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THE PROPERTIES OF CLOSE MULTIPLE STARS

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ABSTRACT

The observational properties of close multiple stars are compared with the results of numerical studies of the formation of close multiple stars by both fission and fragmentation. For 35 systems with a long period of about 100 years or less, the mean ratio of long to short period is 3222 : 1. The few observed systems containing a rapidly rotating single star are consistent with numerical fragmentation models which find that a larger ratio of spin to orbital angular momentum produces a smaller period ratio. Assuming that the fragmentation process produces binaries with mass ratios close to unity while the fission process produces binaries of very unequal mass, the fragmentation process dominates the fission process by a ratio greater than 2 : 1. Comparing the short- and long-period orbital inclinations of 20 systems, 33% of the orbital pairs are not coplanar. For the rest, coplanarity is a permitted possibility. Using analytical stability criteria, all 27 close multiple systems considered are stable if they are corotating. If they are counter-rotating, four systems may be, but are not necessarily, unstable. The ages of the observed solar-type close multiple systems range from 1×10^8 to 2×10^9 years. Thus, all are substantially younger than the Sun.

Subject headings: stars: binaries — stars: formation — stars: visual multiples

I. INTRODUCTION

In recent years numerical simulations of binary and multiple star formation by both fission and fragmentation have become possible. Larson (1969) was the first to examine the collapse in one dimension of an interstellar cloud into a protostar. This was followed by two-dimensional numerical calculations of the collapse of axisymmetric rotating protostars carried out by Larson (1972), Black and Bodenheimer (1975, 1976), and others. Interpreted in three dimensions, those studies found that the collapse leads to the formation of a ring of matter embedded in the cloud.

Norman and Wilson (1978) have taken the numerical calculations one step further and examined the stability of this ring in three dimensions to nonaxisymmetric perturbations. Larson (1972) had postulated that the breakup of the ring into two or more fragments could account for the observed predominance of multiple star systems. Norman and Wilson (1978) found that the ring fragmented into two or three bodies in less than one-half of the rotation period after the perturbation. Since their calculations stopped when the collapse of the bodies became adiabatic, a density of about 10^{13} g cm^{-3}, they could not determine whether these bodies were ejected from the system or sent into more loosely bound orbits. Instead, they suggested a scenario in which a massive cloud undergoes gravitational collapse and forms a ring which fragments into blobs, which in turn collapse and form rings inside them, which fragment, and so on.

Bodenheimer (1978) has explored the consequences of this continued hierarchical ring fragmentation. He has considered schematic fragmentation sequences based on recent two- and three-dimensional numerical hydrodynamical calculations of the early collapse phases of rotating protostars. Although he notes that changing the basic assumptions used to calculate these sequences has substantial effects on the resulting multiple systems, multiple stars can be formed with properties similar to observed systems. The most important parameter determining the period ratio is the fraction of the cloud angular momentum which remains in the spin of a fragment. If the fraction in spin is small, the period ratio is large, and vice versa.

The formation of a close binary by fission during a protostar's final quasi-static contraction to the main sequence has often been suggested. Lucy (1977) tested the fission hypothesis using a three-dimensional gas dynamical calculation of *low* spatial resolution with the protostar initially consisting of only 300 points. From numerous simulations he found that if fission does occur, it leads to binary components of very unequal mass. This conclusion strengthens the belief that close binaries can form by fission, but because of the low spatial resolution used in the study, it could not be conclusively demonstrated.

Triple-star stability has been investigated by Harrington (1972, 1975). His numerical integration results indi-

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cated that the stability of a three-body system depend on the ratio of periastron distance of the long-period orbit to the semimajor axis of the short-period orbit and on the relative sense of orbital motion. Szebehely and Zare (1977) have recently derived a sufficient analytical stability criterion for coplanar, corotational triple systems.

On the observational side, Lucy and Ricco (1979) have examined the distribution of mass ratios of close double-lined spectroscopic systems with total masses of 10 $\mathfrak{M}_\odot$ or less and found a narrow peak at mass ratios very near unity. They argue that this peak is not the result of selection or evolutionary effects. Therefore, the peak implies a formation mechanism for close binaries of small and intermediate mass which in its ideal form would create components of equal mass. Since the ring fragmentation calculations of Norman and Wilson (1978) and Cook and Harlow (1978) can result in binary components of equal mass, Lucy and Ricco (1979) conclude that close binaries having equal masses are probably the result of fragmentation rather than fission.

Abt and Levy (1976, 1978) have examined the frequency of binary and multiple stars in main-sequence B2–B5 and F3–G2 type stars. They transformed the van Rhijn luminosity function, which gives the number of stars of a given luminosity in a given volume of space, into a mass distribution and compared this to their observed distribution of secondary masses. In both spectral-type regions, two groups could be distinguished on the basis of period. Masses of the secondaries of binaries with periods less than about 100 years do not follow the van Rhijn frequency distribution for single stars. However, for periods greater than 100 years, the frequency of secondary masses fits the van Rhijn function reasonably well.

They concluded that the shorter period systems resulted from a protostar associated with a single cloud, while binaries with periods greater than about 100 years have components which contracted in separate but gravitationally bound clouds. These conclusions suggest that the shorter period systems should be compared with the hierarchical fragmentation scenario of multiple stars investigated by Bodenheimer (1978). As a result of these recent numerical, analytical, and observational results, it presently appears to be an excellent time to reexamine the properties of close multiple systems.

II. OBSERVATIONAL DATA

The number of close multiple systems with known short- and long-period elements is presently about 50. The few systems for which both orbits are visual ones (visual multiple systems) are, with one or two exceptions, not suited for comparison with the numerical models, since their long periods are usually greater than 100 years. Thus, nearly all of the known multiple systems considered here have a visual and/or spectroscopic long-period orbit and a spectroscopic short-period one. Not all these systems are equally useful for this investigation since in some only one stellar component is seen having two superposed velocity variations. In most cases, to obtain the fullest amount of information about the multiple system, all of the components must be seen spectroscopically, and visual orbital elements must be known.

To supplement the previously existing material, a spectroscopic search was made to discover new close multiple systems and to detect unseen components in known multiple systems (Fekel 1979). Multiplicity was discovered or confirmed in six systems: HR 266, 64 Ori, HR 3337, HR 3551, HD 140122, and GL 815 and was reinvestigated in eight others: μ Ori, DL Vir, HR 6497, HD 165590, Ψ Sgr, β Cap, HD 202908, and HD 203345. Extensive discussions of these individual systems have been or will be published elsewhere. However, the orbital elements determined for these systems (Fekel 1979) will be used here.

Table 1 lists the long- and short-period orbital elements of 43 presently known multiple systems having at least one set of orbital elements determined spectroscopically. These systems will be called spectroscopic multiples. The first column gives the system and the Aitken double star catalog number. Capital letters indicate visual components, and lowercase letters indicate spectroscopic ones. Unseen spectroscopic components are shown in parentheses. Other columns give P, the period; A, the semimajor axis if the orbit is a visual one or the semimajor axis times $\sin i$ if the orbit is a spectroscopic one; e, the eccentricity; i, the inclination; and T, the time of periastron passage. The L and S subscripts indicate long- and short-period elements, respectively. The final column lists the reference for the data.

Table 2 lists for these same systems the spectral types of the components and the composite Johnson and Strömgren magnitudes and colors. Some of the photometry is previously unpublished and was kindly obtained by Drs. Tom Barnes and Tom Moffett in order to fill in some of the missing data.

These two tables summarize the observational knowledge of close multiple systems as of early 1979. Some of this data will be repeated in other tables for ease of identification.

III. PERIOD RATIOS

The period is the most easily determined parameter of a binary or multiple system. Thus, a quantity often examined in multiple systems is the ratio of the long to short periods. Batten (1973) used data from the spectroscopic and visual orbital element catalogs and found that selection effects heavily biased the observed period ratios. For example, all but one of the visually discovered multiple systems have a period ratio less than 100:1. This is not surprising since few visual systems

TABLE 1

ORBITAL ELEMENTS OF SPECTROSCOPIC MULTIPLE SYSTEMS

System/ ADS No.	P_L (years) P_S (days)	A_L (″ or km) A_S (km)	e_L e_S	i_L (°) i_S (°)	T_L	Reference
HR 142=13 Cet						
490 A,B	6.92	0.23	0.77	45.6	1973.9	1
Aa,(Ab)	2.08	...	0.01	...	...	2
HR 266						
784 A,B	83.4	0.25	0.225	53.5	1952.9	3
Ba+Bb,(Bc)	>1000	...	...	...	...	4
Ba,Bb	4.24	10.9×10^{6}	0.46	...	...	4
HR 604=γ^2 And						
1630 B,C	61.1	0.296	0.93	111.1	1952.1	5
Ba,Bb	2.67	8.9×10^{6}	0.29	...	...	2
HD 14817						
1833 A,B	209.3	0.425	0.30	52.0	1954.58	5
Aa,Ab	1.58	...	0.00	85	...	6
HR 936=β Per						
Aa+Ab,Ac	1.862	3.97×10^{8}	0.23	74	1952.05	2
Aa,Ab	2.867	9.6×10^{6}	0.02	82.4	...	7
HR 1038=ξ Tau						
Aa,Ab+Ac	0.397	14.6×10^{7}	0.15	...	...	8
Ab,Ac	7.15	17.7×10^{6}	0.00	...	...	8
HR 1239=λ Tau						
Aa+Ab,(Ac)	0.09	...	0.00	...	...	2
Aa,Ab	3.95	1.4×10^{7}	0.12	81.3	...	2
HR 1324=b Per						
Aa+(Ab),Ac	1.92	...	0.24	...	...	2
Aa,(Ab)	1.53	...	0.02	...	...	2
HD 27691						
3169 A,B	255.5	1.18	0.29	138.0	1891.6	5
Aa,(Ab)	4.0	...	0.06	...	...	2
HR 1788=η Ori						
Aa+Ab,(Ac)	9.22	...	0.43	...	1971.2	9
Aa,Ab	7.99	3.5×10^{7}	0.00	...	...	2
HR 1868=VV Ori						
Aa+Ab,(Ac)	0.33	...	0.29	...	...	2
Aa,Ab	1.49	8.5×10^{6}	0.00	86.0	...	2
HR 2124=μ Ori						
4617 A,B	18.25	0.28	0.76	95.1	1929.25	5
Aa,(Ab)	4.45	...	0.00	...	...	4
Ba,Bb	4.78	1.1×10^{7}	0.00	...	...	4
HR 2130=64 Ori						
A,Ba+Bb	~12.8	0.05	...	...	...	4
Ba,Bb	14.57	1.4×10^{7}	0.38	...	...	4
HR 2134=1 Gem						
A,B	13.17	0.19	0.32	57.3	1955.6	5
Aa,(Ab)	9.60	...	0.00	...	...	2
HR 2890/1=α Gem						
6175 A,B	420.07	6.30	0.33	115.9	1965.3	5
Aa,(Ab)	9.21	...	0.50	...	...	2
Ba,(Bb)	2.93	...	0.00	...	...	2
HR 3337						
6828 A,B	53.0	0.32	0.45	84.1	1948.4	5
Aa,Ab	2.50	7.2×10^{6}	0.03	...	...	4
Ba,Bb	5.97	8.9×10^{6}	0.00	...	...	4
HR 3482=ε Hya						
6993 AB,C	890.0	4.54	0.29	42.0	1933.0	5
A,B	5489.7	0.23	0.68	50.4	1900.7	5
	5492.0	...	0.61	...	...	2
Ca,(Cb)	9.90	...	0.62	...	...	2
HR 3551						
A,B	7.24	0.10	0.20	33.2	1964.36	11
Aa,Ab	9.07	1.0×10^{7}	0.50	...	...	4

