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MAGNETIC STARS

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OBSERVATIONS OF MAGNETIC FIELDS IN NONDEGENERATE STARS

Instrumentation and Techniques of Observation

MEASURED QUANTITIES Magnetic fields in nondegenerate stars are detected via the Zeeman effect in spectral lines, which leads to changes in absorption line profiles and to linear and/or circular polarization in the lines. Measurement of the splitting or broadening of spectral lines yields the mean surface magnetic field B_s, defined by

$$B_s=\left(\int |B| I \, dA\right)\Big/\left(\int I \, dA\right), \tag{1}$$

where $|B|$ is the local field strength, I is the local surface brightness, dA is a surface area element, and the integrals are carried over the visible disk of the star. Surface fields are measureable only in stars having very sharp spectral lines ($v \sin i \lesssim 10$ km s^{-1}) and large magnetic fields ($|B| \gtrsim 10^3$ G). Such

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measurements have been made for a limited number of stars by Babcock (1960a), Preston (1969a,b,c, 1971b), Wolff & Wolff (1970), and Robinson et al. (1980).

Measurement of the circular polarization in spectral lines yields the mean longitudinal or effective field

$$B_e = \left(\int B \cos \gamma I \, dA\right) \Big/ \left(\int I \, dA\right), \qquad (2)$$

where γ is the angle between the line of force and the line of sight. Because of the cos γ factor, B_e is much more dependent on the geometry of the magnetic field than is B_s.

A measurement of linear polarization in an absorption line yields the transverse component of the magnetic field. In practice, such measurements are difficult because for typical magnetic field strengths and line profiles the linear polarization is expected to be small. Only a few measurements have been published (Kodaira & Unno 1969, Borra & Vaughan 1976).

TECHNIQUES OF MEASUREMENT Virtually all the observations in the literature are of B_e. The detection techniques now used are based on the photographic and photoelectric techniques pioneered by Babcock (1947, 1953). They are reviewed by Babcock (1962), Borra (1980c), and Landstreet (1980).

Internal random errors of the photographic technique can be as small as a few tens of gauss for very sharp-lined stars (Babcock 1958). However, there are subtle systematic errors both in zero-point (Preston 1969c, Landstreet 1980) and in measured field strength (Borra 1974b). In practice, the smallest standard error attainable is probably 150–200 G. Modern automated plate measurement and numerical reduction techniques have been used to improve accuracy (Weiss & Wood 1975, Wood et al. 1977), but it is not clear that the expected increase in accuracy has been achieved (Borra et al. 1981). A variation of this technique utilizes television-type detectors in place of photographic plates (Fahlman et al. 1974, Vogt 1980). The main advantages of the photographic technique are that it is cheap and easy to set up at a coudé spectrograph, and it gives a permanent record of the spectrum that may be used for other purposes. This technique is most effective for detecting fields of a kilogauss or more in stars having $v \sin i \lesssim 30$ km s^{-1}, and for determining periods of magnetic variation in such stars.

Photoelectric field measurements of sufficiently bright stars may be exceedingly precise. Thus, solar magnetograph measurements of the solar field typically have standard errors of less than 1 G, and a few stellar measurements with standard errors of less then 10 G have been reported (Borra et al. 1973, Borra et al. 1981, Brown & Landstreet 1981, Landstreet 1981).

A photoelectric polarimeter can make measurements whose accuracy is not much worse than the theoretical standard error $\sigma = N^{-1/2}$, where N is the number of detected photons. The accuracy of the corresponding magnetic field measurement obtained from a particular spectral line is then given by $\sigma_B = (4\pi mc^2/ze\lambda^2)\,[I\,(dI/d\lambda)^{-1}]\,N^{-1/2}$, where z (Babcock 1962) measures the sensitivity of the spectral line to Zeeman-splitting, $I(\lambda)$ is the *observed* line profile, and the other symbols have their usual meanings. To obtain the smallest value of σ_B in a given time, one must strike a compromise between a wide bandpass that increases N but decreases $dI/d\lambda$, and a narrow bandpass that does the converse.

For stars with $v \sin i \gtrsim 30$ km s^{-1} (e.g. most upper main sequence stars), metal lines are broad, and $dI/d\lambda$ is too small for metal line magnetic measurements to be efficient. It is most efficient to make field measurements of such stars in the very broad Balmer lines of hydrogen (Borra & Landstreet 1977, 1980), using a Cassegrain filter polarimeter. Measurements of field strength in a star with $v \sin i \lesssim 300$ km s^{-1} can be made with approximately the same accuracy in the same time using the Balmer line technique as in a star with $v \sin i \lesssim 30$ km s^{-1} using a single metal line (Landstreet 1981). Balmer line magnetic measurements have several significant advantages over other techniques. Measurement accuracy is only slightly affected by stellar rotational velocity, and interpretation of measurements is not complicated by nonuniform distribution of elements over the stellar surface, as hydrogen is almost certainly uniformly distributed even in those Ap stars in which everything else is patchy. Interpretation is also not complicated by Doppler-mixing of profiles from various parts of the star, a very serious complication in measurements on sharp-line stars (Borra 1980b).

A number of field measurements reported in the literature are based on a coudé variation of the Babcock photoelectric technique, utilizing measurements of a single suitable (metal or helium) spectral line. This method was first explored by Severny (1970) and may be used to search for small fields ($\lesssim 10^2$ G) in sharp-line bright stars. It may also be used to obtain polarization profiles across the spectral lines of magnetic Ap stars, which (in the presence of a modest rotational broadening of $\sim$ 10–50 km s^{-1}) contain considerable information that may be used to extract details of the magnetic field distribution over the star by appropriate computer models (Borra 1980b).

To increase the efficiency of such measurements, it is necessary to observe several lines at the same time. Babcock (1962) accomplished this by placing an opaque template with slits corresponding to spectral lines at the focal plane of a spectrograph, followed by a condensing lens and a photomultiplier. Recently, Borra et al. (1981) and Brown & Landstreet (1981) have explored this technique, and Borra & Mayor (1982) are presently using it (with a Cassegrain instrument) to make a survey of magnetism among late-type stars.

There are many technical difficulties (such as instrumental polarization) associated with the measurement methods discussed above. These are considered in detail by Borra (1976, 1980c). A detailed study of the technique, sources of error, efficiency, etc., is given by Borra (1972).

Occurrence of Magnetic Fields in Various Types of Stars

PECULIAR A AND B STARS Since Babcock (1947) detected a field in the Ap star 78 Vir, fields have been reported in perhaps 200 stars, mostly ones with Ap spectra. Of the known magnetic Ap stars, perhaps 50 have been studied in some detail, mainly by Babcock (1958 and references therein, 1960a,b, 1962, 1967), Preston (1971a and references therein, 1972), a group of astronomers at the University of Hawaii (Wolff 1976 and references therein, Wolff & Hagen 1976, Bonsack 1977, Wolff & Preston 1978), and Borra & Landstreet (1977, 1978, 1980).

Almost all magnetic studies of the magnetic Ap stars have been based either on photographic or on Balmer line measurements of B_e. From these studies a number of general conclusions have emerged. (1) Most—probably all—Ap stars of the Si and Sr-Cr-Eu peculiarity classes (but not those of the Hg-Mn peculiarity class) possess magnetic fields with a longitudinal component of typically a few hundred gauss. The range for measured longitudinal fields is from about 20,000 G down to undetectably small (less than 1–200 G). (2) The longitudinal fields observed, in general, vary with time. In virtually all cases where sufficient data are available, this variation is strictly periodic with periods ranging from about 0.5 day to decades. The median period for the Ap stars is of the order of three days, and most stars have periods in the range of 1–10 days. (3) The observed longitudinal fields vary in roughly sinusoidal fashion. This is only approximately true for fields measured photographically, but photographic B_e measurements are affected by systematic reduction errors, which tend to distort the observed magnetic curves (Borra 1974a). Photoelectric field measurements yield magnetic curves which are fairly sinusoidal in appearance. About two thirds of the well-studied magnetic stars show polarity reversal of the effective field. (4) The period of variation observed for a magnetic field is almost certainly the rotation period of the star. This can be inferred from the fact that only short-period stars have rotationally broadened spectral lines, while long-period stars invariably have sharp lines (Preston 1971a). (5) Most Ap stars also vary in brightness and have spectral lines that vary in strength; the period of variation is always the same as the magnetic period. (6) In a few Ap stars, it is possible to measure the main surface field. In general, this field, B_s, is two or more times larger than the largest observed value of B_e. In the few cases where B_s has been measured throughout the magnetic period, the surface field varies periodically with the same period as other features, but by a factor less than two.

