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GRAVITATIONAL DISTORTION OF THE IMAGES OF DISTANT RADIO SOURCES IN AN INHOMOGENEOUS UNIVERSE[1]

R. D. BLANDFORD AND M. JAROSZYŃSKI[2]
W. K. Kellogg Radiation Laboratory, California Institute of Technology
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ABSTRACT

The images of linear extragalactic radio sources will be distorted through gravitational deflection by inhomogeneity in the universe. The rms bending angle of an intrinsically collinear, high redshift triple source of size θ is shown to be $\phi_{\rm rms} \sim 50° \, (\theta'')^{-0.4} \bar{\Omega}$ for $\theta \gtrsim 10''$, where $\bar{\Omega}$ measures the density clustered as galaxies in units of the critical density. The probability that individual galaxies (with density parameter Ω_g) along the line of sight will induce an observable kink somewhere in a long straight jet is shown to be $\sim 0.3 \Omega_g \theta''$. Jets may therefore provide an important probe of the potential in the outer regions of galaxies. The probability that groups and clusters of galaxies produce an observable distortion in a distant triple source is estimated to be ~ 0.07. Radio jets and triple sources also provide a probe of unseen matter in the universe clustered on scales ~ 100 kpc.

Subject headings: cosmology — galaxies: clusters of — gravitation — radio sources: general

I. INTRODUCTION

Soon after their discovery in the 1950s, extragalactic radio sources were recognized as important probes of the large scale structure of the universe. In order to infer the comoving power and linear size of a distant radio source from its observed flux and angular size, a specific cosmological model must be assumed. Conversely, with a high redshift sample of distant sources it is in principle possible to study the large scale geometry of the spacetime in which we live. Conventionally this is done in terms of a two parameter (H_o, Ω) Friedmann model. In practice it has hitherto been the case that evolutionary effects are so strong that this has not been possible.

Nevertheless, distant radio sources may provide a valuable probe of small scale inhomogeneity in the universe which is known to be strong on length scales of ~ 100 kpc to 10 Mpc. This is because it has recently been demonstrated that many low redshift sources either contain radio jets that are $\gtrsim 200$ kpc long and remarkably straight (e.g., NGC 6251, Waggett, Warner, and Baldwin 1977; NGC 315, Bridle *et al.* 1979) or at least contain three collinear components, two hot spots disposed on opposite sides of an unresolved compact core (e.g., Cygnus A, Hargrave and Ryle 1976). Rays from distant sources will be deflected by galaxies and by groups and clusters of galaxies along the line of sight, and this may destroy the collinearity.

The one-dimensional velocity dispersions in the outer parts of most galaxies appear to be roughly constant and to lie in the range 200–400 km s^{-1} corresponding to nearly constant values of M/r and constant gravitational deflection $4GM/rc^2 \sim 1''$–$4''$ (e.g., Faber and Gallagher 1979). The radii out to which this mass extends and consequently the total galaxy masses are still unknown. Groups of galaxies exhibit a velocity dispersion of similar order, giving comparable deflections on larger angular scales. One quantitative method for describing the clustering of galaxies (and mass distributed as galaxies) involves the two-point correlation function (for reviews, see Peebles 1979; Fall 1979). If the density of total clumped mass is measured in units of the critical density by $\bar{\Omega}$, then the optical depth in clumps of size θ out to a redshift $z \sim 1$ is $\sim \bar{\Omega}(\theta/\alpha)$, where α is the bending angle due to a single clump. For $\theta \gtrsim \alpha \bar{\Omega}^{-1}$ there will be several clumps along the line of sight, and the deflections will add stochastically. $\bar{\Omega}$ is related to the mean field velocity dispersion through some form of cosmic virial theorem, and estimates of $\bar{\Omega}$ range from ~ 0.07 (e.g., Fall 1979) to ~ 0.7 (e.g., Seldner and Peebles 1979).

Roughly a few percent of the mass in galaxies resides in rich clusters for which the velocity dispersion is ~ 1000 km s^{-1} and the core radii are ~ 150 kpc ($H_0 = 100$ km s^{-1} Mpc^{-1}). Radio jets that lie behind rich clusters can show angular displacements up to $20''$, although the probability of this alignment is fairly low: $\lesssim 10^{-3}$.

The angular resolution of the VLA is better than $0\farcs5$, so it is in principle possible to measure these distortions in intrinsically collinear sources. A sample of high redshift jets and triple sources may provide an invaluable probe of

[1]Supported in part by the National Science Foundation [AST78-05484 and AST79-22012].
[2]Permanent address: Astronomical Observatory, Al. Ujazdowskie 4, 00478 Warszawa, Poland.

inhomogeneity within the universe. There is as yet no completely satisfactory quantitative description of cosmological inhomogeneity, and the interpretation of bends in jets as gravitational deflections must be correspondingly difficult. In this paper we confine our attention to three idealized but tractable problems that we believe allow some quantitative deductions concerning the magnitude of these effects to be made. In § II we compute the rms bend angle induced in an intrinsically collinear triple source in terms of the two-point correlation function. In § III we estimate the probability of a measurable deflection in an intrinsically straight jet due to a galaxy lying close to the line of sight. Third, in § IV we compute the probability of a large distortion due to the presence of a foreground cluster and extend these calculations to take account of a continuum of smaller mass scales. In § V we discuss some problems involved in using these observations as a practical probe of the distribution of matter in the universe. We use units in which $G=c=H_o=1$ throughout the remainder of this paper. Numerical values are scaled to a Hubble constant h measured in units of 100 km s^{-1} Mpc^{-1}.

II. BENDING OF A TRIPLE SOURCE IN THE WEAK DEFLECTION LIMIT

We consider first the effect of matter clustered in the same way as galaxies on the image of an intrinsically collinear triple source of total angular size θ at redshift z. That is to say, we assume that there is a mean density of matter satisfying a two-point correlation function in the continuum limit of the form

$$\langle \rho(\boldsymbol{r}'+\boldsymbol{r})\rho(\boldsymbol{r}')\rangle = \bar{\rho}^2\{1+\xi(r)\}, \tag{1}$$

where $\langle\rangle$ denotes a spatial average over $\boldsymbol{r}'$ and $\bar{\rho}$ is the mean density.

