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THE EFFECT OF LOSSES ON ACCELERATION OF ENERGETIC PARTICLES BY DIFFUSIVE SCATTERING THROUGH SHOCK WAVES

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ABSTRACT

The effect of local losses on the acceleration of energetic particles by shocks is discussed considering both energy losses of individual particles and damping processes for the scattering hydromagnetic waves. The calculations are all time asymptotic and steady state. For locally plane and infinitely extended shocks, the requirement for acceleration is that the loss time exceed the acceleration time. The resulting modifications of the spatial structure and of the momentum dependence of the cosmic-ray distribution are described. For acceleration to be a local effect within the Galaxy, the local scattering mean free path must be small compared to the effective overall galactic mean free path as deduced from the cosmic-ray escape time. The required strengths of the scattering wave fields are such that neutral molecular clouds do not allow acceleration; in a partially ionized, warm interstellar medium, quite large shock strengths are needed. Such strong shock discontinuities are surrounded by an ionization layer within which Alfvén wave damping is presumably negligible. Given the spatial extent of the layer for strong shocks propagating into neutral interstellar clouds, the possibility of localized diffusive acceleration is investigated. The estimated strength and extent of the scattering region is not large enough to confine acceleration within the layer. Rather, it will extend across the whole cloud, whose integrated losses then determine the efficiency.
Subject headings: interstellar: molecules — particle acceleration — shock waves — stars: winds

I. INTRODUCTION

The question of cosmic-ray acceleration has received new attention through simple models of the first-order Fermi effect in hydromagnetic shock waves (Axford, Leer, and Skadron 1977; Krymsky 1977; Bell 1978*a*, *b*; Blandford and Ostriker 1978). These models approximate the shock as a transition between two scattering media. The hydrodynamic shock compression can be viewed as a relative flow of these media resulting in a finite divergence of the (shock normal) velocity in which the cosmic-ray particles are accelerated nonadiabatically as they are convected across. Direct reflections (i.e., discontinuous reversals of velocity direction) of energetic particles at the jump in the average magnetic field (e.g., Parker 1963; Fisk 1971; Morfill and Scholer 1975) are neglected in this theory. They would only increase the acceleration effect. Also not considered is the acceleration calculated by, e.g., Chen and Armstrong (1975), which is due to particle drifts parallel to the electric field during multiple shock interactions; these latter acceleration effects are most pronounced for nearly perpendicular shocks. The mechanism discussed here applies, however, equally-well to parallel shocks which exhibit no change in the average magnetic field. This theory has been reviewed recently by Axford (1980).

Particle acceleration proceeds in a succession of scatterings across the shock on irregularities moving relative to each other with a speed that is approximately equal to the velocity discontinuity ΔV at the shock. An energy increment ΔE of order $p\Delta V$ is given to a particle in such individual scattering processes of which a number $N \approx v/\Delta V$ is needed to enhance the particle's energy significantly (here p is the particle momentum and v its velocity). Yet, particles are not always scattered across the velocity discontinuity. On the contrary, most of the collisions occur within the regions upstream and downstream of the shock, respectively. Neglecting second-order Fermi effects, these collisions do not change the particle energy and are, therefore, "neutral." This can be seen from the transport equations describing the average change of the particle energy distribution. They yield a typical acceleration time (e.g., Forman and Morfill 1979), $t_{\rm acc} \approx 4\kappa/V_s^2 \approx 4\kappa/(\frac{1}{3}|v|)^2 \times (\frac{1}{3}|v|/V_s)^2 = t_{\rm scatt} \times 4 \times (\frac{1}{3}|v|/V_s)^2 \approx t_{\rm scatt} \times N^2$, as opposed to the result $t_{\rm acc} \approx t_{\rm scatt} \times N$ which would be expected if there were only energizing collisions (κ, V_s, and $t_{\rm scatt}$ denote the spatial diffusion coefficient normal to the shock, the shock velocity, and the mean scattering time respectively). Thus, diffusive shock acceleration is a slow process, and energetic particles traverse a considerable amount of matter on both sides of the shock, losing energy while being accelerated. Whether or not these energy losses are relevant depends basically on the ratios $t_{\rm acc}/\tau$ and $l/(V_s\tau)$, where τ is the characteristic energy loss time, and l/τ is proportional to the column density of gas sampled during acceleration. For $t_{\rm acc}/\tau \ll 1$ and $l/(V_s\tau) \ll 1$, the acceleration process is almost undisturbed, whereas, for $t_{\rm acc}/\tau > 1$, or $l/(V_s\tau)$, the particles tend to lose energy faster than they gain it, and acceleration is quenched. Depending on energy, nuclear particles will lose energy mainly by ionizing, Coulomb, and nuclear collisions with the gas atoms; for electrons also bremsstrahlung, synchrotron radiation and inverse Compton effect can play a role. The low energy ionization and Coulomb losses play their dominant role at particle injection (Fermi 1949; Ginzburg and Syrovatskii 1964). If the shock is not plane but three-dimensional, e.g., produced by supersonic expansion from a localized source into a gas at rest, there will also be adiabatic energy losses in the downstream region affecting the overall acceleration rate. To lowest order, the adiabatic momentum loss rate $(1/p)(dp/dt) = -\frac{1}{3}\,\mathrm{div}\,V$ simply adds to the collisional loss rate, although in more complicated cases, like shocks in the solar wind, a more detailed treatment is necessary (Fisk and Lee 1980).

The energy loss which accumulates during each acceleration step depends on the mean free path $3\kappa/|\boldsymbol{v}|$ of the particle in the magnetic irregularities (hydromagnetic waves) on both sides of the shock transition. The availability of such waves depends on another form of losses, the dissipation rate of these waves. In cases where the magnetic fluctuations in the medium are produced by powerful nearby sources, like the Sun for the solar wind, the wave damping is not very important. Then, the particle mean free path may be so short that energy losses are negligible. However, in many astrophysical environments, the generation of irregularities may be limited. Indeed, it might be due only to cosmic-ray streaming (Lerche 1967; Kulsrud and Pearce 1969; Wentzel 1974; Bell 1978*a*) in the face of wave dissipation like ion-neutral collisions (Kulsrud and Pearce 1969; Kulsrud and Cesarsky 1971) or nonlinear Landau damping (Lee and Völk 1973; Kulsrud 1978). Then, cosmic-ray energy losses may overpower the acceleration effects.

Associations of shocks with loss regions exist not only by chance. Rather, there is often a physical coupling, like H II regions, supernova remnants, or cloud formation in the wake of the galactic density wave. Particular cases are shocks in and near interstellar clouds which are often the source of shock waves through violent processes in their interiors.

As far as observations distinct from in situ cosmic-ray measurements are concerned, a number of molecular clouds are expected to be observable sources of high energy γ-rays due to π^0 production and decay, or electron bremsstrahlung, especially if they are illuminated by a locally enhanced cosmic-ray flux. In fact, one such source which requires a local cosmic-ray enhancement appears to have been identified with the nearby cloud ρ Ophiuchi (Wills *et al.* 1980; Bignami and Morfill 1980). Morfill *et al.* (1981) have argued that the enhanced γ-ray emission from ρ Ophiuchi is due to an enhanced cosmic-ray intensity associated with an old supernova remnant, the North Polar Spur (Loop I), which is probably interacting with the cloud. Another suggestion has been that stellar wind terminal shocks from young stars in and near the cloud could be responsible for this enhancement (Cassé and Paul 1980; Paul, Cassé, and Montmerle 1980). It has been argued that other γ-ray sources are not only apparently but also physically associated with supernova remnants and OB associations (Montmerle and Cesarsky 1979).

In this paper, we discuss the problem of shock acceleration in a lossy medium in a number of illustrative examples. We do this always in the time asymptotic limit. The effect of losses on the build-up time of the scattering wave field, for example, would be even more pronounced in a situation where a freshly created shock starts to accelerate an existing population of cosmic rays over its finite lifetime. The loss process itself is described in terms of a (energy-dependent) loss time τ for analytical simplicity. Since we are concerned with acceleration rather than, for example, modulation, this approximation seems acceptable.

In § II, we first consider steady state acceleration by a shock in a medium with spatially uniform losses and a background source of cosmic rays. In § III, the conditions on the local diffusion properties of the medium are discussed, and, in § IV, the requirements on the wave field in the face of damping and energy losses are determined for several characteristic environments. Up to this point, the ionization structure of the medium was assumed to be unaffected by the presence of the shock. In § V, we discuss the possibility of local acceleration within the ionization layer accompanying strong shocks in essentially neutral interstellar clouds.