Fields have been discovered in two classes of hotter stars related to the Ap class but peculiar primarily in the appearance of helium lines that are anomalously weak or strong (and often variable) for the stellar temperature. The helium-weak stars occur roughly in the range $14{,}000 \lesssim T_e \lesssim 20{,}000$ K (spectral types B6 to B2.5); the helium-strong stars all have $T_e \sim 22{,}000$ K (spectral type B2). To date, fields have been detected in about 8 helium-weak stars (Wolff & Morrison 1975, Wolff & Wolff 1976, Landstreet et al. 1979, Landstreet & Borra 1982). The observed fields share the characteristics found for magnetic Ap stars, and are about the same amplitude, ranging up to $B_e \sim 7000$ G but typically a few hundred gauss. Surprisingly, more than half the helium-weak stars observed by Landstreet & Borra (1982) are not detectably magnetic even after several observations, and thus have fields of no more than 3–500 G. Of the 10 helium-rich stars so far observed (Landstreet & Borra 1978, Borra & Landstreet 1979, Barker et al. 1982), 7 are clearly magnetic, and 6 have fields of well over a kilogauss. It appears that such stars, on average, have substantially stronger fields then either Ap or helium-weak stars.

NON-AP STARS Some attention has been given to searches for fields in types of stars other than Ap/Bp. The first systematic survey was carried out by Babcock (1958) who, in addition to observing a large number of Ap stars, obtained Zeeman-analyzed spectrograms of more than 100 stars of other spectral classes. Since his work, a number of other papers have appeared that deal with searches for magnetic fields in non-Ap stars. These efforts are briefly summarized in Table 1. For each spectral category, we list the references in which stars of that type are discussed, and the field measurement method used (pg—Zeeman-analyzed photographic spectra, Bl—Balmer line Zeeman analyzer measurement, sl—photoelectric Zeeman measurement of a single spectral line, ml—photoelectric Zeeman measurement of many spectral lines simultaneously, and CID—measurement of B_s via line profile observations). Further columns list the number of stars observed in the category in question, the typical standard errors reported by each reference (typical errors are taken approximately as medians, or as estimates in cases of a large number of observations), and in the remarks column we report any positive results (and other data).

Several photographic surveys report best errors of the order of 50 G. For reasons discussed by Preston (1969c) and Landstreet (1980), we estimate that the best photographic errors are probably of the order of 150–200 G, so fields of less than (perhaps) 50 G amplitude that have been detected only photographically are rather uncertain. Photoelectric observations often suffer from some small unexplained excess scatter (typically of order 1.5σ) about measured mean field curves, but recent null measurements show the expected

Table 1 Searches for magnetic fields in non-Ap stars

Reference	Method	No.	Typ σ(G)	Stars with reported fields; remarks
O and B stars, main sequence and subgiants				
Babcock (1958)	pg	9	300:	HD45677 (B2V); plates not measured
Borra (1974b)	sl	1	500	No field in θ^2Ori (0 9.5Vp)
Borra & Landstreet (1973)	sl	1	320	No field in β Per (B8V)
Conti (1970a)	pg	4	450	No fields in 4 normal B1–B2 Orion stars
Kemp & Wolstencroft (1973a)	Bl	1	360	θ^2 Ori (X-ray source candidate)
Landstreet (1981)	Bl & sl	9	50	No fields in 9 normal B0–B7 stars, including γ Peg
Rudy & Kemp (1978)	Bl	4	600	γ Peg (B2IV)?; survey of β Cep stars
Hg-Mn type Ap stars				
Babcock (1958)	pg	9	100	Fields of $\lesssim$500 G in 7 Hg-Mn stars
Borra & Landstreet (1973)	sl	1	440	No field in α And
Borra & Landstreet (1980)	bl	5	120	No fields detected
Borra et al. (1973)	sl	1	50	No field in ιCrB
Conti (1970b)	pg	4	150	No fields detected in 22 plates of 4 stars
Preston et al. (1969)	pg	1	150	No field above 400 G in 22 plates of κ Cnc
Severny (1970)	sl	1	30	α And?
Borra et al. (1982)	ml	3	30–100	No field in κ Cnc
A and F stars, main sequence and subgiants				
Babcock (1958)	pg	16	150:	γ Vir N(FOV), HD19445(F6sd); $B_e \lesssim 450$ G
Boesgaard (1974)	pg	6	50	Field of $\lesssim$450 G in γVirN
Boesgaard et al. (1975)	pg	1	75	No field in ιPeg (F5V)
Brown & Landstreet (1981)	ml	1	90	No field in RS CVn star Z Her(F4IV–V)
Landstreet (1981)	sl	22	60	No fields in normal AO-F5 stars including Vir N
Severny (1970)	sl	2	15	No field in α Lyr (AOV), αCMi (FSIV)
Borra & Mayor (1982)	ml	3	5–20	No fields in 3 RS CVn stars
Am stars				
Babcock (1958)	pg	29	150:	Fields of $\lesssim$600 G in 8 Am stars
Borra (1975)	sl	1	23	No field in 8 observations of αCMa (A2V)
Borra & Landstreet (1973)	sl	1	100	No field in α CMa
Borra & Landstreet (1980)	Bl	8	140	No fields

Borra et al. (1973)	sl	2	9–135	No fields in αCMa or α^2Lib
Conti (1969)	pg	2	80	No fields
Landstreet (1981)	sl	4	70	No fields
Severny (1970)	sl	1	12	Field of $\lesssim$40 G in αCMa
Borra et al. (1982)	ml	6	30–100	No fields
G, K, and M stars, main sequence and subgiants				
Anderson et al. (1976)	pg	1	?	Field in Hα emission of B Dra?
Babcock (1958)	pg	9	200:	No definite fields
Boesgaard (1974)	pg	2	32	Fields of $\lesssim$150 G in ξBooA (G8V) and 70 Oph A (KOV)
Boesgaard et al. (1975)	pg	1	75	No field in 4 Lick plates of ξBooA
Brown & Landstreet (1981)	ml	12	60	No fields
Preston (1967a)	pg	9	100:	No fields in late G-type dwarfs
Robinson et al. (1980)	CID	2	370	Surface fields in spots on ξBooA and 70 Oph A
Vogt (1980)	ml	19	130	No fields
Borra & Mayor (1982)	ml	8	5–20	No fields in CAII emission dwarfs
O, B, and A stars, giants and supergiants				
Angel et al. (1973)	Bl	2	500	No fields in HD77581 (B0.5Iae), HD153919 (O6.5Iaf)
Babcock (1958)	pg	18	150:	AG Peg (Bep+M)
Borra (1974b)	sl	2	500	No fields in HD31327 (B2Ib), HDE226868 (BOIb)
Kemp & Wolstencroft (1973a)	Bl	1	1300	HD77581 (X-ray source)
Kemp & Wolstencroft (1973b)	Bl	1	10^3:	HD153919 (X-ray source)
Rudy & Kemp (1978)	Bl	5	600	β Cep (B2III)?; survey of β Cep stars
Severny (1970)	sl	1	20	Field of $\lesssim$130 G in β Ori (B8pI)
F, G, K, and M stars, giants and supergiants				
Babcock (1958)	pg	21	150:	Fields in 5 M and S giants, and RR Lyrae (F5)
Borra et al. (1981)	ml	12	10	αUMi (F8Ib), γCyg (F0Ib); δCep (F5Ib)?
Brown & Landstreet (1981)	ml	11	30	No fields
Preston (1967a)	pg	21	150:	No fields in M and S giants or RR Lyrae
Rakos et al. (1977)	pg	1	50	Field of $\lesssim$630 G in αCar (F0Ib)
Severny (1970)	sl	3	30	Field of $\lesssim$200 G in γCyg
Wood et al. (1977)	pg	1	40	Field of $\lesssim$200 G in Cepheid W Sgr.
Borra & Mayor (1982)	ml	8	3	No fields

scatter about zero, and error estimates in searches may usually be taken at face value (Borra & Landstreet 1980,), even when they are as small as a few gauss.

Table 1 contains no really unambiguous evidence for the existence of sizeable coherent fields giving rise to nonzero B_e values in non-Ap stars. In some cases (Hg-Mn stars, Am stars, some X-ray source candidates, some active lower main sequence dwarfs), initial reports of longitudinal fields were not confirmed by subsequent observations. In other cases no confirming observations have been carried out. In most of these cases, the initially reported fields were only detected at the 3–5σ level, and often reflect measuring errors somewhat larger than expected.

