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The Drake Equation Re-examined

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SUMMARY

The Drake equation, which predicts the number of extant technological civilizations in the Galaxy, is considered. A formulation originally due to Kreifeldt, and assumptions about civilization development time and lifetime, are used to set an upper limit to the number of extant civilizations. This number, $N_c(T)$, is found to be of order 10^0–10^2, rather than the 10^6 suggested previously by other authors. The implications of this much-reduced Galactic population on the SETI program and the arguments of Tipler against the existence of extraterrestrial life are discussed.

1 INTRODUCTION

The possible existence of extraterrestrial technological civilizations has intrigued mankind throughout its history. Our civilization has now reached the point at which direct communication over short interstellar distances is feasible. In addition, the detection of marker beacons over galactic distances is now possible. Therefore an evaluation of the number of extant technological civilizations is of prime importance in evaluating the prospects of success for a SETI (Search for Extra-Terrestrial Intelligence) program. In this paper we re-examine the equation, first developed by Drake (1), which is used to predict this number. After evaluating expressions for the development time, and lifetime of a civilization, we are able to reformulate the Drake equation in terms of parameters which describe these expressions. This allows us to examine the dependence of N_c, the number of civilizations, upon the different possible distributions of development time and lifetime.

2 SHORT HISTORY OF THE DRAKE EQUATION

Drake (1) devised an expression for the number of communicative technological civilizations in the galaxy:

$$N_c = R \cdot f_s \cdot L \tag{1}$$

where R is the average rate of 'suitable' star production, f_s is a factor such that Rf_s is the average civilization production rate, and L is the average communicative lifetime of a civilization. Shklovskii & Sagan (2) choose to express f_s as $f_s = f_p\, n_e\, f_L\, f_i\, f_c$; the various parameters composing f_s are defined in Table I. They estimate R to be 10^{11} stars produced in 10^{10} years or 10 per year, and evaluate $f_s \approx 0{\cdot}01$. They then estimate the average lifetime of a technological civilization as $L \sim 10^7$ years, for a total number of civilizations of $N_c \sim 10^6$. As this number of civilizations leads to an average distance between members of 10^2 pc, communication between them is a possibility, and SETI seems reasonable.

TABLE I

Definitions of Drake Equation Parameters

R	Average star production rate.
f_g	Fraction of stars that are single F, G, or K dwarfs.
f_s	Number of civilizations per suitable star.
f_p	Fraction of stars with planets.
n_e	Number of suitable planets per star.
f_L	Fraction of suitable planets which evolve life.
f_i	Fraction of planets possessing life which develop intelligent life.
f_c	Fraction of planets with intelligent life which develop a technological civilization.
H_T	Characteristic time for evolution of a civilization.
H_*	Characteristic decay time for galactic star formation rate.

Kreifeldt (3) has reformulated the Drake equation so that different star generation rates, development time distributions, and lifetime distributions can be considered. According to this formulation, the development time, T_0, is a random variable with probability distribution $p_T(T_0)$, and the lifetime L is a variable with distribution $p_L(L)$. Then the probability that a suitable star of age X currently possesses a civilization which is in the communicative phase is:

$$\Pi(X) = C_{T_0}(X) - \int_{-\infty}^{\infty} C_{T_0}(X-L)\, p_L(L)\, dL \tag{2}$$

in which

$$C_{T_0}(X) \equiv \int_{-\infty}^{X} p_{T_0}(a)\, da \tag{3}$$

is the total probability that a star of age X has evolved a civilization on one of its planets. Then, if $R(t)$, the suitable star production rate, is known, we have:

$$N_c(T) = \int_{0}^{T} R(T-x)\, \Pi(x)\, dx \tag{4}$$

for a galactic age T. With these equations, Kreifeldt (3) points out that $N_c(T)$ can be evaluated for a variety of choices of $R(t)$, $p_T(T_0)$, and $p_L(L)$.

3 AN ESTIMATE OF N_c

We begin by postulating a functional form for $C_{T_0}(X)$. Since the probability that a suitable planet possesses a civilization increases with the passage of time, we can adopt, for $C_{T_0}(t)$:

$$C_{T_0}(t) = f_L f_i f_c\, [1 - \exp(-t/H_T)]. \tag{5}$$

With this assumption, equation (2) becomes:

$$\Pi(X) = f_L f_i f_c \exp(-X/H_T) \left[\int_{-\infty}^{\infty} \exp(L/H_T)\, p_L(L)\, dL - 1 \right]. \qquad (6)$$

In deriving equation (6) we have made use of the fact that $p_L(L)$ is a normalized probability distribution.

