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SELF-SIMILAR SOLUTIONS FOR THE INTERACTION OF STELLAR EJECTA WITH AN EXTERNAL MEDIUM

ROGER A. CHEVALIER
Department of Astronomy, University of Virginia
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ABSTRACT

The interaction of a uniformly expanding gas with a stationary ambient medium is studied in the case where the expanding matter has a power-law density profile ($\rho \propto r^{-n}$), as well as the ambient medium ($\rho \propto r^{-s}$). Self-similar solutions for the structure of the interaction region can be found provided that $s<3$ and $n>5$. Models for Type I supernovae indicate that a $\rho \propto r^{-7}$ profile may occur in the outer part of the expanding star. The $n=7$, $s=0$ solution is applied to the remnant of Tycho's supernova, and it is found to reproduce the basic dynamical and morphological properties of the remnant. The model may also explain the lack of strong Fe line emission in the X-ray spectrum. The solutions with $s=2$ are applicable to Type II supernovae because the expanding supernova envelopes probably interact with regions of presupernova mass loss. The properties of the interaction region are important for the X-ray emission from Type II supernovae.

Subject headings: hydrodynamics — nebulae: supernova remnants — stars: supernovae

I. INTRODUCTION

As a result of a strong explosion, the ejected material tends to develop a velocity gradient in which the velocity is proportional to the distance from the center of the explosion. The expansion of the gas into a stationary ambient medium gives rise to an interaction region which contains shocked ejecta and shocked ambient gas. In the case where the density profile of the ejecta is a steep power law with radius and that of the ambient medium is a shallow power law with radius, the structure of the interaction region tends toward a self-similar solution. It is the purpose of this paper to describe the properties of the self-similar solutions for a variety of power law exponents. Two cases are examined in detail. The first is the situation in which the ambient density is constant. This case has applications to the remnants of Type I supernovae. The other situation is that in which the ambient density drops as r^{-2}, where r is the distance to the center of the explosion. This case has application to the initial interaction of an expanding Type II supernova with a circumstellar medium created by presupernova mass loss.

II. SELF-SIMILAR SOLUTIONS

If a uniformly expanding medium has a power-law density profile, the density is given by

$$\rho = t^{-3}(r/tg)^{-n}, \qquad (1)$$

where g is a constant. I assume that this expanding medium is interacting with a stationary medium which also has a power law density profile, given by

$$\rho = qr^{-s}, \qquad (2)$$

where q is a constant. The interaction region contains an inner shell of shocked ejecta and an outer shell of shocked ambient gas. These shells are separated by a contact discontinuity which is at a radius R_c.

If the interaction region is described by a self-similar solution, the time dependence of R_c can be found from dimensional analysis. It is

$$R_c = [Ag^n/q]^{1/(n-s)} t^{(n-3)/(n-s)}, \qquad (3)$$

where A is a constant. It is necessary to solve for the structure of the interaction region to determine the value of A. There are constraints on the values of n and s. First, we do not expect solutions for $s>3$, because the contact discontinuity would be accelerating while gas elements in the ejecta region move at constant velocity. Second, we note that for $n=5$, $R_c \propto t^{2/(5-s)}$. This is the expansion law for the shock wave resulting from a point explosion (Sedov 1959). For $n \leq 5$, it is expected that the outer shock wave tends to expand with this law and there is a steady separation between the outer shock wave and the contact discontinuity. The self-similar solutions discussed here are only applicable to the case $n>5$.

The structure of the interaction region can be divided into two parts: the structure of the inner shell and that of the outer shell. Self-similar solutions for the structure of the outer region have been determined by Parker (1963), who considered the flow resulting from the motion of a piston with radius increasing as a power law in time into a stationary medium with a power law density profile. Parker's equations apply to the present case if his similarity exponent λ is identified with the quantity $(n-s)/(n-3)$. Parker particularly studied the case $s=2$ for application to explosions interacting with the solar wind.

