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POLARIZED RADIATION FROM HOT PLASMAS AND APPLICATIONS TO AM HERCULIS BINARIES

G. CHANMUGAM
Joint Institute for Laboratory Astrophysics, National Bureau of Standards and University of Colorado
AND
G. A. DULK
Department of Astro-Geophysics, University of Colorado
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ABSTRACT

Absorption coefficients for the ordinary and extraordinary modes are calculated for cyclotron radiation from hot plasmas with temperatures between 0.2 and 20 keV. The results are applied to the accretion columns of the AM Herculis binaries. The linear and circular polarization properties of the observed polarized light can be understood if the emission is at approximately the fifth harmonic of the cyclotron frequency. The magnetic fields in these systems is deduced to be roughly 4×10^7 gauss.
Subject headings: plasmas — polarization — stars: binaries — stars: magnetic

I. INTRODUCTION

The AM Herculis binaries, consisting of AM Her, AN Ursae Majoris, VV Puppis, and 2A 0311–227, are believed to contain a magnetic white dwarf which accretes matter from a lower-main-sequence companion and emits radiation over a wide range of wavelengths from the infrared to hard X-rays (see, e.g., the reviews by Angel 1978; Kruszewski 1978; and especially Chiappetti, Tanzi, and Treves 1980). The main characteristic which distinguishes them from other cataclysmic variables is that they emit strong ($\sim 10\%$) linearly and circularly polarized light (Tapia 1977*a*, *b*, 1979; Michalsky, Stokes, and Stokes 1977; Krzeminski and Serkowski 1977), probably as a result of cyclotron emission in a strong magnetic field $\boldsymbol{B}$. Despite the significance of the polarized radiation, there have been no quantitative calculations of its properties for the parameter regimes of interest in these stars. Previous work has concentrated on applications to fusion reactors (e.g., Trubnikov 1958; Bekefi 1966) and the Sun (Dulk, Melrose, and White 1979). In this paper, we present the first such calculations for the circumstances applicable to the accretion columns of the magnetic white dwarfs.

In § II, we present an efficient and reliable method for calculating the cyclotron absorption coefficients, in the ordinary and extraordinary modes, for a plasma containing isotropic electrons with a Maxwellian distribution. In § III we discuss the polarization of the emitted radiation. The results are discussed in § IV and compared with the observations of the four AM Her type systems in § V. We are able to account for the remarkable linear polarization pulse and the circular polarization observed if the emission is at approximately the fifth cyclotron harmonic and not at the fundamental cyclotron frequency, as had been originally assumed. This implies that the field strength in these systems, for the polarized radiation to be in the optical, is roughly 4×10^7 gauss.

II. CYCLOTRON EMISSION AND ABSORPTION

Consider a homogeneous collisionless plasma with a uniform and static magnetic field $\boldsymbol{B}$ which is inclined at an angle θ with the direction $\hat{k}$ of propagation of the electromagnetic wave. Then the indices of refraction $n_\pm$, where the subscripts $+$, $-$ correspond to the ordinary and extraordinary modes, respectively, are given by (Ginzburg 1964)

$$n_\pm = 1 + \frac{2(\omega_p/\omega)^2(\omega_p{}^2/\omega^2 - 1)}{[\pm\{(\omega_c/\omega)^4 \sin^4\theta + 4(\omega_c/\omega)^2(\omega_p{}^2/\omega^2 - 1)^2 \cos^2\theta\}^{1/2} - 2(\omega_p{}^2/\omega^2 - 1) - (\omega_c/\omega)^2 \sin^2\theta]}, \quad (1)$$

where $\omega_p = (4\pi Ne^2/m)^{1/2}$ is the plasma frequency, $\omega_c = eB/mc$ the cyclotron frequency, and N the electron number density.

If E_x, E_θ are the transverse components of the electric field, with E_θ lying in the plane containing $\boldsymbol{B}$ and $\hat{k}$, the polarization coefficients $a_{\theta\pm}$ for the two modes [with $ia_{\theta\pm} = (E_x/E_\theta)_\pm$] are given by

$$a_{\theta\pm} = -\frac{2(\omega_p{}^2/\omega^2 - 1)\cos\theta}{-(\omega_c/\omega)\sin^2\theta \pm \{(\omega_c/\omega)^2 \sin^4\theta + 4(\omega_p{}^2/\omega^2 - 1)^2 \cos^2\theta\}^{1/2}}. \quad (2)$$

For frequencies $\omega > \omega_c \gg \omega_p$, the component of the electric field parallel to $\hat{k}$ may be neglected. Then the emission from a single electron, for the two modes, is given by (Liemohn 1965; Ramaty 1969; Trulsen and Fejer 1970)

$$\eta_\pm(\omega, \theta, \boldsymbol{\beta}) = \frac{e^2}{2\pi c}\omega^2 \sum_{s=-\infty}^{\infty} \frac{n_\pm}{1 + a_{\theta\pm}^2}\left\{-\beta_\perp J_s'\left(\frac{\gamma\omega n_\pm \beta_\perp \sin\theta}{\omega_c}\right) + \left[a_{\theta\pm}\left(\frac{\cot\theta}{n_\pm} - \frac{\beta_\parallel}{\sin\theta}\right)J_s\left(\frac{\gamma\omega n_\pm \beta_\perp \sin\theta}{\omega_c}\right)\right]\right\}^2 \delta(s\omega_c/\gamma - \omega[1 - n_\pm \beta_\parallel \cos\theta]) , \quad (3)$$

where $\gamma = (1 - \beta^2)^{-1/2}$ is the Lorentz factor, $\boldsymbol{\beta} = \boldsymbol{v}/c$, $\boldsymbol{v}$ is the velocity of the electron and $\beta_\parallel$, $\beta_\perp$ are the components of $\boldsymbol{\beta}$ parallel and perpendicular to the magnetic field. Here, J_s is the Bessel function of the first kind of order s, and J_s' its derivative with respect to its argument.

