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EFFECTS OF PROTON DECAY ON THE COSMOLOGICAL FUTURE

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ABSTRACT

We calculate, for an open universe, the densities of radiation and matter at large times if the proton has a lifetime τ_p of about 10^{30} years; we consider the contributions to these densities from the decay of matter in both clumps and interstellar gas. For times $t < \tau_p$, we show that proton decay keeps dead stars at a few kelvins and neutron stars at about 100 K. For $t \gg \tau_p$, the energy density of the universe, for $k < 0$ (where k is the geometric constant in the Robertson-Walker metric), is eventually dominated by the contribution of a very tenuous e^+e^- plasma, much too thin for direct e^+e^- annihilation and perhaps too thin for gravitational collapse.

For a closed (cyclical) universe, current ideas in particle physics imply that the baryon to photon ratio will be identical for each cycle; thus the effect of entropy production will be to enlarge the cosmic scale, from cycle to cycle, by a cycle expansion factor α. We compute α, taking into account entropy production both by stellar nucleosynthesis and by proton decay.

Subject headings: cosmology — elementary particles — nucleosynthesis

I. INTRODUCTION

In recent stimulating articles, Dyson (1979) and Barrow and Tipler (1978) considered the physical processes that would occur in the course of the indefinite future expansion of the universe which is expected on the basis of current best estimates of the cosmic mass density. Two subsequent developments have raised the possibility that their scenario for the death of the universe may have to be modified: (1) There are some theoretical (see, for example, Mohapatra and Senjanovic 1980) and experimental (Reines, Sobel, and Pasierb 1980; Lyubimso *et al.* 1980) indications that neutrinos have mass. If this were the case, and if the mass values lie in the appropriate range, the cosmic neutrino background could contain an energy density sufficient to close the universe, thereby ruling out the assumption of Dyson and of Barrow and Tipler of indefinite expansion. (2) As Dyson notes, there has been growing speculation that the proton may be unstable (Pati and Salam 1973; Georgi and Glashow 1974; Fritzsch and Minkowski 1975; Gursey, Ramond, and Sikivie 1976), with a lifetime on the order of 10^{30} years (Georgi, Quinn, and Weinberg 1974). The speculation is fueled in large measure by the possibility that, within grand unified models of electroweak and strongly interacting particles, a finite proton lifetime can be used to calculate the observed cosmic ratio of baryons to photons, $10^{-9\pm1}$ (Yoshimura 1978, 1979*a*, *b*; Dimopoulous and Susskind 1978, 1979; Toussaint *et al.* 1979; Weinberg 1979; Nanopoulos and Weinberg 1979; Handa and Yoshimura 1978; Kolb and Wolfram 1980, 1981; Barr, Segre, and Weldon 1980; Yildiz and Cox 1980; Harvey *et al.* 1981*a*, *b*). Several experiments to measure the proton lifetime are currently under way.

The purpose of this paper is to reinvestigate the nature of the end of the universe taking into account these modifications. We consider first the case that development (1) does not obtain while development (2) does. That is, we consider the case of indefinite expansion of a universe in which the proton has a lifetime τ_p on the order of 10^{30} years. For this case, several questions arise, such as: What happens to the heat from proton decay? How does the cosmic energy density ρ vary with time? For what periods is ρ matter dominated, and for what periods is it radiation dominated? What is the fate of the final e^+e^- plasma? What is the effect of the eventual decay of possible central galactic supermassive black holes?

We assume below that the proton decays, perhaps through intermediate stages, into positrons and light neutrals (neutrinos and photons). In § II we first estimate how many electrons and positrons remain immediately after nucleon decay and how the energy is distributed among them. We then consider the effects of proton decay on surrounding matter; in particular we calculate the temperatures at which proton decay will maintain white dwarfs and neutron stars. In § III we calculate $\rho(t)$ for two opposite kinds of matter distribution. In § IV we apply the results to the physical universe and describe briefly the essential properties of the final e^+e^- plasma. Section IV*b* contains our outline of the rest of time [under the above two hypotheses for $\rho(t)$] for the (open) physical universe.

We turn next, briefly, to the possibility that both modifications (1) and (2) obtain; that is, we consider the case that neutrinos have sufficient mass to close the universe (Cowsik and McClelland 1972; Lee and Weinberg 1977; Dicus, Kolb, and Teplitz 1977) and, at the same time, the proton is unstable. We assume that the universe, upon contraction, passes through (or near to) the point at $R \sim T^{-1} \rightarrow 0$, where R is the cosmic scale factor, in a manner consistent with the second law of thermodynamics: that the total entropy of the universe does not decrease upon passage through the minimum radius. With these assumptions, we compute in § V the entropy generated by stellar burning from hydrogen to iron and by proton decay, the expansion of the scale factor, and the increase in the cycle time for successive cycles in a closed universe with unstable protons. Details of this computation are reserved for a separate publication. Finally in § VI we summarize our results.

Several other authors have considered the physics of the closed late universe, although it has perhaps not been as active an area of speculation as one might expect. Important contributions of which we are aware include the following: Tolman (1934) and others (Rees 1969; Novikov and Zel'dovich 1973; Davies 1974) pointed out that in a closed cyclical universe the maximum radius would increase from cycle to cycle. Landsberg and Park (1975) calculated the cycle expansion factor in a model universe with dust and radiation in equilibrium. Dicke and Peebles (1979) have estimated the cycle expansion factor in a manner similar to that of § V below.

The physics of open universes has been studied by a number of authors (Davies 1973; Islam 1977; Barrow and Tipler 1978; Dyson 1979). Of particular interest is the theorem of Collins and Hawking (1973) that the set of parameters for which the universe does not have a growing anisotropy is of measure zero in the six dimensional parameter space that determines the initial data for all homogeneous models. From this result they conclude that the most attractive answer to the isotropy question is the Dicke-Carter conjecture (Dicke 1961; Carter 1974) that the (at least approximate) isotropy of our universe is a result of the fact that out of an ensemble of universes only those which are isotropic and long-lived can develop galaxies and hence life. Thus they conclude that our universe is isotropic because we are here to observe it.

Collins and Hawking show that models with initial data in the five dimensional $k = 0$ subspace tend toward isotropy. Barrow and Tipler (1978), however, have studied $k = 0$ models in detail. They show that radiation dominated $k = 0$ models do not satisfy the hypotheses of Collins and Hawking and, in fact, also enjoy growing anisotropy. Barrow and Tipler point out that a $k = 0$ model becomes radiation dominated as black holes decay and as protons effectively decay through quantum tunneling into micro black holes. They consider a proton lifetime on the order of the 10^{45} years calculated by Zel'dovich (1976, 1977). In the present work we consider the physics of $k = -1$, isotropic models, leaving the modifications caused by anisotropy, if any, to future work.