Parker uses the similarity variables U, P, and C for the velocity, pressure, and sound speed. One equation relating C and U can be obtained from the three fluid equations, making it possible to find the dependence of C on U near R_c. We have

$$C^2 \propto (1-\lambda U)^{b_1}, \tag{4}$$

where

$$b_1 = \frac{2}{3}\frac{(sn-9)}{[(5-s)n+5s-21]}.$$

At R_c, $U=1/\lambda$ and the pressure has a finite value. The sign of b_1 determines whether the sound speed and the gas temperature are 0 or ∞ at R_c. Considering that $n>5$, we see from equation (4) that $T\to\infty$ at R_c for $s=0$ and that $T\to 0$ for $s=2$. For $s=1$, $T\to\infty$ at R_c for $n<9$ and $T\to 0$ for $n>9$. The behavior of the density is the inverse of that of the temperature.

The self-similar solutions for the inner shell have not been previously discussed. I use the similarity variables

$$\eta = t^{-1}r^{\lambda}, \tag{5}$$

$$p = t^{n-5}r^{2-n}P(\eta), \tag{6}$$

$$u = \frac{r}{t}U(\eta), \tag{7}$$

$$\rho = t^{n-3}r^{-n}\Omega(\eta), \tag{8}$$

and

$$C^2(\eta) = \frac{\gamma P}{\Omega}, \tag{9}$$

where γ is the adiabatic index and λ is again equal to $(n-s)/(n-3)$. The fluid equations are then

$$U^2 - U + (\lambda U - 1)\eta\frac{dU}{d\eta} = -\frac{(2-n)}{\gamma}C^2 - \frac{\lambda\eta}{\gamma}\frac{C^2}{P}\frac{dP}{d\eta}, \tag{10}$$

$$(n-3)(1-U) + \eta(\lambda U - 1)\left(\frac{1}{P}\frac{dP}{d\eta} - \frac{2}{C}\frac{dC}{d\eta}\right) + \lambda\eta\frac{dU}{d\eta} = 0, \tag{11}$$

$$P[(n-5)-\gamma(n-3)-U(n-2-n\gamma)] + \frac{dP}{d\eta}\eta(\lambda U-1)(1-\gamma) = -\frac{2\eta\gamma(\lambda U-1)}{C}P\frac{dC}{d\eta}. \tag{12}$$

The boundary conditions at the inner shock wave which is at radius R_2 are

$$U_2 = \tfrac{1}{4}(3/\lambda + 1), \tag{13}$$

$$\Omega_2 = 4g^n, \tag{14}$$

$$P_2 = \tfrac{3}{4}g^n(1-1/\lambda)^2. \tag{15}$$

The boundary condition on the velocity at R_c is that $U=1/\lambda$. The solution of the equations can be accomplished by standard numerical techniques.

As with the solution for the outer shell, the value of C near R_c can be found. It is

$$C^2 \propto (\lambda U - 1)^{b_2}, \tag{16}$$

where

$$b_2 = \frac{2}{3}\frac{(sn-9)}{[(-s+10)n-21]}.$$

For $s=0$, 1, and 2, the asymptotic values of temperature and density at R_c are the same as for the outer shell solution.

The self-similar solutions for the inner and outer shells give values for the physical quantities relative to their values at the inner and outer shock waves. The two solutions can be joined by requiring that the pressure be continuous at the contact discontinuity. This condition yields the value of A:

$$A = \left(\frac{P_c}{P_1}\right)^{-1}\left(\frac{P_c}{P_2}\right)\left(\frac{3-s}{n-3}\right)^2, \tag{17}$$

where the subscript c refers to the contact discontinuity, 1 to the outer shock wave, and 2 to the inner shock wave. The self-similar solutions also yield the thicknesses of the inner and outer shells, which make it possible to calculate the relative values of physical quantities at the two shock waves. We have

$$\frac{\rho_2}{\rho_1} = \alpha\left(\frac{R_1}{R_c}\right)^{s}\left(\frac{R_2}{R_c}\right)^{-n}\left(\frac{n-3}{3-s}\right)^2, \tag{18}$$

$$\frac{p_2}{p_1} = \alpha\left(\frac{R_1}{R_c}\right)^{-(2-s)}\left(\frac{R_2}{R_c}\right)^{-(n-2)}, \tag{19}$$

$$\frac{u_2}{u_1} = \left(\frac{R_1}{R_c}\right)^{-1}\left(\frac{R_2}{R_c}\right)\frac{U_2}{U_1}, \tag{20}$$