TABLE 1—*Continued*

System/ ADS No.	P_L (years) P_S (days)	A_L ('' or km) A_S (km)	e_L e_S	i_L (°) i_S (°)	T_L	Reference
HR 4167=p Vel						
A,B	16.3	0.34	0.73	129.4	1953.0	5
Aa,Ab	10.21	1.2×10^7	0.51	...	...	2
HR 4374/5=ξ U Ma						
8119 A,B	59.8	2.53	0.41	122.6	1935.2	5
Aa,(Ab)	699.18	...	0.53	86.3	...	2
Ba,(Bb)............	3.98	...	0.00	...	...	2
HD 100018						
8189 A,B	86.4	0.41	0.37	56.7	1967.1	5
Aa,Ab	7.40	1.5×10^7	0.38	...	...	2
HD 120901/2=DL Vir						
A,Ba+Bb	6.24	8.0×10^8	0.46	...	...	4
Ba,(Bb)............	1.32	...	0.00	...	...	4
HR 5472						
A,(B)	9.09	...	0.22	...	...	2
Aa,(Ab)	101.56	...	0.04	...	...	2
HR 5618=44 Boo						
9494 A,B	225.0	3.77	0.43	83.9	2021.0	3
Ba,Bb	0.27	1.3×10^6	0.00	68.1	...	2
HD 140122						
9747 A,B	55.24	0.14	0.70	0.0	1934.4	5
Aa,(Ab)	10.86	...	0.18	...	...	4
HR 6063/4=σ^2 CrB						
9979 A,B	1000	6.6	0.78	33.3	1828.0	5
Aa,Ab	1.14	1.9×10^6	0.02	...	...	2
HR 6497						
A,Ba+Bb	3.29	6.8×10^8	0.40	...	...	4
Ba,Bb	3.76	1.1×10^7	0.00	...	...	4
HD 165590						
11060 A,B...........	20.0	0.27	0.97	76.0	1978.2	5
	20.25	1.5×10^9	0.96	82.9	1978.43	10
Aa,(Ab)	0.88	3.2×10^6	0.05	...	...	10
HR 6918=59 Ser						
Aa,Ab+Ac	1.06	2.2×10^8	0.47	...	...	2
Ab,Ac	1.85	4.8×10^6	0.00	...	...	2
HR 7292=ψ Sgr						
12214 A,B...........	18.75	0.15	0.47	83.3	1972.5	11
	20.0	2.1×10^9	0.51	...	1975.1	4
Ba,Bb	10.78	2.1×10^7	0.46	...	...	4
HD 188753						
13125 A,B...........	25.7	0.28	0.52	39.2	1937.1	5
Aa,(Ab)	155.0	...	0.00	...	...	2
HR 7776=β Cap						
A,B................	3.76	0.05	0.43	84	...	12
	3.76	6.2×10^8	0.43	...	...	12
Ba,(Bb)............	8.68	...	0.34	...	...	12
HD 197433=VW Cep						
A,B................	30.4	0.51	0.60	29.2	1966.5	13
Aa,Ab	0.28	1.2×10^6	0.00	60.0	...	2
HR 8034=1 Equ						
14499 A,B...........	101.4	0.66	0.70	92.8	1920.2	5
Aa,(Ab)	2.03	...	0.17	...	...	2
HR 8094=V 389 Cyg						
Aa+Ab,Ac	0.42	...	0.00	...	...	14
Aa,Ab	3.31	...	0.00	...	...	14
GL 815						
A,B................	28.2	0.73	0.55	57.0	1948.2	15
Aa,Ab	3.28	4.4×10^6	0.04	...	...	16

TABLE 1—*Continued*

System/ ADS No.	P_L (years) P_S (days)	A_L ('' or km) A_S (km)	e_L e_S	i_L (°) i_S (°)	T_L	Reference
HD 202908						
14839 A,B..........	76.1	0.52	0.88	99.8	1985.5	5
Aa,Ab.............	3.97	7.4×10^6	0.01	...	...	4
HR 8153						
Aa+(Ab),(Ac)....	0.62	...	0.23	...	...	2
Aa,(Ab)...........	5.41	...	0.11	...	...	2
HD 203345						
14893 A,B..........	6.15	0.10	0.93	110.0	1973.8	17
Aa,Ab.............	2.24	6.0×10^6	0.06	...	...	4
HR 8300=77 Cyg						
A,B................	27.2	0.16	0.33	155.0	1975.9	5
Aa,Ab.............	1.73	5.2×10^6	0.00	...	...	2
HR 8315=κ Peg						
15281 A,B..........	11.6	0.26	0.29	108.4	1979.1	18
Ba,(Bb)............	5.97	...	0.03	...	...	2
HR 8566=37 Peg						
15988 A,B..........	143.0	0.75	0.56	87.6	1911.5	5
Aa,(Ab)...........	372.4	...	0.48	...	...	2
HR 8819=π Cep						
16538 A,B..........	150.0	0.84	0.60	37.6	1934.3	5
Aa,(Ab)...........	556.2	...	0.28	...	...	2

REFERENCES.—(1) Gatewood and Behall 1975; (2) Batten 1967; (3) Heintz 1978*a*; (4) Fekel 1979; (5) Finsen and Worley 1970; (6) Karle 1977, private communication; (7) Tomkin and Lambert 1978; (8) Bolton 1975, private communication; (9) Zizka 1978, private communication; (10) Batten *et al.* 1979; (11) Finsen 1976; (12) Evans and Fekel 1979; (13) Hershey 1975; (14) Gieseking and Seggewiss 1978; (15) Lippincott 1975; (16)Fekel, Bopp, and Lacy 1978; (17) Finsen 1978, private communication; (18) Couteau and Morel 1972.

TABLE 2

PHOTOMETRY OF SPECTROSCOPIC MULTIPLE SYSTEMS

HR/ Name	HD Sp. Ty.	V $b-y$	$U-B$ m_1	$B-V$ c_1	$V-R$ β	References
142...........	3196	5.20	0.08	0.56	0.48	1,2
13 Cet	F7 V+G4 V	0.361	0.179	0.364	2.616	3,4
266...........	5408	5.55	−0.32	−0.07	...	1
...	B7 V, B9 V, A1 V	−0.019	0.118	0.687	2.783	7, 8
604...........	12534	4.84	−0.12	0.03	...	1
γ^2 And BC	B9 V, A0 V	...	...	...	...	...
...	14817	7.01	−0.15	0.25	0.23	9, 10
...	B8 V	...	...	...	...	...
936...........	19356	2.12	−0.37	−0.05	0.07	1, 5
β Per	B8 V, G IV, Am	−0.002	0.094	0.626	2.737	6
1038..........	21364	3.74	−0.33	−0.09	−0.00	1
ξ Tau	2B9 V, B7 V	−0.035	0.131	0.656	2.787	3, 4
1239..........	25204	3.47	−0.62	−0.12	−0.02	1, 5
λ Tau	B3 V, A4 IV	−0.050	0.098	0.371	2.679	6
1324..........	26961	4.65	0.05	0.05	0.15	5
b Per	A2 V	0.029	0.151	1.110	2.854	6
...	27691	6.99	0.56	0.09	...	9
...	F8 IV	0.362	0.199	0.359	2.626	6
1788..........	35411	3.36	−0.92	−0.17	...	1
η Ori	B1 V, B2 V	−0.056	0.066	−0.007	2.608	6
1868..........	36695	5.35	−0.90	−0.19	−0.07	5
VV Ori	B1 V, B7 V	−0.068	0.068	0.022	2.618	3
2124..........	40932	4.14	0.14	0.16	0.20	5
μ Ori	A7m	...	...	...	...	...

