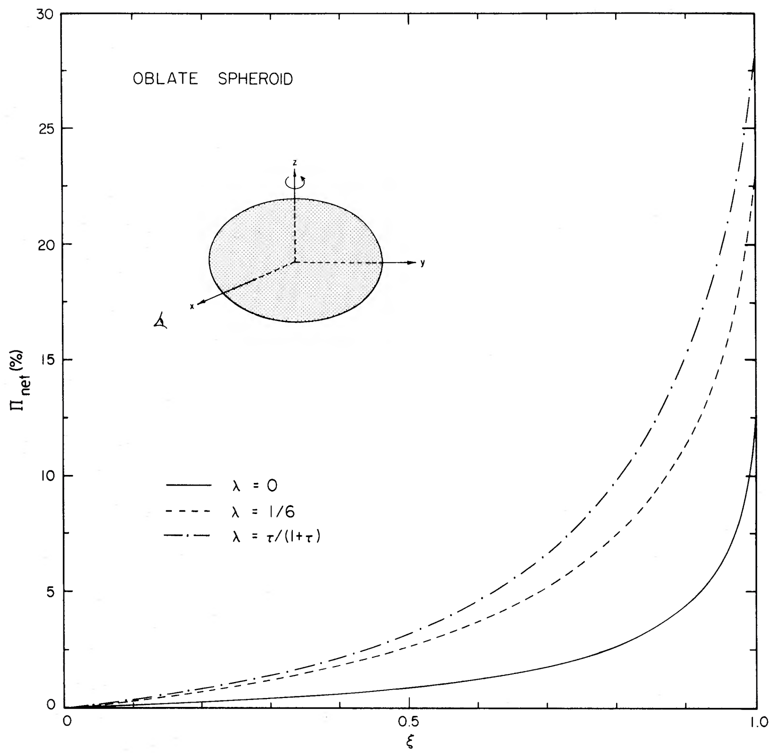

FIG. 2*a*

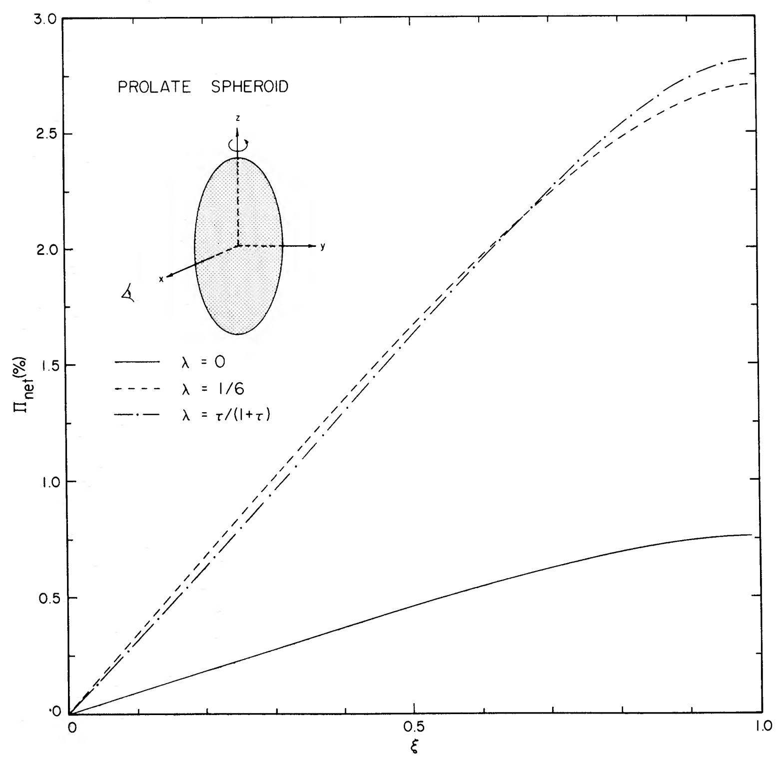

FIG. 2*b*

FIG. 2.—(*a*) Net linear polarization vs. asphericity for an oblate spheroid of uniform surface flux, viewed perpendicular to the symmetry axis. The three curves shown are for the pure Thomson or resonance scattering ($J=0$ to $J=1$ transitions) case ($\lambda=0$) and the mixed scattering-absorption cases ($\lambda=1/6$ and $\lambda=\tau/[1+\tau]$) described in the text. (*b*) Same as (*a*), but for a prolate spheroid.

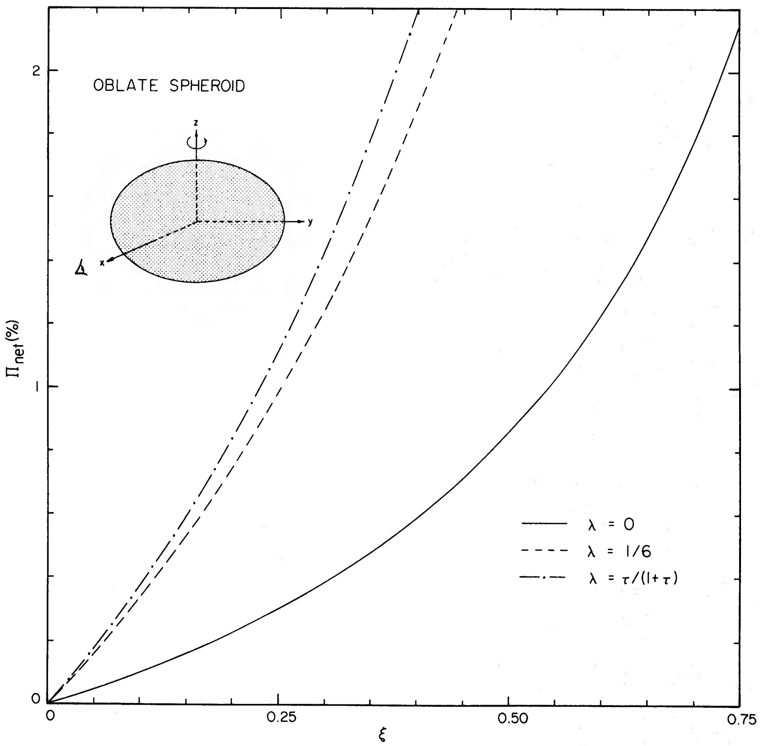

FIG. 2*c*

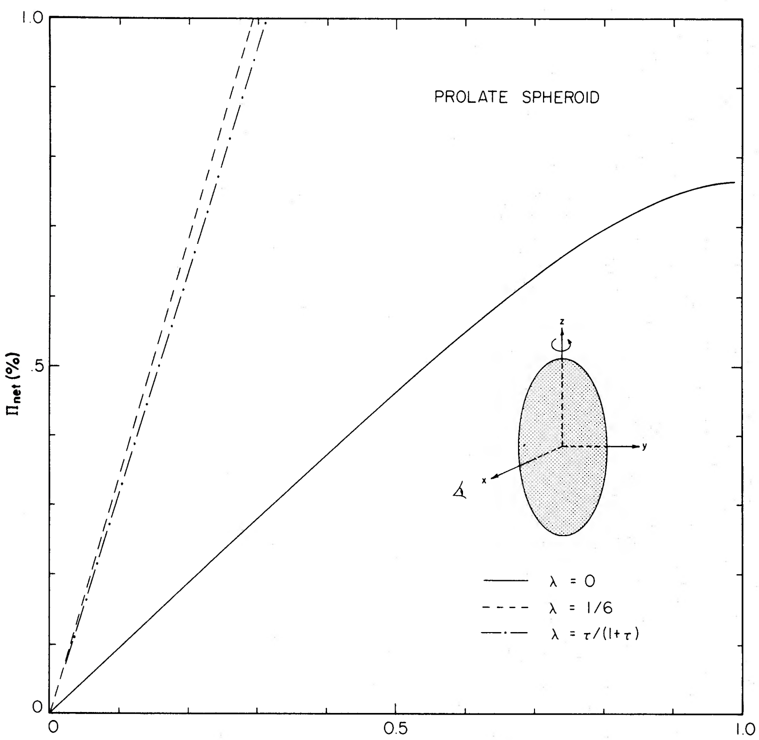

FIG. 2*d*

FIG. 2.—(*c*) Enlargement of a portion of (*a*). (*d*) Enlargement of a portion of (*b*).