Extended radio sources at redshifts $z \gtrsim 1$ have angular sizes θ in the range 5″–60″. Provided that the deflection is weak, we can define an angular deflection field $\boldsymbol{\delta}(\boldsymbol{\chi})$ which measures the displacement of a ray at position $\boldsymbol{\chi}$ on the sky (with respect to some arbitrary origin). If we imagine expanding $\boldsymbol{\delta}(\boldsymbol{\chi})$ in a Taylor series about the center of the source, then the constant term describes a uniform displacement of the sources, and terms containing the first derivatives $d\delta_i/d\chi_j$, describe expansion, shear, and rotation which are probably undetectable. It is the terms containing second, or higher, derivatives which produce curvature that may be detectable. Most of the curvature will be produced by clustering on an angular scale comparable to θ.

We must first compute the relative deflection between two rays separated by θ due to correlated matter along the line of sight. This calculation has been previously carried out by Gunn (1967) using a different form for the correlation function from that now measured (Peebles and Hauser 1974). Further details are given in Gunn (1967).

For generality it is convenient at this stage to introduce a metric function $\mathcal{D}(z, z')$ which is the angular diameter distance along the unperturbed rays for a small object at redshift z observed from redshift z' where the perturbing mass element is located. Let the proper transverse displacement of the perturbing element from the ray be $\boldsymbol{\eta}'$. The element of proper length parallel to the ray can be written

$$dt' = (1+z')^{-2}(1+\Omega z')^{-1/2}\,dz', \tag{2}$$

where Ω measures the *total* mean density in the universe now, and on the large scale the universe is assumed to obey Friedmann dynamics. Ω may differ significantly from $\tilde{\Omega}$ which measures the density clustered as galaxies. A ray with impact parameter $\boldsymbol{\eta}'$ from a mass element $\rho(\boldsymbol{\eta}', z')\,d^2\eta'\,dt'$ will be bent through an angle $4\rho(\boldsymbol{\eta}', z')\,d^2\eta'\,dt'/\eta'$. Integrating over all mass elements along the line of sight, the total angular deflection of a ray from a source at redshift z is

$$\boldsymbol{\delta}(z) = \frac{-4}{\mathcal{D}(z,0)}\int \frac{dz'\,d^2\eta'\,\boldsymbol{\eta}'\mathcal{D}(z,z')\rho(\boldsymbol{\eta}', z')}{\eta'^2(1+z')^2(1+\Omega z')^{1/2}}. \tag{3}$$

Obviously, averaging over all rays from redshift z, $\langle\boldsymbol{\delta}(z)\rangle = 0$. Instead of computing $\langle\delta^2(z)\rangle$ which is unobservable, we compute the mean square relative deflection of two rays 1,2 separated by a small angle $\boldsymbol{\theta} = \boldsymbol{x}(z)/\mathcal{D}(z,0)$ when they are unperturbed, where $\boldsymbol{x}(z')$ is the separation of the unperturbed rays at redshift z'. Then we define

$$V(\boldsymbol{\theta}, z) = \langle\{\boldsymbol{\delta}_1 - \boldsymbol{\delta}_2\}^2\rangle = \langle\{\boldsymbol{\delta}(z,0) - \boldsymbol{\delta}(z,\boldsymbol{\theta})\}^2\rangle$$

$$= \frac{32}{\mathcal{D}^2(z,0)}\int_0^z \frac{dz'\mathcal{D}^2(z,z')\bar{\rho}^2(z')}{(1+z')^4(1+\Omega z')}\int d^2\lambda \int d\Delta\,\xi(\lambda, \Delta, z')I[\boldsymbol{\lambda}, \boldsymbol{x}(z')], \tag{4}$$

where $\boldsymbol{\lambda}=\boldsymbol{\eta}''-\boldsymbol{\eta}'$ is the relative position vector of two elements of mass resolved perpendicular to the line of sight, $\Delta=z''-z'$ is the redshift difference (assumed small for all correlated masses) and

$$I[\boldsymbol{\lambda},\boldsymbol{x}(z')]=\int d^2\eta'\left\{\frac{\boldsymbol{\eta}'\cdot\boldsymbol{\eta}''}{\eta'^2\eta''^2}-\frac{\boldsymbol{\eta}'\cdot[\boldsymbol{\eta}''-\boldsymbol{x}(z')]}{\eta'^2|\boldsymbol{\eta}''-\boldsymbol{x}(z')|^2}\right\}$$

$$=2\pi\ln\left\{\frac{|\boldsymbol{\lambda}-\boldsymbol{x}(z')|}{\lambda}\right\}. \tag{5}$$

Performing one more integral, we obtain

$$V(\boldsymbol{\theta},z)=\frac{128\pi^2}{\mathfrak{D}^2(z,0)}\int_0^z\frac{dz'\mathfrak{D}^2(z,z')\bar{\rho}^2(z')}{(1+z')^4(1+\Omega z')}\int_0^{x(z')}d\lambda\lambda\ln\left(\frac{x}{\lambda}\right)\int_{-\infty}^{\infty}d\Delta\xi(\lambda,\Delta,z'). \tag{6}$$

We next substitute a power law form for the correlation function:

$$\xi(\lambda,\Delta,z')=\left[\frac{R(z')}{\{\lambda^2+\Delta^2/(1+z')^4(1+\Omega z')\}^{1/2}}\right]^{\gamma}. \tag{7}$$

Equation (6) then reduces to the single integral

$$V(\boldsymbol{\theta},z)=\frac{128\pi^2B(1/2,(\gamma-1)/2)\theta^{3-\gamma}}{(3-\gamma)^2\mathfrak{D}^2(z,0)}\int_0^z\frac{dz'\mathfrak{D}^2(z,z')\mathfrak{D}^{3-\gamma}(z',0)R^{\gamma}(z')\bar{\rho}^2(z')}{(1+z')^2(1+\Omega z')^{1/2}}. \tag{8}$$

We must now make some assumptions about the geometry and the evolution of the correlation function in the expanding universe. The degree of convergence of the two unperturbed rays depends upon the mean density of matter along the line of sight. If we can assume that the mean density is distributed uniformly on this angular scale, then

$$\mathfrak{D}(z,z')=\frac{2}{\Omega^2}\frac{\{(1+\Omega z')^{1/2}[2+\Omega(z-1)]-(1+\Omega z)^{1/2}[2+\Omega(z'-1)]\}}{(1+z')(1+z)^2} \tag{9}$$

for a Friedmann universe. For the angular scale in which we are primarily interested (10″–1′), either the universe is low density ($\Omega\ll 1$), in which case all distance measures are similar, or the expected optical depths in lumps out to $z\gtrsim 1$ is greater than unity and equation (9) is appropriate. This is the influence of the constant term in the correlation function (1). However, for universes in which most of the mass is more strongly clustered, $\mathfrak{D}(z,z')$ for small angles is approximately proportional to the increase of affine parameter along the ray (e.g., Press and Gunn 1973). $V(\boldsymbol{\theta},z)$ is relatively insensitive to the choice of $\mathfrak{D}(z,z')$, and we have used equation (9).