We consider a proton lifetime of $\tau_p \sim 10^{30}$ years. For $t < \tau_p$, we calculate the temperatures at which dead stars are maintained by proton decay. For $t > \tau_p$ we calculate the properties of the final e^+e^- plasma. For $k = 0$, as Barrow and Tipler show, the e^+e^- plasma decays to radiation; but, for $k = -1$, as shown below, the e^+e^- plasma is stable. Thus the final dispositions of $k = -1$ and $k = 0$ models are significantly different.

After the completion of this work we received a Pennsylvania State University preprint by Page and McKee (1981) on the late evolution of $k = 0$ Friedmann universes. They discuss baryon decay and black hole evaporation in this context. They calculate the luminosity of massive bodies which absorb their baryon decay fragments. The questions of electron-positron annihilation and gravitational clumping, which are simply answered in our ($k = -1$) model, are explored in detail with careful treatment of positronium formation, decay, and annihilation. They verify the proposition stated by Barrow and Tipler that, for $k = 0$, the final e^+e^- plasma annihilates. Where there is overlap between our papers there does not appear to be any serious disagreement; in particular they also conclude that neutron stars would be maintained at a temperature of approximately 100 K (cf. Feinberg 1981).

II. REMNANTS OF PROTON DECAY

We consider first the proton's decay products and then calculate the temperature at which proton decay will maintain cooling matter.

a) *Decay Products*

The final result of the decay of a proton is one positron, possibly one or more electron-positron pairs, and radiation in the form of neutrinos and photons. The numbers of these final particles and the energy distribution among them depends on the intermediate stages of the decay. For example, in the simplest version of SU(5) the branching ratios are (Machacek 1979) 83% $p \rightarrow e^+X^0$, 13% $p \rightarrow \bar{\nu}_e X^+$, 4% other, where X^0 is $\pi^0, \rho^0, \omega, \eta$, and X^+ is π^+, ρ^+. If SU(6) is used for the flavor-spin structure of the final hadrons, then the exclusive branching ratios are 33% $\pi^0 e^+$, 17% $\rho^0 e^+$, 12% ηe^+, 25% ωe^+, 9% $\pi^+ \bar{\nu}_e$, 4% $\rho^+ \bar{\nu}_e$, where we have rounded off and ignored minor decay modes. This is only one of many possibilities. It could be that the leptons are strongly mixed so that the dominant decay mode is $p \rightarrow \mu^+ X^0$ or $p \rightarrow \bar{\nu}_\tau X^+$. The positron eventually produced would then be much less energetic than those produced from $p \rightarrow e^+X^0$.

Some of the baryons in the universe are neutrons, bound in ^{4}He. If a proton or neutron in this nucleus decays, the nucleus will be broken up, leaving the remaining neutrons to β-decay to a low energy electron and a proton which will eventually decay. One-fourth of the neutrons will decay directly, however, so branching ratios for $n \rightarrow e^+X^-$, $n \rightarrow \bar{\nu}_e X^0$, are also needed.

Our results will not be sensitive to the precise branching ratios of the nucleons. Thus we will use the numbers given above; if they are drastically in error, our qualitative results will not be wrong. Assuming 16% of the nucleons in the universe are neutrons, and the standard branching ratios for the decays of η, ω, ρ^+, ρ^0, π^+, π^0, then, on the average, each nucleon decay gives

0.54 electrons with 86 MeV ,

1.38 positrons with 240 MeV ,

radiation with 562 MeV .

This, combined with the 0.84 electrons per nucleon that exist before decay, gives charge neutrality. The radiation is roughly half neutrinos and half photons; 30% of the proton's energy goes into each, with 35% going into positrons.

It should be noted that positrons from the nuclear decay do not annihilate with the atomic electrons since the annihilation cross section is on the order of 10^{-24} cm^2 while the atomic electrons are spread over an area $\pi a_0{}^2 \approx 10^{-16}$ cm^2.

b) Effects of Proton Decay on Matter Aggregates

If protons decay inside large bodies, the decay energy will tend to heat the body to an equilibrium temperature given by

$$\frac{dE}{dt} = \eta \frac{Mc^2}{\tau_p} = 4\pi R^2 \sigma_{\rm SB} T^4 , \qquad (2.1)$$

where η is the fraction of the energy of the decay products that is absorbed. From the discussion of § II*a*, η is about $\frac{2}{3}$ since, even for large bodies, most of the neutrinos escape. M is the mass of the body, R is the radius, and $\sigma_{\rm SB}$ the Stefan-Boltzmann constant.

The mean free path of the decay positrons which carry the largest single contribution to the total energy is given by

$$\lambda = [n\sigma(e^+e^- \to 2\gamma)]^{-1} , \qquad (2.2)$$

where n is the electron number density and the e^+e^- annihilation cross section is given by

$$\sigma = \tfrac{3}{8}\sigma_{\rm T} \frac{\ln (2\gamma_{e^+})}{\gamma_{e^+}} , \qquad (2.3)$$

where $\sigma_{\rm T} = (8\pi/3)\alpha^2/m_e{}^2$ and the Lorentz factor γ_{e^+} is in the range 100–1000. The electrons and photons from the decay have mean free paths λ' of the same order as that of the positrons.

For a 1 $M_\odot$ white dwarf with radius on the order of 4×10^3 km, λ is about 2×10^{-5} cm. Thus only 10^{-13} of the positrons escape without annihilation so that the result of proton decay is to release photons which heat the white dwarf. White dwarfs, in the absence of proton decay, cool off by radiation according to the relation

$$\frac{dE}{dt} = 4\pi R^2 \sigma_{\rm SB} T^4 . \qquad (2.4)$$

For the white dwarf with the parameters above we have $E \sim 10^{57} kT$ so that (2.4) integrates to

$$\frac{1}{T^3} - \frac{1}{T_0{}^3} = 12\pi R^2 \frac{\sigma_{\rm SB}}{k} 10^{-57} t \sim 10^{-27} t \qquad (2.5)$$

with T in kelvins and t in seconds. Thus, in the absence of proton decay, white dwarfs cool to 1 K in about 10^{20} years. If the white dwarf is heated by proton decay, we have in (2.1)

$$\frac{dE}{dt} = \left(\frac{1 \text{ GeV}}{p}\right) \frac{(10^{57} p\text{'s})}{\tau_p} = 6 \times 10^{16} \text{ ergs s}^{-1} , \quad (2.6)$$

where τ_p is the lifetime of the proton, $\sim 10^{37}$ s. We then see from (2.1) and (2.5) that proton decay should keep white dwarfs at about 5 K during the time 10^{17} years $< t < 10^{30}$ years.