TABLE 1

PROPERTIES OF THE SELF-SIMILAR SOLUTIONS

s	n	R_1/R_c	R_2/R_c	A	ρ_2/ρ_1	p_2/p_1	u_2/u_1	M_2/M_1
0	6	1.256	0.906	2.4	0.75	0.39	1.203	0.28
0	7	1.181	0.935	1.2	1.3	0.47	1.253	0.50
0	8	1.154	0.950	0.71	2.1	0.52	1.263	0.71
0	9	1.140	0.960	0.47	3.1	0.55	1.263	0.93
0	10	1.131	0.966	0.33	4.3	0.57	1.260	1.1
0	12	1.121	0.974	0.19	7.2	0.60	1.255	1.6
0	14	1.116	0.979	0.12	11	0.62	1.250	2.0
2	6	1.377	0.958	0.62	3.9	0.21	1.006	0.44
2	7	1.299	0.970	0.27	7.8	0.27	1.058	0.82
2	8	1.267	0.976	0.15	13	0.31	1.079	1.2
2	9	1.250	0.981	0.096	19	0.33	1.090	1.6
2	10	1.239	0.984	0.067	27	0.35	1.096	1.9
2	12	1.226	0.987	0.038	46	0.37	1.104	2.7
2	14	1.218	0.990	0.025	70	0.38	1.108	3.4

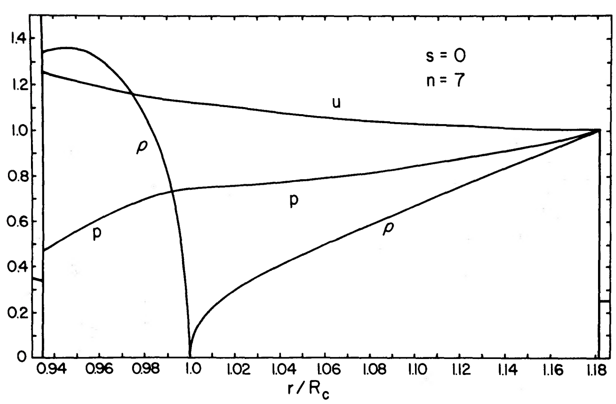

FIG. 1

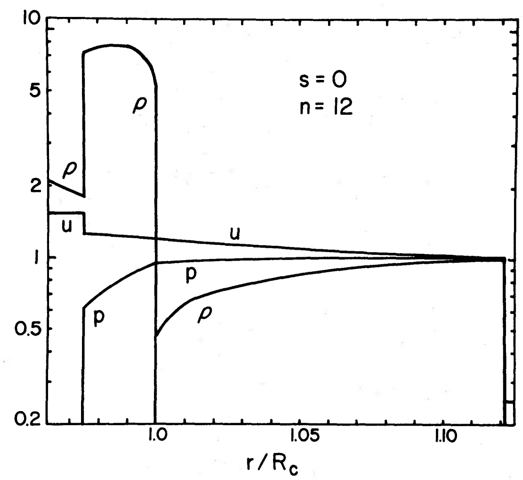

FIG. 2.

FIG. 1.—The variation of density ρ, pressure p, and gas velocity u in the interaction region between uniformly expanding matter and a stationary medium as a function of radius divided by R_c where R_c is the radius of the contact discontinuity between the two media. The self-similar solution is for $s=0$, $n=7$ where s is the power law exponent of the ambient density profile ($\rho \propto r^{-s}$) and n is the power law exponent of the expanding matter density profile ($\rho \propto r^{-n}$). The physical variables have been normalized to their values at the outer shock wave.