Several indirect lines of evidence suggest that large magnetic starspots may exist in many cool dwarf stars, especially those in close binary systems of the RS CVn, BY Dra, and (sometimes) W UMa types (Hall 1976). In such systems, light variations are observed to occur with periods slightly different from the orbital periods. These could plausibly result from the presence on one of the two stars of large, cool regions (starspots), rotating with one of the stars. Such systems also show evidence of activity analogous to solar magnetic activity, including strong (chromospheric?) H and K emission that varies with the spot rotation and (coronal?) soft X-ray emission.

Searches for longitudinal fields on such stars have been unsuccessful. However, Mullan (1979) showed that the combination of concentrated field lines emerging from a spot and re-entering the photosphere nearby, together with the relatively faint radiation expected from a spot 10^3 K cooler than the surrounding photosphere, should lead to observable effective fields of no more than perhaps 3% of the field strength in the spot; this implies that longitudinal fields should be extremely difficult to detect with normal techniques. Recently, Robinson et al. (1980), using a refinement of Preston's technique, detected surface fields of the order of 2000 G on 10–40% of the stellar surface in ξBooA and 70 Oph A. This appears to be a promising approach to the problem of detecting spot fields on such stars. (However, no field was detected in ξBooA by Marcy 1981).

Magnetic Geometries of the Upper Main Sequence Peculiar Stars

BASIC MODEL: THE OBLIQUE ROTATOR Several models have been proposed to explain the magnetic variations of the Ap stars (Babcock 1960b, Ledoux & Renson 1966), but the evidence gathered over the past 30 years now seems heavily in favor of the Oblique Rotator Model (ORM). The magnetic field variations result from a magnetic field frozen into a rotating star; as the star rotates, the observer sees different aspects of the magnetic geometry. The

ORM explains the periodicity of the variations as due to the steady rotation of the star, and the polarity reversals seen in many stars as a simple geometrical effect. Deutsch (1956) found strong evidence in favor of the model: the spectral line widths tend to decrease with increasing periods. The strongest objection against the ORM came from the apparent existence of irregular variables (Babcock 1960b); however, all the irregular variables have been shown to be periodic (Preston 1970a, Borra & Landstreet 1980).

It is possible that there may be small irregular variations superimposed on the basically periodic field variations observed. Bonsack (1977) has found evidence of irregular variations in HD 133029, although a smooth periodic variation was found by Borra & Landstreet (1980). It has also been suggested that the negative polarity extremum of β CrB varies secularly, perhaps with a period of $\sim$ 10 yr (Preston & Sturch 1967, Severny 1970), although other investigators have not confirmed this effect (Wolff & Bonsack 1972, Borra & Dworetsky 1973, Borra & Vaughan 1977). The Balmer line magnetic measurements of Borra & Landstreet (1980) revealed no large irregular variations, but they do not exclude variations of 2–300 G in some stars. Available data do not resolve this question; all that is suggested by these observations is possible small-amplitude irregular variations superimposed on a smooth B_e curve. The fact is that the magnetic periods of the well-studied Ap stars have remained constant over several decades (Borra & Landstreet 1980) and several thousand cycles for some stars (e.g. 78 Vir and αCVn), in agreement with the ORM.

Borra & Vaughan (1976) have obtained further direct confirmation of the validity of the ORM for the Ap star β CrB. They find that the linear polarization vector in an FeII line describes a 360° rotation per magnetic cycle, in agreement with the ORM. Quantitative support for the ORM also comes from computer modeling of observed photoelectric circular polarization profiles across an FeII line of 78 Vir (Borra 1980a,b).

MAGNETIC GEOMETRIES OF THE Ap STARS Most of the observations in the literature are of B_e, so the information used in modeling the magnetic geometries is usually contained in B_e (ψ), where ψ is the phase of variation. What is then sought is a field B (r, θ, φ) such that when r = stellar radius, the integral in Equation 2 yields a B_e that is in agreement with the observations of B_e (ψ) for every phase as the star rotates.

The first Oblique Rotator Model (Stibbs 1950) used a simple dipolar geometry which is, however, incapable of reproducing the strongly anharmonic variations seen in a few photographic B_e curves (Babcock 1960b, Huchra 1972). Adding multipolar components and considering nonuniform element distributions improve the fits (Deutsch 1958, Böhm-Vitense 1965, Pyper

1969). Landstreet (1970) found that he could reproduce the anharmonic photographic B_e curves with a decentered dipole geometry. Serious modeling difficulties arose, however, when both B_e and B_s curves became available for the same star (Wolff & Wolff 1970, Huchra 1972). The Ap star 53 Cam illustrates the situation. Landstreet (1970) fitted the B_e curve with a highly decentered dipole (by 0.67 radii) having a huge polar field (5×10^5 G) at the strong pole. Huchra (1972) found that the B_s curve observed for 53 Cam is in total disagreement with Landstreet's model. He was able to fit the observed B_s curve with a dipole having much smaller decentering (0.15 radii) and a weaker polar field (28,000 G), which gives a nearly sinusoidal B_e curve, quite unlike the strongly anharmonic photographic B_e curve observed. Borra (1974a) showed that the basic difficulty in finding a consistent model of the magnetic geometry of this star could be a result of systematic errors in the photographic measurement of B_e. He showed that a tendency on the part of the person measuring a magnetically analyzed plate to emphasize the line cores at the expense of the shallow diffuse wings would lead to errors in the resulting B_e measurements; this would in turn produce anharmonic magnetic curves of the type observed, even if the true B_e curve was sinusoidal. He predicted that photoelectric Balmer line magnetic measurements would yield sinusoidal magnetic curves, a prediction confirmed by Borra & Landstreet (1977, 1980).

One must not, however, deduce that the magnetic geometries are dipolar. Borra (1974a, 1980b) needed geometries having unequal polar field strengths, as the obserations of B_s also clearly impose. The picture of a typical geometry that emerges at the time of this writing is of a star that has two poles of opposite sign and unequal strength. Typically, one pole should have two to three times the strength of the other. This geometry can be approximated by either a decentered dipole having a moderate decentering (less than 0.2) or by a centered dipole with a comparable quadrupolar component. This is indicated by detailed modeling (Borra 1980b), the observed B_s curves, and the nearly sinusoidal photoelectric B_e curves observed. Note that a decentered dipole geometry and a dipole plus quadrupole geometry with the above parameters are remarkably similar; they cannot be distinguished observationally and are probably idealizations of more complex geometries. There are indications that some geometries are more complex. Borra & Vaughan (1977) find indications of departures from cylindrical symmetry in circular polarization profiles in β CrB; however, a phase-varying blend could be responsible and the observations should be repeated in other lines.

If confirmed, observed phase lags between the B_e and B_s extrema (Wolff & Wolff 1970, Preston 1970b) are also indicative of a breakdown of axial symmetry. Direct modeling of high-quality photoelectric profiles across spectral lines (Borra 1980a,b) is the most promising approach to the more detailed study of individual magnetic geometries.

The inclination between rotation and magnetic axes (obliquity β) is a theoretically important parameter of the geometry. The observations indicate the existence of both low ($< 30°$) and high ($> 60°$) obliquities, with a predominance of the latter. Because of insufficient statistics, we cannot distinguish between a random distribution or a bimodal distribution of low and high obliquities among the Ap stars (Preston 1967b, 1971a, Landstreet 1970, Hensberge et al. 1979, Borra & Landstreet 1980).

A special case of the usual ORM, the symmetric rotator, has been proposed by Krause & Oetken (1976) and Oetken (1977, 1979). In the usual ORM the inclination of the field to the rotation axis is a free parameter, while in the symmetric rotator this inclination is fixed at 90°. The symmetric rotator involves both a dipole and a parallel linear quadrupole, but instead of having both make roughly comparable contributions to B_s, as in the usual ORM [so that the quadrupole makes almost no contribution to B_e (cf. Schwarzschild 1950)], the two components make comparable contributions to B_e, so that the quadrupole dominates B_s. This seems to require larger values of B_s than are observed (Borra & Landstreet 1978, Deridder et al. 1979), but Oetken (1979) has suggested that B_s values predicted by the symmetric rotator model could be substantially reduced by the effects of nonuniform element or brightness distributions over the surface of a star. However, in the only case in which this proposal has been explored, namely HD 215441 (Borra & Landstreet 1978), no acceptable model could be found. Until model calculations show how to produce acceptably small values of B_s from the symmetric rotator, its applicability must remain doubtful.

Preston (1971b) has observed B_s in 28 sharp-lined cool (Sr-Cr-Eu) Ap stars. He finds that the B_s distribution is characterized by a maximum of 2–3 kG with a tail that extends beyond 10 kG. His suggestion that B_s increases with rotational period is weakly supported by B_e observations (Borra & Landstreet 1980). Note, however, that Borra (1981) finds that the young magnetic stars in the Orion OB1 association both have stronger fields than the older magnetic stars, and also seem to be rotating faster (cf. below).

