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ENERGETIC PARTICLE SPECTRA IN FINITE SHOCKS: THE EARTH'S BOW SHOCK

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ABSTRACT

Ipavich *et al.* have reported fast particle spectra near the Earth's bow shock that are exponentials in the quantity energy per charge, E/Q. It is shown that this is expected if the fast particles escape the shock via resonant diffusion to unconnected field lines. The calculated e-folding value of E/Q is, to first approximation, independent of the level of turbulence near the shock and is in good agreement with observation. Predictions of this picture and further evidence supporting it are presented. If the model is correct, then the parallel and perpendicular diffusion coefficients can be determined observationally through combined measurements of gradients and spectra, and the Earth's bow shock can serve as an accurate laboratory for testing theories of diffusion.

Subject headings: Earth: general — shock waves

I. INTRODUCTION

First-order Fermi acceleration at shock fronts is an idea some two decades old (e.g., Schatzman 1962). The physics of the acceleration is very simple; it is just compression. All efforts to compute the spectrum of fast particles produced by such a mechanism have dealt, in one way or another, with the question of how particles escape from the region of acceleration. For in the absence of escape, particles formally attain unrealistically high energies. The power-law spectra usually attributed to shock acceleration (Schatzmann 1962; Fisk 1971; and under more general assumptions, Krymsky 1976, 1977; Axford, Leer, and Skadron 1977; Bell 1978; Blandford and Ostriker 1978) are derived under the assumption that the shock is effectively of infinite extent, and that particles can therefore escape only by getting very far downstream of the shock.

On the other hand, in many instances of shock acceleration the shock can be of sufficiently small extent that the main escape route is lateral convection or diffusion off to the side of the shock; this changes the spectrum drastically. The Earth's bow shock may be one example. Understanding how to interpret spectra of fast particles at the bow shock may lead to diagnostics of particle acceleration in solar flares and interplanetary shocks, where finite shock size may also be critical. Moreover, understanding the escape route of fast particles from the shock helps one estimate, in the case of solar flares, what fraction of high-energy particles rain down on the surface (making γ-ray lines, etc.).

In this paper we discuss particle acceleration and escape at the Earth's bow shock. Ipavich *et al.* (1979) have reported spectra for events during which the solar wind $\boldsymbol{B}$-field was radial, i.e., the field lines were roughly normal to the nose of the bow shock. To within experimental accuracy, the spectra were exponentials in the quantity energy per charge, E/Q. The e-folding energy per charge E/Q_0 may vary significantly over the time scale of a day (e.g., 28 ± 2 keV/e on 1977 October 31 at 0300–0400 to 20 ± 2 keV/e at 1905–1935), yet at any given time they are exponentials, and the variation of E/Q_0 among the different ion species is less than 10%. This suggests that the particular functional form of the spectra is significant at a basic level.

In a previous paper (Eichler 1979*a*, hereafter Paper I) we pointed out that the composition is expected to be constant in E/Q if the mean free path λ scales linearly with gyroradius r_g, assuming the geometry of the diffusion tensor remains the same. This is true regardless of whether escape is primarily via convection or via diffusion. However, the precise spectral form depends on the mode of escape.

Below we propose that the observed spectra can be accounted for if the ions escape the shock by resonant cross field diffusion off to the side, where, at sufficiently low intensity, they stream freely to infinity along unconnected field lines. Moreover, the result obtains regardless of how λ depends on r_g; in this sense the argument is more general than in Paper I.

In § II we list our simplifying assumptions. In § III, we derive the principal result. In § IV, we point out that, given this model, the parallel and perpendicular diffusion coefficients $D_{\parallel}$ and $D_{\perp}$ can be measured rather accurately at the bow shock through indirect means.