The relativistic Maxwellian distribution for electrons of momenta $\boldsymbol{p}$ at temperature T is given by

$$f(\boldsymbol{p}) = N f_0 e^{-\gamma\mu} , \quad (4)$$

where $\mu = mc^2/kT$, $1/f_0 = -i2\pi^2 m^2 ckT \mathrm{H}_2^{(1)}(i\mu)$ and $\mathrm{H}_2^{(1)}$ is the Hankel function of the first kind and order 2.

The emissivities $j_\pm$ for this Maxwellian distribution of electrons can then be derived. The results are

$$\begin{aligned} j_\pm(\omega, \theta) = \int \eta_\pm(\omega, \theta, \boldsymbol{\beta}) f(\boldsymbol{p}')d\boldsymbol{p}' &= 2\pi e^2 N f_0 m^3 c^2 \int_{-1}^{+1} d\beta_\parallel \sum_{s=s_1}^{\infty} \frac{n_\pm}{1 + a_{\theta\pm}^2} \\ &\times \left\{-\beta_\perp J_s'\left(\frac{\gamma\omega n_\pm \beta_\perp \sin\theta}{\omega_c}\right) + \left[a_{\theta\pm}\left(\frac{\cot\theta}{n_\pm} - \frac{\beta_\parallel}{\sin\theta}\right)J_s\left(\frac{\gamma\omega n_\pm \beta_\perp \sin\theta}{\omega_c}\right)\right]\right\}^2 \\ &\times \exp\left[-\frac{\mu s\omega_c/\omega}{1 - n_\pm \beta_\parallel \cos\theta}\right]\frac{\omega(s\omega_c/\omega)^3}{[1 - n_\pm \beta_\parallel \cos\theta]^4} , \end{aligned} \quad (5)$$

with s_1 the minimum value of s, such that

$$\beta_\perp^2 = 1 - \beta_\parallel^2 - \frac{(\omega/\omega_c)^2}{s^2}(1 - n_\pm \beta_\parallel \cos\theta)^2 > 0 . \quad (6)$$

Here, $\beta_\perp^2$ has been obtained by using the δ function in equation (3).

For a Maxwellian distribution of electrons, Kirchhoff's law is applicable so that the absorption coefficients $\alpha_\pm$ are given by

$$\alpha_\pm(\omega, \theta) = \frac{1}{n_\pm{}^r I_{\mathrm{RJ}}} j_\pm(\omega, \theta) , \quad (7)$$

where $I_{\mathrm{RJ}} = kT\omega^2/8\pi^3 c^2$ is the Rayleigh-Jeans intensity (for $kT \gg h\omega$) per polarization mode and $n_\pm{}^r$ is the ray-refractive index (Bekefi 1966); in our case with $\omega \gg \omega_p$, $n_\pm{}^r \approx n_\pm \approx 1$. Note also that equation (7) is a special case of equation (15) of Ramaty (1969), i.e., the case of an isotropic Maxwellian distribution for the electrons.

The principal advantage in evaluating $j_\pm$ (eq. [5]) by working with the variables $\beta_\parallel$, $\beta_\perp$ is that it is easy to take the limit $\theta \to \pi/2$, since $a_{\theta+} \to \infty$, while $a_{\theta-} \to 0$. Hence the cases $\theta = \pi/2$, $\theta \neq \pi/2$ can be treated in an essentially unified manner. Ramaty (1969), on the other hand, works with the variables γ and the pitch angle ϕ so that the case $\theta = \pi/2$ has to be treated in a substantially different manner from the case $\theta \neq \pi/2$. This is computationally cumbersome. The disadvantage in our case is that one has to perform a sum over an infinite number of terms compared to a finite number of terms when using γ and ϕ. However, because of rapid convergence of the series, this does not cause any difficulties.

III. POLARIZED RADIATION

The observable properties of the radiation are conveniently described by the Stokes parameters I, Q, U, and V, and these are determined by the intensities of the magnetoionic modes I_+ and I_- together with the phase relation between them. In the presence of a plasma, the phase relations are not easily determined except in the case of large Faraday rotation in the source region, whence the phase relation varies randomly from one emitting particle to another (e.g., Ramaty 1969). In this context, large Faraday rotation means that the anisotropy of the medium affects the propagation more than does the inhomogeneity of the medium, i.e., the two magnetoionic modes get out of phase over a distance much less than that in which the inhomogeneity causes any significant change in the properties of the modes (Melrose 1980). This circumstance is the usual one in astrophysical situations and, with the parameters used below, is valid for the AM Her binaries.

For a magneto-active plasma with large Faraday rotation, the transfer equations for the intensities I_+, I_- in the ordinary and extraordinary modes decouple and take the form (Ramaty 1969)

$$\frac{dI_\pm}{dz} + \alpha_\pm(\omega, \theta)I_\pm(\omega, \theta) = j_\pm(\omega, \theta) . \quad (8)$$

Hence, for a homogeneous source using equation (7),

$$I_{\pm} = I_{\mathrm{RJ}}[1 - \exp(-\alpha_{\pm} L)] , \tag{9}$$

where L is the path length through the source. The Stokes parameters I, Q, U, and V may then be shown to reduce to (Ramaty 1969):

$$I = I_{+} + I_{-} , \tag{10}$$

$$Q = I_{+} \frac{(1 - a_{\theta +}{}^{2})}{(1 + a_{\theta +}{}^{2})} + I_{-} \frac{(1 - a_{\theta -}{}^{2})}{(1 + a_{\theta -}{}^{2})} , \tag{11}$$

$$U = 0 , \tag{12}$$

$$V = 2\left(\frac{I_{+} a_{\theta +}}{1 + a_{\theta +}{}^{2}} + \frac{I_{-} a_{\theta -}}{1 + a_{\theta -}{}^{2}}\right) , \tag{13}$$

so that the circular polarization $= V/I$, and the linear polarization $= Q/I$.

IV. RESULTS

a) *Cyclotron Absorption Coefficients*

The cyclotron absorption coefficients $\alpha_{\pm}(\omega, \theta)$ were calculated from equations (5) and (7), using the Wild and Hill (1971) approximations for the Bessel functions and their derivatives. In carrying out the integral over $\beta_{\parallel}$, considerable care must be taken, particularly at $\theta \approx \pi/2$, as only a small range of values of $\beta_{\parallel}$ may contribute significantly toward the integral.