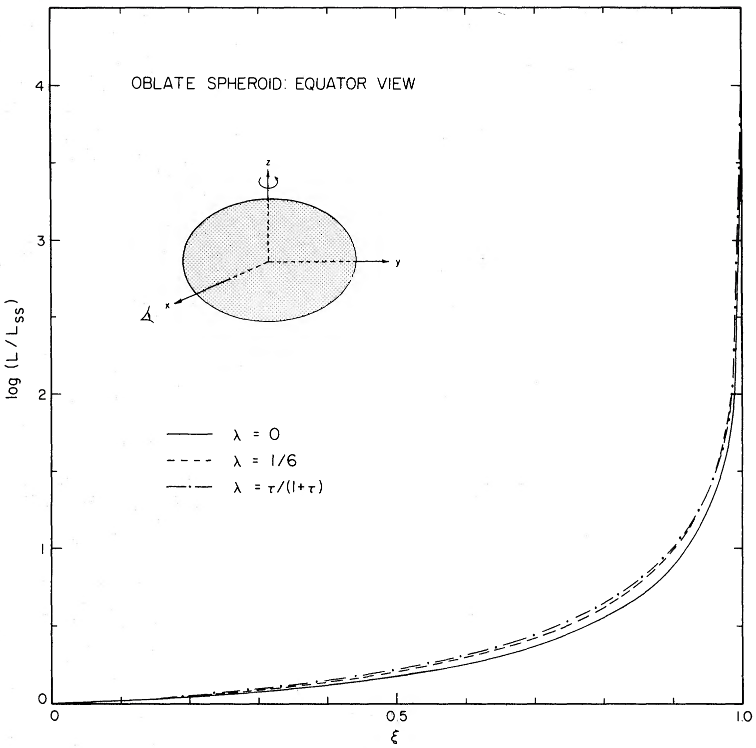

FIG. 3a

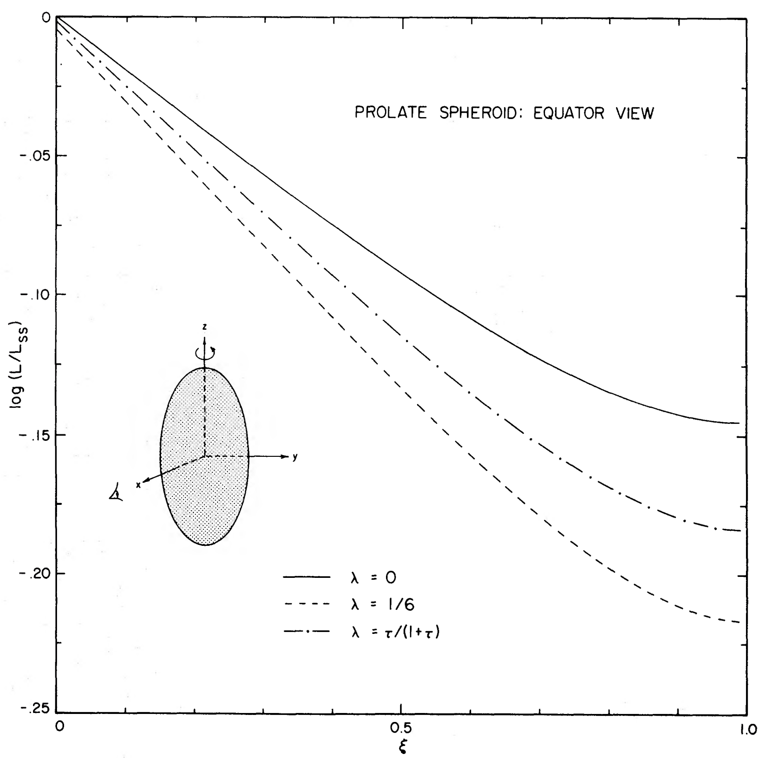

FIG. 3b

FIG. 3.—(a) L/L_{ss} vs. asphericity for an oblate spheroid of uniform surface flux viewed perpendicular to the symmetry axis for the same cases as shown in Fig. 2a. (b) Same as (a), but for a prolate spheroid.

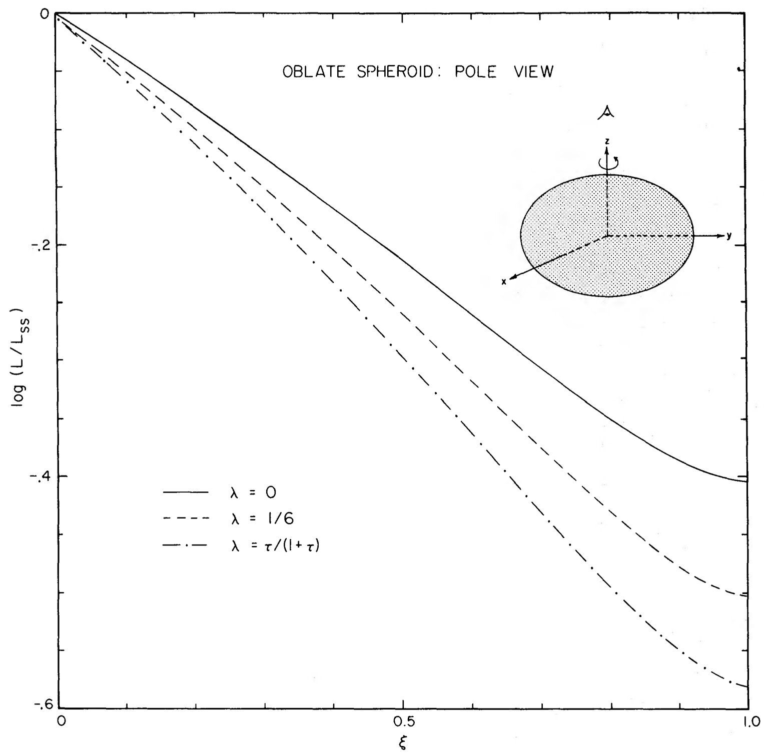

FIG. 3*c*

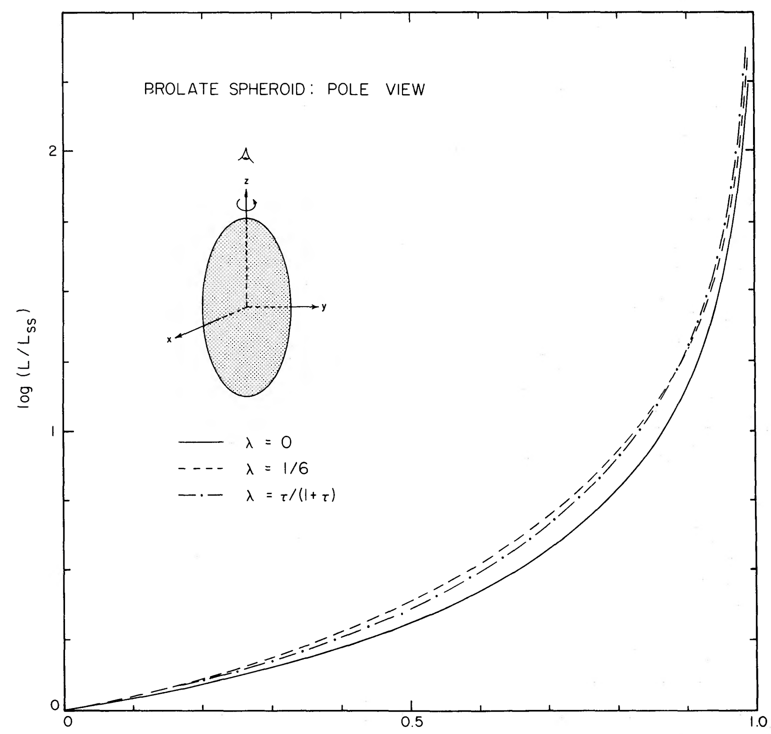

FIG. 3*d*

FIG. 3.—(*c*) Same as (*a*), but for a direction of view *along* the symmetry axis, instead of perpendicular to it. (*d*) Same as (*c*), but for a prolate spheroid.

