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COSMOLOGICAL STRUCTURE PRODUCED BY A PHASE TRANSITION NEAR NUCLEAR DENSITY

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ABSTRACT

This paper shows that a cold expanding universe naturally produces stars at high redshift. A homogeneous universe composed of low-entropy matter ($S \lesssim 1$) spontaneously shatters into large pieces as it passes through a period of metastability during a phase transition. An approximate theory shows that the nucleation of rupture by quantum tunneling always produces cracks that are "large" (i.e., much larger than the mean particle separation) when they meet, of the order of 10^{-3} of the sound horizon length; this gives the size of typical fragments. At nuclear density, matter probably passes through a first-order phase transition as a result of pion condensation or neutron solidification, which produces fragments between $\sim 10^{-10}$ and $10^{-6}\ M_\odot$ at a redshift $z \approx 10^{14}$. This lumpiness generates density fluctuations which grow by the usual gravitational instability on scales larger than the Jeans mass $M_J \approx (2\text{–}10)(S+1)^2\ M_\odot$. Thus, one naturally expects stars in this mass range to form at high redshift, $100 \lesssim z \lesssim 10^7$. This would be a "pregalactic" or "Population III" generation of stars; because $M \gtrsim 2\ M_\odot$, none would be burning now, but their remnants may form the dark matter in galactic halos and clusters. These stars must also be held accountable for the production of the microwave background, galaxies, and, probably, helium. The model makes simple predictions about the expected large-scale anisotropy in the microwave background which differ from standard theories postulating fluctuations in the initial state and/or a primordial origin for the entropy. Other less direct tests are also proposed.

Subject headings: cosmology — dense matter — galaxies: formation

I. INTRODUCTION

The origin of large-scale structure is an enduring problem of big-bang cosmology. We observe a remarkable uniformity on the largest scales: observations of the microwave background radiation show directly that regions much larger than galaxies must have been nearly identical at early times even though the classical causal horizon encompassed much less than even a single star. This uniformity must have originated at some "initial" time t_i, at a cosmic density ρ_i far above that which we can treat with known physics. Nevertheless, one is inspired to ask questions about such an elementary property of space-time: Is this uniformity absolute? Is it fundamental in some sense? Does it apply uniformly on all scales? In other words, we want to know the structural symmetries of the universe near the singularity.

These inquiries lead to the main problem of this paper: Does the observed nonuniformity below ~ 10 Mpc have its origin in the same early epoch t_i, or does it come about from processes at later epochs that we can hope to understand? On small scales, the universe is definitely nonuniform, and, at first sight, this situation seems natural because matter on quantum scales is fundamentally nonuniform. However, it is not clear how microscopic inhomogeneity can explain systems like galaxies with masses $\sim 10^{70}$ times greater. This paper endeavors to show that at least one mechanism exists within the scope of conventional physics which can spontaneously produce structure on astronomical scales from homogeneous initial conditions, essentially by amplifying rare quantum events during a phase transition.

Although the physics herein is entirely conventional and the background cosmology is Friedmann, the model described here postulates that matter at nuclear density is "cold," i.e., that fermions are degenerate, as opposed to the standard "hot" model. The reason is simply that high-entropy matter at densities we understand well ($\rho \lesssim 10^{16}$ g cm^{-3}) probably does not go through a phase transition of the type required. The model described below is therefore acceptable only if it can be shown that the entropy in the microwave background can be created at later epochs—after the structure is introduced—and thermalized to within observational limits. This can probably be accomplished with an early generation of stars formed as a result of the phase transition.

It is indeed unfortunate that the one spectacular prediction of big-bang cosmology—the relationship between observed radiation temperature, the mean cosmic

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density of baryons, and the mass-fraction of primordial helium—cannot be incorporated into the scheme described below. In a cold universe, the mass-fraction Y of helium, like the entropy, is probably not a probe of the very early universe but a relic of a relatively recent, astrophysically active period at $Z \approx 100$. There are so few data in cosmology that it seems worthwhile to consider alternative theories which provide economical explanations for different facts. There are no signposts to tell us which data are explainable and which are not; certainly no simple explanation now exists for the presence of inhomogeneities, but that may be simply because the explanation lies beyond our physical understanding. The origin of structure must be due to a cosmological process of some sort, but it is generally thought to involve either some exotic physics at early times which violates classical horizon constraints or energy-momentum conservation (e.g., Peebles 1968; Harrison 1970; Press 1980; Barrow 1980; Press 1981), or, possibly, exotic new forms of matter such as "strings" (Zel'dovich 1980). Indeed, there is no reason to believe that we have the tools at our disposal to explain cosmological lumpiness at all. However, there are grounds for being suspicious about the standard models of galaxy formation, namely that *they postulate primordial fluctuations at t_i which have a characteristic scale of the order of the scale of the observed structures*. (This is all the more suspicious because the scale of galaxies themselves probably has a natural explanation in gas-dynamical processes which have nothing to do with initial conditions, e.g., Rees and Ostriker 1977.) It was once hoped that a scale-free power law of fluctuations in the initial state would suffice, but detailed work has shown that standard models require either (1) linear adiabatic fluctuations, with a power spectrum peaking at 1–10 Mpc (Peebles 1981*a*), or (2) linear isothermal fluctuations, with $\sim 1\%$ variations in baryons per entropy S^{-1} on galactic scales (see, e.g., Peebles 1980). In this latter case, the spectrum must either turn over or become nonlinear at $\sim 10^6\ M_\odot$–$10^{10}\ M_\odot$ (Hogan 1978), so both of the simple linear-perturbation models have built-in correlations on the scales of observed structures. Because of the horizon problem, these correlations—the essential origin of the observed structural features—must be introduced at t_i. This is very different from the "correlation" necessary to explain *uniformity* at t_i, since uniformity could be explained by a local process if there were (say) a fundamental reason why the expansion rate should be everywhere uniform at a given density (as, for example, the uniformity of a vacuum energy density or "cosmological constant"; see Guth 1981). The case of isothermal fluctuations is especially unattractive because there are now theories which naturally produce a uniform entropy per baryon.

These problems and others have led to alternative theories which entail a radical departure from the simple linear-growth pictures. A common feature of these is a nonlinear, nongravitational instability capable of spontaneously amplifying inhomogeneities. A recent proposal of Ostriker and Cowie (1981) and Ikeuchi (1981) allows a single "seed" star to trigger the formation of a galaxy-sized system through chain-reaction explosions of massive stars. Producing such seeds is much easier than producing a galaxy from scratch, and this paper provides a natural mechanism for forming them. Zel'dovich (1980) has proposed that strings of false vacuum, relics of a high-temperature phase transition at $\sim 10^{15}$ GeV, may be present now, pulling matter around on cosmological scales. This kind of scheme seems to be the only way to generate significant fluctuations in gravitational potential after t_i on scales of the order of the present horizon size; however, it is doubtful whether strings can produce the particular power spectrum of fluctuations needed to produce galaxy clusters without also creating excessive inhomogeneity on scales ~ 10–30 Mpc (Peebles 1981*a*) or excessive anisotropy in the microwave background temperature (Press and Vishniac 1980*a*).

This paper analyzes a basic dynamical mechanism by which a phase transition in an expanding medium can create large-scale structure spontaneously. The theory presented below is applicable in its general features to a whole class of phase transitions but will focus on transitions near nuclear density because they appear most promising in the cosmological case. The process is analogous to shattering. A high-density phase expands into a region of metastability where the pressure is negative, i.e., the medium is under tension. Rare fluctuations on the quantum level permit the uniform medium to tunnel through to a lower-energy state, forming cavities at isolated points. These cavities nucleate cracks or bubbles which propagate until they connect with other cracks or bubbles, and the regions between them become discrete "lumps" within which the expansion is reversed by the cohesive forces of the condensed phase. The expansion literally tears the matter to pieces. The process of nucleating cracks can be formulated in a general way for matter near zero temperature, and it is shown below that fragments always turn out to be very much larger than the interparticle spacing; for a transition near nuclear density, the mass scale of lumpiness is increased by a factor between $\sim 10^{46}$ and 10^{51}.

The first calculation of this sort appears to be that of Peierls, Singwi, and Wroe (1952) who estimated the sizes of lumps formed in pure neutron matter in the context of applying Mayer and Teller's (1949) "polyneutron" theory of nucleosynthesis to a cold universe. Since neutron solidification does not occur with even a small admixture of protons, the work below emphasizes instead the condensation of the pion fields near nuclear density (Baym 1977; Baym and Pethick 1979; Irvine 1978) which probably occurs under more general conditions (although a mixture of a pure neutron solid or liquid and antineutrinos appears plausible in the light of

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grand unified theory; see § V). The idea that shattering could solve the problem of cosmological inhomogeneity was first put forward by Zel'dovich (1963) and enlarged by Layzer (1975) and Hively (1973). These authors discussed more familiar transitions of matter at ~ 1 g cm^{-3}, i.e., liquid-gas or solid-gas transitions due to electronic interactions such as van der Waals or solid-body forces in cold hydrogen. They assumed that the size of the lumps is of the order of (sound speed)$\times$(expansion time), and the detailed theory of nucleation below shows this to be true with the addition of a multiplicative factor which may be as small as 10^{-3}. However, even the maximum possible size of the objects (planetesimals perhaps 10^{2-3} km across, as one expects at $\rho \approx 1$ from the criterion orbital velocity = sound speed) is not nearly sufficient to create structure on galactic scales unless other astrophysical mechanisms amplify the structure further, as these authors suggested.