The evolution of the correlation function is more problematical. For lumps in the angular size range that we are considering, the mean density is substantially greater than $\bar{\rho}$ (i.e., $\lambda\ll R$) and the crossing times are correspondingly shorter than the Hubble time. These lumps were therefore probably formed and virialized when the universe was much younger than now ($z\gtrsim 3$), and their proper sizes and masses should not evolve subsequently. This suggests the prescription that $\xi(r)$ varies inversely as $\bar{\rho}$, i.e., proportional to $(1+z')^{-3}$. In this picture we have treated the lumps as independent and ignored possible agglomeration processes which will tend to convert low-mass lumps into higher mass lumps of larger specific binding energy. Our results are fairly sensitive to this assumption and so we show them for $\xi\propto(1+z')^{n-6}$, $n=2$, 3, 4, i.e., $R\propto(1+z')^{(n-6/\gamma)}$.

We can now use the definition $\bar{\rho}=(3/8\pi)\bar{\Omega}(1+z')^3$ to write equation (8) in the form

$$V(\boldsymbol{\theta},z)=18\frac{B[1/2,(\gamma-1)/2]}{(3-\gamma)^2}\bar{\Omega}^2R^{\gamma}(0)\theta^{3-\gamma}S(z,\Omega,n), \tag{10}$$

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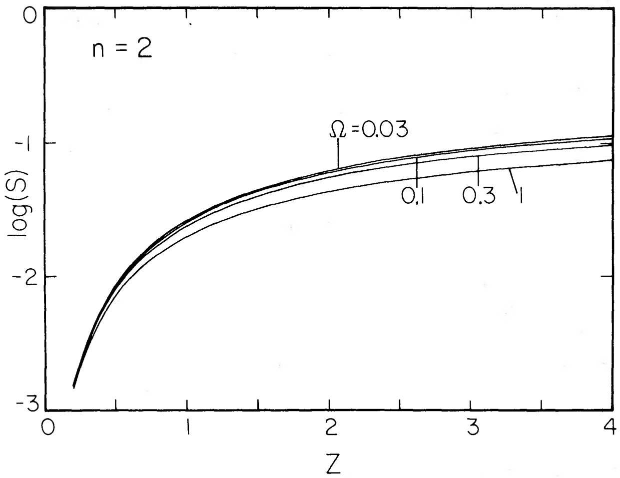

FIG. 1a

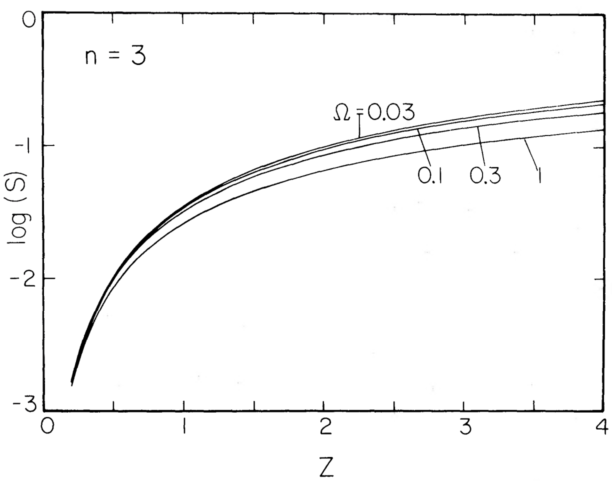

FIG. 1b

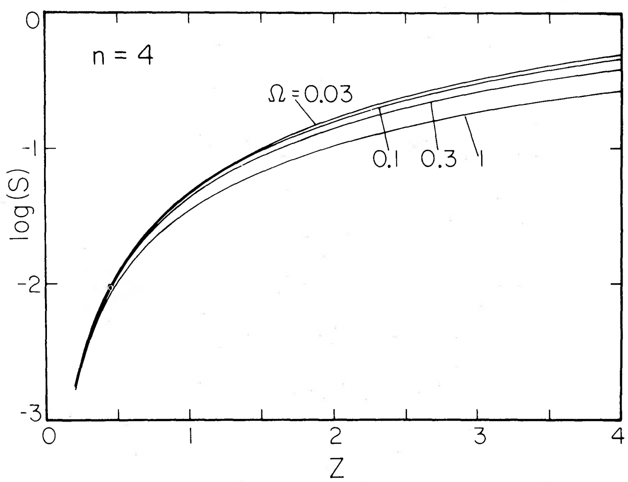

FIG. 1c

FIG. 1.—(a), (b), (c) The dependence of the function $S(z, \Omega, n)$, eq. (11), on redshift z. The curves are for $\Omega=0.03$, 0.1, 0.3 and 1.0 as labeled. The index n describes the evolution of the correlation function.

where

$$S(z,\Omega,n)=\frac{1}{\mathcal{D}^2(z,0)}\int_0^z \frac{dz'\mathcal{D}^2(z,z')\mathcal{D}^{3-\gamma}(z',0)(1+z')^{n-2}}{(1+\Omega z')^{1/2}}. \tag{11}$$

Observationally $\gamma=1.8$, $R(0)=1.8\times10^{-3}$, ($5.3h^{-1}$Mpc) (Fall 1979), and so

$$\{V(\theta,z)\}^{1/2}=3''.0(\theta/1'')^{0.6}\overline{\Omega}S^{1/2}(z,\Omega,n) \tag{12}$$

and

$$S(z,\Omega,n)=0.068z^{2.2};\ z\ll1. \tag{13}$$

$S(z, \Omega, n)$ is plotted in Figure 1 for different values of Ω, n.