FIG. 2.—Same as Fig. 1 for $s=0$, $n=12$

where $\alpha = P_2/P_1$. Values of these quantities for $s=0$ and $s=2$ and for a range of values of n are given in Table 1. The preshock velocity of the gas at the inner shock wave, v_2, is given by

$$\frac{v_2}{u_1} = \frac{4}{3}\left(\frac{R_1}{R_c}\right)^{-1}\left(\frac{R_2}{R_c}\right)\left(\frac{n-s}{n-3}\right). \quad (21)$$

Finally, the ratio of the masses in the inner and the outer shells is given by

$$\frac{M_2}{M_1} = \alpha\left(\frac{R_1}{R_c}\right)^{-(3-s)}\left(\frac{R_2}{R_c}\right)^{-(n-3)}\left(\frac{n-3}{3-s}\right). \quad (22)$$

Values for this quantity are also given in Table 1.

Representative solutions for the interaction region are shown in Figures 1–4. Numerical listings of these solutions are given in Tables 2–5. The pressure and velocity profiles are similar in all the solutions. The pressure

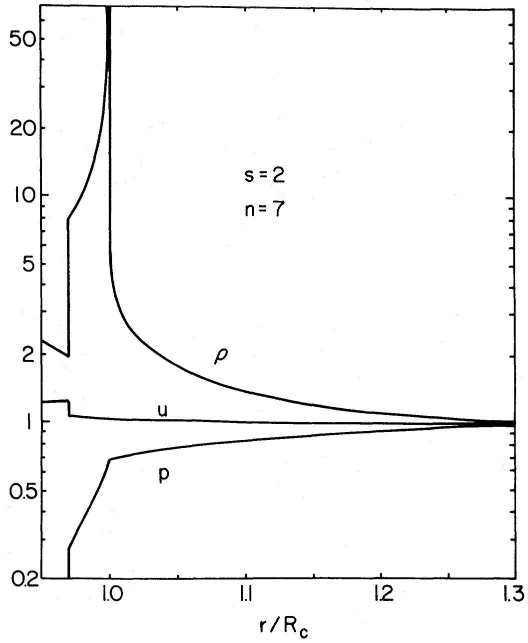

FIG. 3.—Same as Fig. 1 for $s = 2$, $n = 7$

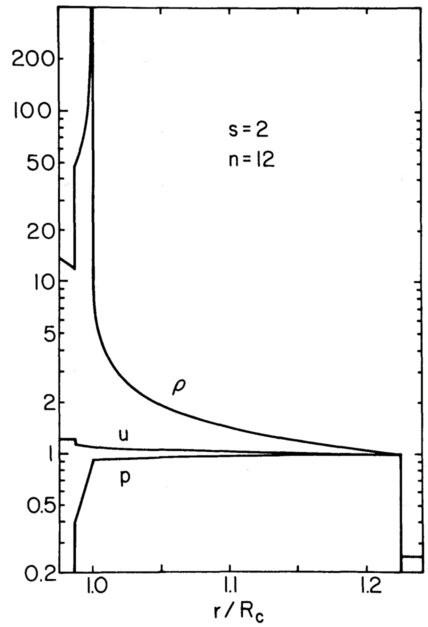

FIG. 4.—Same as Fig. 1 for $s = 2$, $n = 12$

TABLE 2

SELF-SIMILAR SOLUTION FOR $s = 0$, $n = 7$

r/R_c	ρ/ρ_1	p/p_1	u/u_1
0.935	1.336	0.471	1.253
0.94	1.335	0.504	1.237
0.95	1.357	0.564	1.211
0.96	1.315	0.615	1.189
0.97	1.225	0.660	1.171
0.98	1.075	0.697	1.155
0.99	0.826	0.727	1.141
0.995	0.621	0.738	1.135
1.00	0.00	0.744	1.129
1.01	0.219	0.748	1.117
1.02	0.297	0.755	1.105
1.04	0.411	0.774	1.084
1.06	0.503	0.796	1.065
1.08	0.586	0.821	1.049
1.10	0.667	0.849	1.035
1.12	0.746	0.880	1.023
1.14	0.826	0.915	1.013
1.16	0.909	0.954	1.006
1.18	0.995	0.997	1.000
1.181	1.000	1.000	1.000