II. STEADY STATE ACCELERATION OF ENERGETIC PARTICLES WITH SPATIALLY UNIFORM LOSS RATES

In the frame of the shock (assumed to be steady), the transport equation for the isotropic part $f(x, p, t)$ of the cosmic-ray distribution function in coordinate and momentum space is written as

$$\frac{\partial f}{\partial t} + V\frac{\partial f}{\partial x} - \frac{\partial}{\partial x}\left(\boldsymbol{\kappa}\frac{\partial f}{\partial x}\right) - \frac{p}{3}\frac{\partial f}{dp}\frac{\partial V}{\partial x} + \frac{f}{\tau} = \frac{f_\infty}{\tau_\infty}\,, \tag{1}$$

where V denotes the velocity component of the background plasma normal to the shock (approximately equal to the normal phase velocity of the resonantly scattering waves if $|V| \gg |V_{A,n}|$, where $V_{A,n}$ is the normal component of the relevant Alfvén speed in the medium, cf. Kulsrud and Pearce 1969), $\boldsymbol{\kappa}$ is the tensor diffusion coefficient for the energetic particles of momentum p, and $1/\tau$ is their momentum loss rate. The term f_∞/τ_∞ describes a distributed stationary source which maintains the energetic particle distribution $f_\infty \tau/\tau_\infty$ far away from the shock. By analogy with $1/\tau$, $1/\tau_\infty$ is a production rate for the source distribution f_∞.

For continuous energy losses, instead of a catastrophic loss after a mean time τ, the correct form of equation (1) would replace f/τ by a Fokker-Planck term $(1/p^2)\partial/\partial p[(dp/dt)p^2 f]$, where now dp/dt is the mean momentum loss rate (e.g., Skilling 1971). In this form, equation (1) becomes analytically very difficult. For the present discussion, it is, therefore, much more convenient to restrict ourselves to the description in terms of the loss time τ which leads to a simple analytical solution. Whether or not this is a physically reasonable approximation depends on the problem considered. In the case of pure degradation by losses (of a cosmic-ray distribution given either at some initial time, or, in a steady state problem, at some removed boundary), the evolved distribution at a given energy will be determined largely by particles starting at higher energies. A typical example is the modulation of galactic cosmic rays in the solar wind (e.g., Parker 1965; Goldstein, Ramaty, and Fisk 1970). However, the situation where particles are accelerated from lower energies to higher ones while suffering energy losses is difficult. In this case, the main question is whether they can gain a certain amount of energy before they lose it again—or if they are lost entirely from the process. The last alternative is the one described by a loss time and, therefore, appears a reasonable simplification.

The boundary conditions regarding equation (1) are that f remain well-behaved everywhere and be continuous across surfaces of discontinuity of the background plasma. The current density

$$\boldsymbol{S} = C_g \boldsymbol{V} f - \boldsymbol{\kappa}\frac{\partial f}{\partial \boldsymbol{x}} \tag{2}$$

must also be continuous (e.g., Axford 1980). With the so-called Compton-Getting factor $C_g = (-p/3f)\partial f/\partial p$, this requirement for $\boldsymbol{S}$ implies a differential equation in p.

For plane shocks, as treated throughout most of this paper, and an overall steady state, equations (1) and (2) simplify to

$$V\frac{\partial f}{\partial x} - \frac{\partial}{\partial x}\left(\kappa\frac{\partial f}{\partial x}\right) + \frac{f}{\tau} = \frac{f_\infty}{\tau_\infty} \tag{3}$$

on either side of the shock, as well as to continuity of the normal component of the current density,

$$S = C_g V f - \kappa\frac{\partial f}{\partial x} \tag{4}$$

at the shock surface. Here κ denotes the effective diffusion coefficient along the shock normal direction, which defines the x-axis. In fact $\kappa \approx \kappa_\parallel \cos^2 \beta$, where $\kappa_\parallel$ is the diffusion coefficient along the magnetic field, and β is the latter's angle with the shock normal. We shall assume V, κ, τ_∞, and τ to be spatially uniform on either side of the shock. The shock will be taken at $x = 0$, positive x denoting the upstream region. Thus $x > 0$, one has $V = -V_s$, where $V_s > 0$ is the shock normal velocity in the frame of the upstream medium. For $x < 0$ then, $V = -(V_s - \Delta V)$, where $\Delta V > 0$ is the velocity jump across the shock. For strong adiabatic shocks, $\Delta V = 3V_s/4$.

The spatial dependence of f is then given by

$$f_\pm = f_\infty + [f(x=0, p) - f_\infty] \exp(-\alpha_\pm |x|)\,, \tag{5}$$

with

$$\alpha_+ \kappa_+ = V_s/2 + (V_s^2/4 + \kappa_+/\tau_+)^{1/2}\,, \tag{6}$$

$$\alpha_- \kappa_- = (\Delta V - V_s)/2 + [(V_s - \Delta V)^2/4 + \kappa_-/\tau_-]^{1/2}\,, \tag{7}$$

the subscripts $\pm$ referring to the regions $x \gtrless 0$ respectively. For simplicity, it has also been assumed that $\tau_\pm/\tau_{\infty\pm} = 1$. This implies $f_\pm \to f_\infty$ for $x \to \pm\infty$.

In the case where the loss rates are zero, we have $\alpha_- = 0$ and $\alpha_+ = V_s/\kappa_+$. Finite losses increase these α's and steepen the spatial gradients of the accelerated component $[f(x=0, p) - f_\infty] \exp(-\alpha_\pm |x|)$.

Continuity of S implies [writing $f(0, p)$ for $f(x = 0, p)$]

$$(-p/3)(\partial/\partial p)[f(0, p)] = (\kappa_+\alpha_+ + \kappa_-\alpha_-)[f(0, p) - f_\infty]/\Delta V\,, \tag{8}$$

with the formal solution

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$$[f(0, p) - f_\infty] = -\int_0^p \left(\frac{\partial f_\infty}{\partial p}\right)_{p'} \exp\left[-3\int_{p'}^{p} \frac{(\kappa_+\alpha_+ + \kappa_-\alpha_-)}{\Delta V}\frac{dp''}{p''}\right] dp'\,. \tag{9}$$

Writing

$$\frac{\kappa_+\alpha_+ + \kappa_-\alpha_-}{\Delta V} = \frac{V_s}{\Delta V} + \eta\,, \tag{10}$$

with

$$\eta = \frac{V_s}{2\Delta V}[(1 + X_+)^{1/2} - 1] + \frac{V_s - \Delta V}{2\Delta V}[(1 + X_-)^{1/2} - 1]\,, \tag{11}$$

$$X_+ = 4\kappa_+/(V_s^2\tau_+)\,; \qquad X_- = 4\kappa_-/[(V_s - \Delta V)^2\tau_-]\,, \tag{12}$$

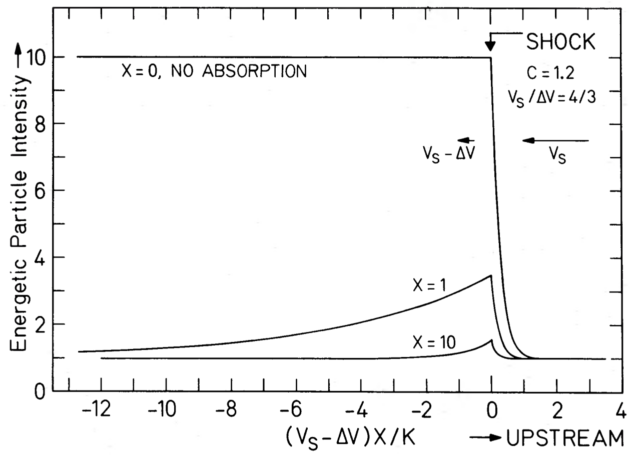

FIG. 1.—The effect of different uniform loss rates τ^{-1} on the acceleration and spatial distribution of energetic particles near a shock. A unit source with a power-law spectrum in rigidity is assumed. The absorption parameter $X = 4\kappa/V_s^2\tau$.

the losses are contained in the quantity η. The parameters X_+ and X_- represent approximately the ratio of the acceleration time $t_{\text{acc}} \approx 4\kappa/V_s^2$ to the loss time $\tau_\pm$. Obviously, losses are important if either X_+ or X_- or both are large compared to 1 (Bulanov and Dogiel 1979*a*, *b*; Völk, Morfill, and Forman 1979). The form of η shows clearly that losses on both sides of the shock are about equally effective; if either X_+ or X_- is large compared to 1, there is little acceleration because acceleration requires scattering between both regions. If the losses in one region (due to ionization or adiabatic deceleration, for instance) are large, then the acceleration is suppressed even if the losses in the other region should be negligible. Effective acceleration requires a combination of large shock velocities, small diffusion coefficients, and small loss rates on both sides of the shock.

To demonstrate this very simply, we choose $f_\infty(p) \approx p^{-3C}$ and $(\kappa_+ \alpha_+ + \alpha_- \kappa_-)/\Delta V = C'$, where C' is simply a large constant, larger than the Compton-Getting factor C of the source spectrum. Then, we get

$$\frac{f(0, p)}{f_\infty} = \frac{C'}{C' - C} > 1 \tag{13}$$

for $C' > C$, which approaches 1 for $C' \gg C$.

Figure 1 illustrates the effect of different values of X on the acceleration near the shock. Note how both the maximum intensity and the spatial scale over which it extends are suppressed by the losses. In this example, τ is the same everywhere.