By using a nuclear-density phase transition (Hogan 1980*a*), as in this paper, the fragments are large enough and form early enough to produce astrophysically active objects at high redshift. The formation of lumps introduces density fluctuations on larger scales which persist even if the lumps disappear, and linear fluctuations above the Jeans mass M_J grow without interruption by pressure forces. The Jeans mass, which thus determines the size of the first objects to form, is approximately $M_J \approx 2\text{–}10(S+\alpha)^2\ M_\odot$ depending on the number of leptons per baryon, $\alpha \approx 1$ or 2, and the process of nonlinear collapse. Energetic considerations show that, almost certainly, $S \lesssim 1$ even after the phase transition, so the first bound systems are stars between $\sim 2\ M_\odot$ and $\sim 100\ M_\odot$. These stars start burning between $Z \approx 100$ and $Z \approx 10^8$, and the massive ones can explode as early as $Z \approx 200\ (\Omega h^2)^{-1/3}$—perhaps triggering galaxy formation, perhaps releasing helium, perhaps producing enough heavy elements to thermalize their radiation. None of these early stars would still be burning today because of their relatively high mass, but their radiation would naturally have an entropy of the order of 10^8 (Rees 1978). Their remnants are natural candidates for the dark matter in galactic haloes.

The most direct test of the whole picture lies in observations of the microwave background anisotropy. On large angular scales, measurements of $(\Delta T/T)_\theta$ ought to show whether fluctuations in curvature were present in the initial state or whether exact uniformity may have prevailed; they may also show direct evidence of nonprimordial entropy production. On smaller angular scales, $(\Delta T/T)_\theta$ should show departures from perfect thermalization which vary from region to region (Hogan 1980*b*). Pregalactic stars also have a host of (mostly desirable) consequences which have been discussed extensively in the literature.

The plan of the paper is as follows. Section II analyzes the process of shattering, defining the conditions for the instability and calculating the size and spectrum of fragments expected. Section III summarizes some results of cosmological statistical dynamics which show how shattering can introduce inhomogeneity (with small amplitude) on scales larger than the individual fragments. These results are used in § IV to show that eventual star formation is an inevitable result of the shattering process for any plausible fragment size. The problems of nucleosynthesis and composition in a cold universe are briefly outlined in § V. Section VI describes how observations of microwave anisotropies will soon reveal unambiguous properties of the initial state and may conclusively falsify the entire picture presented here; an assortment of other, fuzzier observational tests follows. A brief summary in § VIII concludes the work.

II. PROCESS OF SHATTERING

a) *Criteria for Producing Structure*

The process analyzed here for generating large-scale structure in an expanding universe is analogous to the boiling of a superheated liquid or the shattering or tearing of a solid stretched beyond its tensile strength. For the purpose of cosmology, the differences between these processes are unimportant; either a liquid-gas transition or solid-gas transition will serve.

Imagine the universe expanding in the solid or liquid phase, approaching a critical density from above. At a density ρ_0, the pressure becomes negative, there is an effective tension on the medium, and a cavity which forms by fluctuation (either thermal or quantum tunneling type) after t_0 grows as a crack or bubble. A relaxation wave travels into the surrounding medium at the sound speed c_s and relaxes the tension in a region around the initial cavity. The unrelaxed condensed phase is still formally metastable until $\rho_{\rm crit}$ when the isothermal compressibility vanishes (Landau and Lifshitz 1969; Layzer 1975), at which point any material still under tension must rupture on all scales simultaneously.

The structure is generated between densities ρ_0 and $\rho_{\rm crit}$. During this period, a crack system nucleated on the lattice-spacing scale propagates through the medium at the sound speed until it either (1) meets another crack system, leaving the intervening condensed phase in large lumps which relax back to their equilibrium density ρ_0, or (2) comes within the sphere of relaxing (tensionless) matter surrounding another nucleus and heals. If the mean separation between nucleation sites is large enough, cracks propagate for a large fraction of a Hubble time before meeting other cracks, and the resulting lumps are of the order of (sound speed)$\times$(Hubble time) across. (But the separation must not be too large, or the matter ruptures at $\rho_{\rm crit}$ into tiny pieces before the cracks meet; see below.) In the end, one is left with an expanding universe of fragments or lumps (see Fig. 1).

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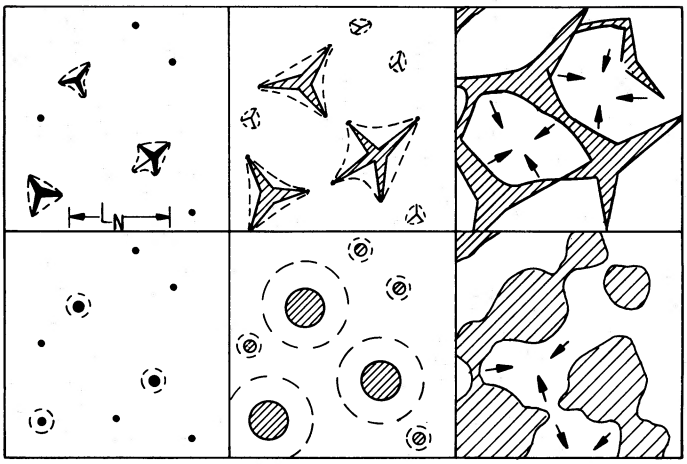


FIG. 1.—Sketch of the spontaneous lumpiness produced in a phase transition, with cracks in an expanding solid (*upper*) and bubbles in an expanding liquid (*lower*). Cavities of low-density phase appear by quantum fluctuation in a medium under tension at a predictable rate and mean separation L_N which decreases rapidly. These cavities become cracks which propagate at the order of the sound speed, relieving components of tension on the dense phase in a surrounding region (*dotted lines*) and allowing cohesive forces in the condensed phase to reverse the expansion. Arrows indicate motion of matter and lumps contract to equilibrium density.

The subsequent fate of these lumps depends on whether they can find an absolute energy minimum, i.e., stable configuration, or whether they must have a confining gas to be energetically favored. In the latter case, they would simply be in phase equilibrium and evaporate after a short time, leaving density fluctuations which eventually cause stars to form on larger scales. In the former case, the lumps persist as bound objects themselves, possibly neutron stars. In either case, the system as a whole goes to a lower energy. The subsequent development of clustering of lumps and the development of fluctuations into stars is discussed in §§ III and IV.

Not every phase transition leads to structure in a cosmological expansion, and it is far from obvious how structure is generated in detail. We can identify several conditions which are necessary for large-scale structure to appear spontaneously (and which, taken together, are also sufficient).

1. There must be a kink in the $E(\rho)$ relation, i.e., the equation of state must pass through a minimum. The internal energy E of the matter must be converted into kinetic energy because the ultimate aim is to move matter around and thereby create inhomogeneity. It is therefore necessary to have two phases with different E, and for the high-density phase to be metastable, in order that E may be released as ordered motion of the matter. This requirement necessitates a first-order phase transition. However, it is clear that not every first-order phase transition gives structure—as in the case where a gas simply condenses into a vapor of droplets. Thus, a second requirement is that internal energy be released as large-scale ordered motions that can do work.

2. For the particular mechanism discussed in this paper, we require a coherent (e.g., solid or liquid) condensed phase with a positive surface tension. It must be energetically favorable for large lumps of condensed phase to exist. An extended surface interface then forms between the two phases that is not unstable to droplet formation when the system is in phase equilibrium. Classically, this requires a positive surface tension, which tends to minimize the surface of the interface. This criterion is necessary to have ordered motions of the matter over large scales—the relaxation wave that propagates out from a crack must not merely trigger droplet formation in adjacent regions. Thus, the matter can do work, either by the condensed phase relaxing to its equilibrium density or by the gaseous phase pushing against an interface, reversing the expansion in localized regions and temporarily accelerating it in others. One can also imagine situations different from, but analogous to, that considered here, e.g., where a "detonation" can cause energy release in isolated regions. In the case of vacuum phase transitions, which are analogous to supercooled systems, this criterion is fulfilled in the sense that ordered motions of bubble walls are automatically produced (as in spontaneously broken gauge theories; Witten 1981). Any process leaving large-scale structure behind must however meet a third condition.

3. The dynamics of the nucleation must be such as to produce lumps of large size—icebergs rather than snowflakes. Even though large pieces might be energetically favored, there is no guarantee that strongly nonequilibrium processes would produce them. Small pieces would result if the nucleation rate is either too large or too small, but it will be shown below that a realistic

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rate produces large pieces if the matter has low entropy.

There are strong indications that a phase transition with the above properties probably occurs in matter near nuclear density as a result of pion condensation (Baym 1977; Baym and Pethick 1979; Irvine 1978), and, in neutron matter, as a result of neutron solidification or liquefaction (Baym 1977; Peierls, Singwi, and Wroe 1952; Mayer and Teller 1949). The sound speed in the pion case may be as great as 0.2 c, and the maximum possible lump size would be $\sim 0.1\ M_\odot$. The sections which follow analyze the process in some detail and show that this is probably an overestimate; the actual size is probably 10^{-10}–$10^{-6}\ M_\odot$.

b) *Nucleation Dynamics and Fragment Size*

It is possible to estimate the size of lumps emerging from a phase transition once we know the probability w per unit time that a lattice site fluctuates enough to produce a crack. One expects w to have the functional form

$$w\,dt = e^{-S}\,dt/\tau, \tag{1}$$

where τ is the characteristic lattice vibration time. The action S is a function of the mean properties of the matter (T, ρ, etc.) which is infinite at $t < t_0$ and then decreases with time until $\rho_{\rm crit}$ when $S \to 0$ and metastability disappears.