It is only recently that magnetic fields have been discovered in helium peculiar stars. The evidence seems to indicate that their geometries are similar to those of the Ap stars. This is indicated by the sinusoidal B_e curves (Landstreet & Borra 1978, Borra & Landstreet 1979, Landstreet et al. 1979, Landstreet & Borra 1982). Note, however, the strange curve of HD 37776 reported by Borra & Landstreet (1979).

RELATIONSHIP BETWEEN MAGNETIC GEOMETRY AND ABUNDANCE PATCHES It is generally believed that the nonuniform distribution of chemical elements over the surface of an Ap or Bp star (which is observed as variation in the line strength of various elements) is produced by the magnetic field. Strong observational evidence for this connection is found in the observed fact that maxima and minima of line strength (and of brightness) almost always coincide with extrema of B_e. The two main theories of the development of such abundance patches are diffusion under the competing influences of gravity and radiation pressure (Michaud 1970, 1980, Vauclair & Vauclair 1982), and selective accretion of elements from the interstellar medium (Havnes & Conti 1971); both suggest the development of abundance patches. It is of considerable theoretical interest to determine observationally the relationship between magnetic geometry and abundance distributions (patches, rings, etc.).

A first attempt to determine abundance distributions and their relationship to the magnetic field was made using harmonic analysis by Deutsch (1958, 1970) for HD 125248; his method has subsequently been applied by Pyper (1969) and Falk & Wehlau (1974) to α^2CVn, by Rice (1970) to HD 173650, and by Megessier (1975) to 108 Aqr. An alternative method involving numerical fitting of line profiles using discrete assumed distributions has been developed by Khoklova & Ryabchikova (1975) and applied to χ Ser. Unfortunately, no definitive results have yet been obtained. There are serious doubts about the uniqueness of the inferred element distributions (Borra & Landstreet 1977, Megessier et al. 1979), especially about the latitude of patches. Furthermore, the available analyses either involve stars for which the magnetic geometries are not unique [α^2CVn, HD 125248, HD 173650; see discussions by Borra & Landstreet (1977) and Landstreet (1980)] or the necessary magnetic observations are simply not available yet (χ Ser, 108 Aqr). Thus at present the relationship between abundance distributions and magnetic geometry is not well established for a single star, in spite of the importance of the problem. Probably the key star for future study is 53 Cam, which has strong abundance variations (Farragiana 1973) and a very well-determined magnetic geometry (Borra & Landstreet 1977).

The effect of inhomogeneous element distributions on the magnetic measurements has not been extensively explored, in spite of its evident importance for measurements made using metal lines. This effect has been examined by Pyper (1969) for α^2CVn and by Oetken (1979) for a family of models; Oetken's results clearly indicate that quite varied measurements of B_e can result from a given magnetic geometry with various assumed abundance distributions.

THEORY OF MAGNETIC STARS

Introduction

The Oblique Rotator Model is the universally accepted frame for interpreting the observations of magnetic stars: the rotation of the star makes manifest a structure (magnetic and spectral) that is nonsymmetric about the rotation axis

(Stibbs 1950, Deutsch 1956, 1958). Most comparisons with observation adopt surface field distributions that are specialized to be symmetric about a magnetic axis, which is inclined to the rotation axis at a finite angle (referred to either as β or χ). In these phenomenological studies this obliquity angle is usually a parameter adjustable for the star under study, and attempts have been made to infer statistical properties of the χ-distribution. The earlier suggestion (Preston 1971a) of a marked preference for either small or large χ has now been eroded, and most observers argue either for a completely random distribution (Hensberge et al. 1979), or (as above) that the apparent marginal nonrandomness (Borra & Landstreet 1980, Figure 19) is inconclusive because of the meagerness of the statistics. However, as already noted, there is one group who on the contrary argue for the most extreme nonrandomness: the Krause-Oetken "symmetric rotator" has $\chi = \pi/2$ and the field mirror-symmetric with respect to the equatorial plane defined by the rotation axis. The surrender of freedom in the choice of χ makes these models *prima facie* more vulnerable to attack (Borra & Landstreet 1978, Deridder et al. 1979). Oetken (1977) and Krause & Oetken (in preparation) have responded by emphasizing that the mapping of the surface magnetic field does depend on knowing the distribution of the elements producing the line under study. The lesson for the theorist is that it is very difficult to be certain that observation rules out one model or another, although at present the Krause-Oetken proposal does look somewhat *ad hoc*.

The theorist is faced with a large number of problems, some arising immediately from the observations, others from the necessary generalizations of the theory of stellar structure. For instance, what is the origin of the field, and why are only a subclass of the early-type stars observably magnetic? What can theory say about the distribution of obliquities? How does the field strength vary through the star, and what is the relation between poloidal and toroidal components? What determines the amount of observable flux that appears above the stellar surface? How do the magnetic field and the star's rotation field interact? What is the braking process that is presumably responsible for the tendency towards slow rotation, and during which period of the star's history did the braking occur? What is the cause of the abundance peculiarities, and why are they different in the observably magnetic and non-magnetic Ap-stars?

These questions are interrelated, and suggest theoretical studies (e.g. on hydromagnetic stability) that would in any case excite workers with a nodding familiarity with laboratory plasmas. The last question is particularly important: perhaps we must wait for a detailed understanding of the surface layers before the ambiguities in the phenomenology are eliminated, and the issue of the generally oblique rotator versus the perpendicular model is resolved. For reviews of this part of the theory, see Michaud (1980) and Vauclair & Vauclair (1982).

Origin of the Field; Stability

The "fossil" field theory (Cowling 1945, 1953) derives from the very long decay times ($\simeq 5 \times 10^9$ years) of the largest scale modes in stars with an extensive stable radiative zone. Thus it is argued that a star is magnetic now because it acquired a magnetic field earlier in its life. In its simplest form the theory appeals to the Galactic magnetic field threading the gas from which the star formed, but the field could also be a relic of a field generated spontaneously in an earlier phase of the star's life. By definition, a "fossil" is a field that at the present epoch is not being maintained against spontaneous decay.

The main alternative theories have the fields being continuously supplied with energy by dynamo action, e.g. within the convective cores of the upper main sequence stars. (For discussions of dynamo theory in general, the reader is referred to Krause et al. 1971, Krause & Rädler 1980, Moffatt 1978, and Parker 1979). There is also the possibility of flux generation from the "battery" effect (Biermann 1950), the proximate source of energy being now not the bulk turbulent motions, but the thermal motions which sometimes yield an electron partial pressure gradient per charge—$\nabla p_e/n_e e$—with a nonvanishing curl, and so analogous to a technological battery exerting an emf.

The astronomical advantage of the fossil theory is primarily its flexibility. By allowing the magnetic flux within a star to be within fairly wide limits a consequence of its history, it bypasses embarrassing confrontations with observation (cf. below). In fact, one can invert the statement of the theory and ask instead why it is that more stars do not show a relic of the interstellar field, and indeed why even the stars with the highest surface fields are almost certainly magnetically "weak," in the sense that the magnetic energy is a very small fraction of the gravitational. If (as seems probable) the freezing of magnetic flux into condensing gas breaks down at the molecular cloud phase and is not re-established until the opaque phase, say at a density near 10^9 particles/cm^3, then the flux within a main sequence star would be quite modest. One can also hazard a guess that even if proto-stars do reach the pre-main-sequence phase as magnetically "strong," either dynamical or secular instability (similar to those discussed below) will ensure that most of the flux is destroyed within a Kelvin-Helmholtz time scale. And in fact if the star is of mass $< 2M_\odot$ and so passes through a lengthy fully convective Hayashi phase, the question is now whether any primeval flux survives, or whether the star suffers a complete "magnetic brainwash." No definitive answer can be given, but numerical studies such as those by Galloway et al. (1978) and Galloway & Weiss (1981) lend tentative support to the picture in which some of the primeval flux survived the turbulent phase in the form of concentrated ropes surrounded by field-free, fully convecting regions; once the turbulence decays, the field would diffuse back into a more uniform distribution through the star. Other workers (e.g. Schüssler 1975, 1980) argue that in a star with mass $< 2M_{\odot}$, the Hayashi convection will destroy the primeval flux but that a field built up by dynamo action during this phase will survive the turbulence decay. Stars more massive suffer only a short Hayashi phase and may preserve some of their primeval flux. Either way, it is not implausible to postulate that stars arrive on the main sequence with a magnetic flux that is sometimes at a potentially observable level. The crucial question is that of stability. It is known (Wright 1973, Tayler 1973, Markey & Tayler 1973, 1974, Goossens & Tayler 1980, Goossens et al. 1981) that even in subadiabatic stellar zones, purely poloidal or purely toroidal fields are unstable near their neutral lines to adiabatic motions that are primarily perpendicular to the gravitational field. To remove these obvious instabilities, one needs to construct topologically complex fields, with toroidal loops linking the poloidal loops. Dynamo-built fields will have such a structure. An initially purely poloidal field (e.g. a field that has detached from the local galactic field) may be destroyed by its spontaneous instability, or it may (with the help of finite resistivity) transform itself into such a complex structure.