In general, equation (9) can be written as

$$f(0, p) - f_\infty = -\int_0^p dp' \left(\frac{\partial f_\infty}{dp}\right)_{p'} \left(\frac{p}{p'}\right)^{-3V_s/\Delta V} \quad \exp\left[-3\int_{p'}^{p} \frac{\eta(p'')dp''}{p''}\right]. \tag{14}$$

Expression (14) shows that the accelerated particles at momentum p come from all lower momenta p' in the source spectrum, attenuated by the exponential factor involving $\eta(p)$. If the losses are large at some p'', then very few particles of momentum $p' < p''$ in the source can contribute to the accelerated particles.

The different types of possible loss mechanisms will cause η to vary with momentum or energy in characteristic ways. Ionization and Coulomb losses will always make η large at low energies, but these losses may be small enough to permit acceleration above a certain threshold energy. Nuclear collisions will make η finite and constant at relativistic energies, and large, if the medium is dense, so that acceleration would not be possible in dense media at relativistic energies. Finally, it is possible that losses due to adiabatic deceleration behind an expanding shock, or simply a combination of all three effects, will make η large at all energies for some shock environments. Then, there will be no appreciable acceleration at any energy. Figure 2 shows the steady state spectra according to equation (14), which this acceleration mechanism produces for these representative types of variation of η with energy. More realistic, smoother variations of η with energy will produce smoother spectra.

Exact evaluation of the exponent in equation (14) will not affect these general conclusions but only describe more precisely how $f(p)$ varies near momenta where $X(p)$ is changing from small to large or from large to small values.

Power-law spectra with $C > 0$ cannot extend to arbitrarily low energies. We chose for illustration in Figure 2 a spectrum which has a low-energy cutoff at momentum p_0, although any spectrum with $C \to 0$ as $p \to 0$ with some characteristic momentum p_0 would be equally realistic at small p and show similar effects. Note that the accelerated spectrum in Figure 2*a* becomes a power law independent of p_0 only for $p \gg p_0$. Similarly, in Figure 2*b*, the accelerated

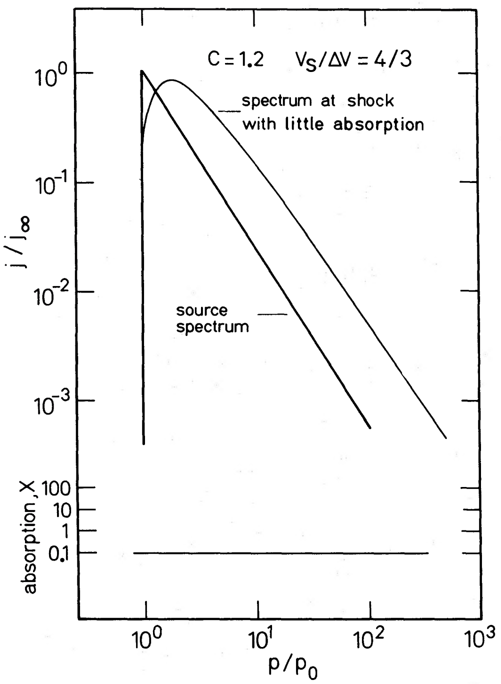

FIG. 2*a*

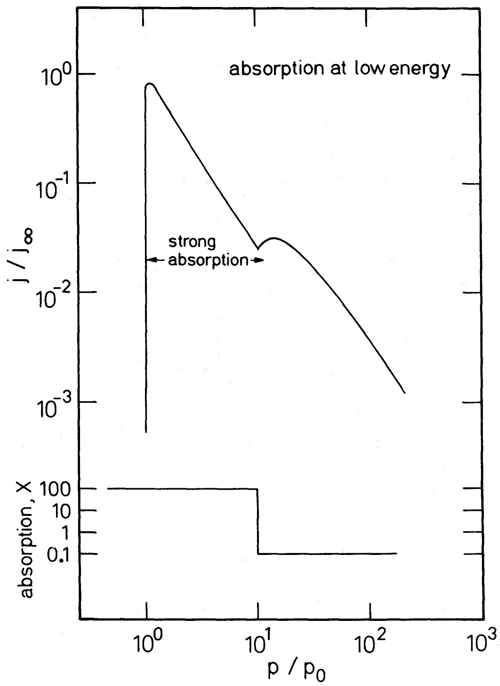

FIG. 2*b*

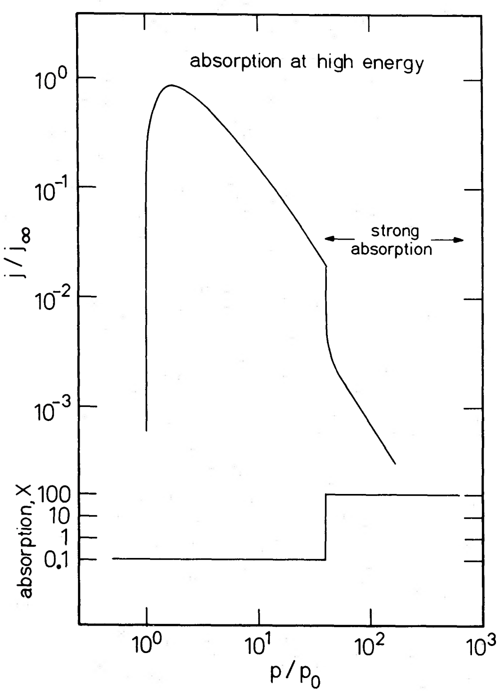

FIG. 2*c*

FIG. 2.—Spectra resulting from acceleration by diffusive scattering across a shock, for a power-law source spectrum, for different representative variations of $X = 4\kappa/V_s^2\tau$ with momentum: (*a*) X small at all energies; (*b*) X small above some critical momentum; (*c*) X small below some critical momentum.

spectrum just above p_1 (the momentum where X becomes small, $p_1 = 10p_0$ in the figure) is not a power law, but it becomes a power law when $p \gg p_1$.

In Figures 1 and 2, the Compton-Getting factor of the source spectrum was chosen to be hard enough so that $C = 1.2 < V_s/\Delta V = \frac{4}{3}$. Then, the spectrum of accelerated particles at $p \gg p_1$ retains the same form as the source spectrum, and the amplification at $p \gg p_1$ becomes independent of energy and is given by equation (13). This was done to illustrate the effects of losses without other complications. In fact, $C = 1.2$ is an unrealistically hard spectrum. The local high-energy cosmic-ray background, which is a likely source for his mechanism, has $C = 1.53$. Other conceivable, locally preaccelerated sources for this mechanism, such as flare-type spectra, would probably have even larger C.

The steady state solutions we have been discussing are applicable only when $C < V_s/\Delta V + \eta$. Otherwise, equation (13) makes no sense, and even equation (14) (which is correct for the truly steady state) incorrectly predicts larger amplification, proportional to $(p/p_1)^{3C - V_s/\Delta V}$ at high energies. The problem is that the steady state spectrum in the case $C > V_s/\Delta V + \eta$ is only reached after a long time at high energies, $p \gg p_1$. The time-dependent solutions (Krymsky *et al.* 1979; Forman and Morfill 1979) will be discussed in detail for various source spectra in another paper, but we present a few relevant results from that work here. At energies just above p_1 (or p_0 if $p_1 < p_0$), the steady state form, equation (14), applies. At p rather larger than p_1, a characteristic spectrum, $f(0, p) - f_\infty = f_\infty(p_1)(p/p_1)^{-3V_s/\Delta V}$, is produced. This spectrum extends from a few times p_1 to, near the highest momentum, $p_h \approx p_1$ exp $\{CV_s^2t/4\kappa[(C - V_s/\Delta V]\}$, where the steady state has been reached in the age of the accelerating region, t. Above p_h, the spectrum falls off more steeply, parallel to the source spectrum. At momenta so large and at times so small that the steady state has not been reached at that momentum, the amplification does not depend on the particle momentum but only on the age, t, as exp $[3CV_s^2t/(4\kappa)]$.

III. CONDITIONS ON DIFFUSION COEFFICIENTS IN SHOCK ACCELERATION REGIONS

In § II we discussed the formal changes that occur in the general shock acceleration picture if losses are included. This was done for a homogeneous medium. In order to see whether shock acceleration can indeed be considered locally in the Galaxy, and whether the propagation characteristics of cosmic rays in a shock acceleration region must be similar to or different from the overall galactic propagation, as inferred from the leakage lifetime τ^{CR} and a linear galactic scale L, we compare the relevant time scales.

Assuming overall galactic storage as well as leakage of cosmic rays to be described by a diffusion approximation with an effective diffusion coefficient κ_1, the diffusion time of a particle across the galaxy is given by $\tau^{CR} \approx L^2/\kappa_1$. The time an accelerating shock needs to cross the Galaxy is L/V_s (if the shock itself lives for such a long time). In order to apply diffusion theory everywhere in the Galaxy, we must have

$$L/V_s \gg L^2/\kappa_1 \ . \tag{15}$$

Then, the shock is a local phenomenon, and the conditions at the galactic boundary are unimportant. On the other hand, for acceleration to be faster than leakage, we need to have

$$4\kappa_2/V_s^2 \ll L^2/\kappa_1 \ , \tag{16}$$

where κ_2 is the diffusion coefficient in the acceleration region. Inequalities (15) and (16) are compatible only if

$$\kappa_2 \ll \kappa_1 \ , \tag{17}$$

i.e., the scattering mean free path in the acceleration region must be small compared to the effective overall galactic mean free path.

