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THE FORMATION OF GALAXIES FROM MASSIVE NEUTRINOS

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ABSTRACT

Neutrinos with nonzero rest mass strongly influence galaxy formation in the early universe. If stable neutrinos have rest masses on the order of 100 eV, they close the universe, but they erase initial perturbations on mass scales less than $4\times10^{15}\ M_\odot$. However, if in addition there exist unstable neutrinos with rest masses on the order of 100 keV, they preserve and amplify initial perturbations on galactic mass scales ($10^{12}\ M_\odot$). These perturbations are picked up and further amplified by the lighter, stable neutrinos, as long as the heavy neutrinos decay somewhat after the lighter neutrinos go nonrelativistic. If the heavy neutrinos decay into light neutrinos, the decay products contribute about one-half of the present mass density in a hot unclustered background. The only alternative method of retaining initial perturbations until the light neutrinos become nonrelativistic is to introduce large amplitude initial fluctuations such as primordial black holes. If the light neutrinos close the universe, black hole seeds of size $10^9\ M_\odot$ would be required for galaxies of $10^{12}\ M_\odot$ to form. We point out that the neutrino damping mass is a steep function of the present neutrino temperature and that galaxy sized fluctuations would be preserved if $T_\nu < 1.0$ K. However, the only model we can devise to effect this cooling is shown to be in serious violation of astrophysical constraints.

Subject headings: black holes — galaxies: formation — neutrinos

I. INTRODUCTION

Most grand unified theories of the strong, weak, and electromagnetic interactions predict that neutrinos have nonzero rest masses. It is plausible that the τ-neutrino is the most massive and the electron neutrino the least massive.

To set the mass scale of interest to cosmology, recall that the critical density (ρ_c) required to close the universe is:

$$\begin{aligned}\rho_c &= \frac{3}{8\pi}\frac{H^2}{G} = 1.88\times10^{-29}\,h^2\ \mathrm{g\ cm^{-3}}\\ &= 1.12\times10^{-5}\,h^2\,m_p\ \mathrm{cm^{-3}}\\ &= 10{,}540\,h^2\ \mathrm{eV\ cm^{-3}},\end{aligned}\tag{1}$$

where we set the Hubble constant $H=(100h)$ km s^{-1} Mpc^{-1}, and m_p is the mass of a nucleon. The number of blackbody photons is

$$n_\gamma = 8\pi\left(\frac{kT}{hc}\right)^3 2\zeta(3) = 399\ \mathrm{cm^{-3}}\tag{2}$$

with $T=2.7$ K.

The number of neutrinos (plus antineutrinos) of each type is

$$n_\nu = \tfrac{3}{4}\cdot\tfrac{4}{11}\cdot n_\gamma = 109\ \mathrm{cm^{-3}},\tag{3}$$

where the factor of 3/4 comes from Fermi statistics, and the factor of 4/11 is due to a heating of the γ's (but not the ν's) when the $e^+ - e^-$ annihilated.

Therefore, if the most massive, stable neutrino has a mass of $96.8h^2$ eV, it closes the universe.

Massive neutrinos offer appealing possibilities for resolving a number of astrophysical problems:

1. Neutrino oscillations could account for the anomalously low counting rates in Davis's solar neutrino experiment (Bahcall *et al.* 1980).

2. Big bang nucleosynthesis restricts the baryon density to less than about 3% of the closure density (Wagoner 1974; Yang *et al.* 1979; Schramm and Steigman 1981). More baryons would increase the helium and decrease the deuterium relative to what is observed.

On the other hand, measurements of distortions of the Hubble flow on scales of the Virgo supercluster indicate that the mean density of the universe is more than 40% of the closure density (Davis *et al.* 1980). Massive neutrinos could comprise the bulk of the gravitating mass without violating the nucleosynthesis constraints.

3. Individual galaxies are surrounded by massive dark halos, which have as much as 10 times the mass of the visible matter (Ostriker, Peebles, and Yahil 1974; Einasto, Kaasik, and Saar 1974). The density of the galactic halo is distributed like that of a self-gravitating isothermal sphere [$M(r) \propto r$] in the outer regions. If that distribution is continued into the center, the central density (ρ_H) is

$$\rho_H = \frac{3}{4\pi}\frac{v^2}{Gr_c^2} = 3.38 \times 10^{-24} \frac{(v/300 \text{ km s}^{-1})^2}{(r_c/10 \text{ kpc})^2} \text{ g cm}^{-3}, \tag{4}$$

where r_c is the core radius.