Other forms of matter will have temperatures that can be similarly computed. For example, for the Earth, we have $M \approx 6 \times 10^{27}$ g, $R \approx 6 \times 10^8$ cm, so that (2.1) gives $T \sim 0.16$ K. The densest known objects are neutron stars; their radii are on the order of $\frac{1}{400}$ as large as white dwarf radii, but they have similar masses. From (2.1) we see that, for fixed M, T scales as $R^{-1/2}$. Thus we have

$$T_{\rm NS} \approx 20 T_{\rm WD} \approx 100 \text{ K} . \qquad (2.7)$$

They would cool to this temperature, in the absence of proton decay, in about 10^{19} years.

Other weak phenomena that would violate baryon number conservation, such as n-$\bar{n}$ oscillations or $n + n \to \pi$'s or $p + p \to \pi$'s (Mohapatra and Marshak 1980*a*, *b*; Cowsik and Nussinov 1981), if constrained to lifetimes on the order of 10^{30} years, will heat matter aggregates to about the same temperature as proton decay.

III. OPEN UNIVERSE CALCULATIONS

In this section we first review the kinematics of expansion with k close to -1, then address two extreme models. These models are: (1) all matter is found in clumps; (2) all matter is found in gas. In § IV we apply the calculations to a model of the real universe.

a) Expansion of the Universe

Einstein's equations reduce in the standard model (Weinberg 1972) to

$$\dot{R}^2 + k = \frac{8\pi G}{3} \rho R^2 , \qquad (3.1)$$

where R is the scale length in the Robertson-Walker metric. The present value of the Hubble constant is

$$H_0 = \dot{R}_0 / R_0 . \qquad (3.2)$$

The critical value of the present energy density ρ such that k in (3.1) is zero is

$$\rho_c = \frac{3H_0{}^2}{8\pi G} . \qquad (3.3)$$

Let $\rho_0 = \Omega\rho_c$. The present best estimate (Schramm and Steigman 1981) for Ω is $\Omega \approx 0.06$, provided there is not a lot of hidden matter in the universe, such as a cosmic background of *massive* neutrinos, as referred to above. We assume, for this section, that Ω is sufficiently small that the right-hand side of (3.1) can be neglected, relative to $k = (\Omega - 1)H_0{}^2 R_0{}^2$, today and in the future. R is given by (taking the present time t_0 as zero)

$$R = \dot{R}t + R_0 , \tag{3.4}$$

where, neglecting Ω,

$$\dot{R} = H_0 R_0 \tag{3.5}$$

is constant.

b) *Clumpy Universe*

We consider first the case that most of the matter in the universe is, by $t \sim 10^{30}$ years, in the form of clumps, e.g., principally dead stars, and, to a lesser extent, planets, and rocks. The e^+ from proton decay goes, to 1 part in 10^{13}, into heating matter, i.e., to a good approximation the result of proton decay is strictly thermal radiation from matter clumps and neutrinos. We now write down, and solve, the equations for the cosmic matter and radiation densities for this case. Note that the equations are independent of the wavelengths of the emitted radiation. We have, for baryons,

$$\frac{d\rho_B}{dt} = -\frac{\rho_B}{\tau_p} - 3\rho_B \frac{\dot{R}}{R} . \tag{3.6}$$

The solution is

$$\rho_B = (\rho_B)_0 \exp(-t/\tau_p)/R^3 , \tag{3.7}$$

in units where today's radius is 1. The energy density of electrons is $\rho_e = (m_e/m_B)\rho_B$ so that the total matter density ρ_M is roughly just ρ_B in (3.7).

The radiation energy density is given by

$$\frac{d\rho_R}{dt} = \frac{\rho_M}{\tau_p} - 4\rho_R \frac{\dot{R}}{R} , \tag{3.8}$$

whose solution is

$$R^4\rho_R = (\rho_R)_0 + (\rho_M)_0 \dot{R}\tau \times [1 - (1 + t/\tau_p) \exp(-t/\tau_p)] . \tag{3.9}$$

c. Diffuse Universe

Let us now ignore matter in clumps and ask about the final disposition of atomic matter. Consider gaseous matter with density $\rho_M = \rho_e + \rho_B$. If this matter is uniformly spread throughout the universe, a positron from proton decay will not annihilate, as one can see from the following: The rate for annihilation is given by

$$\frac{dN_+}{dt} = -\langle v\sigma \rangle \frac{N_+ N_-}{V} \tag{3.10}$$

where V is the volume and σ is the annihilation cross section. Taking the number of positrons N_+ equal to the number of electrons, N_-, using (3.5) for dt, and taking

$$\langle v\sigma \rangle = \frac{\alpha^2 \pi}{\beta m_e^2} (\beta c) = \tfrac{3}{8} c\sigma_T ,$$

the ratio of the number of free positrons left to the total number produced is

$$X_e = \frac{1}{1 + \xi} ,$$

$$\xi = \frac{\langle v\sigma \rangle n_s R_s^3}{2R_0 H_0} \left(\frac{1}{R_s^2} - \frac{1}{R^2} \right) , \tag{3.11}$$

where R_s is the radius and n_s is the number density at 10^{30} years. With the present number density of 10^{-5} Ω, we find, at $R = \infty$,

$$\xi \sim 10^{-42}\Omega . \tag{3.12}$$

That is, none of the positrons annihilate.

It is instructive to compare this result with that of Barrow and Tipler who show that, in a universe with k equal to zero, the positrons and electrons will annihilate. Page and McKee find that, for $k = 0$, the dominant process is $e^+ + e^- + e \to P_n + e$, where P_n is positronium with principal quantum number n; they show that (3.10) for this process becomes

$$\frac{dN_n}{dt} \approx \frac{(N_\pm)^3}{V^2} \frac{n^4}{T^2 m^3} + \cdots , \quad n < n_{\rm Max} , \tag{3.10$'$}$$

where the omitted correction terms take into account ionization and $n \leftrightarrow n'$. N_n is the number of positronium systems with principal quantum number n. In (3.10′) for $k = 0$ we have $T/T_s \sim (R_s/R)^2$, $n_{\rm Max} \sim (R/R_s)^{1/2}$, and with matter domination by the e^+e^-, $t/t_s \sim (R/R_s)^{3/2}$. Thus we have

$$-\int \frac{dN_\pm}{N_\pm^3} = \sum_{n=1}^{n_{\rm Max}} \int \frac{dN_n}{N_\pm^3} \sim \int^\infty R^{-6+5/2+4+1/2} dR .$$

The divergence of the integral on the right-hand side "proves" that $N \to 0$ for the $k = 0$ case. This argument does not, of course, hold for the $k = -1$ case because the estimates of T, $n_{\rm Max}$, and t in terms of R are much different.