This condition can be fulfilled under two circumstances. The overall galactic escape may be based on turbulent diffusion and convection of the interstellar medium as a whole, carrying the fields and cosmic rays with it on scales which are large compared to the cosmic-ray scattering mean free path along the magnetic field lines (e.g., Jokipii and Parker 1969; Parker 1969; Axford 1980). Then, κ_1 can indeed be much larger than the microscopic scattering diffusion coefficient and inequality (17) may not have much significance. Alternatively, κ_1 may not be entirely dominated by large-scale stochastic mass motions. In this case, the acceleration region must be associated with strongly enhanced wave activity compared to the average galactic situation. These waves must then be produced either directly by the shock or by the accelerated cosmic rays themselves (e.g., Lerche 1967; Kulsrud and Pearce 1969). There is no positive guarantee, however, that the necessary wave power exists for efficient acceleration in every convenient type of shock configuration, nor that this can then be used as a kind of a black box for cosmic-ray production. On the contrary, for shocks in regions with strong wave damping, one expects much less scattering power of the background medium than that which exists on the average in the Galaxy, unless the waves stem ultimately from the shock itself. In § IV, we make some estimates for the shock strengths required in different environments where wave damping and particle energy losses occur.

IV. WAVE FIELD REQUIREMENTS AGAINST ION-NEUTRAL DAMPING AND PARTICLE ENERGY LOSSES

Considering first molecular clouds, we use the estimates which Cesarsky and Völk (1978) made for a so-called standard cloud of density $n_H = 2 \times 10^3 N$ atoms cm^{-3}, ion density $= 2 \times 10^{-6} N_i$ cm^{-3}, radius $R_c = 8.2R$ pc, mass $M_c = 10^5\ M_\odot$, and magnetic field $B_c = 50B$ micro-gauss; the normalization parameters N, N_i, R, and B equal unity for the standard case. Two types of criteria are used to estimate the scattering mean free path $\lambda_{scat} = vt_{scat}$ and the shock velocities required for particle acceleration. First of all, lower bounds on λ_{scatt} are set by estimating the maximum level of fluctuations W_w which the available energy sources for waves can maintain against ion-neutral friction. The most important energy sources for waves considered are gravitational collapse of the cloud and hydromagnetic motions induced by expanding H II regions. Asking, then, for the necessary speeds V_s of shocks that can accelerate cosmic rays with the estimated λ_{scatt} against particle energy losses (i.e., for which $X \leq 1$ from equation [12]), we obtain lower bounds on V_s. The strongest energy source, at least averaged over the entire cloud, is presumably the gravitational energy released in a free-fall time. In this case, we obtain $V_s/RN \leq 10^8$ cm s^{-1} for mildly relativistic particles $[p/(mc)] \leq 1$, and nuclear losses dominant] and $V_s/RN \leq 10^{10}$ cm s^{-1} for 10 MeV protons (ionization or Coulomb losses dominant) respectively.

In reality, we must assume wave production to be only a minor by-product of gravitational collapse. Since $V_s^2 \propto \lambda_{\text{scatt}} \propto W_w^{-1}$ for $t_{\text{scatt}} = \tau$, such a reduction in the efficiency of wave production will correspondingly raise the lower bound for V_s. The wave intensity to be expected from the volume oscillations of H II regions in the cloud's overall magnetic field (Arons and Max 1975) is about two orders of magnitude smaller than that possible from free-fall collapse. Thus, formally, the required values for V_s would have to be an order of magnitude larger than those given above. Such unreasonably large numbers, of course, simply show that, from energy arguments regarding wave production by processes not directly related with the shock, typical molecular clouds are to be considered as scatter-free, as far as acceleration is concerned, *on the average*, over the whole cloud.

On the other hand, it is only near the shock where a high level of magnetic fluctuations is required for acceleration. Thus, even without cosmic rays present, the *local* influence (e.g., the ionization precursor) which a strong shock will have on the ambient medium may be important. This question is the subject of § V. Here, we continue to assume that the ionization structure of the upstream medium is not altered by the presence of the shock. In order to obtain the second criterion mentioned above, the possible self-excitation of upstream waves through the accelerated particles is considered in competition with ion-neutral friction in the upstream cloud. From the kinematics of the acceleration process (e.g., Bell 1978*a*), it follows that the accelerated particles diffuse with (normal) speed V_s through the upstream medium. This determines a critical wave growth rate which must exceed the damping rate to lead to a self-consistent mechanism in the absence of other sufficiently strong wave sources. For V_s below the critical value, there is no acceleration of any kind in this picture. The critical condition is $V_s/V_{A,n} > 1 + [p/(m_H c)]^{3/2} \times 4.2 \times 10^2/A$ for nonrelativistic shock speeds (cf. Kulsrud and Cesarsky 1971), where m_H is the proton mass. Neglecting geometrical factors, for most parameters of interest (see § V), $V_{A,n}$ equals $V_A{}^* = B_c(4\pi n_i m_i)^{-1/2} \approx 2.4 \times 10^8\ BN_i{}^{-1/2}$ cm s^{-1}, the Alfvén speed of the ionized cloud component alone (Kulsrud and Pearce 1969), where m_i is the mean ion mass; $A > 1$ is the amplification factor of the cosmic-ray spectrum relative to the galactic nucleon spectrum (Morfill, Völk and Lee 1976). Formally evaluating the numbers, we obtain $V_s \gtrsim 10^{11} BN_i{}^{-1/2}/A$ cm s^{-1} for mildly relativistic protons, and $V_s \gtrsim 5.5 \times 10^9 BN_i{}^{-1/2}/A$ cm s^{-1} for 10 MeV protons. From these numbers, we conclude that, unless the upstream ionization structure is strongly modified by the shock itself, molecular clouds should be scatter-free ($\kappa \approx \infty$) from both criteria.

In particular, the criterion employing self-excited waves can also be applied to other media where ion-neutral friction is the dominant wave damping process. In the so-called warm intercloud medium, for the parameters of Kulsrud and Cesarsky (1971; see also McKee and Ostriker 1977)—$n_H = 0.2N$ atoms cm^{-3}; $n_i = 3 \times 10^{-2} N_{i,\infty}$ cm^{-3}; and $B = 3B$ micro-gauss—the condition reads $V_s/V_A{}^* > 1 + [p/(m_H c)]^{3/2} \times 10.4/A$, with $V_A{}^* \approx 3.8 \times 10^6 BN_{i,\infty}{}^{1/2}$ cm s^{-1}. Thus, one obtains $V_s \gtrsim 4 \times 10^7 BN_{i,\infty}{}^{-1/2}/A$ cm s^{-1} for mildly relativistic protons, and $V_s \gtrsim 3.8 \times 10^6 BN_{i,\infty}{}^{-1/2}/A$ cm s^{-1} for 10 MeV protons. Neither requirement appears prohibitive for strong shocks.

From the consideration of self-excited upstream waves, it is clear that $V_s/V_{A,n} > 1$ is a necessary condition for acceleration in a medium without a sufficiently strong background of preexisting magnetic irregularities. In this case, the (self-excited) waves travel in one direction into the upstream medium and, for their excitation, the drift speed V_s must exceed $V_{A,n}$. Indeed, under these circumstances the approximate transport equations (1) and (2) are exact to order v/c, if it is understood that $\boldsymbol{V} = \boldsymbol{V}^{\text{gas}} + V_{A,n}$ where now $\boldsymbol{V}^{\text{gas}}$ is the (normal) mean flow velocity of the gas carrying the waves (e.g., Skilling 1971). In order that the waves do not leave the shock upstream, and for $\boldsymbol{V}(\boldsymbol{x}, t)$ to describe a shock transition in the first place, we must therefore have $|\boldsymbol{V}^{\text{gas}}| > |V_{A,n}|$ everywhere in the region of the shock.

This is a slightly more general condition than the one required by the instability criterion alone. Particularly in the case of a partially ionized medium with strong wave damping, this can be a severe restriction: on the one hand, there only exists a negligible wave background; on the other hand, the phase velocity $V_A{}^* \propto (n_i)^{-1/2}$ can be quite large if $n_i/n_H \ll 1$. Even with a strong wave source providing a large background of waves propagating in both directions, equations (1) and (2) with $\boldsymbol{V} = \boldsymbol{V}^{\text{gas}}$ are meaningful only for the case $|\boldsymbol{V}^{\text{gas}}| \gtrsim |V_{A,n}|$ since, otherwise, second-order Fermi acceleration dominates shock acceleration (see also Blandford and Ostriker 1978). Thus, quite generally, shock acceleration requires rapidly propagating shock fronts. Large mass, momentum, and energy fluxes are only of indirect importance.