We measure comoving distances in units of mean nucleon separation l. The mean separation between cracks which have already nucleated up to time t is

$$L_N = W^{-1/3}, \tag{2}$$

where

$$W = \int_{t_0}^{t} w\,dt. \tag{3}$$

The other length in the problem is the comoving distance traveled by a sound wave or crack between t_1 and t_2. For simplicity, assume the sound speed c_s is constant; then

$$L_{\rm sound}(t_1, t_2) = a_2 \int_{t_1}^{t_2} dt' C_s / a(t')$$

$$= 3(t_2/\tau)\left[1 - (t_1/t_2)^{1/3}\right], \tag{4}$$

where we have used the Friedmann relations for the scale factor $a(t)$ with $p = 0$. For the pion phase transition ($\rho_{\rm crit} \approx 3 \times 10^{14}$ g cm^{-3}), we can use the approximation

$$L_S \approx 10^{19}\left(\frac{t_2 - t_1}{t_1}\right). \tag{5}$$

As the tension grows after ρ_0, the separation L_N decreases catastrophically. To find the size of the lumps, we want to find $L_N(t_1)$, where t_1 is the *first* time for which a sound wave $L_S(t_1, t_2)$ can travel the mean separation between cavities at t_1. Thus, the size of the final lumps is the size of the first crack system which "connects," i.e., which travels the mean separation between its nucleation sites. This can be described diagrammatically as in Figure 2: given a nucleation curve L_N and a family of sound curves L_S, the crack length is given by the tangent point T. The model can be solved analytically once w is known. We have two equations in

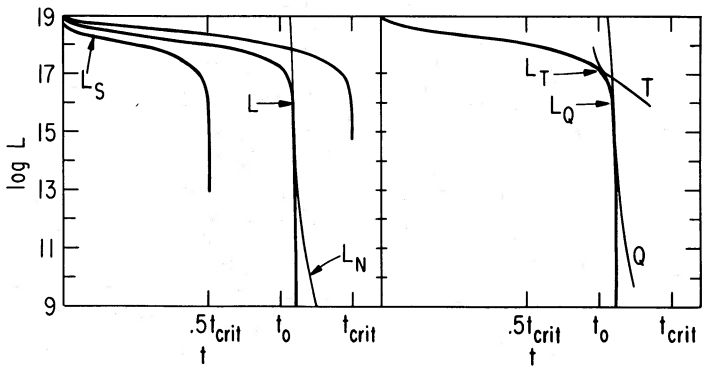


FIG. 2.—Calculation of fragment size from nucleation rate. The vertical axis is comoving length expressed in units of mean internucleon separation; the horizontal axis is time. The family of curves L_s represents propagation of sound waves; the curve L_s which is tangent to the instantaneous distance between nucleation sites L_N determines the size L of the final lumps. If L_N includes contributions from thermal fluctuations T as well as quantum fluctuations Q, the mean size L may be increased (*right*) or the number density of larger lumps may increase. For cold matter the lumps are always large, of the order of 10^{-3}–10^{-2} of the sound horizon distance ($\sim 10^{19}$ at nuclear density). The time $t_{\rm crit}$ corresponds to the density at which rupture would occur if no cavitation took place, i.e., the time when the dense phase is no longer metastable.

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(t_1, t_2):

$$L_{\text{fragments}} = L_S(t_1, t_2) = L_N(t_1), \tag{6}$$

$$\partial_1 L_S = \partial_1 L_N. \tag{7}$$

These are solved below for plausible nucleation rates.

To calculate the size of fragments and the coherence length of the nongravitational force giving structure, we first need to know the nucleation probability function w. In this section, we calculate the rate w for both classical thermal fluctuation at finite temperature and for spontaneous fracture by quantum tunneling by matter at zero temperature. The latter case gives a reliable minimum estimate which is insensitive to parameters as long as the matter is fairly cold.

To create a cavity of radius a, a surface area $4\pi a^2$ must be created; this requires an energy expenditure

$$\Delta E_{\text{surface}} = 4\pi a^2 \sigma, \tag{8}$$

where σ is the surface tension. On the other hand, we gain energy by releasing the tension K on the corresponding volume:

$$\Delta E_{\text{tension}} = -(4\pi/3) a^3 K. \tag{9}$$

For this calculation, we ignore contribution to ΔE by the pressure of a new gaseous phase in the cavity. The cavity has zero energy at radius a_0, given by

$$4\pi a_0^2 \sigma - \frac{4\pi}{3} a_0^3 K = 0 \tag{10}$$

or

$$a_0 = 3\sigma/K. \tag{11}$$

Thus, there is an energy barrier to be overcome in the formation of a cavity, roughly of the order of

$$\Delta E_0 \approx 36\pi\sigma^3/K^2. \tag{12}$$

Matter at finite temperature forms cavities by thermal fluctuation; the thermal nucleation rate is

$$w_T = \frac{1}{\tau} \exp(-\Delta E_0/kT), \tag{13}$$

$$w_T = \frac{1}{\tau} \exp(-36\pi\sigma^3/K^2 kT), \tag{14}$$

where τ is the characteristic lattice vibration time. If we assume that σ is constant, and approximate the tension by

$$K \approx C_S^2(\rho_0 - \rho) = C_S^2 \rho_0 x, \tag{15}$$

where

$$x \equiv (\rho_0 - \rho)/\rho_0, \tag{16}$$

then

$$w_T = \frac{1}{\tau} \exp\left(-\frac{36\pi\sigma^3}{C_S^4 \rho_0^2 kT} x^{-2}\right). \tag{17}$$

We see that, for large values of T, the nucleation rate can be large; indeed, when $w \approx 1$ the phase "melts." For small T, the thermal rate becomes very small and, in particular, it is likely to be much smaller than the quantum rate calculated below. Since equation (17) has a different functional form from the quantum rate, it may still be important in some situations (e.g., Fig. 2, *right*). However, in calculating the lump size we assume that T is small enough to ignore w_T.

Peierls, Singwi, and Wroe (1952) have calculated the wave function ψ for a cavity in a fluid under tension, using the WKB approximation $\psi = \exp[\phi(a)]$ to solve the Schrödinger equation. The amplitude they calculate yields a probability for the formation of a particular cavity per unit time

$$w_Q = (2\sigma/\rho l^3)^{1/2} \exp[-2\phi(a_0)]$$

$$\approx \tau^{-1} \exp[-2\phi(a_0)], \tag{18}$$

where

$$\phi(a_0) = 168(\sigma^4/\hbar)(\rho^{1/2}/K^{7/2}). \tag{19}$$

We have approximated the constant $(\rho l^3/2\sigma)^{1/2}$ as being the "lattice" vibration time scale $\tau = l/c_S$. The probability $W(t_1)$ of a cavity forming up to t_1 in a unit volume is

$$W(t_1) \equiv b^{-3} \int_0^{t_1} w\, dt$$

$$\approx \frac{2}{7} b^{-3} \tau^{-1} (\partial_1 K)^{-1} K^{9/2} \left(\frac{\hbar}{336\sigma^4 \rho^{1/2}}\right)$$

$$\times \exp\left[-K^{-7/2}\left(\frac{336\sigma^4\rho^{1/2}}{\hbar}\right)\right], \tag{20}$$

where b is a length of the order of unity ($1 < b < a_0 \approx 3\beta x^{-1} \lesssim 100$) which depends on the interference between wave functions of overlapping cavities. We parameterize the surface tension by a variable β,

$$\sigma = C_S^2 l_0 \rho_0 \beta \approx \text{constant}, \tag{21}$$

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and define a parameter γ of the order of unity,

$$\gamma \equiv \hbar/\left(C_s l_0^4 \rho_0^{1/2} \rho_1^{1/2}\right) \approx 1. \tag{22}$$

Variables like ρ_1 are approximately constant because ρ varies by only a small amount during the period of interest. The main time dependence of W comes from the tension K, which, as before, can be replaced by the perturbative variable $x \equiv (\rho_0 - \rho)/\rho_0$, which varies from 0 to 1 but is small in the region of interest:

$$K \approx C_s^2(\rho_0 - \rho) = C_s^2 \rho_0 x, \tag{23}$$

$$\partial_1 K = 2C_s^2 \rho_0 / t_1. \tag{24}$$

Thus, we write

$$W(t_1) = b^{-3} \frac{1}{7 \cdot 336} \left(\frac{t_1}{\tau}\right) \frac{\gamma}{\beta^4} x_1^{9/2} \exp\left(-x_1^{-7/2} 336 \frac{\beta^4}{\gamma}\right). \tag{25}$$