TABLE 2—*Continued*

HR/ Name	HD Sp. Ty.	V $b-y$	$U-B$ m_1	$B-V$ c_1	$V-R$ β	References
2130...........	41040	5.14	−0.43	−0.11	...	1
64 Ori	B5 V, B7 V, B8 V	−0.042	0.109	0.603	2.722	7
2134...........	41116	4.16	0.52	0.82	0.68	1, 5
1 Gem	G8 III, K0 III	...	...	...	...	...
2890/1	60178/9	1.59	0.03	0.01	...	1
α Gem	Am+A1 V	...	...	...	...	...
3337...........	71663	6.48	0.05	0.39	0.34	2, 9
...	Am+F1 V	0.202	0.192	0.678	2.742	3
3482...........	74874	3.38	0.36	0.68	0.61	1, 5
ε Hya	G0 III, F5	...	...	...	...	...
3551...........	76360	5.33	...	0.26	...	1
...	A9 V	0.160	0.181	0.833	2.763	3
4167...........	92139/40	3.84	0.07	0.30	...	1
p Vel	F3 IV, F0 V, A6 V	...	...	...	...	...
4374/5	98230/1	3.79	0.05	0.59	0.52	5
ξ UMa	G0 V+G0 V	...	...	...	...	...
...	100018	6.95	−0.03	0.47	...	1
...	2F2 V, F5 V	...	...	...	...	...
...	120901/2	6.98	0.33	0.61	0.58	2,11
DL Vir	A5, G8 III	...	...	...	...	...
5472...........	129132	6.14	...	0.40	0.39	2
...	F1 V	...	...	...	...	...
5618...........	133640	4.76	0.11	0.65	...	1
44 Boo	2G1 V, F5 V	...	...	...	...	...
...	140122	7.20	0.16	0.22	0.19	12
...	Am	0.130	0.216	0.966	2.850	6
6063/4	146361/2	5.22	0.06	0.58	...	1
σ^2 CrB	2F8 V	...	...	...	...	...
6497...........	157978/9	6.06	0.28	0.58	0.57	1,2
...	2A0 V+G5 III	0.408	0.114	0.771	2.696	3,4
...	165590	7.07	0.13	0.66	...	9
...	G0 V+G5 V	0.420	0.198	0.309	...	6
6918...........	169985/6	5.21	0.21	0.50	0.49	5
59 Ser	2A2 V, G8 III	0.328	0.142	0.814	2.730	6
7292...........	179950	4.85	0.32	0.56	0.46	1,5
ψ Sgr	A9 III, A7 V, G5 III	0.348	0.228	0.711	2.699	6
...	188753	7.43	0.42	0.79	...	1
...	G9 V	...	...	...	...	...
7776...........	193495	3.08	0.28	0.79	0.55	1,5
β Cap	B8 V+K0 II	0.518	0.226	0.347	2.602	6
...	197433	7.38	+0.43	+0.85	...	1
VW Cep	G5 V	...	...	...	...	...
8034...........	199766	5.23	+0.02	+0.46	...	1
1 Equ	F6 V	0.306	0.169	0.474	...	...
8094...........	201433	5.59	−0.26	−0.10	...	1
V389 Cyg	B9 V	−0.025	0.133	0.704	2.788	6
...	...	10.12	1.10	1.50	1.47	13,14
GL 815	dM3e	...	...	...	...	...
...	202908	7.02	...	0.58	0.51	2
...	F9 V, G0 V, G1 V	0.378	0.196	0.329	...	6
8153...........	203025	6.42	−0.50	0.20	...	1
...	B2 III	0.221	−0.031	0.272	2.553	6
...	203345	6.75	0.00	0.53	...	9
...	F7 V+F8 V	0.336	0.164	0.359	...	6
8300...........	206644	5.69	−0.01	0.01	...	1
77 Cyg	2A0 V	0.030	0.170	0.982	2.911	6
8315...........	206901	4.13	0.03	0.43	0.43	1,5
κ Peg	F4 V+F6 V	0.273	0.150	0.558	...	6
...	213235	5.48	0.09	0.38	...	1
37 Peg	F5 IV	0.233	0.223	0.637	2.711	3,4
8819...........	218658	4.41	0.46	0.80	0.65	1,5
π Cep	G2 III	...	...	...	...	...

REFERENCES.—(1) Nicolet 1978; (2) Barnes and Moffett 1978; (3) Grønbech and Olsen 1976; (4) Grønbech and Olsen 1977; (5) Iriarte *et al.* 1965; (6) Lindemann and Hauck 1973; (7) Crawford *et al.* 1973; (8) Crawford 1963; (9) Blanco *et al.* 1970; (10) Barnes and Moffett 1979; (11) Schoffel 1977; (12) Mendoza, Gomez, and Gonzalez 1978; (13) Krzeminski 1969; (14) Veeder 1974.

have been observed for over 150 years and, therefore, there is an insufficient time base to determine the long period of visual multiple systems with large-period ratios.

On the basis of his limited sample of about 30 systems, Batten (1973) suggested that the period ratio of Algol, 250 : 1, is probably typical of those triple systems in which both periods have been determined. For 21 multiple systems, most of which were spectroscopically discovered, Evans (1977) found an average period ratio of nearly 1000 : 1.

Table 3 lists all spectroscopic multiple systems for which both the short and long periods are known. Also listed in the table are the logarithm of the period ratios, and, if it could be reasonably estimated, the mass ratio of the short-period secondary to the short-period primary, the ratio of the mass of the single star to that of the spectroscopic binary, and the total mass of the system.

Figure 1 is a histogram of log P_L/P_S for 50 multiple systems, both visually and spectroscopically discovered, of which seven systems are quadruple and contribute two period ratios. Relative to that given by Batten, the peak of the combined distribution has been shifted toward larger period ratios. This is because an extensive number of spectroscopically discovered systems, which often have large-period ratios, have been added, but no new visually discovered systems, which have small-period ratios, have been added.

Although the size of this sample is much larger than Batten's, it still suffers seriously from selection effects. Multiple stars with periods less than 100 years form a more homogeneous subset without the incompleteness problems of the visually discovered binaries. For this group of 35 systems the average period ratio is 3222 : 1.

Bodenheimer's (1978) models have suggested that the most important theoretical parameter which determines this period ratio is the fraction of orbital angular momentum which remains in the spin of the protostar after fragmentation of the ring. The orbital angular momentum was computed for 27 systems in Table 3 which have a total mass given. In nearly all of these, the angular momentum of the long-period orbit dominates. The specific angular momenta of these long-period orbits ranged from 4.6×10^{18} to 2.2×10^{20} cm^2 s^{-1}, while those for the short-period orbits were about a factor of 10 smaller.

Unfortunately, the spin angular momentum is much more difficult to determine. Components in the short-period orbit usually cannot be used since tidal friction tends to synchronize their rotational and orbital periods, causing a redistribution of spin and orbital angular momenta. Stars later than F5 cannot be used since angular momentum lost by a magnetically coupled stellar wind substantially reduces their rotational velocities. Spectral lines in early-type stars are weak and, if broadened by substantial rotation, become more difficult to detect and are also weakened by the continua of the other stars in the system.

Instead of attempting to compute the spin angular momenta of the systems, the stars of Table 3 were examined to see whether there was a correlation between multiple systems containing a broad-lined single star and a small-period ratio. Figure 2 is a plot of log P_L/P_S versus total mass. Of the systems hypothesized to have formed from a single cloud ($P \lesssim 100$ years), HR 266, 64 Ori, λ Tau, HR 3551 and κ Peg contain a rapidly rotating single star. Except for 64 Ori, the period ratios of these stars are the smallest found at their respective total masses. The few observations available appear consistent with Bodenheimer's (1978) result that a larger ratio of spin to orbital angular momentum produces a smaller period ratio.

The numerical calculations of Norman and Wilson (1978) suggest that the large masses have larger spin angular momenta. Figure 2 shows that for systems with periods less than 100 years, the ratio of the periods tends to decrease with increasing total mass. Following Bodenheimer, this can be interpreted as a tendency for more massive systems to have larger ratios of spin to orbital angular momentum and is consistent with the tentative result of Norman and Wilson (1978).

IV. MASS RATIOS

The mass ratios of the components of multiple systems are determined by the formation mechanism. Lucy (1977) has shown that fission probably produces quite unequal masses, with a mass ratio, $\mathfrak{M}_{\rm sec}/\mathfrak{M}_{\rm pri}$, usually toward the lower end of the range 0.1–0.5. According to the models of Norman and Wilson (1978) and Cook and Harlow (1978), fragmentation often results in components of similar mass or a ratio close to 1.0. These fragmentation calculations begin with some particular imposed perturbations which predetermine the mode of fragmentation. Thus, although these models show that a binary with components of equal mass is a probable outcome, other outcomes are still possible. As a result of the above studies, a mass ratio of 0.6 has been chosen to distinguish between the fission and fragmentation processes. It should be emphasized that this mass ratio is used only as a convenient, possible dividing line between the two processes. It is hoped that future numerical investigations will further establish this mass ratio division.

The mass ratios of short-period pairs were examined using only those systems in Table 3 which have long periods of about 100 years or less. Systems undergoing mass transfer were eliminated from consideration since their initial mass ratios are not known. Similarly, systems containing a W UMa binary were not used since post-formation evolution may have changed their mass ratios (Lucy 1976). Several systems with extremely small

TABLE 3

PROPERTIES OF SPECTROSCOPIC MULTIPLE SYSTEMS

System	P_S (days)	P_L (years)	$\log P_L/P_S$	$\mathcal{M}_{\rm SB2}/\mathcal{M}_{\rm SB1}$	$\mathcal{M}_{\rm sin}/\mathcal{M}_{\rm SB}$	$\mathcal{M}_{\rm TOT}$ $\mathcal{M}_\odot$
HR 142	2.1	6.9	3.08	<0.6	0.50	2.7
13 Cet						
HR 266	4.2	6.0	2.72	0.72	0.20	12.3
	6.0	83.4	1.14	...	0.53	...
HR 604	2.7	61.1	3.92	0.80	0.45	9.1
γ^2 And BC						
HD 14817	1.6	209.3	4.68	1.00	0.50	12.0
HR 936	2.9	1.9	2.38	0.22	0.38	6.2
β Per						
HR 1038	7.1	0.4	1.31	0.96	0.91	11.2
ξ Tau						
HR 1239	4.0	0.1	0.92	0.27	0.06	8.2
λ Tau						
HR 1324	1.5	1.9	2.66	<0.60	...	...
b Per						
HD 27691	4.0	255.5	4.37	...	...	...
HR 1788	8.0	9.2	2.62	0.82	0.50	41.1
η Ori						
HR 1868	1.5	0.3	1.90	0.45	0.14	12.6
VV Ori						
HR 2124	4.4	18.2	3.18	<0.60	...	5.6
μ Ori	4.8	...	3.14	0.99	...	...
HR 2130	14.6	12.8	2.51	0.92	0.77	13.5
64 Ori						
HR 2134	9.6	13.2	2.69	...	...	...
1 Gem						
HR 2890/1	9.2	420.1	4.22	...	...	...
α Gem	2.9	...	4.72	...	...	...
HR 3337	6.0	53.0	3.51	0.63	...	6.0
	2.9	...	3.89	0.69	...	...
HR 3482	9.9	890	4.52	...	...	...
ε Hyd						
HR 3551	9.1	7.2	2.47	0.90	0.56	5.3
HR 4167	10.2	16.3	2.77	0.80	0.62	6.3
p Vel						
HR 4374/5	669.2	59.8	1.51	...	...	...
ξ UMa	4.0	...	3.74	...	...	...
HD 100018	7.4	86.4	3.63	0.90	0.42	4.4
HD 120901/2	1.3	6.2	3.24	...	0.56	6.1
DL Vir						
HR 5472	101.6	9.1	1.51	...	...	...
HR 5618	0.3	225.0	5.54	0.50	0.71	2.7
44 i Boo						
HD 140122	10.9	55.2	3.27	...	...	...
HR 6063/4	1.1	1000.0	5.51	0.88	0.43	3.3
σ^2 Cr B						
HR 6497	3.8	3.3	2.50	0.98	0.67	9.3
HD 165590	0.9	20.2	3.92	<0.60	0.56	2.5
HR 6918	1.8	1.1	2.32	0.90	0.67	9.3
59 Ser						
HR 7292	10.8	20.0	2.83	0.79	0.72	7.0
ψ Sgr						
HD 188753	155.0	25.7	1.78	...	...	...
HR 7776	8.7	3.8	2.20	...	0.91	7.9
β Cap						
HD 197433	0.3	30.4	4.60	0.33	0.40	2.1
VW Cep						
HR 8034	2.0	101.4	4.26	<0.60	...	...
1 Equ						
HR 8094	3.3	0.4	1.67	...	...	...
V 389 Cyg						
GL 815	3.3	28.2	3.50	0.69	0.36	0.7