Our results were checked by comparing them with the analytic results of Engelmann and Curatolo (1973, hereafter denoted EC). For the case $kT = 1$ keV, line center for the harmonic $n = 6$ occurs at $\omega/\omega_c \approx n(1 - n/\mu) \approx 5.93$ (Hirshfield, Baldwin, and Brown 1961; Chanmugam 1980). The results of EC, in Figure 1, were obtained after including the (small) contributions from the neighboring harmonics. We note that there is excellent agreement for both modes except for $\theta \approx \pi/2$ for the ordinary mode. In this case the EC value is $\alpha_{+}(\omega, \pi/2) = 0$, whereas our value is finite with $\alpha_{+}(\pi/2, \omega) \approx \alpha_{-}(\pi/2, \omega)/\mu$, as expected. This is because the EC result includes only terms up to order v/c.

For $kT = 5$ keV, we see that the agreement is poorer (Fig. 2). This is because the nonrelativistic approximations used by EC, which are good for $(\mu)^{1/2} \gg 1$, begin to break down. This has also been noted for the total absorption coefficient which was calculated by a different method (Chanmugam 1980; Beard and Baker 1962).

At 10 keV, very good agreement is obtained with the results of Tamor (1978), who used a different formulation to determine the absorption coefficients for applications to nuclear fusion reactors. Excellent agreement is also obtained

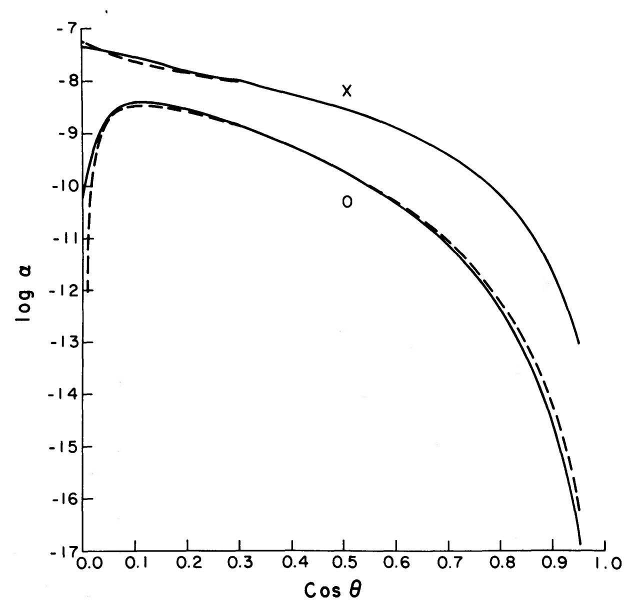

FIG. 1.—Plot of $\log \alpha$, with α in units of $\omega_p{}^2/\omega_c c$, for the ordinary (O) and extraordinary (X) modes as a function of $\cos \theta$, for the sixth harmonic. The line center is at $\omega/\omega_c = 5.93$ for $kT = 1$ keV. The *dashed curves* correspond to the nonrelativistic approximation of Engelmann and Curatolo (1973).

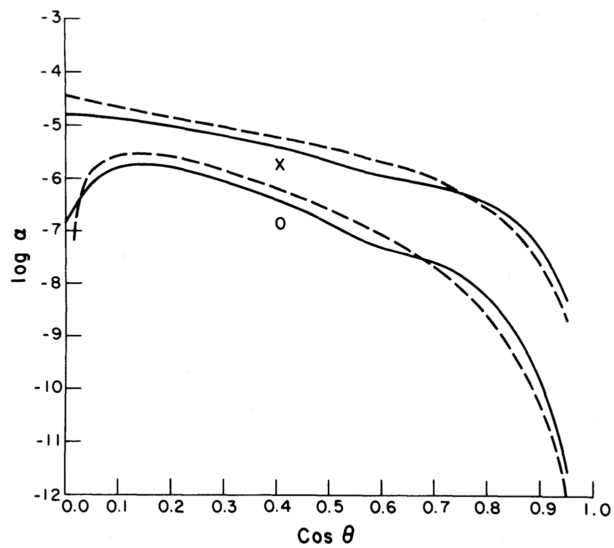

FIG. 2.—Same as Fig. 1, except that $kT = 5$ keV and $\omega/\omega_c = 5.67$.

with the analytic results of Bornatici and Englemann (1979) for the ordinary mode at $\theta = \pi/2$ and ω close to ω_c. This is a sensitive test of the calculations (Tamor 1978).

In Figure 3, $\alpha_{\pm}(\omega, \theta)$ is plotted as a function of ω/ω_c for the angle $\theta = 60°$ and a temperature $kT = 1$ keV. This temperature is roughly that of the accretion column above the shock-heated region in the AM Her binaries (to be discussed later). Since the absorption coefficients are given in units of $\omega_p{}^2/\omega_c c$, for a given value of the dimensionless constant $\Lambda \equiv \omega_p{}^2 L/\omega_c c = 4\pi eNL/B$, one can determine which harmonics are optically thin or thick. This has important consequences for the polarization of the radiation.

b) *Polarization of the Radiation*

We now consider the linear and circular polarization of the radiation emitted by an isothermal, homogeneous, infinite slab of thickness l with the magnetic field parallel to the surface of the slab (Fig. 4). We choose a value of Λ which makes the slab optically thick at the first few harmonics and optically thin at high harmonics. We calculate the intensity and polarization, as a function of the angle θ between the line of sight and the magnetic field, by using equations (9)–(11) and