To estimate the suitable star formation rate, we can use Fig. 1 of Oliver (4), which leads to a rate which increases linearly for $0 \leq T \leq 3\times10^9$ yr, and decreases exponentially thereafter. We write:

$$R_*(t) = f_g f_p n_e \begin{cases} (7\times10^{-9})\, t & 0 \leq t \leq 3\times10^9 \\ (44) \exp(-t/H_*) & 3\times10^9 \leq t \leq 10^{10} \text{ yr} \end{cases} \qquad (7)$$

where $H_* = 4\times10^9$ years. The fact that the total integral of $R_*(t)$ from $t = 0$ to $t = 10^{10}$ years is $N_{tot} = f_g f_p n_e (10^{11})$ was used to evaluate the constants in equation (7). In the above, f_g is the fraction of stars that are single F, G, or K main-sequence stars. Oliver (4) estimates $f_g \approx 4\times10^{-2}$.

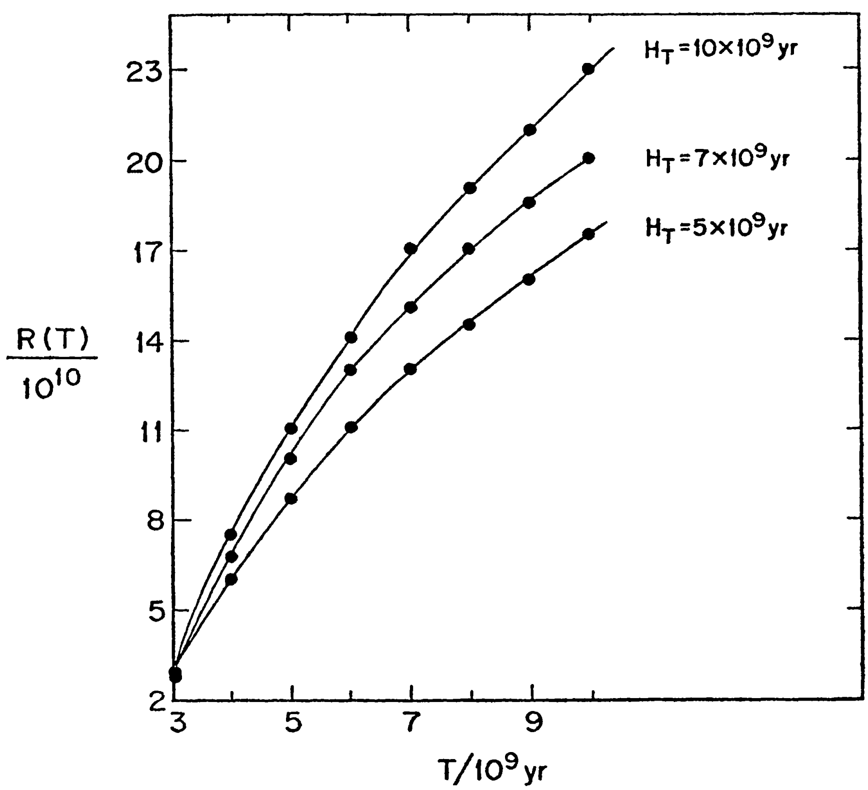

FIG. 1. A plot of the function $R(T)$ v. galactic age, T, for various choice of H_T, the characteristic time for civilization development. The galactic population grows monotonically, with $R\ (T = 10^{10}$ years) equal to 2×10^{11} to within a few tens of per cent, and only slightly dependent on H_T.

It remains to specify a form for $p_L(L)$. We consider two cases: (a) $p_L(L)$ is an exponentially decreasing function of L, with a characteristic time H_L, and (b) $p_L(L)$ is a Gaussian function characterized by a most probable lifetime L_{MAX} and a characteristic time H_L, as before. We will see that these two assumptions lead to very different forms for $N_c(T)$.