Even then, new unsuspected instabilities can arise. For example, an axisymmetric mixed poloidal-toroidal field with a finite current density along the axis has an unstable mode (Tayler 1980). Furthermore, the toroidal component of an initially stable field will diffuse into such an unstable structure in less than the star's age, provided there is no compensating input of energy (cf. below). This is just one example of how difficult it is to be sure that a stellar magnetic field is dynamically stable. And even if dynamical stability is assured, there is the more difficult question of secular stability. In a subadiabatic zone, individual flux-tubes tend to float to the surface via "magnetic buoyancy" (Parker 1979) at a rate determined by heat diffusion into the cool gas within the tube—an "Eddington-Sweet" effect (cf. below). It may be that global fields with mutually linking poloidal-toroidal loops are subject to similar instabilities, perhaps depending also on the finite resistivity which allows changes of field topology. Effective stabilization over a stellar lifetime may thus impose upper limits on the total magnetic flux of the star.

If, however, it should turn out that there is no plausible way to stabilize magnetic fields of the required strength over stellar lifetimes, then one would be forced to look for a process of regeneration of the flux, which is presumably being continually destroyed because of the instabilities. Dynamo action within the core is the obvious candidate, whether a steady dynamo (as in the Krause-Oetken model) or possibly an oscillating model (Moss 1980). The advocate of the fossil explanation does not deny that dynamo action can occur; the question is whether the fields that we see at the surface are part of a field generated in the convective core. (Likewise, accepting that we are seeing an oscillatory dynamo at work in the Sun and other late-type stars is not inconsistent with postulating a fossil field within the radiative core, prevented from appearing at the surface by the violent convection in the outer envelope.) We do not have anything like a complete kinematic dynamo theory, let alone a dynamical theory, which would in turn require an understanding of the rotation field in a turbulent zone. Advocates of both the fossil and the dynamo theory have similar tasks—given their differing initial assumptions, can they show that what we know about stellar structure yields at least qualitative understanding of the observations?

The Biermann "battery" process is now usually thought of just as a means of supplying the "seed" magnetic field necessary for starting dynamo action. Biermann's (1950) original application assumed an initially nonmagnetic, chemically homogeneous star to have a nonconservative centrifugal field; the associated battery $\nabla p_e / n_e e$ would then build up a toroidal field $\mathbf{B}_t$ against self-induction, in a time of the order of the Cowling decay time (Cowling 1953). Fields $\simeq 10^3$ G could be built up even in a slow rotator like the Sun. However, the minimal steady-state constraints imposed on the rotation field by even a weak poloidal field are sufficient to greatly reduce the effect (Mestel & Roxburgh 1962); and if the star's rotation is uniform, as in a quasi-steady oblique rotator, then the effect vanishes. More recently, Dolginov (1977) has appealed to the nonspherical distribution of mean molecular weight μ associated with the local abundance anomalies on the stellar surface to yield a battery that is ultimately supposed to be responsible for the whole of the observed field. A less ambitious scheme (Mestel & Moss 1982) assumes a primeval poloidal field, but asks what toroidal fields can be maintained by the battery, again appealing to a nonspherical μ-distribution. The motivation comes from noting that the poloidal currents maintaining the toroidal field, and flowing along the poloidal loops near the magnetic axis, must flow through the low-conductivity surface regions of the star, and will suffer much greater Ohmic dissipation than the toroidal currents maintaining $\mathbf{B}_p$ (which tend to flow deep in the star). This could lead to an embarrassingly short decay time for the whole of the $\mathbf{B}_t$ field (without which the $\mathbf{B}_p$ field will suffer the topological instabilities discussed earlier). Preliminary results show that the nonspherical μ-distribution capable of suppressing the Eddington-Sweet circulation (cf. below) can yield $\mathbf{B}_t \simeq 10^3$ G over the bulk of a star with a typical magnetic star rotation period. This result gains in significance from the tentative conclusion that the Eddington-Sweet circulation may in fact be suppressed by a μ-gradient. Furthermore, the input of energy from the battery should offset the diffusion of a stable toroidal field into an unstable structure (Tayler 1982). It is also confirmed that near the surface, the toroidal field is small, giving support to phenomenological analysis of observed fields in terms of purely poloidal components.

The Structure of Rotating Magnetic Stars; Magnetic Braking

The magnetic field—whether a slowly decaying fossil or dynamo-maintained—like the rotation field must affect at least locally the thermal-gravitational equilibrium of the star; likewise, any mass motions will tend to redistribute flux through the star, and so could shed light on the crucial question as to when a large-scale magnetic field is observable at the surface. To be strictly relevant, the stellar models studied should have magnetic fields with the linked poloidal-toroidal structure that is a necessary (though not sufficient) condition for dynamical stability. However, some insight can be gained from models that are mathematically simpler. Most of the published work also assumes both symmetry about the rotation axis and a steady state (extra reasons for caution in application to observed magnetic stars). Uniform rotation is also assumed, for only in a strictly axisymmetric state is time-independence consistent with the generalization to isorotation (with a different constant value of Ω along individual field-lines); if the work is to be extrapolated to oblique rotators, "quasi-steadiness" requires uniform rotation, identical with that of the frame in which the observer sees no rotatory motions.

The stellar models mainly studied are appropriate to an upper main sequence star, with a convective core and a radiative envelope. Nonuniformities in mean molecular weight μ are assumed absent from the envelope (but cf. below). A linear perturbation treatment breaks up the density-temperature-pressure field into $\rho = \rho_0 + \rho_\Omega + \rho_B$ etc., in an obvious notation. In general the resulting nonradial radiation flux-vector $\mathbf{F} = \mathbf{F}_0 + \mathbf{F}_\Omega + \mathbf{F}_B$ does not satisfy radiative equilibrium; in particular, the quantity $\nabla \cdot \mathbf{F}_\Omega \neq 0$ in a uniformly rotating star (von Zeipel's result). However, as a first example one can construct a model of a rotating magnetic star in which $\nabla \cdot \mathbf{F}_\Omega$ and $\nabla \cdot \mathbf{F}_B$ do mutually cancel throughout the whole of the radiative envelope. Not surprisingly, this requires that the magnetic field be much more condensed towards the high-density interior than, for example, in Cowling's principal decay modes (Wright 1969, Moss 1973, 1975). The result can be stated in this form: for a prescribed total magnetic flux F_t, then with strict radiative equilibrium always presumed, a steady increase in Ω yields a steady decrease in the flux F_s that emerges from the surface. Beyond a certain value of Ω, local radiative equilibrium breaks down; the most relevant steady-state models then have the familiar Eddington-Vogt-Sweet circulation flowing over the bulk of the radiative zone, driven by the centrifugally caused breakdown in radiative equilibrum (Mestel & Moss 1977, Moss 1977a). The conductivity is high enough for the magnetic Reynolds number to be large; the flow, though slow, is "inexorable" and will tend to distort the field into a structure satisfying approximately $\rho v/B$ = constant, again implying a concentration of flux into the higher density regions.[1] Near the surface, however, the density is low enough for the magnetic force per gram not to be negligible compared with the centrifugal, and also the conductivity is low enough for the largely horizontal flow (written as $\mathbf{v}_\Omega + \mathbf{v}_B$) to cross the field-lines, so allowing some flux to emerge. The general conclusion for aligned models is that **B** is again strongly concentrated towards the high-density regions—ratios $F_t/F_s \sim 10^3$ are typical—and the ratio again increases with Ω. Note however that none of the models suggest that the total magnetic energy should be more than a small fraction ϵ of the star's thermal and gravitational energy.