When considering the distortion of a linear feature, it is the mean square relative deflection normal to $\boldsymbol{\theta}$, $V_\perp(\boldsymbol{\theta})$, that is important. In fact, it is easier to compute the parallel component $V_\parallel(\boldsymbol{\theta})$ analogously to $V(\boldsymbol{\theta})$ defined in equation (4) with

$$I_{\parallel}(\boldsymbol{\lambda}, \boldsymbol{x}(z'))=\int d^2\eta' \frac{(\boldsymbol{\eta}'\cdot\boldsymbol{x})(\boldsymbol{\eta}''\cdot\boldsymbol{x})}{\eta'^2\eta''^2x^2}-\frac{(\boldsymbol{\eta}'\cdot\boldsymbol{x})[(\boldsymbol{\eta}''-\boldsymbol{x})\cdot\boldsymbol{x}]}{\eta'^2|\boldsymbol{\eta}''-\boldsymbol{x}|^2x^2}. \tag{14}$$

Performing three integrals, the analog of equation (6) is

$$V_{\parallel}(\theta, z)=\frac{32\pi^2}{\mathfrak{D}^2(z,0)}\int_0^z \frac{dz''\mathfrak{D}^2(z,z')}{(1+z')^4(1+\Omega z')}\int_0^{x(z')}d\lambda\lambda\left\{2\ln\left(\frac{x}{\lambda}\right)+\left(1-\frac{\lambda^2}{x^2}\right)\right\}\int d\Delta\xi(\lambda,\Delta,z')$$

$$=\{(4-\gamma)/(5-\gamma)\}V(\theta,z). \tag{15}$$

We then obtain

$$V_{\perp}(\theta,z)=V(\theta,z)-V_{\parallel}(\theta,z)=V(\theta,z)/(5-\gamma)=0.31V(\theta,z). \tag{16}$$

There are two simply related measures of the distortion of a linear source of total angular size θ. First, we can measure the perpendicular displacement $\boldsymbol{\varepsilon}$ of the line center from the displaced line:

$$\boldsymbol{\varepsilon}=\left[\boldsymbol{\delta}(0,z)-\frac{1}{2}\{\boldsymbol{\delta}(\boldsymbol{\theta}/2,z)+\boldsymbol{\delta}(-\boldsymbol{\theta}/2,z)\}\right]\times\boldsymbol{\theta}/\theta \tag{17}$$

Again $\langle\boldsymbol{\varepsilon}\rangle=0$, but the variance is nonzero:

$$\langle\varepsilon^2\rangle=V_{\perp}(\theta/2,z)-\frac{1}{4}V_{\perp}(\theta,z)=0.057V(\theta,z), \tag{18}$$

where (θ, z) is given by equation (12).

Second, we can measure the bending angle ϕ between the two (equal) arms of a triple source of total angular size θ. In the linear approximation $\varepsilon \ll \theta$ which is adequate for present purposes $\langle\phi\rangle=0$ and

$$\langle\phi^2\rangle^{1/2}=4\langle\varepsilon^2\rangle^{1/2}/\theta=160°(\theta/1'')^{-0.4}\bar{\Omega}\delta^{1/2}(z,\Omega,n). \tag{19}$$

Equation (19) is valid for sources of angular size θ compatible with the range of the measured covariance function (i.e., $10'' \lesssim \theta \lesssim 10'$, for $z \gtrsim 1$). From Figure 1*b*, we see that $S(2,0.1,3)\approx 0.1$ and therefore sources at redshift $z\approx 2$ of size $\theta \approx 30''$ will be bent through an angle $\phi \sim 13°\bar{\Omega}$. Changing the exponent n from 2 to 4 roughly doubles the size of the bend in a high redshift source. This distortion is likely to be measurable only if $\bar{\Omega}\sim 1$ and could constitute evidence of a high mean density of clustered matter in the universe if commonly observed.

Our equation (1) is valid only in "continuum limit," i.e., as long as we can neglect the Poissonian contribution from individual galaxies. Treating galaxies as point masses (cf. Fall 1979), we should add one extra term, $\langle\eta_g m^2\rangle\delta(\boldsymbol{r})$, to equation (1), where n_g is the galaxy number density and their mass is m. Since point masses give large deflections for small impact parameters of the rays, this leads to divergences in expressions for $V(\theta,z), V_{\perp}(\theta,z)$, etc. Real galaxies, however, are not pointlike, and we should replace the delta function by a function involving the effective size of a galaxy, r_0; the bending is fairly insensitive to the function assumed, and we use a step function, $3/(4\pi r_0^3)$ for $r<r_0$. The contribution of individual galaxies to the perpendicular component of the variance is given by

$$V_{p\perp}(\theta,z)=\frac{32\pi\langle n_{g0}m^2\rangle}{\mathfrak{D}^2(z,0)}\int_0^z dz'\frac{\mathfrak{D}^2(z,z')(1+z')}{(1+\Omega z')^{1/2}}$$

$$\times\min\left[\frac{1}{4}\left\{\frac{\theta\mathfrak{D}(z',0)}{r_0}\right\}^2, \ln\left\{\frac{\theta\mathfrak{D}(z',0)}{r_0}\right\}+\frac{1}{4}\left\{\frac{r_0}{\theta\mathfrak{D}(z',0)}\right\}^2\right]. \tag{20}$$

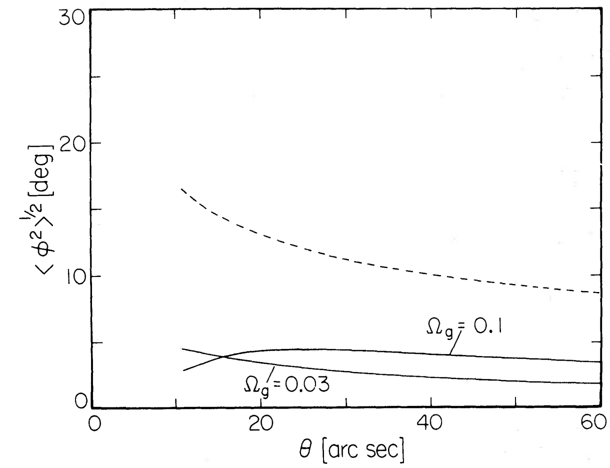

FIG. 2.—Root mean square bend angle $\langle \phi^2 \rangle^{1/2}$ resulting from the action of a correlated matter distribution (density Ω) and individual galaxies (density Ω_g) as a function of the total angular size θ of a triple radio source at a redshift $z=2$. The equivalent galaxy mass-to-light ratio is $(M/L)_\odot \sim 2000\Omega_g h$. The use of the covariance function is unjustified for $\theta \lesssim 10''$. The case $\bar{\Omega}=0.1$, with $\Omega_g=0.03$, 0.1, is represented by the solid line. The case $\bar{\Omega}=1$, which is insensitive to the value of Ω_g, is represented by the dashed line.

