THE ASTROPHYSICAL JOURNAL, **246**:278–291, 1981 May 15


# THE CARBON STAR MYSTERY: WHY DO THE LOW MASS ONES BECOME SUCH, AND WHERE HAVE ALL THE HIGH MASS ONES GONE?[1]

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*Received 1980 August 11; accepted 1980 November 20*

## ABSTRACT

Comparison between an observed number versus magnitude ($N$, $M_{\rm Bol}$) distribution of carbon stars in the Magellanic Clouds and various theoretical distributions, each of which is a function of a different assumed "dredge-up" law, suggests that a significant fraction of the carbon that is freshly synthesized in an asymptotic giant branch (AGB) star during thermal pulses is brought to the surface of *every* AGB star in the Clouds, whatever the mass of the underlying carbon-oxygen core. This conclusion conflicts with extant theoretical estimates which limit convective dredge-up to the pulse power-down phase in AGB stars with core masses greater than about 0.6 $M_\odot$, and it is suggested that one or more additional mixing processes may be of crucial importance in AGB stars possessing small cores. Alternatively, nuclear reaction rates and neutrino-loss rates which influence pulse strength and, concomitantly, the extent of dredge-up during pulse power-down, may have been improperly estimated.

Whatever the dredge-up law adopted, theoretical ($N$, $M_{\rm Bol}$) distributions of carbon stars exhibit a substantial fraction of stars with bolometric magnitudes brighter than $M_{\rm Bol} = -6$, whereas the observed distributions show none. Since, with current estimates of the $^{22}$Ne($\alpha$, $n$)$^{25}$Mg cross section, AGB stars must reach a magnitude brighter than $M_{\rm Bol} \sim -6.5$ before producing significant quantities of $s$-process isotopes in the solar-system distribution, it is suggested that carbon stars bolometrically brighter than $M_{\rm Bol} \sim -6$ may surround themselves with a thick dust shell and thereby escape detection in the near-infrared. If this is not the case, one might infer that the effective cross section (at 30 keV) for the $^{22}$Ne($\alpha$, $n$)$^{25}$Mg reaction has been considerably underestimated. Another possibility is that, for large enough core masses, the temperatures near the base of the convective envelope are sufficiently large that $^{12}$C is converted into $^{14}$N at least as rapidly as fresh $^{12}$C is dredged up; stars bright enough to produce $s$-process elements might thus not be carbon-rich at their surfaces.

*Subject headings:* nucleosynthesis — stars: carbon — stars: evolution — stars: interiors — stars: stellar statistics

## I. INTRODUCTION

Recent observational studies of carbon stars (hereinafter C stars) in the Magellanic Clouds (Blanco, Blanco, and McCarthy 1978; Westerlund *et al.* 1978; Richer, Olander, and Westerlund 1979; Mould and Aaronson 1979, 1980; Frogel, Persson, and Cohen 1980; Blanco, McCarthy, and Blanco 1980; Richer 1981) have demonstrated that most C stars achieve their carbon-rich characteristics while in the asymptotic giant branch (AGB) phase of evolution at magnitudes that can be perhaps as low as $M_{\rm Bol} \sim -3.5$ but are perhaps not much brighter than $M_{\rm Bol} \sim -6.0$.

Theoretical calculations (Iben 1974, 1975, 1976) have shown that, during pulse power-down, a "dredge-up" process can bring to the surface of an AGB star some of the fresh carbon that is formed in the helium- and carbon-rich convective shell during a thermal pulse, and that this process will ultimately lead to the development of C star characteristics. In the explicit case of a star with an initial main-sequence mass of 7 $M_\odot$ and with a corresponding initial (on first reaching the AGB) C-O core mass of $M_c \sim 0.96$ $M_\odot$, C star characteristics first appear at a magnitude of $M_{\rm Bol} \sim -6.5$. Calculations by Sugimoto and Nomoto (1975) and by Fujimoto, Nomoto, and Sugimoto (1976) have shown that the simple dredge-up process operates in stars with core masses larger than 0.96 $M_\odot$. However, calculations by Gingold (1975), Sweigart (1973, 1976), Sackmann (1980), and Wood and Zarro (1981) suggest that simple dredge-up does not occur following thermal pulses for core masses as small as or smaller than $M_c \sim 0.6$ $M_\odot$.

Adopting a specific dredge-up law that is consistent with the theoretical data then available (zero for $M_c \leq$

[1]Supported in part by the National Science Foundation, grant AST 78-20124.

$0.6 M_\odot$ and monotonically increasing as core mass increases above 0.6 $M_\odot$) and adopting a semiempirical surface mass-loss rate (Reimers 1978), Iben and Truran (1978, hereafter IT) have shown that the luminosity at which a theoretical AGB star will first become a C star (if at all) increases with increasing initial main-sequence mass. Thus the H-R diagram (in the vicinity of AGB stars) can be broken into two parts by a carbon star formation line (CSFL) which intersects individual evolutionary tracks at a brightness which increases with increasing initial main-sequence stellar mass. For any given dredge-up law there is a minimum initial mass and a minimum brightness below which C star characteristics are not expected to be formed. The CSFL extends from this point upward to a brightness of $M_{\rm Bol} \sim -7.3$.

Choosing a "conservative" dredge-up law, IT find a minimum C star brightness of $M_{\rm Bol} \sim -6$, whereas a "less conservative" dredge-up law yields a minimum brightness of $M_{\rm Bol} \sim -5$. Renzini and Voli (1981, hereinafter RV), choosing still another dredge-up law which restricts dredge-up to core masses $M_c \geq 0.6\ M_\odot$, find a minimum brightness of $M_{\rm Bol} \sim -4.9$. Clearly, theoretical expectations are not consistent with the observed distribution of C stars in several carefully surveyed regions in the Magellanic Clouds. The observations show a distribution in number versus magnitude that peaks in the neighborhood of $M_{\rm Bol} \sim -4.6$ to $-5.0$ (Blanco, McCarthy, and Blanco 1980, hereinafter BMB; Richer 1981), whereas the theory predicts a peak at least a full magnitude brighter than this and predicts that there are very few C stars (due to simple dredge-up connected with thermal pulses) in the neighborhood of the observed peak.

It is one of the major purposes of this paper to demonstrate that, if the observations are to be understood, significant dredge-up *must* occur in thermally pulsing AGB stars of low metal abundances ($Z \leq 0.01$) when core mass is considerably smaller than 0.6 $M_\odot$ and to suggest several possible deficiencies in the theory that may account for the failure of models with small cores to experience dredge-up. The discussion here, which focuses on the properties of a homogeneous and complete sample of C stars whose distances are well known, is complementary to a recent study by Scalo and Miller (1979), which focuses on the properties of C stars in our Galaxy. The basic conclusions are identical: some mixing process (referred to loosely in this paper as a "dredge-up" process) that has not yet been accounted for by theory must be operating in AGB stars of low core masses and of low metallicity.

Another puzzling feature of the BMB C star distributions vis-à-vis the theory is the apparent almost total absence of C stars brighter than $M_{\rm Bol} \sim -6$. It might be inferred either that such stars are surrounded by a thick dust shell that redistributes energy into the far infrared, where it has escaped detection in the current near-infrared surveys, or that they actually do not exist in the fields that have been carefully studied. It may simply be that the BMB fields, selected for their "quietness," contain no stars young enough to have ultimately evolved brighter than $M_{\rm Bol} \sim -6$. However, the paucity of bright C stars in the general field (Richer, Olander, and Westerlund 1979) suggests that most stars of intermediate mass may terminate their existence on the AGB before reaching $M_{\rm Bol} \sim -6.0$ or that, on evolving through magnitudes brighter than this, carbon is destroyed in the stellar envelope at least as rapidly as it is dredged up (Iben 1975; RV).

## II. APPROXIMATE AGB EVOLUTIONARY MODELS

The basic algorithms for constructing approximate theoretical evolutionary tracks of AGB stars and for constructing distributions from them are given in IT. In the study pursued here, several improvements in the details have been incorporated. The composition of intershell matter that is dredged up and the extent of surface mass loss prior to the AGB phase are taken from RV, including an approximation to the composition of this matter found by Sackmann (1980) for small core masses. The relationship between surface temperature, luminosity, mass, and composition plus the initial mass in the C-O core, as a function of initial main-sequence mass and composition, are taken from Becker and Iben (1979, 1980). For $Y=0.28$, $Z=0.02$, this latter dependence is

$$\begin{aligned} M_c &= 0.56, \text{ if } M_0 \leq -1.35; \\ M_c &= 0.60 + M_0(-0.0601 + M_0 \times 0.0226), \\ &\qquad \text{if } 1.35 < M_0 < 4.7; \\ M_c &= 0.535 + 0.060\ M_0, \text{ if } M_0 > 4.7. \end{aligned} \tag{1}$$

Here $M_c$ is the C-O core mass and $M_0$ is the main-sequence mass, both in solar units. For $Y=0.28$, $Z=0.01$,

$$\begin{aligned} M_c &= 0.538, \text{ if } M_0 < 1.00; \\ M_c &= 0.524 + M_0(-0.00988 + M_0 \times 0.0240), \\ &\qquad \text{if } 1.00 < M_0 < 3.90; \\ M_c &= 0.654 + 0.0506\ M_0, \text{ if } M_0 > 3.90. \end{aligned} \tag{2}$$

The final modification is a (surface luminosity, core mass)-relationship for AGB stars that joins together the Paczyński (1970)–Uus (1970) relationship, which is valid for small stellar masses, and the Iben (1977) relationship, which is valid for large stellar masses. The adopted law is

$$L_s/10^4 L_\odot = (5.925 + 0.415x)(M_c - 0.495 + 0.0505x),$$

$$\text{where} \quad x = \left[M_* - M_c)/6.04\right]^{1.83} \qquad (3)$$

and $M_*$ is the total stellar mass. The number 0.495 is taken from Wood and Zarro (1981).