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II. MODEL ASSUMPTIONS

a) *Morphology*

We consider, for simplicity, a square defined by the plane segment $z = 0$, $-a < x < a$, $-a < y < a$. In the shock frame, the fluid flows from $z = -\infty$ to $z = +\infty$. It is assumed that the shock is a flat plane segment, rather than bowed, and that the compression ratio ξ is a constant over the shock surface. The fluid velocity u is assumed to change only at the shock itself (see Fig. 1). The magnetic field is assumed to be perpendicular to the shock.

b) *Diffusion*

We assume that the parallel and perpendicular diffusion coefficients, $D_{\parallel}$ and $D_{\perp}$, are inversely proportional to each other. This is roughly equivalent to the assumption that, for each reversal of direction, the particle takes a step perpendicular to the magnetic field of one gyroradius (to within a geometric factor of order unity). This follows from quasi-linear theory (Jokipii 1971, eqs. [46], [47], [55], and [56]) when the "field meandering" term is ignored, and the turbulence is not terribly anisotropic (or, at any rate, when the anisotropy is not very scale dependent). The field meandering term, we argue, may be justifiably ignored near the Earth's bow shock. It is derived from perturbation theory, assuming that the zeroth-order particle trajectory points in a given direction along the field. The lateral displacement thus contains a term proportional to the random walk of the field line itself over a distance $v_{\parallel} t$.

However, in a time much longer than one scattering time but smaller than the MHD time scale, a fast particle travels back and forth over the same segment of field lines many times. On such a time scale, quasi-linear theory greatly overestimates the lateral transport of a particle riding a given field line. Moreover, it is the scattering to different field lines, rather than lateral displacement in absolute space, that allows a particle to escape the shock, so the accuracy of the model used here is increased by ignoring whatever real contribution to $D_{\perp}$ field meandering makes.

We further assume that $D_{\parallel}$ and $D_{\perp}$ do not depend on x or y. That is the assumption under which equations (55) and (56) of Jokipii (1971) are formally valid. In the equations of the following section, $D_{\parallel}$ and $D_{\perp}$ are treated as constants in z as well. However, it is straightforward to show, through the transformation $z \rightarrow \int_0^z D_{\parallel}^{-1} dz$, that variation in D with z does not affect the results.

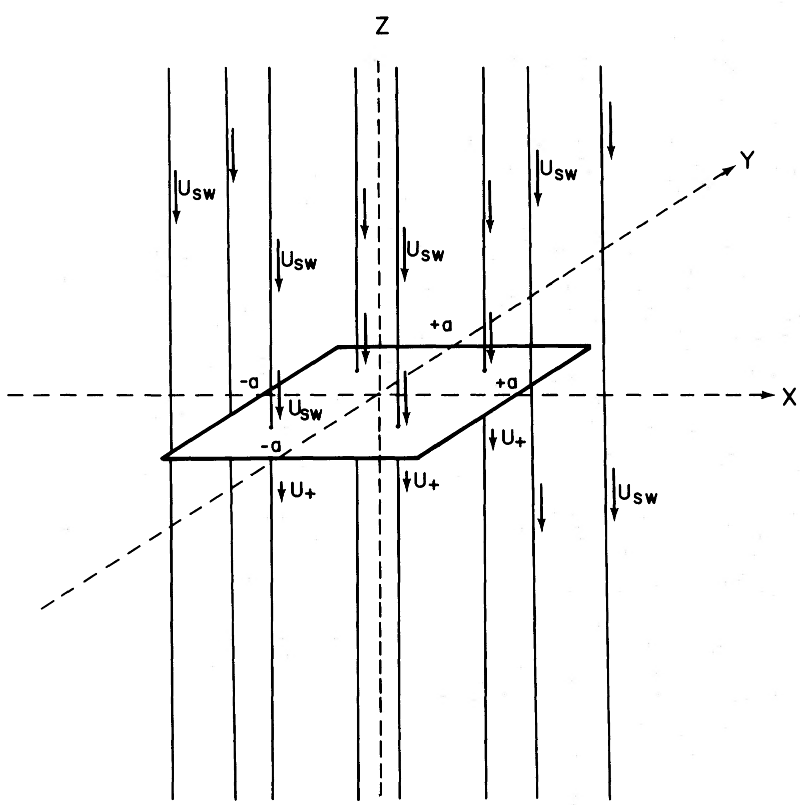

FIG. 1.—Idealized flow pattern is shown. Solid vertical lines are magnetic field lines.

