

TECHNETIUM AND MIXING IN SIMULATED POPULATIONS OF DOUBLE-SHELL PHASE STARS¹

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ABSTRACT

Simulated populations of double-shell phase stellar models are compared with recent observations of technetium in red giants. It is assumed that Tc is mixed to the surface during helium shell flashes and that mixing commences at some minimum core mass $M_c^{\min}$ and ceases for core masses greater than some value $M_c^{\max}$. The results show that the observed fraction of M giants in which Tc is present can be matched only if mixing begins at small core masses ($M_c^{\min} \lesssim 0.6\text{--}0.7 M_\odot$) in stars of small total mass ($\sim 1 M_\odot$). The observed correlations of Tc with spectral type can be reproduced by mixing during helium shell flashes, but only if $M_c^{\min} \lesssim 0.7$, consistent with the constraint imposed by the fraction of M stars with Tc present. These results are in agreement with independent arguments based on the observed masses and luminosities of peculiar red giants. The surprisingly large fraction ($\sim 30\%$) of S and N stars which exhibit *s*-process enhancements but no detectable Tc implies either that mixing of *s*-process material does not occur in consecutive shell flashes (i.e., large numbers of flashes must be skipped), or that mixing of *s*-process elements occurs during consecutive flashes but ceases for core masses greater than $M_c^{\max} \sim 0.8 M_\odot$. None of these results agree with current stellar evolution calculations, indicating that the processes by which red giants are mixed are not yet understood. Areas for future theoretical research are suggested.

Subject headings: convection — stars: abundances — stars: interiors

I. INTRODUCTION

It is well known that several types of red giant stars possess anomalous chemical compositions with respect to “normal” K and M giants. The anomalies found in these stars (hereafter referred to collectively as *peculiar red giants* or *P giants*) include C/O abundance ratios in excess of the solar value, low $^{12}\text{C}/^{13}\text{C}$ ratios, and enhancements of *s*-process elements and lithium, although not all anomalies occur in all types of P giants. Based on comparison of observed temperatures and luminosities with evolutionary tracks (Scalo 1973*a*, 1976*b*), there is a strong suspicion that mixing occurs during (or earlier than) helium shell flashes for cool P giants of types MS, S, SC, and N in our galaxy. This conclusion for the N stars is supported by the LMC data of Crabtree, Richer, and Westerlund (1976) and Richer, Olander, and Westerlund (1979), which do not contain significant uncertainties in the relative luminosities. The fact that current stellar evolutionary models for masses consistent with the observed properties of P giants (Scalo and Miller 1979, hereafter SM) are so far unable to account for these abundance anomalies indicates that our knowledge of advanced evolution of low-mass stars is seri-

ously incomplete. It is the purpose of the present paper to use recent observations of technetium line strengths in cool giants by Little-Marenin and Little (1979, hereafter LL) to shed some light on the theoretical problems which require investigation.

The presence of Tc in red giants, particularly M giants (Merrill 1952; see LL for further references), provides a unique discriminant of evolutionary mixing models for a number of reasons.

1. All isotopes of Tc have decay lifetimes less than 3×10^6 yr and the isotope produced by the *s*-process, ^{99}Tc , has a half-life of 2×10^5 yr. Therefore, observation of Tc in a star gives evidence concerning mixing which occurred recently, and we can be confident that the Tc is not a relic from some previous phase of evolution. Since Tc is found in red giants of types M, MS, S, and N, all of which can be identified with the double-shell phase of evolution on the basis of their luminosities and temperatures, we can infer that Tc production and mixing is related to helium shell flashes.

2. Technetium is found in “normal” M giants. Mixing of a *stable s*-process element into a red giant atmosphere would go unnoticed until the mass of the element mixed roughly exceeds that already in the envelope, so the unstable Tc serves as a probe of mixing in stars which otherwise would be thought unmixed, in much the same way as certain molecular band strength ratios can

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be used to indicate variations in the C/O ratio in M giants (Scalo 1974).

3. Although M giants can be segregated spectroscopically according to relative effective temperatures, temperature classification is much more difficult for S stars (Ake 1979; Piccirillo 1980) and N stars (Scalo 1973*b*). Therefore, Tc can be used to investigate correlations of mixing with effective temperature, a property which we exploit below. In addition, from the known correlation of dissociation energy with position in the periodic table (Tsuji 1973), it is likely that the TcO molecule does not lock up much Tc in the atmospheres of cool stars, and so Tc line strengths should not be influenced by the unknown C/O ratio.

Our goals are the answers to the following three questions: (1) Can the hypothesis of mixing during helium shell flashes account for the observed correlation between Tc line strength and spectral type in M giants? (2) How early during double-shell phase evolution (i.e., by what core mass value) must mixing occur in order to explain the observed fraction of M giants which have undergone mixing as judged by the presence of Tc lines? (3) What is the significance of the absence of Tc in stars which are known to possess enhancements of other s-process elements?

Our approach involves computed simulations of the evolution of a large sample of stars. This technique, which we refer to as *population simulation*, consists of creating a sample of model stars in the relevant evolutionary stage by randomly choosing a mass from an initial mass function and an age from a stellar birthrate history. The present evolutionary phase of the model star is identified knowing its mass and age, and the star is rejected if it is not in the phase of interest. This process is repeated until a desired number of model stars is accumulated. The properties of each model star are then calculated using analytic expressions which accurately represent the results of stellar evolution calculations, or, in the areas where stellar evolution calculations are uncertain (mixing, mass loss), properties of model stars are parametrized. With this information, statistical properties of the sample as a whole are obtained, such as distribution functions of luminosities, temperatures and abundances, as well as correlations between these properties. Since population simulation predicts the properties of a group of stars, more observational constraints can be applied to constrain theories than in the case of single stars. Data for large samples of giant stars have become available because of advances in instrumentation over the last decade, with the result that a population simulation offers a powerful tool for the study of stellar evolution which is not afforded by theoretical models of single stars. Previous work in this direction can be found in Scalo (1977) and Scalo and Miller (1979, 1981).

In § II we give details of the simulations, including prescriptions we have adopted for evolution, mass loss, and mixing during helium core flashes, and give the relation between Tc abundance and core mass for various total masses. Results of the simulations for a range of parameters are presented in § III, and our conclusions are discussed and summarized in § IV.

II. METHOD OF CALCULATION

a) Description of Double-Shell Phase Evolution

Shortly after stars exhaust helium in their cores, nuclear burning moves to a thin helium shell surrounding an inert carbon-oxygen core. The core mass gradually increases as matter is processed through the shell source. Overlying the helium shell is a thin hydrogen-burning shell; the two shell sources are separated by an inert radiative "buffer zone." The ensuing evolution is referred to as the "double-shell phase" (DSP, sometimes called the asymptotic giant branch). The helium shell undergoes a periodic, short-lived instability, during which the nuclear burning rate increases dramatically, and a convection zone advances through the buffer zone, approaching the hydrogen shell. The detailed behavior of these helium shell flashes for models of various total masses and core masses has been studied in a number of papers; for references, see Sackmann (1980).

An important feature of DSP evolution is that the variation of the mass of the intershell buffer zone, the time between shell flashes, the stellar luminosity, and other properties are primarily functions of the core mass M_c , with only a weak dependence on total stellar mass and chemical composition. There are, however, two DSP parameters which do depend on total mass. First, the luminosity, or core mass, M_c^0 , at which DSP evolution begins is an increasing function of total mass; the core mass at the beginning of DSP evolution varies from about $0.5 M_\odot$ at $1 M_\odot$ to $0.95 M_\odot$ at $7 M_\odot$. Second, because of steady mass loss, the final luminosity, or final core mass, M_c^f , at which a star terminates its red giant evolution (leaving a remnant or exploding as a supernova) is also an increasing function of total mass. For example, using typical mass loss rate parametrizations (see below), a $1 M_\odot$ star only attains a core mass of about $0.6 M_\odot$ and a $2 M_\odot$ star reaches a core mass of about $0.9 M_\odot$, while stars more massive than about $4 M_\odot$ are able to reach the limiting (or Chandrasekhar) core mass of $1.4 M_\odot$. All these features play an important role in the calculations presented here.

The formulae used for calculating DSP evolution are given below. These are used in the simulation calculations described in § II*d*. These formulae, taken for the most part from Iben and Truran (1978, hereafter IT), are a fit to the results of detailed stellar evolution

calculations by many authors. The formulae are uncertain because: (1) they involve interpolation and extrapolation into ranges of core mass which have not been explored in detail; and (2) no published results have found mixing for core masses $M_c \lesssim 0.8 M_\odot$, while several independent arguments, including the present work, strongly suggest that mixing occurs much earlier in a star's evolution, and it is not known how the mixing affects subsequent evolution. Given these uncertainties, we assume that the adopted analytic expressions are a reasonable representation of DSP evolution.