TABLE 3—*Continued*

System	P_S (days)	P_L (years)	$\log P_L/P_S$	$\mathfrak{M}_{SB2}/\mathfrak{M}_{SB1}$	$\mathfrak{M}_{sin}/\mathfrak{M}_{SB}$	$\mathfrak{M}_{TOT}$ $\mathfrak{M}_\odot$
HD 202908	4.0	76.1	3.84	1.00	0.45	3.3
HR 8153	5.4	0.6	1.62	...	...	...
HD 203345	2.2	6.2	3.00	0.64	0.67	3.0
HR 8300 .... 77 Cyg	1.7	27.2	3.76	1.00	0.50	9.0
HR 8315 .... κ Peg	6.0	11.6	2.85	<0.60	0.67	2.9
HR 8566 .... 37 Peg	372.4	143.0	2.15	<0.60	...	...
HR 8819 .... π Cep	556.2	150.0	1.98	...	...	...

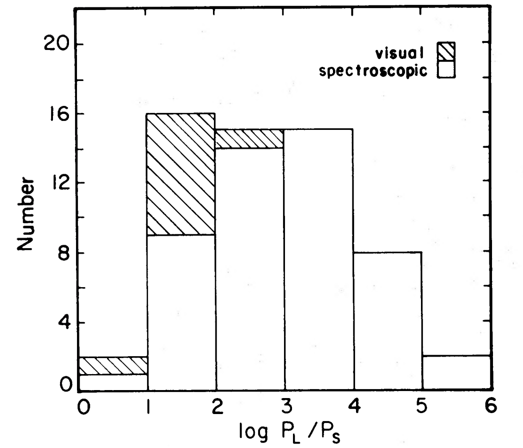

FIG. 1.—Histogram showing the observed frequency distribution of the ratio of the long period to the short period in multiple systems discovered visually or spectroscopically.

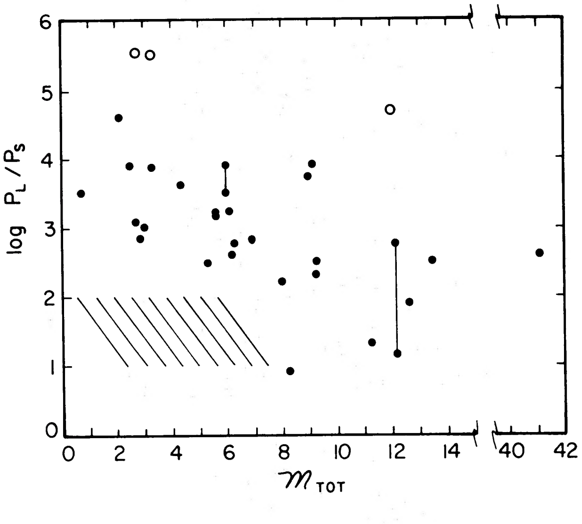

FIG. 2.—The ratios of the long to short period of multiple systems vs. total mass. Dots with a connecting line are quadruple systems. Open circles $= P_L > 100$ years. Dots $= P_L \leq 100$ years. Hatched area is the region occupied by visual multiple systems.

velocity variations, whose short-period pair, if double-lined, would not be resolved, were also not considered. Of the remaining 31 short-period pairs, 18 are double-lined, and all of these have mass ratios greater than 0.6 and, therefore, probably were formed by fragmentation.

Using the McDonald Observatory 2.7 m telescope and coudé Reticon system, double-lined main-sequence binaries have been detected with mass ratios as small as 0.6 (Fekel 1979). This is a substantially smaller mass ratio than that detectable using photographic plates in the blue region of the spectrum. For 7 of the remaining 13 short-period binaries, Reticon observations failed to detect the secondary component (Fekel 1979). Therefore, their mass ratios are assumed to be less than 0.6, and they may have been formed by fission.

For the other 6 short-period pairs the formation mechanism is unknown since, although they are single-lined according to photographic observations, Reticon observations in the red, which can detect components with mass ratios as small as 0.6, have not been made.

Of the 25 short-period pairs whose mass ratios are known, 18 have mass ratios greater than 0.6. Thus, for this limited sample the fragmentation process appears to dominate by a ratio greater than 2 : 1.

It is also of interest to note that two substantially different mass ratios can occur in the same system. In the quadruple system μ Ori, the visual component *A* consists of a short-period binary with a mass ratio less than 0.6, while *B* consists of a pair of stars with a mass ratio of 1.0. From spectroscopic observations there is no evidence that the short-period binary of *A* might be a mass-transfer system and, thus, might originally have had a substantially different mass ratio. Therefore, the different mass ratios suggest that different formation mechanisms may occur in the same system.

For triple systems the mass ratio of the single star to the spectroscopic binary is given in Table 3. Ten of the 25 systems, or 40%, have mass ratios greater than 0.6 and were presumably formed by fragmentation. But, only 2 of the 10 systems have mass ratios greater than 0.9. Eighteen of the 25 triple systems, or 72%, have

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ratios between 0.4 and 0.67. This clustering around 0.5 may be a selection effect since three stars with equal masses and, therefore, equal luminosities would have a mass ratio of 0.5.

Probably of greater interest is the fact that these ratios are less than 1.0, indicating that the more massive protostar has subdivided to produce the short-period binary in each case. This result also appears to be affected by selection effects. For example, suppose that the mass of the single star, component *A*, were equal to the mass of the short-period binary, component *B*. Then the luminosity of the single star would be roughly 5 times the sum of the luminosities of the short-period binary components, if these two components had equal mass. This results in a magnitude difference, $\Delta m(B-A)$, of 1.7 mag which would make visual detection of *B* extremely unlikely if it had a separation as small as $0''.3$ (Heintz 1978*b*). Likewise, even if such a visual pair were detected, it would be extremely difficult to detect the short-period velocity variations of the components of *B*. The luminosity ratio of the single star plus one short-period binary component to that of the other short-period component would be roughly 10:1 or a magnitude difference of 2.5 mag. Making the mass of the single star larger than the sum of the masses of the short-period pair simply increases these luminosity differences. Only if the mass ratio of the short-period pair were substantially different from unity, would there be an improved chance of the multiplicity of the system being discovered. This general problem is discussed further by Batten and Scarfe (1977).

In Table 3 there are four quadruple systems with long periods less than 100 years. Three of these consist of a visual binary, each component of which is also a binary. The visual binary components of all three of these systems, μ Ori, HR 3337, and ξ UMa have mass ratios very close to 1.0.

V. ORBITAL COPLANARITY

The relative inclination of orbital planes in a multiple system poses a problem of particular interest because of its relation to the formation of these systems. The theory used by Bodenheimer (1978) predicts that the orbits should be coplanar at all fragmentation stages. However, physical effects, such as encounters or magnetic fields, could be expected to disrupt some levels of the hierarchy and change orbital properties. The only observational data previously analyzed (Worley 1967) were for nine visual systems, nearly all of which had periods greater than 100 years. These systems showed little tendency toward coplanarity (van Albada 1968). According to the conclusions of Abt and Levy (1976, 1978), this result is not surprising since the components of the long-period orbit came from two gravitationally bound clouds whose angular momentum vectors and other properties would not necessarily be the same.

The vast majority of the multiple systems listed in Table 3 have outer periods less than 100 years and, according to the conclusion of Abt and Levy, were formed from the same cloud. The most obvious example of a multiple system forming from a single cloud is the solar system, and nearly all its major bodies have orbits close to the ecliptic and rotating in the same direction. Thus, coplanarity and corotation of shorter period multiple systems might be expected if the analogy with our solar system holds.

Figure 3 shows the inclination, *i*, and the position angle of the ascending node in the plane of the sky, Ω, for a visual binary. The relative inclination, ϕ, between two orbital planes, defined by angles (i_S, i_L) and (Ω_S, Ω_L), where subscripts *S* and *L* are the short- and long-period orbits, is given by

$$\cos\phi = \cos i_S \cos i_L + \sin i_S \sin i_L \cos(\Omega_L - \Omega_S).$$

If the ascending node of either or both orbits is unknown, then $\Omega_L - \Omega_S$ is uncertain by 180°, and the + sign in the second term is replaced by a ± sign. Then two values of ϕ are possible.