c) *Boundary Conditions*

The boundary conditions at $x = \pm a$, $y = \pm a$ are a complicated matter. They depend strongly on how the diffusion tensor on a given field line varies as a function of how that field line connects to the shock. On field lines connected to the shock, there are fast particles that generate MHD turbulence. Clearly the larger the amplitude of the turbulence, the smaller $D_{\|}$ and the larger $D_{\perp}$. Far from the shock, on the other hand, the magnetic field is nearly turbulence-free on the scale of the Earth's bow shock. The particles can stream freely along the field lines, and $D_{\perp}$ is very small. Such a region acts as a particle sink because a particle that is scattered onto a quiet field line is more likely to stream freely along that field line to infinity than to be scattered back to a connecting field line. It appears from observation that the transition from one region to the other is rather abrupt, for the presence of fast particles and turbulence on a particular field line is a very reliable function of whether the field line connects to the bow shock (e.g., Fairfield 1969). (Note that the abruptness of the transition does not necessarily imply a small $D_{\perp}$ everywhere, only on unconnected field lines.) We thus assume the walls $x = \pm a$ and $y = \pm a$ to be perfectly absorbing.

d) *Injection*

Injection at low energies is assumed to come from thermal particles that are scattered from the postshock region back ahead of the shock (Jokipii 1966; Eichler 1979*b*). This process is expected to be most effective when the magnetic field lines are perpendicular to the shock because that is when the condition for particle reflection is most easily satisfied. Specifically, in order for a post shock particle more than a gyroradius downstream of the shock to get back upstream it must satisfy

$$(\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{B}})(\boldsymbol{v} \cdot \hat{\boldsymbol{B}}) \geq u_+ \,, \tag{1}$$

where $\hat{\boldsymbol{n}}$ is the shock normal, u_+ is the velocity of the shocked fluid in the frame of the shock, $\hat{\boldsymbol{B}}$ is the unit vector in the direction of the postshock magnetic field, and $\boldsymbol{v}$ is the particle's velocity. Since the postshock thermal velocity is only comparable to u_+, equation (1) is most easily satisfied by thermal particles when the magnetic field is quasi-parallel to $\hat{\boldsymbol{n}}$. The strong Alfvén wave turbulence on scales of thermal gyroradii or larger that is expected from thermal particle backscatter is indeed observed when the angle between $\hat{\boldsymbol{n}}$ and the upstream magnetic field is less than 45° and seldom observed when that angle exceeds 50° (Fairfield 1974; Greenstadt and Russel 1979). This is consistent with the correlation between thermal particle injection and $\hat{\boldsymbol{B}} \cdot \hat{\boldsymbol{n}}$ that is expected according to the above considerations.

When $\boldsymbol{B}$ is perpendicular to the nose of the bow shock, equation (1) is satisfied most readily at the nose of the bow shock, so injection is assumed here to peak near the middle of the shock and taper off toward the edges. We adopt an injection function proportional to $\cos(\pi x/2a)\cos(\pi y/2a)$; i.e., we keep only the lowest harmonic. Assuming the injection function is even around the origin, the next highest nonvanishing harmonics would be $\cos(n\pi x/2a)\cos(n\pi y/2a)$ with $nm \gtrsim 3$. It is pointed out in the next section that these higher harmonics, even if nonvanishing at the injection energy $E_{\rm inj}$, have much softer energy spectra than the lowest one, so they are, in any case, unimportant at the energies of interest.

e) *Time Dependence*

The events observed by Ipavich *et al.* (1979) are generally flattopped, i.e., the count rate, after an initial rise, reaches a constant level for the remainder of the event, suggesting that the event reached steady state. As discussed by these authors, the particle spectrum is steady only after the intensity has leveled off, further supporting the steady state picture. We thus assume steady state in the calculations below.

III. BASIC CALCULATION

The equation that describes the acceleration at $E \gg E_{\rm inj}$ is (Fisk 1971)

$$\frac{\partial f}{\partial t} = -\nabla \cdot uf + \frac{\partial}{\partial z} D_{\|} \frac{\partial}{\partial z} f + \frac{\partial}{\partial x} D_{\perp} \frac{\partial}{\partial x} f + \frac{\partial}{\partial y} D_{\perp} \frac{\partial}{\partial y} f + \frac{2}{3}(\nabla \cdot u)\frac{\partial f}{\partial \ln E}\,, \tag{2}$$

where f is the particle number density per unit logarithm in energy E. In the limit where the perpendicular spatial transport is dominated by diffusion, we may neglect terms containing u_x and u_y. Assuming f vanishes at the boundaries $x = \pm a$, steady state, and for simplicity that $D_{\|}$ and $D_{\perp}$ are constant in space, equation (2) allows formal solutions on either side of the shock