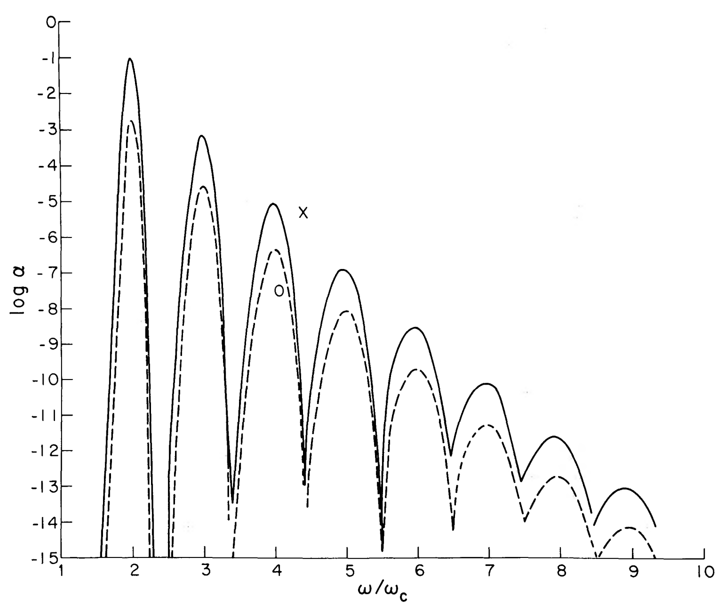

FIG. 3.—Plot of log α, with α in units of $\omega_p{}^2/\omega_c c$, for the ordinary (O) and extraordinary (X) modes, as a function of ω/ω_c, for the case $kT = 1$ keV, $\theta = 60°$.

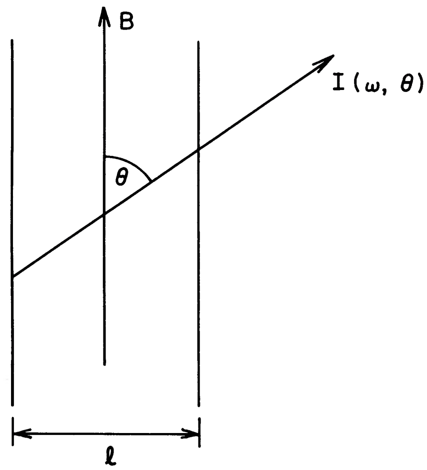

FIG. 4.—The accretion column idealized as a homogeneous plasma slab of thickness l with the magnetic field $\boldsymbol{B}$ parallel to the slab. The radiation of intensity I travels at an angle θ with respect to the magnetic field.

(13). The thickness of the slab along the line of sight is $L = l/\sin\theta$. (This model is motivated by our picture of the accretion column above the shock in the AM Her binaries for which $l \approx 10^8$ cm, $\Lambda \approx 10^8$; the applications will be discussed below.)

Figures 5*a*, 5*b*, 6*a*, and 6*b* give our results for $kT = 1$ keV and $kT = 0.2$ keV, respectively. To understand the shapes of the curves, let us examine the fifth harmonic in Figures 5*a* and 5*b*. For $\cos\theta \approx 0$, there is a dip in log I. This is because of the decrease in the absorption coefficient in the ordinary mode so that the slab becomes optically thin as $\cos\theta \to 0$, while remaining optically thick in the extraordinary mode (cf. Figs. 1 and 2); this is the cause of the sharp rise in the linear polarization as $\cos\theta \to 0$. At angles $0.05 \lesssim \cos\theta \lesssim 0.5$, the slab is optically thick in both modes ($\tau_\pm > 1$). At $\cos\theta \approx 0.5$, the ordinary mode becomes optically thin, the intensity decreases slightly with increasing $\cos\theta$, and the circular

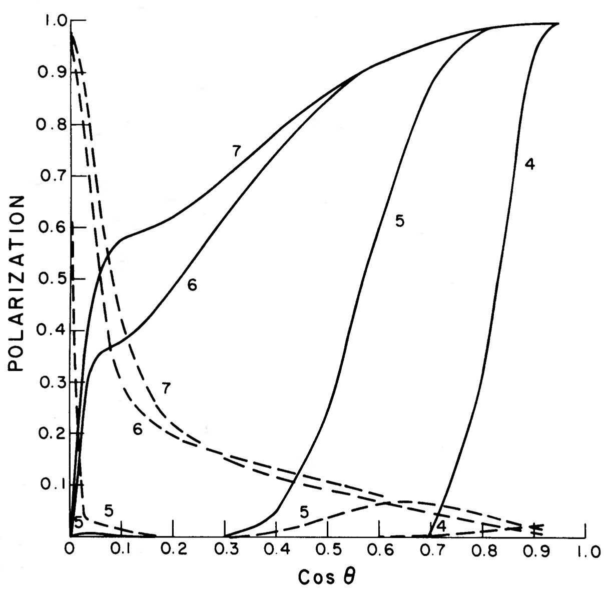

FIG. 5*a*

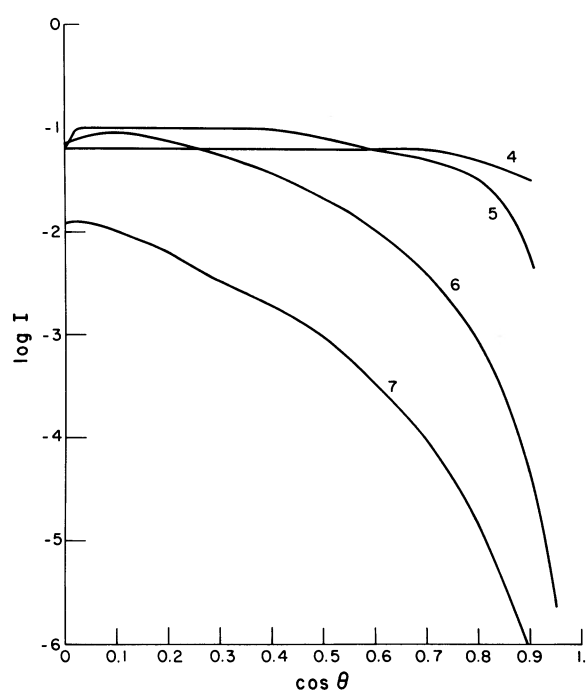

FIG. 5*b*

FIG. 5.—(*a*) The circular (*solid line*) and linear (*dashed line*) polarization from a plasma slab (as in Fig. 4) as a function of $\cos\theta$ for cyclotron harmonics 4, 5, 6, and 7. The line centers occur at $\omega/\omega_c \approx 3.97$, 4.95, 5.93, and 6.90, respectively. The temperature of the plasma $kT = 1$ keV, the dimensionless parameter $\Lambda = 10^8$, and $l = 10^8$ cm. (*b*) The log of the intensity [in units of ergs cm^{-2} s^{-1}(rad s^{-1})$^{-1}$ sr^{-1}, i.e., ergs cm^{-2}] as a function of $\cos\theta$ for a plasma with the same parameters as in (*a*).