(a) *Exponentially Decreasing* $p_L(L)$. If we write, for $p_L(L)$:

$$p_L(L) = (1/H_L)\exp(-L/H_L) \tag{8}$$

Equations (4), (6), (7) and (8) lead to:

$$N_c(T) = f_g f_s \left(\frac{H_L}{H_T - H_L}\right) R(T) \tag{9}$$

with

$$R(T) = 7\times 10^9 \left[T H_T(1-e^{-3/H_T}) + H_T^2\left(\frac{3}{H_T}+1\right)e^{-3/H_T} - H_T^2 + \frac{6{\cdot}3\, e^{-T/4}}{(1/4-1/H_T)}\left\{e^{T(1/4-1/H_T)} - e^{3(1/4-1/H_T)}\right\}\right]. \tag{10}$$

In the above, we have defined $f_s = f_p n_e f_L f_i f_c$ after Shklovskii & Sagan (2), expressed all times in billions of years, and set $H_* = 4$.

(b) *Gaussian* $p_L(L)$. If we write, instead:

$$p_L(L) = \frac{2}{H_L\sqrt{\pi}}\exp\left[-\left(\frac{L-L_{MAX}}{H_L}\right)^2\right], \tag{11}$$

the above equations yield:

$$N_c(T) = f_g f_s\left\{\exp\left[\left(\frac{H_L}{2H_T}\right)^2 + \frac{L_{MAX}}{H_T}\right] - 1\right\} R(T). \tag{12}$$

The function $R(T)$ contains all of the information pertaining to the gradual change in the galactic population of communicative civilizations. Fig. 1 is a plot of $R(T)$ *vs* time for various choices of H_T. It is apparent from the figure that even if H_T varies from 5 to 10 billion years, the number of expected civilizations at $T = 10^{10}$ years varies by only about 50 per cent. So we do not err appreciably in taking a value of R ($T_0 = 10^{10}$ years) $= 2\times 10^{11}$. After Oliver (4), we choose $f_g \approx 4\times 10^{-2}$, and $(f_p n_e) \approx 10^{-1}$. Also from Shklovskii & Sagan (2), we have $(f_L f_i f_c) \approx 10^{-2}$. This gives $f_g f_s = 4\times 10^{-5}$. With these substitutions, equations (9) and (12) become:

$$N_c(T_0) = (8\times 10^6)\begin{cases} H_L/H_T & \text{(13a)} \\ \left(\frac{H_L}{2H_T}\right)^2 + \frac{L_{MAX}}{H_T} & \text{(13b)}\end{cases}$$

for cases (a) and (b), respectively. So we see that the value of $N_c(T_0)$ depends critically on the shapes of the probability distributions used in the calculation of equations (13).

It now remains to estimate H_T, H_L and L_{MAX}. If we hypothesize that the probability of a civilization evolving upon a suitable planet in 6×10^9 years is about 1/2, then equation (5) leads to $H_T = 8{\cdot}7\times10^9$ years. This is in keeping with our earlier assumption about the value of $R(T)$. In addition, we suspect that civilization lifetimes are always small as compared to cosmic timescales; with the assumptions of $H_L \ll H_T$ and $H_L \approx L_{MAX}$, equations (13) become:

$$N_c(T) = (10^{-3})\begin{cases} H_L & \text{(14a)} \\ L_{MAX} & \text{(14b)} \end{cases}.$$

H_L and L_{MAX} are the most difficult parameters to estimate in this formulation. We will use the standard *ad hoc* assumption that most civilizations die out almost immediately, while a few (1/1000) survive for 10^6 years or longer. We can then take L_{MAX}, the most likely civilization time, to be equal to or less than 10^5 years. If this is true, then a Gaussian probability distribution for civilization lifetimes leads to a number of civilizations presently in the communicative phase of order 10^2. The assumption that 1 in 1000 civilizations persists for 10^6 years or greater leads to a value of $H_L = 1{\cdot}5\times10^5$ years, and a value of $N_c(T) = 150$. It should be noted at this point that even if we require that 1/10 of all civilizations last longer than 10^6 years, the resulting values of H_L and $N_c(T)$ are only increased by a factor of 3.

If the number of communicative civilizations is ~ 150, rather than 1 million, then their average separation is 2 kpc, making the success probability of a SETI program rather small. With present equipment, a technological civilization may be detectable at kiloparsec distances, so that SETI programs are not ruled out by this estimate of $N_c(T)$. However, a sphere of radius 2 kpc contains a volume of 3×10^{10} pc^3. As the average mass density in the galaxy is $\approx 1\ M_\odot$ pc^{-3}, this implies 3×10^{10} stars in our sphere. Assuming that we can pick out the single F, G and K dwarfs, we are left with $1{\cdot}2\times10^9$ stars (here we have used $f_g \approx 4\times10^{-2}$). Now, at some time in the past, a fraction $f_s \approx 10^{-3}$ of these target stars possessed a technical civilization, for a total of 1 million nearby fossil civilizations! However, only one or two of these target stars are at present owners of planets inhabited by technical civilizations. Even if we have fortuitously selected the most likely wavelengths to monitor, and are able to recognize an extraterrestrial signal when encountered, the chances for a successful SETI must be regarded as very low.