Returning to the $k = -1$ case, the positrons (and electrons), when first produced, will have energies of several hundred MeV so that they will appear as high energy radiation until the universe has expanded by a factor of the decay energy divided by the electron mass; at this time they will appear as low energy matter. Let α be the electron mass divided by the average decay energy per electron and β be the same ratio for positrons. Also let f_α be the average number of electrons produced per nucleon decay and f_β be the same quantity for positrons. Then from § II*a* we have

$$\alpha = \tfrac{1}{172} , \quad f_\alpha = 0.54 ;$$

$$\beta = \tfrac{1}{480} , \quad f_\beta = 1.30 . \tag{3.13}$$

The equations governing the behavior of ρ_M and ρ_R in this scenario are:

$$\frac{d\rho_B}{dt} = -\frac{\rho_B}{\tau_p} - \frac{3\rho_B \dot{R}}{R} , \tag{3.14a}$$

$$\frac{d\rho_{e^-}}{dt} = -\frac{3\rho_{e^-}\dot{R}}{R} + \frac{m_e}{m_B} f_\alpha \rho_B(t_\alpha) \frac{\alpha^4}{\tau_p} \theta(t_\alpha) , \tag{3.14b}$$

$$\frac{d\rho_{e^+}}{dt} = -\frac{3\rho_{e^+}\dot{R}}{R} + \frac{m_e}{m_B} f_\beta \rho_B(t_\beta)\frac{\beta^4}{\tau_p}\theta(t_\beta)\,, \quad (3.14c)$$

$$\frac{d\rho_R}{dt} = \frac{\rho_B}{\tau_p} - \frac{4\rho_R \dot{R}}{R} - \frac{m_e}{m_B}\frac{1}{\tau_p} \times [f_\alpha \rho_B(t_\alpha)\alpha^4\theta(t_\alpha) + f_\beta \rho_B(t_\beta)\beta^4\theta(t_\beta)]\,. \quad (3.14d)$$

In (3.14), $\rho_B(t_\alpha)$ is the baryon density evaluated at

$$t_\alpha = \alpha t - (1-\alpha)/\dot{R}\,; \quad (3.15)$$

t_α is the time at which the universe as $1/\alpha$ times smaller than it will be at time t; in (3.14) and (3.15), as above, lengths are measured in units of the present scale length $R(t_0)$, and the present time, t_0, is taken to be zero.

The term

$$\frac{m_e}{m_B} f_\alpha \rho_B(t_\alpha)\frac{\alpha^4}{\tau}\theta(t_\alpha)$$

needs explanation. Electrons (and positrons) from proton decay can initially be considered radiation, but if we say their energy averages m_e/α, they will become matter when the universe is $1/\alpha$ times expanded. Thus the rate of electron formation is equal to the rate of proton decay when the universe was $1/\alpha$ smaller; reduced by $1/\alpha$ because the expansion is slower by this factor at the later time and by an additional $1/\alpha^3$ because the density is smaller by this much at the later time. Since the increase in nonrelativistic electrons in ρ_{e^-} must be matched by a decrease in relativistic electrons in ρ_R, the term must be added to ρ_{e^-} and subtracted from ρ_R.

If $\rho_B(0)$ is the baryon density today, and we take $\rho_{e^+}(0) \sim 0$, $\rho_R(0) \sim 0$, and

$$\rho_{e^-}(0) = (m_e/m_B)\rho_B(0)(f_\beta - f_\alpha)\,,$$

then the solutions to (3.14) are

$$\frac{R^3\rho_B}{\rho_B(0)} = e^{-t/\tau} \quad (3.16a)$$

$$\frac{R^3\rho_{e^-}}{\rho_B(0)} = \frac{m_e}{m_B}\left\{f_\beta - f_\alpha + f_\alpha\left[1 - \exp\left(-\frac{\alpha t}{\tau_p}\right)\right]\right\}, \quad (3.16b)$$

$$\frac{R^3\rho_{e^+}}{\rho_B(0)} = \frac{m_e}{m_B} f_\beta\left[1 - \exp\left(-\frac{\beta t}{\tau_p}\right)\right], \quad (3.16c)$$

$$\frac{R^3\rho_R}{\rho_B(0)} = \frac{\tau_p}{t}\left\{1 - \left(1 + \frac{t}{\tau_p}\right)e^{-t/\tau_p} - \frac{m_e}{m_B}\frac{f_\alpha}{\alpha}\left[1 - \left(1 + \frac{\alpha t}{\tau_p}\right)e^{-\alpha t/\tau_p}\right] - \frac{m_e}{m_B}\frac{f_\beta}{\beta}\left[1 - \left(1 + \frac{\beta t}{\tau_p}\right)e^{-\beta t/\tau_p}\right]\right\}. \quad (3.16d)$$

IV. OPEN UNIVERSE RESULTS

a) Results in the Two Extreme Models

In Figure 1 we give the results for ρ_R and for $\rho_M = \rho_B + \rho_{e^+} + \rho_{e^-}$ for the two cases discussed above, the clumpy and diffuse universes. The graphs show plots

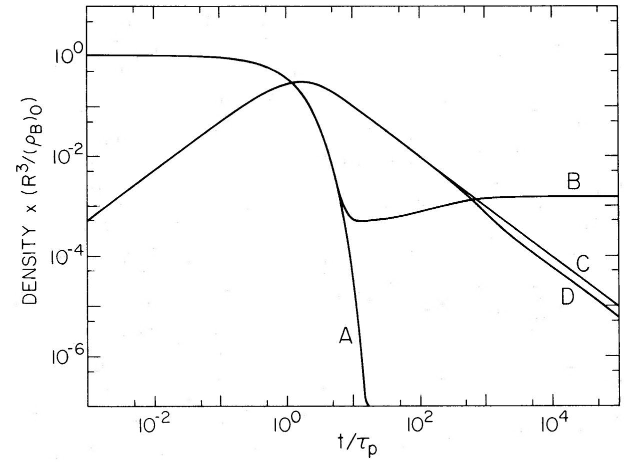

FIG. 1.—Radiation and matter densities in the clumpy and diffuse universes. Curve A is the matter density in the clumpy universe. Curve B is the matter density in the diffuse universe. These curves coincide before proton decay. Curves C and D are the radiation densities in the clumpy and diffuse universes respectively. There is eventually less radiation in the diffuse universe because the positrons do not annihilate the electrons; they become matter when they become nonrelativistic, starting at $t \approx 100\tau_p$. A transition from matter to radiation domination is indicated at $t \approx \tau_p$, and a transition from radiation to matter domination (occurring only in the diffuse universe) at $t \approx 500\tau_p$.

of curves for $R^3\rho_M/(\rho_M)_0$ and $R^3\rho_R/(\rho_M)_0$ for $10^{-2} \leq t/\tau_p \leq 10^4$. Curves for both the diffuse and clumpy universes begin the same. The upper curve is ρ_M. All matter decays away exponentially in the clumpy universe. In the diffuse universe, electrons remain as part of the matter density. Decay positrons, as they become nonrelativistic, gradually add to the matter density. The curve that begins lower is ρ_R. The two sets of curves (clumpy and diffuse) are identical until $t \sim 10^3\tau_p = 10^{33}$ years. At this time, in the diffuse universe, positrons drop out of the radiation density so that ρ_R takes the lower fork and the atomic universe becomes matter dominated.

b) The Physical Universe

Dyson shows that in times (on the order of 10^{20} years) short compared to the lifetime, $\tau_p \sim 10^{30}$ years, of the proton, galaxies largely dissolve by means of close collisions between constituents. By the virial theorem, constituents have about 70% of escape velocity; a close collision can provide the rest. Barrow and Tipler point out that as galaxies evaporate at their surfaces, their centers must condense. Dyson estimates, taking into account this effect, that 90–99% of a galaxy probably evaporates.