TABLE 3

SELF-SIMILAR SOLUTION FOR $s = 0$, $n = 12$

r/R_c	ρ/ρ_1	p/p_1	u/u_1
0.974	7.18	0.602	1.255
0.980	7.61	0.701	1.235
0.985	7.77	0.777	1.222
0.990	7.71	0.844	1.210
0.995	7.24	0.904	1.199
0.9975	6.63	0.930	1.194
1.00	0.00	0.951	1.189
1.005	0.583	0.954	1.180
1.01	0.651	0.958	1.170
1.02	0.729	0.966	1.152
1.03	0.781	0.974	1.135
1.04	0.821	0.980	1.119
1.05	0.855	0.986	1.103
1.06	0.883	0.991	1.087
1.07	0.908	0.995	1.072
1.08	0.930	0.998	1.057
1.09	0.950	1.000	1.043
1.10	0.968	1.001	1.029
1.11	0.984	1.001	1.015
1.121	1.000	1.000	1.000

gradient is compensated by the deceleration of the shell. The density profiles show the greatest variation, particularly because of the difference in the asymptotic value of ρ at R_c for $s = 0$ and for $s = 2$. The asymptotic values are presumably never attained in a real situation because the initial conditions are important for the matter near R_c. The $s = 2$ solutions have similar gradients of p and ρ in the inner shell, indicating that this region is approximately isothermal.

The solutions assume that the flow is adiabatic in the interaction region and that heat conduction can be neglected. One difficulty with the solutions is that they clearly contain regions that are subject to the Rayleigh-Taylor instability. In a decelerating shell, the unstable

TABLE 4

SELF-SIMILAR SOLUTION FOR $s=2$, $n=7$

r/R_c	ρ/ρ_1	p/p_1	u/u_1
0.970	7.83	0.273	1.058
0.975	9.00	0.319	1.050
0.980	10.4	0.366	1.043
0.985	12.2	0.416	1.038
0.990	14.8	0.474	1.033
0.995	19.9	0.543	1.030
0.9975	26.1	0.588	1.028
1.00	∞	0.670	1.027
1.02	2.31	0.722	1.018
1.04	1.86	0.752	1.010
1.08	1.496	0.799	0.999
1.12	1.319	0.838	0.992
1.16	1.207	0.874	0.989
1.20	1.127	0.909	0.988
1.24	1.066	0.945	0.991
1.28	1.019	0.982	0.997
1.299	1.000	1.000	1.000

TABLE 5

SELF-SIMILAR SOLUTION FOR $s=2$, $n=12$

r/R_c	ρ/ρ_1	p/p_1	u/u_1
0.987	45.8	0.367	1.104
0.990	54.8	0.437	1.099
0.992	63.5	0.494	1.096
0.994	75.3	0.557	1.094
0.996	93.8	0.629	1.091
0.998	132.	0.719	1.090
0.999	181.	0.781	1.089
1.000	∞	0.904	1.088
1.01	3.66	0.926	1.081
1.02	2.70	0.937	1.074
1.04	2.10	0.951	1.062
1.06	1.78	0.961	1.051
1.08	1.58	0.968	1.042
1.10	1.44	0.973	1.033
1.12	1.33	0.978	1.026
1.14	1.24	0.982	1.019
1.16	1.17	0.986	1.013
1.18	1.11	0.990	1.008
1.20	1.06	0.994	1.004
1.22	1.01	0.999	1.001
1.226	1.00	1.000	1.000

regions are those in which the density decreases with radius, excluding shock waves.