The core mass M_c^0 at the beginning of DSP evolution is

$$M_c^0 = 0.95 + 0.075(M_i - 7), \quad (1)$$

where M_i is the initial total mass. (All masses are given in units of $M_\odot$.) The surface luminosity L (in units of $L_\odot$) is

$$L = 63400(M_c - 0.44)(M/7)^{0.19}, \quad (2)$$

where M_c and M are, respectively, the current core mass and current total mass. The luminosity of the hydrogen-burning shell L_H is

$$L_H = L - 2000(M/7)^{0.19} \exp[3.45(M_c - 0.96)]. \quad (3)$$

The period between successive flashes is

$$t_{\text{IF}} = 1.0 \times 10^{10} \Delta M_c X / L_H \text{ yr}, \quad (4)$$

where ΔM_c is the increase in core mass during the interflash period (see eq. [7] below), and X is the mass fraction of hydrogen at the surface. The relation between effective temperature, T_{eff} , and other parameters was taken from Becker and Iben (1979):

$$\log T_{\text{eff}} = 3.515 - 0.0643[\log(L) - 4.5] + 0.120 \log(M/7) - 0.0440 \log(Z/0.02) - 0.0431 \log(Y/0.28), \quad (5)$$

where Y and Z are the surface mass fractions of helium and metals, respectively, at the surface. These theoretical temperatures are uncertain because they depend sensitively on the treatment of convection in the upper layers of the envelope (i.e., in the most widely used formulation, the ratio of mixing length to pressure scale height) and the treatment of the radiative atmospheric structure and opacity (see Scalo and Ulrich 1975). The corresponding formula given by Scalo (1976*a*), which is a fit to the evolutionary tracks of Scalo, Despain, and Ulrich (1975) calculated with a more detailed treatment of the atmosphere, gives about the same dependence on L , M , and Z . Although the absolute calibration of effective temperatures is therefore uncertain, we have some confi-

dence that the functional dependence of effective temperature on luminosity and mass is more trustworthy and, hence, that *relative* effective temperatures are reasonably well determined. We shall refer to theoretical effective temperatures as "model temperatures" to emphasize the uncertainty in the absolute calibration.

b) Prescriptions for Mixing and Mass Loss

During helium shell flashes, a significant quantity of triple- α carbon is deposited in the buffer zone between the shells, and it is believed that the mixing of this carbon into the convective envelope is responsible for the enhanced carbon abundances in the cool S and N stars. These stars can also be identified with DSP evolution on the basis of their luminosities and temperatures (Scalo 1973*a*, 1976*b*; Crabtree, Richer, and Westerlund 1976; Richer, Olander, and Westerlund 1979). It is likely that *s*-process nucleosynthesis and mixing also occur during DSP evolution. Either the helium shell may become so hot that $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$ releases neutrons to fuel the *s*-process (Iben 1975), or the intershell convection zone may reach the base of the overlying hydrogen shell, allowing the $^{12}\text{C}(\rho, \gamma)^{13}\text{N}(\beta^+ \nu)^{13}\text{C}(\alpha, n)^{16}\text{O}$ neutron-producing chain to operate. The former process is only likely to occur in fairly massive ($\gtrsim 5 M_\odot$) stars (Iben 1975), while the ^{13}C source probably occurs only in low-mass stars (see Sackmann 1980), if it occurs at all.

A major uncertainty concerns the physical mechanism by which the processed material in the intershell region is mixed to the surface. There are at least two possibilities which have been suggested. Iben (1976) has shown that, after the peak of the flash, when the hydrogen shell is extinguished, the base of the outer convective envelope can dip down into the intershell zone, dredging up processed material. Unfortunately, this dredge-up mechanism has only been found to occur for core masses greater than about $0.8 M_\odot$ (Paczynski 1977), while there is strong evidence from luminosities (Scalo 1976*b*; Crabtree, Richer, and Westerlund 1976; Richer, Olander, and Westerlund 1979; Richer 1980), masses (SM), and the incidence of Tc lines (this paper) that the mixing must occur by the time the core mass reaches 0.5 – $0.7 M_\odot$. Alternatively, the convection zone driven by the shell flash may reach the hydrogen shell, driving some form of convection into the envelope. Although such contact between the convective shell and the hydrogen-rich layers has not been definitely established, the work of Sackmann (1980) suggests that contact may occur at core masses around $0.6 M_\odot$, especially considering the probability of convective overshoot. The ensuing events are undoubtedly quite complicated, and the physics of such contact mixing is currently an open question. A specific scenario for contact mixing was proposed by Scalo and Ulrich (1973). Contact mixing is precluded at

much larger core masses because of the "entropy barrier" which arises as a result of the growing importance of radiation pressure (Iben 1976).

In view of the severe uncertainties concerning the mixing process and of the lack of stellar model calculations in most ranges of core mass, we have chosen to use a simple prescription for mixing and allow the observations to constrain the parameters of the prescription. The prescription is that a constant fraction λ of the increase in core mass during an interflash period, ΔM_c , is mixed to the surface following each flash if the core mass is greater than some value $M_c^{\min}$ but less than some value $M_c^{\max}$. The amount of mass mixed at a given flash is then

$$\Delta M_{\text{mix}} = \lambda \Delta M_c, \quad (6)$$

where the increase in core mass during the interflash period is given by

$$\Delta M_c = (1-r) M_{\text{CSH}}. \quad (7)$$

The mass contained in the convective helium shell at maximum extent M_{CSH} and the fractional mass overlap of successive shells, r , are taken from the formulae of IT (their eqs. [6] and [8]). The mixing parameter λ is given by

$$\begin{aligned} \lambda &= \text{constant} & (M_c^{\min} < M < M_c^{\max}) \\ \lambda &= 0 & (\text{otherwise}). \end{aligned} \quad (8)$$

The abundances in this material are taken from IT (their § III), with the addition that the flashed material contains Tc and, for the purposes of comparison, a nondecaying element with the same abundance as Tc. The Tc and nondecay abundances in the flashed material are set equal for all flashes. In practice, the adopted abundances have no effect on our results since we are only interested in relative Tc abundances. The adopted inter-shell carbon abundance does affect the fraction of stars which become carbon stars, but the adopted abundance is irrelevant for our purposes because this fraction is so small ($\lesssim 1\%$). Following mixing, the model's core mass is increased by ΔM_c at the expense of the envelope mass because of nuclear burning, while the envelope reclaims a mass ΔM_c^{mix} from the core because of mixing, with the net effect that the core increases by $(1-\lambda) \Delta M_c$. We were concerned that our results might depend on the functional form of this mixing formula, so we also studied a formula where a constant fraction of the convective helium-flashing shell is mixed, as well as the more complicated mixing formula of IT (their eq. [13]). Note that this latter formula, though based on stellar evolution calculations for large core masses, is essentially ad hoc for core masses less than about 0.9. We have verified that the main features of our conclusions

do not depend on the functional form of the mixing formula.

The envelope is assumed to lose mass because of a steady wind. We have adopted Reimers's (1975) semi-empirical formula for the mass loss rate:

$$\dot{M} = -1.34 \times 10^{-5} \alpha L^{1.5} T_{\text{eff}}^{-2} M^{-1}, \quad (9)$$

where α is a parameter which sets the scale of the mass loss rate and has a value near unity. The envelope mass is decreased by $\dot{M} t_{\text{IF}}$ after each flash, where t_{IF} is given by equation (4). We are fully aware of the uncertainties involved in this functional form for the mass loss, but some expression had to be used and sample calculations using the mass loss prescription of Sanner (1976) did not lead to significantly different results. The parameter α is uncertain by a factor of about 3 (Reimers 1977; Kudritzki and Reimers 1978), so we have used values of 0.1, 0.3, and 1.0 in the calculations, although most of the simulations reported below were calculated for $\alpha = 0.3$.

The initial metal abundances of the model stars are specified as follows:

1. The relation between metallicity Z and age A was taken from Twarog's (1980) study of the solar neighborhood. The following relation is a good representation of his results:

$$\begin{aligned} \log Z &= -1.599 - 0.030 A \quad (A < 6 \text{ Gyr}) \\ &= -1.383 - 0.066 A \quad (A \geq 6 \text{ Gyr}). \end{aligned} \quad (10)$$

(The present metallicity is 0.025, while the oldest DSP stars, which are about 10 Gyr old, have a metallicity of 0.009.)

2. Following Twarog, we take the helium abundance Y as $0.2 + 3.33(Z - 0.001)$ up to a maximum of 0.3.

3. The relative abundances of the metals, as well as the change in abundances prior to the DSP, are taken from IT (their § III).

c) Technetium in Individual Theoretical Models

The implications of the basic features of DSP evolution for the mixing of Tc to the surfaces of stars of various masses are illustrated in Figure 1, which shows the surface Tc abundance (arbitrary units) as a function of core mass for different initial masses, assuming s -process nucleosynthesis and mixing begin at $M_c^{\min} = 0.5 M_{\odot}$. Also shown in Figure 1 is the ratio of the time between flashes, t_{IF} , to the e -folding decay time of ^{99}Tc (3×10^5 yr) as a function of core mass. This ratio is always significantly less than unity, so it is clear that decay of Tc between flashes is unimportant as long as fresh Tc is added at every flash.