As Tremaine and Gunn (1979) pointed out, neutrinos, being collisionless, preserve their maximum phase-space density. Thus,

$$\rho_\nu \leq \frac{4\pi}{3}\left(\frac{mv}{h}\right)^3 m$$
$$= 4.78 \times 10^{-22}\left(\frac{m}{100 \text{ eV}}\right)^4\left(\frac{v}{300 \text{ km s}^{-1}}\right)^3 \text{ g cm}^{-3}. \tag{5}$$

The scaling factor is $4\pi/3$ rather than $8\pi/3$ because only half the states were occupied. Therefore, if the halos are neutrinos,

$$\left(\frac{m}{100 \text{ eV}}\right) \gtrsim 0.3\left(\frac{v}{300 \text{ km s}^{-1}}\right)^{-1/4}\left(\frac{r_c}{10 \text{ kpc}}\right)^{-1/2}. \tag{6}$$

Note that for an isothermal sphere, the circular velocity is $\sqrt{2/3}\, v \approx 248$ km s^{-1}. We would find this coincidence (that the neutrino mass which just closes the universe is the same as needed to make galactic halos) compelling, if we could explain why $v \approx 300$ km s^{-1}.

Unlike baryons that are frozen in the radiation field prior to recombination, the neutrinos are free to cluster as soon as they become nonrelativistic. A 100 eV neutrino becomes nonrelativistic at $z \approx 2 \times 10^5$ and, if it did not cluster, would have a thermal velocity of ~ 1 km s^{-1} today.

Although galaxy formation with neutrinos can begin at an earlier epoch than in the standard scenario, massive neutrinos impose a severe constraint on all theories. As shown in the next section, 100 eV neutrinos erase their primordial perturbations on scales less than 10^{15} $M_\odot$. Unless large baryon number fluctuations are postulated ab initio on galactic sizes, neutrino streaming results in very low density contrast on galactic scales and an extremely slow growth rate for the residual baryon fluctuations, thus making galaxy formation nearly impossible (Bond, Efstathiou, and Silk 1980).

We take note of the suggestion by Zel'dovich (1978), Doroshkevich, Saar, and Shandarin (1978), and Doroshkevich *et al.* (1980) that 10^{15} $M_\odot$ is the mass scale of rich clusters of galaxies, and that they would form directly by Jeans instabilities. However, on such scales, the neutrino velocities would be thousands of kilometers per second. Even if, later, baryon galaxies were formed by hydrodynamic shocks, we see no way to trap neutrinos around the baryon galaxies to form galactic halos.

To preserve and amplify perturbations on a galactic scale (10^{12} $M_\odot$) requires much more massive neutrinos. But if these more massive neutrinos existed today, they would overclose the universe by a large factor. Therefore, the more massive neutrino must be unstable.

To focus the discussion we will identify the massive unstable neutrino with ν_τ having a mass of keV to MeV energies. We identify the stable light neutrino with ν_μ, having a mass on the order of 100 eV. We assume ν_e is stable, and we neglect its mass.

In the following sections we discuss three separate means to form galaxies with the aid of the ν_τ and ν_μ, and the constraints imposed on these models by big bang nucleosynthesis, the requirement that the decay products not contradict present observations, and the requirement that the universe be matter dominated today.

The third scenario is presented as an obituary to discourage future investigations, because, while attractive, our particular model seriously violates several observational constraints.

II. THE GROWTH OF PERTURBATIONS

As is usual in studies of galaxy formation, we assume an initial perturbation spectrum on all scales and ask what scales grow and what scales decay. In a matter dominated universe, the Jeans mass is the dividing line; larger mass scales grow and smaller mass scales decay as damped sound waves. As the Jeans mass in a matter dominated universe decreases with time (as t^{-1}), one is assured that if ever a mass scale grows, it will continue to grow at later times. In a universe that alternates between radiation and matter domination, the Jeans mass is not a monotonic function of time. Furthermore, we are interested in the rest mass in galaxies today; but if one calculates the Jeans mass in a radiation dominated era, one includes a substantial contribution of energy density that has, by now, redshifted away.

A reliable estimate of the critical scale is the free-streaming neutrino horizon, or diffusion length, which is given by

$$d(t) = R_0 \int_t^{t_0} \frac{v(t')\, dt'}{R(t')}, \tag{7}$$

where R is the scale factor of the universe, v is the neutrino velocity, t is the time when the neutrinos begin free-streaming, and t_0 is the present time. The mass contained in a diffusion length stops growing shortly after the neutrinos become nonrelativistic, and so d is insensitive to the upper limit. The mass scale, M, *above* which perturbations grow, is

$$M = \frac{4\pi}{3}\rho_0 d^3. \tag{8}$$

Since $(4\pi/3)\rho_0 = 1.16\times10^{12}\ \Omega h^2\ M_\odot\ \mathrm{Mpc}^{-3}$ (Ω is the cosmological density parameter), we require $d \lesssim 1$ Mpc for $M \lesssim 10^{12}\ M_\odot$.