For spectroscopic orbits, Ω is totally unknown. In this case

$$+1 \geq \cos(\Omega_L - \Omega_S) \geq -1,$$

so ϕ lies between

$$i_S - i_L \leq \phi \leq i_S + i_L,$$

and coplanarity cannot be completely determined. Thus, equal inclinations are a necessary but not a sufficient condition for coplanarity. The inequalities also are ap-

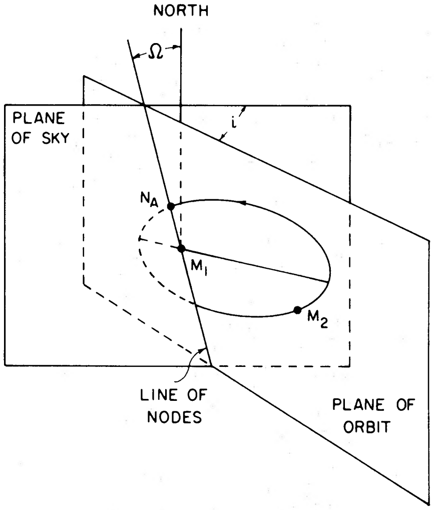

FIG. 3.—Orbital plane and plane of the sky illustrating the angles *i* and Ω for a visual orbit. N_A is the ascending node of the secondary star M_2, and M_1 is the primary star.

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propriate for spectroscopic-visual triple systems, where one component of a visual pair is a spectroscopic binary, since Ω is unknown for the spectroscopic system.

If the angle of the relative inclination is acute, the system is corotating; if it is obtuse, the system is counter-rotating. For all but one system in Table 4 the short-period orbit is spectroscopic and Ω is undetermined, so that co- or counter-rotation cannot be determined for these multiple systems. Since the limiting values of ϕ are symmetric and simply indicate co- or counter-rotation, only one limit of the relative inclination needs to be examined.

Although coplanarity cannot be completely determined for the systems in this study, a lower limit for the number of systems which are not coplanar may be found from examining the minimum or maximum limits of ϕ. Table 4 lists short and long periods and inclinations, and the minimum values of the relative inclination for 23 systems. For most of these close multiple systems, the long-period orbital inclination is obtained for visual binary elements. For short-period eclipsing systems, denoted by e, the inclination is known from the photometric solution. For noneclipsing double-lined binaries, a spectroscopic inclination was determined by estimating the mass of the primary from its spectral type and then solving the minimum mass, $\mathfrak{M}\sin^3 i$, for the inclination. For a few systems no visual orbit is known, but inclinations were obtained by estimating the mass of the short-period primary and solving the minimum masses of the short- and long-period orbits.

The estimated uncertainty of the spectroscopic inclinations depends on the spectral type of the short-period primary. Allen's (1973) values for the masses of stars of various spectral types were used. The assumed mass uncertainty for an M0 V star is $\pm 0.1\ \mathfrak{M}_\odot$, for G0 V, $\pm 0.2\ \mathfrak{M}_\odot$, for F0 V, $\pm 0.3\ \mathfrak{M}_\odot$, and for A0 V, $\pm 0.5\ \mathfrak{M}_\odot$. The uncertainty of the inclination also is dependent upon the inclination itself, because as i is increased the same uncertainty in the assumed mass produces a larger uncertainty in the inclination.

TABLE 4

RELATIVE INCLINATIONS OF CLOSE MULTIPLE SYSTEMS

System	P_S (days)	P_L (years)	i_S (degrees)	i_L (degrees)	$\phi_{\min}$ (degrees)
HR 266	4.2	83.4	48±4	54	6
HD 9770	1665.1	115.2	25	32	7
HR 604 .... γ^2 And BC	2.7	61.1	118±7	111	7
HD 14817	1.6	209.3	85*e*±3	52	33
HR 936 .... β Per	2.8	1.9	82*e*	74	8
HR 1038 .... ξ Tau	7.2	0.4	57±5	48±4	9
HR 2124 .... μ Ori	4.8	18.2	100±7	95	5
HR 3337	2.5	53.0	87*e*±3	84	3
	6.0	53.0	45±3	84	39
HR 3551	9.1	7.2	33±3	33	0
HR 4167 .... p Vel	10.2	16.3	149±4	129	20
HD 100018	7.4	86.4	69±3	57	12
HD 120901/2 ... DL Vir	1.8	6.2	84*e*±2	61±7	23
HR 5618 .... 44 Boo	0.3	225.0	68*e*	84	16
HR 6063/4 ...... σ^2 Cr B	1.1	1000.0	30±2	33	3
HR 6497	3.8	3.3	65±4	79±7	14
HD 165590	0.9	20.2	77*e*±7	83	6
HR 6918 .... 59 Ser	1.8	1.1	39±3	43±3	4
HR 7292 .... ψ Sgr	10.8	20.0	71±5	85	14
HD 197433 ...... VW Cep	0.3	30.4	60*e*	29	31
GL 815...........	3.3	28.2	55±8	57	2
HD 202908	3.9	76.1	129±4	100	29
HD 203345	2.2	6.1	104±4	110	6
HR 8300 .... 77 Cyg	1.7	27.2	140±3	155	15

By convention for visual binaries, if the position angle increases with time, $i < 90°$; if the position angle decreases with time, $i > 90°$. Since the direction of motion of the spectroscopic binary in the plane of the sky is unknown, its inclination may be i or $180° - i$. Although both cases are considered in determining the limiting values of ϕ, only the value in the same quadrant as the long-period inclination is listed in Table 4.

The maximum uncertainty of the short-period inclinations is 8°. A similar uncertainty in the visual inclinations seems reasonable since the close visual pairs are difficult to observe. Thus, the maximum determined relative inclination consistent with coplanarity has been chosen as 15°.

Three systems, HD 14817, 44 Boo, and σ^2 Cr B, have periods appreciably greater than 100 years and are eliminated from consideration, since presumably they were formed in separate clouds. Of the 20 remaining systems one, HR 3337, is quadruple and has two relative inclinations, making a total of 21 minimum relative inclinations.

For $\phi_{\min}$ 5 of the 21 relative inclinations are greater than 15° and, therefore, coplanarity is excluded. Coplanarity can also be excluded for two systems, ξ Tau and HR 6497, with relative inclinations less than 15°. In these systems, both the long- and short-period orbits are spectroscopic. Since the sum of the minimum masses in the short-period orbit is substantially different from the minimum mass of this pair in the long-period orbit, the inclinations are different and the orbits are not coplanar. Thus, considering only masses and inclinations, 7 of the 21 orbital pairs, or 33%, are not coplanar.

HD 9770 is the only system in Table 4 for which visual elements are known for both the long- and short-period orbits. From a comparison of the inclinations alone, coplanarity is a possibility, but the values of Ω for the two orbits differ substantially. Of the two possible values of ϕ, the smallest is $34\overset{\circ}{.}2$ and the system is not coplanar.

If the percentage of orbital pairs which are not coplanar according to their values of Ω is similar to that found for the values of i and these noncoplanar orbits are distributed randomly among the sample, then 44% of the orbital pairs are not coplanar. Although this result indicates that there are a substantial number of systems whose orbits are not exactly coplanar, it does not rule out the possibility, suggested by the values of $\phi_{\min}$ in Table 4, that there is a tendency toward coplanarity in multiple systems formed from a single protostar. Is this suggested tendency toward coplanarity a selection effect due to the predominantly different methods of discovering and observing the short- and long-period systems? Visual binaries with inclinations greater than 60° gradually become more difficult to discover (Heintz 1969) as their orbits become more nearly edge-on. On the other hand, the observed velocity variation of a spectroscopic binary increases with increasing inclination, making it easier to discover.

To determine whether there is a resultant bias in Table 4 either for or against coplanarity, those systems were examined which are in the inclination range where *both* the spectroscopic and visual binaries are least influenced by selection effects. The discovery of visual binaries was assumed to be reasonably complete to a maximum inclination of 70°. The limiting inclination for spectroscopic binaries was somewhat more difficult to determine.

Except for eclipsing systems, the spectroscopic inclinations in Table 4 were estimated from the minimum masses of double-lined systems. In the survey by Fekel (1979), who has discovered or determined orbital elements for one-third of these systems, double-lined components can be measured with velocity separations as small as 30 km s^{-1}. Assuming a semiamplitude of 15 km s^{-1}, a period of 11 days, and an eccentricity of zero, the minimum detectable inclination is 10° for a pair of 3 $\mathfrak{M}_\odot$ stars. If the period is less than 11 days the detectable inclination is smaller. Abt used dispersions of 13 and 17 Å mm^{-1} in most of his spectroscopic binary surveys and was able to resolve double-lined components with velocity separations as small as 40 and 60 km s^{-1}, respectively. As a compromise among the three dispersions, a semiamplitude of 25 km s^{-1} was chosen. This implies that the discovery of spectroscopic binaries is reasonably complete down to an inclination of 25°.

Of the eight systems with periods less than 100 years which have visual and spectroscopic orbital inclinations between 25° and 70°, two, or 25%, are definitely not coplanar. Considering the small size of the sample, this value is in reasonable agreement with the value of 33% found when all inclinations are considered. Thus, the conclusion that there is a tendency toward coplanarity in close multiple systems does not appear to be particularly biased by selection effects.

VI. STABILITY

The question of the stability of multiple stellar systems has not been answered satisfactorily. However, for the special case of triple stars, Harrington's (1968, 1969) analytical perturbation studies have shown that if the separation of the third star from the center of the binary's mass is great compared to the binary separation, the system is stable, in the sense that there are no secular terms in the expression for the semimajor axis.

Harrington (1972, 1975) next attempted to establish sufficient dynamical stability criteria for triple stars using numerical integration of three-body point masses and found two important parameters. Stability depended on the ratio of periastron distance of the long-period orbit to the semimajor axis of the short-period orbit, q_L/a_S. In addition, it did not depend on the relative inclinations of the two orbits, but only on the relative sense, co- or counter-rotation, of orbital motion. However, the orbits did appear to be unstable when the relative inclination between the two orbits was within a few degrees of 90°. From these studies Harrington found a lower limit of 3.5 for the distance ratio of a triple system of equal masses for the corotation case and 2.75 for counter-rotation. For unequal masses Harrington (1975) found that the lower limit of the distance ratio varied roughly as the logarithm of the ratio of total mass of the system to the mass of the close pair.