The general trend of these results was considered both intuitively reasonable and encouraging for comparison with observation, especially if interpreted using the fossil theory. One expects magnetic stars to be slow rotators because of magnetic braking, but perhaps the interaction between **B** and Ω is two-way, with the slow but inexorable circulation tending to steadily bury flux deep down, removing from the surface the means by which angular momentum is lost. Both processes are comparatively slow, and in fact discussion in terms of strictly steady states can be misleading (cf. below). One need not be surprised to find some rapidly rotating magnetic stars among the Ap stars and the newly discovered earlier-type magnetic stars. The theory (so far) would predict that, given sufficient time, the stars would either be magnetically braked, or that the flux would be concentrated below and the star would cease to be observably magnetic. The steady-state models could also be applied to yield an anticorrelation between F_s and Ω within the class of magnetic stars, as was tentatively suggested by the observations (e.g. Preston 1971b, Borra & Landstreet 1980). Note also that the same effects would occur if the field were dynamo-maintained, with the difference now that the total flux would itself depend on Ω. If F_t decreases with Ω (e.g. as in Moffatt's model—see below), then this could explain a decrease of F_s with Ω; but even if F_t increases moderately with Ω, the effects just discussed could still yield a decrease of F_s with Ω (Mestel & Moss 1977).

It should be emphasized that in much of this work, there are severe mathematical difficulties, which enforce the premature truncation of expansions in orthogonal polynomials. This is probably the reason for the failure of Mestel & Moss to cover all the physically allowed (and much of the astronomically

[1]Note that although in this class of model the weak magnetic field hardly affects hydrostatic equilibrium over the bulk of the star, it remains crucial to the picture, since it offsets advection of angular momentum by the circulation and so keeps the rotation effectively uniform: all that is required is that the very slow circulation be well below the Alfvén speed. This mixed "passive-active" role of the field was recognized as essential in the earlier studies (Sweet 1950, Mestel 1953) and more explicitly in Mestel (1961, 1965) and Roxburgh (1963). The problems of the evolution of the rotation and circulation fields in a strictly nonmagnetic star (e.g. Busse 1981) largely disappear in a star with even a very weak general magnetic field.

interesting) parameter range. The attempts on the perpendicular rotator have had markedly less success, probably for similar reasons (Monaghan 1973, Moss 1977b). The limited results found now show a surprising slight *positive* correlation between Ω and F_s. Moss (1981) notes that this is contrary to intuition but possibly more in line with recent observations of Borra (1981) (but see below). However, the small parameter range covered and the mathematical difficulties encountered make it difficult to say more than that the conclusions for the axisymmetric models in radiative equilibrium have not been shown to hold for rotators of high obliquity.

Furthermore, a crucial assumption of all the analysis is that of a zero μ-gradient through the radiative envelope. Even if the envelopes of upper main sequence stars begin their lives as chemically homogeneous (a plausible consequence of a pre-main-sequence phase with an extensive outer convective zone), the steady increase in μ during stellar evolution may not be limited to regions that are convective during at least part of their main sequence life. The models of magnetic stars with the unmodified Eddington-Sweet circulation deep down assume that at the surface of the convective core there is a "μ-barrier" where the vertical velocity is reduced to zero; the simple theory outlined above predicts large horizontal velocities from continuity, with an associated large local concentration of magnetic flux. The μ-barrier region is thus similar to the surface regions, in that locally the magnetic forces also contribute to hydrostatic and thermal balance. To validate this picture, one should look for a local self-consistent steady state in which the combined effect of centrifugal and magnetic forces reduces the thermally driven vertical velocity to zero at the barrier. Rough estimates suggest that the magnetic contribution should be significant in a boundary layer of thickness $D \simeq r \cdot(\bar{B}^2/8\pi\bar{\rho}\Omega^2 r^2)^{1/3}$, where $\bar{B}$ is of the order of the field deep in the star but outside the layer. However, the attempted construction of such a model has run into even more severe truncation difficulties than in the surface regions. More seriously, it is not clear that one can satisfy the conditions of zero slip and zero stress at the barrier that are strictly required, even though the viscosity (radiative and ionic) of stellar material is so small. It may be that the picture of a μ-barrier separating the domain with a steady increase in μ from an envelope with a zero μ-gradient should be questioned (cf. the nonmagnetic study of Huppert & Spiegel 1977). Instead, perhaps a μ-gradient steadily extends itself through the radiative zone. The Eddington-Sweet circulation would then try to establish a nonspherical μ-distribution, which in turn would react back on the temperature distribution, yielding a "μ-current" $\mathbf{v}_\mu$ (Mestel 1953, 1965, Moss 1980). From the computations done to date, it appears that near the core $\mathbf{v}_\mu$ will oppose $\mathbf{v}_\Omega$, and will prevent a continual mixing of matter between core and envelope (except possibly for stars rotating at the centrifugal limit). This was the reason for the original picture of the μ-barrier separating the constant μ envelope with its steady circulation from a homogeneous core, with μ steadily increasing through normal thermonuclear processes. However, it may be that the Eddington-Sweet circulation suffers from a "creeping paralysis," with $\mathbf{v} = \mathbf{v}_\Omega + \mathbf{v}_\mu \rightarrow 0$. If so, then the tendency of the magnetic field to be concentrated into the high-density regions will be at most temporary; as the circulation dies out, the field over the bulk of the radiative envelope would diffuse back into something closer to a Cowling mode, with a much smaller ratio of F_t/F_s than that given by the various models discussed above. With μ-effects ignored, the previous results were applied to stars with F_s of the order observed, implying a much larger F_t than that given by the Cowling modes. One can now argue that the total flux F_t is not, in general, large, so that the initial concentration of flux deep down by an unimpeded Eddington-Sweet circulation in a rapidly rotating star would ensure that the surface field is unobservably small. As the μ-distribution spreads through the envelope and kills off the circulation, the consequent diffusion of the field will cause F_s to increase to an observable level. Subsequent magnetic braking would then ensure that a magnetic star would rotate at the normal A star rate only for a limited period.

A μ-gradient is likely to be established near the surface of A stars, especially if a magnetic field assists in stabilizing the atmosphere against circulation and any weak turbulence (cf. Michaud 1970, 1980, Vauclair & Vauclair 1982). In their model of the surface regions of a rotating magnetic star, Mestel & Moss (1977) find that $\mathbf{v}_\Omega + \mathbf{v}_B \rightarrow 0$ near the surface (with $\mathbf{v}_\Omega \neq 0$). With a $\mathbf{v}_\mu$ contribution present, the total velocity is forced by the hydromagnetic condition to vanish at the (mathematical) surface, but now it is possible for $\mathbf{v}_\Omega + \mathbf{v}_\mu \rightarrow 0$ and $\mathbf{v}_B \rightarrow 0$ separately. However in some unpublished work, Moss finds that the consequent effect on the ratio of internal to external field is likely to be small: one needs a μ-gradient through a large fraction of the envelope for these effects to be important.

The aligned ($\chi = 0$) and perpendicular ($\chi = \pi/2$) rotators are special in that the only internal motions expected inside the stars are the thermally driven circulation and the turbulence in superadiabatic domains. The generally oblique magnetic rotator has some properties in common with the classical problem of a body with unequal axes of inertia rotating about a general axis (Spitzer 1958). In slowly rotating stars, the departures from spherical symmetry of the density-pressure field are the linear superposition of a part symmetric about the rotation axis **k** and a part symmetric about the magnetic axis **p**. (Even for a rapid but *uniform* rotator, such a linear superposition is tolerable over the bulk of the star, where central condensation ensures that the ratio of centrifugal force to gravity must be well below unity.) The motions within such a body can be most simply analyzed into (1) the basic rotation $\Omega\mathbf{k}$; (2) the Eulerian nutation $\omega\mathbf{p}$, with $\omega \sim \Omega\epsilon$, enforced by the condition that the angular momentum vector remain invariant in space; and (3) a field of "ξ-motions" with frequency ω, which ensures that the star remains in hydrostatic equilibrium (Mestel & Takhar 1972, Mestel et al. 1981, Nittmann & Wood 1981). To order of magnitude $\xi \sim l(\Omega^2 r^3/Gm(r))$, where l is the local scale height. The nutation period $2\pi/\omega$ is much greater than the rotation period $2\pi/\Omega$, but can easily be well below the stellar lifetime or the Kelvin-Helmholtz time. In slow rotators ξ/l is small, but the generalization to rapid rotators strongly suggests that the motions could yield mixing of matter between a convective core and at least part of a stellar envelope, so modifying tracks in the H-R diagram; in fact, Nittman & Wood (1981) have suggested ξ-motion mixing in rapid rotators as an explanation of the blue straggler phenomenon. And as already noted, the presence of a μ-gradient in a radiative zone can affect indirectly the magnetic flux distribution through the star, and so the observability of a field at the surface.