$$f = \sum_{n,m} g_{nm}(E) \exp(k_{\|} z) \cos\left(\frac{n\pi x}{2a}\right) \cos\left(\frac{n\pi y}{2}\right), \tag{3}$$

where

$$k_{\|}^{-} = \frac{u_{\rm sw} + [u_{\rm sw}^2 + (\pi^2/a^2)D_{\|} D_{\perp}(n^2 + m^2)]^{1/2}}{2D_{\|}} \text{ upstream} \tag{4a}$$

and

$$k_{\parallel}{}^{+} = \frac{u_{+} - [u_{+}{}^{2} + (\pi^2/a^2)D_{\parallel} D_{\perp}(n^2 + m^2)]^{1/2}}{2D_{\parallel}} \text{ downstream} . \tag{4b}$$

As injection peaks at the center of the shock, we take the lowest harmonic to be a reasonable approximation to $f(E_{\rm inj}, z = 0)$, so that

$$g(E_{\rm inj}) = 1 , \quad \text{for} \quad n = 1, m = 1 ,$$
$$0 , \quad \text{for} \quad nm > 1 , \tag{5}$$

where $g_{11}(E_{\rm inj})$ is normalized to unity for convenience.

Integrating equation (2) across the shock from $z = 0+$ to $z = 0-$ gives, with equations (3) and (4),

$$\tfrac{2}{3}(u_{+} - u_{\rm sw})\frac{dg_{11}}{d \ln E} = \left[\frac{[u_{\rm sw}{}^{2} + (\pi^2/a^2)(2D_{\parallel} D_{\perp})]^{1/2}}{2} - \frac{u_{\rm sw}}{2} + \frac{[u_{+}{}^{2} + (\pi^2/a^2)(2D_{\parallel} D_{\perp})]^{1/2}}{2} + \frac{u_{+}}{2}\right] g_{11} \tag{6a}$$

As $a \to \infty$, equation (6a) just gives the familiar power-law spectrum. In the opposite limit, the terms proportional to $u_{\rm sw}$ and to u_{+} are unimportant and

$$\tfrac{2}{3}(u_{+} - u_{\rm sw})\frac{dg_{11}}{d \ln E} = (2D_{\parallel} D_{\perp})^{1/2}(\pi/a)g_{11} , \tag{6b}$$

whence

$$g_{11} = \exp \int_{E_{\rm inj}}^{E} \frac{3\pi(2D_{\parallel} D_{\perp})^{1/2}}{2a(u_{+} - u_{\rm sw})} d \ln E . \tag{7}$$

Equation (6b) is valid when the spectral index, $d \ln g/d \ln E$, is equal to or greater in absolute magnitude than unity—$(2D_{\parallel} D_{\perp})^{1/2}\pi/a \gtrsim \frac{4}{9}u_{\rm sw}$ for $u_{\rm sw}/u_{+} = 3$. It can be verified by numerical computation that over this range, the right-hand sides of equations (6a) and (6b) differ by only $\sim 10\%$.

Now $(D_{\parallel} D_{\perp})^{1/2}$ is known to much better accuracy than either $D_{\parallel}$ or $D_{\perp}$ separately. Assume that a particle moves cross-field by about one gyroradius (r_g) for each reversal of direction. This is formally the case in quasi-linear theory if the "field meandering" term is neglected (see above) and the geometry of the turbulence is not sensitive to scale. It then follows (or see Jokipii 1971 for a formal, quasi-linear derivation) that the diffusion tensor is of the form

$$D_{\parallel} = \frac{v^2}{3}\tau ,$$
$$D_{\perp} = \frac{\eta r_g{}^2}{t} = \tfrac{2}{3}\eta \frac{m^2c^2v^2}{Q^2B^2\tau} , \tag{8}$$

where τ is the mean free "collision" time, v is total particle velocity, m is mass, Q is charge, c is the speed of light, B is magnetic field strength, and η is a dimensionless number of order unity depending on the geometry of the turbulence. The major uncertainty in either $D_{\parallel}$ or $D_{\perp}$ is in τ, but $(D_{\parallel} D_{\perp})^{1/2}$ is independent of τ and is given by

$$(D_{\parallel} D_{\perp})^{1/2} = \left(\frac{2\eta}{9}\right)^{1/2} \frac{c}{B}\frac{mv^2}{Q}$$
$$= \left(\frac{8\eta}{9}\right)^{1/2} \frac{c}{B}\left(\frac{E}{Q}\right) . \tag{9}$$