It should be noted that the parameter β^4/γ is fixed once the equation of state is known precisely, in particular by the value $x_{\rm crit}$ at which the compressibility vanishes. The parameter thus measures roughly how far the medium can be stretched without breaking. Approximately, we may fix β^4/γ by requiring that $w(t_{\rm crit}) = 1$, or, taken literally, that

$$\frac{\beta^4}{\gamma} \approx \frac{x_{\rm crit}^{7/2}}{336}, \tag{26}$$

although the perturbative WKB approximation should not really be trusted when $w \approx 1$. We now write the mean separation (in lattice units) between cracks at t_1,

$$L_N \equiv W^{-1/3} = 6 \times 10^{-6} b \left(\frac{\beta^4}{\gamma}\right)^{1/3} x^{-3/2} \times \exp\left(x^{-7/2} 112 \frac{\beta^4}{\gamma}\right), \tag{27}$$

and the distance (eq. 5) traveled by a sound wave between x_1 and x_2, for small $x_1 - x_2$:

$$L_S(t_1, t_2) \approx C_S(t_2 - t_1) \approx \frac{t_1}{2\tau}(x_2 - x_1), \tag{28}$$

where $t_1/\tau \approx 10^{19}$. As before, we ask for the size L of the first crack system to connect with itself, which is given by $L_S(t_1, t_2)$ where t_1 and t_2 are determined by

$$L_N = L_S \tag{29}$$

and

$$\partial_1 L_N = \partial_1 L_S. \tag{30}$$

Equations (29) and (30) become

$$b\left(\frac{\beta^4}{\gamma}\right)^{1/3} x_1^{-3/2} \exp\left(x_1^{-7/2} 112 \frac{\beta^4}{\gamma}\right) = 8 \times 10^{23}(x_2 - x_1), \tag{31}$$

$$b\left(\frac{\beta^4}{\gamma}\right)^{4/3} x_1^{-5} \exp\left(x_1^{-7/2} 112 \frac{\beta^4}{\gamma}\right) = 2 \times 10^{21}. \tag{32}$$

Equation (32) determines x_1, and equation (31) then determines $x_2 - x_1$ and hence L. Table 1 shows solutions for various values of β^4/γ over the plausible range (i.e., for matter which cannot be "stretched" more than a factor of 2 in density). The scale of the fragments is certainly of astronomical size, although it seems doubtful that quantum nucleation alone could produce lumps larger than $\sim 10^{-5}\ M_\odot$. The answer might be different if thermal nucleation is important, so that $W = W_T + W_Q$ and

$$L_N = (W_T + W_Q)^{-1/3}, \tag{33}$$

which may produce curves like the construction in Figure 2 (*right*). However, we may treat these numbers as minimum estimates of total structure produced if the matter is fairly cold.

c) *Spectrum of Lump Sizes*

A simplifying approximation allows us to estimate the size spectrum of fragments $N(>L)$, the fraction of matter which ends up in lumps of linear scale greater than L. We assume that all cracks meet simultaneously at a time t_2, so that all lumps of size L are nucleated at the same time $t_L \equiv t_2 - L/C_s$. The probability P of a fragment in the size interval $(L, L + C_s dt_L)$ nucleating

TABLE 1

LUMP SIZES FROM A NUCLEAR DENSITY PHASE TRANSITION

β^4/γ	x_1	$x_2 - x_1$	$\left(\frac{L}{C_s t_1}\right)^3 \approx \left(\frac{L}{10^{19}}\right)^3 \approx \left(\frac{M}{M_\odot}\right)$
10^{-4}	0.09	8.3×10^{-4}	7×10^{-11}
10^{-3}	0.18	1.2×10^{-3}	2×10^{-10}
10^{-2}	0.34	2×10^{-3}	10^{-9}
10^{-1}	0.65	4.5×10^{-3}	10^{-8}
0.2	0.79	1.6×10^{-2}	5×10^{-7}

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at a point p is approximately

$$P_L dL = w(t_L)\left[1 - N(>L)\right] dt_L, \tag{34}$$

where w, as before, is the nucleation rate per lattice site per time, and the factor $[1-N(>L)]$ is roughly the fraction of nucleation sites still under tension at t_L which will not heal before t_2, i.e., which will not be incorporated into lumps larger than L. This probability $P_L dL$ is just the number density (in lattice units) of fragments in the size interval $(L, L + C_s dt_L)$ which is

$$\partial_L N L^{-3} \frac{dL}{dt_L} dt_L; \tag{35}$$

so, equating equations (34) and (35), we derive a first-order differential equation for N:

$$\partial_L N - NF(L) + F(L) = 0, \tag{36}$$

where

$$F(L) \equiv w(t_2 - L/C_s) L^3 / C_s, \tag{37}$$

and w is derived from quantum theory as in the previous sections.

The solution is

$$N = 1 - \exp\left[-\int_L^\infty F(L')\, dL'\right]. \tag{38}$$

The differential size function

$$\eta \equiv -\partial_L N = F(L) \exp\left[-\int_L^\infty F(L')\, dL'\right] \tag{39}$$

is strongly peaked in the region where $F(L) \approx 1$, which is the characteristic size derived in the previous section. For large sizes, as one expects, $\eta \approx w(t_L) L^3 / c_s$, i.e., the frequency of lumps is just proportional to the nucleation probability at the appropriate time.

III. GRAVITATIONAL EFFECTS OF PHASE TRANSITION

Even when nongravitational forces shut off, there is the well-known gravitational instability of Lifshitz which causes the amplitude of density fluctuations to grow with time. Moreover, even if the lumps themselves disappear, there is a residual nonzero amplitude in the Fourier components of growing density fluctuations on larger scales, produced by the nonadiabatic processes operating during the phase transition. This section applies some key results of more sophisticated analyses (e.g., Peebles 1974, 1980; Bardeen 1980; Press and Vishniac 1980*b*) to estimate the structural effects of the phase transition at later epochs.

We need to know the power spectrum of growing density perturbations introduced at wavenumber k when matter is moved about by nongravitational forces on scale $k' \gg k$. In the Newtonian case (Zel'dovich 1965), there is a unique answer which does not depend on the forces involved, but only on energy and momentum conservation:

$$|\delta_k^2| = (k/k')^4 |\delta_{k'}^2|. \tag{40}$$

The same law can be expressed with masses. Suppose that matter pulls itself into lumps of mass m. Then the growing modes of density fluctuation on a mass-scale M have the spectrum

$$(\delta\rho/\rho)_M = (M/m)^{-7/6} \tag{41}$$

This equation is sufficient to calculate the time t_2 at which bodies of mass M collapse. If the lumps m form at t_1, then

$$(M/m) = (t_2/t_1)^{4/7} = (a_2/a_1)^{6/7}. \tag{42}$$

This equation determines the lines A, B, C, and D drawn in Figure 3. Given a fragment mass, these curves show the mass which would be collapsing at z if no pressure forces intervened to halt the growth of fluctuations; above the maximum Jeans mass M_J, growth is always unimpeded, and it turns out that M_J gives the mass of the first systems to collapse. They are small enough ($\sim 10\ M_\odot$) that the simple Newtonian description is adequate.

Now equation (42) also holds in the relativistic case, the only difference being that a different set of coordinates might describe differently a situation that is exactly the same. A full relativistic treatment has been made by Bardeen (1980). A gauge-invariant quantity is the "nonadiabatic stress amplitude" η_M, which measures the dispersion in work done by nongravitational forces on mass-scale M compared to the total rest energy. If the perturbing process is uncorrelated on scales larger than m, then the dispersion in perturbation has a Poisson spectrum,

$$\eta_M \approx \eta_m (M/m)^{-1/2}. \tag{43}$$

When M comes within the horizon M_H, residual peculiar motions and pressure gradients cause density fluctuations to form with amplitude η. Thus, the fluctuations in potential, $\delta\phi = G\delta M/Rc^2$, have a Poisson spectrum:

$$\delta\phi = (M/M_H)^{2/3} (\delta\rho/\rho) \approx \eta_M \propto M^{-1/2}. \tag{44}$$

A mass-scale becomes nonlinear when $(M/M_H)^{2/3} =$

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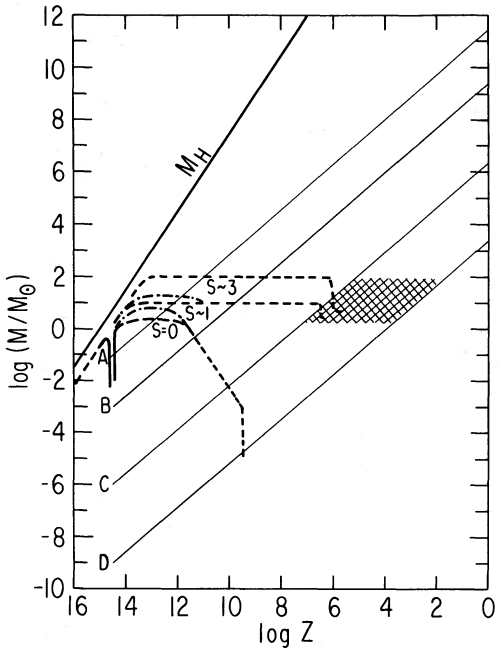


FIG. 3.—Star formation caused by shattering near nuclear density, $z_N \approx 3\times10^{14}$, $\rho_N \approx 3\times10^{14}$ g cm^{-3}. The horizon mass is M_H. Curves A, B, C, and D represent density fluctuations introduced on larger scales by stress perturbations at ρ_N for various fragment masses; they show the mass of fluctuations which would just be collapsing ($\delta\rho/\rho \approx 1$) at z if no pressure forces intervened to stabilize their growth in amplitude. Pressure is unimportant above the Jeans mass M_J, which is shown by heavy dashed curves for various values of entropy S, with dash-dot curves showing the effect of larger lepton-to-baryon ratios. We expect $S<3$ in the early stages; thus, the shaded region shows the mass of the first system to collapse and the redshift at which collapse would occur for the most plausible range of initial fragment sizes, $10^{-9} \lesssim m/M_\odot \lesssim 10^{-6}$. Note that fragments of any mass 10^{-9} to 0.1 $M_\odot$ would produce stars in the mass range from 2 to 100 $M_\odot$ at a high redshift $z \gtrsim 100$. The expected clustering scale of remnants today ($\sim 10^{4-6}$ $M_\odot$) is too small to make galaxies from scratch, so stellar explosive energy must also be invoked to trigger collapse of galaxy-mass gas clouds.