However, such applications depend on the motions persisting over all or most of the star's lifetime. As pointed out by Spitzer (1958), these dynamically driven motions will be subject to dissipation, which acts as a drain on the rotational kinetic energy of the star. This has the form $\mathbf{h}^2/2I$, where $\mathbf{h}$ is the angular momentum and I the moment of inertia about the instantaneous axis of rotation. Thus if dissipation is efficient enough, and if nothing else intervenes to affect the flux distribution, the magnetic axis should rotate in space (and the rotation axis simultaneously precess through the star) until the star is rotating about its maximum moment of inertia and the ξ-motions cease. Originally the theory was tentatively connected with the requirement that stable magnetic fields must have linked poloidal and toroidal flux of comparable magnitudes. The suggestion was that stars with dominant poloidal flux would tend to be dynamically oblate about the field axis, and would tend to approach $\chi = 0$, while those with dominant toroidal flux would be dynamically prolate and would approach $\chi = \pi/2$; this in turn appeared to fit in with earlier reports that the obliquity angles seemed to be concentrated towards either small or large χ values. We have seen that the observational evidence is now less clear. The Krause-Oetken dynamo model (with its rather special element distribution) does have $\chi = \pi/2$; and as remarked by Moss (1981), if this model were to be phenomenologically acceptable, then it could alternatively be explained using the fossil theory, with a sufficiently powerful dissipation mechanism acting on systems with dominant toroidal flux. If on the other hand one accepts that the observations imply a random or only marginally nonrandom χ-distribution, then to fit in with the fossil theory in its simplest form, we require that the time τ_χ in which dissipation causes χ to approach one asymptotic value or another should in fact be too long to be of interest, so that an initial χ value and the associated ξ-motions would persist over a main sequence time τ_{ms}. In the most recent papers τ_χ is computed for a number of model fields. The dominant dissipative process in radiative envelopes is apparently Ohmic dissipation of the currents associated with the distortion of the basic magnetic field by the $\boldsymbol{\xi}$-motions. The nutation frequency ω and the time τ_χ are sensitive to the internal concentration of the magnetic flux. It appears that for fields with a modest ratio of internal to surface field, such as the principal Cowling mode, then $\tau_\chi > \tau_{\rm ms}$; but for virtually all the (zero μ-gradient) models constructed earlier, with strong inward increase of B, τ_χ is embarrassingly short. The results may force advocates of the fossil theory to accept that the flux $F_{\rm t}$ is indeed low, but that there is also a μ-gradient through much of a radiative envelope that suppresses the Eddington-Sweet circulation, which would otherwise concentrate flux deep down and render the star magnetically unobservable.

We emphasize again that much of this theoretical work is also relevant to steady dynamo-generated fields. For example, if the Krause-Oetken perpendicular rotator has a dominantly toroidal field deep down, so that the star is dynamically prolate about **p**, then it is already in a state of minimum energy: any slight deviations from strict perpendicularity will lead to $\boldsymbol{\xi}$-motions, but dissipation of their energy will re-establish the perpendicular state. If the model is dynamically oblate about **p**, then an equally slight departure from perpendicularity will be steadily increased, but at a rate dependent on the dissipation. The new factor is that in the Krause-Oetken picture, the motions in the core preferentially generate a perpendicular rotator. If the non-perpendicular part of a field spontaneously decays (due not so much to Ohmic resistivity alone, but to some as yet unanalyzed instability; cf. above), then a complete theory of the perpendicular rotator will need to estimate and contrast the different time scales. Note that in Mestel et al. (1981), it is assumed that the fossil field is excluded from the convective core, so that the $\boldsymbol{\xi}$-motions are primarily in the envelope, and dissipation occurs mainly in the high-resistivity outer regions of the star; whereas with a dynamo-generated field, the $\boldsymbol{\xi}$-motions in the core will be subject to a turbulent viscous drag, yielding a shorter time τ_χ. A link-up between the Krause-Oetken model and the $\boldsymbol{\xi}$-motion theory would be of interest.

The surface magnetic field is presumably the means by which the magnetic Ap stars are braked. Contrary to the results of Hartoog (1977), there is now observational support for angular momentum loss on the main sequence (Abt 1979, Borra 1981, Wolff 1981). Efficient braking via a magnetically controlled stellar wind requires the star to be surrounded by a corona hot enough for the sonic point to be in a region of reasonably high density at the coronal base. It is not clear how such a hot corona would arise in the absence of a strong outer convection zone (cf. Nakajima 1979). A rapidly rotating star with a cooler corona will drive a centrifugal wind (Mestel 1968a), but the braking rate declines rapidly as soon as the star slows up a little and the sonic point begins moving out. Furthermore, if the corona is hot and the surface flux F_s is fixed, then the Alfvénic radius R_A—the radius of effective corotation of the outflowing gas—depends only weakly on Ω, so that the simple wind theory yields for Ω an uncomfortable exponential decrease with time; one would require very special parameter values to produce rotation periods close to those observed in the majority of Ap stars. There is also the danger of a wind removing the outer regions too quickly, embarrassing the theory by which the abundance anomalies appear through diffusion (Michaud 1970, 1980). A magnetic accretion model (Mestel 1975, Arons & Lea 1976a,b, Wolff 1981) similar to that discussed for neutron star X-ray source models looks more promising. Accreted material piles up near the magnetospheric boundary of radius R_A, where now the accretion velocity equals the Alfvén speed. The configuration is Rayleigh-Taylor unstable; material falling into the magnetosphere will pick up angular momentum from the magnetic pressures, and as long as $\Omega^2 R_A{}^3 > GM$ it will be shot out to infinity by the centrifugal slingshot process. Even if the accretion is from the low-density interstellar medium, rather than from a companion star, the process appears efficient enough to yield a characteristic time scale of $\simeq 10^7$–10^8 yr; the process spontaneously cuts off when $\Omega^2 R_A{}^3 \simeq GM$, yielding periods of the right order for most of the magnetic stars. A more detailed study of the problem would be welcome.

Magnetic braking probably occurs also during the pre-main-sequence phase. Wolff (1981) concludes that whereas the Si group are braked predominantly on the main sequence, the cooler, less massive Sr-Eu-Cr group suffer extra braking during their much longer Hayashi phase. If it could be shown that there are in fact no *normal*, rapidly rotating A stars in the mass range of the non-Si group, then one could argue that the slow rotations are simply the expected consequence of the longer Hayashi phase. On the other hand, if only a fraction of this mass range end up as magnetic Ap stars, one will have to look for parameter differences: for example, the future non-Si Ap stars may have retained enough primeval magnetic flux to suffer excess braking through coupling with the pre-main-sequence wind. With accretion braking on the main sequence cutting off at a period near that of the moderately slow Ap stars, one certainly needs a more efficient process to produce the much longer periods also found. Since braking via a wind coupled to the star by a surface field of fixed flux—as opposed to a dynamo-built field declining with Ω (cf. below)—yields an exponential rather than an algebraic braking law, it is not impossible that the small number of stars with observed magnetic periods of the order of years have indeed suffered exceptionally efficient pre-main-sequence braking.

A secondary effect of magnetic braking is the associated precessional torque, which also tends to alter the obliquity angle χ (Mestel 1968b). Only one case has been worked out in detail when braking is via a wind and with a specially simple magnetic field structure (Mestel & Selley 1970). The effect was found to be small, but it should be greater for more realistic fields. An acceptable braking process must be required simultaneously not to establish a χ-distribution in conflict with observation (cf. above).

Confrontation with Observation

Both fossil and dynamo theories have to cope with the fact that stars which are apparently similar in other respects are not magnetically similar. Not all slowly rotating A stars are observably magnetic, even though they all seem to be either of Am- or Ap-type. There is the minority of magnetic Ap stars that do rotate rapidly, even though most rapidly rotating A stars are both spectroscopically normal and not observably magnetic. Within the class of magnetic Ap stars, there is a bewildering lack of correlation where (perhaps naively) one might expect it (e.g. pairs of stars with very similar Zeeman curves but widely differing periods).

The answer of the fossil theory (e.g. Mestel 1976) is to have the initial magnetic flux possessed by a star as an extra parameter. The observations are then to be interpreted in terms of a competition between the rate of flux decay, the rate of magnetic braking, and the tendency of rotationally driven circulation to bury the field deep down. As outlined above, the presence of a μ-gradient through the star, though not yet fully explored, is likely to affect both the general picture and some of the numbers deduced. Such a scheme could be appropriate for the more massive (Si) Ap stars that appear to reach the main sequence rotating rapidly. Schüssler (1980) has applied a similar scheme to the pre-main-sequence phase, where it appears that the less massive Ap stars have suffered much (if not all) of their excess braking, perhaps via a wind coupled to a primeval field. This may be appropriate for the non-Si Ap stars. (We should note that there is no generally agreed-upon theory for rotating convective zones of the large-scale laminar circulation—analogous to the Eddington-Sweet circulation for rotating magnetic radiative zones—that would presumably be responsible for the tendency of flux to be buried in the dense regions below.)