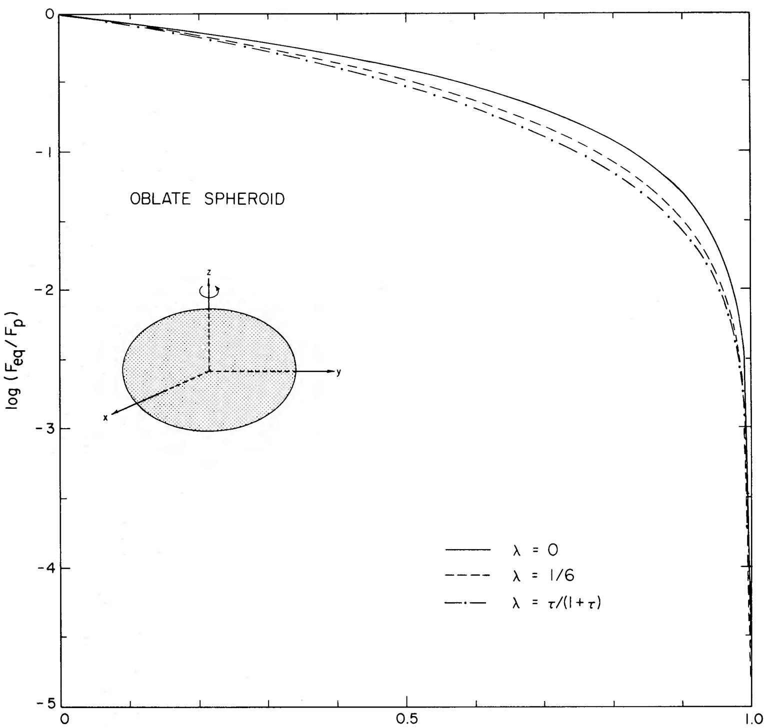

FIG. 4a

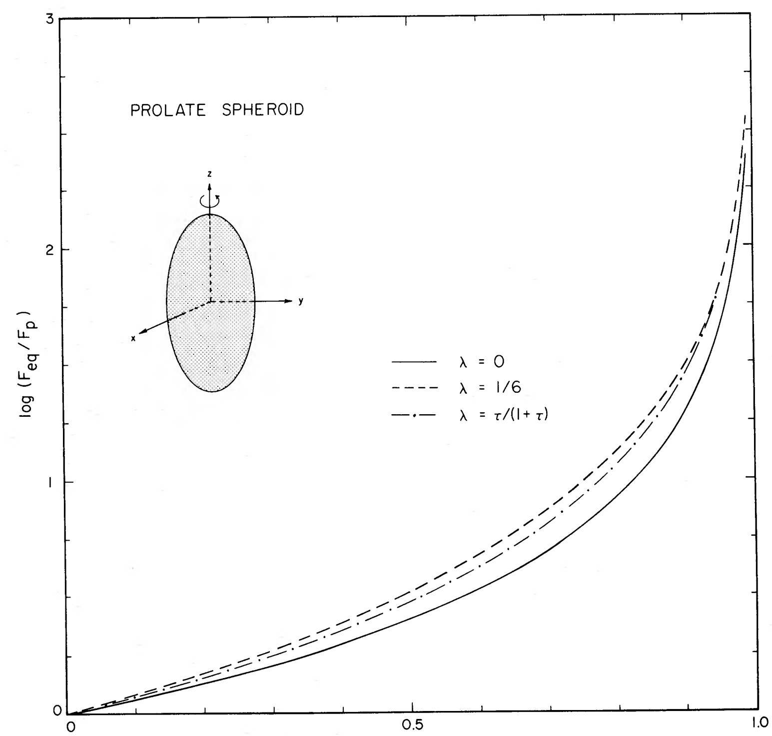

FIG. 4b

FIG. 4.—(a) Ratio of net flux observed in a direction perpendicular to the symmetry axis to that observed along the symmetry axis vs. asphericity for an oblate spheroid of uniform surface flux for the same cases as shown in Figs. 2 and 3. (b) Same as (a), but for a prolate spheroid.

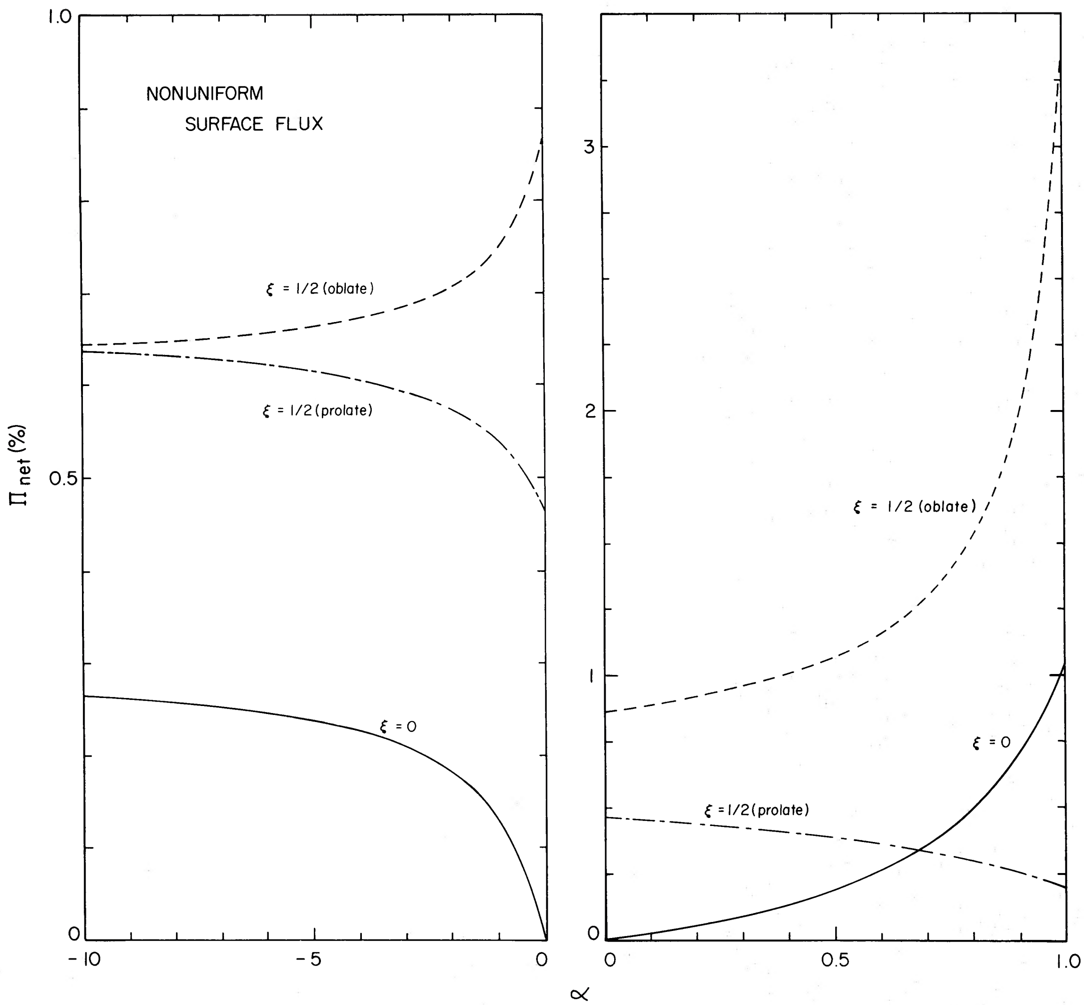

FIG. 5.—Net linear polarization vs. α for the pure scattering atmosphere ($\lambda = 0$) with surface flux varying according to $F(\theta) = F_{\text{pole}}(1 - \alpha \sin^2\theta)$. The three curves shown correspond to spherical ($\xi = 0$) and nonspherical prolate and oblate ($\xi = 1/2$) atmospheres, as labeled.

integral in equation (39) is evaluated numerically by quadrature, where dA is given by equation (22) and F can vary according to equation (25).