$\delta\phi = \eta$, and since $M_H \propto t$, one ends up with

$$(M/m) \propto (t_2/t_1)^{4/7} \tag{45}$$

as before.

Now it is possible to calculate the present-day, nonlinear mass M_0 produced by a phase transition near nuclear density, 3×10^{14} g cm^{-3}, or a redshift

$$z = 2.6\times10^{14}(\Omega h^2)^{-1/3}, \tag{46}$$

where $h = H_0/100$ km s^{-1} Mpc^{-1}. The total growth factor in mass is $\sim(\Omega z_1)^{6/7}$ (because growth stops at a redshift $1+z \approx \Omega^{-1}$), so

$$\frac{M_0}{M_\odot} = 2.3\times10^{12}(m/M_\odot)\Omega^{4/7}h^{-4/7}, \tag{47}$$

where m is the mass of the fragments. Since m is substantially less than a solar mass in the general case, some other astrophysical mechanisms must be responsible for galaxy formation; nevertheless, it is clear that astronomical scales have certainly been reached, and, as shown below, one expects massive stars to form at a sufficiently high redshift to initiate an astrophysical instability if one exists.

IV. SUBSEQUENT EVOLUTION AND STAR FORMATION

After shattering takes place, what happens to the lumps and what effects do they have? The most likely outcome is that the lumps themselves disappear after approximately a Hubble time, leaving acoustic waves. However, the shattering process also introduces small-amplitude growing modes of density fluctuation, on larger scales, with the spectrum given in equation (41). Above the maximum Jeans mass $M_{J_{max}}$, these fluctuations grow without interruption and reach $(\delta\rho/\rho) \approx 1$ at a redshift z which depends on the fragment size but is high enough ($z > 200$) for stars to complete their evolution near the earliest possible redshift, i.e., when the age of the universe is roughly 3×10^6 yr. These stars are held responsible for the high entropy and helium abundance in the present universe; and, as shown by Rees (1978), they would naturally be expected to generate an entropy of the order of 10^8. No stars would form with $\leq 2M_\odot$, the lowest possible $M_{J_{max}}$. We thus expect all of these stars to be extinct by $t \approx 10^9$ yr, $z \approx 3$. The remnants are naturally identified with the dark matter now present between visible galaxies.

At the moment of shattering, the phase-equilibrium mass fraction in lumps is

$$f_{\rm lumps} = \left(\frac{\rho_c}{\rho_g} - \frac{\rho_c}{\langle\rho\rangle}\right) \Big/ \left(\frac{\rho_c}{\rho_g} - 1\right),$$

where ρ_c and ρ_g are the densities of the condensed and gaseous phases respectively in pressure equilibrium, and $\langle\rho\rangle$ is the mean density of the whole system. If (say) $\rho_c \approx 2\langle\rho\rangle \approx 3\rho_g$, then $f_{\rm lumps} = 1/2$; one generally expects $f_{\rm lumps}$ to be substantial at the "moment" of shattering. If classical thermodynamics determines the phase equilibrium from then onwards, eventually the fragments evaporate because a confining gas pressure is probably necessary for the condensed phase to exist (although this may not be true of some nuclear fluids or solids; see Baym 1977). If the mass of the lumps is large enough, then their self-gravity can stabilize them without external pressure. The minimum mass of neutron stars is not reliably known and is clearly important to an understanding of how much matter ends up bound in relativistic objects. For lumps $\ll 0.1$ $M_\odot$, the fragments probably evaporate. Note that black holes would not form because their mass would have to be close to ~ 3 $M_\odot$ (the Oppenheimer-Volkoff limit), i.e., a substantial fraction of the horizon mass.

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At $\rho \approx 10^{14}$ g cm^{-3}, we thus expect gas with growing density fluctuations with the power spectrum $|\delta_k^2| \propto k^4$, or $(\delta\rho/\rho)_M \propto (M/m)^{-7/6}$ (eq. [41]). On scales smaller than the Jeans mass, the fluctuations are stabilized by pressure forces and oscillate as acoustic vibrations, but, on larger scales, the Lifshitz instability causes fluctuations to grow gravitationally without interruption (see, e.g., Rees 1975). The Jeans mass during this period thus determines the size of the first systems to separate out as bound objects with $(\delta\rho/\rho) \approx 1$.

In cold matter ($S < 1$), the Jeans mass is determined by the degeneracy pressure of electrons and neutrinos. During the period when e^- and ν_e are relativistic

$$M_J \approx \alpha^2 \left(\frac{Gm_p}{hc}\right)^{-3/2} m_p \approx 2\alpha^2 M_\odot,$$

where $\alpha \equiv |n_\ell / n_\mathcal{B}|$ denotes the number of relativistic leptons per baryon (of the order of 1 or 2), and m_p is the proton mass. At density

$$\rho \approx 10^{16} \left(\frac{m_e}{m_p}\right)^3 \approx 10^6 \text{ g cm}^{-3}, \quad z \approx 5 \times 10^{11},$$

the neutrinos decouple and the electrons become nonrelativistic, so the Jeans mass $M_J \approx (c_s/c)^3 M_H \approx (p/\rho)^{3/2} \times M_H \approx a^{-3/2}$ after that time. This behavior is illustrated in Figure 3.

However, there might be entropy generated during or after the shattering which would warm the matter. For $S \geq 1$, M_J is determined largely by radiation pressure; from standard formulae given in Peebles (1980) and Weinberg (1972), a reasonable approximate formula for low entropy is $M_J \approx 2\text{–}10(s+\alpha)^2 M_\odot$, where the uncertainty is due largely to ambiguity in going from wavenumber to mass. (After decoupling of radiation from matter, if matter then remains in temperature equilibrium with radiation, $M_J \approx 10 S^{1/2} M_\odot$.) Simple energetic considerations show that the entropy generated at such high density cannot be too large: the fraction ε of the matter rest mass which must be converted to radiation to create entropy S at density ρ is $\varepsilon \approx S^{4/3}(\rho/10^{16} \text{ g cm}^{-3})^{1/3}$. Thus, at 10^{14} g cm^{-3}, we know $S < 3$ and expect $S < 1$. Of course, if neutron stars are present in larger numbers they would make a continuously growing contribution to S by accretion possibly leading to higher M_J, but, if the gas is fairly smooth, $S \leq 1$ is a good upper limit until the first energy sources collapse. Thus, the first systems to collapse always have a mass between 2 $M_\odot$ and ~ 100 $M_\odot$ (generally < 40 $M_\odot$) and end up as ordinary stars. This conclusion is independent of the size of the original fragments over a wide range. The situation is illustrated in Figure 3.

The time at which these stellar mass systems collapse does, however, depend on the lump size at t_0. For initial lumps as large as 10^{-3} $M_\odot$, 10 $M_\odot$ stars collapse and start nuclear burning at $\rho \geq 1$ g cm^{-3}, and then inflate out of the degenerate region to enter the main sequence from greater than equilibrium density (Carr 1977). Smaller lumps cause stellar mass clouds to collapse at a lower density and start burning in a more conventional way, from below equilibrium density (see Rees 1976). As long as the initial fragments are $\gtrsim 10^{-9}$ $M_\odot$, stars < 100 $M_\odot$ collapse at $z > 100$. Stars greater than ~ 20 $M_\odot$ would finish their evolution after $\sim 3 \times 10^6$ yr, so the earliest explosions occur when the universe is about that age, at $z \approx 180\,(\Omega h^2)^{-1/3}$. These explosions may trigger formation of much more massive stars, $\sim 10^3$ $M_\odot$ (Ostriker and Cowie 1981). The smallest possible pregalactic stars, 2 or 3 $M_\odot$, would leave the main sequence at $t \approx 10^9$ yr, or $1 + z \approx 4$, and so would now be extinct.

Pregalactic stars are a natural source for the entropy of the microwave background—one can even write down a dimensionless number,

$$S \approx \left(\frac{e^2}{Gm_p^2}\right)^{1/4} \frac{m_p}{m_e} \left(\frac{e^2}{hc}\right)^{3/4} \varepsilon \approx 10^8,$$

for the entropy expected from a closure density of pregalactic sources radiating near the Eddington limit with stellar efficiency $\varepsilon \approx 10^{-2}$ (Rees 1978). The energetic requirements of the microwave background are thus easily met.

The thermalization of the microwave background is more difficult. It must take place at $z \gtrsim 50$ (Rees 1978; Layzer and Hively 1973) and requires a mass-fraction in dust grains or molecules of perhaps 10^{-3}, somewhat less if thermalization at longer wavelengths is accomplished with free-free opacity. Stars in the region of 10–100 $M_\odot$ would be able to produce these heavy elements at a sufficiently high redshift. It might be necessary to form observed low-metallicity Population II stars before the grains get mixed with gas (Rowan-Robinson, Negroponte, and Silk 1979). However, an initial enrichment of the order of 10^{-3} is desirable in models of galactic chemical evolution (Audouze and Tinsley 1976; Ostriker and Thuan 1975; Truran and Cameron 1971).