Szebehely and Zare (1977) have derived a sufficient analytical stability criterion for coplanar, corotational triple systems. They find a limiting value of 3.2 for the ratio of the semimajor axis of the long-period single star to that of the short-period pair for circular orbits ($a_L/a_S=q_L/a_S$ in this case) and stars of equal mass. This value is in good agreement with the ratio of 3.5 found by Harrington (1972) from numerical integration. They also confirm Harrington's finding that the eccentricity of the long-period orbit through the ratio q_L/a_S is important in determining stability of the triple system, but a moderate short-period orbital eccentricity has little effect on stability.

On the other hand, Szebehely and Zare (1977) found that the critical value of q_L/a_S for stability is much larger for retrograde or counter-rotational systems than for direct or corotational systems. This is the opposite of Harrington's (1972) result of a smaller q_L/a_S for counter-rotational stability than for corotational. This difference may be traced to their different definitions of stability. In Harrington's (1972) numerical analysis, a system is considered unstable if there are secular trends in the elements, particularly in the semimajor axes or eccentricities. The analytical method of Szebehely and Zare (1977) uses the two-body approximation and surfaces of zero velocity to determine stability criteria for triple systems. This method demonstrates stability for a system rather than instability. If q_L/a_S of a system is greater than the limiting value for stability, the system is stable and the long-period single star cannot exchange position with one of the stars of the short-period binary. If it is less than the limiting value, exchange of the components may, but does not have to, occur, which implies instability. Thus if a counter-rotational system is stable according to the criterion of Szebehely and Zare, it is also stable according to the criterion of Harrington, but the reverse is not necessarily true.

The analytical method of Szebehely and Zare (1977) has been used to examine the stability of as many of the systems in this study as possible. Using two-body approximations, the angular momentum, c, and the total energy of the system, H, were computed for each triple system for which short and long periods and eccentricities were known and individual masses could be estimated.

The parameter c^2H controls the topology of the surface of zero velocity and also determines the regions of possible motion, since bodies may not cross zero velocity surfaces. Therefore, an observed value of c^2H was compared with the critical value of c^2H obtained from the surfaces of zero velocity of the planar three-body problem tabulated by Szebehely and Zare (1977).

The dimensionless stability parameter $s=(c^2H)/(G^2\overline{m}^5)$, where G is the gravitational constant and $\overline{m}$ is the average mass of the system, is computed for the actual, $s(\mathrm{ac})$, and critical, $s(\mathrm{cr})$, cases. Two values of the stability parameter are possible for the observed systems, depending on the assumed sense of rotation: $s(d,\mathrm{ac})$ for direct or corotating systems and $s(r,\mathrm{ac})$ for retrograde or counter-rotating systems.

For comparison purposes Szebehely and Zare (1977) also introduced a measure of stability parameter $S=[s(\mathrm{ac})-s(\mathrm{cr})]/s(\mathrm{cr})$. If S is positive, then there is no exchange of bodies between the short- and long-period orbits and the system is stable. When S is negative, exchange may, but does not have to, occur, and the system is considered unstable. If both stability measures, $S(d)$ and $S(r)$, are positive, then all relative inclinations between the two orbits are allowed. If $S(r)$ is negative, the system is unstable if the relative inclination is greater than a critical value. The results for 27 systems, listed in order of decreasing total mass, are given in Table 5 with columns similar to that of Table 3 in Szebehely and Zare (1977). The mass of the short period primary is $\mathfrak{M}_1$, the short period secondary is $\mathfrak{M}_2$, and the single star is $\mathfrak{M}_3$. Subscripts S and L refer to parameters of the short and long periods, respectively, for e, the eccentricity, a, the semimajor axis, and q, the periastron distance.

The short-period orbital elements of the systems in Table 5 have been determined spectroscopically and, therefore, usually have periods of a few days. On the other hand, the long periods, determined spectroscopically or visually, are generally a few to several hundred years. Thus, the ratio of the semimajor axes, a_L/a_S, is generally much greater than the values for the six visual multiple systems listed in Table 3 of Szebehely and Zare (1977), which range from 9 to 19. For nearly all the systems in Table 5, q_L/a_S, the critical parameter found by Harrington, also remains large. Therefore, it is not surprising that all values for $S(d)$ are positive.

One way for Harrington's stability parameter q_L/a_S to become small enough to make a system unstable is for the outer orbital eccentricity e_L to be very large. Four systems in Table 5 have outer orbital eccentricities $\gtrsim 0.9$. For each system the corotational and counter-rotational measures of stability are positive, indicating that these systems are stable despite the large outer orbital eccentricity.

This is also shown in Figure 4 where the logarithm of the period ratio is plotted against the long-period eccentricity. The dashed line represents the stability

TABLE 5

Measures of Stability for Triple Systems

System	$\frac{\mathfrak{M}_1}{\mathfrak{M}_\odot}$	$\frac{\mathfrak{M}_2}{\mathfrak{M}_\odot}$	$\frac{\mathfrak{M}_3}{\mathfrak{M}_\odot}$	e_S	e_L	a_L/a_S	q_L/a_S	$S(d)$	$S(r)$
η Ori	15.0	12.4	13.7	0.00	0.43	64.4	36.7	5.8	3.9
64 Ori	4.0	3.5	6.0	0.38	0.35	57.0	37.1	4.8	3.7
VV Ori	7.6	3.4	1.6	0.00	0.32	19.5	13.2	1.6	−0.5
HR 266	6.5	1.6	4.2	0.00	0.22	6.6	5.2	0.3	−0.3
HD 14817[a]	4.0	4.0	4.0	0.00	0.30	1519.6	1063.7	151.6	141.9
ξ Tau	3.2	3.1	5.5	0.00	0.40	9.2	5.5	0.4	−0.2
HR 6497	2.8	2.8	3.7	0.00	0.39	55.4	33.8	4.9	3.4
59 Ser	2.9	2.7	3.7	0.00	0.47	41.7	22.1	3.2	2.0
γ^2 And	3.5	2.8	2.8	0.29	0.93	465.6	32.6	6.8	4.6
77 Cyg	3.0	3.0	3.0	0.05	0.33	367.1	246.0	36.5	31.8
λ Tau	6.1	1.6	0.5	0.00	0.00	4.2	4.2	0.1	−1.0
β Cap	3.3	0.9	3.7	0.36	0.42	36.1	21.0	2.2	1.6
ψ Sgr	2.3	1.7	2.9	0.47	0.51	92.5	45.3	6.8	5.4
p Vel	2.1	1.8	2.4	0.51	0.73	81.8	22.1	3.8	2.6
β Per	3.7	0.8	1.7	0.01	0.23	42.6	32.8	4.3	3.0
DL Vir	2.4	1.1	2.0	0.00	0.46	167.7	90.6	14.0	11.8
HR 3551	1.8	1.6	1.9	0.50	0.20	51.0	40.8	5.2	3.7
HD 100018	1.6	1.5	1.3	0.38	0.37	295.5	186.2	28.6	24.1
σ^2 Cr B[a]	1.2	1.1	1.0	0.00	0.78	5281.0	1161.8	225.6	212.3
HD 202908	1.1	1.1	1.0	0.00	0.88	414.9	49.8	10.5	7.7
HD 203345	1.1	0.7	1.2	0.06	0.91	119.6	10.8	1.6	0.7
κ Peg	1.2	0.5	1.2	0.00	0.30	94.6	66.2	8.6	7.1
13 Cet	1.1	0.7	0.9	0.01	0.77	130.3	30.0	5.6	3.8
HD 165590	1.0	0.6	0.9	0.04	0.96	477.4	19.1	3.6	2.3
44 Boo[a]	1.0	0.5	1.0	0.00	0.36	5416.2	3466.3	474.5	462.9
VW Cep	1.1	0.4	0.6	0.00	0.59	1304.4	528.4	93.8	86.4
GL 815	0.3	0.2	0.2	0.00	0.55	238.3	107.2	18.4	14.2

[a] $P_L > 100$ years.

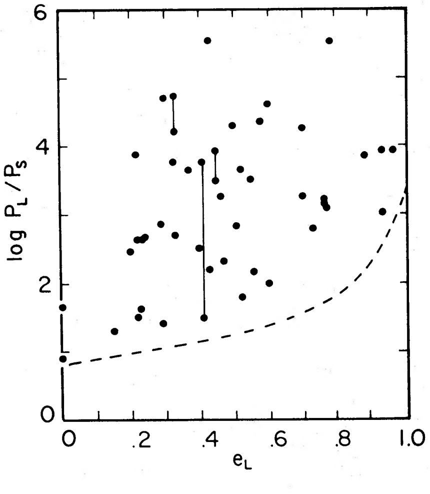

Fig. 4.—The ratio of the long to short period of multiple systems vs. long-period eccentricity. Dots with a connecting line are quadruple systems. The dashed line is the stability boundary for a corotating triple system with equal mass components.

boundary for a corotation equal-mass system, with the area below the line being unstable according to the criteria of Szebehely and Zare. Although few of the observed systems have equal mass components, this is a reasonable approximation in most cases. The figure shows that the lower limit of the observed period ratios increases with increasing eccentricity, roughly following the stability boundary, and that even the large eccentricity systems are in the stable region.

For all but 4 of the 27 systems in Table 5, $S(r)$ also is positive. For VV Ori, HR 266, ξ Tau,and λ Tau, $S(r)$ is negative, implying instability according to this criterion if the orbital motions are counter-rotational. Although λ Tau has been included in the lists of close multiple stars, Batten, Fletcher, and Mann (1978) question the reality of the third component because of newly reported observations of the system. As a result, the system with the largest index for instability may not even be a multiple system.

But, even if λ Tau is a triple system, there are several reasons for believing that it and the other three systems with negative $S(r)$ values are stable. Among visual-triple systems, Worley (1967) found that 78% appear to be corotating. In addition, the long orbital periods are very short, much less than one year, for three of the four systems. From analogy with our solar system, we might expect these systems to be corotational and, therefore, stable. Also, except for ξ Tau, the mass of at least one component in the other three systems is a lower limit, and stability in the counter-rotational case might result if this mass were increased. Finally, as noted earlier, Harrington's (1972) numerical stability analyses for components with equal masses in the counter-rotational case permit much smaller values of q_L/a_S than the analytical criterion of Szebehely and Zare (1977). Thus, numerical analysis of the four systems might show them to be stable in the counter-rotational case. Therefore, it appears likely that all the systems in Table 5 are stable.