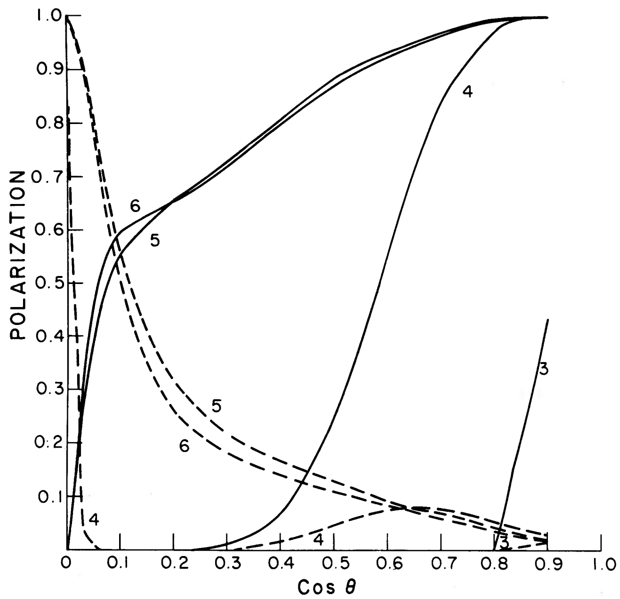

FIG. 6a

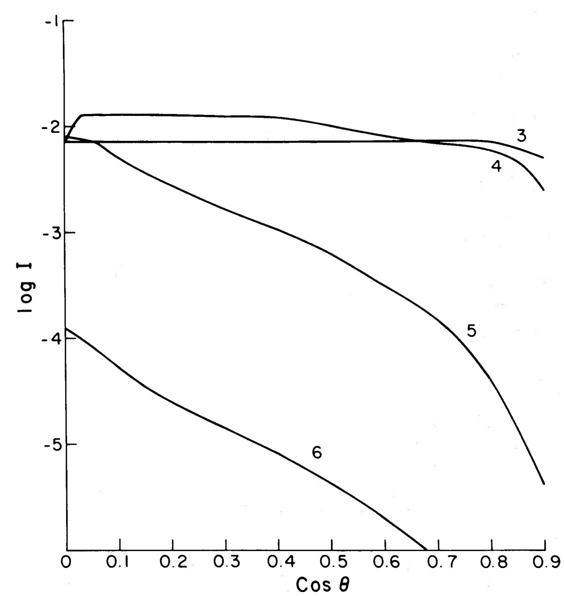

FIG. 6b

FIG. 6.—(*a*) Same as in Fig. 5*a* except that $kT = 0.2$ keV. Here, $\omega/\omega_c = 2.996$, 3.994, 4.99, and 5.986, which are the approximate line centers for harmonic numbers 3, 4, 5, and 6. (*b*) The log of the intensity as a function of cos θ for the same parameters as in (*a*).

polarization increases. At cos $\theta \approx 0.8$, the extraordinary mode also becomes optically thin (but retaining $\tau_- \gg \tau_+$), the intensity decreases greatly, and the circular polarization tends to unity. A similar sequence occurs for the other harmonics, but the lower harmonics remain optically thick except at very small θ, and the higher harmonics are optically thin even for cos $\theta \approx 0$.

Linear polarization is confined exclusively to $\theta \approx \pi/2$. At low harmonics there is no significant linear polarization because the slab is optically thick in both modes. (Recall that for $\theta = \pi/2$, the ordinary and extraordinary modes are linearly polarized parallel and perpendicular to the field, respectively.) At intermediate harmonics the slab is optically thin for the ordinary mode only, and at high harmonics it is optically thin for both modes (but $\tau_- \gg \tau_+$). Note that we have ignored mode coupling and possible boundary effects at the edge of the slab; under some circumstances these might affect the emergent polarization (Melrose 1978).

We consider next the radiation emitted by an isothermal homogeneous slab of thickness h with the magnetic field perpendicular to the slab (Fig. 7). This model is motivated by the shock-heated region of AM Her for which $h \sim 10^6$ cm, $\Lambda \approx 10^6$ (Masters 1978; Lamb and Masters 1979), and $kT \approx 20$ keV (Staubert *et al.* 1978; Swank *et al.* 1977; Raymond *et al.* 1979). The path length through the slab is then $L \approx h/\cos\theta$.

The results for the polarization and intensity are given in Figures 8*a* and 8*b*. We note from Figure 8*a* that circular polarization is mostly confined to cos $\theta \gtrsim 0.6$, except at high harmonics ($\gtrsim 10$). There is a negligible amount of linear polarization, even at the twelfth harmonic, because the opacity is high for both modes near $\theta = \pi/2$; this can be deduced from Figure 8*b* since the intensity is at the Rayleigh-Jeans limit for all cos $\theta \lesssim 0.1$.

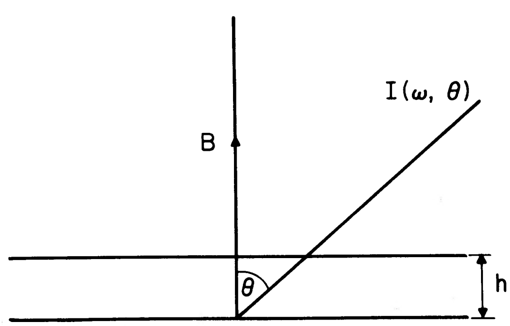

FIG. 7.—The shock-heated region idealized as a thin homogeneous plasma of thickness h. The magnetic field $\boldsymbol{B}$ is perpendicular to the slab, and the radiation travels at an angle θ with respect to the field.