Thus, as a rough guess, the universe at 10^{30} years can be taken as a smooth background, composed mostly of dead stars, with 1% in the form of atomic hydrogen or helium. There may, in addition, be large black holes left over from galactic centers. A very rough estimate of the distribution of mass among these constituents at 10^{30} years may be: (1) dead stars (maintained between a few kelvins and 100 K by the heat from proton decay—see § II*b*), 90%; (2) central galactic supermassive black holes, 9%; (3) atomic matter (mostly hydrogen), 1%.

If we assume this is the composition of the universe, homogeneously distributed at 10^{30} years, we can deduce some features of the subsequent evolution of the universe: (1) The decay of protons in dead stars will produce a nonthermal cosmic background of photons and one of neutrinos with higher energies. (2) At $t \sim \tau_p$ to $10\tau_p$ this background will dominate the mass density of the universe. (3) For $t > 10\tau_p$, the redshift of radiation energies will make the energy density of the central-galactic supermassive black holes dominant. (4) For $t > 10^3\tau_p$, the positrons from proton decay form, with the spectator electrons from the original interstellar neutral H, a low density nonrelativistic e^+e^- plasma. (5) The lifetime of a spherical, nonrotating black hole radiating as a blackbody from its surface is about $10^{66}(M/M_\odot)^3$ years. Thus the supermassive black holes of $10^{10}\ M_\odot$ decay away in about 10^{96} years. The temperature available to make decay products is about $5 \times 10^{-6}(M_\odot/M)$ MeV, so most of the energy goes into radiation. (6) The radiation from the black holes is redshifted so that for $t > 10^{100}$ years, the tenuous e^+e^- plasma is the dominant feature of the universe.

c) *Properties of the e^+e^- Plasma*

1. Its antecedent neutral hydrogen is not captured by the supermassive black holes: If N is the number of hydrogen atoms in the galaxy, then we have

$$\frac{dN}{dt} = -\frac{v\sigma_{\text{cap}}}{V_{\text{gal}}} . \tag{4.1}$$

We take $\sigma_{\text{cap}} = \pi a^2$, where a is the black hole Schwarzschild radius, $2GM/c^2$, the relative velocity v is 100 km s^{-1}, and we take the volume V to be that of our Galaxy, about 10^{67} cm^3. The Schwarzschild radius a, for $M = 10^{10}\ M_\odot$, is about 10^{15} cm. Thus we have

$$\frac{dN}{dt} = 10^{-30}\ \text{s}^{-1} . \tag{4.2}$$

If the gas were confined in the galaxy for the lifetime of the proton, it would be captured. Dyson, however, says that gravitationally bound systems dissolve in about 10^{19} years by means of close collisions which provide sufficient escape veolcity. In this case, the gas is distributed throughout the universe.

2. The e^+e^- plasma does not annihilate: We showed, in (3.11), that there is, by a large margin, no annihilation in our $k = -1$ universe. The density of the plasma at a time t is

$$n(t) \approx 10^{-60}\frac{R^3(\tau_p)}{R^3(t)}n_0 , \tag{4.3}$$

where $n_0 \sim 10^{-5}\Omega$ cm^{-3} is today's number density.

3. The e^+e^- plasma is too tenuous to thermalize photons: The photon mean free path is

$$\lambda \sim \frac{1}{n\sigma} \sim 10^{90}\frac{R^3(t)}{R^3(\tau_p)}\ \text{cm} , \tag{4.4}$$

where we have taken $\sigma \sim 10^{-25}$ cm^2. Since λ is much larger than 10^{48} cm, the age of the universe at $t \sim \tau_p \sim 10^{30}$ years times the velocity of light, essentially no photons ever scatter off the electrons and positrons that make up the plasma.

4. the e^+e^- plasma frequency is too small to affect anything we can think of:

$$\omega_p{}^2 = 4\pi n\left(\frac{e^2}{\hbar c}\right)\frac{\hbar c}{m} \sim 10^{-56}\frac{R^3(\tau_p)}{R^3(t)}\ \text{s}^{-2} . \tag{4.5}$$

Thus the plasma frequency is less than 10^{-28} Hz. In comparison, the lowest frequency electromagnetic waves should be the present 3 K background, redshifted to $t = \tau_p$; their frequency would be

$$\omega_{3\,\text{K}} \sim 10^{11}\ \text{Hz}\left[\frac{\tau_p(\text{yr})}{10^{10}\ \text{yr}}\right]\frac{R(\tau_p)}{R(t)} \sim 10^{-9}\frac{R(\tau_p)}{R(t)}\ \text{Hz} \gg \omega_p . \tag{4.6}$$

Thus, even the present 3 K microwave background is not affected by the e^+e^- plasma.

On the other hand, the plasma frequency $10^{-28}[R(\tau_p)/R(t)]^{3/2}$ Hz is larger than the expansion rate of the universe, $10^{-37}R(\tau_p)/R(t)\,\text{s}^{-1}$ for $t/\tau_p < 10^{18}$. Thus, between about 10^{30} yr and 10^{48} yr, linear plasma oscillations are possible; after 10^{48} yr the universe is expanding too fast for linear plasma oscillations to exist.

5. The e^+e^- plasma does not radiate: We estimate roughly the radiation rate as follows: A beam of charged particles of mass m (and charge e) incident on a charged particle of mass m (and charge $-e$) radiates with a total effective radiation on the order of

$$\langle E\sigma\rangle \sim \alpha^2\frac{(\hbar c)^2}{mc^2} \sim 4\ \text{MeV}\ f^2 . \tag{4.7}$$

The rate of energy loss for one particle, through collision, is then

$$\frac{dE}{dt} \sim vn\langle E\sigma\rangle \sim 10^{-81}\left[\frac{R(\tau_p)}{R}\right]^4\ \text{MeV s}^{-1} . \tag{4.8}$$

This should be compared with the rate of energy loss through expansion of the universe:

$$\frac{dE}{dt} \sim \frac{\frac{1}{2}mv^2}{t} \sim 10^{-38}\left[\frac{R(\tau_p)}{R}\right]^3\ \text{MeV} . \tag{4.9}$$

The rate of energy loss of the plasma into radiation is negligible and falling.