We evaluate d in detail in the Appendix, but we note here that if the bulk of the mass density is in the form of one type of massive neutrino, then $d \approx 11\ (\Omega h^2)^{-1}$ Mpc.

If the mass of the τ-neutrino is increased by a factor of η, then its diffusion length decreases by a factor of η. But, as mentioned earlier, these more massive neutrinos must decay. In the next section, we follow the perturbations in a universe with a massive, unstable ν_τ.

III. A UNIVERSE WITH A SUPERMASSIVE, UNSTABLE ν_τ AND A MASSIVE STABLE ν_μ.

Suppose the mass of the τ-neutrino is η times the mass of the μ-neutrino. For $\eta \gtrsim 15$, the primordial τ-neutrino perturbations will be preserved on galactic scales. However, the diffusion length for the μ-neutrino is larger, and so it will have erased its own perturbations on galactic scales. However, after they go nonrelativistic, the ν_μ will be Jeans unstable to clustering in the potential field of the ν_τ fluctuations on galactic mass scales. The Jeans length r_J of the ν_μ is given by

$$r_J = \frac{1}{2}\left(\frac{\pi}{3}\right)^{1/2} v_\mu/\sqrt{G\rho_\mu\eta}\,, \tag{9}$$

where the factor of $\eta^{-1/2}$ is due to the presence of the ν_τ. The mass of ν_μ within this Jeans length is

$$M_J = \frac{4\pi}{3}\rho_\mu r_J^{\,3} \approx 5\times10^{14}\left(\frac{1+z}{10^5}\right)^{3/2}\eta^{-3/2}\,M_\odot, \tag{10}$$

so that shortly after the ν_μ go nonrelativistic ($z \approx 10^5$) (and with $\eta \gtrsim 100$), they will be Jeans unstable on sufficiently small mass scales and will quickly pick up the existing ν_τ perturbations.

For example, let δ_τ be the density contrast of the ν_τ and δ_μ be the density contrast of the ν_μ. Then

$$\ddot{\delta}_\mu + \frac{4}{3}\frac{1}{t}\dot{\delta}_\mu \approx 4\pi G\rho_\tau\delta_\tau = \frac{2}{3}\frac{\delta_\tau}{t^2} \tag{11}$$

(since the ν_τ dominate the mass). Let the growing mode begin at $t = t_\mu$ with initial values $\delta(t_\mu) = \dot{\delta}(t_\mu) = 0$. Then the solution of equation (11), with $\theta = t/t_\mu$, is

$$\begin{aligned}\delta_\mu(\theta) &= \delta_\tau(t_\mu)\left(\theta^{2/3} - 2\theta^{-1/3} - 3\right)\\ &= \delta_\tau(\theta)\left(1 + \frac{2}{\theta} - \frac{3}{\theta^{2/3}}\right).\end{aligned} \tag{12}$$

When $t/t_\mu = 8$, $\delta_\mu = (1/2)\delta_\tau$ and is growing as $\theta^{2/3}$. Thus, the ν_μ readily cluster on scales smaller than their initial diffusion length, if the ν_τ preserve perturbations on smaller scales.

Suppose the ν_τ decay at epoch $z = z_d$. The decay products will be relativistic, and so, since the ν_τ dominated the mass density prior to decay, the universe will become radiation dominated again. Until the universe again becomes matter dominated at $z = z_M$, the perturbations in ν_μ stop growing. In fact, the perturbations damp slightly because the slower Hubble expansion rate of the radiation dominated epoch causes d_μ to slowly increase again even though the ν_μ are by now nonrelativistic.

To give the perturbations in ν_μ time to mimic the perturbations in ν_τ, we calculate d_μ starting at epoch $z = 4z_D$ to the present. The result is approximately

$$\begin{aligned}d_\mu &\approx \frac{c}{H_0}\frac{p_0}{\Omega_\nu^{3/2}}(1+z_M)^{1/2}\left(\ln 4\eta + 2\Omega_\nu^{1/2}\right)\\ &\approx .016(1+z_M)^{1/2}\Omega_\nu^{-3/2}\left(\ln 4\eta + 2\Omega_\nu^{1/2}\right)\ \mathrm{Mpc},\end{aligned} \tag{13}$$

where we have allowed for the possibility that, at present, the mass density of the clusterable ν_μ is $\Omega_\nu\rho_c$. The resulting M_D of clusterable matter versus z_D and η is plotted in Figure 1*a* for $\Omega_\nu = 1$ and Figure 1*b* for $\Omega_\nu = 1/2$. On scales smaller than M_D, the cutoff is not exponential but will scale approximately linearly with mass. We allow for the possibility of