The fossil theory must find a place for the nonmagnetic slow rotators—the Am stars, the Hg-Mn group, and their possible extrapolation to higher surface temperatures (Wolff 1981). This reduces to the question: how did these stars acquire the slow rotation that is apparently necessary for the abundance peculiarities to develop? For stars that are or have been in moderately close binary systems, spin-orbit synchronization is probably the answer. One may have to appeal to as yet unknown special conditions that enabled some stars to suffer excess loss of angular momentum in the later stages of star formation. It is also possible that we are witnessing the effect of small differences between the decay times of fossil fields in stars that have extended nonuniform μ-distributions and those that do not.

The gross anticorrelation shown (at least by the A stars) between rotation and appearance of field above the surface is in striking constrast to the growing body of observational material on the late-type stars with strong outer convection zones, where we would expect solar-type activity to appear. We may plausibly accept that Ca II emission is a good diagnostic of solar-type dynamo activity. The pioneering work of Wilson (1978) has shown convincingly the occurrence of periodicity reminiscent of the solar cycle in a number of G, K, and M stars. The general positive correlation between Ca activity and stellar rotation has been demonstrated both for late-type main sequence stars (Wilson 1966, Kraft 1967) and for giants (Middelkoop & Zwaan 1981). Most spectacular are the RS CVn close binary systems, where tidal interaction can be expected to ensure rapid rotation through synchronization of spin and orbital motion. These stars show what appears to be greatly enhanced solar activity, with strong chromospheric, radio, and X-ray emission, and evidence that a large fraction of their surfaces is covered by dark spots (Hall 1976, papers in Plavec et al. 1980). These observations do *prima facie* fit into a simple picture, in which the strength of the dynamo-built field increases with rotation. One can certainly expect such an increase with Ω of the flux generated in steady dynamos as well, at least as long as the reaction of the rotation on the turbulence is not important. *Prima facie*, this is a problem for the dynamo theory of magnetic A stars.

However, there are several possible answers within the framework of dynamo theory. Moffatt (1970, 1972) has produced a dynamo model in which increasing rotation makes the motions conform closer to the Taylor-Proudman theorem and to become more nearly two-dimensional, reducing the "α-effect" and also the strength of the field generated. Other workers emphasize that flux generated in a convective core has to reach the surface to be visible. Schüssler & Pähler (1978) showed that if flux migrates purely by Ohmic diffusion, then the time scale is so long that the more massive stars would have left the main sequence before the field became visible. Moss (1980) pointed out a further consequence: if the field generated were in fact oscillatory rather than steady, then the surface field would be unobservable. Noting a tendency for faster rotating dynamos to be oscillatory (Parker 1979), he suggested this as the reason why rapidly rotating A stars are normally nonmagnetic. Thus if the transition from an oscillatory to a steady dynamo occurs at rotation periods typical of the Ap stars, then one could perhaps argue that the fields of the rapidly rotating magnetic stars are indeed fossils, but with a decay time of the order 10^8 years, as suggested by Borra's (1981) observations. This field is able to brake the star before it decays: subsequently, the steady dynamo field is generated and diffuses to the surface.

A different explanation has been proposed by Krause (1982), assuming his dynamo-maintained perpendicular rotator is the correct model for the Ap star fields. Faced with the question of why there are any normal, rapidly rotating nonmagnetic A stars, i.e. of why they do not all build up magnetic fields which subsequently brake them, he effectively replaces the suggestion of a fossil magnetic field by a fossil rotation field. Rädler (1982) has argued that differential rotation destroys the part of the magnetic field that does not have symmetry about the rotation axis: the winding of the field leads to the juxtaposition and mutual annihilation of oppositely directed field-lines. Krause suggests that the magnetic Ap stars are those for which the differential rotation in the envelope is zero or weak, so that the dynamo-generated (perpendicular) field in the core can rise to the surface; if there is a strong differential rotation, the nonaxisymmetric core field is effectively screened. He suggests further that there may not be any differences in total angular momentum between different types of A star, but merely in its distribution: the normal A stars with their observed rapid rotations may have a markedly nonuniform Ω-field.

This picture clearly depends on the nonuniform Ω-field in the radiative zone persisting over all or most of the star's lifetime. A nonconservative centrifugal field is known to be subject to the Goldreich-Schubert-Fricke instabilities, but debate continues on their nonlinear development and on the time scale in which they cause a significant change in the angular velocity field. A recent study (Kippenhahn et al. 1980) agrees essentially with the earlier estimates of James & Kahn (1970) that the time scale is of the order of the Eddington-Sweet time, which is at least comparable with the Kelvin-Helmholtz time, and will be markedly longer if the internal rotation period is 5 days or so. However, these treatments explicitly ignore any magnetic reaction on the rotation field. There should be continual generation of flux by dynamo action in the core, followed by twisting of the field lines emanating into the radiative zone by the nonuniform rotation. The stresses exerted by the distorted field must react back on the rotation field. It is the energy of the differential rotation that is continually converted into magnetic energy and dissipated. The presence of the turbulent core may actually assist the process, for any Alfvén waves propagated into the core will be subject to turbulent viscous drag. Under the most favourable conditions, a shear may be effectively ironed out within a few Alfvén travel times across the sheared region, and this will be far less than a stellar lifetime. A detailed study is clearly desirable; meanwhile one feels that for the nonuniform rotation field to persist in spite of the continual generation of flux in the core is a very severe requirement.

Even if dynamo theory can explain why most of the rapid rotators are not observably magnetic, there remains the question of the slowly rotating but nonmagnetic Ap stars and the Am stars. Again, one wonders whether the slow diffusion of a dynamo-built field from below is being impeded by circulation; perhaps only when a μ-gradient halts the circulation is a spontaneously generated field able to appear at the surface. And is the μ-distribution perhaps not fixed uniquely by the structure of the star, but instead has some "fossil element?"

The interpretation of observations is frustratingly ambiguous, and it may be particularly misleading to think in terms of steady-state models if the time scales of the active processes are long. For example, we have referred to Borra's (1981) observations, and given a possible interpretation, which involved both fossil and dynamo-maintained fields. Borra studied the magnetic fields of 13 Ap stars and helium-weak stars in the Orion OB1 association, concluding that (1) young magnetic stars rotate faster than the older magnetic stars, confirming that magnetic braking occurs on the main sequence; and that (2) the fields of the magnetic stars in Orion are stronger by a factor of 3 than those in the older magnetic stars. Can these observations be interpreted in terms of the fossil theory alone? If the second result is an indication of decay of the total flux F_t, then the e-folding time is $\simeq 10^8$ yr—shorter by more than an order of magnitude than the classical Cowling estimate, but not to be ruled out if decay results from slow, thermally controlled magnetic buoyancy effects. But is the decay conclusion enforced? Perhaps the field has a flux that is essentially fixed, but is adjusting steadily to the condition of radiative equilibrium: maybe we are seeing a manifestation of the surprising result found for perpendicular rotators, that over some parameter range, if radiative equilibrium holds, the reduction of Ω actually decreases the ratio F_s/F_t (Monaghan 1973, Moss 1977b). Alternatively if the stars have not yet reached a strictly steady state, could we be witnessing just the tendency of a slow circulation to drag primeval flux beneath the surface, which continues (at a slower rate) even though the star is being braked?

Directions of Future Theoretical Work

It is clear that we need to know much more about basic stellar hydromagnetics, such as dynamo theory (especially the full dynamical problem) and the related problems of magnetic stability (both dynamical and secular). Some of the work summarized above, stimulated originally by the fossil theory, could advantageously be linked up with current dynamo models. More subtle studies are required of the low-density surface regions. A modest departure from a curl-free structure for the field can make a significant difference to the subphotospheric density field. This may result in an observable shift in the H-R diagram, and also in a periodically varying projected disk, which could account for the small magnitude variations that accompany the magnetic and spectral variations. It is encouraging that observations of B_s enforce the construction of decentered field models, so as to avoid the prediction of two B_s maxima per B_e cycle: the magnitude variations due essentially to the effects of the Lorentz forces exerted by a displaced dipole should likewise have the rotation period, rather than the half-period to be expected in a strictly centered dipole. A challenging problem is that of the oscillations of magnetic stars (e.g. Biront et al. 1982), made topical by observations of fields in some Cepheids, by the possibility that RR Lyrae is an oblique rotator with a period of several weeks, and by Kurtz's (1981) discovery of very-high-frequency oscillations in magnetic stars.

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