III. TYPE I SUPERNOVAE

A specific model which had some success in reproducing the light curves of Type I supernovae was proposed by Chevalier (1981*a*). In this model, the outer 3/7 of the star (by mass) have a density profile $\rho \propto r^{-7}$ (see Colgate and McKee 1969). This applies to material velocities greater than

$$u_t = \left(\frac{5E}{3M}\right)^{1/2}, \qquad (23)$$

where E is the total kinetic energy in the explosion and M is the total mass. For velocities less than u_t, the density is constant. The density profile in the power law region has

$$g^7 = \frac{25}{21\pi}\frac{E^2}{M}. \qquad (24)$$

For a Type I supernova expanding into a uniform medium, the $s=0$, $n=7$ self-similar solution should apply to the interaction region provided that the reverse shock wave is within the power law section of the density profile. Using the properties of the self-similar solution, the condition that this be the case is $t<t_c$, where

$$t_c = 0.36\left(\frac{M^5}{E^3\rho_a^2}\right)^{1/6} \qquad (25)$$

and ρ_a is the ambient density. The expansion law during this time is

$$R_1 = 1.06\left(\frac{E^2}{M\rho_a}\right)^{1/7} t^{4/7}. \qquad (26)$$

The light curve models indicate that $E=1\times10^{51}$ ergs and $M=1.4\ M_\odot$. For these values, $u_t=7.7\times10^8$ cm s^{-1} and $t_c=204\ n_H^{-1/3}$ yr, where n_H is the ambient hydrogen number density in cm^{-3}.

I now apply this model to the remnant of Tycho's supernova, assuming that the reverse shock wave is close to the turnover point in the density profile. For the age of Tycho's remnant, this requires that $n_H=0.13$ cm^{-3}. This density may be typical of the warm, neutral component of the interstellar medium at a galactic altitude of 85 pc, and there is independent evidence from the optical emission that the remnant is expanding into such a medium (Chevalier and Raymond 1978). The adopted supernova model has $g^7=1.35\times10^{68}$ in cgs units. Given g, ρ_a, and t, the self-similar solution yields a remnant radius of 4.0 pc. The angular diameter of Tycho's remnant is 8′, corresponding to a distance of 3.3 kpc for this radius. While the distance to Tycho's remnant is controversial, this value is well within the commonly accepted range.

The model has the outer shock wave radius increasing as $t^{4/7}$; that is, if $R_1 \propto t^m$, $m=0.57$. The data on the expansion of the optical filaments yields $m=0.4$ (Kamper and van den Bergh 1978). However, the optical filaments cover only a small fraction of the remnant and the expansion of the radio shell, which is complete, indicates that m is approximately 0.5 (Strom, Goss, and

Shaver 1982). The expansion of the radio shell and of the optical filaments agree where they overlap, showing that there are irregularities in the expansion of the remnant. The radio data imply that the density profile may be somewhat less steep than r^{-7} (an r^{-6} profile yields $m = 0.5$). This might be expected if the inner shock wave is close to the transition between the steep and the flat parts of the density profile.

The model indicates that two X-ray emitting shells should be present, with the inner edge being at 0.79 times the radius of the outer edge (Fig. 1). The X-ray picture of Tycho's remnant does appear to show the presence of two X-ray shells (Seward 1981). As mentioned in § II, the density structure of the inner shell is Rayleigh-Taylor unstable. The dominant length scale of the instability should be comparable to the thickness of the unstable region; this is 5% of the outer shock radius, or 0.20 pc. The X-ray image of Tycho's remnant shows that the inner shell is clumpy, with a dominant size of 0.15 pc (Seward 1981).

In this model, the mass of shocked interstellar matter is 1.2 $M_\odot$, the mass of shocked supernova matter is 0.6 $M_\odot$, and the mass of unshocked supernova matter is 0.8 $M_\odot$. The outer shock velocity is 5.57×10^8 cm s^{-1} and the mean particle weight is 0.61 $m_{\rm H}$, where $m_{\rm H}$ is the hydrogen mass, so that the postshock temperature is 4.3×10^8 K. The shocked supernova gas probably does not contain a substantial amount of hydrogen. Spectra of Type I supernovae near maximum light indicate that Si and Ca are present (see discussion in Chevalier 1981*a*). The X-ray spectrum of Tycho's remnant shows strong lines of the Si group (Becker *et al.* 1980). If the shocked supernova material is composed of mostly ionized heavy elements, the mean particle weight is about 2 $m_{\rm H}$ and the postshock temperature is 5.0×10^8 K. In the present model, the unshocked supernova matter is primarily composed of Fe, which explains the absence of strong Fe lines in the X-ray spectrum (Becker *et al.* 1980). A similar suggestion for the absence of strong Fe line emission has been made by Arnett (1980), based on a different supernova model.