$$\nu_\tau \to \nu_\mu + \nu_e + \bar{\nu}_e \quad (\Omega_\nu = \tfrac{1}{2}) \tag{14a}$$

or

$$\nu_\tau \to \nu_e + \nu_e + \bar{\nu}_e \quad (\Omega_\nu = 1). \tag{14b}$$

The decay μ-neutrinos will be nonthermal as opposed to their primordial counterparts and would have average velocity $v = c/(1+z_M)$, far too hot to cluster in galaxies or clusters of galaxies. They could constitute an unclustered background field with perhaps half the total mass density of the universe.

The radiative decay

$$\nu_\tau \to \nu_\mu + \gamma \tag{14c}$$

is ruled out for ν_τ lifetimes greater than 10^{13} s and branching ratio $\gtrsim 10^{-2}$ by the absence of an optical or near-infrared background at the expected high flux levels (Kimble, Bowyer, and Jakobsen 1981; Hayakawa *et al.* 1978). Particle physics models in which triple neutrino decays predominate by this factor or more have been discussed by Dicus, Kolb, and Teplitz (1978) and by de Rújula and Glashow (1980).

After the decay products have redshifted by a factor η, the universe will again become matter dominated, at $z = z_M \approx z_D/\eta$. Once the universe becomes matter dominated for the last time, the neutrino streaming ceases, and perturbations again grow. However, if only a fraction f of the mass density is in clusterable form, then the growth rate of perturbations becomes $\delta\rho/\rho \sim t^{2m/3}$, $m = [(1+24f)^{1/2} - 1]/4$. If $f = \frac{1}{2}$, $m = 0.65$, so the growth rate can be slowed moderately. This scheme must obey several constraints:

1. If the ν_τ are not to change the rate of expansion during the time when the neutron-proton ratio is frozen-in ($T \sim 1$ MeV), the τ_ν must not dominate the mass at $z \sim 10^{+10}$. Since they will dominate the mass when

$$z \leq \frac{m_{\nu_\tau} c^2}{3.15\, kT_{\nu_0}} \approx \frac{\eta}{5.4\times 10^{-6}}, \quad (15)$$

we require $\eta < 5.4\times 10^4$.

2. A lower limit to the decay times for the ν_τ is obtained from scaling the decay of the muon (cf. Dicus, Kolb, and Teplitz 1978):

$$t_d \geq 2.2\times 10^{-6}\,\mathrm{s}\left(\frac{106\ \mathrm{MeV}}{m_{\nu_\tau} c^2}\right)^5 = \frac{3.5\times 10^{24}}{\eta^5 \Omega_\nu^5}\ \mathrm{s}. \quad (16)$$

The age of the universe (t_u) is

$$t_\mu \approx \tfrac{2}{3}(1+z_M)^{-3/2} H_0^{-1} \Omega_\nu^{-1/2} \eta^{-2}$$

$$= 2.1\times 10^{17}\,\mathrm{s}\,(1+z_M)^{-3/2} \Omega_\nu^{-1/2} \eta^{-2}. \quad (17)$$

Since $t_u \geq t_d$,

$$\eta > 256(1+z_M)^{1/2}(\Omega_\nu)^{-3/2}. \quad (18)$$

Some typical values of η and z_D are given in Table 1. As can be seen in Figure 1, the larger values of η lead to damped masses $> 10^{12}\ M_\odot$, so we are forced to small z_m. (The universe stays relativistic until $z \sim 10$.)

Note that in spite of this, the total time available for perturbations to grow is from $z \approx 2\times 10^5\ \eta$ to z_d and from $z_M = z_d/\eta$ to the present. So perturbations grow by a factor of $\sim 2\times 10^5$.

The lifetime of the ν_τ is obviously critical for this scenario, and it is not certain that equation (16) is applicable to the reactions that do not conserve tau lepton number. If the lepton has a longer lifetime than expected from equation (16), η is required to increase and/or z_m must decrease.