The stars with smallest initial mass, $0.9 M_{\odot}$, begin mixing at a small core mass when the intershell mass is relatively large. Therefore, relatively large quantities of

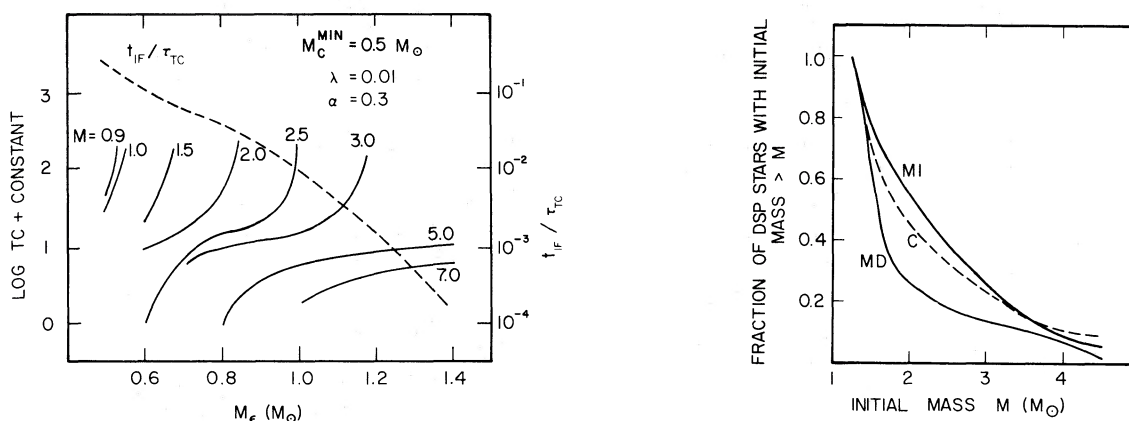


FIG. 1.—Solid line: Tc abundance (arbitrary units) as a function of core mass for various initial total masses. Minimum core mass for mixing $M_c^{\min} = 0.5 M_\odot$, mixing parameter $\lambda = 0.01$, and mass loss parameter $\alpha = 0.3$. Dashed line: ratio of time between helium shell flashes t_{IF} to Tc decay e -fold time τ_{TC} as a function of core mass.

FIG. 2.—Fraction of DSP stars with initial masses greater than M as a function of M for three different birthrate histories (see text). Mass loss parameter $\alpha = 0.3$, metal abundance $Z = 0.013$.

Tc are mixed to the surface per flash, and the surface Tc abundance rapidly attains large values. However, the Tc abundance does not increase indefinitely because mass loss terminates evolution quickly for low-mass stars. The behavior of the surface Tc abundance at $2.5 M_\odot$ is more complicated. Initially the intershell mass is fairly large and the Tc abundance increases rapidly. However, by $M_c \approx 0.7 M_\odot$ the intershell mass has decreased significantly, and so the surface Tc abundance increases much more slowly. The sharp rise at $M_c \approx 1 M_\odot$ is because mass loss has reduced the envelope mass to a very small value, so that, even though the intershell mass is small, the dilution in the envelope is very small, and each injection of Tc increases the surface Tc abundance significantly. At an initial mass of $5 M_\odot$, mass loss never reduces the envelope mass to small values, the intershell mass is very small (because the core mass during DSP evolution is large for such a massive star), and dilution is always significant; thus, the surface Tc abundance remains at a low level. These illustrative calculations were repeated using a different value of λ and also using the mixing rule given by IT; the qualitative behavior remains the same. The results obviously depend on the adopted value of $M_c^{\min}$ since, for larger values of this parameter, the lower-mass stars will not have mixed.

Of course the fraction of Tc stars in a given mass range depends on additional factors, such as the time spent in the DSP (larger for smaller masses) and the relative number of stars of different masses as determined by the initial mass function. The fraction of DSP stars with initial masses greater than some value is shown in Figure 2 for initial mass functions corresponding to the allowed range of stellar birthrate histories determined by Miller and Scalo (1979). The “maximum

decreasing” (MD) birthrate and the “maximum increasing” (MI) birthrate have an average past birthrate about 5 times larger or 3 times smaller than the present birthrate, respectively. The constant (C) birthrate is probably the best representation for the solar neighborhood (Scalo and Miller 1981; Twarog 1980). It is seen that, for any birthrate, very few DSP stars have initial masses greater than about $4 M_\odot$. The duration of the DSP and the final core mass are shown as a function of initial mass in Figures 3 and 4, respectively. Because of the relatively long time between flashes at small core masses, low-mass stars spend the greatest amount of time in the DSP even though mass loss terminates their red giant evolution at smaller core masses than for massive stars. Figures 2–4, coupled with the fact that the intershell mass is large for small core masses, suggest

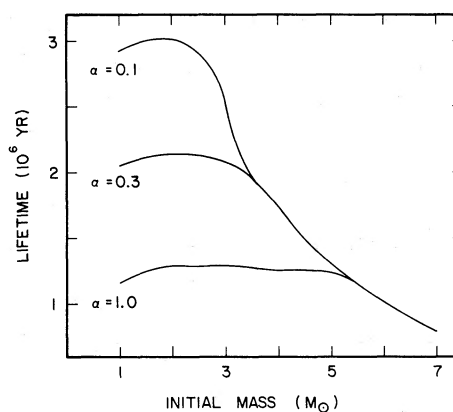


FIG. 3.—Duration of DSP evolution as a function of initial mass for selected values of the mass loss parameter α .

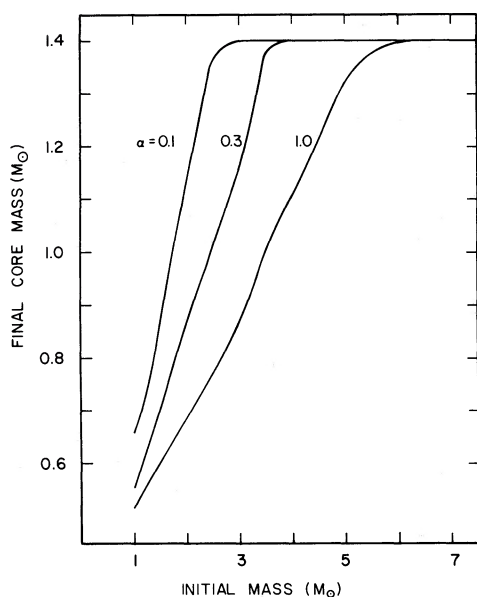


FIG. 4.—Terminal core mass as a function of initial mass for selected values of the mass loss parameter α .

that most red giants with strong Tc lines are stars with initial masses such that $M_c^0 \approx M_c^{\min}$, where M_c^0 is the core mass when the star enters the DSP of evolution (eq. [1]). The detailed simulations reported below confirm this result.

d) Simulation of Red Giant Populations

The computer simulation can be divided into two parts. The first part is the generation of a sample of simulated stars in the proper evolutionary stage (DSP here), while the second part is the calculation of various properties of each star (luminosity, temperature, Tc abundance, etc.).

Generation of a sample of simulated stars requires knowledge of: (1) the rate at which stars are formed as a function of time (the birthrate); (2) the number of stars which are formed as a function of mass (the initial mass function or IMF); and (3) the duration of the various stages of evolution as a function of mass. Each of these are discussed below. Once these functions are specified, a sample of model stars is generated as follows. A “star” is created by drawing an age randomly from the birthrate function and drawing a mass randomly from the IMF. Next, the evolutionary phase of the star is identified by comparing its age with the duration of the various evolutionary phases for its mass. For reasons given earlier, it is believed that the cool, luminous giants of interest here are stars in the double-shell phase, so only stars in this stage are retained for further calculation. The process is repeated until a sample of 1000 DSP stars is accumulated.

Miller and Scalo (1979) review the birthrate and IMF in the solar neighborhood. They find that the average birthrate in the solar neighborhood has been at most 5 times larger or 3 times smaller than the present birthrate, and favor a variation of less than a factor of 2. Applying the population simulation technique to lithium abundances in red giants, Scalo and Miller (1981) were able to tighten this constraint and confirm a variation of less than a factor of 2. Because of this result and the insensitivity of our general conclusions to the adopted birthrate (within these limits), and in order to avoid redundant calculations, the simulations reported here all employ a constant birthrate. The age of the solar neighborhood is taken as 12 Gyr. In this case, the stellar IMF, $\xi(\log M)$ which gives the number of stars per unit $\log M$ per square parsec, is (Miller and Scalo 1979)

$$\xi(\log M) = 1.53 - 0.96 \log M - 0.47(\log M)^2. \quad (11)$$

For the main-sequence, subgiant, first red giant branch, and helium core-burning phases of evolution, the amount of time spent in each stage T_i is taken to be a function of initial mass only:

$$\log T_i = A_0 + A_1 \log M + A_2 (\log M)^2, \quad (12)$$

where the constants A_i are given in SM (their Table 1). However, the DSP lifetime has a significant dependence on factors other than mass, the most important being the mass loss rate. This leads us to a technical point of the simulations. The most straightforward approach would be to generate a sample of stars for each value of all the parameters. Yet the generation of a sample of stars requires considerably more computer time than the subsequent analysis of the sample, so the following, more efficient, procedure was adopted. In generating a sample of stars, the DSP lifetime is taken as a constant (independent of mass) greater than the maximum possible DSP lifetime for any star, shown in Figure 3. Therefore, the population sample contains some stars which have become remnants. Just which stars are remnants depends on the particular choice of parameters, for example, the mass loss parameter α (eq. [9]).