Using Schechter's (1976) form for the galaxy luminosity function: $dN/dL \propto L^{-5/4} e^{-L/L_*}$, we obtain

$$\langle n_{g0} m^2 \rangle = \frac{3}{4} \rho_L L^* (M/L)^2 = 3 \times 10^{-12} \Omega_g^2, \tag{21}$$

where $\rho_L = 1.5 \times 10^8 h L_\odot$ Mpc^{-3} is the luminosity density (e.g., Felten 1977), $L^* = 10^{10} h^{-2} \mathfrak{L}_\odot$, and Ω_g is the galaxy density parameter. For a galaxy with $L \sim \frac{3}{4} L^*$, and $v_r \sim 300$ km s^{-1}, the effective size is $r_0 \sim 8 \times 10^{-5} \Omega_g (250 \Omega_g h^{-1}$ kpc), and this is adequate for substitution in equation (20). Finally, we obtain the Poissonian contribution to $\langle \phi^2 \rangle^{1/2}$ using equation (19). In Figure 2 the sum of both contributions to the rms bending angle is plotted for different values of $\bar{\Omega}$ and Ω_g. We see that, as expected, the Poissonian contribution is appreciable relative to expression (19) only if $\bar{\Omega} \lesssim 0.1$.

The quantities $\langle \varepsilon^2 \rangle$ and $\langle \phi^2 \rangle$ are the lowest nonzero moments of the probability distribution functions of ε and ϕ. Higher moments are furnished by higher order correlation functions (odd moments are zero by symmetry). For example, the four-point correlation function could be used to compute $\langle \varepsilon^4 \rangle$, and in principle the complete distribution of ε can be reconstructed from a knowledge of all the even moments. However, from an observational point of view, these higher order correlation functions are difficult to determine and it seems more useful to use an approach based on the multiplicity function to determine the tail in the distribution of ε. This we attempt in § IV.

III. BENDING OF A JET IN THE WEAK DEFLECTION LIMIT

We have seen that measurable distortion ($\phi \gtrsim 10°$) of a typical triple source is only to be expected if $\bar{\Omega} \gtrsim 0.5$. A somewhat related problem is the detectability of distortion of a long jet. This is generally easier to detect because there are effectively several triples of collinear points that can be sampled in an individual object. Long straight jets also constitute a better probe of unseen matter than distant point sources such as quasars. Press and Gunn (1973) showed that the probability of multiple image formation by sufficiently compact objects is roughly Ω_c, their density in units of the critical density. For an individual radio jet the probability of an observable effect is larger because it is unnecessary to form a multiple image and because a larger solid angle of sky is sampled by a single object.

Consider a straight line of angular length θ at a redshift z. Now let there be a distribution of compact objects of mass m_c. The probability that any part of the line be deflected through angle larger than δ is then

$$P(\delta) = \frac{3}{\pi} \Omega_c \left(\frac{\theta}{\delta} \right) U(z, \Omega), \tag{22}$$

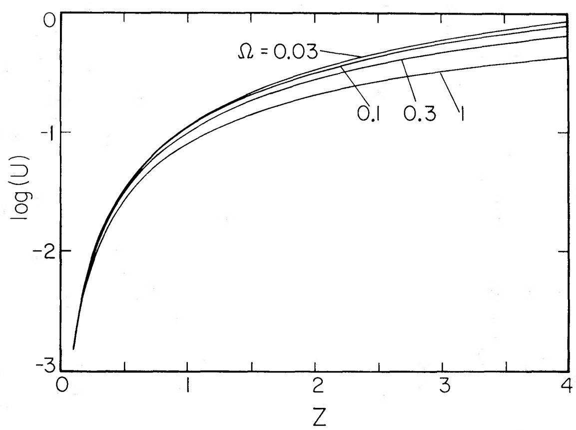

FIG. 3.—The dependence of the function $U(z, \Omega)$, eq. (23), on redshift. The curves are labeled by the total dimensionless mean mass density Ω.

where

$$U(z,\Omega)=\int_0^z dz' \frac{(1+z')}{(1+\Omega z')^{1/2}} \frac{\mathfrak{D}(z',0)\mathfrak{D}(z,z')}{\mathfrak{D}(z,0)}. \tag{23}$$

$U(z, \Omega)$ is plotted in Figure 3. Equation (22) is independent of m_c.

Deflections $\delta \gtrsim \delta_{\min} \sim 0''.3$ should be detectable using the VLA if the jet is intrinsically straight and narrow. Jets that lie behind galaxies (i.e., those regions with velocity dispersions ~ 250 km s^{-1}) will be deflected through angles larger than $\delta_{\min}$. The main contribution to equation (22) from galaxies will therefore arise from jets that do not lie outside the galaxies, and so the point mass approximation should be adequate. For validity of equation (22) rays deflected by $\delta_{\min}$ should pass neither closer to the deflector than $\delta_{\min}$ nor farther away than θ, the jet length. This condition limits the masses of objects that can be detected to the range $2\times10^{10} M_\odot \lesssim M \lesssim 2\times10^{12} M_\odot$ (for $\theta = 30''$ and a deflector at a typical redshift $0.5 \lesssim z \lesssim 1$). On this basis we conclude that Ω_c in equation (22) refers to masses in the above range with high enough central velocity dispersion. Ω_c includes most of the density apparently contained in galaxies, Ω_g. Unlike the probability of forming multiple images, $P(\delta)$ is sensitive to mass in extended galactic halos as well as a hypothetical population of point masses.