IV. GROWTH OF PERTURBATIONS AROUND A BLACK HOLE

The advantage of introducing initial perturbations via a black hole is to prevent them from being erased by neutrino diffusion. It has been suggested by a number of authors, independent of massive neutrinos (see, for example, Carr 1977 and Ryan 1972) that galaxies formed around "seed" black holes. As we show below, because the Jeans mass for 100 eV neutrinos does not drop to $10^{12}\ M_\odot$ until $z = 1570$, which is close to the decoupling z of 1000, the seed black hole scenario is not much modified by massive neutrinos. However, massive neutrinos, in this scenario, also do not inhibit galaxy formation. We take for the Jeans mass the expression

$$M_J = \frac{4\pi}{3} r_J^3 = \frac{\pi}{6}\rho\left(\frac{\pi}{3}\frac{v^2}{G\rho}\right)^{3/2}. \quad (19)$$

We are interested only in nonrelativistic neutrinos in a matter dominated universe, so we let

$$\rho = \rho_0(1+z)^3$$

and

$$V = cp_0(1+z),$$

where $p_0 = 3.15kT_{\nu_0}/m_\nu C^2$, whence

$$M_J = 2^{1/2}\left(\frac{\pi}{3}\right)^3 \frac{(cp_0)^3}{GH_0}(1+z)^{3/2}$$

$$= 1.6\times 10^7\ M_\odot\ (1+z)^{3/2}.$$

Thus the Jeans mass is $10^{12}\ M_\odot$ when $z = 1570$.

TABLE 1

TYPICAL VALUES OF η AND z_D

z_m	$\eta(\Omega_\nu=1)$	$z_d(\Omega_\nu=1)$	$\eta(\Omega_\nu=1/2)$	$z_d(\Omega_\nu=1/2)$
1 ...	2.56×10^2	2.56×10^2	7.25×10^2	7.25×10^2
10 ...	8.10×10^2	8.10×10^3	2.29×10^3	2.29×10^4
25 ...	1.28×10^3	3.20×10^4	3.62×10^3	9.05×10^4
50 ...	1.81×10^3	9.05×10^4	3.13×10^3	2.57×10^5

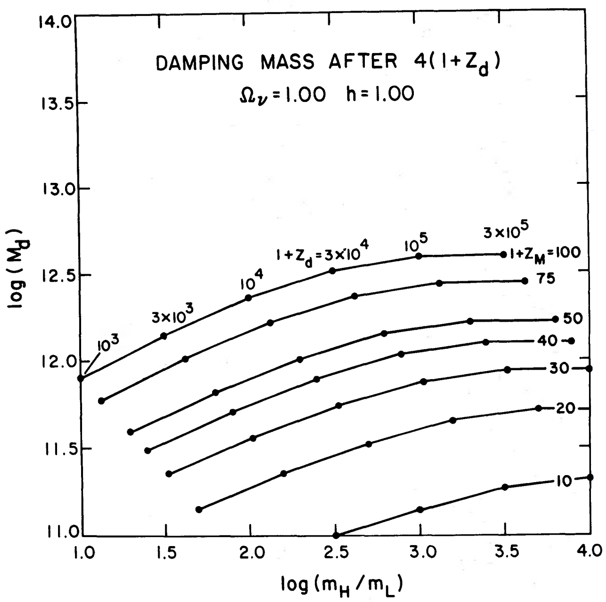

FIG. 1*a*

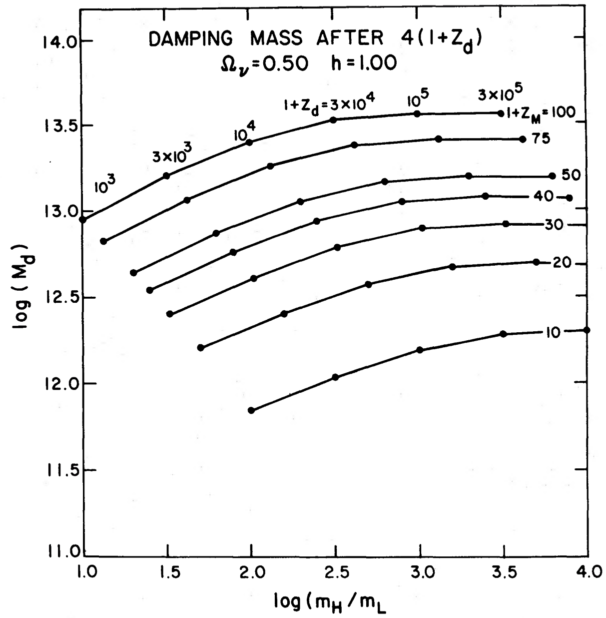

FIG. 1*b*

FIG. 1.—The damping mass is plotted versus $\eta = m_{\rm H}/m_L$ for various z_m, the epoch when the universe became matter dominated for the second time. The curves are also labeled by the epoch of ν_τ decay, $z_D = z_m \eta$. Fig. 1*a* is for $\Omega h^2 = 1$, if the ν_τ decay results only in massless particles. Fig. 1*b* applies if $\Omega h^2 = 1$, but only half of the matter density is today cool enough to cluster, as would result if the ν_τ decays into $\nu_\mu + \nu_e + \bar{\nu}_e$.