Results of a calculation of model evolution for $Z = 0.02$, $Y = 0.28$, are shown in Figure 1. The "less conservative," or quadratic IT dredge-up law has been adopted, and the coefficient in the Reimers (1978) mass loss rate has been chosen as $\alpha = \frac{1}{3}$. The quadratic IT dredge-up law permits no dredge-up for core masses smaller than $M_c = 0.64$. The curve marked "begin" in Figure 1 represents, as a function of initial main-sequence mass, the "quiescent" luminosity which a star adopts when first beginning to thermally pulse. Evolutionary tracks for several representative initial masses are shown.

The termination of theoretical tracks occurs along the curve labeled "end" and is due to three different physical effects, each of which contributes a segment (WIND, PN, SN) to the curve. Stars initially less massive than $M_0 \sim 1.23$ lose their hydrogen-rich envelopes via a wind both before ejecting a planetary nebula and before developing a core mass $M_c = 1.4$. They then become white dwarfs. Stars in the range $1.38 \lesssim M_0 \lesssim 5$ eject a planetary nebula before becoming white dwarfs. Finally, stars initially more massive than $M_0 \sim 5$ develop a core mass of $M_c \sim 1.4$ before losing their hydrogen-rich envelopes. These stars presumably become supernovae. It must be emphasized that the termination curve is highly speculative. In particular, the PN segment of the termination curve might extend both below the wind-termination segment (RV 1981) and below the SN-termination segment (Tuchman, Sack, and Barkat 1978, 1979), with the consequence that all intermediate-mass stars end their lives (as AGB stars) by ejecting planetary nebulae.

An additional set of curves in Figure 1 shows how the surface carbon abundance increases as a result of dredge-up. Beginning with an initial main-sequence abundance that is about half of the oxygen abundance, the number abundance of carbon becomes equal to that of oxygen at the stellar surface along the curve labeled "S star." With the algorithms chosen, only stars initially more massive than $M_0 \sim 1.23$ can become S stars (defined as stars with $^{12}C \sim {}^{16}O$) and immediately thereafter evolve into C stars.

Of critical importance for our understanding of the synthesis of *s*-process elements is the fate of an AGB star after the mass in its carbon-oxygen core reaches and exceeds $M_c \sim 1.0$. It is thought that only in such stars can *s*-process elements be made *both* in large quantities

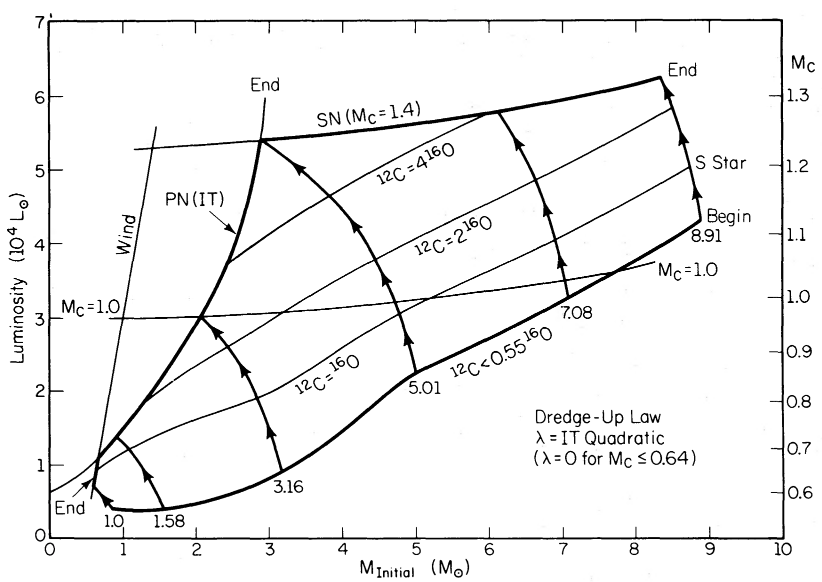


FIG. 1.—Mass-luminosity properties of evolutionary models when the IT quadratic dredge-up law is adopted: $\lambda = 0$ when $M_c \le 0.64$; $\lambda = 0.33 + 0.73(M_c - 0.73) - 0.531\ (M_c - 0.96)^2$ when $M_c > 0.64$. Models begin the thermally pulsing AGB stage along the curve marked "begin" and terminate along a curve made up of three segments: a supernova (SN) segment, a planetary nebula (PN) segment, and a wind segment. Initial mass for several evolutionary tracks is marked along the "begin" curve. Also shown are curves where the surface $^{12}C$ number abundance reaches the $^{16}O$ number abundance, reaches double and then quadruple the $^{16}O$ abundance. Core masses for stars that are near the $^{12}C = {}^{16}O$ curve are shown along the right-hand vertical axis. Finally, the location at which core mass reaches $M_c = 1.0$ is shown. As each star crosses this curve, *s*-process elements are made prolifically in the convective helium-burning shell during thermal pulses and thereafter brought to the surface via convective dredge-up.

and in the solar-system distribution (e.g., IT). From Figure 1, curve "$M_c = 1.0$," it is evident that (with the chosen termination algorithm) only those AGB stars that become more luminous than $L_s \sim 3.0 \times 10^4 L_\odot$ ($M_{\rm Bol} \sim -6.4$) and that have an initial main-sequence mass greater than $M_0 \sim 3$ can contribute substantially to the Galactic nucleosynthesis of *s*-process elements. One might tentatively consider the curve "$M_c = 1.0$" to be a very conservative lower limit to the location of the high-luminosity segment of the termination curve.

Further properties of theoretical models are shown in Figure 2*a*. For any given initial mass, an evolutionary track is a vertical line extending upward between the two curves marked "begin" and "end." As it crosses the curve marked "S star," a model metamorphoses from an M star into a C star. The total number of thermal pulses experienced by a model of specified initial mass increases with increasing initial mass. For example, as $\log M_0$ increases from 0.0 to 0.7 in steps of 0.1, the number of pulses is 13, 27, 42, 61, 99, 215, 840, and 9399. Beyond $\log M_0 = 0.7$, the number of pulses does not vary significantly from about 9000.

In first approximation, the rate at which a star evolves in magnitude per unit time along its track is independent of core mass, so that, for any given main-sequence mass, the time a model spends as a C star relative to the time it spends as an M star is simply the ratio of the magnitude intervals spent in the two phases. The actual situation is slightly more complicated, since dredge-up reduces the rate of evolution in magnitude per unit time by a factor $(1-\lambda)$, where $\lambda = \Delta M_{\rm dredge}/\Delta M_{\rm H}$ is the ratio of the amount of freshly processed matter dredged up following a pulse to the amount of matter traversed by the hydrogen-burning shell between pulses. Since it has been assumed that $\lambda$ increases with increasing core mass, the actual ratio of times spent in the C star and M star phases is slightly greater than the ratio of corresponding magnitude intervals.