The total mass of the systems in Table 5 ranges from 41.1 $\mathfrak{M}_\odot$ for η Ori to 0.7 $\mathfrak{M}_\odot$ for GL 815. Excluding these two systems, the other 25 systems only range from 13.5 $\mathfrak{M}_\odot$ to 2.1 $\mathfrak{M}_\odot$. The range of the measure of stability for corotational systems appears to be similar throughout the range of total mass. Thus, over this limited range of mass, the total mass of the system appears to have little effect on stability.

VII. LITHIUM ABUNDANCES AND AGE ESTIMATES

The question of stability in multiple systems may be approached from a direction independent of dynamical considerations. The ages of multiple systems and, thus, the length of time the systems have presumably been stable can be estimated from spectroscopic and photometric properties.

For multiple systems containing early-type stars, it is possible to estimate the age of the systems using the main-sequence lifetime of the star with the earliest spectral type. But F and later type stars spend a large amount of time, well over 10^9 years, on the main sequence so that this procedure is not as satisfactory for them. However, it appears that the ages of solar-type stars do show a statistical correlation with several observable spectroscopic phenomena: the strength of Ca II H and K emission lines, rotational velocity, and the strength of the lithium line at 6708 Å.

Wilson's (1963) inverse relationship between Ca II H and K emission intensity and age has been examined quantitatively by Skumanich (1972) and Blanco *et al.* (1974). Skumanich's result that the intensity of H and K emission decreases as the inverse square root of the star's age was conformed by Blanco *et al.* However, in short period binaries it appears that tidal coupling may enhance the emission intensity (Young and Koniges 1977), so that these binaries do not conform to the above relation. In addition, chromospheric activity and emission are at a maximum near K0 (Blanco *et al.* 1974) but are weak in late F stars, since the development of an extensive convective zone, which is believed to drive the chromosphere, only begins at about F5. Thus, unless the F stars are very young, the emission will be very weak and difficult to detect.

Kraft's (1967) extensive high-dispersion study of rotation among solar-type stars indicated that rotational velocity decreases with increasing age. Skumanich (1972) found that the rotational velocity of solar-type stars also goes as the inverse square root of age. This velocity decrease is believed to be caused by angular momentum lost by a magnetically coupled stellar wind and is related to the chromospheric phenomenon and to the development of an extensive convective envelope. For most multiple systems in this study, rotational velocity cannot be used as an age indicator. The components of the short-period binaries are usually synchronously rotating, and the velocity of the single star in the systems is usually less than the resolution limit of 10 km s^{-1}.

Another potential indicator of age in solar-type stars is the abundance of lithium. In four T Tauri stars, Bonsack and Greenstein (1960) found equivalent widths of Li I at 6708 Å ranging from 150 mÅ (GW Ori) to 450 mÅ (T Tau). These very young pre-main-sequence stars appear to be rich in lithium compared to normal dwarfs of late spectral type. From cluster and field dwarf data, Herbig (1965) suggested a correlation between lithium abundance and stellar age. He also found a correlation between the strength of a star's H and K emission intensity and its lithium abundance. Wallerstein, Herbig, and Conti (1965) found that Hyades stars cooler than the Sun show a progressive decrease in lithium abundance with later spectral type. In the Pleiades, a younger cluster, this decrease with later spectral type is much less marked (Danziger and Conti 1966). Zappala (1972) confirmed these results and estimated an *e*-folding time of 1.5×10^9 years for depletion of lithium in solar-type stars.

Of the three phenomena which appear to correlate with age in solar-type stars, the measurement of lithium abundances appears to be the most suitable for use here. Observations covering 100 Å centered on the lithium doublet 6707.8 Å were made for five solar-type multiple systems using the McDonald 2.7 m telescope and coudé Reticon detector. Six late F-type field stars from the list of Herbig (1965) were also observed.

If more than one component of a multiple star is visible in the same spectrum, the lines of one component appear weakened by the addition of the continua of the others. Because of uncertainties introduced in correcting for this effect, the lithium abundances were not determined from a curve-of-growth analysis but rather from the method of Danziger and Conti (1966). They used model atmospheres to predict the change in the weak-line equivalent width ratio, $R = \mathrm{Li}(\lambda 6707)/\mathrm{Ca}(\lambda 6717)$, with effective temperature and stellar colors

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to estimate the temperatures. In order to reduce the sensitivity of the ionization correction to temperature, Ca was used as the standard element. The Li/Ca abundance ratio relative to the Sun is then given by the formulae:

$$[\mathrm{Li/Ca}]=\log R-1.1(B-V)+2.2;$$

$$0.45\leq(B-V)\leq0.90,$$

$$[\mathrm{Li/Ca}]=\log R-1.4(B-V)+2.3;\ (B-V)<0.45,$$

where

$$[\mathrm{Li/Ca}]=\log(R)_{*}-\log(R)_{\odot}.$$

These equations underestimate the lithium abundance in stars hotter than the Sun and overestimate it in stars cooler than the Sun. However, the effective temperatures of the program stars are not very different from the Sun's and the error introduced is usually small.

Danziger and Conti (1966) estimated uncertainties of ±0.3 in their logarithmic abundances but used eye estimates of the equivalent width ratio of the lithium to calcium line. Because of the higher quality of the present observations, it is expected that the uncertainties in this study are somewhat less.

Figure 5 shows the lithium region in ξ Peg, and Table 6 gives the lithium abundances of the components of the five solar-type multiple stars and the six field stars. For comparison, the average lithium abundance (Li/Ca) for late F stars in the Hyades is about 1.6 (Danziger and Conti 1966). The Δm column of the table gives the amount by which each field star lies above the zero-age main sequence (ZAMS) and is also an indicator of age. These values were obtained from Crawford (1975) or computed using the Strömgren and Hβ indices and the standard relations from Crawford's (1975) Table 1.

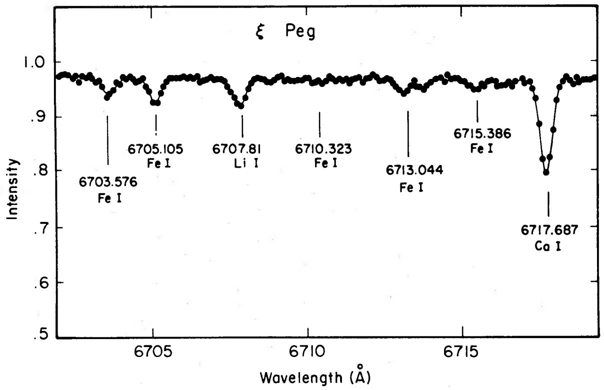

FIG. 5.—A portion of a Reticon spectrogram of ξ Peg showing the region around the lithium line in a late-type F star.

In this small sample of field stars there appears to be a correlation between the lithium abundance and the Strömgren c_1 index. For example, Φ^2 Cet, which has the largest lithium abundance, is nearly on the ZAMS, while β Vir and ι Psc, which have the lowest abundances, are the most evolved. The values of Δm for o Aql and ξ Peg

TABLE 6

LITHIUM ABUNDANCES AND AGES

Star	Sp.T.	$B-V$	W(Li) (mÅ)	W(Ca) (mÅ)	R	[Li/Ca]	Δm	Age (years)
13 Cet Aa	F7 V	0.51	71	107	0.66	1.5	...	1.4×10^9
B	G4 V	0.65	8	47	0.17	0.7	...	...
HD 165590 Aa ...	G0 V	0.59	106	102	1.04	1.6	...	$\leq1.0\times10^8$
B	G5 V	0.66	77	60	1.28	1.6	...	...
HD 202908 Aa ...	F9 V	0.57	67	57	1.18	1.6	...	$<1.4\times10^9$
Ab ...	G0 V	0.59	58	62	0.94	1.5	...	...
B	G1 V	0.61	27	33	0.82	1.4	...	...
HD 203345 Aa ...	F7 V	0.51	56	39	1.44	1.8	...	$<1.4\times10^9$
B	F8 V	0.54	26	51	0.51	1.3	...	...
κ Peg A	F4 V	0.41	51	57	0.89	1.7	...	2×10^9
Ba	F6 V	0.46	31	38	0.82	1.6	...	...
HR 235 .... ϕ^2 Cet	F8 V	0.50	72	105	0.69	1.5	0.1	$<1.4\times10^9$
HR 1101 .... 10 Tau	F8 V	0.58	47	106	0.44	1.2	0.4	3×10^9
HR 4540 .... β Vir	F9 V	0.56	19	118	0.16	0.8	0.7	4×10^9
HR 7560 .... o Aql	F8 V	0.55	57	133	0.43	1.2	0.7:	4×10^9:
HR 8665 .... ξ Peg	F5 V	0.50	30	83	0.36	1.2	0.5:	3×10^9:
HR 8969 .... ι Psc	F7 V	0.51	28	113	0.25	1.0	0.7	4×10^9

are somewhat uncertain because no values of Hβ were available for them. The last column of Table 6 gives the ages of the observed systems which have been determined from the spectroscopic and photometric considerations and Iben's (1967) evolutionary tracks.

From the evolutionary tracks of Iben (1967), a 0.7 mag increase as a star evolves off the main sequence corresponds to an age of 3.0×10^9 for a 1.25 $\mathfrak{M}_\odot$ star and 6.7×10^9 for a 1.0 $\mathfrak{M}_\odot$ star. Thus, the ages of β Vir and ξ Psc are probably about 4×10^9 years while that of ϕ^2 Cet is probably less than 1.4×10^9 years. The lithium abundances of the components of HD 202908 and HD 203345 are all large and their composite values of the c_1 index (Table 2) are also consistent with a maximum age similar to ϕ^2 Cet.

Stars of spectral type F5 and earlier apparently do not suffer pre-main-sequence or main-sequence depletion of lithium (Danziger and Conti 1966). The components of κ Peg are about spectral type F5 and both have high lithium abundances, although the composite c_1 index (Table 2) suggests that the components are about 1 mag above the main sequence leading to an age from Iben's (1967) evolutionary tracks of about 2×10^9 years. Thus, the lithium abundances in this system are consistent with the idea that there is no main-sequence depletion of lithium in stars earlier than about F5.