There is a similar expression which can be derived for the ratio $(L/L_{ss})_p$, the corresponding quantity when the direction of view for the spheroid is down along the pole. In this case, we must replace I_{net} as calculated for the equator view by the value for the pole view. To do this, we simply replace $\mu = \hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{x}}$ as it is given by equation (30) by $\mu = \hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{z}}$, given by

$$\mu = \frac{(1 - e_2)\cos\theta}{\left[1 - e_2(2 - e_2)\cos^2\theta\right]^{1/2}}. \tag{40}$$

With this replacement for μ, equation (35) is still the correct expression for I_{net} for the pole view, and equation (39) is then the correct expression for $(L/L_{ss})_p$ if this I_{net} is used.

Finally, it is useful to compare the net fluxes which would be observed for the equator and pole views, respectively, of the same spheroid. The ratio of these fluxes F_{eq}/F_p is just given by

$$F_{\text{eq}}/F_p = (I_{\text{net}})_{\text{eq}}/(I_{\text{net}})_p. \tag{41}$$

IV. RESULTS

Results for the quantities Π_{net}, $(L/L_{ss})_{\text{eq}}$, $(L/L_{ss})_p$, and (F_{eq}/F_p) versus ξ are shown in Figures 2–4 for a variety of cases in which the surface flux F is assumed uniform. These cases include both oblate and prolate spheroids for each of the laws of darkening and polarization tabulated in Table 1. As expected, in the limit of oblateness each of the curves for the net polarization approaches the maximum polarization which would be seen from a plane-parallel atmosphere viewed edge-on.

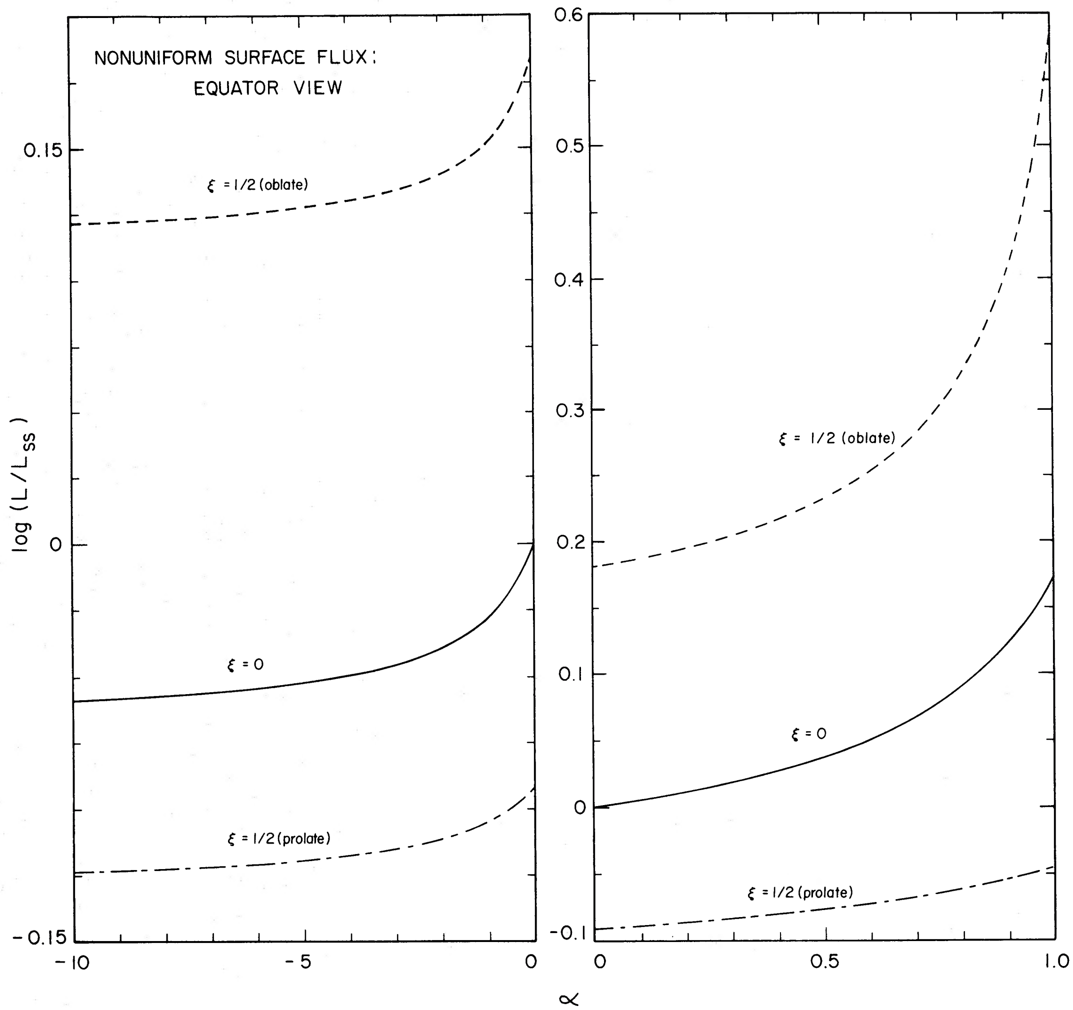

FIG. 6*a*.— L/L_{ss} vs. α for the direction of view perpendicular to the symmetry axis for the same cases as shown in Fig. 5

In the prolate case, however, the asymptotic limit of the polarization is considerably smaller than this maximum. This can be understood as follows.

The net polarization is determined, in crude terms, by two effects. The light from the limbs near the poles is polarized in an opposite sense to that from the limbs near the equator. The departure from spherical symmetry makes the cancellation of these opposite polarizations incomplete. In addition, the light emitted from the part of the disk which is away from the limbs emits light which is much less polarized than that from the limbs (recall that the polarization from a face-on plane-parallel atmosphere is zero). The extent to which the total light is dominated by the limbs rather than the regions away from the limbs will therefore also affect the amount of net polarization. In the prolate limit (the cigar shape), the face-on part of the disk dominates the light from the limbs, thus diluting the net polarization compared to the maximum, edge-on value. Hence, in the pure scattering case of Figure 2*b*, for example, the polarization maximum is only 0.77%, rather than 11.7% as in the oblate case.

This result suggests that a detection of polarization larger than the prolate maximum may serve to discriminate clearly between oblateness and prolateness. On the other hand, it also suggests that a null detection will serve to place a much more stringent upper limit on the asphericity of an oblate supernova than on that of a prolate one.