V. COSMIC-RAY ACCELERATION BY IONIZING SHOCK WAVES IN CLOUDS

The estimates of acceleration efficiencies in § IV assume that the ionization structure of the medium remains unchanged by the shock. In sufficiently strong shocks, however, not only collisional ionization becomes important in the hot postshock region, but also a radiation field is set up which photoionizes the gas ahead of the shock. This produces an "ionization precursor" (see, e.g., Raymond 1979; Shull and McKee 1979) and presumably an associated wave field. Behind the shock, radiative cooling eventually lowers the gas temperature and recombination takes place. Thus, even in a medium in whose unperturbed neutral state hydromagnetic waves are strongly damped, a strong shock carries along with it an ionization layer which exhibits very different wave propagation characteristics, in particular, little wave damping from neutrals. Compressive waves will still be damped heavily, on account of transit-time damping (Barnes 1966), but Alfvén waves are subject only to nonlinear damping mechanisms, which we disregard here. Concentrating on Alfvén waves, the question then is whether this ionization layer can serve as a local acceleration region irrespective of the large-scale environment.

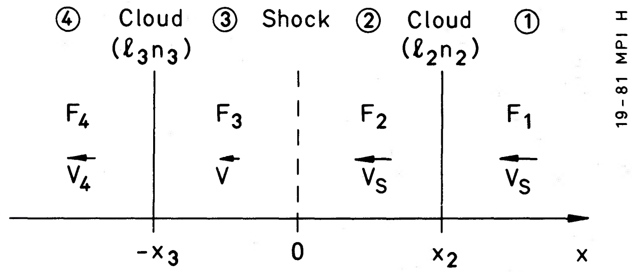

FIG. 3.—Schematic configuration of an ionizing shock of speed V_s in a neutral cloud, which is embedded in external media (1) and (4). The frame of the shock is assumed to describe a steady state, and the shock is located at $x = 0$. It creates upstream and downstream ionized regions (2) and (3), of extent x_2 and x_3. The remaining neutral portions of the cloud are assumed essentially scatter free with spatial extents l_2 and l_3, and densities n_2, and n_3 respectively. In the framework of a diffusion convection theory, they can therefore be treated as δ-functions located at $x = x_2$ and $x = -x_3$. The cosmic-ray distributions in the various regions are F_1 to F_4; the arrows denote flow speeds in the shock frame.

a) *Macroscopic Model*

A typical scenario is the interaction of a supernova shock with a neighboring interstellar cloud. The shock propagates into the cloud and is slowed down compared with the intercloud propagation speed. The postshock intercloud gas streams into the cloud and—for sufficiently dense and massive clouds—condenses onto the cloud so that we may disregard the possibility of a backward traveling shock. Figure 3 shows the situation schematically. The shock is inside the cloud; the ionized region around the shock extends the distances x_2 upstream and x_3 downstream. The two surrounding, almost neutral, and therefore practically scatter-free portions of the cloud are formally concentrated into infinitely thin slabs at $x = x_2$, and $-x_3$ with column densities $l_2 n_2$, and $l_3 n_3$ respectively. The shock is at rest in the comoving coordinate system. We make the assumption that the time scales relevant for the cosmic rays (e.g., cloud traversal times, l_2/c and l_3/c, and acceleration time, $4\kappa/V_s^2$) are small compared with the time scale for the shock passage through the cloud, $(l_2 + l_3)/V_s$, and that losses in the ionization region are small compared with losses in the main body of the cloud.

With these conditions, which do not seem unreasonable when substituting typical cloud parameters, the solution of the cosmic-ray transport equation is derived in Appendix A. This solution is rather unwieldy in its general form because there are two velocity jumps whose acceleration effects have the typical spatial range κ/V_s. Thus, the two scale parameters $x_2 V_2/\kappa$ and $x_3 V/\kappa$ determine whether or not the two acceleration regions overlap with each other and with the residual parts of the cloud.

There are simple limits, however. The first limiting case, $V_s x_2/\kappa \gg 1$, where the upstream cosmic-ray effects due to the shock are confined to the precursor region is indicated in Appendix A. The opposite limiting case, where the shock at $x = 0$ does not play a direct role any more, is obtained by the requirements

$$V_s(x_2/\kappa) \ll 1 \; ; \qquad V(x_3/\kappa) \ll 1 \; . \tag{18}$$

They imply a spatial extent of the acceleration region κ/V_s which is large compared to the size of the ionization layer. Then, the solutions (A15)–(A20) of Appendix A give

$$K_1 = K_2 = 0 \; ; \qquad K_3 = K_4 = \tfrac{1}{3}(V_s - V_4) \; ; \qquad K_5 = V_s + (l_2 + l_3)/\tau \; ; \qquad K_6 = K_5 - V_s \; ; \tag{19}$$

where the parameters K_1 through K_6 determine the p-dependence of the particle distribution.

The spectral exponent of the amplified component of the cosmic-ray flux is then

$$\alpha = -\frac{K_5}{K_3} = -\frac{3V_s}{V_s - V_4} - \frac{3(l_2 + l_3)}{\tau(V_s - V_4)} \, . \tag{20}$$

If there are no losses, this reduces to the standard result (e.g., Axford, Leer, and Skadron 1977). The solution for the cosmic-ray distribution inside the cloud is (for $p > p_0$)

$$f_c = f_0 + A_1\left(\frac{p}{p_0}\right)^{-3C} + A_0\left(\frac{p}{p_0}\right)^{\alpha} , \tag{21}$$

where $f_0 = G(p/p_0)^{-3C}$ is the cosmic-ray distribution at $x = \infty$ upstream, and A_1 and A_2 are given as

$$A_1 = G\left(\frac{C - Q}{C - Q - L}\right) \tag{22}$$

$$A_0 = G\left(\frac{C - Q}{C - Q - L} - \frac{Q}{L + Q}\right) \tag{23}$$

with

$$Q \equiv (l_2 + l_3)/[\tau(V_s - V_4)] \; ; \qquad L \equiv V_s/(V_s - V_4) \tag{24}$$

This result was considered (not as a special case of a more general solution) by Morfill *et al.* (1981), who investigated the possible interaction of the North Polar Spur, Loop I, supernova remnant with the interstellar cloud ρ Ophiuchi. We do not, therefore, repeat the conditions under which the cosmic-ray intensity may become enhanced even in the presence of losses, and refer the reader to this paper.

b) *Waves in the Ionization Layer*

The above solution and its limiting cases describe the typical overall situation from the point of view of the cosmic-ray transport equations. The central question, however, concerns the wave activity surrounding the shock since it determines the spatial scale κ/V of the acceleration region.

The waves in the upstream ionization zone are generated by photon emission and, possibly, resonant cosmic rays from the shock. Therefore, there is a tendency for the waves to propagate forward, in the same direction as the shock. Assuming wave production to be ultimately due to photoionization, we expect a broad-band frequency spectrum to be established around $x \approx x_2$. If the shock is super-Alfvénic, the forward waves also will encounter the shock since they are eventually overtaken. Also, apart from amplification, reverse waves are then generated though with smaller amplitude (e.g., McKenzie and Westphal 1969; Morfill and Scholer 1977). For linear, forward waves with amplitudes $\delta B_0 \ll B_0$ in the upstream medium, and a strong shock, the wave amplitudes downstream are given by:

$$\delta B^+ = \frac{3}{2}\frac{V_s - V_{A_0}}{V_s/2 - V_{A_0}}\,\delta B_0 \,, \tag{25}$$

$$\delta B^- = \frac{1}{2}\frac{V_s - V_{A_0}}{V_s/2 + V_{A_0}}\,\delta B_0 \,, \tag{26}$$

where the + and − signs denote forward and backward traveling waves respectively, and V_{A_0} is the Alfvén speed ahead of the shock based on the average field strength B_0. For $V_s \gg V_{A_0}$, $\delta B^+/\delta B^- \to 3$, which is the minimum ratio allowed, and $\delta B^- \leq \delta B_0$.

However, it is unlikely, from energy arguments, that propagating shocks inside dense molecular clouds can have velocities V_s much in excess of 10^7 cm s^{-1}. Furthermore, V_{A_0} is typically of the order of 10^6 cm s^{-1}. Since $\lambda_{\rm scatt} \approx |(\delta B^+)^2 + (\delta B^-)^2|^{-1}$, the contribution of the shock-generated δB^- to $\lambda_{\rm scatt}$ is, therefore, typically of the order of a few percent and may be neglected. The same argument applies to shock-generated forward waves from reverse waves. Thus, unless the expected asymmetry between the generation of forward and reverse waves upstream should be very large, it remains basically unchanged through the shock. The amplification for both waves is still given by equation (25).

As the wave disturbances travel further downstream the flow velocity may become smaller than the relevant Alfvén velocity. Also, recombination begins to set in, and the medium becomes increasingly more neutral. A detailed calculation is given in Appendix B. As a consequence, for slowly varying background parameters, the fate of the waves is characterized by four possibilities.