After multiplying the time spent in the C star phase by an assumed birthrate, the relative contribution of

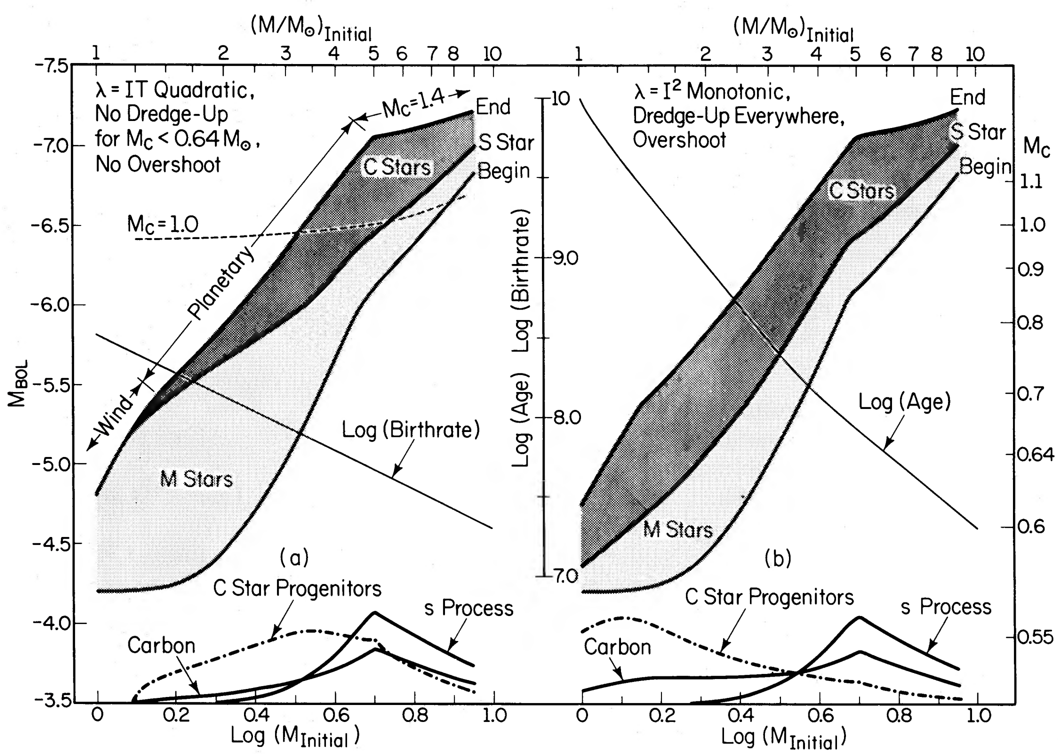


FIG. 2.—In the left half of this figure (2*a*) is shown the extension in bolometric magnitude versus initial (main sequence) mass of model stars on the asymptotic giant branch when the IT quadratic dredge-up law is assumed. Stars evolve between the "begin" and "end" curves and become C stars (C>0) on crossing the "S star" curve. Core masses for stars reaching the S star curve, when applicable, are shown along the vertical axis at the far right (2*b*), and where core mass reaches 1.0 is also shown for every model. The birthrate function $S(M_0) = M_0^{-1.35}$ is shown for reference. The scale for this curve is arbitrarily normalized to log(birthrate) = 8.5 when $M_0 = 1.0$. Along the base of the graph are shown the contribution of stars of different mass (per logarithmic mass interval) to the Galactic production of $^{12}$C and of *s*-process elements. Also shown is the contribution per logarithmic mass interval to the production of C stars. In all of the latter three cases, normalization is uniform but arbitrary. The area under the "Carbon" curve is 0.725 times the area under the "*s*-process" curve.

In the right half of the figure (2*b*), $\lambda = 0.33 + 0.56667(M_c - 0.96)$ or 0.5, whichever is smaller. The area under the "Carbon" curve is 1.06 times the area under the "*s*-process" curve.

stars in various (logarithmic) mass intervals is obtained. For most of the distributions in this paper, a birthrate derived from the Salpeter (1955, 1959) mass function,

$$S(M_0)=\frac{d(dN/dt)}{d\log M_0}=\text{BIRTHRATE}=M_0^{-1.35}, \quad (4)$$

will be used. The choice of a time-independent birthrate is somewhat arbitrary (see, e.g., Searle, Sargent, and Bagnuolo 1973). A feeling for how a birthrate that declines monotonically with time will affect the distribution can be achieved by using a time-independent rate with a steeper mass-dependence than given by equation (4). This will be done in § VI.

Using equation (4), the curve labeled "C star progenitors" appearing near the base of Figure 2*a* results. The normalization is arbitrary. The maximum contribution to the distribution of C stars is predicted to come from stars with an initial main-sequence mass slightly in excess of $M_0 \sim 3$. In a similar fashion, as detailed by IT, one can derive the contribution of model stars in various logarithmic mass intervals to the Galactic nucleosynthesis of $^{12}$C and *s*-process elements. If the area under the curve labeled "*s* process" near the base of Figure 2*a* is taken to be 1.0, then the area under the curve labeled "carbon" is 0.725. Thus, if it is assumed that the entire nucleosynthesis of Galactic *s*-process elements is confined to stars of mass in the range $1.00 < M_0 < 8.91$, then these same stars also contribute 72.5% of the $^{12}$C produced in the Galaxy.

## III. ALGORITHMS FOR THE PREPARATION OF THEORETICALCARBON-STAR AND M-STAR DISTRIBUTIONS

Only interpulse "quiescent" magnitudes are represented in Figures 1 and 2. On an evolutionary time scale, the magnitude of an AGB star varies appreciably from the quiescent value during and for some time after a pulse. During the pulse, the surface magnitude drops precipitously by about 0.5 mag, and then recovers slowly over about 20% of the interpulse phase, thereafter remaining near the quiescent value (Weigert 1966; Gingold 1975; Iben 1975; Wood and Zarro 1981). Thus, in rough approximation, the probability of finding a star at its interpulse quiescent magnitude $M_{\text{Bol}}^{\text{quiescent}}$ is about 0.8, and the probability of finding it at some other magnitude in the interval $M_{\text{Bol}}^{\text{quiescent}}$ to $M_{\text{Bol}}^{\text{quiescent}}+0.5$ is about 0.2 and is approximately independent of location within this interval. For masses smaller than about 2 $M_\odot$, the precipitous dip by $\Delta M_{\text{Bol}} \sim 0.5$ is preceded by a brightening which is of such short duration that it need not be taken explicitly into account (e.g., Wood and Zarro 1981).

A second variation in magnitude is of much shorter duration (hundreds of days) and is undoubtedly due to a dynamical pulsation of the stellar envelope. It is established by the observations. From a comparison of plates taken 310 days apart, BMB conclude that at least one-third of all Magellanic Cloud C stars in their survey are variable, with an amplitude that can reach 0.7 mag. They conclude further that the data are not inconsistent with *all* C stars being small-amplitude variables. For purposes of illustration, it will be assumed in this paper that all AGB stars are sinusoidally variable with a half-amplitude of 0.35 mag.

The construction of a theoretical number-versus-magnitude distribution for C stars proceeds in the following fashion. Acoustical variations in magnitude are at first assumed not to occur. For any given initial mass $M_0$, the probability $dP_C$ that a star is a C star with a magnitude between $V$ and $V+dV$ is taken to be (arbitrary normalization)

$$dP_C(M_0,V)\approx\frac{T_C(M_0)\,dV}{|V_E(M_0)-V_S(M_0)|}\times\Big[0.8\delta(M_0,V)+0.4\{\delta_1(M_0V)[V_1(M_0)-V]+\delta_2(M_0,V)\Delta(M_0)+\delta_3(M_0,V)[V-V_E(M_0)]\}\Big], \quad (5)$$

where

$$V_1(M_0)=V_S(M_0)+0.5,\ V_2(M_0)=V_1(M_0)-\Delta(M_0),$$

$$V_3(M_0)=V_E(M_0)+\Delta(M_0),$$

$$\Delta(M_0)=\min[0.5,|V_E(M_0)-V_S(M_0)|],$$

$$\delta(M_0,V)=1,\ \text{if}\ V_S(M_0)\geq V\geq V_E(M_0),$$

$$\delta_1(M_0,V)=1,\ \text{if}\ V_1(M_0)\geq V\geq V_2(M_0),$$

$$\delta_2(M_0,V)=1,\ \text{if}\ V_2(M_0)\geq V\geq V_3(M_0),$$

$$\delta_3(M_0,V)=1,\ \text{if}\ V_3(M_0)\geq V\geq V_E(M_0).$$

Here $T_C(M_0)$ is the time spent by the model star with a quiescent magnitude between $V_S(M_0)$ and $V_E(M_0)$, and all $\delta$'s are zero outside of the interval within which they are unity. In exactly the same fashion one may construct the probability $dP_M$ that a star is a thermally pulsing M star between magnitudes $V$ and $V+dV$.

Next, the $(M_{\text{Bol}}, \log M_0)$-plane in Figure 2 is broken into a grid with spacings of $\delta M_{\text{Bol}}=0.01$ and $\delta \log M_0=0.01$. Actually, an evolutionary track has been constructed at every value of $\log M_0$ in the grid. At every grid point $J$ along the $M_{\text{Bol}}$ axis, the quantity

$$(dN/dM_{\text{Bol}})_J=\sum_I S(M_{0I})\,dP(M_{0I},V_J)/dV \quad (6)$$

is constructed, both for C stars and for thermally pulsing M stars. Here $V_J$ is the value of $M_{\rm Bol}$ at the $J$th grid point and $M_{0I}$ is the main-sequence mass at the $I$th grid point. The resulting distributions are applicable if one assumes that AGB stars are not pulsating on an acoustical time scale. In order to take into account the fact that such stars do in fact pulsate, an average over the pulsation cycles of stars at different mean $M_{\rm Bol}$ must be constructed.