The effects of nonuniformity in the surface flux distribution are illustrated by the results shown in Figures 5–7. These results are for a pure scattering atmosphere (i.e., the $\lambda = 0$ scattering case in Table 1) for two values of asphericity, $\xi = 0$ and $\xi = 0.5$, and with $\beta = 1$ in equation (25). Positive values of α then correspond to a *prolate* variation of surface flux (i.e., $F_{\rm pole}/F_{\rm eq} = [1-$

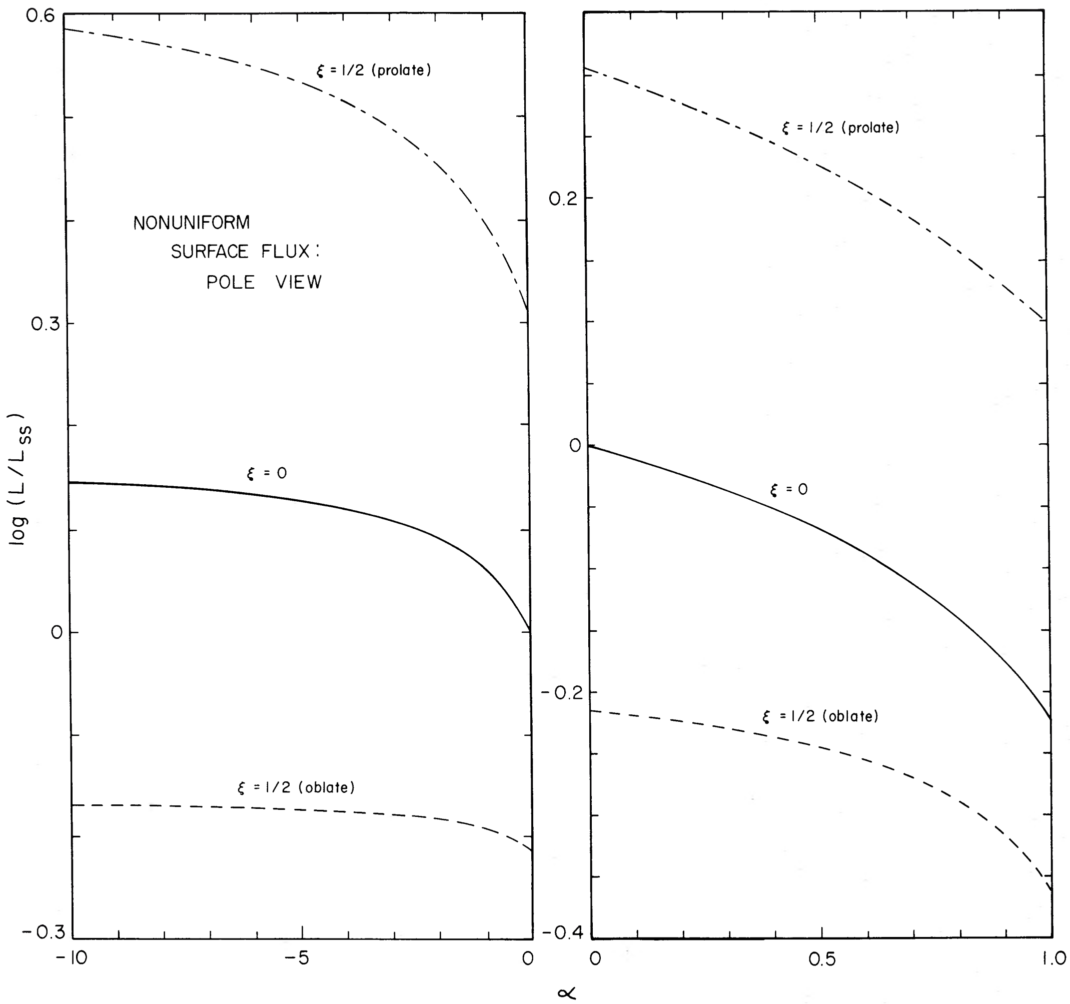

FIG. 6*b*.—Same as (*a*), but for the direction of view *along* the symmetry axis

$\alpha]^{-1} > 1$), while negative values of α correspond to an *oblate* variation (i.e., $F_{\rm pole}/F_{\rm eq} = [1-\alpha]^{-1} < 1$).

These results show that a perfectly spherical supernova which has a nonuniform distribution of surface flux can emit polarized light. In fact, the prolate brightness distribution which we have chosen for illustrative purposes here results in a degree of polarization shown in Figure 5 which is comparable to that shown in Figure 2*b* for the case of uniform surface flux but prolate shape. Measurements of a number of supernovae which show polarization of the order of that shown in Figure 5 and which also show very little scatter in the value of the photospheric velocity as determined from P Cygni profiles might be taken as an indication that the shape is *not* asymmetric while the surface flux distribution is.

The results shown in Figure 5 for the sphere with an oblate flux distribution, however, indicate that an oblate distribution yields a substantially smaller polarization than a prolate distribution. Moreover, the sphere with an oblate flux distribution exhibits a much smaller degree of polarization for the same degree of departure from symmetry than that of either the oblate or the prolate spheroid of uniform surface flux. This suggests that, when we predict the polarization emitted by an aspherical supernova, we can treat an oblate-type nonuniformity in the surface flux as a second-order effect in comparison with the dominant effect resulting from shape anisotropy.

This conclusion is reinforced when we examine the effect of the oblate surface flux distribution on the prolate shaped case with $\xi = 0.5$ shown in Figure 5. Let us compare the polarization at the point $(1-\alpha)^{-1} = 1 - \xi = 0.5$ (i.e., the degrees of departure of shape and flux distribution from symmetry are roughly the same) with that at the point $\alpha = 0$ (uniform flux distribution). The extra polarization introduced by the flux variation is