Adopting as illustrative numbers $z \gtrsim 1, \theta \sim 30''$, we find $P(0''.3) \sim 10\Omega_c$. This probability is appreciable even for conservative estimates of mass contained in galaxies. Most long, straight, high redshift jets should show a measurable distortion somewhere along their length, and if there is sufficient suitable dark mass, gravitational distortion of these sources will be pronounced.

IV. DISTORTION BY CLUSTERS AND GROUPS OF GALAXIES

Rich clusters of galaxies can produce substantial ($\gtrsim 30''$) deflections of rays passing through them. However, only a few percent of galaxies are found within such tightly bound environments, so the chance of this occurring is rather small. In other words, the distribution of distortions contains a long and possibly significant tail, which we now attempt to estimate.

Unfortunately, clusters are a highly heterogeneous class of objects. They span a wide range of masses, sizes, and velocity dispersions (e.g., Bahcall 1977); some appear regular and relaxed, others are amorphous and presumably still undergoing virialization. A further uncertainty arises out of the unknown evolutionary properties of clusters. Presumably they were less regular and condensed in the past, but we cannot describe this quantitatively.

We idealize clusters' density distribution using the law:

$$\sigma = \sigma_0 (1 + r^2/a^2)^{-p}, \tag{24}$$

where σ is the surface mass density, σ_0 is its value in the center, r the distance from the center, and a is a scale length. We use an index $p = 0.85$ in agreement with that postulated by Fry and Peebles (1980). Our results are not especially sensitive to the choice of p in the range 0.6 to 1 or to the radius at which the density law is truncated.

Observations give measurements of a cluster core radius r_c, where the surface density drops to half its central value, and central velocity dispersion v_r^2. They are related to the parameters of equation (24) by

$$r_c=(2^{1/p}-1)^{1/2}a,$$

$$v_r{}^2=8\left[\frac{\Gamma(p+1/2)}{\Gamma(p)}\right]^2\sigma_0 a\int_0^\infty \frac{dx}{x(1+x^2)^{p+1/2}}\int_0^x \frac{y^2dy}{(1+y^2)^{p+1/2}}. \tag{25}$$

(The double integral can be calculated with the help of a hypergeometric series.) The central surface density obtained using equation (4.2) is $\sigma_0=1.42v_r{}^2/r_c$. (The same relation holds to within 2% for $0.6<p<1$ and also for King models of the density distribution.) Estimates of these parameters for rich clusters are: $r_c=4\times10^{-5}(120h^{-1}$ kpc) and $v_r=3\times10^{-3}$ 900 km s^{-1}) (Bahcall 1977), equivalent to $\sigma_0=0.58h$ g cm^{-2}. The spatial density of rich clusters is roughly $n_{\mathrm{cl}_0}=5\times10^5(2\times10^{-5}\,h^3$ Mpc^{-3}) (Gunn 1980).

For a ray with impact parameter b, the deflection angle is

$$\alpha=4\pi\sigma_0 a\frac{(1+b^2/a^2)^{1-p}-1}{(1-p)(b/a)}. \tag{26}$$

We can now calculate what the impact parameter would be if the rays were not deflected, b', using

$$b'=b\left[1-4\pi\sigma_0\frac{\mathfrak{D}(z,z')\mathfrak{D}(z',0)}{\mathfrak{D}(z,0)}\frac{(1+b^2/a^2)^{1-p}-1}{(1-p)(b/a)^2}\right], \tag{27}$$

where z, z' are redshifts of the source and the cluster, respectively. Solving equation (27) for (b/a), we obtain possible image positions for a known source position. The solution depends on two dimensionless numbers: $W\equiv 4\pi\sigma_0\mathfrak{D}(z,z')\mathfrak{D}(z',0)/\mathfrak{D}(z,0)$ and (b'/a). A collinear triple source can be described by three dimensionless parameters: (b'/a) for the midpoint; (d/a), the scaled length of the source as projected onto the cluster plane; and Ψ, the position angle of the source line. Solving equation (27) for each source component, we obtain the shape of its image. The bend angle now depends only on dimensionless numbers:

$$\phi=\phi(W, b'/a, d/a, \Psi). \tag{28}$$

Averaging over the angle Ψ, we found that, to sufficient accuracy,

$$\langle\phi\rangle_\Psi\approx\frac{2}{\pi}\phi(W, b'/a, d/a).$$

Now we can ask what range of (b'/a) gives $\langle\phi\rangle_\Psi\geq\phi_0$, thus defining a set of source positions leading to an average distortion greater than some given value. Let the cross section $\Sigma(\phi_0, W, d/a)$ be the surface area of this set. The probability of a bend greater than ϕ_0 due to the action of a single cluster can be calculated from

$$P(\phi_0; z,0)=n_{\mathrm{cl}_0}\int_0^z\frac{dz'(1+z')}{(1+\Omega z')^{1/2}}\Sigma(\phi_0, W, d/a), \tag{29}$$

where we have assumed a cluster density evolution $n_{\mathrm{cl}}(z)=n_{\mathrm{cl}_0}(1+z)^3$. The angle θ is related to d/a by

$$d/a=\theta\mathfrak{D}(z',0)/a. \tag{30}$$

Numerical calculations of $P(\phi_0; z,0)$ are plotted in Figure 4 for $\phi_0=10°$, $10''<\theta<60''$, and a few combinations of Ω, z. As we see even for large sources $\theta>30''$, at high redshift, $z\approx3$, in a low density universe the probability of bending through an angle $\phi>10°$ is low; $P\lesssim2\times10^{-3}$.