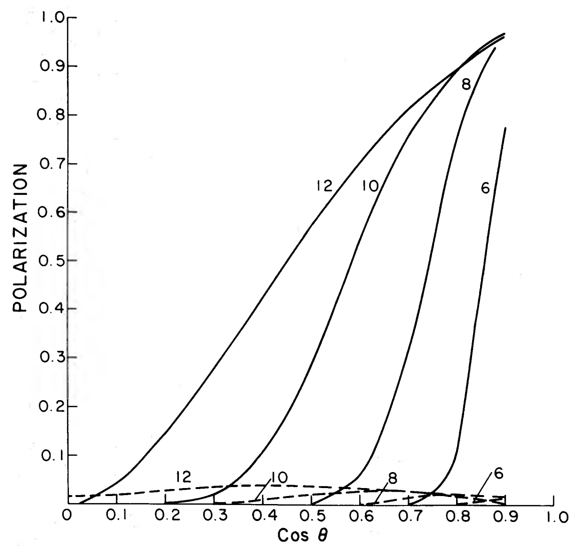

FIG. 8*a*

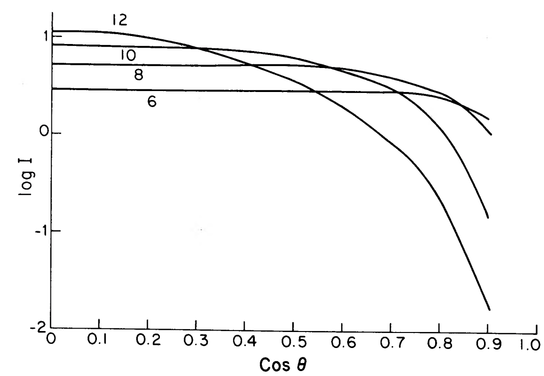

FIG. 8*b*

FIG. 8.—(*a*) the circular (*solid line*) and linear (*dashed line*) polarization from the shock-heated region (Fig. 7) as a function of cos θ for $\omega/\omega_c = 6, 8, 10$, and 12. The temperature of the plasma $kT = 20$ keV, $\Lambda = 10^6$, and $h = 10^6$ cm. (*b*) The log of the intensity as a function of cos θ for the same parameters as in (*a*).

V. APPLICATIONS TO THE AM HERCULIS BINARIES

The AM Her binaries are characterized by the strongly polarized light they emit. Typically, they exhibit $\sim 10\%$ circular and linear polarization, the latter appearing as a pulse, usually once per orbital cycle. The variations of the polarization and the visual flux are plotted schematically in Figure 9. We now discuss the systems separately.

a) *AM Herculis*

The polarized light could arise in either the shock-heated region or the overlying accretion column. Because circularly polarized radiation is observed near phase 0.2, when the X-ray emission is eclipsed, it is unlikely that the shock-heated region is involved (Chanmugam and Wagner 1977). In addition, the hot shock-heated region does not produce linear polarization (Fig. 9*a*), except possibly at rather high harmonics, in which case the radiation would be in the UV instead of the visible range. Therefore, we assume that the polarized light arises in the cooler plasma ($kT \lesssim 1$ keV) of the accretion column. We suggest, therefore, that the region of the accretion column emitting the polarized light is considerably extended in height (contrary to the shock-heated region at its base) and that for AM Her much of it is never eclipsed, hence the finite polarization at optical wavelengths at X-ray eclipse.

Our results in Figures 5*a* and 6*a* suggest that the observed sharp pulse of linear polarization in AM Her (Fig. 9*a*) corresponds to that calculated for harmonic number ≈ 5 at θ close to $\pi/2$, i.e., when the active pole and accretion column are on the stellar limb. The harmonic number corresponding to the visible radiation cannot be too low or no linear polarization would result. We note, however, that our slab model cannot explain all features of the observations. First, we have not included any unpolarized radiation from the star or shock region; this would dilute the observed polarization considerably. Second, we would expect another pulse of linear polarization near phase 0.3 when the accretion column is on the limb and the circular polarization again reverses sense; this pulse is, in general, not observed, although Tapia (personal communication) sometimes detects some linear polarization at that phase. Possibly the plasma associated with the flow from the companion star wipes out the polarization. Third, a complete model would not be a slab as assumed here, but possibly a cylinder, and nonuniformities associated with the edges of the cylinder could be important in some circumstances.

Regarding the circular polarization, again our simple slab model can explain many, but not all, of the observed features. The calculated degree of polarization is larger than observed, probably because of dilution by an unpolarized background. The low polarization and low visual flux near phase 0.6, where the X-ray flux is a maximum, probably results from our viewing the accretion column at a small angle, with a correspondingly low optical depth, small apparent area, and low intensity of the cyclotron emission (Figs. 5*a*, 6*a*).

It seems likely that most, or all, of the optical variability (with phase) in the *V* band is due to the varying cyclotron emission (Kruszewski 1978). The relative lack of variation in the *U* band then must be explained. We postulate that cyclotron emission from the accretion column contributes little intensity in the blue and UV because of the steep spectrum at high frequencies. The flux of cyclotron emission is given by $S = 8\pi^2 kT(\omega^2/c^2)[1 - \exp(-\tau)]\Omega$, where Ω is