The second part of the simulation consists of calculating properties of interest for each model star. Our procedure is taken chiefly from IT, with the exceptions noted above. The properties of a star at the onset of the DSP are specified above in this subsection (§ II d) along with the initial core mass relation of equation (1). Each model star is then evolved through the DSP, according to the following loop cycle. First, the stellar properties are calculated from equations (2)–(5). Next, the element abundances are altered according to the mixing given by equations (6)–(8). Then, the core and envelope masses are changed by the amounts required by mixing and by the outward progress of the nuclear burning shells. The

envelope mass is then decreased by the mass loss prescription of § IIb, equation (9). Finally, to complete a flash cycle, a model star is tested for the following:

1. If M_c exceeds the maximum core mass of 1.4, the star is flagged as a remnant and the properties of the next model star are calculated.

2. If the star has lost its envelope ($M \leq M_c$), it is flagged as a remnant and the program proceeds to the next star.

3. If the luminosity exceeds dex ($3.22 + 0.64 M$) (taken from IT), the star is marked as a Mira variable and assigned the first harmonic radial mode period of $3.36 \times 10^7 L^{0.5} T_{\text{eff}}^{-2}$ days. This period must be interpreted cautiously, for there is no assurance that real stars pulsate in the first harmonic nor that these formulae reproduce the properties of real stars. Our conclusions do not depend on these formulae; we keep track of variable stars only for completeness.

4. If the luminosity exceeds dex ($3.8 + 0.325 M$) (taken from IT), the star ejects its envelope as a planetary nebula.

5. If the surface abundance by number of ^{12}C exceeds that of ^{16}O , the star is a carbon star.

6. The time since the onset of the DSP is incremented by t_{IF} (eq. [4]) and compared to the age of the model star in the DSP, where the latter is given by the difference of the age of the star and the amount of time spent in all previous stages. If the time since the onset of the DSP is less than the age of the star in the DSP, another flash cycle is calculated. Otherwise, the properties of the model star are stored, and the DSP properties of the next model star are calculated, the process being repeated for all stars in the sample.

The simulated populations of DSP stars contain about 500 stars. To create such a sample of DSP stars requires the creation of about 6×10^6 total stars, most of which are either main-sequence stars or remnants. This takes about 1600 seconds on the University of Texas CDC Dual Cyber 170/750 computer system.

A summary of the parameters describing DSP evolution in the adopted formulation follows. Mixing is described by the parameter λ which gives the fraction mixed to the surface of the growth in core mass during an interflash period. Actually, none of our results for Tc depend on the value of λ chosen, but only on the assumption that λ is a constant. Mixing is assumed to begin at a core mass M_c^{min} and to cease at M_c^{max} . We have varied M_c^{min} between 0.5 and 1.0 and M_c^{max} ($> M_c^{\text{min}}$) between 0.6 and 1.4. Steady mass loss is regulated by the parameter α , which we varied between 0.1 and 1.0. Our main results are insensitive to α in this range, and most of the simulations presented below use $\alpha = 0.3$. Therefore, given the assumption of mixing due to helium shell flashes with constant λ , the only major parameters in the present work are M_c^{min} and M_c^{max} , whose values will be varied to obtain the best agreement

between the properties of the simulated populations and the Tc observations of LL.

Figures 5a and 5b show synthetic H-R diagrams for two simulated populations of stars generated for mass loss parameters (a) $\alpha = 0.1$ and (b) $\alpha = 1.0$. The mixing parameters ($M_c^{\text{min}} = 0.5$, $M_c^{\text{max}} = 1.4$, and $\lambda = 0.05$ in Fig. 4) do not affect the distribution of stars in the diagrams, but only the fraction which become carbon stars. The carbon stars are shown as open circles, while M stars which have become Mira variables according to the rule given above are shown as five-pointed stars. Superposed are evolutionary tracks for different individual initial masses, including the effect of mass loss. Note that the tracks are for $Z = 0.01$, while the simulated stars have a variety of metal abundances. The lower boundary of the simulated samples reflects the evolutionary track of the smallest mass star which has had time to reach DSP evolution ($1.0 M_\odot$), while the left-hand boundary reflects the luminosity at which stars of different masses enter DSP evolution. For the lower mass loss rate, more stars have time to become carbon stars before being stripped of their envelopes. For the larger mass loss rate, no higher-luminosity (larger core mass) stars have time to become carbon stars, but some of the lowest-mass stars still become carbon stars before being stripped because of the large intershell masses and relatively small dilution factors in the envelope. A detailed discussion of the dependence of the relative number and luminosity function of the carbon stars on mass loss rate, metal abundance, mixing parameters, and birthrate history is given in Miller and Scalo (1981).

III. RESULTS

In this section we examine four possible methods for using the observations of Tc by LL in conjunction with the results of our simulations to constrain the values of M_c^{min} and M_c^{max} .

a) Correlation of Technetium with Initial Mass

Figures 6a–6d show the Tc abundance as a function of initial mass for four choices of the parameter pair ($M_c^{\text{min}}, M_c^{\text{max}}$). In Figures 6a and 6b, the value of M_c^{min} is fixed at $0.5 M_\odot$ while M_c^{max} varies from $0.6 M_\odot$ to $0.8 M_\odot$. The lower mass limit of the plotted points is controlled by M_c^{min} and represents the smallest initial mass which can mix, $1.0 M_\odot$ in these two cases. The upper mass limit is determined by M_c^{max} . In Figure 6a, stars with an initial mass greater than $2.5 M_\odot$ cannot have mixed because such stars enter the DSP with $M_c > 0.6 M_\odot$. With $M_c^{\text{max}} = 0.8 M_\odot$ (Figure 6b), stars with masses up to $5 M_\odot$ have mixed. Also, with a larger range of core masses during which mixing can occur, the Tc abundances of the lower-mass stars are larger. Note that carbon stars (*open circles*) are predicted to have the largest Tc abundances, since we have assumed in the

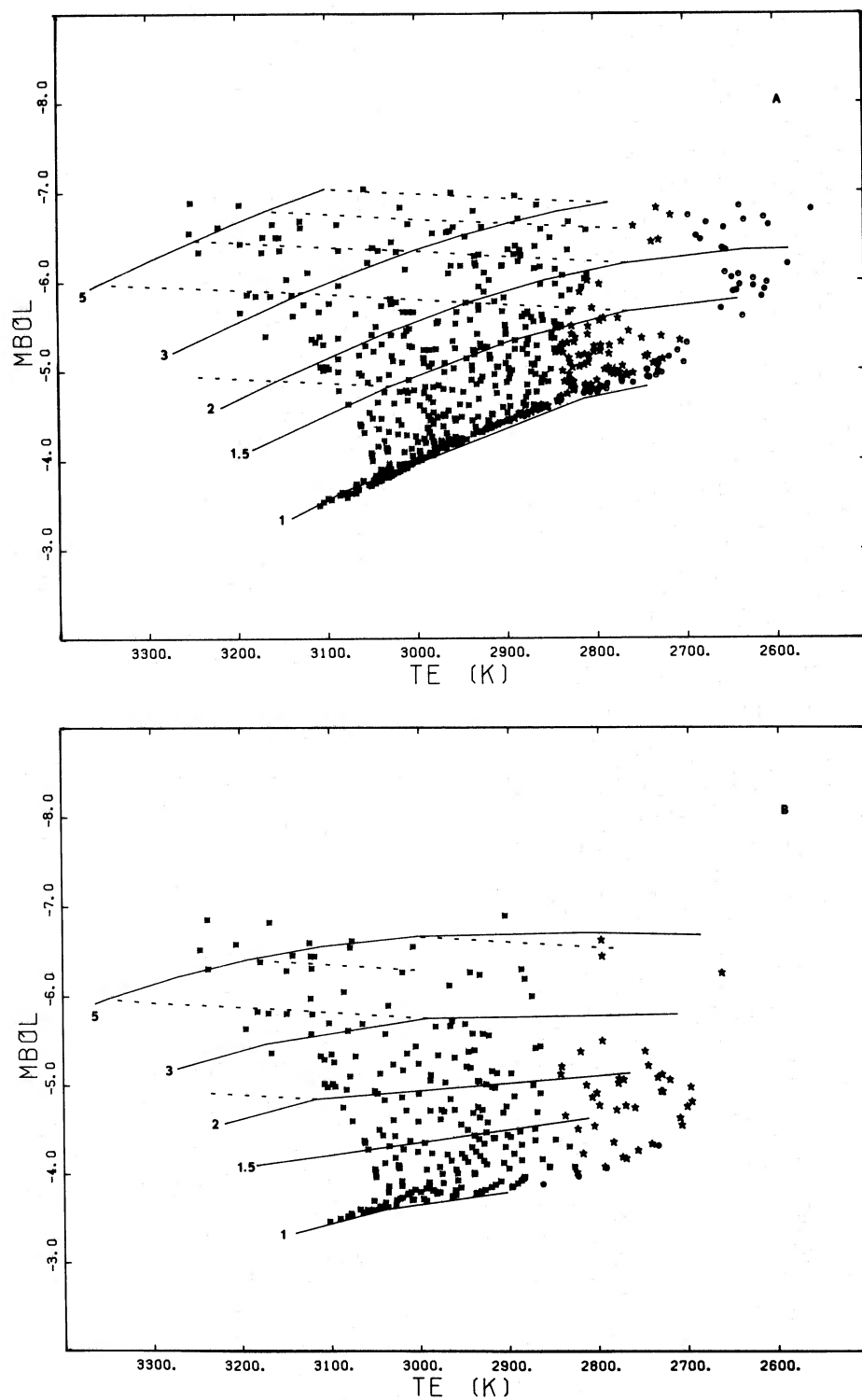


FIG. 5.—Synthetic H-R diagrams of DSP stars with $M_c^{\min}=0.5$ and $\lambda=0.05$. The mass loss parameter is (a) $\alpha=0.1$ and (b) $\alpha=1.0$. Circles, carbon stars; five-pointed stars, noncarbon Mira stars; and squares, noncarbon, non-Mira DSP stars. Solid lines, evolutionary tracks for single stars, labelled by total initial mass. Dashed lines, loci of constant core mass. The dashed lines differ by 0.2 in core mass, with the line at lowest luminosity giving the locus for a core mass of $0.6 M_{\odot}$.