Not knowing individual stellar periods or pulsation amplitudes as a function of $M_{\rm Bol}$, a very simplified approach is adopted in this paper. At any value of $M_{\rm Bol}$, it is assumed that every star with mean magnitude $\bar{V}$ in the range $M_{\rm Bol} - V_0 < \bar{V} < M_{\rm Bol} + V_0$ pulsates sinusoidally at the same frequency $\omega$ and with the same half-amplitude $V_0$, so that $V = \bar{V} + V_0 \sin \omega t$. Each star in the interval makes excursions in magnitude, crossing a magnitude interval $\Delta V$ at a magnitude distance between $(I-1)\Delta V$ and $I\Delta V$ from its mean location $\bar{V}$ in a time $t_I - t_{I-1}$ given by

$$\sin \omega t_I - \sin \omega t_{I-1} = \Delta V / V_0 \equiv N^{-1}, \qquad (7)$$

where, for convenience, $V_0/\Delta V$ is chosen to be an integer $N$ (=35 in the case at hand), $I$ is an integer ranging from 0 to $N$, and $\sin \omega t_I = I/N$. Clearly, $\omega t_N = \pi/2$ and the value of $\omega$ (chosen here as $\omega = 1$) is irrelevant in determining the probabilities $(t_1 - t_{I-1})/t_N$. At every grid point $J$ in $M_{\rm Bol}$, a revised distribution function is then constructed, taking into account contributions from all grid points $K$ in the range $J - N \leq K \leq J + N$:

$$(dN/dM_{\rm Bol})_J{}' = \frac{1}{4t_N} \sum_{K=J-N+1}^{J+N} |t_K - t_{K-1}| \times \{(dN/dM_{\rm Bol})_K + (dN/dM_{\rm Bol})_{K-1}\}. \qquad (8)$$

This final distribution may be normalized in any way that appears convenient.

## IV. THE DISTRIBUTIONS

The distributions of C stars and thermally pulsing M stars that result from the model properties summarized in Figures 1 and 2*a* are shown in Figure 3*a*. The distributions have been normalized to 100 C stars. Note that the M star distribution extends 0.85 mag dimmer than the location of the theoretical minimum of $M_{\rm Bol} \sim -4.2$ for an AGB star during its quiescent phase and that the C star distribution also extends significantly below the theoretical minimum of $M_{\rm Bol} \sim -5.2$ for a C star during its quiescent phase.

For comparison, shown also in Figure 3*a* is the distribution, in number versus the magnitude indicator $M(0.81)$, that is defined by the 320 Magellanic Cloud C stars studied by BMB. The observed distribution has been shifted very slightly by $-0.1$ mag, simply to dramatize the exceedingly remarkable fact that, although it bears absolutely no quantitative resemblance to the theoretical C star distribution, the observed distribution bears a striking resemblance to the theoretical distribution of thermally pulsing M stars! As remarkably, the theoretical distributions predict that thermally pulsing M stars should outnumber thermally pulsing C stars by 4 to 1; the BMB observations suggest that C stars outnumber M stars that are as bright as C stars by about 5 to 3. It is as though only those real thermally pulsing AGB stars with small core masses, $M_c \lesssim 0.64$ ($M_{\rm Bol}^{\rm quiescent} > -5.1$) dredge up carbon, whereas most real thermally pulsing AGB stars with large core masses, say $M_c \gtrsim 0.7$ ($M_{\rm Bol}^{\rm quiescent} \lesssim -5.5$), do not dredge up carbon!

At least a dozen experiments have been performed with different dredge-up laws, the conclusion being that it is simply not possible to account for the observed distribution of C stars (assuming that the application of appropriate bolometric corrections does not *significantly* brighten the observed distribution) *unless dredge-up occurs at a substantial rate for all AGB stars of small core mass*. In particular, the recent results of Wood and Zarro (see Wood 1981) for the extreme case of $Z = 0.001$ and $l/H = 1.5$ do not reproduce the observations (Iben 1981). An example of a dredge-up law that provides theoretical distributions more in accord with the observational evidence is

$$\lambda = {\rm Min}[0.5,\, 0.33 + 0.56667(M_c - 0.96)]. \qquad (9)$$

With this law, dredge-up is substantial for *all* core masses achieved by AGB stars and increases monotonically through the calculated point at $M_c = 0.96$, reaching a maximum of $\lambda = 0.5$ at $M_c = 1.26$.

The results are summarized in Figures 2*b* (model evolutionary tracks) and 3*b* (distributions). From both figures it is clear that thermally pulsing C stars outnumber thermally pulsing M stars (by about 10 to 7), a result which is certainly in better accord with the observational evidence (a ratio of about 10 to 6 in one field of the Large Magellanic Cloud [LMC]. Note from Figure 3*b* that, although the greatest contribution to the theoretical distribution of C stars now comes from stars with an initial main-sequence mass near $M_0 \sim 1.6$, the bulk of the carbon contributing to Galactic nucleosynthesis still comes from those stars that contribute in accordance with the first (IT quadratic) dredge-up law adopted here. Further, meeting the requirements for the Galactic nucleosynthesis of *s*-process elements still demands dredge-up in stars initially more massive than $M_0 \sim 3$. In Figure 3*b*, the theoretical distributions have been