The temperature, density, and age of the remnant indicate that the X-ray emitting gas is in a regime where nonequilibrium effects are important. In addition, the irregularities in the expansion of the remnant indicate that there is a significant clumping of the gas. These considerations may account for the low-temperature thermal components that Becker *et al.* (1980) derived in their analysis of the X-ray spectrum. A discussion of the detailed properties of the X-ray emission in the present model is beyond the scope of this paper. The model should be further investigated because it does appear to reproduce the basic dynamical and morphological properties of Tycho's remnant. As detailed observations become available on Kepler's remnant, they should also be examined in the context of the present model.

IV. TYPE II SUPERNOVAE

Type II supernovae probably have a steep outer density profile which can be approximately described by a power law distribution; the evidence for this is summarized by Chevalier (1982). The fraction of the supernova mass in the steep power-law region is considerably smaller than it is for Type I supernovae. The medium surrounding the supernova is expected to be mass loss from the progenitor star. If the mass loss is steady, the self-similar solutions with $s = 2$ should apply to the interaction region between the expanding supernova and the circumstellar matter.

The importance of this interaction region for radio and X-ray observations of Type II supernovae was discussed by Chevalier (1982). In that paper, I used an approximate model for the interaction region based on the assumption that the region is thin compared to its radius. The results of the approximate solution are sufficiently close to the more accurate results presented here that the discussion in that paper is not changed. However, the self-similar solutions do show that the ratio of the density in the inner shell to that in the outer shell is about 30% higher than it is in the approximate models. This enhances the X-ray emission from the shocked supernova material; the emission from this gas may be responsible for the X-ray emission observed from SN 1980k (Canizares, Kriss, and Feigelson 1982). As more detailed observations of X-rays from Type II supernovae become available, the present models should be useful in interpreting the results.

V. DISCUSSION

Previous numerical calculations have followed the propagation of a reverse shock wave through an expanding supernova as it interacts with the interstellar medium (Gull 1973, 1975; Itoh 1977). Gull (1975) and Itoh (1977) present detailed results for the case in which the freely expanding supernova material has a constant density. Under these circumstances a thin shell forms which initially has a peak density 300 or more times the ambient density. This density contrast is larger than that found in the models presented here. This result can be understood in that the models of Gull and Itoh have a sharp edge to the supernova ejecta which corresponds to the limit $n\to\infty$. In this limit, the density contrast does become large.

Jones, Smith, and Straka (1981) presented the results of numerical calculations which do include a tail on the supernova density distribution. As expected, these calculations do not show a large density contrast between the shocked supernova matter and the ambient medium. Jones, Smith, and Straka analyze the structure of the interaction region, but they do so on the assumptions that the density, pressure, and velocity are constant through the region. While these assumptions lead to an approximate expression for the expansion of the interaction region, they are not exact.

The explosion of a realistic stellar structure probably does result in the formation of a tail to the density distribution. While the reverse shock wave is within the steep power-law part of the density profile, a density contrast like that given in Table 1 is expected. When the shock wave moves into the flatter portion of the density profile, the density contrast should decrease. Considering the small pressure gradient across the interaction region, the temperature in the peak density region can be estimated. For a supernova expanding into the interstellar medium, this leads to an estimate of the cooling time for the shocked gas. The cooling time is much larger than the age of the remnant even if the shocked gas is primarily composed of heavy elements. The conclusion is that optical emission is not expected from the shocked supernova gas if the expanding supernova material is uniformly distributed. If optical emission is observed from shocked ejecta, as appears to be the case for the Cas A supernova remnant, the implication is that there were clumps in the expanding gas before it was shocked. Chevalier (1981*b*) describes how the clumps may have formed in Cas A (see also Jones, Smith, and Straka 1981).