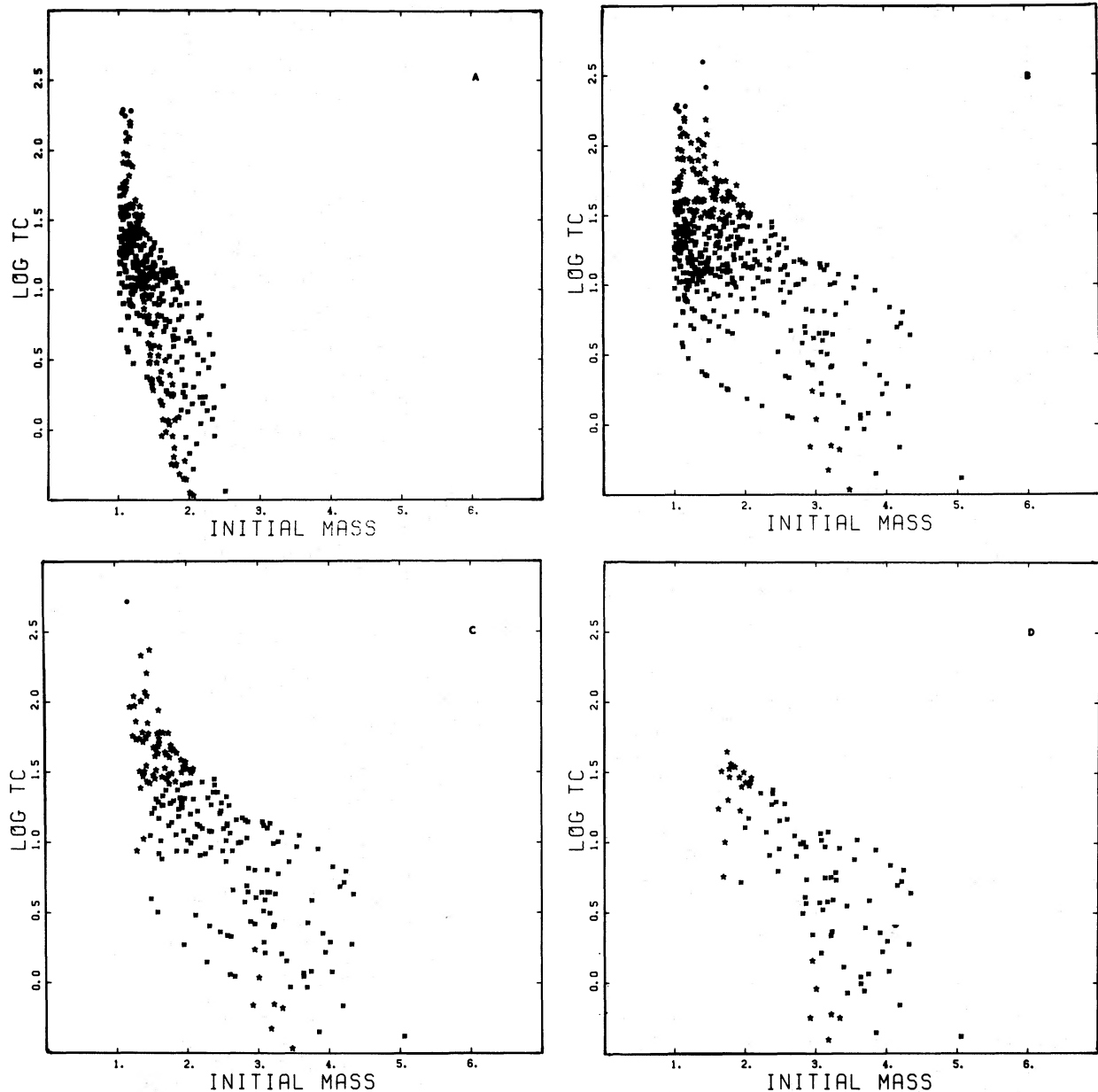


FIG. 6.—Tc abundance (arbitrary units) as a function of initial total mass for simulated populations of DSP stars. Parameters are: $\lambda=0.05$; $\alpha=0.3$; and in (a) $(M_c^{\min}, M_c^{\max})=(0.5, 0.6)$, in (b) $(0.5, 0.8)$, in (c) $(0.6, 0.8)$, and in (d) $(0.7, 0.8)$. Circles, carbon stars; five-pointed stars, noncarbon Mira stars; and squares, noncarbon, non-Mira DSP stars.

calculations that carbon is mixed up with Tc. Figures 6b, 6c, and 6d show the effect of increasing $M_c^{\min}$, while keeping $M_c^{\max}$ fixed at $0.8 M_{\odot}$. The lower mass limit for the plotted points moves to progressively larger values as $M_c^{\min}$ is increased ($1.5 M_{\odot}$ in Figure 6d), since lower-mass stars terminate their evolution before a core mass of $M_c^{\min}$ is attained.

An important feature of Figures 6a–6d, and of all the cases we have studied, is that most of the stars with

large Tc abundances have initial masses M such that $M_c^0(M)=M_c^{\min}$, where $M_c^0(M)$ is the core mass at the beginning of the DSP and is given by equation (1). This result suggests one possible method of observationally constraining $M_c^{\min}$. If $M_c^{\min}$ were as large as, say, $0.9 M_{\odot}$, Tc stars should all be more massive than about $6.3 M_{\odot}$ and should have considerably different kinematical properties than non-Tc M giants, which have initial masses $\sim 1 M_{\odot}$ (see Fig. 2). Unfortunately there is

insufficient radial velocity data for such a comparison at present. The fact that LL find Tc preferentially in longer-period Mira variables does suggest such an effect, since longer periods are known to correspond to an earlier population type (e.g., Feast 1963). However, the mean initial masses corresponding to each period group are not accurately known for Miras (i.e., a variation in mass from $1 M_{\odot}$ to $1.5 M_{\odot}$ may be sufficient to account for the variation of population characteristics with period); thus, we cannot use this data to constrain $M_c^{\min}$, except that its value must exceed 0.5 and must be less than about 1.0 (or else the longer-period Miras would have very young population characteristics). We show below that the observed fraction of M stars which show Tc lines offers a much more stringent constraint on $M_c^{\min}$.

b) Correlation Between Technetium Line Strength and Spectral Type

On the basis of their spectral survey, LL were able to divide stars into the categories: Tc definitely present, probably present, possibly present, doubtful, and absent. They found that the stars with definite, probable, and possible Tc lines were mostly confined to variables with longer periods. Since kinematical properties of M variables are correlated with period, they suggested that the presence of Tc is anticorrelated with age. However, we note that period is also correlated with spectral type (effective temperature), so a more fundamental relation may exist between Tc line strength and spectral type. Spectral type correlations are also much more easily compared with our simulations, since we feel that relative effective temperatures of the theoretical stars are trustworthy while theoretical periods are quite uncertain. In this section we show that the assumption of DSP mixing can reproduce the observed correlation of Tc with spectral type.

The observed correlation of the presence of Tc with spectral type is shown in Figure 7. This histogram shows the fraction of M and MS stars (these are the only spectral types which give an unambiguous indication of T_{eff}) with definite, probable, and possible Tc lines versus spectral type at maximum light. The number of stars in each bin is indicated. Fractions instead of numbers of stars at each spectral type are plotted in order to remove the sample bias toward later spectral types, which is present because the stars chosen to be observed were mainly Mira variables. Although the number of stars is small, there is a clear indication that Tc is found predominantly among the cooler stars. If the "possible" Tc stars are omitted, the trend is not changed significantly.

Figure 8 shows the line strengths estimated by LL as a function of spectral type at maximum light. All stars with doubtful or absent Tc lines were assigned zero line strength (i.e., we assume that the nonzero line strengths tabulated by LL for these stars only reflect the presence

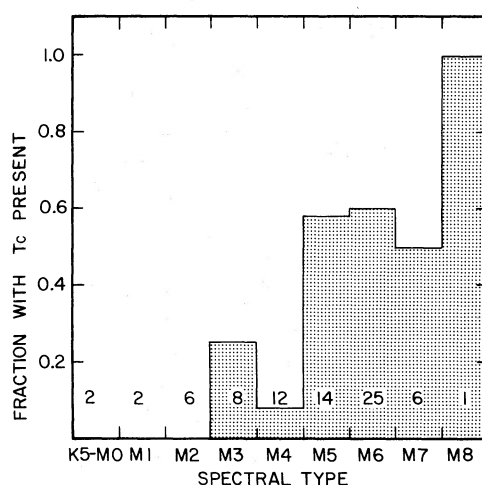


FIG. 7.—Observed correlation between presence of Tc with spectral type at maximum light. Histogram shows the fraction of stars in each spectral type interval with Tc present. Data from Little-Marenin and Little (1979).

of blends). It is seen that, among stars with Tc present, the strongest lines occur for the coolest stars. On the other hand, it should be noted that no correlation exists between Tc line strength and period for stars with Tc present, suggesting that T_{eff} may be a more fundamental parameter than is period.