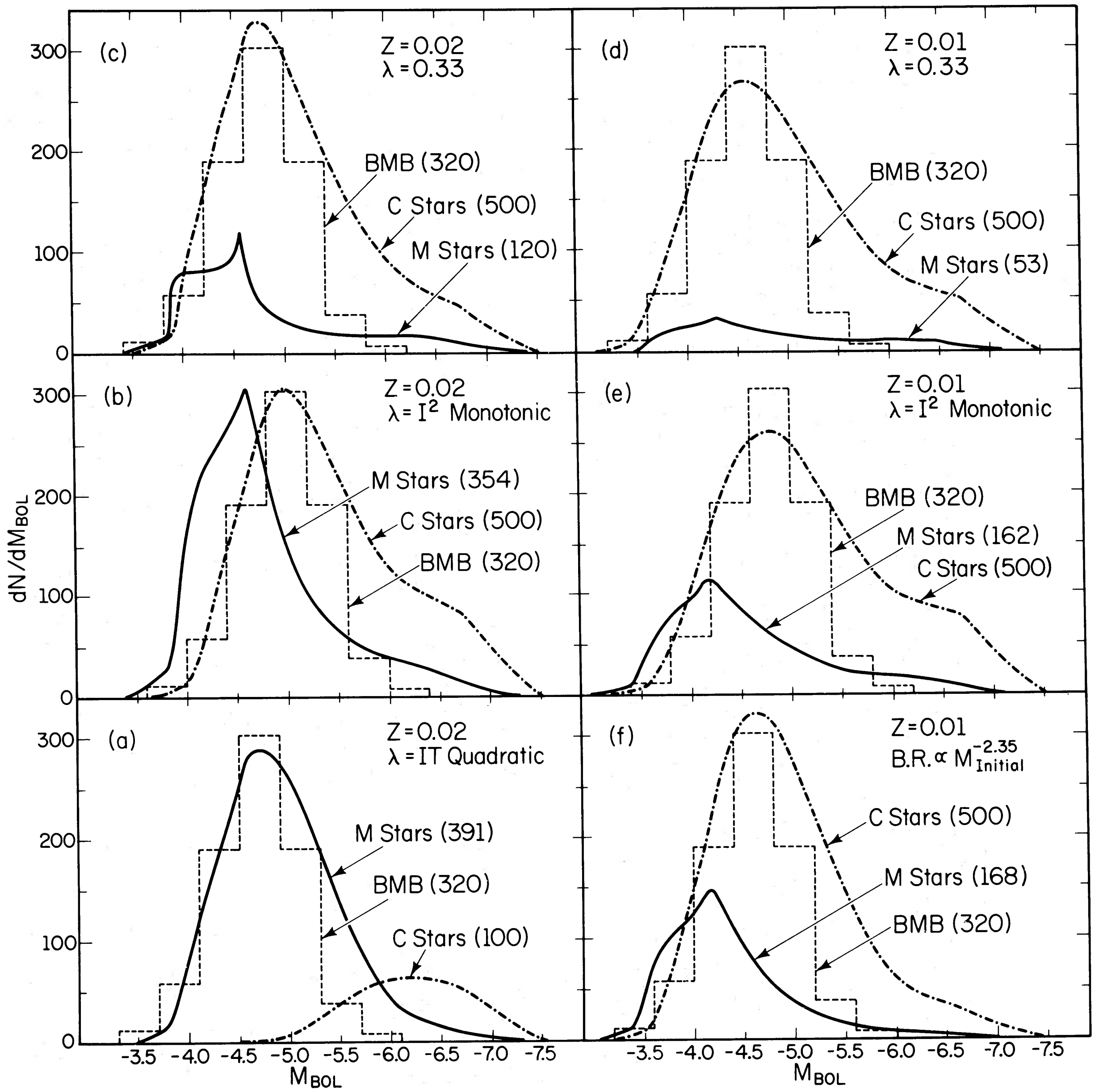


FIG. 3.—Distributions in number versus bolometric magnitude for thermally pulsing M stars and C stars for various dredge-up laws and choices of metallicity. Alphabetic labeling begins in the lower left-hand corner and proceeds in a clockwise direction from *a* through *f*. In segment *a*, $Z=0.02$ and the quadratic IT dredge-up law is assumed. The Blanco, McCarthy, and Blanco distribution for Magellanic Cloud stars in number versus $M(0.81)$ is also shown, but shifted brighter by 0.1 mag. In segment 3*b*, $Z=0.02$ but dredge-up is assumed to occur for all core masses according to the law given by equation (9) in the text. The BMB distribution has been shifted 0.4 mag brighter than given by BMB. In segment 3*c*, $Z=0.02$, but a $\lambda=0.33$ dredge-up law has been assumed. The BMB distribution is shifted by $-0.2$ mag. In segment 3*d*, $Z=0.01$ and $\lambda=0.33$ everywhere. The BMB distribution has not been shifted. Segment 3*e* is the same as segment 3*b*, except that $Z=0.01$. The BMB distribution has been shifted by $-0.2$ mag. Segment 3*f* is the same as segment 3*e*, except that the birthrate function has been chosen as $S(M_0)=M_0^{-2.35}$. The BMB distribution has not been shifted.

normalized to 500 C stars, and the observed BMB distribution has been shifted another 0.3 mag brighter so that maxima in both observed and theoretical C star distributions coincide. The most important feature in Figure 3*b* to be emphasized is that, although there is now a considerable resemblance between theoretical and observed C star distributions, the theoretical C star distribution extends to substantially brighter magnitudes (over 20% are brighter than $M_{\rm Bol}=-6.0$) than does the observed one [no stars with $M(0.81)<-6.0$ and less than 6% of the entire distribution with $-6.0<M(0.81)<-5.2$].

One possible way of remedying this last deficiency might be to increase the extent of dredge-up at low core mass relative to that at large core mass. Distributions that result from choosing $\lambda=0.33$ for all core masses are shown in Figures 3*c* and 3*d*. The effect of decreasing the opacity parameter from $Z=0.02$ (Figure 3*c*) to $Z=0.01$ (Figure 3*d*) is also demonstrated. In neither theoretical instance is the excess frequency of bright C stars ($M_{\mathrm{Bol}} \leq -6$) relative to the frequency of bright stars in the observed C star distribution significantly reduced, although the peak in each theoretical C star distribution is somewhat more consistent with that in the observed distribution (peak at $M_{\mathrm{Bol}} \sim -4.7$ for $Z=0.02$ and at $M_{\mathrm{Bol}} \sim -4.6$ for $Z=0.01$). This is not of high significance, particularly since the observed magnitudes have not been converted into bolometric magnitudes. Richer (1981) estimates bolometric corrections for 71 LMC carbon stars in the BMB fields and obtains a distribution that peaks near $M_{\mathrm{Bol}} \sim -5.0$. This, then, would tend to argue (but weakly) against the $\lambda=0.33$ dredge-up law.

The major qualitative effect of using the $\lambda=0.33$ dredge-up law is to substantially reduce the ratio of thermally pulsing M stars to C stars (a ratio of about 1 to 4 for $Z=0.02$ and of about 1 to 10 for $Z=0.01$). This is not necessarily totally undesirable, since in any observed distribution of M stars there will be a contribution from AGB stars that have not yet begun to pulse thermally. This contribution may be considerable at magnitudes near and below the faint end of the distribution defined by thermally pulsing AGB stars (see Becker and Iben 1980 for relative lifetimes of prepulsing and pulsing AGB stars). However, more than a factor of 2 contribution by prepulsing stars to the M star distribution at magnitudes above the location of the faintest thermally pulsing AGB stars seems excessive. Since a value of $Z=0.01$ is probably more appropriate than $Z=0.02$ for stars in the Magellanic Clouds, the very small number of M stars (that are as bright as C stars) relative to the number of C stars in the $\lambda=0.33$, $Z=0.01$ case, even on taking into account the contribution of the prepulsing M stars, would seem to argue against a dredge-up law as adventuresome as $\lambda=\text{const.} \approx 0.33$.

As a next approximation, distributions have been prepared for $Z=0.01$ (more appropriate for the Clouds than $Z=0.02$) using the more conservative dredge-up law defined by equation (9). The results are shown in Figure 3*e*. The ratio of C stars to thermally pulsing M stars in the theoretical distributions is now about 3 to 1 and, allowing for a doubling of the bright M star population by contributions from prepulsing stars, the theoretical ratios are not inconsistent with the overall ratio in the BMB sample.

The effects of lowering $Z$ (compare Fig. 3*b* with Fig. 3*e* or Fig. 3*c* with Fig. 3*d*) can be easily understood. For the lowest mass stars that contribute to the distributions, initial core masses are smaller for smaller $Z$. Hence, the faint edge of the combined M star and C star distributions becomes dimmer (by about 0.4 mag) as $Z$ is lowered from 0.02 to 0.01. Since (by assumption) the absolute initial abundances of both $^{12}\mathrm{C}$ and $^{16}\mathrm{O}$ are smaller for smaller $Z$, the dredge-up of $^{12}\mathrm{C}$ leads to an excess of $^{12}\mathrm{C}$ over $^{16}\mathrm{O}$ sooner in the low $Z$ models. This is why, all other things being equal, the ratio of C stars to thermally pulsing M stars is larger by about a factor of 2 in the smaller $Z$ cases and why the peak in the C star distribution occurs at a dimmer magnitude (by about 0.2 mag) in the smaller $Z$ cases. The effect of altering $Z$ on the dredge-up law has not been taken into account. Wood and Zarro (see Wood 1980) have shown that, at least for stellar masses substantially larger than the core mass, the theoretical minimum core mass for dredge-up decreases with decreasing $Z$. However, the dependence appears to be far too modest to account for the dramatic difference in the incidence of C stars in the Clouds relative to their incidence in the Galaxy and for the difference in the incidence of C stars in the direction of the Galactic center as opposed to their incidence in the direction away from the center (Blanco, Blanco, and McCarthy 1978).

## V. WHAT'S MISSING IN THE THEORY?

The observational evidence appears to demonstrate unequivocally that carbon is brought to the surface of AGB stars of small core mass (when $Z \lesssim 0.01$) and the theoretical evidence appears almost as unequivocally to show that simple dredge-up—the process that convectively transports freshly synthesized carbon outward during pulse power-down—is confined to AGB stars of large core mass.

What is being overlooked in the theoretical calculations? One possibility is that *convective overshoot* may be of vital importance during pulse power-down. In the explicit case of a C-O core of mass $M_c=0.96\ M_{\odot}$, overshoot is known to occur at the base of the convective region actively involved in dredge-up (Iben 1976), but its occurrence does not significantly alter the extent of the outward mixing of freshly produced matter that is found when overshoot is neglected and the base of the fully convective envelope is allowed to advance inward in mass according to standard criteria. The major effect of overshoot in this case is to spread out the composition profile at the inner edge of the inward-advancing convective region until overshoot ceases. However, the conditions near the base of the convective region in stars with much smaller cores may be significantly different from those prevailing in the case of large core masses. In particular, the width of the composition profile at the base of the inward-moving convective region during pulse power-down may increase with decreasing core mass to such an extent that, ultimately, this width accounts for most of the dredge-up of freshly processed matter.

The recent calculations of Wood and Zarro (1981) suggest that, surprisingly enough, some of the problem may be associated with the treatment of convection in the outer layers of the star. They find that the threshold mass for dredge-up ($M_c^{\min}$) is a function both of the mass of the envelope above the core ($M_e$) and of the value of the ratio of mixing length to pressure scale height ($l/H$) chosen in the mixing-length treatment of convection in the sense that $M_c^{\min}$ decreases both with increasing $M_e$ or increasing $l/H$. Unfortunately, for "reasonable" values of $l/H$, the minimum values of $M_c^{\min}$ achievable are still too large, by a significant margin, to account for the BMB observations. However, the demonstrated sensitivity to $l/H$ means that $M_c^{\min}$ is sensitive to the entropy distribution in the outer parts of the envelope, and this fact permits one to hope that a more sophisticated treatment of convection might give a quite different result for $M_c^{\min}$ than is given by the mixing-length treatment when $l/H$ is restricted to "reasonable" values.