Substituting into equation (7),

$$g \propto \exp [-(E/Q)/(E/Q)_0] , \tag{10}$$

where $(E/Q)_0$ is given by

$$\left(\frac{E}{Q}\right)_0^{-1} = \frac{3\pi}{2a(u_{\rm sw} - u_{+})}\left[\frac{(2D_{\parallel} D_{\perp})^{1/2}}{E/Q}\right] , \tag{11a}$$

$$= \frac{2\pi(\eta)^{1/2}}{a(u_{\rm sw} - u_{+})}\frac{c}{B} \tag{11b}$$

It is straightforward to show that the e-folding E/Q for higher harmonics, $nm \geq 3$, is lower than $(E/Q)_0$ by a factor of $[(n^2 + m^2)/2]^{1/2}$. We are concerned here with the range $E/Q > (E/Q)_0$, so even if higher harmonics do not vanish at $E_{\rm inj}$, they are in any case unimportant in the range of interest.

The simple form of the spectrum and its independence of many details can be understood by considering cosmic ray diffusion on a lattice in the vicinity of the shock. Assume the particle takes a step (equal to one lattice point spacing) in the z-direction for each step in the x-direction. If the shock is n lattice points wide, the particle makes on the order of n^2 steps before reaching the boundary. The average distance (in units of lattice points) from the shock, prior to escape, scales as n, so the number of shock crossings scales as n. The percentage gain in energy for each shock crossing, by the familiar arguments of first-order Fermi acceleration, scales as $1/v$. The "lattice spacing" in the x-y plane scales as gyroradius, or mv/Q, so for a given shock, n scales as $(mv/Q)^{-1}$. Finally, the average further percentage gain in energy of a particle of energy E scales as n/v or $(E/Q)^{-1}$, implying a spectrum proportional to $\exp(-E/Q)$. The point is that this result is independent of the lattice spacing in the z-direction, or its variation with z, because the only relevant information regarding a particle's location is how many mean free paths separate it from the shock and from the boundary. *Thus, the spectrum of fast particles is, to first approximation, independent of the properties of the hydromagnetic turbulence near the shock.*

We may obtain a fair estimate of $(E/Q)_0$ from quasi-linear theory. If the turbulence is assumed, as a first approximation, to be isotropic, quasi-linear theory gives a value for η of $\frac{1}{4}$ (see Jokipii 1971, eqs. (46), (47), (55), and (56)]. The solar wind velocity is taken to be 400 km s^{-1}, and the average value of u_+ behind the shock is taken to be $\frac{1}{3}u_{\rm sw}$. $\boldsymbol{B}$ is taken to be 4×10^{-5} gauss, and a is taken to be the radius of curvature of the Earth's bow shock, 6×10^9 cm. These values, when used in equation (11b), yield an e-folding energy per charge of

$$\begin{aligned}(E/Q)_0 &= 67(\text{cgs}) \\ &= 20 \text{ Kev per electron charge} . \end{aligned} \tag{12}$$

(E/Q_0) is observed to range from 10 to 28 Kev/e (Ipavich *et al.* 1979). The variation can be caused by variations in $|\boldsymbol{B}|$, $u_{\rm sw}$, and a. Given that these quantities vary, as well as the theoretically uncertain factors of order unity, the agreement is fine.

We now return and check the validity of neglecting convective losses. The diffusion time scale for particles to escape the shock is

$$\tau_D = \frac{D_\perp^{-1}}{2}\left(\frac{2a}{\pi}\right)^2 = \left(\frac{D_\parallel}{2D_\parallel D_\perp}\right)\left(\frac{2a}{\pi}\right)^2 \tag{13}$$

By equations (11a) and (13), with $u_{\rm sw}/u_+ = 3$, the diffusion time scale for particles at, say, $E/Q = \frac{3}{2}(E/Q)_0$, is

$$\tau_D = \frac{9D_\parallel}{u_{\rm sw}^2} \tag{14}$$

By contrast, a given field line is connected to the shock for a time of

$$\tau_c \approx \frac{2a}{u_{\rm sw} \sin\theta}, \tag{15}$$

where θ is the angle between the solar wind velocity and $\boldsymbol{B}$-field vectors. Thus, our neglect of convective losses is justified for $\tau_D \ll \tau_c$

$$\theta \ll \theta_c \equiv \sin^{-1}\left(\frac{2a}{u_{\rm sw}\tau_D}\right). \tag{16}$$