One might suspect that the observed correlation of Tc line strength with spectral type is simply an excitation or gravity effect, rather than being related to mixing. Since we feel that the ability of our simulations to reproduce the correlation is an important test of our assumptions, we devoted considerable effort to disentangling effects of atmospheric structure from effects due to mixing. Our

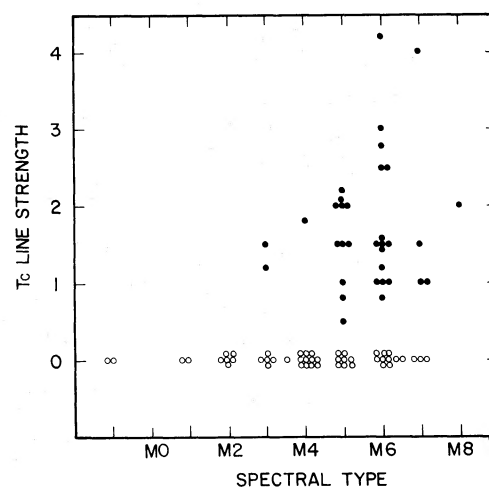


FIG. 8.—Observed Tc line strengths versus spectral type at maximum light. Data from Little-Marenin and Little (1979). Stars with doubtful or absent Tc lines were assigned zero line strengths.

approach was to compute theoretical Tc equivalent widths for a range of Tc abundances in a grid of model atmospheres spanning the relevant portion of the (T_{eff} , $\log g$) plane. Details of these calculations are given in the Appendix. No significant dependence of Tc equivalent width on T_{eff} was found for $T_{\text{eff}} < 4000$ K; we conclude that the correlations with spectral type shown in Figures 7 and 8 reflect the process by which the stars were mixed.

Wood (1980, private communication) has pointed out the possible danger of including the non-Miras with the Miras in Figures 7 and 8 because of the large variation of spectral type during a light cycle and the uncertainty in comparing spectral types of nonvariables, semi-irregular variables, and Miras. For this reason we cannot put much quantitative weight on these figures; we note, however, that the correlations do persist even when only the Mira variables are included in the sample.

Figures 9a–9d show Tc abundances as a function of model temperature for all mixed stars in four different, simulated samples with different values of M_c^{min} and M_c^{max} . In the (0.5, 1.4) case, the plotted points have a sharp, lower boundary, which can be understood with reference to Figure 5. At a given T_{eff} , there is a lower limit to the luminosity, and hence core mass, of stars in the sample. A line of constant T_{eff} intercepts higher-mass evolutionary tracks at much larger core masses than the lower-mass tracks, so this lower core mass limit refers to the stars of smallest mass. For the adopted mixing prescription, the Tc abundance increases with core mass, so there can be no Tc abundance smaller than that corresponding to the lower limit on core mass at the given T_{eff} . The correlation of Tc with T_{eff} occurs because, as T_{eff} is decreased, the minimum core mass intercepted in the H-R diagram increases. Comparison with Figure 8, however, shows that the (0.5, 1.4) case fails to reproduce the range of Tc at later spectral types.³ The situation is greatly improved by taking $M_c^{\text{max}} < 1.4 M_{\odot}$, so that some stars have time for their Tc to decay, as shown in Figures 9b–9d. Note that by $M_c^{\text{min}} = 0.7 M_{\odot}$ the correlation has nearly disappeared, suggesting that M_c^{min} cannot be much larger than this value.

It is also important to examine the theoretical analog of Figure 7, the fraction of stars with detectable Tc in each interval of model temperature. Unfortunately we have no quantitative way to normalize the theoretical abundances, so we define “detectable” as any Tc abundance larger than some fraction of the largest Tc abundance encountered in the simulated populations. The results for a “detectability factor” (=detectable abundance/maximum abundance) of 1/30 are given in Ta-

ble 1 for most of the cases we have calculated. (A mass loss parameter $\alpha = 0.3$ is used throughout Table 1.) The main results are unchanged if this factor is increased or decreased by a factor of 5. Examination of Table 1 shows that the observational data of Figure 7 can be qualitatively reproduced, but only if $M_c^{\text{min}} \lesssim 0.7 M_{\odot}$. The tendency for larger fractions of Tc stars at lower model temperatures, as observed, has disappeared by $M_c^{\text{min}} \gtrsim 0.8 M_{\odot}$. The table also shows that the theoretical fractions and their dependence on model temperatures are not very sensitive to M_c^{max} .

We conclude that mixing by helium shell flashes is capable of accounting for the observed dependence of Tc line strengths on spectral type, but only if $M_c^{\text{min}} \lesssim 0.7 M_{\odot}$.

c) Fraction of Stars Which Have Mixed

Perhaps the most interesting result found by LL was that Tc is present in about 40% of the M giants studied (41% including MS stars, 38% excluding MS stars; these ratios are reduced to 33% if all of the stars with “possible” Tc actually do not contain Tc). Ignoring selection effects for now, this result suggests that, if mixing is due to helium shell flashes, mixing must occur relatively early (i.e., at small core masses) during DSP evolution, since the fraction of DSP stars with core masses greater than M_c^{min} is a decreasing function of M_c^{min} . We quantify this statement in Table 2, which shows the fraction of stars that have mixed, $f(\text{Tc})$, for a number of simulated populations. (This includes all stars which have mixed, not just those with an abundance exceeding the detectability factor.) It is seen that, as expected, $f(\text{Tc})$ is controlled primarily by M_c^{min} , with a much weaker dependence on M_c^{max} (and α). The observed value of $f(\text{Tc})$ appears to imply that M_c^{min} must be around 0.6 or slightly greater. [Cases with $M_c^{\text{min}} \geq 0.8$ and $M_c^{\text{max}} < 1.4$ are not presented because they only give smaller values of $f(\text{Tc})$.]

However, there are three effects which must be taken into consideration. First, the observed sample of LL is weighted toward later spectral types, since most of the stars chosen were Mira variables. This effect tends to overestimate the observed fraction of M stars which have mixed Tc since, as discussed earlier, Tc is more commonly found among cooler stars. We have no way of quantitatively estimating the weights which should be applied to the observed sample, but it can be seen from Table 1 that, if the theoretical results were recalculated including only stars cooler than a given temperature in order to imitate this selection effect, the value of $f(\text{Tc})$ would not be increased significantly for $M_c^{\text{min}} \geq 0.8$. For this reason we are hopeful that the selection effect is not a major problem. Second, the theoretical sample refers only to DSP stars, while the observed sample may, at the earlier spectral types, contain some stars which have not yet reached DSP evolution. For comparison

³We have no accurate means of relating the intensity scale of LL with actual Tc abundances, but the synthetic spectra calculations described in the Appendix suggest that the intensity range from 1 to 5 may correspond to a change in Tc abundance by a factor on the order of 50 to 100. This estimate is very uncertain.