Although only a small fraction (<5%) of galaxies can be found in rich clusters, the majority probably belong to some kind of bound systems, ranging from small groups with luminosity $L_1=8\times10^9h^{-2}\,L_\odot$ to rich clusters with

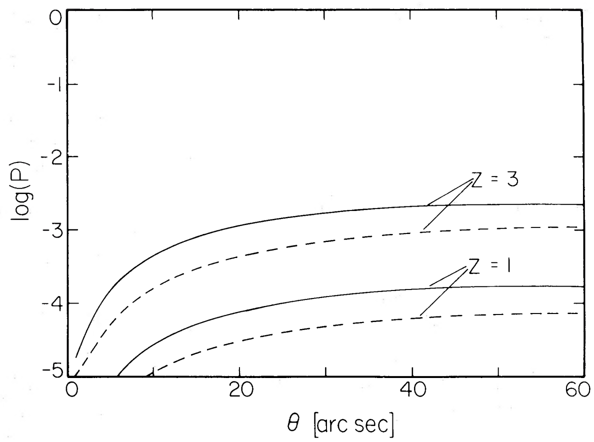

FIG. 4.—The probability of a source being bent by an angle larger than 10° due to clusters. The solid lines are for $\Omega=0.1$; dashed lines are for $\Omega=1$. The probabilities with source redshifts $z=1$, 3 are shown.

luminosity $L_2=3\times10^{12}h^{-2}\,L_\odot$. The observed luminosity function of groups in this range has a form $\eta(L)\propto L^{-7/3}$ (Gott and Turner 1977). Using a luminosity density in the universe of $\rho_L=1.5\times10^8 h\,L_\odot$ Mpc^{-3} and assuming that all the light is emitted by group members, we obtain a normalized luminosity function in the form

$$\eta(L)\,dL=3\times10^{-6}(L/L_2)^{-7/3}(dL/L_2)h^3\,\mathrm{Mpc}^{-3};\,L_1<L<L_2. \quad (31)$$

For the purpose of *estimating* distortions of images by groups, we assume (i) that the mass-to-light ratio is independent of L; (ii) that groups may be represented by density distributions given by equation (24); (iii) that the central velocity dispersion in groups $v_r^{\,2}\propto L^{1/3}$—this law is in rough agreement with measurements in small groups and rich clusters and with the correlation function. The group model given by equation (24) uses two parameters—σ_0 and a— which depend on the group mass (luminosity). Our assumptions (i)–(iii), equation (25), and the proportionality $M\propto\sigma_0 a^2$ imply:

$$\sigma_0\propto L^{-1/3},\,a\propto L^{2/3}. \quad (32)$$

Now we can calculate the probability of a source being distorted by more than ϕ_0 due to action of groups:

$$P_g(\phi_0;z,\theta)=\int_{L_1}^{L_2}dL\eta(L)\int_0^z dt'\sum(\phi_0,W,d/a), \quad (33)$$

where a and W (which is proportional to σ_0) are functions of L according to equation (32). The results of calculating integrals (33) for $10''<\theta<60''$ and a few combinations of z,Ω are shown in Figure 5. The probability of bending through an angle $\phi_0\gtrsim10°$ by groups is substantially higher (up to 0.07 for $\theta=10''$) than by clusters alone due to the great number of groups and their larger central densities (cf. eq. [32]). From Figures 4 and 5 we see that a group (cluster) most effectively distorts high redshift sources of size a few times its scale length. Larger sources, $30''\lesssim\theta\lesssim60''$, are distorted mainly by clusters. Smaller sources, $10''\lesssim\theta\lesssim30''$, are distorted mainly by groups. In our calculation of the bending of triple sources by rich clusters we have probably seriously underestimated the distortion particularly for small sources. This is because the potential in the outermost ($3-5r_c$) parts will probably not be as smooth as given by equation (24). These regions have probably not had time to achieve full virial equilibrium even in apparently regular clusters, and, as noted above, roughly half of rich clusters have an irregular morphology.

There is a complementary problem. Given a high redshift cluster, what is the probability of finding a background source that is substantially distorted? To answer this question requires a knowledge of the radio luminosity function, together with its evolution. For a simple estimate of this probability we assume that there exists a population of radio sources whose comoving density increases as $(1+z)^6$ for $z<2$, with spectral index $\alpha=0.5$ and a luminosity function $dN/dL\propto L^{-2}$. Our results are not especially sensitive to these assumptions. We can then compute the number density of sources brighter than a given observed flux limit, $n_{\mathrm{ob}}(z)$.

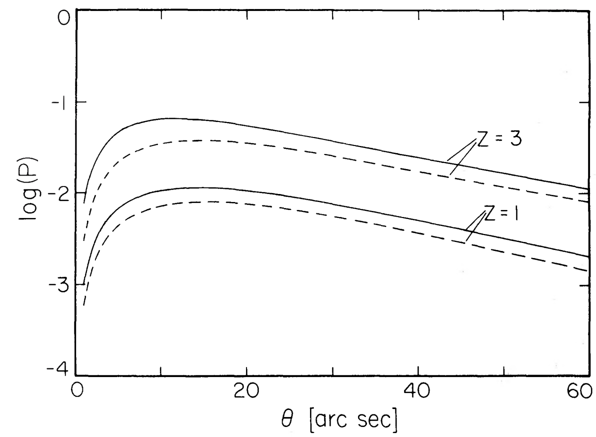

FIG. 5.—The same as in Fig. 4 but now including groups of galaxies, according to the prescription described in § IV.

For a source of given linear sized θ at redshift z' behind a cluster at redshift z we define the solid angle:

$$\mathbb{S}(\phi_0, d_0, z, z') = \sum\left[\phi_0, W, (d_0/a)\mathfrak{D}(z,0)/\mathfrak{D}(z',0)\right]. \quad (34)$$

If the source lies within $\mathbb{S}$, it will be bent through an angle larger than ϕ_0. Averaging over sources of fixed size behind the cluster, we obtain

$$\langle \mathbb{S} \rangle(\phi_0, d, z) = \frac{\int_z^2 dt' \mathfrak{D}^2(z',0) n_{\mathrm{ob}}(z') \mathbb{S}(\phi_0, d_0, z, z')}{\int_z^2 dt' \mathfrak{D}^2(z,0) n_{\mathrm{ob}}(z')}. \quad (35)$$

The dependence of $\langle \mathbb{S} \rangle$ on the cluster redshift z for $\Omega=0.1$ and 1 and for sources of sizes $1^{\infty}h^{-1}$ kpc and $300h^{-1}$ kpc is shown in Figure 6. $\langle \mathbb{S} \rangle$ is maximized for clusters with redshift $z\sim0.5$. We estimate the number of mappable radio sources with flux $\gtrsim 1$ mJy at 600 MHz is $\sim 10^5$ sr^{-1} (Willis 1977). In this case for a single cluster at $z\sim0.5$, the