Stars also furnish a source of hydrodynamical energy for moving gas around on supercluster scales, as suggested by Ostriker and Cowie (1981), Ikeuchi (1981), and Doroshkevich, Zel'dovich, and Novikov (1967). Recent observations (Davis *et al.* 1980) in fact suggest that dark and light matter have been segregated up to ≥ 10 Mpc. Preexisting compact objects such as stars would be unaffected by pressure gradients or shocks in gaseous matter. The clustering of dark matter appears to be weaker than the observed clustering of the luminous galaxies, which is the expected situation in this type of model: the mass-to-light ratio should increase with scale.

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If explosions or ionization fronts caused gas to move over large scales and collapse into galaxies, these systems would "accrete" haloes of dark matter even if the dark matter were relatively uniform to begin with. The expected radial dependence of $\rho \approx r^{-2.25}$ agrees reasonably well with observations of halo rotation curves and mass-to-light ratios in clusters (Gunn 1977).

V. COMPOSITION OF MATTER NEAR NUCLEAR DENSITY AND NUCLEOSYNTHESIS IN THE COLD UNIVERSE

Although properties of cold matter at subnuclear density are fairly well understood (Zel'dovich and Novikov 1978), the process of nucleosynthesis in a completely degenerate medium has never been analyzed comprehensively in the literature (although see Kaufman 1970; Peierls, Singwi, and Wroe 1952; Zel'dovich 1965; Carr 1977). We will not present a detailed discussion, but it is essential to understand the general features.

Zel'dovich (1965) and Jacubov (1964) showed that a sufficiently high lepton-to-baryon ratio—$n_{\mathfrak{L}}/n_{\mathfrak{B}} \approx 2$—would convert nearly all neutrons into protons via $n + \nu \rightarrow p + e^-$ because of the energy gained by draining the degenerate neutrino sea. The composition emerging from such a universe would be pure hydrogen.

However, it is no longer clear that $n_{\mathfrak{L}}/n_{\mathfrak{B}}$ is a free parameter if grand unification fields permit conversion of leptons to baryons. Indeed, a general grand unified theory (GUT) has no conserved quantities carried only by fermions, so that the ground state at $T = 0$ is always a vacuum—a universe of cold matter is impossible. However, it is interesting that in the SU(5) and SO(10) GUTs, the most economical unifications of known fields, $\mathfrak{B} - \mathfrak{L}$ is conserved in all reactions. Nonvacuum matter at zero temperature is therefore possible if there exists a net $\mathfrak{B} - \mathfrak{L}$ charge per comoving volume. When particle degeneracy energies are $\sim E_{\rm GUT} \approx 10^{15}$ GeV, mass differences between quarks and leptons are unimportant, and the system would find an equilibrium state with $\mu_q \approx \mu_{\mathfrak{L}} \approx E_{\rm GUT}$. With three quark colors, and both left-handed and right-handed neutrinos [as in SO(10)], this yields equal numbers of baryons and antileptons or $n_{\mathfrak{L}}/n_{\mathfrak{B}} = -1$. At lower energies, leptons and hadrons can no longer be transformed into each other. The equilibrium concentration at $\rho \lesssim 10^{15}$ g cm^{-3} would be an equal admixture of neutrons and electron-type antineutrinos. Almost the entire chemical potential is due to the degeneracy energy of $\bar{\nu}_e$, and almost all the density is due to the neutrons. Formation of (p, e^-) pairs would be suppressed by

$$p + e^- + \bar{\nu}_e \rightarrow n$$

because the Fermi energy of the $\bar{\nu}_e$ is much greater than the decay energy of neutrons.

This stabilization of neutrons takes place until the ν_e decouple. Subsequent production of elements depends on where the neutrons are and how warm the matter is. A free neutron decays at $t \approx 1000$ s into $p + e + \bar{\nu}$, and heating from $p + n \rightarrow D$ provides enough energy to overcome the Coulomb barrier into helium (Carr 1977). Thus, one is left with helium only. If neutron matter is cold, neutrons may stick in large, dense, solid or liquid chunks ("polyneutrons," Mayer and Teller 1949) which evaporate heavy nuclei at $t \approx 1000$ s as they heat up from decays. This yields *r*-process–type elements. Peierls, Singwi, and Wroe (1952) concluded that excessive heavy elements would be produced and ruled out this version of the cold universe; however, their conclusion seems premature if warming occurs during the pion transition. Furthermore, it is not clear what fraction of the heavy nuclei produced actually ends up in the diffuse pregalactic gas because, by $t \approx 1000$ s, stars are already forming, presumably with polyneutrons inside them. In fact, if many polyneutrons existed, they would be bound in self-gravitating clusters with high velocity dispersion ($\sim 10^{-3}c$), so only a small fraction of their products might end up in what is now visible matter. But it is difficult to see how a universe with negative $n_{\mathfrak{L}}/n_{\mathfrak{B}}$ could avoid overproducing helium.

The whole scenario for cold-universe nucleosynthesis appears to depend on $n_{\mathfrak{L}}/n_{\mathfrak{B}}$, and hence ultimately on the grand unification scheme, which is still very uncertain. It is not implausible, however, that the bulk of matter would be hydrogen. Possible mechanisms exist for producing deuterium and heavy elements in small amounts for values of $n_{\mathfrak{L}}/n_{\mathfrak{B}}$ between 1 and 2. The cosmic helium abundance, however, must probably be explained as an outcome of pregalactic nucleosynthesis in stars. Perhaps this is a blessing; observations do not clearly show the minimum abundance Y_p predicted by the cosmological hypothesis, and some of the helium abundances that have been observed in extragalactic H II regions (Peimbert 1975; French and Grandi 1981; French 1980; Lequeux *et al.* 1979), are rather lower than allowed by the standard model. (However, most observations to date can be accomodated if the bulk of the dark gravitating mass is in massive neutrinos [Olive *et al.* 1981]). In any case, there is evidence for a substantial amount of nonprimordial helium synthesis (Peimbert 1975).

At present, it appears that normal stars can create large amounts of helium without overproducing heavy nuclei, if they lie in the right mass range—either of order $\sim 5\ M_\odot$ or $\gtrsim 100\ M_\odot$ (Arnett, Bond, and Carr 1982). In the present model, it would not be unnatural for the initial mass-function of pregalactic stars to be sharply peaked in the region from 4 to 8 $M_\odot$, or for pregalactic explosions to trigger formation of supermassive stars. However, the understanding of stellar mass loss, especially in massive stars, is still at a primitive

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stage, and it would be presumptuous to "predict" Y for a stellar mass function.

VI. OBSERVATIONAL TESTS

The model described above is subject to a battery of observational tests, some of which should be feasible in the near future.

a) Angular Dependence of Anisotropy in the Microwave Background

Measurements of large-scale anisotropy at more than one angular scale are an important test of the origin of fluctuations. Certain kinds of results would imply that fluctuations in gravitational potential on large scales have a spectrum different from $\delta\phi \propto M^{-1/2}$ (eq. [44]); since that spectrum is produced by *any* small-scale, nongravitational process, such observations would imply either (i) fluctuations in large-scale curvature exist in the initial state, contradicting the observations which inspired this paper, or (ii) nongravitational forces are indeed being broadcast over cosmological scales at present, implying the action of "strings" of false vacuum or similar objects. Other observational results would support the hypothesis that anisotropies at large θ are due to random fluctuations in entropy generation at late epochs: a nonprimordial origin of the microwave background produces a characteristic angular power spectrum of temperature fluctuations which is always flat ("white noise") at large θ.

The key test is to measure the slope of the $(\Delta T/T)_\theta$ relation at large enough angles that we know scattering is not affecting the result. Essentially this means measuring the ratio of quadrupole to octopole anisotropy because, at $\theta \lesssim 20°$, which is the angle subtended by the horizon at $z \approx 10$, we cannot be sure that the slope is not affected mainly by the blurring effect of the last scattering surface. The scattering optical depth to a redshift z,

$$\tau_{cs} = \int_0^z \sigma n_e c \, dz'$$

$$= 0.06 \Omega_{\mathrm{H\,II}} \left[(1+z)^{3/2} - 1 \right] h \quad (\text{for } \Omega = 1)$$

is less than unity out to $z \approx 20$ if the ionization $\Omega_{\mathrm{H\,II}} < 0.2$. The angle subtended by the horizon ct_z,

$$\theta_H(z) = \frac{(1+z)(1+\Omega z)^{-1/2}\Omega^2}{2\left[(1+\Omega z)^{1/2} + \Omega - 1\right]\left[(1+\Omega z)^{1/2} - 1\right]},$$

$$\theta_H = \frac{1}{2}\left[(1+z)^{1/2} - 1\right]^{-1} \quad (\text{for } \Omega = 1),$$

at $\tau = 1$ is less than about 10°. The angular spectrum of anisotropy at $\theta > 20°$ is thus a clean test of the present model and a simple and direct constraint on initial conditions.