We chose the mass of the seed black hole to make sure that a $10^{12}\ M_\odot$ patch of the universe reaches maximum expansion in the interval associated with $z = 1570$. Consider a patch of the universe of mass M and radius R_1, and with critical density. Insert, in the center of the patch, a black hole of mass m. Then, the energy per unit mass of a shell at R_1 is

$$\varepsilon = \frac{-Gm}{R_1}, \tag{20}$$

its maximum radius is $R_M = (M/m)R_1$, and the time to maximum expansion is

$$T_M = \frac{\pi}{2}\left(\frac{R_1^3}{2GM}\right)^{1/2}\left(\frac{M}{m}\right)^{3/2} = \frac{\pi}{2}\frac{1}{H(R_1)}\left(\frac{M}{m}\right)^{3/2}. \tag{21}$$

In the time interval T_M, the universe expands by a factor

$$\frac{R_2}{R_1} = 1 + z = \left(\frac{3\pi}{4}\right)^{2/3}\frac{M}{m}. \tag{22}$$

If we want to start when $M_J \lesssim 10^{12}\ M_\odot$, $z \lesssim 1570$ and $m \gtrsim 2 \times 10^9\ M_\odot$. This is about the same size black hole Carr required for a baryon universe.

V. INCREASING THE MASS OF THE μ NEUTRINO

In principle, a straightforward way to decrease the diffusion scale of the μ-neutrino is to increase its mass. If a very massive ν_τ were to go nonrelativistic and decay soon after the neutrinos had decoupled from the matter, and if this decay produced photons which thermalized, then the neutrino temperature today could be much lower than that of the microwave background. This would allow the ν_μ to be more massive for a given mass density, since their number density would be reduced. The effect is analogous to the heating of the photons when the electrons and positions annihilated, which in the standard model results in $T_\nu = (4/11)^{1/3} T_\gamma$.

If when the neutrinos go nonrelativistic the universe has the age $t_{\rm nr}$, scale $R_{\rm nr}$, and density $\rho_{\rm nr}$, then their diffusion length is approximately given by

$$d_\mu \approx (R_0/R_{\rm nr})(ct_{\rm nr}) \approx (c/v_{\nu,0})\left(c/(G\rho_{\rm nr})^{1/2}\right), \tag{23}$$

where $v_{\nu,0}$ is the neutrino velocity today. If the present mass density in neutrinos is held constant that velocity varies with $T_{\nu,0}$ as

$$v_{\nu,0} \propto p_{\nu,0}/m_\nu \propto T_{\nu,0}/T_{\nu,0}^{-3} \propto T_{\nu,0}^4. \tag{24}$$

Because the universe is radiation dominated at the time

$$\rho_{nr} \propto (R_0/R_{nr})^4 \propto T_{\nu,0}^{-16}. \tag{25}$$

If $T_{\nu,0}=r(4/11)^{1/3}T_{\gamma,0}$, then d_μ scales as r^4 and the diffusion mass scales as r^{12}. Since we have to reduce the diffusion mass by a factor of $\sim 10^3$, we require $r<1/2$. (A more careful calculation yields $r \lesssim 0.46$.)

A number of constraints combine to make this much heating of the photons relative to the neutrinos impossible. The neutrinos decouple right at the beginning of nucleosynthesis ($T \approx 2.6$ MeV according to Dicus *et al.* 1978), and thus the decay must occur after element production. For neutrinos with a mass of 5–10 MeV the Dicus *et al.* (1978) calculations show that it is just possible to get enough heating and preserve reasonable abundances. The effects of a photon-to-baryon ratio that is smaller by a factor of r^3 are offset by the more rapid expansion resulting from the presence of the nonrelativistic neutrino. These neutrinos also satisfy the requirement that the decay occur before $z \approx 10^7$ in order for the photons to thermalize.

However, Lindley (1979) has shown, with very conservative assumptions, that if the photons produced in the decay have an energy greater than 2.225 MeV, there are easily enough of them to destroy all the deuterium by photofission.

The scheme could survive with the boundary value of 5 MeV. But Falk and Schramm (1978) and Cowsik (1977) have pointed out that any neutrino less massive than ~ 10 MeV should be produced in large quantities in supernovae. If the neutrinos have a lifetime in the range of interest here (10^3–10^4 s), they escape from the star, and their decay would produce (from all supernovae taken together) more of a gamma ray background than is observed. It should be noted that this constraint can be tightened and made independent of the controversial supernova rate by the fact that supernovae are not observed as gamma ray burst sources.