Another possible deficiency in the theory lies in the estimation of pulse strength. In AGB stars of large core mass, the extent of simple dredge-up is directly related to the strength of the thermal pulse which precedes it. That is, the larger the rate at which energy is injected into the He-C region below the H-He envelope during the peak of a pulse, the greater the store of excess energy that becomes available during pulse power-down to drive fresh carbon outward by convection. When this store is below a critical threshold value, as during the initial buildup of pulse strength in AGB model stars of large core mass or as during pulses even of limiting strength in AGB model stars of small core mass, dredge-up does not extend inward as far as the region of freshly synthesized carbon.

In real stars of small core mass, however, it would appear that pulse strength may in fact exceed the critical value. One way to increase pulse strength above the limiting value calculated for a specific fuel is to change the nature of the fuel (e.g., Iben 1976; Despain and Scalo 1976). If, for example, the cross sections for the relevant helium-burning reactions were reduced by many orders of magnitude, the onset of the thermal pulse would be delayed to higher densities and temperatures, with the consequence that the limiting pulse amplitude would be increased. Or, if the neutrino loss rates involving annihilation of real or virtual electron-positron pairs were to be increased (by, say, neutral-current contributions), the onset of the thermal pulse could be delayed to higher densities, again with the consequence of increasing the violence of the thermal pulse.

Finally, if somehow hydrogen were introduced into the He-C convective shell during the pulse, the ignition of the $^{12}C(p, \gamma)^{13}N(\beta^+\nu)^{13}C$ and subsequent reactions could inject significant energy over and above that being supplied by the helium-burning reactions.

It is this latter possibility which has intrigued theoreticians for over a decade but which has eluded attempts to explore it in a quantitatively satisfactory way. In the very first study of thermal pulses occurring in a model of small core mass, Schwarzschild and Härm (1965, 1967) found that, during one pulse, the outer edge of the He-C convective shell actually engulfed a region near the base of the hydrogen-rich envelope. However, subsequent calculations have not reproduced this behavior (Sweigart 1973, 1976), and it has been shown (Iben 1976) that it is the neglect of radiation pressure in the Schwarzschild and Härm calculations that is partly responsible for the apparent discrepancy. Radiation pressure contributes significantly to an "entropy barrier" that prevents contact between the He-C convective region and the outer convective envelope. Nevertheless, in all thermal pulse calculations, the outer edge of the formal He-C convective shell comes tantalizingly close (within a fraction of a scale height) to the base of the H-He profile that is defined by hydrogen-burning reactions during the interpulse quiescent phase. It does not seem unreasonable to suppose that real convective motions, as opposed to those calculated formally, extend (as precursors) into a portion of the H-He profile.

Ulrich and Scalo (1972) and Scalo and Ulrich (1973) have speculated that contact between the outer edge of the He-C convective shell and hydrogen-rich matter will lead to the formation of convective plumes. They envision that very little hydrogen will diffuse into the He-C convective shell ($X_H < 10^{-4}$), being prevented by "buoyancy" forces energized by the $^{12}C(p, \gamma)^{13}N(\beta^+\nu)^{13}C$ reactions, but that carbon will be efficiently carried outward by the plumes, also fueled by the $^{12}C(p, \gamma)^{13}N(\beta^+\nu)^{13}C$ reactions. Thus, in the Ulrich-Scalo (US) scheme, carbon is brought to the surface during the height of a thermal pulse rather than during pulse power-down. One troublesome feature of the US scheme is that, as a consequence of burning in plumes, the surface ratio of $^{12}C$ to $^{13}C$ drops to a value of 6 or below, once C/O exceeds unity (see Fig. 10 in Scalo and Ulrich 1973). Such low ratios do not appear to be the rule in LMC C stars (Richer 1981).

In model accreting white dwarfs (Fujimoto 1977) and in model stars that have left the AGB to approach the white-dwarf sequence (Schönberner 1979), the entropy barrier inhibiting contact between envelope convection and shell convection is, in some instances, sufficiently small that explicit contact is made between the hydrogen-rich envelope and the shell convective region. Sugimoto and Fujimoto (1980) find that, in these instances, large quantities of hydrogen are mixed into the outer portion of the He-C convective region before the onset of the hydrogen-burning thermal runaway prevents further inward mixing. Plumes do not occur. Instead, hydrogen-burning causes a new H-He-C convective shell of higher entropy to form at the outer edge of the original He-C convective shell. Sweigart (1973) finds the same phenomenon in an AGB star of low core mass when hydrogen is artificially inserted at a sufficiently rapid rate into the He-C convective shell.

An amalgam of all of these processes may be occurring in AGB stars of low core mass, thanks, perhaps, to some form of overshoot at and beyond the formal outer edge of the advancing He-C convective shell. As the outer edge of the He-C convective shell approaches the H-He profile, some hydrogen may be mixed deeply enough into the convective shell to initiate a hydrogen-burning thermal runaway. Energy released by the $^{12}C(p, \gamma)^{13}N(\beta^+\nu)^{13}C$ reaction could then force the convective shell to grow even larger, engulfing more hydrogen, which would then be swept downward to enhance the runaway. The entropy of the convective region at and beyond the point of most intense hydrogen-carbon burning would rise to a value intermediate between that in what is left of the He-C convective shell and that in the envelope. The two convective shells would thus become separated by a narrow radiative region.

The entropy barrier between the H-He-C shell and the envelope may not be destroyed by this process but may act only to confine inward mixing to that hydrogen which was once contained in the inner portion of the hydrogen-helium profile existing at the start of the thermal pulse. The mass in this entire profile is typically about 1% of the mass in the convective shell. An upper limit to the energy released by the $^{12}C(p, \gamma)^{13}N(\beta^+\nu)^{13}C$ reactions can therefore be estimated as

$$\Delta E_H \lesssim 10^{-2} \frac{X_H 9.5 \text{ MeV}}{(\Delta X_{12}/12) 7.3 \text{ MeV}} \Delta E_{He}, \qquad (10)$$

where $\Delta X_{12}$ is the abundance by mass created by the $3\alpha$ process during a given pulse, $X_H$ is the average abundance by mass of the ingested hydrogen, and $\Delta E_{He}$ and $\Delta E_H$ are the energies released via helium and hydrogen burning, respectively, during the double pulse. With $\Delta X_{12} \sim 0.25$ and $X_H \sim 0.35$, $\Delta E_H < 0.2 \Delta E_{He}$. Thus, the energy released by the hydrogen-burning reactions is not expected to significantly increase the total nuclear energy that would have been released if hydrogen had not been mixed inward. However, the circumstance that this additional energy will be released over a much shorter time span (hydrogen is swept into a region where temperatures are much higher than during quiescent burning) may lead to conditions that promote the dredge-up of fresh carbon during pulse power-down.

Whatever the details, mixing by plumes during a pulse or mixing by dredge-up during pulse power-down, the transport of freshly synthesized carbon to the surface in AGB stars of small core mass would appear to be enhanced by the burning of carbon with hydrogen that follows if hydrogen is mixed into the He-C convective shell. Should it take place, will this mixing occur with the same probability in AGB stars of all core masses? Two factors will favor its operation in stars of small core mass. First, the entropy barrier decreases with decreasing core mass, and one might expect that the fraction of the H-He profile that is mixed into the He-C convective shell increases with decreasing entropy barrier. Second, the smaller the core mass, the less steep the H-He profile becomes, thereby enhancing, in stars of small core mass, initial contact between the outer edge of the convective shell and the inner tail of the H-He profile.

As shown by Sanders (1967) and developed by US and by Ulrich (1973), *s*-process elements will be made by capture of neutrons released by the $^{13}C(\alpha, n)^{16}O$ reaction following the injection of hydrogen into the He-C convective shell. In the past, one weakness of this scheme for the production of the bulk of Galactic *s*-process isotopes has been thought to be the fact that, in contrast to the scheme that is based on the $^{22}Ne(\alpha, n)^{25}Mg$ source of neutrons, a solar-system distribution of *s*-process isotopes requires the injection of hydrogen in "just the right" amount and that achieving "just the right" amount of hydrogen might be a total accident in any given real star. In the light of the observational evidence presented by AGB stars in the Magellanic Clouds, this reservation needs reexamination. Could it be that the "fine tuning" needed to bring about "just the right" amount of injection is related to the fact that the ratio of matter in the H-He profile (matter contained in the hydrogen-burning shell during the quiescent phase) and the matter in the He-C convective shell during a pulse is a near invariant with core mass?

One final comment is in order. Suppose that dredge-up occurs not during the AGB phase, but before, perhaps during the helium-core flash? Here, too, theory makes a definite statement: it does not occur. Fortunately, several simple observational tests can be applied. If the establishment of a surface ratio of C to O which is geater than unity occurs during the core flash, and not after, then either the distribution of thermally pulsing AGB M stars will be essentially identical to the distribution of AGB C stars (if only a fraction of core-flashing stars enjoy the appropriate initial conditions to lead to surface C greater than O) or *all* AGB stars will be C stars. It is most definitely established that there are a multitude of M stars in the AGB phase. From the limited information that has thus far been published, the C star distribution and the bright M star distribution do not in general appear to resemble one another. This is particularly true for distributions in the SMC and in the Galaxy. It is therefore reasonably sure that C star characteristics appear predominantly during the AGB phase, and not before.