1. For sufficiently low wave frequencies ω in the frame of the shock, collisional friction between ions and neutrals enforces coherent mass motion. The wave propagation in the rest frame of the medium is $\pm V_{A,c} \approx \pm B/[4\pi(\rho_i + \rho)]^{1/2}$, where ρ_i, ρ, and B are the mass densities of ions and neutrals, and the field strength respectively. Wave damping, i.e., spatial decay, is very small until, behind the shock, $\epsilon = \rho_i/\rho$ ultimately becomes small compared to unity.
2. The forward, low-frequency waves are quite likely to reach a resonance $V - V_{A,c} \approx 0$ before they are damped away if we assume partial flux freezing $B \propto (\rho_i + \rho)^{1/2}$ (see, e.g., Mouschovias 1976); then $V_{A,c}$ remains roughly constant, although $(\rho_i + \rho)$ rises strongly in the relaxation zone behind the shock (e.g., Shull and McKee 1979). In resonance, the forward waves cannot propagate further away from the shock. The associated, strong, resonant damping dissipates the wave energy around that point. However, part of the forward wave energy might also be transformed to reverse wave energy which can propagate freely.
3. If ω is sufficiently high compared to the ion-neutral collision frequency ν_0, the waves propagate with the rest frame phase speed $\pm V_A{}^* \approx \pm B/(4\pi\rho_i)^{1/2}$ and a damping length proportional to ${\nu_0}^{-1}$.
4. Forward, high-frequency waves will reach resonance $V - V_A{}^* \approx 0$ with decreasing ϵ and get damped there. Again, production of reverse wave energy might be associated with this resonance.

c) *Estimate of the Cosmic-Ray Interaction Scales*

The acceleration of cosmic rays by an ionizing shock propagating through a largely neutral cloud will be inhibited if losses inside the cloud are important. In order to estimate the effects which the cloud may have, the typical cosmic-ray interaction scale length κ/V_s must be compared with the extent of the region of wave activity surrounding the ionizing shock, i.e., with x_2 and with x_3 (see Appendix A). The extent of the region of wave activity upstream of the shock must be the same as the extent of the ionization, since it is reasonable to assume that the waves are generated by the ionization precursor itself. From numerical calculations (Shull and McKee 1979), a column density of ionized material $L_i \approx 10^{17}$ cm^{-2} both upstream and downstream of the shock is obtained [shock speed $O(10^7$ cm s$^{-1})$, upstream gas density 10–100 cm^{-3}]. Downstream, the extent of wave activity is more difficult to determine. Wave convection and propagation into the neutral cloud is, in principle, possible (see Appendix B), albeit with some damping. Thus, the downstream extent of the region of wave activity is larger than the extent of the downstream ionization region. Furthermore, when the waves enter the partially neutral recombination region, cosmic rays may interact (under certain conditions) with two types of waves. First, if the resonant interaction frequency $f_{\text{res}} \gg \nu_0$ (the ion-neutral collision frequency), cosmic rays interact with waves which are "uncoupled," i.e., ions and neutrals do not move together. Second, if the resonant interaction frequency $\omega_{\text{res}} \ll \nu_0$, cosmic rays interact with waves which are "coupled," i.e., ions and neutrals move coherently. The damping and the propagation velocities of these two wave modes are different (see Appendix B).

In order to obtain quantitative estimates we make the following assumptions:

1. The cosmic-ray diffusion mean free path $\lambda \geq 10 r_g$, where r_g is the particle's gyroradius. (In the solar system, where wave activity $\delta B/B$ is very strong, of the order of 1, this relationship is roughly measured for relativistic cosmic rays. Since the available wave power is generally distributed over a broad frequency range, a minimum value $\lambda = 10 r_g$ appears reasonable [also on theoretical grounds] even though, by adiabatic effects, the wave amplitudes may be strongly amplified near resonances.)
2. The background field strength B scales with the total particle density as $(n_i + n)^{1/2}$, and, as reference values, we use $B_{\text{ref}} = 3$ micro-gauss, $(n_i + n)_{\text{ref}} = 1$ cm^{-3}. The quantities n_i and n are ion and neutral particle densities respectively.
3. In the fully ionized region surrounding the shock, wave damping is negligible.
4. For definiteness only, we assume that the chemistry in the recombination region has not proceeded very far, so that the dominant ions are H^+ and He^+, and the dominant neutrals are H and He. Under these circumstances, we may use a constant mean molecular weight 1.4 m_H throughout. The ion-neutral collision frequency $\nu_0 \approx 10^{-9} n$(Hz) (see, e.g., Kulsrud and Cesarsky 1971).

The gyroradius of a cosmic ray with rigidity R(GV) is given by $r_g = R/(3 \times 10^{-7} B)$, where B is in gauss. Applying our scaling for B, we obtain

$$\lambda \geq 10 r_g = 1.1 \times 10^{13} \frac{R}{(n + n_i)^{1/2}} \text{(cm)} . \tag{27}$$

For relativistic particles, $\kappa = \frac{1}{3}\lambda c$, so that the cosmic-ray scale length κ/V_s is given by (using $V_s = 10^7$ cm s^{-1} as a typical value):

$$\frac{\kappa}{V_s} \geq 1.1 \times 10^{16} \frac{R}{(n + n_i)^{1/2}} \text{(cm)} . \tag{28}$$

In the upstream ionized region, we have $n \approx 0$, and the extent x_2 of the ionized region is given by $x_2 = L_i/n_i$, i.e.,

$$x_2 \approx 10^{17}/n_i \text{(cm)} . \tag{29}$$

It follows that, for rigidities > 1 GV and for an upstream density $n_i + n \geq 100$ cm^{-2}, conditions (28) and (29) are not compatible any more and, thus, the whole upstream portion of the cloud influences the acceleration at the shock. We shall come back to this point later.

The condition that cosmic rays should be able to interact resonantly with both "coupled" and "uncoupled" waves is given by

$$f_{\text{res}} \ll \nu_0 , \tag{30}$$

where the resonant interaction frequency for a cosmic ray of rigidity $R(GV)$ is given by

$$f_{\text{res}} = \frac{3 \times 10^{-7} B(V + V_{A,n})}{2\pi R} . \tag{31}$$

For "uncoupled" waves, we must use $V_{A,n} \approx V_A{}^*$, for "coupled" waves $V_{A,n} \approx V_{A,c}$.

We may differentiate two regions in the downstream medium from the work of Shull and McKee (1979):

1. $V \gg V_A{}^* \geq V_{A,c}$, i.e., the region before which cooling and recombination become important. This region is of no consequence for wave damping and is assumed to contain the full, undisturbed wave field.
2. $V_A{}^* > V_{A;c} > V$, i.e., the region of recombination. Here, neutral particles are able to damp wave activity. The damping lengths for $V \to 0$ and $\epsilon = \rho_i/\rho \ll 1$ are, for low-frequency "coupled" waves ($\omega \ll \nu_0$),

$$L_L \approx \frac{2\nu_0 \epsilon(1 + \epsilon) V_{A,c}}{\omega^2} \tag{32}$$

and, for high-frequency "uncoupled" waves ($\omega \ll \nu_0$),

$$L_H \approx \frac{2V_A^*}{\nu_0} \tag{33}$$

(see Appendix B, eqs. [B16] and [B12]).

Using the scaling for B and equations (30) and (31), we obtain the result that cosmic rays may interact resonantly with *both* types of waves if the rigidity exceeds a critical value

$$R \gg R_{\rm crit} = \frac{78.8(n + n_i)^{1/2}}{n} . \tag{34}$$

For $n_i \ll n$, $R_{\rm crit} = 1$ GV if $n = 6.2 \times 10^3$ cm^{-2}. In the postshock recombination region, such values for the gas density are not unusual even for relatively modest, upstream gas densities (see, e.g., Raymond 1979; Shull and McKee 1979), so that, in general, we have to consider both types of waves.

Using the resonant frequency (31) and substituting in equation (32), we obtain the damping length for those "coupled" waves with which a cosmic ray of rigidity R(GV) may resonantly interact

$$L_L = 4.5 \times 10^9 \epsilon(1 + \epsilon)R^2(\text{cm}) . \tag{35}$$

In other words, the result is independent of density, and, for small ϵ, it is proportional to $\epsilon = \rho_i/\rho$. Damping scale lengths comparable with the mean free path (eq. [27]) can only be obtained for rigidities far in excess of 1 GV.

Similarly, for "uncoupled" waves, we have (substituting $V_A^*[\epsilon/(1 + \epsilon)]^{1/2} \equiv V_{A,c}$)

$$L_H \approx 1.1 \times 10^{15} \frac{1}{n} \frac{[(1 + \epsilon)]^{1/2}}{\epsilon} (\text{cm}) . \tag{36}$$

For example, with $n = 10^3$ and $\epsilon = 0.1$, this is 3.6×10^{12} cm. L_H is independent of particle rigidity.

In both cases, we conclude that the wave damping lengths for the particle rigidities of interest (1–100 GV) and for small ϵ (~ 0.1) are less than 10^{13} cm.