One or the other of the above constraints can be evaded but not both at once. If a 10 MeV neutrino decays just before nucleosynthesis, it can produce only $r \approx 0.75$ and a diffusion mass of $\sim 2\times 10^{14}\ M_\odot$, a very minor reduction.

VI. CONCLUSIONS

If neutrinos have nonzero rest masses, and if the most massive neutrino (the τ-neutrino) has a mass on the order of 50–100 keV, it has a lifetime on the order of 100–3000 years and would have decayed by now according to our optimistic lifetime estimates.

Prior to its decay, it would have amplified initial perturbations on galactic mass scales, and 50–100 eV neutrinos (μ-neutrinos) would have had time to mimic these perturbations. Until the present, the μ-neutrinos could have amplified the initial perturbations on galactic mass scales by a factor of 1–2×10^5. This is to be compared with a baryon dominated universe where the growth factor is about 10^3. In addition, there is no need, in a neutrino dominated universe, to require isothermal (or entropy) perturbations on galactic mass scales. In the standard cosmological model, Silk damping during the recombination epoch erases adiabatic perturbations on galactic mass scales. Thus, it has been argued that primordial perturbations must have been isothermal, yet isothermal perturbations are quite contrary to the spirit of grand unified field theory in the early universe (Turner and Schramm 1979; Weinberg 1980).

The detailed nature of the decay of the heavy neutrino is the chief uncertainty in this model. If the lifetime is too long, the universe would be radiation dominated today, and if the decay occurs before the ν_μ become nonrelativistic, galactic size perturbations would be damped out. If the decay produces photons, the resulting background, which would not be thermalized if $z_D < 10^7$, would contain far more energy than the microwave background radiation and would have characteristic energies somewhere in the optical or near-infrared, where it would be observable. Present limits at 2 μm, for example, are some two orders of magnitude below the expected flux. If the decay of the heavy tau neutrino yields muon neutrinos, then these neutrinos, containing roughly half the mass density of the present universe, will today be too hot to cluster on any observed scale and will form a smooth background. They could help explain why estimates of mass-to-light ratios of galactic systems are monotonic functions of measurement scale, even for scales larger than clusters of galaxies.

If the initial perturbation was in the form of a massive ($10^9\ M_\odot$) black hole, it would have clustered by infall galactic masses of 100 eV neutrinos by now. In this scenario, we do not require an unstable neutrino.

Finally, we have shown that the damping mass is a very strong function of the present neutrino temperature, a quantity not subject to direct observation. If some mechanism can be found for the cooling of the neutrino temperature to approximately 0.9 K, then neutrino streaming will not erase galactic size perturbations. However, a mechanism we once thought promising is in complete violation of a variety of astrophysical constraints. Perhaps there are alternative processes to further cool neutrinos, but for the present, this process is ruled out.

Our motivation has been to preserve fluctuations of $10^{12}\ M_\odot$, which is the upper limit usually associated with one galactic mass. We note, however, that the space density of bright galaxies ($M < -18$) is $\sim 10^{-2}$ Mpc^{-3} and that the mass per bright galaxy is $2.7\times 10^{13}\ \Omega\ M_\odot$. Even with massive galactic halos, there is a large excess mass per galaxy if $\Omega > 0.1$. A mechanism for producing $1/r^2$ halos by tidally shearing off the outer regions of initial protogalaxies has been suggested by Dekel, Lecar, and Shaham (1978), and if this process is important, then perhaps only fluctuations of $10^{13}\ M_\odot$ and higher need be preserved. This would significantly ease the constraints on all the scenarios described here.

In summary, we have described scenarios that, by including an unstable τ-neutrino, facilitated galaxy formation. Although we chose the unstable particle to be the τ-neutrino, we note that another particle (perhaps not a neutrino at all) with similar mass, lifetime, and decoupling time would serve as well. However, without the massive, unstable particle, the lighter neutrinos by themselves seem to make galaxy formation on scales $\leq 10^{12}\ M_\odot$ almost impossible.

Future developments in high energy physics will hopefully provide solid information on the viability of these speculations.

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APPENDIX

THE DIFFUSION LENGTH d

The diffusion length is the horizon of free-streaming neutrinos. In the interval $t_1 \leq t \leq t_0$, a neutrino with a random velocity $v(t)$, moves a physical distance d, given by

$$d = R_0 \int_{t_1}^{t_0} \frac{v(t)\,dt}{R(t)} = R_0 \int_{R_1}^{R_0} \frac{v(R)\,dR}{R\dot{R}(R)} . \tag{A1}$$