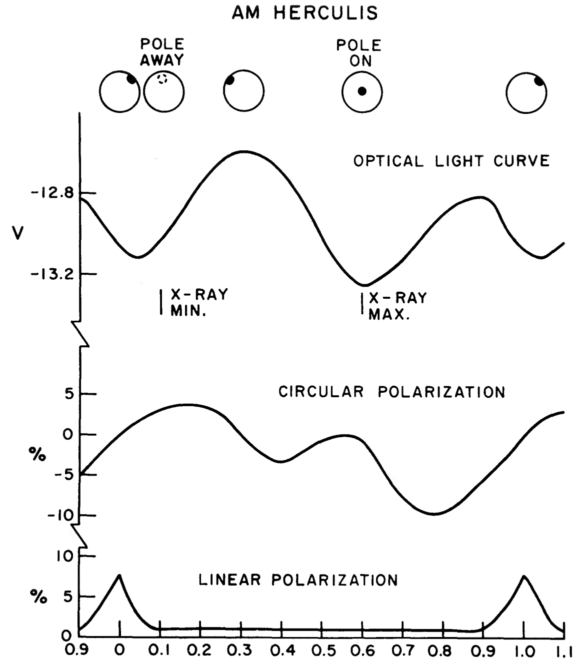

FIG. 9*a*

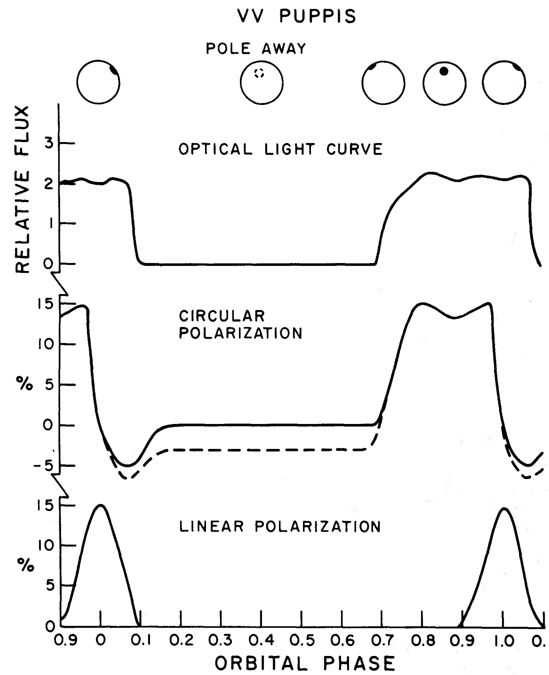

FIG. 9*b*

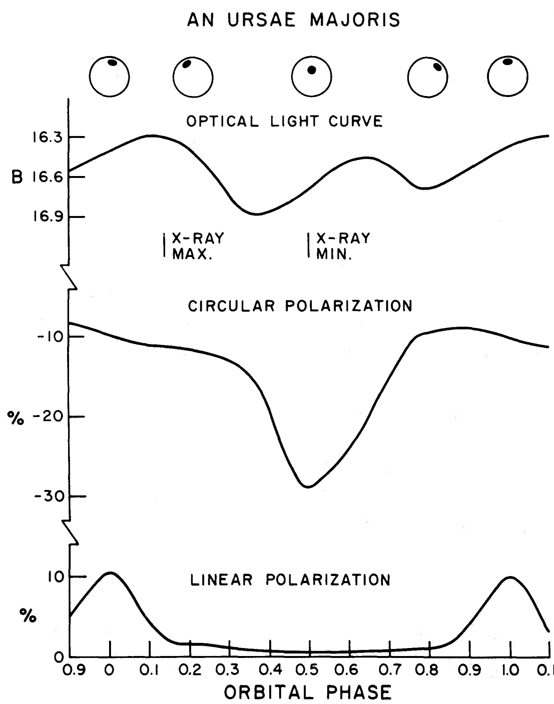

FIG. 9*c*

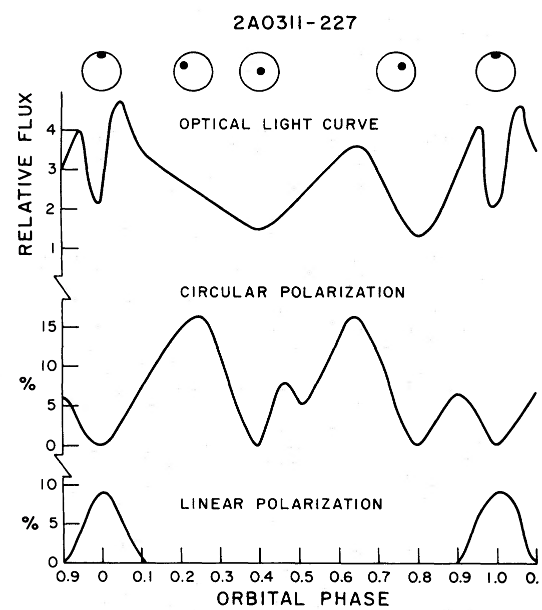

FIG. 9*d*

FIG. 9.—(*a*) Schematic sketch of the visual light curve (Priedhorsky *et al.* 1978), polarization curves (Tapia 1977*a*), and phases of the soft X-ray maximum and minimum (Hearn and Richardson 1977) for AM Her. The diagrams show schematically the position of the active pole of the magnetic white dwarf (as projected onto the stellar disk) at the various phases. (*b*) Same as in (*a*) but for VV Pup. The optical light and polarization (*solid lines*) curves are from Tapia (1977*b* and private communication). The circular polarization (*dashed line*) is from Liebert and Stockman (1979) during a different (bright) phase. No X-rays have yet been observed from VV Pup. (*c*) Same as in (*a*) but for AN UMa. The light and polarization curves in the blue band are from Krzeminski and Serkowski (1977). The X-ray observations (Hearn and Marshall 1979) and optical observations were made about a year apart. (*d*) Same as in (*a*) but for 2A 0311-227. The polarization curves are from Tapia (1979 and private communication). The optical light curve is from Bond, Chanmugam, and Grauer (1979). As in the other systems, there is considerable flickering which is smoothed over in the schematic light curve.

the solid angle subtended by the source. For Maxwellian electrons, the frequency dependence of the optical depth is large, i.e., $\tau \propto \omega^{-\alpha}$ where α is between about 5 and 15, depending on T and Λ (Bekefi 1966; Mätzler 1978; Dulk, Melrose, and White 1979). Therefore, at low frequencies the cyclotron emitting source is optically thick and $S \propto \omega^2$, whereas above some critical frequency ω^*, at which $\tau \approx 1$, the source is optically thin and $S \propto \omega^{\alpha+2}$, i.e., the flux sharply decreases with frequency. We suggest that, for AM Her, the critical frequency for the accretion column is somewhere in the visible frequency range, and for this reason the cyclotron emission contributes little flux in the blue and UV. The shock-heated region, on the other hand, should be optically thick throughout the visible range and well into the UV, to about 2000 Å, if the critical harmonic number is about 10 (Chanmugam 1980; see also Fig. 8*b*). The eclipse of the shock-heated region may be the cause of the secondary minimum of V flux near phase 0.1.