6. The e^+e^- gas may not collapse gravitationally: The Jeans length is the size of a volume of plasma which would have a gravitational attraction too large for particles at the gas temperature to escape. It is given by (Weinberg 1972)

$$L_J \sim \frac{v_s}{(4\pi G m_e n)^{1/2}} , \tag{4.10}$$

where n is the number density of electrons and v_s is the speed of sound which we can approximate by (for $t > \tau_p$)

$$v_s \sim c\frac{R(\tau_p)}{R(t)} . \tag{4.11}$$

L_J is then approximately

$$L_J \sim 10^{59}\left[\frac{R(t)}{R(\tau_p)}\right]^{1/2} \text{ cm} . \quad (4.12)$$

This is at first larger than the distance to the event horizon, L_c:

$$L_c \sim 10^{48}\frac{R(t)}{R(\tau_p)} \text{ cm} , \quad (4.13)$$

so that gravitational collapse is not possible. However, at $t = 10^{22}\tau_p$, L_J becomes smaller than the distance to the event horizon, so that gravitational collapse becomes possible, in principle, at that time. Only a small amount of collapse is possible by "violent relaxation"; some mechanism for losing kinetic energy is necessary. Electromagnetic radiation is a possibility, but the rate (4.8) appears to be too much slower than the rate (4.9) to be effective in radiating away enough energy to allow gravitational collapse. However, even without collapse, if gravitationally bound clumps form of a size on the order of the Jeans length, positronium formation, as discussed by Page and McKee (1981) for the $k = 0$ universe, might proceed much faster than evaporation from the clumps and could lead to eventual annihilation.

V. CLOSED CYCLICAL UNIVERSE

We treat some of the physics of a closed cyclical universe in a separate paper (Dicus *et al.* 1981). Here, for completeness, we review briefly the main ideas of that treatment. We note at the outset that we have nothing to contribute to the question of whether and/or how the universe bounces.

a) *Assumptions*

Our assumptions are:

1. The universe is homogeneous and isotropic, and Einstein's equations reduce to

$$\dot{R}^2 + 1 = \frac{8\pi}{3}G\rho R^2 . \quad (5.1)$$

In Dicus *et al.* (1981) we note some of the modifications to the results below that could result from significant anisotropy. We assume that 1 is much less than the right-hand side of (5.1) for $t < t_1$, where t_1 is the time at which the energy density of matter is equal to that of radiation.

2. We assume that the universe reaches a sufficiently high temperature in each cycle that n_p/n_γ, the baryon to photon ratio ($\sim 10^{-9}$), is generated by $\Delta CP \neq 0, \Delta B \neq 0$, nonequilibrium reactions (see references quoted in the Introduction) and its value is determined by elementary particle parameters. Thus n_p/n_γ has the same value for every cycle, and galaxy and star formation are similar in each cycle of period τ_c greater than, say, 10^9 years.

3. We take $H_0 = \dot{R}_0/R_0 = 10^{-10}h_0$ yr^{-1} with $0.4 \leq h_0 \leq 1.0$ (Schramm and Steigman 1981). The subscript zero refers to a present day value. We close the universe by taking

$$\rho_\nu = \rho_c = 2 \times 10^{-29}h_0{}^2 \text{ g cm}^{-3} . \quad (5.2)$$

The motivation for this stems from recent theoretical and experimental indications of nonzero mass for neutrinos. We take $\Omega_N = 0.1$ in $\rho_N = \Omega_N \rho_c$. $\Omega = \Omega_N + \Omega_\nu = 1.1$ implies for the present cycle $\tau_c \approx 10^{12}$ years. Our numerical results for the cycle expansion factor are not too sensitive to, and can be easily modified to other values of, Ω_N and Ω_ν.

4. We assume that radiation is generated by hydrogen burning in stars and by proton decay. We have for the entropy density $S(t)$,

$$S(t) = \rho_\gamma(t)/T_{\text{eq}} , \quad (5.3)$$

where T_{eq} is the temperature the photon sea would have if all photons were thermalized. Note that *photons generated when the universe is near its maximum radius are the most important source of entropy*. This follows from the fact that when a photon is eventually (during contraction) thermalized, it contributes an amount of entropy equal to $h\nu/kT_p$, where T_p is the blackbody temperature at the time it was produced.

5. Consistent with assumption (4) above, we assume no entropy is produced (or destroyed) during the time the universe is radiation dominated at the end of one cycle and the beginning of the next. In Dicus *et al.* (1981) we note other possible mechanisms for generating entropy such as by neutrino or pair production viscosity from anisotropies (Misner 1968, 1969*a b*; Matzner 1969, 1971*a*, *b*, 1972; Matzner and Misner 1972; Zel'dovich 1970; Stewart 1969; Collins and Stewart 1971; Barrow and Matzner 1977; Hartle and Hu 1980; Hu and Parker 1978) or by supercooling at phase transitions (Guth and Tye 1980; Guth 1981). Our result for the cycle expansion factor should therefore be considered a lower bound.

b) *Calculations*

We calculate here in an abbreviated form, reserving details for the other paper. Suppose that, starting at $t = t_1$, when $\rho_M = \rho_R = \rho(t_1)/2 = \rho_1/2$ and $R = R_1$, the universe expands to R_{Max} and then contracts. As it expands and contracts, we have that $\rho_M \sim R^{-3}$ and $\rho_R \sim R^{-4}$. If no stars burn, then by Einstein's equation,

$$\dot{R}^2 + 1 = \frac{8\pi G}{3}\rho R^2 ,$$

the universe will get back to density ρ_1 when the scale length R gets back to R_1. Note that at $R = R_{\text{Max}}$ we have $\rho_R(R_{\text{Max}}) = (R_1/R_{\text{Max}})\rho_M(R_{\text{Max}})$. Now let us see what happens if stars burn. Let us make the simplifying approximation that the stars all burn exactly at $R = R_{\text{Max}}$. The density ρ_R is then augmented by the addition of

$$\rho_{\text{St}} = 10^{-3}\rho_M(R_{\text{Max}}) = 10^{-3}(R_1/R_{\text{Max}})\rho_1 . \quad (5.4)$$

The energy in radiation from stellar burning, ρ_{St}, is negligible (10^{-3}) compared to that in ρ_M, but it is much larger than ρ_R, as long as $R_1/R_{\text{Max}} \ll 10^{-3}$ which is certainly true for our cycle.