Another test has to do with the composition of stars on the horizontal branch, which immediately precedes the AGB phase. If C exceeds O in a goodly fraction of such stars, the necessity of discovering physical processes that produce C star characteristics during the thermally pulsing phase would be reduced. However, the search for C star producing processes that might be active during the core helium-flashing phase would have to be intensified. Whatever the result, the theory of stellar interiors would appear to be deficient.

## VI. WHERE ARE THE MASSIVE C STARS?

In every theoretical distribution that has been described in previous sections, a much larger fraction of the total C star population consists of very bright C stars ($M_{\rm Bol} < -6$) than is the case in the observed BMB and Richer (1981) distributions, which are consistent with *no* C stars with $M_{\rm Bol} \lesssim -6$! The seriousness of the problem cannot be overemphasized. On the one hand, the production of *s*-process elements in the solar-system isotopic distribution and the phenomenon of simple dredge-up are theoretically well established for stars with large cores corresponding to $M_{\rm Bol} < -6$. On the other hand, our understanding of the rate at which mass is lost from such stars via a wind and our understanding of the precise conditions that lead to the ejection of a planetary nebulae is not at all as well developed.

If we interpret the absence in the observed BMB distribution of stars with $M_{\rm Bol} < -6$ as an indication that stars initially more massive than, say, $M_0 \sim 2.5-3.0$ eject a planetary nebula or suffer total envelope evaporation via a wind before they develop carbon-star characteristics, we contribute some insight into one set of theoretical questions (e.g., Tuchman, Sack, and Barkat 1978, 1979). However, we also eliminate the one stellar source thus far discovered that can produce *s*-process isotopes in the solar-system distribution in quantities that meet the requirements of Galactic nucleosynthesis —*provided* that the frequency of such sources is, over the long haul, roughly consistent with a Salpeter-like birthrate function plus the assumption that such stars survive until they have evolved to magnitudes substantially brighter than $M_{\rm Bol} \sim -6.5$.

One possible way of alleviating the problem is to assume that the birthrate function that currently characterizes star formation in the Magellanic Clouds is anomalous relative to the time-averaged birthrate function that has characterized star formation in our Galaxy during its major period of nucleosynthesis. Recently, Dennefeld and Tammann (1980) have interpreted the analysis of Cepheid distributions in the Magellanic Clouds by Becker, Iben, and Tuggle (1977) to mean that, in the LMC, the birthrate of intermediate-mass Cepheids ($4 \lesssim M_0 \lesssim 9$) is roughly 4 times smaller and, in the SMC, is roughly 2 times smaller than expected on the basis of an interpolation between the birthrates of more massive stars and the birthrates of less massive stars in the two Clouds. A simple way to implement this inference in the construction of M star and C star distributions is to adopt a birthrate function that is appropriately steeper than given by expression (4). The choice of $S(M_0) = M_0^{-2.35}$ alters the distributions shown in Figure 3*e* to those shown in Figure 3*f*. The fraction of C stars with $M_{\rm Bol} < -6$ has been reduced from 17% ($S = M_0^{-1.35}$) to 7% ($S = M_0^{-2.35}$). However, 7% of the BMB sample is 22 stars, and so the mystery of the missing bright C stars has not been solved by this expedient.

Relative to the entire Magellanic Cloud stellar population, the number of stars in the BMB fields is small indeed. Furthermore, the BMB fields contain no Cepheids or M supergiants; they have in fact been selected for their "quietness" or lack of evidence for recent star formation. Since the maximum main-sequence lifetime of a Cepheid progenitor is on the order of a few times $10^8$ yr, one might interpret the absence of Cepheids in the BMB fields to mean that these fields contain no stars younger than this limit. Once we accept the possibility of a lower limit to the age, it is reasonable to argue that the true limit corresponds to the total lifetime of a star that will terminate its existence on the AGB (either via a wind or via the ejection of a planetary nebula) just before it reaches $M_{\rm Bol} \sim -6$. By adopting the birthrate function of equation (4), but cutting it off at a mass $M_0 \sim 2.5$ (corresponding to a total lifetime of $\sim 10^9$ yr), it is possible to reproduce reasonably well the C star distribution in the BMB fields (see Iben 1981).

For the Clouds as a whole, however, an age cutoff cannot be invoked and the problem of the missing C stars remains. There are at least 2000 known Cepheids in the Clouds, and it is estimated that the total number of Cepheids is nearer 4000 than 2000. Most of these Cepheids are intermediate-mass stars (masses $M_0 \gtrsim 3$), and many of them should become thermally pulsing AGB stars with *initial* magnitudes brighter than $M_{\rm Bol} \sim -6$. The typical total lifetime of a parent star in this mass range is between a few times $10^7$ yr and a few times $10^8$ yr. The time that an intermediate-mass star spends as a Cepheid is on the order of $10^6$ yr, and, since the typical theoretical lifetime of an AGB star with an intermediate-mass progenitor is also on the order of $10^6$ yr, the number of bright AGB stars in the Clouds should be comparable to the number of Cepheids, namely about 4000. In contrast, on the basis of studies such as those of Richer, Olander, and Westerlund (1979), Richer (1980) estimates that, in the Clouds, there are no more than a *total* of a half dozen C stars as bright as $M_{\rm Bol} \sim -6.5$. The evidence appears to show unambiguously that many intermediate-mass stars simply either do not become C stars or do not remain as such once they exceed $M_{\rm Bol} \sim -(6-6.5)$ in brightness.

Does the assumption of a birthrate function that is constant in time seriously overestimate the number of bright AGB stars to be expected in the Clouds? One may use the Cepheid and C star statistics to argue that this is not the case. BMB estimate a discovery rate of about 500 C stars per square degree. This translates into roughly 35,000 ($=500 \times 72$) C stars in the LMC and a comparable number in the SMC. Crudely accounting for the dropoff in frequency of C stars toward peripheral regions of the Clouds, one may guess a total C star population of perhaps 50,000. Since there are perhaps 4000 Cepheids, there is approximately one Cepheid progenitor of a bright AGB star ($M_{\rm Bol} < -6$) for every 10 C stars of typical magnitude ($M_{\rm Bol} > -6$). The analysis presented in this paper predicts that there should be one Cepheid (one bright C star) for every six ordinary C stars if $S(M_0) = M_0^{-1.35}$ or one Cepheid for every 14 ordinary C stars if $S(M_0) = M_0^{-2.35}$. Since steepening the mass dependence of a time-independent birthrate function has the same effect as using a birthrate function that decreases with time, the apparent lack of bright C stars can therefore not be attributed to a birthrate function which decreases significantly over the age of the Clouds.

Are perhaps the estimates of bolometric magnitude significantly in error? In none of the extant estimates of bolometric luminosity has the possibility of circumstellar reddening been taken into account. Perhaps C stars more luminous than $M_{\rm Bol} \sim -6$ become enveloped in a cloud rich in carbon grains that are at a sufficiently high number density to redistribute most of the near-infrared radiation emitted by the central star into the far infrared, where it escapes detection by the BMB and related near-infrared sensitive detection schemes. Cahn (1980) has estimated that the brightest carbon stars in the Magellanic Cloud cluster samples of Mould and Aaronson (1980) and of Frogel, Persson, and Cohen (1980) are reddened considerably more than would be expected if the reddening originated solely in interstellar matter. The reddening appears to increase with increasing luminosity, and Cahn suggests that the bulk of the reddening originates in a circumstellar shell of matter in which the number density of carbon grains increases with the luminosity of the central star.

Qualitatively, this picture is extremely attractive. First, due to dredge-up, the more bolometrically luminous the central AGB star becomes, the richer in carbon the matter at its photosphere should become. Second, the bolometrically more luminous the central star becomes, the more rapidly it should lose mass, forming a denser circumstellar shell. Finally, the bolometrically more luminous the central star becomes, the cooler its photosphere and the lower the mean temperature in its circumstellar shell should become. All three phenomena should enhance (selectively in bolometrically brighter stars) the rate of formation of carbon-rich grains and the redistribution of near-infrared radiation emitted by the central star into the far-infrared.

Gallagher (1980) has suggested that some of the far-infrared sources in the AFCRL survey (Lebovsky and Rieke 1977; Gehrz, Hackwell, and Briotta 1978; Cohen 1978) may be Galactic analogs of the missing bright carbon stars in the Magellanic Clouds, and it is to be hoped that an attempt will be made to search for such sources in the Clouds. If the stellar distribution in the BMB fields were representative of the Clouds as a whole, rather than being modified by an age cutoff, and if the solution just described were correct, one might anticipate finding roughly 30 of these sources in the BMB fields. The absence of Cepheids (there should also be roughly 30 of these) in the BMB fields suggests that one should concentrate on the bright stars in more active regions.