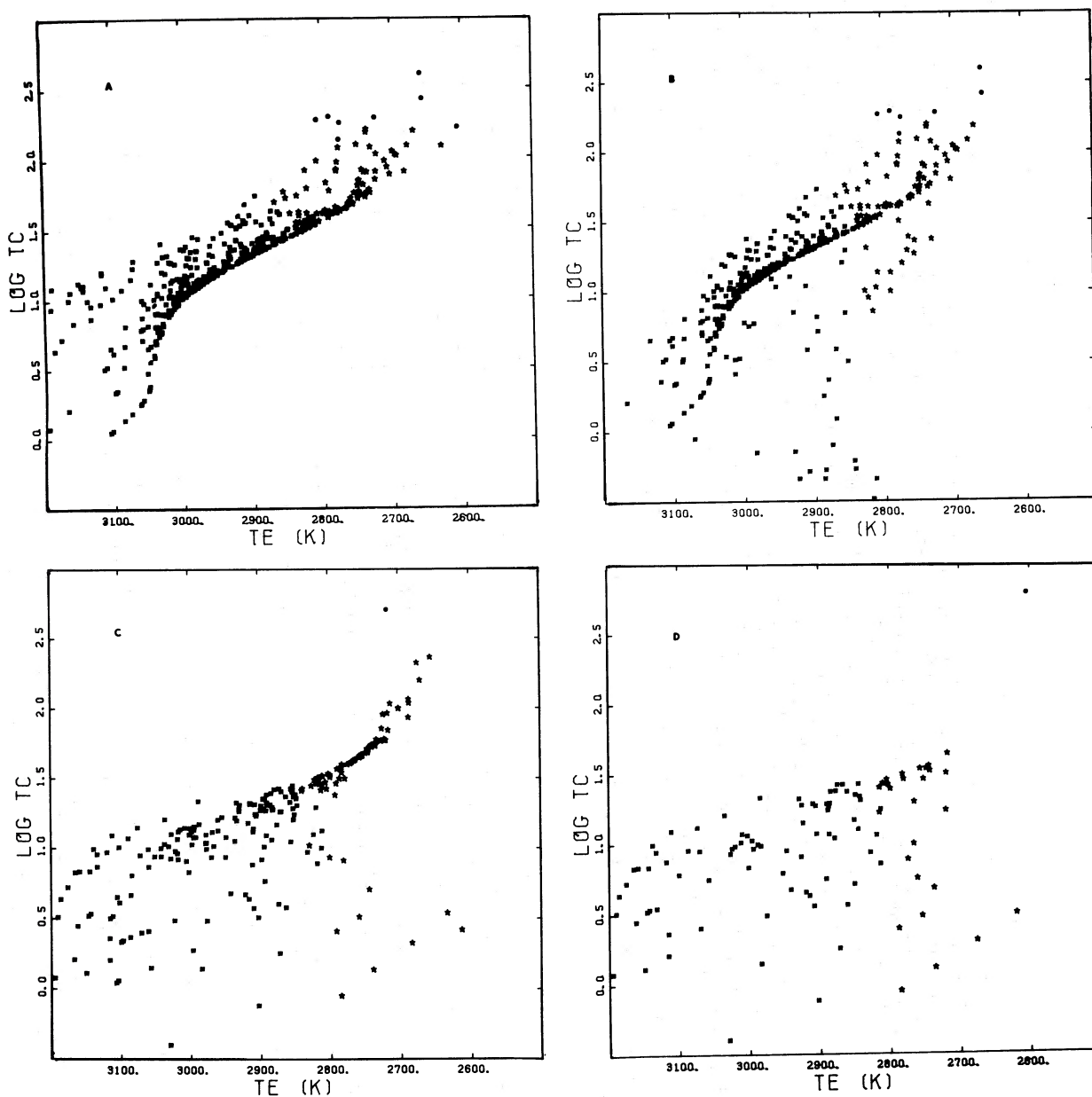


FIG. 9.—Tc abundance (arbitrary units) as a function of model temperature for all stars in the simulated samples; $\lambda=0.05$ and $\alpha=0.3$ in all cases. In (a) $(M_c^{\min}, M_c^{\max})=(0.5, 1.4)$, in (b) $(0.5, 0.7)$, in (c) $(0.6, 0.9)$, and in (d) $(0.7, 0.9)$. Circles, carbon stars; five-pointed stars, noncarbon Mira stars; and squares, noncarbon, non-Mira DSP stars.

with the observed sample, the theoretical fraction is then only an upper limit. Third, the theoretical fraction includes all DSP stars with *any* Tc, while actually the predicted Tc abundances may show a range of several orders of magnitude, independent of $M_c^{\min}$. Therefore, Tc will be present but undetectable in some DSP stars, again giving an upper limit to the theoretical fraction. Both of these last two effects cause an *overestimate* of

the minimum core mass for mixing.

Using Table 2 and considering the above effects, we conclude that the minimum core mass for mixing of *s*-process material must be no more than 0.6–0.7. This result is consistent with the value of $M_c^{\min}$ required to match the correlation between Tc and spectral type and with two other independent estimates based on luminosities and masses of P giants, as discussed in § IV below.

TABLE 1
PREDICTED FRACTION OF STARS IN EACH MODEL TEMPERATURE
RANGE WITH DETECTABLE TECHNETIUM ABUNDANCE

CORE MASS		MODEL TEMPERATURE						
$M_c^{\min}$	$M_c^{\max}$	3250	3150	3050	2950	2850	2750	2650
0.5	0.6	0	0	0.15	0.68	0.46	0.37	0
0.5	0.7	0	0	0.22	0.88	0.77	0.91	0.71
0.5	0.8	0	0.03	0.29	0.92	0.85	0.91	0.71
0.5	0.9	0	0.10	0.29	0.95	0.95	0.90	0.62
0.5	1.4	0	0.41	0.34	1.0	1.0	1.0	1.0
0.5	1.4	0	0.38	0.43	1.0	1.0	1.0	1.0 ($\alpha=0.1$)
0.5	1.4	0	0.36	0.16	0.96	1.0	1.0	1.0 ($\alpha=1.0$)
0.6	0.7	0	0	0.05	0.17	0.36	0.73	0.75
0.6	0.8	0	0.03	0.11	0.21	0.47	0.75	0.75
0.6	0.9	0	0.10	0.12	0.24	0.57	0.74	0.67
0.6	1.4	0	0	0.04	0.03	0.14	0.22	0
0.7	0.8	0	0	0.04	0.03	0.14	0.22	0
0.7	0.9	0	0.07	0.05	0.06	0.24	0.22	0
0.7	1.4	0	0.38	0.10	0.11	0.30	0.31	0.17
0.7	1.4	0	0.34	0.10	0.16	0.32	0.26	0 ($\alpha=0.1$)
0.7	1.4	0	0.36	0.08	0.12	0.11	0.20	0.14 ($\alpha=1.0$)
0.8	1.4	0	0.32	0.06	0.09	0.16	0.10	0.17
0.9	1.4	0	0.14	0.04	0.06	0.13	0.10	0.17
1.0	1.4	0	0.07	0.02	0.04	0.04	0.10	0.17

NOTE: Refer to text for definition of detectable Tc abundance with regard to the simulations. Except where noted, mass loss parameter $\alpha=0.3$.

TABLE 2
INTEGRATED MIXING FRACTIONS FOR SIMULATED POPULATIONS

$M_c^{\min}$	$M_c^{\max}$	$f(\text{Tc})^a$	$f(\text{decay})^b$
0.5	0.6	0.76	0.22
0.5	0.7	0.89	0.086
0.5	0.8	0.94	0.066
0.5	0.9	0.96	0.034
0.5	1.4	0.97	0
0.5	1.4	0.98	0 ($\alpha=0.1$)
0.5	1.4	0.94	0 ($\alpha=1.0$)
0.6	0.7	0.45	0.17
0.6	0.8	0.48	0.13
0.6	0.9	0.51	0.065
0.6	1.4	0.52	0
0.7	0.8	0.21	0.24
0.7	0.9	0.22	0.12
0.7	1.4	0.25	0
0.7	1.4	0.27	0 ($\alpha=0.1$)
0.7	1.4	0.19	0 ($\alpha=1.0$)
0.8	1.4	0.13	0
0.9	1.4	0.12	0
1.0	1.4	0.07	0
observed	...	0.4	~ 0.3

^a $f(\text{Tc})$ is the fraction of stars in a population which have mixed Tc.

^b $f(\text{decay})$ is the fraction of stars in a population which have a Tc abundance less than 0.1 of the abundance of a nondecaying element mixed along with Tc.

NOTE: Except where noted, mass loss parameter $\alpha=0.3$.

d) Absence of Technetium in Stars With Enhanced s-Process Abundances

A crucial constraint on the mixing process for P giants which has been overlooked in the past is that a *relatively large number of stars with strong enhancements of s-process elements have no detectable Tc lines*. Using the results summarized by LL, out of thirty MS, S, SC, and N stars which have been searched for Tc, nine have undetectable Tc lines. Furthermore, the fraction of s-process-enhanced stars which have no detectable Tc is about the same among the MS, S, and N stars (only one SC star has been checked). Technetium clearly has had time to decay in a significant fraction of stars which have already mixed s-process elements to their surface. We denote this fraction $f(\text{decay})$. However, the interflash-period-core mass relation (eq. [4]) shows that, even at a core mass as small as $0.5 M_{\odot}$, the ratio of interflash period to Tc decay time is small (about $1/3$) and decreases rapidly with increasing core mass (see Fig. 1). The cumulative effect of all the partial Tc decays which occur between flashes is also insignificant; the maximum decrease in the ratio of the Tc abundance to the abundance of a stable element which is mixed is only about a factor of 3.

This result immediately rules out any model in which mixing occurs during strictly consecutive shell flashes and continues until $M_c^{\text{max}} = 1.4$. We can think of only two plausible explanations.⁴ First, it is possible that mixing (at least mixing of s-process elements) does not occur during consecutive flashes but that, instead, mixing occurs sporadically, with large numbers of flashes

not leading to mixing of s-process elements. The contact mixing described in § II might lead to such a result because the extent of the convective intershell region is expected to fluctuate from flash to flash because of turbulent intermittency (see Laufer 1975) or other apparently nondeterministic turbulent phenomena, for example gravitational potential fluctuations (Edwards 1980). If contact occurs, the details of the hydrodynamical events at the contact interface, where protons would be mixed with large quantities of ^{12}C at high temperatures, are also likely to vary from flash to flash.

An alternative explanation is that mixing commences at M_c^{min} and occurs during strictly consecutive shell flashes but then ceases at $M_c^{\text{max}} < 1.4$. (This was, in fact, our initial motivation for considering M_c^{max} as a parameter.) We define a theoretical $f(\text{decay})$ as the fraction of stars in the simulated population which have a surface Tc abundance less than 0.1 times the abundance of a stable s-process element whose abundance in the intershell region was the same as that of Tc; i.e., we assumed Tc becomes unobservable when it has decayed to an abundance 10 times smaller than the stable s-process element. The values of $f(\text{decay})$ so computed for the simulated populations are given in Table 2. Although the observed value of $f(\text{decay})$ is only based on 30 stars, and the theoretical value depends somewhat on the criteria used in its calculation, the results indicate that, if the " $M_c^{\text{min}}, M_c^{\text{max}}$ " model is valid, then any value of M_c^{max} in excess of 0.8 will not produce enough P giants with Tc absent.