b) *VV Puppis*

VV Puppis is similar to AM Her in many respects, and therefore most of the above remarks apply. For both, the active magnetic pole is eclipsed during some parts of the orbital cycle, giving rise to the sharp minimum in soft X-rays and also to the reversal of sense of circular polarization near the time when the accretion column passes over the limb. However, contrary to AM Her, for VV Puppis the eclipse covers most of the accretion column and produces the minimum in the light curve at optical wavelengths from phase $\approx$0.2–0.7 (Fig. 9*b*). During epochs when VV Puppis is inactive, it appears that the circular polarization is zero during eclipse, whereas during bright, active epochs, a small amount of circular polarization remains (Fig. 9*b*). This has been interpreted by Liebert and Stockman (1979) (see also King and Lasota 1979) as evidence for the second magnetic pole becoming active. We suggest an alternative explanation: During bright, active epochs, the emitting region in the accretion column extends to greater heights and hence is incompletely eclipsed.

c) *AN Ursae Majoris*

Unlike AM Her and VV Pup, the circular polarization of the radiation from AN UMa does not change sign during the orbital cycle (Fig. 9*c*). This suggests that the accretion column is never perpendicular to the line of sight, i.e., never passes over the limb, and that the shock region is always in view. The fact that linear polarization is observed near phase zero implies that θ comes close to $\pi/2$, i.e., the accretion column is near the limb; this is consistent with the low degree of circular polarization at that phase. It is also consistent with the maximum in the light curve being near phase zero, when the accretion column is near the limb and least foreshortened. The minimum in light should occur near phase 0.5 when the accretion column is nearest disc center and, thus, most foreshortened; the observed minimum is near phase 0.4, and there is a secondary maximum in the light curve near phase 0.65 which cannot be explained in this simple manner.

An apparent discrepancy in this scenario comes from soft X-ray data, where the flux was reported to be at a maximum near phase zero and at a minimum near phase 0.5 (Hearn and Marshall 1979; Fig. 9*c*). From our model we would expect the minimum in X-rays to occur near phase zero, when the shock-heated region is nearest the limb and thus foreshortened, and the maximum to occur near phase 0.5. One explanation, suggested by Hearn and Marshall, is that the X-rays come from the other pole, in which case it is difficult to understand why the sense of circular polarization never reverses. It should be emphasized, however, that the X-ray and optical observations were made a year apart; not only does that make the phase comparison difficult, but there may have been a phase shift due to a departure from synchronous rotation (Joss, Katz, and Rappaport 1979).

d) *2A 0311–227*

The situation with 2A 0311–227 is similar to that of AN UMa in that the circular polarization does not change sign (Fig. 9*d*). However, for 2A 0311–227 the degree of polarization has a double instead of a single peak, with a deep minimum in between; this minimum (at phase 0.4) coincides with a minimum in the light curve. As suggested by Bond, Chanmugam, and Grauer (1979), the minimum probably indicates that we have a near "pole-on" view of the accretion column, with the attendant foreshortening and inefficient emission of cyclotron radiation at θ near 0°.

VI. CONCLUSIONS

In this paper, we have presented an accurate and efficient method for calculating the cyclotron absorption coefficients, in the ordinary and extraordinary modes, for a three-dimensional Maxwellian distribution of electrons. These results are applied to the accretion columns of the AM Her binaries, and the first quantitative calculations for the emergent polarized radiation are presented. The characteristic polarized light observed in these systems can be accounted for, if the radiation is emitted from the cooler regions of the accretion column overlying the shock-heated region, and if the emission is at approximately the fifth cyclotron harmonic. The field strength in the emitting region is deduced to be $\approx 4 \times 10^7$ gauss. This is consistent with the value of $\approx 3 \times 10^7$ gauss deduced from the cyclotron absorption lines observed in VV Puppis (Viswanathan and Wickramasinghe 1979), and a value of $\lesssim 5 \times 10^7$ gauss deduced (Chanmugam 1980) for AM Her in order to account for the observed UV flux (Raymond *et al.* 1979; Tanzi *et al.* 1980). The results confirm the speculation (Stockman 1977; Chanmugam and Wagner 1978) that the characteristic linear polarization pulse observed in all of these systems occurs at the time when the magnetic field in the accretion column is nearly perpendicular to the line of sight and when Faraday depolarization is unimportant. We emphasize that it is extremely difficult to explain the observed polarized light if it were emitted at the fundamental cyclotron frequency with a field $\approx 2 \times 10^8$ gauss, as had originally been assumed. This is because the optical depths in both modes at the fundamental cyclotron frequency are too high by several orders of magnitude (Fig. 3). These optical depths could be reduced if the temperature of the emitting region is substantially lower ($kT \ll 0.1$ keV), but the intensity would then be far too low.

Note added 1980 October 21.—It has recently been reported that the circular polarization of 2A 0311–227 shows a reversal of sign, lasting from about phase 0.9 to 1.0 (Bailey, Hough, and Axon 1980). Hence, the pole (Fig. 9*d*) should be hidden from view during this time. The schematic optical light curve shows a dip at approximately this time (Fig. 9*d*), but its significance is uncertain because of the rapid flickering. The pole should be closest to being pole-on at phase ~ 0.45. This is consistent with the discovery of an X-ray maximum at phase ~ 0.5 (White *et al.* 1980) which had been predicted by Bond, Chanmugam, and Grauer (1979).

The surface magnetic field of AM Her has recently been deduced from Zeeman spectroscopy to be $\sim 1.5 \times 10^7$ gauss (Schmidt, Stockman, and Margon 1980; Hutchings, Crampton, and Cowley 1980). This implies that the peak polarization which occurs at 0.9 μm (Michalsky, Stokes, and Stokes 1977; Bailey, Hough, and Axon 1980) corresponds to cyclotron harmonic number $\lesssim 8$, since the field at the pole may be slightly higher. These observations are therefore consistent with our results.

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G. Chanmugam: Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803

G. A. Dulk: Department of Astro-Geophysics, University of Colorado, Boulder, CO 80309