The components of 13 Cet are substantially different in spectral type and have substantially different lithium abundances as well. This effect was seen in the Hyades stars and presumably is due to a main-sequence lithium depletion mechanism which operates more efficiently in later type stars. The abundance of the later type star, *B*, is less than that of Hyades' stars of similar spectral type, suggesting that the age of 13 Cet is probably greater than the Hyades' age of about 5×10^8 years. But since the abundance of the early-type component *Aa* is still high, the system is probably not appreciably older than 1.4×10^9 years.

The two components of HD 165590 are also substantially different in spectral type, but in this case have the same lithium abundance. If the system is as old as the Hyades, about 5×10^8 years, the G5 star would be expected to have a somewhat lower lithium abundance than the G0 star. Since the two abundances are the same, the age of the components is probably closer to that of the stars in the Pleiades, about 5×10^7 years, and the triple system is quite young.

This conclusion is consistent with several other properties of the components. Emission lines of Ca II H and K are seen in both *Aa* and *B*. In addition, the absorption lines of component *B* in HD 165590 appear to be broadened. To obtain the $v \sin i$ of both *Aa* and *B*, a solar model atmosphere was used to compute a theoretical profile for the Ca I 6439 Å line. This profile was broadened by various amounts using the algorithms of Smith (1978) and adjusted for dilution effects. These broadened theoretical profiles were compared to the Reticon observations. Best fits for the projected rotational velocities are 75 ± 5 km s^{-1} for *Aa* and 18 ± 2 km s^{-1} for *B*. The large velocity of *B* is extremely rare for a G5 V field star. Possibly it was tidally induced by the short-period pair during the close (≤ 0.4 AU) periastron passages. However, Conti (1968) has suggested a relationship between the lithium destruction and rotational breaking mechanisms. For several stars with spectral types close to G0 from each of the Pleiades and the Hyades, he finds average equatorial rotational velocities of 19 km s^{-1} and 8 km s^{-1}, respectively, and a higher average lithium abundance for the Pleiades stars than those in the Hyades. Since *B* has a similar rotational velocity to these Pleiades stars, but is of even later spectral type, it appears that HD 165590 AB is quite young with an age of perhaps 1×10^8 years or less.

The lithium region was not observed in five other close multiple systems having a solar-type or later primary star. However, previous observations have been made of three of the five systems by other observers. Herbig (1965) has observed the lithium region in 44 Boo and ξ UMa. He estimated a moderate lithium abundance from the combined spectrum of 44 Boo, which suggests an age of perhaps 1×10^9 years or less. For ξ UMa the visual components, both G0 V stars, were observed separately. Component *A*, a long-period spectroscopic binary, had a high lithium abundance and weak H and K emission. However, *B*, a short-period spectroscopic binary, had no detectable lithium feature on a 4 Å mm^{-1} plate but did have strong H and K emission, making this component quite anomalous. Tidal interactions of a short-period binary system (Young and Koniges 1977) may be the cause of the enhanced emission. Component *B* is 0.5 mag fainter than *A*, so its spectral type might be later than G0. If so, the increased rate of lithium destruction found in late-type Hyades stars might account for the lack of lithium in *B*. As in the case of 13 Cet, this would make the age of the system somewhat greater than the Hyades cluster. The only system which is much later than solar-type is the dMe flare star and BY Dra-type variable GL 815. This star was included by Bopp (1974) in a program of lithium detection in flare and dMe stars. Bopp found no lithium line with an equivalent width greater than 50 mÅ in GL 815. However, lithium is believed to be depleted very rapidly in M dwarfs, much faster than in solar-type stars, because their totally convective atmospheres rapidly mix surface lithium to hotter temperature regions where it is destroyed. Thus, this system may still be relatively young despite its lack of lithium. Although no lithium observations have been made of the fourth star, HD 27691, it is a member of the Hyades cluster and, thus, has an age of 5×10^8 years. No lithium observations have been made of the final system, VW Cep, which contains a W UMa binary.

In conclusion, ages of known, close multiple systems in the solar neighborhood indicate that nearly all systems, even those of solar type, are substantially younger than the Sun. The ages range from 1×10^8 years for late B star systems to $1-2\times10^9$ years for the oldest solar-type systems. Although it is not possible to infer extremely long-term stability for any of the solar-type systems, these systems presumably have been stable for their entire lifetimes.

The result that all the solar-type multiple systems observed in this survey are substantially younger than the Sun should not be assumed to be a general conclusion. Certainly the current sample size is small and should be doubled or tripled before one might begin to think seriously about such a conclusion. Nevertheless, this result raises the question of whether age estimates suggested by lithium abundances give a similar result for binaries.

Herbig's (1965) extensive survey of F and G dwarfs included a few visual binaries. Only two of his six visual binaries appear to have a very small lithium abundance and, thus, may have an age approaching that of the Sun. The other four visual binaries, as well as the two spectroscopic binaries he observed, appear to be much younger than the Sun. The only other survey of lithium in binaries is that of Wallerstein (1966). Although he strongly biased his survey toward a search for young visual binaries, he unexpectedly found 6 stars in his sample of 19 systems which had larger lithium abundances than any of the approximately 100 stars in Herbig's survey. Wallerstein (1966) noted that systematic differences in the reduction procedures might be the cause of the larger abundances in his survey. However, he could not rule out the possibility that there is a tendency for young stars to be found preferentially in binary systems. Thus, the suggestion that, in general, solar-type binary and multiple systems are significantly younger than similar-type single stars should be investigated further.

VIII. FUTURE OBSERVATIONS

Close multiple systems continue to be discovered, primarily as a result of high dispersion spectroscopic and speckle interferometric observations. Table 7 is a list of two dozen suspected or confirmed close multiple systems whose long periods are about 100 years or less. The first three columns list the HD number, spectral type, and visual magnitude of the system. The columns *SS*, *LS*, *LV*, and Phot denote short-period spectroscopic elements, long-period spectroscopic elements, long-period visual elements, and photometric magnitudes and colors, respectively. A + indicates that these quantities are known, while a 0 indicates that extensive observations are currently being obtained at various observatories. Despite the fact that over half of these systems are brighter than 6.5 mag extensive spectroscopic observations are needed to determine the short-period elements and at least the mass ratio of the stars in the long-period orbit if a long-period spectroscopic orbit cannot be determined. Unfortunately, about one-quarter of the systems have large southern declinations making it difficult if not impossible to observe them from northern observatories. In addition, many of the systems have components with moderate rotational velocities making the measurement of radial velocities more difficult. Nevertheless, because they are so bright, it is hoped that extensive observations of many of them will be made in the near future as new detectors and techniques of observation become widely available. This will provide a broader observational base for comparison with the numerical studies.

IX. CONCLUSIONS

The observational properties of close multiple stars have been compared with the results of numerical studies of their formation by both fission and fragmentation. It has been assumed that components of a multiple system with a long period of about 100 years or less form from a single cloud and can be compared with these studies. For 37 such systems, the mean ratio of long to short period is 3222 : 1. According to numerical fragmentation models, this period ratio depends on the fraction of orbital angular momentum which remains in the spin of the protostar after the ring of accreting matter has fragmented. The few observed systems containing a rapidly rotating single star are consistent with numerical fragmentation models which find that a larger ratio of spin to orbital angular momentum produces a smaller period ratio.

Numerical studies indicate that fragmentation produces binary components with mass ratios usually close to 1.0, while fission usually produces mass ratios in the range 0.1–0.5. In the observed short-period systems of close multiple stars, fragmentation appears to dominate by a ratio greater than 2 : 1. For the long-period systems, the observed ratio is uncertain because of selection effects. No matter what the formation mechanism, the more massive long-period component of the triple systems is observed to be the binary. This result also may be influenced strongly by selection effects.

Only limits can be placed on the orbital coplanarity of a multiple system if one or both of its orbits are spectroscopic. Thus, only the number of systems which are definitely not coplanar could be determined. Considering only the inclinations of the short- and long-period orbits of 20 systems, 33% of the orbital pairs are not coplanar. For the rest, coplanarity is a permitted possibility. Although a number of systems are not exactly coplanar, there does appear to be a tendency toward orbital coplanarity in systems with periods $\lesssim 100$ years.

Using analytical stability criteria, all 27 close multiple systems considered are stable if they are corotating. If they are counter-rotating, four systems are possibly un-

TABLE 7

CLOSE MULTIPLE SYSTEMS WITH INCOMPLETE DATA

HD	Sp.T.	M_v	*SS*	*LS*	*LV*	Phot.	Remark
2885	A2 V	4.5	...	...	+	+	...
4676	F8 V	5.2	+	0	...	+	...
28363	F8 V	6.6	0	...	+	...	...
30869	F7 V	6.3	0	...	+	...	...
35652	B2 V	8.4	+	...	...	+	...
37468	O9.5 V	3.8	0	...	+	+	triple lined
38735	A4 V	6.0	0	...	...	+	triple lined
107259	A2 V	3.9	+	...	0	+	...
118261	F7 V	5.6	...	...	+	+	triple lined
133955	B3 V	4.0	...	...	+	+	...
137844	F0 V	8.1	0	...	+	+	...
141556	B9 IV	3.9	+	...	...	+	possible triple
157482	F8 V	5.5	0	0	...	+	...
162724	B9 V	6.0	+	...	...	...	triple lined
172743	Am	7.8	...	...	+	...	...
174343	F2 V	7.2	+	...	0	...	...
187949	A2 V	6.5	+	...	...	...	possible triple
191095	A1 V	6.4	...	...	...	+	triple lined
194882	A3 V	6.4	...	...	+	+	...
202260	A1 V	7.5	...	...	+	...	...
216608	Am	5.6	...	...	+	...	possible triple
219018	G2 V	8.2	0	...	+	...	...
219815	A7 m	5.9	+	...	...	...	possible triple
221264	F7 V	7.4	...	...	+	+	possible quadruple

stable. However, other factors suggest that even these four are probably stable.

The ages of solar-type close multiple systems were estimated from the lithium abundances of the individual components. These ranged in age from 1×10^8 to 2×10^9 years. The possibility that solar-type binary and multiple systems are younger than single stars of similar spectral type should be investigated further.

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