IV. SUMMARY

The use of a population simulation technique applied to DSP models has allowed us to reach some strong conclusions concerning mixing in red giants, using the Tc survey of LL. We have assumed that Tc is mixed to the surface during consecutive helium shell flashes, that a constant fraction of the increase in core mass between successive flashes is mixed, and that mixing commences at some minimum core mass M_c^{min} and ceases for core masses greater than M_c^{max} . While the adopted mixing model is idealized, the results do not depend sensitively on any of the parameters except M_c^{min} and M_c^{max} . Our main results are:

1. Mixing must begin at $M_c^{\text{min}} \sim 0.6\text{--}0.7 M_{\odot}$ in order to reproduce the fraction of M giants which exhibit Tc lines and the observed correlations of Tc with spectral type. This result is in agreement with two other independent arguments relating to the minimum core mass for mixing: (a) the mean initial masses of P giants deduced from kinematics and binary and cluster membership are small ($\sim 1\text{--}1.5 M_{\odot}$), requiring small core masses (SM); (b) the luminosities of galactic P giants of all types (Scalo 1976b) and Magellanic Cloud carbon stars (Crabtree, Richer, and Westerlund 1976; Mould and Aaronson 1979; Richer 1980) are such that small core

⁴One other possible explanation of the lack of Tc in some MS, S, and N stars involves the recent evidence (McClure, Fletcher, and Nemec 1980; Böhm-Vitense 1980) that all strong Ba stars are members of wide binaries with degenerate companions, so that Ba stars are merely former main-sequence stars which received processed material transferred from the companion when the companion was a cool P giant. Indirect support for this idea comes from the recent abundance analysis of Kollatschny (1980) in which a K dwarf was found to have s-process abundance enhancements similar to those found in Ba stars. If this is true, then we must ask whether the 30% of cool P giants with no Tc are simply former Ba stars which have evolved to the DSP, giving sufficient time for Tc decay since the mass transfer event. This explanation can be ruled out as follows. Say that some fraction y_B of main-sequence stars are binaries, independent of mass. The fraction of G and K giants which are Ba stars, $\lesssim 0.01$, indicates that it must be very difficult for binary members to become P giants. If the period is too small, mass transfer will occur before the primary reaches DSP evolution. The number 0.01 suggests that only about 1% of binaries have periods large enough to undergo such mass transfer. Therefore the fraction of P giants which were once mass transfer recipients (i.e., Ba stars) can be no more than $0.01 y_B / [(1 - y_B) + 0.01 y_B]$, which is much less than 1 for any reasonable y_B . Thus nearly all observed cool P giants cannot have participated in mass transfer, and we cannot explain 30% of MS, S, and N stars as evolved Ba stars. Perhaps more fundamentally, there is no apparent reason why such an evolved Ba star should not undergo the same mixing processes during DSP evolution as single stars, so we would expect Tc even in evolved Ba stars.

masses are required. The most convincing evidence in this regard is the LMC carbon star luminosity function constructed by Richer (1980) which shows that mixing must set in at $M_c \sim 0.5\text{--}0.6 M_\odot$ (see also Iben 1980), in excellent agreement with the present results. The corresponding luminosity is $M_{\text{bol}} \sim -3.5\text{--} -4$.

This result cannot be accounted for by current evolutionary models. In a recent paper, Wood (1980) has shown that "dredging" of the type found by Iben (1975) can occur at smaller core masses than previously thought, but only if the total mass in the DSP is large enough. Using an approximate technique, Wood estimates that mixing occurs at $M_c \approx 0.6 M_\odot$ for a total mass $M \approx 4 M_\odot$, $M_c \approx 0.7$ at $M \approx 3.2 M_\odot$, and larger values of M_c for smaller masses. However, the fraction of DSP stars with initial masses greater than $4 M_\odot$ is very small (see Fig. 2); thus, if we were to allow mixing at small core masses in these intermediate-mass stars but not in low-mass stars, our results would not be noticeably affected. Several galactic carbon stars in binaries and clusters are known observationally with $M \sim 1\text{--}1.5 M_\odot$, and the mean galactic carbon star mass determined from kinematical properties is certainly no larger than $1.5 M_\odot$ (SM). One needs mixing at small core masses in *low-mass* stars.

Wood (1980) points out that an increase in the adopted ratio of mixing length to scale height, l/H_p , will reduce the minimum total masses for mixing, but a very large increase in this ratio would be required in order to reduce the predicted minimum total masses to $\sim 1 M_\odot$. Even in this case, it is not clear that increasing l/H_p would decrease the predicted minimum luminosities sufficiently (see Wood 1980, Table 1).

The severe disagreement with current stellar-evolution calculations strongly suggests that our knowledge of DSP evolution at small core masses is seriously incomplete. Either "dredging" occurs at such a very early phase for stars of small total mass, or contact between

the intershell convection zone and the hydrogen shell occurs ("contact mixing") as suggested by the recent work of Sackmann (1980).

2. The large fraction of S and N stars which show *s*-process enhancements but no detectable Tc implies that either mixing of *s*-process material does not occur in consecutive flashes, with a large number of flashes skipped, or that mixing of *s*-process elements does occur during consecutive flashes but ceases at $M_c^{\text{max}} \sim 0.8 M_\odot$. The former possibility might arise if contact mixing occurred, since then one might expect large flash-to-flash variations because of the nature of convective overshoot. The latter possibility is suggested by the LMC carbon star luminosity function of Richer (1980), which turns over at a luminosity that corresponds to a core mass of about $0.8 M_\odot$. (We note that, while some carbon stars with larger luminosities may exist, this does not imply that they were mixed at their current luminosity.) The cessation of mixing at some core mass could be explained if mixing depended on total mass, such that, when mass loss reduced the total mass below some limit, mixing could not occur, as suggested by Wood (1980, private communication). However, the limiting mass must be around $1 M_\odot$.

It is hoped that the present results will motivate detailed flash calculations in the core mass range $0.5\text{--}0.8 M_\odot$ for low-mass stars (see Wood and Zarro 1980) and, especially, a renewed examination of the albeit complicated physical processes which may result from shell flashes, such as convective overshoot, nonspherical contact mixing, and stochastic processes.

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APPENDIX

We calculated theoretical Tc equivalent widths using the synthetic spectrum program described by Scalo and Ross (1974). Model atmospheres from Johnson, Bernat, and Krupp (1980) spanning the range $2500 \leq T \leq 4000$ K, $0 \leq \log g \leq 2$ were used for the runs of temperature, gas pressure, electron pressure, and standard continuous opacity κ_0 as a function of standard optical depth τ_0 . The emergent flux at each wavelength was computed assuming that the lines are formed in local thermodynamic equilibrium and pure absorption. The monochromatic optical depth τ_λ was calculated from the total (continuous plus line) monochromatic opacity. The continuous opacity included absorption by H, H⁻, H₂⁺, H₂⁻, He, He⁺, He⁻, Mg, and Si, and scattering by H, H₂, and electrons, while the line absorption coefficient was computed by folding a Lorentz profile into a Doppler profile. The radiation damping constant was set equal to the classical value, and Stark broadening was ignored. The van der Waals damping constant was computed assuming a mean square radius of $25a_0^2$. A depth-independent microturbulent parameter of 2 km s^{-1} was used, and the emergent flux was folded over a Maxwellian macroturbulent profile with rms velocity 3 km s^{-1} (this parameter does not affect the equivalent widths). In order to calculate equivalent widths of Tc I lines, we used the ionization potentials and partition functions of Ru I since they are very similar and Tc was not included in the library of our standard synthesis program. The *gf*-values were taken from Hackos (1970), using correction factors suggested by Garstang (1979).

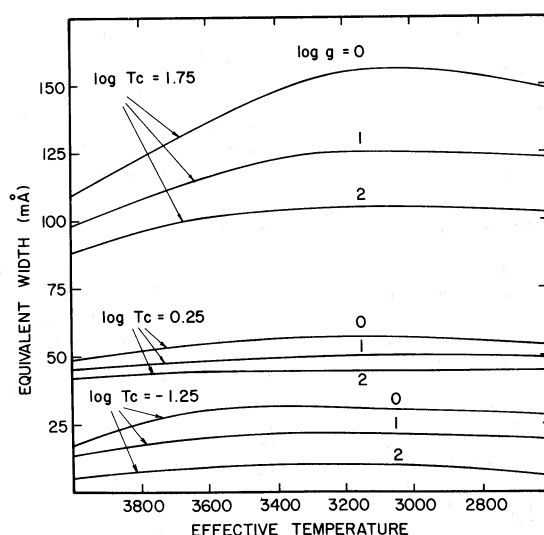


FIG. 10.—Calculated equivalent width of Tc I $\lambda 4297$ line as a function of T_{eff} for a range of surface gravities and Tc abundances (on the logarithmic scale $\log H = 12$).

Figure 10 shows the equivalent width of the Tc I $\lambda 4297$ line as a function of model temperature for 3 gravities and 3 Tc abundances. The abundances are logarithmic on the scale hydrogen = 12. The temperatures of the simulated models are in the range from 2600 to 3300 K, and the mean gravity varies by less than a factor of 10 over this range; thus, we expect the predicted variation of Tc line strength with spectral type to be less than a factor of ~ 3 due to effects of temperature and gravity alone.

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