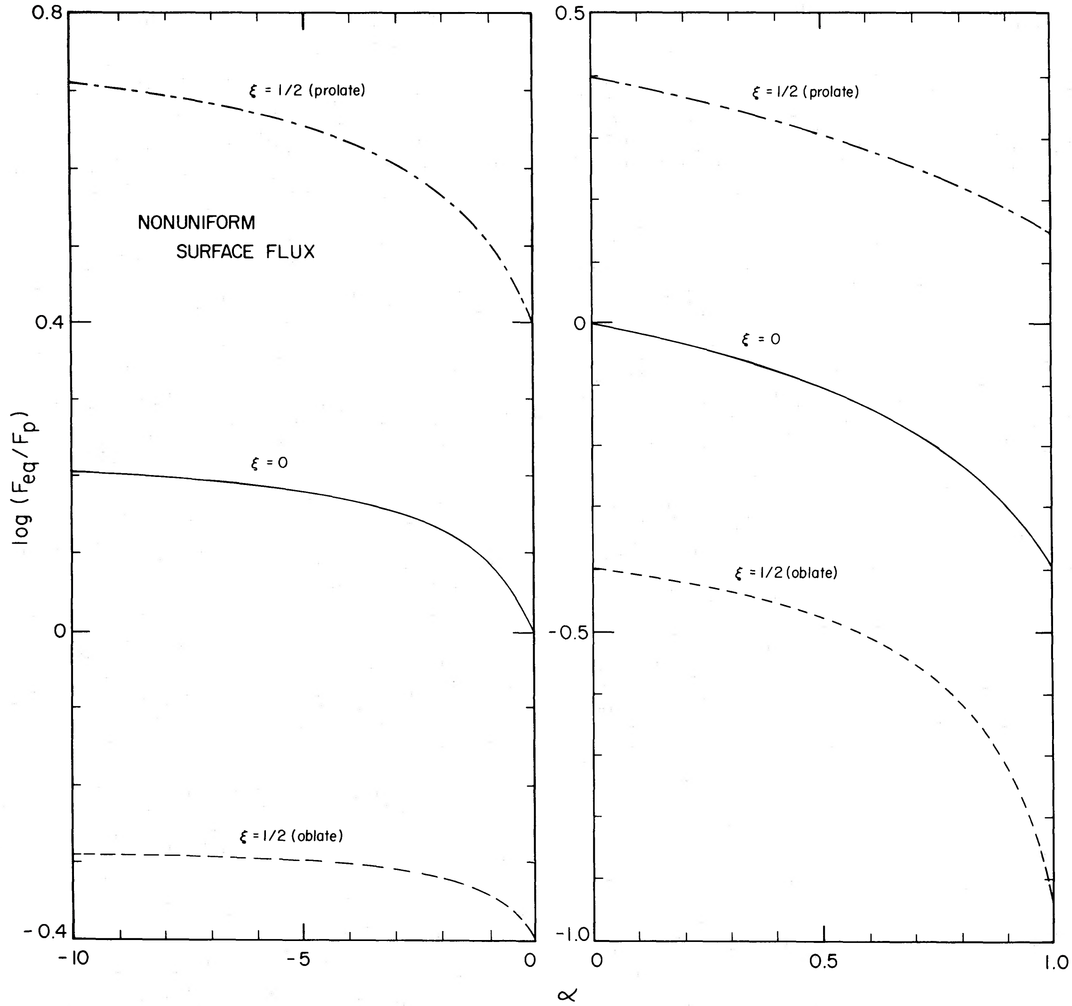

FIG. 7.—Ratio of net flux observed in a direction perpendicular to the symmetry axis to that observed along the symmetry axis vs. α for the same cases as shown in Figs. 5 and 6.

only $\sim 15\%$ of that due to the shape anisotropy alone. The oblate flux distribution with $F_p/F_{eq} = 0.5$ makes a similarly weak (albeit negative) contribution to the polarization of the oblate spheroid with $\xi = 0.5$ shown in Figure 5.

In general, changing the flux distribution from uniform to prolate *decreases* the net polarization emergent from a prolate atmosphere. Similarly, changing the flux distribution from uniform to *oblate* decreases the net polarization from an *oblate* atmosphere. *Prolate* flux variations with *oblate* shapes produce increased polarization, however, as do *oblate* variations with *prolate* shapes. For all of these combinations, it is generally true that when the departures from symmetry of shape and of brightness distribution are of the *same* order, the emergent polarization is determined predominantly by the effect of the shape asymmetry.

V. APPLICATION AND CONCLUSION

a) *Summary*

Our goal in this paper has been to express the dependence of the polarization emergent from an aspherical supernova atmosphere on the asphericity of that atmosphere. Our purpose has been to provide an observational measure of the degree of departure from spherical symmetry. As the results in § IV indicate, there is a range of values expected for the polarization, depending upon both the *nature* of the asymmetry (e.g., shape vs. surface flux distribution, prolate vs. oblate) and the detailed structure of the atmosphere (e.g., pure scattering opacity vs. mixed scattering-absorption opacity). Our hope is that these results can be used in conjunction with a systematic program of future supernova observation in order to deduce information about both the departure from symmetry and the characteristics of the supernova atmosphere.

For example, an observation of large polarization ($\Pi_{net} \gtrsim$ a few percent) could be taken to indicate that (1) the supernova is highly asymmetric, (2) the asymmetry is oblate, not prolate, (3) the atmospheric opacity is, indeed, scattering dominated, (4) the estimates of the luminosity which are based upon the assumption of spherical symmetry are likely to significantly underestimate the actual luminosity. If this observation happened to show the polarization increasing with time toward shorter wavelengths, then this could be taken as indirect evidence that the kind of mixed scattering-absorption effect discussed in § III is occurring in which the Planck function has a steep gradient and the steepness increases with decreasing wavelength and temperature. At the other extreme, a statistically meaningful sample of observed supernovae, all of which exhibit less than some very small amount of polarization ($\Pi_{net} \lesssim$ 0.1%, say) can be taken to indicate that either (1) supernova atmospheres are spherically symmetric to a high degree, both in shape and in surface flux distribution, or (2) the atmospheres are not, as generally thought, scattering dominated. For measured polarizations which fall between the two extremes mentioned above, it will be necessary to consider the entire range of possibilities presented in § IV.

b) *The Effect of Intervening Matter on the Observed Polarization*

The work presented here refers, of course, to the *intrinsic* polarization of supernova light. It is well known that the dust component of the interstellar medium of our own Galaxy can polarize starlight by an amount up to $(P/A_V)_{max} = 0.03$, where P is the degree of interstellar polarization at the visual wavelength (~ 5500 Å) and A_V is the interstellar extinction in magnitudes at this wavelength (see Spitzer 1978). Values of P have been observed to range up to a maximum of $\sim 7\%$. Hence, it is quite possible that the passage of light from an extragalactic supernova through the interstellar medium of our own Galaxy and that of the parent galaxy can introduce a linear polarization comparable to or greater than the intrinsic polarization.

The contribution from our own interstellar medium may be quite small for supernovae which occur in directions near the galactic pole. For other directions, observations of the interstellar polarization of *starlight* along lines of sight through the Galaxy which are very close to the line of sight to the supernova may allow a subtraction of the interstellar contribution. It will be more difficult to deduce and subtract off the possible contribution from the interstellar medium of the external galaxy containing the supernova, however. It might be possible to compare the polarization of the supernova light with that of light from the same region of the external galaxy either before or well after the supernova outburst. Even in this case, however, the contributions to the measurement, in the absence of the supernova, from different parts of the line of sight through the parent galaxy, including points which are further away than the position of the supernova, may make a proper subtraction difficult.

The most effective way to identify the intrinsic part of any positive detection of supernova polarization involves the measurement of the time and wavelength dependence of both the polarization and its position angle. Interstellar polarization does not vary in time if the source is intrinsically unpolarized, so time variations in either the amount or position angle of polarization imply that the supernova light is intrinsically polarized. Since the form of the wavelength dependence of interstellar polarization is well known, a measured wavelength dependence for the supernova polarization which departs significantly from this form is also an indication that a large part of the polarization is intrinsic. Lastly, although variations of position angle of the interstellar polarization of starlight with wavelength in the optical are known to occur, they occur in only 13% of the stars examined and average only $\sim 10°$ (Coyne 1974). Hence, large measured rotations of the position angle of supernova polarization with wavelength may also indicate intrinsic polarization.

A prediction of the time dependence of the intrinsic supernova polarization would be highly model dependent. Our assumption that the emission spectrum is primarily a thermal continuum which is formed deep within and reprocessed by a scattering-dominated atmosphere is most likely correct only during the first month or so after maximum. At later times, the spectrum is affected by emission lines as well as by scattering, and our assumption must break down. Hence, a comparison of the polarization measured within the first month with that measured a few months later at a wavelength which clearly corresponds to an emission rather than a scattering line should provide a good indication of the intrinsic contribution to the first measurement. Other effects, such as a possible diminishing of the asymmetry with time, might reduce the polarization at late times as well.

The observed *wavelength* dependence of the supernova polarization should be compared with the well-known shape of the interstellar curve. Serkowski, Mathewson, and Ford (1975) have derived an empirical formula for the wavelength dependence of interstellar linear polarization, given by

$$P(\lambda)/P_{max} = \exp[-1.15 \ln^2(\lambda_{max}/\lambda)], \quad (42)$$

where P_{max} is the peak optical polarization and λ_{max} is the wavelength at which it occurs. This λ_{max} varies from ~ 4500 to 8000 Å, with a mean value of ~ 5500 Å. No such wavelength dependence is expected for the intrinsic polarization of supernova light calculated here, for any values of P_{max} or λ_{max}. If the opacity is purely Thomson scattering, the intrinsic polarization should be independent of wavelength. Where resonance scattering dominates, the wavelength dependence will depend upon the values of $\bar{E}_1$ and $\bar{E}_2$ described in § III*a*. For wavelengths at which the opacity is dominated by $J=0$ to $J=1$ resonance scattering transitions, for example, for which $\bar{E}_1=1$ and $\bar{E}_2=0$, the intrinsic polarization may achieve a relative maximum compared to other wavelengths. Wavelengths dominated by transitions with $E_2 > E_1$, on the other hand, should show relatively less polarization. Lastly, if the mixed scattering-absorption opacity effect described in § III*a* and illustrated by the results in § IV occurs, it should be evidenced by an increase in the intrinsic polarization toward shorter wavelengths, becoming most pronounced in the near-UV, where the interstellar polarization is dropping.