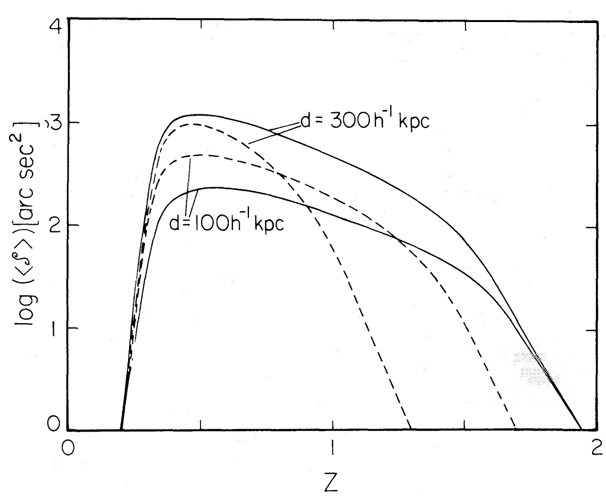

FIG. 6.—The average solid angle in which background sources are bent by more than 10° due to the action of a cluster at a given redshift z. The average is over a flux-limited sample of sources with redshift greater than that of a cluster. The solid lines are for $\Omega=0.1$; the dashed lines $\Omega=1$. The curves are labeled by the total source size.

probability of observing a background source of typical size $\sim 250h^{-1}$ kpc, bent through an angle $\phi \gtrsim 10°$, is $\lesssim 10^{-2}$. It is probably not feasible to search a sample of distant clusters for distorted background radio sources.

V. OBSERVATIONS OF GRAVITATIONAL DISTORTION

In this paper we have made an attempt to quantify gravitational distortion of distant radio sources in an inhomogeneous universe. Useful observation of this effect is handicapped by our ignorance of the cosmological distribution of mass and the knowledge that many nearby radio sources, for which these effects are definitely ignorable, are intrinsically bent. Nevertheless, the fact that the first known example of a multiply-imaged object also contains a jet whose image is probably curved by the gravitational field of an intervening cluster encourages us in our belief that these effects may be generally useful (Roberts, Greenfield, and Burke 1979; Young *et al.* 1980). The discovery of a probable second example of a gravitational lens (Weymann *et al.* 1980) already leads one to suspect that there may be a higher density of lensing objects than is known to be associated with the innermost ~ 10 kpc of brighter galaxies (Sanitt 1971).

In § II, we considered the distortion of a linear triple source and showed that unless $\bar{\Omega}$, which measures the density clustered as galaxies on linear scales in the range $0.1-10h^{-1}$ Mpc is substantially larger than indicated by dynamical studies of groups ($\bar{\Omega} \sim 0.1$) then this distortion is probably not detectable. If distant sources are found to be significantly less linear than local sources, then this may indicate that there is a substantial component of "dark matter" (e.g., massive neutrinos, black holes) that is clumped in a manner different from galaxies. (One physically possible way in which this may occur is if galaxies form in potential wells dominated by dark matter [cf. White and Rees 1979] with a probability decreasing with distance from the nearest galaxy [e.g., if the cooling of hot intergalactic gas is triggered by energetic processes in the neighboring galaxy; cf. Cowie and Ostriker 1980]. The correlation function for the dark matter may then have a flatter slope than the galaxy correlation function.)

As argued in § III, radio jets provide a more useful probe than triple sources or quasars; and most long, distant jets should show measurable kinks caused by intervening galaxies. Deep optical photometry in the neighborhood of these kinks may be able to reveal the presence of such a galaxy. The amplitude of the kinks would then provide a direct measure of the gravitational potential in the outermost parts of the galaxy. We note that the best known example of an isolated quasar with a radio jet, 4C 32.69 (Potash and Wardle 1980), does indeed contain at least two bends. At the redshift of the quasar ($z = 0.69$), galaxy-induced bending of the magnitude and shape observed is relatively improbable, but any such galaxy should be definitely detectable.

In § IV we attempted to estimate the tail of the distribution of bend angles for triple sources due to clusters and groups. In this instance we have normalized to the measured velocity dispersions rather than $\bar{\Omega}$ as in § II. This involves the use of a smooth model cluster potential that is at best a fair representation of only a minority of rich clusters and neglects any inhomogeneity in the unvirialized outer regions. Nevertheless, we were able to demonstrate that the probability of an individual radio source undergoing a strong distortion induced by a rich cluster is extremely small ($\lesssim 2\times 10^{-3}$). Significant distortion induced by small groups is more probable ($\lesssim 0.07$), although the results are sensitive to the assumed velocity dispersion and the assumption of spherical symmetry in large groups. The probability that there is a distorted radio source behind a given cluster is likewise shown to be small.

The calculation of the probability of lensing of quasars by galaxies or optical quasars and of galaxies by clusters is complicated by two strong selection effects. First, the production of multiple images will be accompanied by magnification. The number of lensed galaxies must be compared with the density of apparently fainter unlensed galaxies. Second, the presence of the galaxy may lead to a lower detection efficiency for the quasar. Both of these selection effects will be lessened in the case of extended radio sources. The surface brightness of an image is unchanged by gravitational distortion. Provided that the deflection is relatively weak over most of the source, the total flux of an extended component will not be seriously altered. If the source is taken from a low frequency radio survey, the sample should be fairly homogeneous. As most sources are much larger than galaxies, radio emission associated with the galaxies is unlikely to be of importance. A further difficulty associated with the lensing of optical quasars is that the distribution of matter (i.e., stars) in a galaxy may not be smooth on the scale of a source. This should not be a problem for radio sources.

In this paper we have made an attempt to estimate the importance of gravitational deflection in distorting the images of distant radio sources and to show how radio jets may provide a unique probe of the distribution of matter in the universe. However, calculations of the magnitude of this distortion show that it is generally somewhat smaller than given by simple estimates (cf. § I) and will probably not be easily detectable.

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R. D. Blandford and M. Jaroszyński: Theoretical Astrophysics 130-33, California Institute of Technology, Pasadena, CA 91125