Recently, there has been some discussion as to whether galactic colonization is a reasonable course for civilizations to pursue. Hart (5) argues that interstellar colonization is so simple that the entire galaxy can be occupied in a time of order 10^6 years. Since no solid evidence exists for the presence of extraterrestrial life on Earth, he concludes that we are the only advanced technological civilization in the galaxy. Both Cox (6) and Walters *et al.* (7) have pointed out possible limitations to expansion which might prevent a civilization from filling the galaxy. However, Jones (8) has studied the interstellar colonization process and concludes that the sphere of colonized stellar systems that a colonizing civilization would produce expands at a rate $v_{eff} \approx 0{\cdot}06 v_{ship}$. If the ship speed is 0·1 c, the 'boundary of empire' crosses the 25 kpc galactic diameter in only 15 million years, a short time compared with the galactic age.

Another compelling argument that galactic colonization can be accomplished quickly has been made by Tipler (9), who points out that Von Neumann machines, capable of replicating themselves and (if necessary) human beings, could colonize the galaxy in less than 300 million years. Since these machines (or the beings who could design them) have not reached the Earth, Tipler (9) argues that no extraterrestrial life exists. However, several counter-arguments present themselves. First, we have assumed in our analysis that only a small fraction of civilizations survive for more than 1 million years. If expansion of empire ceases when the founding civilizations ends, as seems likely, then the fact that there are no extraterrestrials on the Earth means only that there are no *ancient* civilizations in the galaxy, not that we are alone. Secondly, we have not yet considered the likelihood of a civilization's attempting to colonize the galaxy. Suppose that there are 10^6 civilizations in the galaxy, and that they are *all* long-lived. If even one of them had attempted colonization, the galaxy would be full by now. So the probability that a civilization will attempt colonization must be much less than $p_{crit} \sim \frac{1}{f_{old} N_c}$ to maintain an empire-free galaxy. Here N_c is the number of civilizations, and f_{old} is the fraction of them which live to ages older than $\sim 10^6$ years. If $N_c \sim 10^6$ and $f_{old} \sim 1$, then $p_{crit} \sim 10^{-6}$, a very strict requirement. However, if $N_c \sim 150$ and $f_{old} \sim 0{\cdot}001$, then $p_{crit} > 1$, and we would expect the galaxy to be empty even if all civilizations that survive for any length of time attempt colonization. So the smaller N_c is, the less surprising is the fact that no one has colonized the galaxy yet.

There is another reason why we expect the galaxy to be empire-free, if N_c is small. It has to do with the fact that the value which each civilization assumes for N_c will itself influence the decision to colonize. If we concluded that there are many civilizations in the galaxy, we might be inclined to colonize the galaxy and contact them. We might do this despite our knowledge that contact between civilizations leads, on occasion, to the destruction of one. If a few civilizations were destroyed, there would be plenty more in the galaxy to take their place and communicate with us (this philosophy has certainly been apparent in the dealings of human society with animal society here on Earth, as the growing number of extinct species indicates). However, if we concluded that only 150 civilizations exist in the galaxy, we would perhaps be less inclined to risk disturbing one by overzealous exploration. Finally, we draw attention to the 'Stagnation Hypothesis' of Von Hoerner (10), which asserts that planetary civilizations must stagnate economically while they are still young in order to avoid destroying themselves by over-consumption and over-population. If this stagnation is reached before the civilization is mature enough to commit itself to interstellar voyages, then no colonization will ever take place. Therefore, it is plausible that less than 1 in 100 civilizations attempts to colonize, and a galaxy with only 100 civilizations may remain largely empty, no matter what the ages of the civilizations inhabiting it.