It is possible to express the contaminating effects of interstellar polarization on the observed supernova polarization quantitatively as follows. Let us assume that the interstellar medium of either our own or the parent galaxy is uniformly filled with partially aligned nonspherical grains. Let P_2 be the degree of linear polarization which initially unpolarized light would exhibit after passage through this dust layer, with position angle χ measured clockwise with respect to the intrinsic supernova polarization direction. Let P_1 be the intrinsic supernova polarization (i.e., $P_1=\Pi_{net}$). If P_{net} is the degree of linear polarization observed in the supernova light after its passage through the dust layer, then it can be shown (see Appendix) that

$$P_{net}=\left(P_1^2+2P_1P_2\cos 2\chi+P_2^2\right)^{1/2}, \qquad (43)$$

assuming that the dust grain alignment direction is constant along the line of sight. The observed polarization direction in this case will be rotated with respect to the intrinsic supernova polarization angle by an amount χ_{net} which can be shown (see Appendix) to be given by

$$\chi_{net}=\tfrac{1}{2}\tan^{-1}\left[(P_2\sin 2\chi)/(P_1+P_2\cos 2\chi)\right]. \qquad (44)$$

Let us define $w=P_2/P_1$ and a "depolarization factor" $d_p=P_{net}/P_1$. According to equation (43), for $0°<\chi<45°$ or $135°<\chi<180°$, $d_p\geq 1$ for all values of w, while for $45°<\chi<135°$ and $w<1$, $d_p<1$.

In addition to its effect on the linear polarization observed in light from the supernova, the interstellar dust can also introduce a net circular polarization V_{net}/I_{net} as a result of its linear birefringence (e.g., Martin, Illing, and Angel 1972; Martin 1974). This circular polarization can be shown (see Appendix) to be given by

$$V_{net}/I_{net}=-\Delta\varepsilon P_1\sin 2\chi, \qquad (45)$$

where positive V corresponds to right-handed circular polarization. The quantity $\Delta\varepsilon$ is the relative phase lag which two orthogonal, plane-polarized waves with electric vectors parallel and perpendicular, respectively, to the mean alignment direction of the dust grains projected onto the plane of the sky will exhibit after they pass through the dust grain medium (assuming that their initial relative phase lag was zero). The value of $\Delta\varepsilon$ depends upon the nature of the dust and the degree of alignment.

According to equation (45), the possible net interstellar circular polarization of supernova light is likely to be quite small (i.e., of the order of P_1P_2). Nevertheless, values of interstellar circular polarization as small as less than 0.01% have been successfully detected in the past in the optical light from bright stars (e.g., Kemp and Wolstencroft 1972). Hence, for a range of possible supernova and interstellar linear polarizations, the circular polarization predicted here should be observable.

To the extent that our approximation that χ is constant along the line of sight (i.e., that the direction of grain alignment does not rotate with position along this line) is good, starlight which travels the same path through the interstellar medium as the supernova light should exhibit *no* circular polarization. If this starlight, too, is seen to be circularly polarized, however, the direction of grain alignment must rotate *substantially* with position along the line of sight. In this case, our simplifying assumption of constant χ must be replaced by a more general one.

c) *Preliminary Observational Results*

We are aware of only four reports in the literature of attempts to measure supernova polarization. Serkowski (1970) lists, without comment, a measurement of supernova 1968l, classified by Wood and Andrews (1974) as Type II, which appeared in the galaxy NGC 5236 (M83). The polarization was measured only once, ~10 days after maximum, in the blue, and found to be quite low, at only 0.2%. Wood and Andrews estimate the reddening within our own Galaxy along the line to this supernova (at longitude $b_{II}=32°$) to be $E_{B-V}=0.11$ mag, or $A_V=0.33$. This suggests that the interstellar polarization could conceivably be as large as 1%, larger than the observed amount. Moreover, the position of the supernova was coincident with the nucleus of the parent galaxy, which is itself a supergiant Sc galaxy viewed face-on. The internal reddening due to this parent galaxy was estimated to be as large 0.2 mag, implying the possibility of an even larger interstellar polarization than that due to our own Galaxy. Unfortunately, therefore, with no time or wavelength dependence measured for the polarization and its position angle, even the small polarization measured by Serkowski cannot be unambiguously identified as intrinsic to the supernova. Indeed, according to equation (43), the effect of contamination by interstellar polarization could be *either* to increase or *decrease* the observed supernova polarization. Hence, we cannot be sure that Serkowski's measurement is even an upper limit to the intrinsic polarization.

A second supernova polarization measurement is reported by Shakhovskoi and Efimov (1972), who performed multicolor polarimetry on the Type II supernova 1970g in the face-on spiral NGC 5455 (M101) for several weeks after its discovery. The observations were made in four colors (U, B, V, and R, or roughly at $\lambda = 3630$, 4340, 5450, and 7440 Å) and without filters ($\lambda \approx 5000$ Å).

Within the uncertainties of the data, the unfiltered light showed no significant time variation in either the amount or direction of polarization, with the amount centered on 0.5%. This amount of polarization, if entirely intrinsic, could be explained by a range of possibilities for the supernova as discussed in the previous sections. For example, a prolate, spheroidal, Thomson-scattering atmosphere would have to have an asphericity at least as great as $\xi = 0.5$ (axis ratio 2:1) in order to achieve such a polarization, even if viewed perpendicular to its symmetry axis. Other orientations require larger asphericities. On the other hand, if the same spheroid is oblate, $\xi \gtrsim 0.35$ (axis ratio 1.5:1) is required.

The four color data reported by Shakhovskoi and Efimov indicate a definite *wavelength* dependence for both the polarization amount and its position angle. In fact, the monotonic increase of the polarization from 0.3% to 1.0% as the wavelength decreased from 7500 to 3600 Å is qualitatively what is predicted here for the intrinsic polarization if the mixed scattering-absorption effect discussed in § III*a* and illustrated by the results in § IV occurs. The rotation of the position angle by some 40° across this wavelength interval may be attributed in part to the change with wavelength of the relative contributions of intrinsic and interstellar polarization, respectively, to the total. However, if the report that the position angle rotates by 40° *just between the B and V bands alone* is correct, then the requirements placed on the wavelength dependence of the intrinsic polarization by equations (42)–(44) are quite severe. As an illustration, equations (43) and (44) indicate that the values $\chi = 67.5°$, $w(V) = 10$, and $w(B) = 0.5$ will result in an observed rotation $\Delta\chi_{\rm net}$ between the V and B bands of $\sim 40°$, with $P_{\rm net}(V) \approx 0.4\%$ and $P_{\rm net}(B) \approx 0.6\%$, just as the observations require. This satisfies the requirement on P_2 imposed by equation (42) by allowing P_2 to remain at $\sim 0.4\%$. However, the intrinsic polarization P_1 must then vary over the same wavelength interval from $P_1(V) \approx 0.04$ to $P_1(B) \approx 0.8$, a factor of increase of the order of 20 while the wavelength varies by only 25%! This seems unlikely and indicates that great care should be taken in interpreting the polarization observations in terms of the combined effect of intrinsic and interstellar polarization.