Assume for the moment that $\Omega = 1$. On angular scales larger than $\theta_H(\tau = 1)$, the theoretical predictions are unambiguous. Fluctuations in gravitational potential generated by the Sachs and Wolfe (1967) effect produce an angular spectrum

$$(\Delta T/T)_\theta \propto \theta^{1/2 - n/2},$$

where $(\Delta T/T)_\theta$ is the dispersion in temperature between adjacent beams of width θ, and n is the power-spectrum index of growing modes of density perturbation, $\delta_k^2 \propto k^n$. Spontaneous theories predict $n = 4$; strings produce $n = 1$ (Zel'dovich 1980), and a white-noise Poisson distribution of matter gives $n = 0$. If the microwave background is nonprimordial, then uncorrelated "shot-noise" fluctuations in the amount of entropy

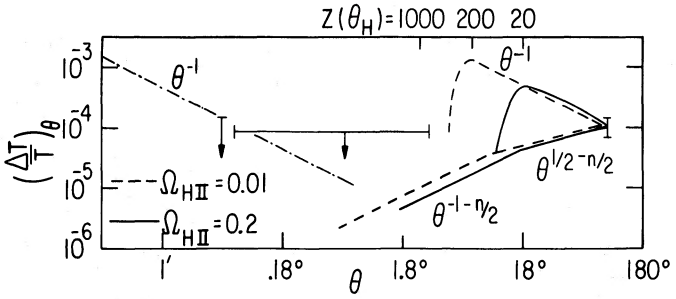


FIG. 4.—Anisotropy of the microwave background as a function of angular scale. Measurements are those of Partridge (1979, 1980), Parijskij (1977), and Boughn, Cheng, and Wilkinson (1981). At sufficiently large θ [$\theta \gtrsim 20°$ for $\Omega_{\mathrm{H\,II}} = 0.2$, $\theta \gtrsim 7°$ for $\Omega_{\mathrm{H\,II}} = 0.01$; these are angles $\theta_H(z)$ subtended by $2ct(z)$ at the redshift where $\tau = 1$, $z(\tau = 1) = 20$ and 200 respectively], the blurring effect of the last scattering surface does not affect $(\Delta T/T)_\theta$, and the slope of $(\Delta T/T)_\theta \propto \theta^{1/2 - n/2}$ is a clear measure of the power spectrum of growing density fluctuations $|\delta_k^2| \propto k^n$, or of fluctuation in nonprimordial entropy production $(\Delta T/T)_\theta \propto \theta^{-1}$. Below $\theta_H[z(\tau = 1)]$, the relativistic fluctuation steepens by one-half power but random entropy fluctuations are exponentially blurred, becoming invisible on smaller scales; this is a characteristic signature not shared by either primordial isothermal or adiabatic fluctuations. At very small angular scales, unthermalized radiation from sources at high z would have a wavelength-dependent anisotropy scaling like $(\Delta T/T)_\theta \propto \theta^{-1}$, which may be observable at $\theta \approx 1''$.

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produced would give $(\Delta T/T)_\theta \approx \theta^{-1}$. Figure 4 shows these predictions normalized to the quadrupole observation of Boughn, Cheng, and Wilkinson (1981). A more detailed discussion of these predictions is given by Hogan (1982).

The amplitude of the quadrupole moment together with upper limits on $\delta T/T$ at smaller θ already constrain initial conditions severely. If one excludes primordial fluctuations and relies on a nonlinear process for galaxy formation, then the Sachs-Wolf fluctuations (with $n=4$) yield a quadrupole less than $\sim 10^{-8}$; therefore, Ostriker-Cowie type models require tepid or cold universes where a large quadrupole can be generated by random entropy production. Conversely, if the standard model is true, then primordial fluctuations are needed, and we already have information about their spectrum (Peebles 1981*a*, *b*; Silk and Wilson 1981; Kaiser 1982). The question may be resolved by more sensitive measurements now in progress.

b) *Small-Scale Anisotropies*

The thermalization of the background radiation in a cold bang is likely to be imperfect. This may be observed either by direct measurements of the spectrum or by measurements of anisotropies at very small angular scales ($\sim 1''$). The fluctuations in $(\Delta T/T)$ created by discrete sources of radiation at high redshift are easily distinguishable from contamination by "local" sources (at $z \lesssim 5$) and have a characteristic θ-dependence (Fig. 4) and a wavelength dependence which depends on the radiation mechanism (Hogan 1980*b*). Centimeter wavelengths are most promising for this type of test because of the relative lack of thermal infrared sources or nonthermal radio sources contributing to anisotropy. These observations are being pursued with increasing sensitivity on large interferometers (e.g., Martin, Partridge, and Rood 1980; and Partridge 1981, private communication). It should also be noted that the Doppler-type fluctuations predicted by pancake theory (Zel'dovich 1978) have a characteristic signature which would be difficult to mimic.

c) *Physical Tests*

There are a number of ways that the existence or absence of a pion-condensate phase in neutron stars might be tested, for example, by observations of core cooling rate (Baym 1977). It is also conceivable that laboratory experiments could reveal condensates inside nuclei (Migdal 1978).

d) *Consequences of Pregalactic Stars*

The consequences of pregalactic stars might include the following:

1. The observed (Woody and Richards 1979) distortion of the microwave background spectrum might be due to imperfect thermalization of starlight by grains (Rowan-Robinson, Negroponte, and Silk 1979; Puget and Heyvaerts 1980).

2. Stellar mass objects in galactic haloes would produce a characteristic time variability of gravitational lenses (Chang and Refsdal 1979; Gott 1981).

3. Line emission from lumpy ionized gas at $z \approx 10$ would have a characteristic angular and spectral structure (Hogan and Rees 1979).

4. An infrared background might exist with energy density comparable to the microwave, which could be observed by planned infrared satellites.

5. Supernovae at $z \approx 50$–100 provide a natural explanation for the amplitude and spectrum of the X-ray and γ-ray background (Hogan and Layzer 1977).

6. Models of galactic evolution generally require substantial amounts of initial enrichment (Truran and Cameron 1971; Ostriker and Thuan 1975; Audouze and Tinsley 1976).

VII. DISCUSSION

The above theory can be generalized to "tepid" (Carr and Rees 1977) or "hot" models with initial $S > 1$, if one assumes that a suitable phase transition exists at (say) slightly supernuclear density (see Carr 1981; Carr and Silk 1981). However, several effects make it doubtful whether any bound systems would form by now through the process analyzed in this paper.

1. The nucleation of cavities is much more efficient if the temperature is high (eq. [14]), so one might expect the initial scale of lumpiness relative to the horizon to be much reduced; in particular, black hole formation appears implausible. This objection does not apply to first-order vacuum phase transitions, which may be nucleated by tunneling even for $S \gg 1$ (e.g., Witten 1981).

2. The comoving volume (i.e., net baryon mass) encompassed by a sound wave is reduced for $S \gg 1$ because the mass density is dominated by photons and particle-antiparticle pairs. The size of lumps, of course, also automatically decreases as one goes to higher z in search of a phase transition.

3. The Jeans mass $\propto S^2$, which means that the stress perturbation from the phase transition must have larger amplitude to allow the more massive systems to collapse before now. But, in fact, the stress perturbations are reduced because of effects (1) and (2) above.

If one *could* form black holes at high density, then the second two problems would be alleviated because lumpiness of holes would not be wiped out by gas pressure. Then, as shown in § VI*a*, the main problem would be producing a quadrupole anisotropy as large as that observed ($\sim 10^{-4}$) with the tiny curvature fluctuations produced by the $n=4$ spectrum (eq. [40]). Of course, in a tepid model, most of the background is nonprimordial, so one would still have recourse to fluctuations in ent-

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ropy generation. But, then, the problem of excessive helium production remains—if GUTs operate, it is not possible to introduce the degenerate sea of ν_e or $\bar{\nu}_e$ that are the standard ways to avoid helium production, and the predictions seem at variance with observations because, in tepid models, massive neutrinos are no help; even the observed density of baryonic matter leads to high helium.

If the helium problem could somehow be circumvented, a model similar to that sketched above might be constructed if a strongly first-order phase transition occurs in high-entropy matter. The general condition is that a phase transition at temperature T must produce enough perturbation on large scales to cause systems of mass $M_J \approx S^2\ M_\odot$ to collapse by the present epoch. The perturbation $\delta\varphi$ is conserved at a fixed comoving scale; for collapse at t_0, we require

$$\delta\phi_{M_J} > (L_0/ct_0)^2 \approx 10^{-15}(M_J/M_\odot)^{2/3} \approx 10^{-15}S^{4/3}.$$

According to equation (44), the $\delta\varphi$ produced in the phase transition is

$$\delta\varphi_{M_J} \approx (M_J/m)^{-1/2}\delta\varphi_m,$$

where m is the baryon charge of the fragments or bubbles produced,

$$m \approx 1 M_\odot S^{-1} T_{\rm GeV}^{-3}(L_m/ct)^3,$$

and $\delta\varphi_m \approx (L_m/ct)^2$ where t^{-1} is the expansion rate at T. As in the model above, the size of fragments is determined by the coherence length L_m when bubbles collide. [A general argument along the lines of § II shows that typically $L_m/ct \approx (\log L_m)^{-1} \lesssim 10^{-1}$ (with L_m in microscopic units) for any transition nucleated by tunneling.] The condition for obtaining collapsed systems of any sort can then be written

$$S < 10^5\ T_{\rm GeV}^{-9/17}(L_m/ct)^{21/17}, \qquad (48)$$

where S is the entropy after the phase transition. This means, for example, that a first-order transition at electroweak symmetry breaking ($\sim$300 GeV), as described by Witten (1981), might cause systems of $M_J \lesssim 10^6\ M_\odot$ to collapse by the present if the entropy after the transition is less than $\sim 10^3$. It also means that transitions above 10^{10} GeV cannot cause systems to collapse by the Lifshitz instability for any value of S.