For particles at $E/Q \approx 30$ Kev/$e \approx \frac{3}{2}(E/Q)_0$, the scale height of the distribution upstream of the shock is reported by Ipavich *et al.* 1981 to be ~ 7 earth radii ($\approx 0.8a$), so that $D_\parallel \approx 0.9u_{\rm sw}a$ by equation (4a), and

$$\theta_c \approx 14° . \tag{17}$$

On an hourly average, θ was less than 6° (Interplanetary Medium Data Book, Suppl. 1) for the two 1977 October 31 events studied by Ipavich *et al.* (1979), so neglect of convective losses is justified at $E/Q \gtrsim \frac{3}{2}(E/Q)_0$, and probably at even lower E/Q. In the limit of an infinite plane shock with a compression ratio of 4, the spectral index for non-relativistic particles is $-\frac{1}{2}$ (Bell 1978), so convection in the z-direction can be neglected only at $E/Q > \frac{1}{2}(E/Q)_0$. Just before and after the events, θ exceeds 20°, strongly suggesting (a) that the decrease in θ is responsible for the sharp increase in particle flux (Paper I) and (b) that $\theta_c \lesssim 20°$.

IV. THE BOW SHOCK AS A DIFFUSION LABORATORY

If the model presented here is assumed to be correct, then $D_{\parallel}$ and $D_{\perp}$ can be determined separately by measuring $k_{\parallel}$ and $(E/Q)_0$. Measurement of the $(E/Q)_0$ immediately gives $D_{\parallel} D_{\perp}$ by equation (11a). Given $k_{\parallel}$ and equation (4a), $D_{\parallel}$ and $D_{\perp}$ can then be determined individually with no further assumptions. Given the simple relations between $D_{\parallel} D_{\perp}$ and measured quantities, $D_{\parallel}$ and $D_{\perp}$ should be determinable to good accuracy.

Thus, the Earth's bow shock may be a useful laboratory for studying cosmic ray diffusion theory. In such a role, it offers a number of distinct advantages. As $D_{\parallel} D_{\perp}$ can be obtained directly from the spectrum, $D_{\parallel}$ and $D_{\perp}$ can surely be determined to far better accuracy than by merely measuring gradients. As the length scale is so small compared to 1 AU, there is no possibility that ∇B drifts complicate particle transport, and there are no uncertainties caused by possible motion out of the ecliptic. Steady state analysis can be used, and the geometry is very simple, so little if any modeling is required to unravel the observations.

In view of the failure of quasi-linear theory in explaining cosmic ray diffusion in the solar wind at large, the Earth's bow shock seems especially attractive as a diffusion laboratory. Various explanations of the failure of quasi-linear theory in interplanetary space might be tested by applying them to the Earth's bow shock.

V. CONCLUSIONS AND DISCUSSION

The predicted spectrum is in excellent agreement with observation, and on these grounds we propose that cross field diffusion is the primary means by which particles escape the Earth's bow shock in events like that observed by Ipavich *et al.* (1979). That is, we claim that the escape time from the shock of particles at $E/Q \gtrsim 20$ Kev/e is shorter than the lifetime of a given magnetic field line on the shock.

We emphasize, however, that our conclusions apply only to the event reported on by Ipavich *et al.*, (where θ is less than θ_c, as defined in eq. [16]), and other "radial field" events. When $\theta > \theta_c$, the finite lifetime of a given field line on the shock becomes significant, and convection can be the dominant means of escape. Indeed, we have predicted in Paper I that, precisely for this reason, particle acceleration should be most effective when θ is small. To this prediction, we now add that at $\theta < \theta_c$, further decreases in θ should not enhance the acceleration appreciably.

Another prediction of the model is that for $\theta \ll \theta_c$, there should be no dawn-dusk asymmetry in the particle intensities and spectra. When, on the other hand, convection is the primary cause of escape—say the field lines make contact on the dusk side and exit on the dawn side—there should be harder, more intense spectra on the dawn side where the fast particle population is oldest.

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