Compared with the extent of the fully ionized region in the postshock regime, which can again be derived from the "canonical" ion column density of $\sim 10^{17}$ cm^{-2}, this is small. Thus x_3, the postshock extent of wave activity is, for all practical purposes, given by the extent of the postshock ionization region. Equations (28) and (29) apply in the postshock region also; however, the mean ion density in the whole postshock ionized region, $\epsilon \geq 1$, is much greater than that upstream (due to shock compression and subsequent cooling). A typical scaling factor is ~ 50, as can be deduced from the work of Shull and McKee (1979). This implies that the scaling factors for cosmic-ray acceleration are too large (using sensible upstream gas densities $n > 10$ cm^{-3}), compared with x_3 (at least for particles with rigidity > 1 GV), and the whole downstream portion of the cloud can inhibit the shock acceleration.

A likely scenario, and one which is probably quite common, is the interaction of supernova shocks with interstellar clouds. A typical feature of such events, which we may derive from the above analysis, is that, initially, while the shock is propagating through the more tenuous outer layers of the cloud (density $\lesssim 10$ cm^{-3}), cosmic-ray acceleration may still occur and lead to enhanced γ-ray emission. Later on, the cloud will inhibit further local cosmic-ray acceleration, both because the shock slows down and because the acceleration scale length becomes larger than the ionization regions.

The authors would like to thank W. I. Axford for valuable discussions.

APPENDIX A

COSMIC-RAY INTERACTION WITH IONIZING SHOCK WAVES

The physical situation is depicted schematically in Figure 3, which is also described in the main text. For convenience, we choose the same κ in all the ionized regions and assume a cosmic-ray distribution function f_0 at $x = \infty$ of the form

$$f_0 = G\left(\frac{p}{p_0}\right)^{-3C} \qquad (p > p_0) \tag{A1}$$

$$f_0 = G \qquad (p \leq p_0) . \tag{A2}$$

In regions $i = 1$ through 4, we have to solve the diffusion convection equation for the distribution functions f_i

$$\frac{\partial}{\partial x}\left(-V_i f_i - \kappa \frac{\partial f_i}{\partial x}\right) = 0 , \tag{A3}$$

where the V_i are indicated in Figure 3. The solution is of the general type,

$$f_i = f_{0i} + A_i \exp\left[-V_i(x/\kappa)\right], \tag{A4}$$

with $\partial f_{0i}/\partial x = \partial A_i/\partial x = 0$, and the boundary conditions are

$$f_1 \to f_0 \qquad (\text{for } x \to \infty) \tag{A5}$$

$$f_4 \text{ finite} \qquad (\text{for } x \to -\infty), \tag{A6}$$

if V_4 is directed in the same sense as V_s, or

$$f_4 = f_- \qquad (\text{for } x \to -\infty). \tag{A7}$$

Then, f_- is the (arbitrary) intensity at $-\infty$. Continuity of f requires

$$f_1(x_2) = f_2(x_2) \equiv f_{c2} \qquad (\text{for } x = x_2) \tag{A8}$$

$$f_2(0) = f_3(0) \equiv f_{c3} \qquad (\text{for } x = 0) \tag{A9}$$

$$f_3(-x_3) = f_4(-x_3) \qquad (\text{for } x = -x_3). \tag{A10}$$

In addition, the streaming across the cloud portions and the shock must be continuous (if we include absorption). This yields three further equations:

$$\left(V_s C_g f_1 + \kappa \frac{\partial f_1}{\partial x}\right)_{x_2} - \left(V_s C_g f_2 + \kappa \frac{\partial f_2}{\partial x}\right)_{x_2} = \frac{l_2}{\tau_2} f_{c2} \tag{A11}$$

$$\left(V_s C_g f_2 + \kappa \frac{\partial f_2}{\partial x}\right)_0 - \left(V C_g f_3 + \kappa \frac{\partial f_3}{\partial x}\right)_0 = 0 \tag{A12}$$

$$\left(V C_g f_3 + \kappa \frac{\partial f_3}{\partial x}\right)_{-x_3} - \left(V_4 C_g f_4 + \kappa \frac{\partial f_4}{\partial x}\right)_{-x_3} = \frac{l_3}{\tau_3} f_{c3}, \tag{A13}$$

where $(1/\tau_2) = \alpha n_2$ and $(1/\tau_3) = \alpha n_3$ are the loss rates of cosmic rays due to nuclear interactions in the preshock and postshock region of the cloud (with densities n_2 and n_3 and column densities $l_2 n_2$ and $l_3 n_3$ respectively). The Compton-Getting operator is $C_g = -(p/3)(\partial/\partial p)$.

The process of solving these equations is rather cumbersome and will not be repeated here. By suitable substitutions, one arrives at two coupled, first-order, ordinary differential equations in p. These can be combined to a second-order differential equation for the amplitude A_1 (see equation A4):

$$K_1 p^2 \left(\frac{\partial^2}{\partial p^2}\right) A_1 + K_2 p^2 \left(\frac{\partial^2}{\partial p^2}\right) f_0 + K_3 p \left(\frac{\partial}{\partial p}\right) A_1 + K_4 p \left(\frac{\partial}{\partial p}\right) f_0 + K_5 A + K_6 f_0 = 0, \tag{A14}$$

where, putting

$$a_1 \equiv \left\{1 - \exp\left[V\left(\frac{x_3}{\kappa}\right)\right]\right\}, \qquad a_2 \equiv \left\{1 + \frac{l_2}{\tau_2 V_s} - \frac{l_2}{\tau_2 V_s} \exp\left[-V_s\left(\frac{x_2}{\kappa}\right)\right]\right\},$$

$$a_3 \equiv \left\{1 - \frac{l_2}{\tau_2 V_s} + \frac{l_2}{\tau_2 V_s} \exp\left[V_s\left(\frac{x_2}{\kappa}\right)\right]\right\}, \qquad a_4 \equiv \left\{-\frac{l_3}{\tau_3} + V \exp\left[V\left(\frac{x_3}{\kappa}\right)\right] + \frac{l_3}{\tau_3} \exp\left[V\left(\frac{x_3}{\kappa}\right)\right]\right\},$$

we have

$$K_1 \equiv (V_4 - V)(V_s - V) \frac{1}{qV} a_1 a_2 \tag{A15}$$

$$K_2 \equiv (V_4 - V)(V_s - V) \frac{1}{qV} a_1 a_3 \tag{A16}$$

$$K_3 \equiv \frac{1}{3}\left[(V - V_4) a_2 - (V - V_4) \frac{V_s}{V}\left(1 + \frac{l_2}{\tau_2 V_s}\right) a_1 - 3K_1 + \frac{V_s - V}{V} a_2 a_4\right] \tag{A17}$$

$$K_4 \equiv \frac{1}{3}\left\{(V - V_4) a_3 - (V - V_4)\left(\frac{V_s}{V}\right)\left(\frac{l_2}{\tau_2 V_s}\right) \exp\left[V_s\left(\frac{x_2}{\kappa}\right)\right] a_1 - 3K_2 + \frac{V_s - V}{V} a_3 a_4\right\} \tag{A18}$$

$$K_5 \equiv \frac{V_s}{V}\left(1 + \frac{l_2}{\tau_2 V_s}\right) a_4 + \frac{l_3}{\tau_3} a_2 \tag{A19}$$

$$K_6 \equiv \frac{V_s}{V} \frac{l_2}{\tau_2 V_s} \exp\left[V_s\left(\frac{x_2}{\kappa}\right)\right] a_4 + \frac{l_3}{\tau_3} a_3. \tag{A20}$$

The cosmic-ray intensity in the upstream portion of the cloud then becomes (for $p > p_0$)

$$f_{c2} = f_0(1 - Q_0) + G(Q_0 - Q_1)\left(\frac{p}{p_0}\right)^{\alpha}, \tag{A21}$$

where f_0 is given in (A1) and

$$Q_0 = \frac{K_2 3C(3C+1) - K_4 3C + K_6}{K_1 3C(3C+1) - K_3 3C + K_5} \exp\left[-V_s\left(\frac{x_s}{\kappa}\right)\right] \tag{A22}$$

$$Q_1 = \frac{K_6}{K_5} \exp\left[-V_s\left(\frac{x_2}{\kappa}\right)\right], \tag{A23}$$

and α is defined by the simple quadratic

$$K_1 \alpha(\alpha - 1) + K_3 \alpha + K_5 = 0\,. \tag{A24}$$

Similarly, the cosmic-ray intensity in the downstream portion of the cloud is (for $p > p_0$)

$$f_{c3} = f_0\left\{K_7 - K_9 Q_0 \exp\left[V_s\left(\frac{x_2}{\kappa}\right)\right]\right\} + K_8 G(Q_0 - Q_1) \exp\left[V_s\left(\frac{x_2}{\kappa}\right)\right]\left(\frac{p}{p_0}\right)^{\alpha}, \tag{A25}$$

where α is given by (A24) and

$$K_7 \equiv a_3 - \frac{l_2}{\tau_2 V} \exp\left[V_s\left(\frac{x_2}{\kappa}\right)\right] a_1 + C\frac{V_s - V}{V} a_1 a_3 \tag{A26}$$

$$K_8 \equiv a_2 - \frac{V_s}{V}\left(1 + \frac{l_2}{\tau_2 V_s}\right) a_1 - \frac{\alpha}{3}\frac{V_s - V}{V} a_1 a_2 \tag{A27}$$

$$K_9 \equiv a_2 - \frac{V_s}{V}\left(1 + \frac{l_2}{\tau_2 V_s}\right) a_1 + C\frac{V_s - V}{V} a_1 a_2\,. \tag{A28}$$

We see quite generally from (21) and the expressions for Q_0 and Q_1 that, with increasing x_2, the shock begins to be more and more isolated from the upstream portion of the cloud. The critical scale factor is $V_s(x_2/\kappa)$. In the limit where this is large compared to 1, we obtain $f_{c2} = f_0/(1 + l_2/\tau_2 V_s)$, which now acts as the convected input from the local acceleration within the ionization layer. The opposite limiting case is described in the main text.