VIII. SUMMARY

The most important conclusion of this paper is that a certain type of phase transition in an expanding, uniform universe of cold matter naturally produces structure many orders of magnitude greater than the interparticle separation. If one then asks what would happen if the *actual* universe were cold and uniform at nuclear density, the following sequence of events suggests itself:

1. At $\sim 3\times 10^{14}$ g cm^{-3}, the system shatters into large pieces containing between 10^{46} and 10^{51} nucleons (i.e., between 10^{-10} and $10^{-6}\ M_\odot$). Large enough pieces may stabilize as neutron stars or "polyneutrons."

2. Whether or not fragments survive, they introduce enough fluctuation in stress to cause stars to form at $10^7 \gtrsim z \gtrsim 100$. The first stars must have a mass $\gtrsim 2\ M_\odot$ and probably less than 100 $M_\odot$.

3. The most massive stars of the initial generation explode early ($z \approx 200$) and trigger the collapse of much more massive systems, eventually causing collapse of galaxy-sized clouds of gas.

4. Pregalactic objects produce an entropy S of the order of 10^8, enough heavy elements to thermalize their radiation at $z \gtrsim 100$, and helium enrichment.

5. The remnants of the pregalactic stars form galaxy haloes by dissipationless infall onto the visible parts of galaxies and later collapse with galaxies into clusters, where they comprise the bulk of the virial mass.

A similar sequence of events may occur in a moderately tepid universe ($S \ll 10^5$), with systems of mass $S^2\ M_\odot$ collapsing first in place of stars, if a first-order phase transition takes place in high-entropy matter at a temperature consistent with equation (48). The problem with such schemes is excessive helium production.

Galaxy formation is thus viewed as a series of nonlinear amplification processes originating at the microscopic level. This somewhat involved model for galaxy formation has claim to simplicity in its initial conditions, where no fluctuations in the initial state are required except on the scale of individual particles. Whether or not this assumption is justified will become apparent as accurate measurements of the various moments of anisotropy in the microwave background become available.

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REFERENCES

Arnett, W. D., Bond, J. A., and Carr, B. J. 1982, in preparation.
Audouze, J., and Tinsley, B. M. 1976, *Ann. Rev. Astr. Ap.*, **14**, 43.
Bardeen, J. M. 1980, *Phys. Rev. D*, **22**, 1882.
Barrow, J. 1980, *Phil. Trans. R. Soc. London, A*, **296**, 273.

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Baym, G. 1977, *Neutron Stars and the Properties of Matter at High Density* (Copenhagen: Nordita).
Baym, G., and Pethick, C. 1979, *Ann. Rev. Astr. Ap.*, **17**, 415.
Boughn, S. P., Cheng, E. S., and Wilkinson, D. T. 1981, *Ap. J. (Letters)*, **243**, L113.
Carr, B. J. 1977, *Astr. Ap.*, **60**, 13.
_____. 1981, *M.N.R.A.S.*, **195**, 669.
Carr, B. J., and Rees, M. J. 1977, *Astr. Ap.*, **16**, 705.
Carr, B. J., and Silk, J. 1981, preprint.
Chang, K., and Refsdal, S. 1979, *Nature*, **282**, 561.
Davis, M., Tonry, J., Huchra, J., and Latham, D. W. 1980, *Ap. J. (Letters)*, **238**, L113.
Doroshkevich, A. G., Zel'dovich, Ya. B., and Novikov, I. D. 1967, *Soviet Astr.—AJ*, **11**, 233.
French, H. B. 1980, *Ap. J.*, **240**, 41.
French, H. B., and Grandi, S. A. 1981, *Ap. J.*, **244**, 493.
Gott, J. R. 1981, *Ap. J.*, **243**, 140.
Gunn, J. E. 1977, *Ap. J.*, **218**, 592.
Guth, A. H. 1981, *Phys. Rev. D.*, **23**, 347.
Harrison, E. R. 1970, *Phys. Rev. D.*, **1**, 2726.
Hively, R. M. 1973, Ph.D. thesis, Harvard University.
Hogan, C. J. 1978, *M.N.R.A.S.*, **185**, 889.
_____. 1980*a*, *Nature*, **286**, 360.
_____. 1980*b*, *M.N.R.A.S.*, **192**, 891.
_____. 1982, *Ap. J. (Letters)*, submitted.
Hogan, C. J., and Layzer, D. 1977, *Ap. J.*, **212**, 360.
Hogan, C. J., and Rees, M. J. 1979, *M.N.R.A.S.*, **188**, 791.
Ikeuchi, S. 1981, *Pub. Astr. Soc. Japan*, **33**, 211.
Irvine, J. M. 1978, *Neutron Stars* (Oxford: Clarendon).
Jacubov, V. 1964, *Soviet Astr.—AJ*, **41**, 884.
Kaiser, N. 1981, *M.N.R.A.S.*, in press.
Kaufman, M. 1970, *Ap. J.*, **160**, 459.
Landau, L. D., and Lifshitz, E. M. 1969, *Statistical Physics* (Reading, Mass.: Addison-Wesley)
Layzer, D. 1975, in *Stars and Stellar Systems*, Vol. **9**, *Galaxies and the Universe*, ed. A. Sandage, M. Sandage, and J. Kristian (Chicago: University of Chicago Press), p. 665.
Layzer, D., and Hively, R. M. 1973, *Ap. J.*, **179**, 361.
Lequeux, J., Peimbert, M., Rayo, J. F., Serrano, A., and Torres-Peimbert, S. 1979, *Astr. Ap.*, **80**, 155.
Martin, H. M., Partridge, R. B., and Rood, R. T. 1980, *Ap. J. (Letters)*, **240**, L79.
Mayer, M. G., and Teller, E. 1949, *Phys. Rev.*, **76**, 1226.
Migdal, A. B. 1978, *Rev. Mod. Phys.*, **50**, 107.
Olive, K. A., Schramm, D. N., Steigman, G., Turner, M. S., and Yang, J. 1981, *Ap. J.*, **246**, 557.
Ostriker, J. P., and Cowie, L. L. 1981, *Ap. J. (Letters)*, **243**, L127.
Ostriker, J. P., and Thuan, T. X. 1975, *Ap. J.*, **202**, 353.
Parijskij, Yu. N., Petrov, Z. E., and Cherkov, L. N. 1977, *Soviet Astr.—AJ (Letters)*, **3**, 263.
Partridge, R. B. 1979, in *Physics of the Expanding Universe*, ed. M. Demianski (Berlin: Springer-Verlag), p. 131.
Partridge, R. B. 1980, *Ap. J.*, **235**, 681.
Peebles, P. J. E. 1968, *Nature*, **220**, 227.
Peebles, P. J. E. 1974, *Astr. Ap.*, **32**, 391.
_____. 1980, *The Large-Scale Structure of the Universe* (Princeton: Princeton University Press).
_____. 1981*a*, *Ap. J.*, **248**, 885.
_____. 1981*b*, *Ap. J. (Letters)*, **243**, L119.
Peierls, R. E., Singwi, K. S., and Wroe, D. 1952, *Phys. Rev.*, **87**, 46.
Peimbert, M. 1975, *Ann. Rev. Astr. Ap.*, **13**, 113.
Press, W. H. 1980, *Phys. Scripta*, **21**, 702.
Press, W. H. 1981, preprint.
Press, W. H., and Vishniac, E. T. 1980*a*, *Ap. J.*, **236**, 323.
_____. 1980*b*, *Ap. J.*, **239**, 1.
Puget, J. L., and Heyvaerts, J. 1980, *Astr. Ap.*, **83**, L10.
Rees, M. J. 1975, in *The Structure and Evolution of Galaxies*, ed. G. Setti (Dordrecht: Reidel), p. 285.
_____. 1976, *M.N.R.A.S.*, **176**, 483.
_____. 1978, *Nature*, **275**, 35.
Rees, M. J., and Ostriker, J. P. 1977, *M.N.R.A.S.*, **179**, 541.
Rowan-Robinson, M., Negroponte, J., and Silk, J. 1979, *Nature*, **281**, 635.
Sachs, R. K., and Wolfe, A. M. 1967, *Ap. J.*, **147**, 73.
Silk, J., and Wilson, M. L. 1981, *Ap. J. (Letters)*, **244**, L37.
Truran, J. W., and Cameron, A. G. W. 1971, *Ap. Space Sci.*, **14**, 179.
Weinberg, S. 1972, *Gravitation and Cosmology* (New York: Wiley).
Witten, E. 1981, *Nucl. Phys.*, **B 177**, 477.
Woody, D. P., and Richards, P. L. 1979, *Phys. Rev. Letters*, **42**, 925.
Zel'dovich, Ya. B. 1963, *Soviet Phys.—JETP*, **16**, 1395 (original 1962, *JETP*, **43**, 1982).
_____. 1965, *Adv. Astr. Ap.*, **3**, 241.
_____. 1978, in *The Large-Scale Structure of the Universe*, ed. M. S. Longair and J. Eimasto (Dordrecht: Reidel), p. 409.
_____. 1980, *M.N.R.A.S.*, **192**, 663.
Zel'dovich, Ya. B., and Novikov, I. D. 1978, *Relativistic Astrophysics*, Vol. **1**, ed. K. S. Thorne, and W. D. Arnett (Chicago: University of Chicago Press, Midway Reprints).

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