Two recent studies of stars in active regions (Glass 1979; McGregor and Hyland 1980) indicate that AGB stars as bright as $M_{\rm Bol} \sim -7$ may occur. However, the suspected AGB stars appear not to exhibit C star characteristics, and it is not clear that one can distinguish unambiguously between an AGB star originating from an intermediate-mass star and a supergiant (of the same magnitude) originating from a more massive progenitor. If further exploration permits one to resolve the ambiguity and if the bright AGB stars turn out not to be C stars, one may infer at once that either the dredge-up mechanism does not operate in AGB stars of large core mass or that $^{12}$C is converted first into $^{13}$C and thence into $^{14}$N at least as rapidly as it is dredged up.

The first alternative would mean that *s*-process elements, though made in the stellar interior, are not brought to the surface and then injected into the interstellar medium. It would mean, further, that even the model calculations for stars with large masses which show that the dredge-up mechanism works for them with great facility are completely wrong. The second alternative would preserve intermediate-mass stars as major sources of *s*-process isotopes and would not require serious modification of theoretical models. This is because, in a sufficiently bright and massive AGB stellar model, an increase in $l/H$ leads to an increase in the average temperatures in the convective envelope and consequently to an increase in the rate at which carbon is converted into nitrogen. This effect, evident in many calculations (e.g., Uus 1970; Iben 1975), has been exploited briefly by Becker and Iben (1980) and very extensively by RV, who derive conversion rates for a wide range of core masses, total masses, and values of $l/H$. A distinction between the two alternatives obviously requires estimates of the CNO abundances and of the $^{12}$C/$^{13}$C ratio in bright, red stars in the Clouds that have a bolometric magnitude in the range $M_{\rm Bol} \sim -7 \pm 0.5$. If many of these stars are "N stars" (N>O>C), then the problem of the missing high-mass C stars may be solved.

## VII. COMMENTS ON CLUSTER AGE ESTIMATES THAT ARE BASED ON PROPERTIES OF THE MOST LUMINOUS CLUSTER MEMBERS

Mould and Aaronson (1979, 1980) and Aaronson and Mould (1980) have made estimates of the ages of several intermediate-age clusters in the Magellanic Clouds and in Fornax, and have placed upper limits on these ages by assuming that the brightest (or only) carbon star in such clusters has essentially completed its AGB phase of evolution. This upper limit might be expected to approach the actual age in a cluster with a well populated AGB that contains many carbon stars, but it is not a very stringent limit when only a few carbon stars occur in the cluster. This is so because the thermally pulsing AGB phase is of such short duration ($\sim 10^6$ yr) compared with the typical pre–AGB phase lifetime ($\sim 2\times 10^7$ yr to $10^{10}$ yr) that there is no way of knowing whether the few carbon stars that are in a given cluster have just developed their carbon-rich characteristics or are just ending the AGB phase. A further complication is that, due to the influence of a previous thermal pulse, there is a distinct probability (20%) that any given carbon star is less luminous than it is during its quiescent phase; it is also quite probable that any given C star is undergoing acoustical pulsations and, at the time of observation, is at something other than its mean (cycle-averaged) magnitude.

Using the dredge-up law of equation (9), and choosing $Y=0.28$, $Z=0.01$, it turns out that the bolometric magnitudes along the appropriate "S star" and "end" curves (see Fig. 2*b*) are related to the pre–AGB lifetime [see the log (Age) vs. log ($M_0$) curve in Fig. 2*b*] by

$$V_E = -5.86 + 1.20 \log t_9,$$

$$V_S \approx V_E + 0.86, \tag{11}$$

where $t_9$ is the total pre–AGB (main-sequence plus core helium-burning phase) lifetime in units of $10^9$ yr. This means that, given the average magnitude $\bar{V}$ (averaged over an acoustical pulsation cycle) of only one carbon star, the best one can say is that the total pre–AGB lifetime of this star is 80% likely to be given by

$$\log t_9 = 0.83(\bar{V} + 5.43) \pm 0.36. \tag{12}$$

The probability is 20% that the age is less than this (by as much as $\Delta \log t_9 \sim 0.4$).

Even if one were fortunate to find a well populated AGB, it is safer to estimate the age using the mean magnitude of the entire population than to do so using the magnitude of the brightest C star. This can be demonstrated by constructing theoretical distributions for C stars and thermally pulsing M stars in hypothetical clusters of various ages. Choosing $Z=0.01$, $Y=0.28$, and the dredge-up law of equation (9), and assuming in every case a spread in stellar formation times equal to $10^8$ yr, the distributions in Figure 4 result. The magnitude at the peak of any C star distribution is roughly halfway between the quiescent extrema $V_S$ and $V_E$ and

$$\log t_9 \sim 0.83(V_{\rm peak} + 5.4). \tag{13}$$

It is clear from Figure 4 that, in a cluster with a very poorly populated AGB, it is much more probable that the magnitudes of the few C stars present are near the peak of the distribution (roughly halfway between $V_S$ and $V_E$) than they are near the bright edge. Hence, the age inferred by assuming that the magnitude of the brightest C star is near $V_E$ tends to be larger than the actual age. Comparing equations (11) and (13), it is seen that the typical overestimate is by roughly a factor of 2.5.

It is interesting that, for the acoustical pulsation amplitude we have chosen, the bright edge of the thermally pulsing M star distribution coincides approxi-

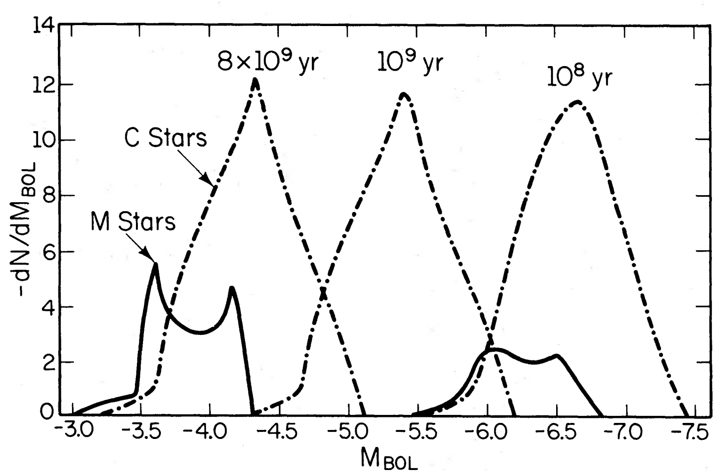


FIG. 4.—Distributions of C stars and thermally pulsing M stars for clusters of different ages. Each C star curve is normalized to 10 stars, and each M star curve is properly scaled to the appropriate C star curve. A spread of $10^8$ yr in formation times has been assumed in each case.

mately with the peak of the C star distribution. Given the paucity of AGB stars in most clusters, a good rule of thumb might be to average the mean (cycle averaged) magnitude of the brightest M star with the mean (cycle and number averaged) magnitude of the C stars and then enter an equation such as equation (13) to estimate age.

## VIII. SUMMARY

The properties of carbon star distributions in the Magellanic Clouds appear to demonstrate that carbon is brought to the surface of AGB stars of low core mass and low metallicity despite theoretical estimates to the contrary. This may mean that convective overshoot plays a prominent part in the operation of the dredge-up phenomenon in such stars.

The apparent near absence of C stars of bolometric magnitude brighter than $M_{\mathrm{Bol}} \sim -6$ indicates that a star of intermediate mass which reaches the AGB with $M_{\mathrm{Bol}} \lesssim 5.5$ (1) spends an order of magnitude less time as an AGB star than is predicted by theory, (2) becomes shrouded by a circumstellar shell that diverts much of the near-infrared radiation emitted by the central star into the far-infrared, or (3) converts carbon into nitrogen in its envelope at least as rapidly as carbon is dredged up into the envelope.

It is a pleasure to thank Steve Becker, Victor Blanco, Julius Cahn, Jay Frogel, Masayuki Fujimoto, Jay Gallagher, Martin McCarthy, Jeremy Mould, John Scalo, Alvio Renzini, Harvey Richer, and Peter Wood for several useful conversations and to thank Drs. Blanco, Frogel, Mould, Renzini, and Richer for providing me with preprints of their recent work on carbon stars in the Magellanic Clouds. I am particularly grateful to the AURA board for giving me the opportunity to visit, as a member of the KPNO/CTIO Visiting Committee, the observatory where the recent exciting discoveries about carbon star distributions have been made. Revisions to the originally submitted manuscript were considerably influenced by discussions held at a workshop on "Physical Processes in Red Giants" that was held in Erice, Sicily, in September, 1980. I thank the participants in this workshop for their contributions.

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