Now, as the universe contracts, we will have $\rho_{\text{St}}(R) = \rho_1$ at a value of R such that

$$10^{-3}(R_1/R_{\text{Max}})^3\rho_1(R_{\text{Max}}/R)^4 = \rho_1 , \quad (5.5)$$

i.e., at

$$R = R_2 = R_1(10^{-3}R_{\rm Max}/R_1)^{1/4} . \qquad (5.6)$$

Thus the universe will be (at $R = R_2$) bigger than it was in the expansion phase; it has the same density ρ_1 that it had at $R = R_1$ but it now has a radius $R = R_2 > R_1$. (By our assumption [5] it will have $R = R_2$ when $\rho = \rho_1$ on the expansion phase of the next cycle.)

c) Results

We call

$$\alpha = R_2/R_1 \approx (10^{-3}R_{\rm Max}/R_1)^{1/4} \qquad (5.7)$$

the cycle expansion factor. In a better approximation we obtain

$$\alpha^4 = 1 + \frac{10^{-3}}{\tau_{\rm H} R_1}\int_{t_1}^{t_2} dt' R(t')\theta(\tau_{\rm H} - t') \approx 170 h_0^{8/3} \approx 25 , \qquad (5.8)$$

where $\tau_{\rm H}$ is the time over which stars convert protons to iron ($\tau_{\rm H} \approx 10^{11}$ yr $\approx \tau_c/10$ for the present cycle). One can show that in the next cycle the Hubble parameter takes on the same value as in this cycle, but the deceleration parameter q approaches the critical value (where $k = 0$) as

$$q_2 - q_{2c} \approx \frac{1}{\alpha^2}(q_1 - q_{1c}) \qquad (5.9)$$

and hence the cycle time grows from cycle to cycle as

$$\tau_c(2) = \alpha^3 \tau_c(1) \approx 50 h_0^2 \tau_c(1) . \qquad (5.10)$$

The above results obtain for 10^{20} yr $> \tau_c > \tau_{\rm H}$. For $\tau_p > \tau_c > 10^{20}$ yr, however, proton decay becomes an important source of entropy and we find

$$\alpha^4 \approx 10^{17}(\tau_c/\tau_p)^{5/3} . \qquad (5.11)$$

For $\tau_c > \tau_p$, α grows to values on the order of 10^3–10^4.

Using (5.8) one can write a recursion relation for α

$$\alpha_{n+m}^4 = 1 + \alpha_{n+m-1}^5 \alpha_{n+m-2}^5 \cdots \alpha_n^5(\alpha_n^4 - 1) . \qquad (5.12)$$

Thus, looking back in time, each cycle generated less entropy, had a smaller cycle time, and had a smaller cycle expansion factor then the cycle that followed it. The growth in the cycle expansion factor is at first slow; for example, 40 cycles are necessary for α^4 to go from 1.01 to 1.02 and 35 more cycles are needed for α^4 to reach 1.10. Within our approximations it only makes sense to trace cycles back to where τ_c is just large enough for hydrogen burning to occur; this would very roughly be in the range between $\tau_c = 10^8$ yr, for which α^4 is 1.0005, and $\tau_c = 10^9$ yr, for which α^4 is 1.02.

d) Effects of Black Holes

If, as expected, there exist black holes, then, as a closed universe contracts, they will (1) accrete matter at an increasing rate and (2) coalesce to form one large black hole coextensive with the "entire" universe. We calculate (Dicus *et al.* 1981) that this will occur at about $T = 10^{14}$ K (or less if there are supermassive black holes at the centers of galaxies). Penrose (1979) has conjectured that the Bekenstein-Hawking formula (Bekenstein 1973, 1974; Hawking 1975) for black hole entropy could be applicable to the entire universe. In this case the entropy would be given by

$$S = \frac{kAc^3}{4\hbar G} , \qquad (5.13)$$

where

$$A = 4\pi(GM/c^2)^2$$

and

$$M = \Omega\rho_c V ,$$

with V the volume of the universe. If true, this would lead to a tremendous increase in entropy and hence scale length from cycle to cycle,

$$R_2 \approx 10^{-15} N_B^{1/3} R_1 \gtrsim 10^{12} R_1 , \qquad (5.14)$$

with $N_B \geq 10^{80}$ for this cycle. It is not, however, clear that the Bekenstein-Hawking formula should apply to the entire universe since its basis is the loss of information from outside the region of formation of a horizon and, for the collapse of a closed universe, the "observers" are inside the horizon that results from black hole coalescence.

On the other hand, it is not at all clear that a (presently unknown) mechanism that could cause a bounce could also prevent the anisotropy and inhomogeneity that would be expected from the existence of individual accreting black holes within the global contracting region.

e) Discussion

Our quantitative assumptions, $\Omega_\nu = 1$ and $\Omega_N = 0.1$, lead to $\tau_c \approx 10\tau_{\rm H}$ for the present cycle and hence relatively large cycle expansion factors $\alpha^4 \geq 25$ from now on. Since the present age of the universe is at least $\tau_{\rm H}/10$, α^4 must be at least 2.9. For $\tau_c < \tau_{\rm H}$, the chance of formation of life would seem to increase from cycle to cycle like $R^3\tau_c \sim \alpha^6$, and for $\tau_c > \tau_{\rm H}$ like $R^3 \sim \alpha^3$. In future cycles with $\tau_c \lesssim \tau_p$ those phenomena discussed by Dyson that take place at times less than τ_p—detachment of planets from stars and of stars from galaxies—can occur.

The total energy of the universe at $T = T_1$, $\frac{8}{3}\pi R^3(T_1)aT_1^4$, grows from cycle to cycle like α^3. This results from the fact that the energy density of radiation varies as R^{-4}, while that of matter varies as R^{-3}; these factors, in turn, follow from conservation of the energy-momentum tensor (together with the equation of state, $p_M = 0$, $p_R = \frac{1}{3}\rho_R$). Thus the increase from cycle to cycle in total energy appears to violate no conservation law (see note added in proof).

VI. CONCLUSIONS

If the universe is open and if the proton is unstable, then eventually the energy density of the universe will be dominated by a tenuous e^+e^- plasma. "Eventually" means at about (*a*) 10^{35} yr or (*b*) 10^{98} yr according to whether (*a*) the main constituents of the universe are stars and interstellar gas, or (*b*) supermassive black holes exist or form in galactic centers. For times less than τ_p, proton decay will keep white dwarfs at a few kelvins and neutron stars close to 100 K. The final e^+e^- plasma will be too tenuous for e^+e^- annihilation and perhaps for gravitational collapse.

If, on the other hand, the universe is closed and cyclical, its maximum radius will increase in each cycle by the cycle expansion factor $\alpha = R_2/R_1$. The factor α is calculated in § V in the approximation that hydrogen burning in stars and proton decay are the chief sources of entropy. (If there are other sources of entropy, then our calculation provides a lower bound on α.) Under the assumptions $\rho_v = \rho_c$ and $\rho_N = 0.1\rho_c$ this approximation gives $\alpha \sim 2$ for the present cycle.