The only data we possess pertaining to the possible existence of interstellar colonization is that, at present, we do not believe that extraterrestrials have visited the Earth. An additional fact that should be mentioned, however, is that we have no evidence that small, nearby empires exist. We suspect that when empires grow to the point where their size is a significant fraction of the separation between them, the likelihood of detection is greatly increased. Since the mean separation of civilizations is 2 kpc, we ask what sort of empire stretches $\sim$ 100 pc in radius. Assuming a $v_m \approx 0{\cdot}006$ c, we find that such an empire is produced by a civilization in 54 000 years. With $H_L = 1{\cdot}45 \times 10^5$ years, a full 70 per cent of all civilizations have lifetimes in excess of this number! In the Local Solar Neighbourhood the mass density is $\rho_{LSN} \sim 0{\cdot}1\ M_\odot\ \mathrm{pc}^{-3}$, and if $f_g \approx 4 \times 10^{-2}$ and $f_p\ n_e \approx 10^{-1}$, our 100 pc–radius empire contains 1600 inhabitable planets. Now, according to Jones (8), less than half of these planets will have been colonized. However, an area of space this large which is densely inhabited would not escape detection for long, provided that we were smart enough to know how to search for it. Since the galaxy apparently is not teeming with life, and our corner of it apparently does not contain small empires, we must conclude that either $N_c \sim 150$ and colonization on any scale is unlikely, or $N_c \sim 1$ and we are alone.

4 CONCLUSIONS

We have examined the Drake equation, as formulated by Kreifeldt (3). It was found that, for simple choices of $C_{T_0}(T)$ and $p_L(L)$, $N_c(T)$ scales inversely with the development characteristic time H_T, and linearly with the lifetime characteristic time H_L. If a Gaussian distribution of $p_L(L)$ is chosen, $N_c(T)$ scales linearly with the most probable lifetime, as noted by Drake (1). We have evaluated N_c ($T = 10^{10}$ years) $\sim 10^2$ rather than $N_c \sim 10^6$, as hypothesized by Shklovskii & Sagan (2).

The true number of extant civilizations may, however, be even smaller. Following the argument of Oliver (4), we took $(f_p\ n_e) \sim 10^{-1}$ for about one habitable planet for every 10 suitable stars. However, this may be an overstatement. Hart (11) has attempted to estimate the size of the 'habitable zone' about F, G and K dwarfs, and finds it to be extremely narrow, with width of order 10^{-1} AU at maximum. The probability that a planet falls right on this narrow strip is expected to be small, leading to the conclusion that $(f_p\ n_e) \sim 10^{-1}$ must be viewed as an upper limit. With regard to this point, Bond & Martin (12, 13) have made a conservative estimate of the number of habitable planets in the galaxy. They consider only single, G dwarf stars. Only disc stars are selected, and Pop. II and obviously young Pop. I stars are discarded. They find $f_g \approx 10^{-3}$, a factor of 40 lower than Oliver's estimate. They then use numerical models developed by Dole (14) to estimate the fraction of these stars which contain a planet with Earthlike gravity ($\pm$10 per cent) within the thin habitable zone of 0·95–1·01 AU. Surprisingly, they obtain $f_p\ n_e \cong 1/40$. Bond & Martin's conservative numbers are lower than Oliver's (4) by a factor of 150, so that if we use their conservative estimate we arrive at $N_c(T_0) = 1$. If this estimate is true, there is, of course, no problem in accounting for an empty galaxy. However, as mentioned previously, a small number of civilizations could still meet the observational criteria we have imposed, while at the same time giving us an opportunity to contact an alien civilization (provided we have the patience to sift the heavens looking for them).

A final problem concerns our assumption that $(f_L f_i f_c) \approx 10^{-2}$, i.e. 1 per cent of planets suitable for life develops a technological civilization. Tipler (9) discusses the fact that while most astronomers prefer the above optimistic value for $(f_L f_i f_c)$, most evolutionary biologists prefer a value several orders of magnitude smaller. If these probabilities are in fact lower, we may not only be the only civilization in our Galaxy, but we may find ourselves alone in the Universe.

What then, we must ask, of the prospects for SETI? Recent reports concerning Project Cyclops (15) and SETI (16) have pointed out that the possibility of detecting remote civilizations is a reasonable one, providing that we know where to look, what wavelengths to use, etc. However, an attempt to sift the nearby stars for the one or two civilizations within a kiloparsec is, at best, a difficult task. Our civilization is in its infancy at the present time. As it matures, our SETI strategy will become more efficient. Eventually, we should be able to detect the attempts at contact which other civilizations may make. However, we should not be surprised if no one ever calls.

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