Wolstencroft and Kemp (1972) report an observation of both linear *and* circular polarization for a bright Type I supernova discovered in 1972 May in the irregular galaxy NGC 5253. They measured the polarizations with no filter (λλ3500 to 6500) on a single night a month after the supernova was discovered. Since the discovery of this supernova occurred *after* maximum light, the Wolstencroft and Kemp observation was made an uncertain number of days past maximum. They found an amount of linear polarization given by $0.35 \pm 0.2\%$, while the degree of circular polarization was found to be $-0.08 \pm 0.04\%$. The lack of time or wavelength dependence in this measurement makes a determination of the magnitude of the intrinsic part of the linear polarization difficult. However, Lee *et al.* (1972) mention unpublished polarization measurements of this supernova by Serkowski and Wamsteker which indicated short-term variations in both the magnitude and the position angle of the linear polarization. This would imply an intrinsic origin for a substantial part of the polarization.

The linear polarization of a Type I supernova in NGC 7723 in 1975 was measured by Shakhovskoi (1976). The observations were made in three colors, B, V, and O (where O corresponds to ~ 6190 Å) at epochs corresponding to maximum light and a month past maximum, respectively. The mean linear polarization of the supernova was found to be close to 1.5%, with a weak wavelength dependence indicating an increase toward shorter wavelength. The time variation was found to be small. Shakovskoi also observed the light from the *nucleus* of NGC 7723 and found no appreciable pollarization. This fact, together with the galactic latitude of the parent galaxy ($b = -68°$), the location of the supernova relative to the nucleus of the parent galaxy, and the inclination of the galaxy, made Shakovskoi conclude that a large part of the observed supernova polarization was intrinsic.

Finally, a Type I supernova discovered in 1981 in NGC 4536 was observed in linear polarization by M. Breger (unpublished) on 1981 May 4, about 2 months past maximum light. The observation was in a broad band with width of ~ 1000 Å, centered on the visual wavelength. A polarization of 0.41% (with 1 σ error corresponding to 0.14%) was detected, including the sky contribution.

There is little that can be concluded from this measurement. de Vaucouleurs, de Vaucouleurs, and Corwin (1976) estimate that the extinction due to our own Galaxy combined with the internal extinction expected for the morphological type [SAB(rs)bc] and inclination of NGC 4536 is $A_V \approx 0.35$. This much extinction could easily correspond to an interstellar polarization of the order of 1%. In the meantime, the timing of the measurement, 2 months past maximum light, is such that the scattering-dominated character of the atmosphere, a prerequisite for intrinsic polarization, may no longer be a good assumption. It is difficult, therefore, to use this measurement even to place a significant upper limit on the departure of the supernova atmosphere from spherical symmetry.

The five observations of supernova polarization discussed above must be viewed as preliminary. Four of the five showed linear polarization which was less than 1%, and two could conceivably be accounted for entirely by the effects of interstellar polarization. However, the only observations which included the time and wavelength dependence of the polarization, that of supernova 1970g in M101 and that of the supernova in 1975 in NGC 7723, which showed as much as 1.5% polarization, are strongly indicative of the presence of intrinsic polarization in addition to the interstellar contribution. Only one of the five supernova observations described above included an attempt to detect the interstellar *circular* polarization which, we have argued, may commonly occur if the supernova is intrinsically linearly polarized. In the one case in which it was looked for, however, a significant amount of circular polarization *was* detected, and unpublished results on the time variability of the *linear* polarization of this supernova support the view that the linear polarization was intrinsic. Only a systematic study of the polarization of future supernova, one which includes time and wavelength dependence of the polarization and its position angle, as well as a careful attempt to measure the interstellar polarization along the same line of sight, will enable reliable conclusions to be drawn or limits to be set. A measurement of the interstellar *circular* polarization of supernova light predicted here, in comparison with that of some unpolarized source (e.g., a bright star) along the same line would also be a valuable additional piece of information.

We thank M. Breger for useful discussion and for observing the polarization of the supernova in NGC 4536 at our request.

APPENDIX

The effect of the interstellar dust-grain medium of our own Galaxy or of the parent galaxy on the observed supernova polarization is described by the following set of differential equations (Martin 1974):

$$I^{-1}\,dI/ds = -\tfrac{1}{2}n_g[(C_{\rm ext})_l + (C_{\rm ext})_r], \tag{A1}$$

$$d(Q/I)/ds = \tfrac{1}{2}n_g[(C_{\rm ext})_l - (C_{\rm ext})_r]\cos 2\chi, \tag{A2}$$

$$d(U/I)/ds = \tfrac{1}{2}n_g[(C_{\rm ext})_l - (C_{\rm ext})_r]\sin 2\chi, \tag{A3}$$

$$d(V/I)/ds = \tfrac{1}{2}n_g[(C_p)_l - (C_p)_r][(\cos 2\chi)U/I - (\sin 2\chi)Q/I], \tag{A4}$$

where ds is the differential path length along the line of sight, n_g is the number density of grains, $(C_{\rm ext})_{l,r}$ and $(C_p)_{l,r}$ are mean extinction and phase lag cross sections for the grains, respectively, for radiation with electric vector parallel (subscript "l") and perpendicular (subscript "r") to the mean alignment direction of the grains as projected onto the plane of the sky, and χ is the angle between the mean alignment direction at position s and the polarization angle of the supernova radiation incident on the far side of the medium.

Assume that χ is constant along the line of sight. Let P_1 be the intrinsic supernova polarization (i.e., $P_1 = \Pi_{\rm net}$), and let Q_1 and I_1 be the net intrinsic Stokes parameters for the supernova, with $U_1 = V_1 = 0$. Let P_2 and ϕ be the amount and position angle of the linear polarization which initially unpolarized light, incident on the far side of the medium, would exhibit after passage through the medium. In that case,

$$P_2 = \tanh(\Delta\tau/2), \tag{A5}$$

where

$$\Delta\tau = n_g L[(C_{\rm ext})_l - (C_{\rm ext})_r], \tag{A6}$$

where L is the total path length through the medium. The position angle ϕ is just χ.

The quantity $\Delta\tau$ is small for all cases of interest to us here. This fact can be used in conjunction with equations (A1)–(A6) to show that the net observed Stokes parameters for the supernova light after passage through the dust medium are given by

$$I_{\rm net}/I_1 = e^{-\tau}, \tag{A7}$$

$$Q_{\rm net}/I_{\rm net} = P_1 + P_2 \cos 2\chi, \tag{A8}$$

$$U_{\rm net}/I_{\rm net} = P_2 \sin 2\chi, \tag{A9}$$

$$V_{\rm net}/I_{\rm net} = -\Delta\varepsilon P_1 \sin 2\chi, \tag{A10}$$

where

$$\tau = \tfrac{1}{2} n_g \left[(C_{\rm ext})_l + (C_{\rm ext})_r \right] L, \tag{A11}$$

and

$$\Delta\varepsilon = \tfrac{1}{2} n_g \left[(C_p)_l - (C_p)_r \right] L. \tag{A12}$$

Therefore, the net observed linear polarization $P_{\rm net}$ and its position angle $\chi_{\rm net}$ are given by

$$P_{\rm net} = \left[(Q_{\rm net})^2 + (U_{\rm net})^2 \right]^{1/2} / I_{\rm net}, \tag{A13}$$

or

$$P_{\rm net} = \left(P_1^2 + 2 P_1 P_2 \cos 2\chi + P_2^2 \right)^{1/2}, \tag{A14}$$

and

$$\tan 2\chi_{\rm net} = U_{\rm net}/Q_{\rm net}, \tag{A15}$$

or

$$\chi_{\rm net} = \tfrac{1}{2} \tan^{-1} \left[(P_2 \sin 2\chi)/(P_1 + P_2 \cos 2\chi) \right]. \tag{A16}$$

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