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REFERENCES

Barr, S., Segre, C., and Weldon, H. 1980, *Phys. Rev.*, **D20**, 2494.
Barrow, J., and Matzner, R. 1977, *M.N.R.A.S.*, **181**, 719.
Barrow, J., and Tipler, F. 1978, *Nature*, **276**, 453.
Bekenstein, J. 1973, *Phys. Rev.*, **D7**, 2333.
———. 1974, *Phys. Rev.*, **D9**, 3292.
Carter, B. 1974, in *Confrontation of Cosmological Theories with Observational Data*, ed. M. Longair (Dordrecht: Reidel).
Collins, C., and Hawking, S. 1973, *Ap. J.*, **180**, 317.
Collins, C., and Stewart, J. 1971, *M.N.R.A.S.*, **153**, 419.
Cowsik, R., and McClelland, J. 1972, *Phys. Rev. Letters*, **29**, 669.
Cowsik, R., and Nussinov, S. 1981, University of Maryland preprint.
Davies, P. 1973, *M.N.R.A.S.*, **161**, 1.
———. 1974, *The Physics of Time Asymmetry* (Berkeley: University of California Press).
Dicke, R. 1961, *Phys. Rev.*, **126**, 1875.
Dicke, R., and Peebles, P. 1979, in *General Relativity: An Einstein Centenary Survey*, ed. S. Hawking and W. Israel (Cambridge: Cambridge University Press).
Dicus, D., Kolb, E., and Teplitz, V. 1977, *Phys. Rev. Letters*, **39**, 168.
Dicus, D., Letaw, J., Ruderman, M., Teplitz, D., and Teplitz, V. 1981, in preparation.
Dimopoulous. S., and Susskind, L. 1978, *Phys. Rev.*, **D18**, 4500.
———. 1979, *Phys. Letters*, **81B**, 416.
Dyson, F. 1979, *Rev. Mod. Phys.*, **51**, 447.
Feinberg, G. 1981, *Phys. Rev.*, **D23**, 3075.
Fritzsch, H., and Minkowski, P. 1975, *Ann. Phys.*, **93**, 193.
Georgi, H., and Glashow, S. 1974, *Phys. Rev. Letters*, **32**, 438.
Georgi, H., Quinn, H., and Weinberg, S. 1974, *Phys. Rev. Letters*, **33**, 451.
Gursey, F., Ramond, P., and Sikivie, P. 1976, *Phys. Letters*, **60B**, 177.
Guth, A. 1981, *Phys. Rev.*, **D23**, 347.
Guth, A., and Tye, S.-H. 1980, *Phys. Rev. Letters*, **44**, 631.
Handa, M., and Yoshimura, M. 1978, *Progr. Theor. Phys.*, **62**, 1704.
Hartle, J., and Hu, B. 1980, *Phys. Rev.*, **D21**, 2756.
Harvey, J., Kolb, E., Reiss, D., and Wolfram, S. 1981*a*, *Phys. Rev. Letters*, **47**, 391.
———. 1981*b*, Caltech preprint 68–850.
Hawking, S. 1975, *Comm. Math. Phys.*, **43**, 199.
Hu, B., and Parker, L. 1978, *Phys. Rev.*, **D17**, 933.
Islam, J. 1977, *Quart. J.R.A.S.*, **18**, 3.
Kolb, E., and Wolfram, S. 1980, *Phys. Letters*, **91B**, 217.
———. 1981, *Nucl. Phys.*, in press.
Landsberg, P., and Park, D. 1975, *Proc. Roy. Soc. London*, **A346**, 485.
Lee, B., and Weinberg, S. 1977, *Phys. Rev. Letters*, 39, 165.
Lyubimso, V., *et al.* 1980, preprint from Institute of Theoretical and Experimental Physics, Moscow.
Machacek, M. 1979, *Nucl. Phys.*, **B159**, 37.
Matzner, R. 1969, *Ap. J.*, **157**, 1085.
———. 1971*a*, *Ann. Phys.* (*NY*), **65**, 438.
———. 1971*b*, *Ann. Phys.* (*NY*), **65**, 482.
Matzner, R., and Misner, C. 1972, *Ap. J.*, **171**, 415.
Misner, C. 1968, *Ap. J.*, **151**, 431
———. 1969*a*, *Phys. Rev. Letters*, **22**, 1071.
———. 1969*b*, *Phys. Rev.*, **186**, 1319.
Mohapatra, R., and Marshak, R. 1980*a*, *Phys. Rev. Letters*, **44**, 1316.
——. 1980*b*, *Phys. Letters*, **94B**, 183.
Mohapatra, R., and Senjanovic, G. 1980, *Phys. Rev. Letters*, **44**, 912.
Nanopoulous, D., and Weinberg, S. 1979, *Phys. Rev.*, **D20**, 2484.
Novikov, I., and Zel'dovich, Ya. 1973, *Ann. Rev. Astr. Ap.*, **11**, 387.
Page, D., and McKee, M. 1981, *Phys. Rev.*, **24**, 1458.
Pati, J., and Salam, A. 1973, *Phys. Rev.*, **D8**, 1240.
Penrose, R. 1979, in *General Relativity: An Einstein Centenary Survey*, ed. S. Hawking and W. Israel (Cambridge and New York: Cambridge University Press).
Rees, M. 1969, *Observatory*, **89**, 193.
Reines, F., Sobel, H., and Pasierb, E. 1980, University of California at Irvine preprint and discussion in *Phys. Today*, Vol. **33**, No. 7, p. 17.
Schramm, D., and Steigman, G. 1981, *Ap. J.*, **243**, 1.
Stewart, J. 1969, *M.N.R.A.S.*, **145**, 347.
Tolman, R. 1934, *Relativity*, *Thermodynamics*, *nd Cosmology* (Oxford: Oxford University Press).
Toussaint, D., Treiman, S., Wilczek, F., and Zee, A. 1979, *Phys. Rev.*, **D19**, 1036.
Weinberg, S. 1972, *Gravitation and Cosmology* (New York: Wiley).
———. 1979, *Phys. Rev. Letters*, **42**, 850.
Yildiz, A., and Cox, P. 1980, *Phys. Rev.*, **D21**, 906.
Yoshimura, M. 1978, *Phys. Rev. Letters*, **41**, 281.
———. 1979*a*, *Phys. Rev. Letters*, **42**, 746.
———. 1979*b*, *Phys. Letters*, **88B**, 294.
Zel'dovich, Ya. 1970, *JETP Letters*, **12**, 307.
———. 1976, *Phys. Letters*, **59A**, 254.
———. 1977, *Soviet Phys.—JETP*, **45**, 9.

Note added in proof.—It should be emphasized that black hole effects and anisotropy growth in a coalescing universe make these results for α a (possibly very low) lower bound, if indeed a cyclical universe is possible at all.