As we will demonstrate shortly, $(R_1/R_0)d$ is interpreted as a Jeans length, i.e., the distance a neutrino with random velocity $v(t_1)$ travels in a "Hubble time." Thus, linear perturbations on scales smaller than $(R_1/R_0)d$ are damped by the excess neutrinos diffusing out of the region. It is convenient to set $R = xR_0$, where $x = 1/(1+z)$. Then d is written

$$d = \int_{x_1}^{1} \frac{v(x)\,dx}{x\dot{x}} . \tag{A2}$$

We set $v = \beta c$, and we approximate β by

$$\beta = (1 + p^{-2})^{-1/2} , \tag{A3}$$

with $p = \alpha k T_\nu / mc^2$ (α is a constant which we will specify below). Since $T_\nu = T_{\nu_0}/x$, we write

$$\beta = \left[1 + (x/p_0)^2\right]^{-1/2} ,$$

where

$$p_0 = \frac{\alpha k T_{\nu_0}}{mc^2} .$$

We also take $\dot{x}^2 = (8\pi/3) G\rho(x) x^2$. Finally, we let $\rho(x) = \rho_0 g(x)$ and write

$$d = \frac{c}{\left[(8\pi/3)G\rho_0\right]^{1/2}} \int_{x_1}^{1} \frac{dx}{\left[1 + (x/p_0)^2\right]^{1/2} \left[g(x)x^4\right]^{1/2}} . \tag{A4}$$

If the universe is matter dominated, $g \approx x^{-3}$; and if it is radiation dominated, $g \approx x^{-4}$. To approximate δ when the universe contains both matter and radiation, we set

$$g = \frac{1}{x^3} + \frac{\varepsilon}{x^4} . \tag{A5}$$

In a matter dominated universe ($\varepsilon = 0$), if the neutrinos are nonrelativistic ($x_1 \gg p_0$),

$$d = \frac{2cp_0}{\left[(8\pi/3)G\rho_0\right]^{1/2}} \frac{1}{x_1} . \tag{A6}$$

Since $v(x_1)=cp_0x_1$ and $\rho(x_1)=\rho_0x_1^{-3}$,

$$x_1d=\frac{2v(x_1)}{\left[(8\pi/3)^{1/2}G\rho(x_1)\right]^{1/2}}\approx r_J(x_1), \tag{A7}$$

where r_J is the "Jeans length." The mass contained in d (evaluated at $x=x_0=1$) is the mass contained in r_J evaluated at $x=x_1$); i.e.,

$$M\sim\rho_0d^3\sim\rho_0\frac{r_J^3(x_1)}{x_1^3}\sim\rho(x_1)r_J^3(x_1). \tag{A8}$$

In general, when the neutrinos are relativistic, or when the universe is radiation dominated, this equality does not hold, and we use d.

We now evaluate $\rho(x)/\rho_0=\delta$. To avoid integrating over the neutrino distribution function, we approximate the density in the following manner. For each species (ν_τ, ν_μ, ν_e, and photons), we set

$$\rho=nm_{\text{eff}},$$

where m_{eff} is the "effective mass" for this species and

$$n=8\pi\left(\frac{kT_\nu}{hc}\right)^3\frac{3}{2}\zeta(3). \tag{A9}$$

For a neutrino with rest mass m, we set

$$m_{\text{eff}}=m\sqrt{1+p^2}. \tag{A10}$$

When $p\gg1$, $m_{\text{eff}}=pm$, and since for a relativistic fermion,

$$\rho=8\pi\left(\frac{kT_\nu}{hc}\right)^3\frac{7}{8}\frac{\pi^4}{15}\frac{kT_\nu}{c^2}, \tag{A11}$$

we require

$$\frac{3}{2}\zeta(3)mp=\frac{3}{2}\zeta(3)m\frac{\alpha kT_\nu}{mc^2}=\frac{7}{8}\frac{\pi^4}{15}\frac{kT_\nu}{mc^2},$$

whence

$$\frac{3}{2}\zeta(3)\alpha=\frac{7}{8}\frac{\pi^4}{15}\quad(\alpha=3.15). \tag{A12}$$

We take m as the mass of the μ-neutrinos, ηm as the mass of the τ-neutrinos, and zero as the mass of the electron-neutrinos. Then we have the effective masses shown in Table 2.

The values of d used to crease Figure 1 were obtained by numerical integration of the appropriate density for the scenarios where ν_τ decays into either 3 massless ν_e's or half ν_e's and one half massive ν_μ's.

TABLE 2
EFFECTIVE MASSES

Particle	m_{eff}
Tau neutrino	$\eta m\sqrt{1+(p^2/\eta^2)}$
Muon neutrino	$m\sqrt{1+p^2}$
Electron neutrino ...	$\alpha kT_\nu/c^2=mp$
Photon	$\frac{8}{7}[T_\gamma/T_\nu]. \alpha kT_\nu/c^2$

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