The solutions given here assume that the flow in the postshock region is adiabatic and that a single fluid dominates the flow. Yet it is well known that electron conduction is important in young supernova remnants if it is not impeded by magnetic fields and that the electron-ion equilibrium time is longer than the hydrodynamic expansion time. The conduction time scale is

$$\tau_{\text{con}} \approx \frac{L n_e k T_e}{\min(q_u, q_s)}, \tag{27}$$

where

$$q_u = \frac{0.6\times 10^{-6} T_e^{7/2}}{L},$$

$$q_s = 0.4(n_e k T_e)\left(\frac{2kT_e}{\pi m_e}\right)^{1/2},$$

L is the length scale for variations in the electron temperature T_e, n_e is the electron density, k is Boltzmann's constant, m_e is the electron mass, q_u is the unsaturated heat flux, q_s is the saturated heat flux from Cowie and McKee (1977), and cgs units are used. The time scale for electron-ion energy equipartition is (Spitzer 1962)

$$\tau_{\text{eq}} \approx 8.4 T_e^{3/2}/n_e, \tag{28}$$

where cgs units are used. The hydrodynamic expansion time is

$$\tau_{\text{hyd}} \approx R/v_{\text{sh}}, \tag{29}$$

where R is the radius of the remnant and v_{sh} is the velocity of the outer shock wave.

If electron-ion energy equipartition is achieved in the shock wave,

$$T_e = \frac{3}{16}\frac{\bar{m}}{k} v_{\text{sh}}^2,$$

where $\bar{m}$ is the mean particle weight. If the conduction is saturated, we have

$$\frac{\tau_{\text{con}}}{\tau_{\text{hyd}}} \approx 0.2\frac{L}{R}. \tag{30}$$

In the self-similar solutions, L is of order $0.1R$ and conduction is important. For Tycho's remnant, $v_{\text{sh}} = 5.6\times 10^8$ cm s^{-1}, $n_e = 0.5$ cm^{-3}. $T_e = 4\times 10^8$ K, and $R = 1.2\times 10^{19}$ cm. These parameters imply that conduction is saturated and that equation (30) applies. They also imply $\tau_{\text{eq}} \approx 4\times 10^6$ years, so that the ions and electrons are not in equilibrium in the postshock region. For a young Type II supernova, $\tau_{\text{hyd}} = 10^7$ s, $R = 10^{16}$ cm, $n_e = 8\times 10^5$ cm^{-3}, and $T_e = 1\times 10^9$ K. These parameters again imply that conduction is saturated and that the ions and electrons are out of equilibrium. If conduction is not impeded and if equilibration is achieved at the shock wave, then the self-similar solutions do not apply and a time-dependent two-fluid hydrodynamic calculation must be performed.

In the absence of energy equilibration in the shock front, the electrons are expected to be cooler than the ions at the shock. Then the similarity solutions do apply with only the ions contributing to the gas pressure, regardless of whether conduction is impeded. In this case, the solutions give the properties of the ions, while the electron temperature distribution must be calculated separately. The effects of ion heat conduction are expected to be small. If conduction is impeded by magnetic fields and if equilibration is achieved at the shock, then the present solutions apply, with both the electrons and the ions contributing to the gas pressure.

The present results show that the initial interaction of a supernova with the surrounding medium may be described by a self-similar solution if the expanding matter has a power-law density distribution. If the density distribution is not a power law, the solutions still provide a guide to the structure of the interaction region. Once the reverse shock wave moves into a region with $n < 5$, the interaction region can no longer maintain the same structure. The region of shocked ambient material then evolves toward the self-similar solution for a blast wave resulting from a point explosion.

The solutions described here may have applications to explosions other than supernovae. Parker (1963) investigated solutions like the $s=2$ outer shell solution in the context of disturbances in the solar wind generated by solar flares. The present solutions for the complete interaction region would only apply to cases where the amount of swept up solar wind gas is less than the amount of original flare ejecta. This restricts the applicability of the solutions to regions close to the solar surface.

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ROGER A. CHEVALIER: Department of Astronomy, P.O. Box 3818, University Station, Charlottesville, VA 22903-0818