APPENDIX B

CONVECTION AND PROPAGATION OF ALFVEN WAVES IN A PARTIALLY IONIZED MEDIUM

In § V*b* we stated that waves are locally produced in the fully ionized region ahead of the shock and then are convected (in the shock frame) downstream into the cloud which, through recombination, becomes progressively more neutral. In the steady state frame of the shock, the waves are characterized by a (real) frequency ω, which is determined by the physical process responsible for wave generation and remains constant during propagation. The question then concerns the spatial dependence of the wave field. This is in contrast to a situation discussed by Kulsrud and Pearce (1969) who solved an initial value problem. The steady state case considered here allows the discussion of waves propagating in (quasi) stationary flows like supernova remnants interacting with clouds, stellar winds in a cool interstellar medium, radiation-dominated H II regions, or high-velocity clouds. Therefore, it has a different and perhaps wider application. We assume all background parameters to vary slowly over a wavelength so that a lowest order WKB approximation may be employed.

The local dispersion relation for Alfvén waves (Kulsrud and Pearce 1969) is Galilean invariant. It reads

$$(\tilde{\omega}^2 - \omega_k{}^2)\tilde{\omega} + i\nu_0[(1+\epsilon)\tilde{\omega}^2 - \epsilon\omega_k{}^2] = 0\,, \tag{B1}$$

where $\tilde{\omega} \equiv \omega - \boldsymbol{k} \cdot \boldsymbol{V}$, $\omega_k \equiv \boldsymbol{k} \cdot \boldsymbol{V}_A{}^*$, $V_A{}^* = B/(4\pi\rho_i)^{1/2}$ with B denoting the field strength, $\epsilon = \rho_i/\rho$ is the ratio of ion to neutral mass density, $\nu_0 \equiv \rho\langle\sigma v_t\rangle m_{\text{red}}/(m \times m_i)$ is the ion-neutral collision frequency for momentum exchange, $\langle\sigma v_t\rangle \approx 10^{-9}$ cm^3 s^{-1} (e.g., Kulsrud and Cesarsky 1971) is the collision rate, and m_{red} is the reduced mass for ions and neutrals with the masses m_i and m respectively. In the one-dimensional aligned case considered here, we have $\tilde{\omega} \equiv \omega - kV$, $\omega_k \equiv kV_A{}^*$. The parameters ω, V, and $V_A{}^*$ are assumed positive.

Equation (B1) determines the wave number k. Introducing instead

$$V_\phi = \frac{\omega}{k} - V\,, \tag{B2}$$

the phase velocity in the frame of the medium, we get the two alternative forms

$$(V_\phi^2 - V_A{}^{*2})\left[V_\phi + i\frac{\nu_0}{\omega}\epsilon(V_\phi + V)\right] + i\frac{\nu_0}{\omega}V_\phi{}^2(V_\phi + V) = 0 \tag{B4}$$

$$(V_\phi^2 - V_A{}^{*2})V_\phi + i\frac{\nu_0}{\omega}(1+\epsilon)(V_\phi + V)\left[V_\phi{}^2 - \frac{\epsilon}{(1+\epsilon)}V_A{}^{*2}\right] = 0 \, . \tag{B5}$$

We solve the cubic equation (B4) or (B5) in several asymptotic limits.

$$\text{a)} \quad \nu_0/\omega \ll 1 \, . \tag{B6}$$

In this case, to lowest order, independent of ϵ

$$V_\phi = \pm V_A{}^* \; ; \qquad \omega/k = V \pm V_A{}^* \, , \tag{B7}$$

corresponding to a fully ionized medium or large frequencies ω in a partially ionized medium. The $\pm$ sign describes reverse (+) and forward (−) waves. Using the zeroth order (B7) in equation (B4), the first-order result is

$$\frac{\omega}{k} = (V \pm V_A{}^*)\left\{1 - \frac{i}{2}\frac{\nu_0}{\omega}\frac{(\pm V_A{}^*)}{[\pm V_A{}^* + i\epsilon(\nu_0/\omega)(V \pm V_A{}^*)]}\right\} \tag{B8}$$

If, in addition to inequality (B6), we assume

$$\epsilon\frac{\nu_0}{\omega}\,|(V \pm V_A{}^*)| \ll V_A{}^* \, , \tag{B9}$$

equation (B8) simplifies to

$$\frac{\omega}{k} = (V \pm V_A{}^*)\left(1 - \frac{i}{2}\frac{\nu_0}{\omega}\right) . \tag{B10}$$

Inspection of equation (B8) shows that this last result remains essentially valid up to $\epsilon(\nu_0/\omega)\,|(V \pm V_A{}^*)| = O(V_A{}^*)$. Thus, for $V = O(V_A{}^*)$ also, large values of $\epsilon = O(\omega/\nu_0)$ are described basically by equation (B10).

The wave number is given by

$$k \approx \frac{\omega}{(V \pm V_A{}^*)}\left(1 + \frac{i}{2}\frac{\nu_0}{\omega}\right) , \tag{B11}$$

showing a strongly damped resonance of forward waves for $V - V_A{}^* = 0$. In general, the damping length is given by

$$\frac{1}{\mathrm{Im}\,(k)} \approx \frac{2}{\nu_0}(V \pm V_A{}^*) \, . \tag{B12}$$

$$\text{b)} \quad \nu_0/\omega \gg 1 \, . \tag{B13}$$

Since $\epsilon \geq 0$ this also means $(\nu_0/\omega)(1+\epsilon) \gg 1$. Therefore, away from the resonance $V + V_\phi = 0$, we obtain to lowest order for $\epsilon \gg (\nu_0/\omega)^{-1}$:

$$V_\phi = \pm\left(\frac{\epsilon}{1+\epsilon}\right)^{1/2} V_A{}^* \equiv \pm V_{A,c} \, , \tag{B14}$$

where the subscript c in $V_{A,c}$ implies coupled motion of ions and neutrals in the wave. Many collisions occur during a wave period.

The first-order solution for $(\nu_0/\omega)(1+\epsilon)\,|V \pm V_{A,c}| \gg V_{A,c}$ is then

$$\frac{\omega}{k} = (V \pm V_{A,c}) - i\frac{V_{A,c}{}^2}{2(\nu_0/\omega)(1+\epsilon)(V \pm V_{A,c})\epsilon} \, . \tag{B15}$$

As long as $\epsilon \gg (\nu_0/\omega)^{-1}$, this leads to the damping length

$$\frac{1}{\mathrm{Im}\,(k)} \approx \frac{2(\nu_0/\omega)(1+\epsilon)(V \pm V_{A,c})^3}{\omega V_{A,c}{}^2} \, . \tag{B16}$$

The solution (B15) is inappropriate near the resonance $V - V_{A,c} = 0$ since the ordering $(\nu_0/\omega)(1+\epsilon)\,|V - V_{A,c}| \gg V_{A,c}$ does not apply for forward waves. However, one can show that the physical expectation of a strongly damped resonance near $V = V_{A,c}$ holds for forward waves, as in case (*a*) for $V = V_A{}^*$. Putting $V_\phi = -V_{A,c} + ix + y$ in equation (B5), with $x/V_{A,c} \ll 1$, $y/V_{A,c} \ll 1$, but neglecting terms $\sim (V - V_{A,c})$, results in

$$x = y \approx (-1)\left[\frac{1}{4}\frac{V_{A,c}{}^2}{\epsilon(v_0/\omega)(1+\epsilon)}\right]^{1/2} . \tag{B17}$$

This implies strongly damped resonance at $V = V_{A,c} - x$.

$$\text{c)} \quad V \ll |V_\phi| . \tag{B18}$$

In this case, convection is negligible and equation (B5) reduces to a quadratic equation for V_ϕ with the solution

$$k = \frac{\omega}{(\pm V_A{}^*)}\left\{1 + \left[\frac{v_0}{\omega}(1+\epsilon)\right]^2\right\}^{1/2}\left\{1 + \frac{\epsilon}{(1+\epsilon)}\left[\frac{v_0}{\omega}(1+\epsilon)\right]^2 - i\frac{v_0}{\omega}\right\}^{1/2} . \tag{B19}$$

This of course comprises the asymptotic cases (*a*) and (*b*) for $V = 0$. For $(v_0/\omega) \gg 1$, but $\epsilon(v_0/\omega) = 0(1)$, or smaller, the wave becomes critically damped, ie., Im $(k) \gtrsim$ Re (k).

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