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LATE CORE COLLAPSE IN STAR CLUSTERS AND THE GRAVOTHERMAL INSTABILITY

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ABSTRACT

Numerical Fokker-Planck computations of core collapse in a one-component star cluster are presented. The evolution of the cluster has been followed to the point where the central density has increased by a factor of 10^{20}. During the late stages of the core collapse, nonisothermal self-similar structure develops in the region which lies between the rapidly shrinking isothermal core and the halo. In this region, the radial profiles of the stellar density, the gravitational potential, and the velocity dispersion are characterized by the power laws $\rho \propto r^{-2.23}$, $\phi \propto r^{-0.23}$, and $v_m{}^2 \propto r^{-0.23}$. Late core collapse proceeds at a rate $d \ln \rho(0)/dt = 3.6 \times 10^{-3}\, t_r(0)^{-1}$. The central velocity dispersion increases with the central density according to $v_m{}^2(0) \propto \rho(0)^{0.10}$. These results provide strong new evidence for the identification of the late phase of core collapse with the gravothermal instability of Lynden-Bell and Wood.

Subject headings: clusters: globular — stars: stellar dynamics

I. INTRODUCTION

A considerable amount of theoretical effort has been applied to the study of the development of core collapse in an idealized star cluster consisting of a large number ($>10^4$) of uniform point masses. One goal of this work has been the determination of the structure of the cluster at the time of complete core collapse. Lynden-Bell (1975) has argued that the central regions of star clusters should evolve homologously and independently of the outer regions. While the central cores of evolving clusters are expected to be isothermal, the gravothermal instability picture of Lynden-Bell and Wood (1968) indicates that the isothermal region cannot extend out to an arbitrarily large number of core radii. Press and Lightman (1978) have argued heuristically that the structure of the region of the cluster between the core and the halo should be characterized by a power law $\rho \propto r^{-\beta}$ with β in the range $2 \leq \beta \leq 2.5$. A similar argument has recently been presented by Lynden-Bell and Eggleton (1980).

Numerical evidence for self-similarity is provided by the hydrodynamic computation of cluster evolution performed by Larson (1970), which produces a cluster density profile well approximated by a power law of index $\beta = 2.4$. This same value is also obtained by Hachisu *et al.* (1978) on the basis of a thermally conducting gas sphere model. The equations used by Hachisu *et al.* (1978) can be obtained from Larson's hydrodynamic equations in the approximation that the velocity distribution is isotropic and that certain high-order velocity moments are negligible.

Lynden-Bell and Eggleton (1980) have recently applied the assumption of self-similarity to derive the density profile of a highly evolved cluster using the equations of Hachisu *et al.* (1978) with a somewhat different form for the coefficient of thermal conductivity than was adopted in this previous study. Lynden-Bell and Eggleton obtain a power-law index $\beta = 2.21$ for the density profile between the core and the halo. This result depends on the assumption that the rate of core collapse is constant relative to the central relaxation time. While supported by the results of the present study, this assumption is not consistent with the findings of Heggie (1979) as discussed in § IV.

The present paper reports the results of a stellar dynamical calculation of the development of core collapse in a one-component star cluster. The numerical procedure for treating the Fokker-Planck equation described by Cohn (1979, hereafter Paper I) has been applied to this problem in the approximation that the velocity distribution is isotropic and thus the stellar distribution function depends only on the stellar energy. The evolution of a cluster which begins as Plummer's model has been followed over an interval during which the central density increases by a factor of $\sim 10^{20}$. In comparison the density growth factors for the fluid dynamical computations of Larson (1970) and Hachisu *et al.* (1978) are 10^8 and 10^{13}, respectively. The Monte Carlo Fokker-Planck calculation of cluster evolution recently reported by Marchant and Shapiro (1980) achieves a density growth factor of 10^4.

The core collapse observed in the present calculation is characterized by two phases. During the early phase, the cluster becomes more nearly isothermal as the depth of the potential well increases relative to the central velocity dispersion. This early evolutionary phase resembles the King (1966) sequence of cluster models. During the late phase, the central isothermal region shrinks to zero radius, and nonisothermal self-similar structure develops in the region between the core and halo. The density profile in this region is characterized by a power law of index $\beta = 2.23$. Thus during the late stages of core collapse the initial approach to global isothermal structure is reversed. This two-phase development of the core collapse is in good agreement with the gravothermal instability picture of Lynden-Bell and Wood (1968).

II. THE ONE-DIMENSIONAL FOKKER-PLANCK EQUATION

Paper I describes a method for following the evolution of a one-component star cluster by means of numerical integration of the orbit-averaged energy-angular momentum (E, J) space Fokker-Planck equation. This procedure had been applied to a study of the evolution of a cluster which begins as Plummer's model; the central density of the model increased by a factor of 10^3 over the course of the computation. In order to follow the core collapse to significantly larger density growth factors with a reasonable expenditure of computation time the two-dimensional (E, J) Fokker-Planck equation was reduced to a one-dimensional form by integrating over the J dependence. Since the two-dimensional computation reported in Paper I indicated that the velocity distribution in the central regions of the star cluster remains nearly isotropic, this angular momentum averaging should provide a useful first approximation for studying late stages of core collapse. The close agreement between the results of Hachisu *et al.* (1978), who assume velocity isotropy, and those of Larson (1970), who does not, supports the validity of this approximation. Additional support is provided by the good agreement of the results of the two-dimensional calculations of both Paper I and Marchant and Shapiro (1980) with those presented here.

The one-dimensional Fokker-Planck equation gives the time evolution of the number density $N(E)$ in E-space which is related to the position-velocity phase space distribution function $f(E)$ by

$$N(E) = 4\pi^2 p(E) f(E) . \qquad (1)$$

The statistical weight factor $p(E)$ is given by

$$p(E) = \int_0^{J_c^2(E)} d(J^2) P(E, J) = 4\int_0^{\Phi^{-1}(E)} dr\, r^2 v , \qquad (2)$$

where $J_c(E)$ is the angular momentum of a circular orbit of energy E, $P(E, J)$ is the orbital period, $\phi^{-1}(E)$ is the inverse of the potential function $\phi(r)$, and $v = [2\phi(r) - 2E]^{1/2}$. It is useful to define a second function,

$$q(E) = \int_0^{J_c^2(E)} d(J^2) Q(E, J) = \frac{4}{3}\int_0^{\phi^{-1}(E)} dr\, r^2 v^3 , \qquad (3)$$

where $Q(E, J)$ denotes the radial action variable defined in Paper I.

The functions $p(E)$ and $q(E)$ are related by

$$q(E) = \int_E^{\phi(0)} dE' p(E') , \qquad (4)$$

from which it can be seen that $4\pi^2 q(E)$ is the phase space volume available to stars of energy E or greater. If we define an integral distribution $\mathcal{N}(E)$ to be the number of stars of energy E or greater, then f, q, and $\mathcal{N}$ are related by

$$f = \frac{1}{4\pi^2}\frac{d\mathcal{N}}{dq} . \qquad (5)$$

The angular-momentum averaged Fokker-Planck equation for a fixed potential may be written in a flux conservation form,

$$\frac{\partial N}{\partial t} = -\frac{\partial F_E}{\partial E} , \qquad (6)$$

where the energy space particle flux F_E is given by

$$F_E = -D_{EE}\frac{\partial f}{\partial E} - D_E f . \qquad (7)$$

The two flux coefficients are given by the expressions,

$$D_{EE} = 16\pi^3\Gamma q(E) \int_0^E dE' f(E') + 16\pi^3\Gamma \int_E^{\phi(0)} dE' q(E') f(E') ,$$

$$D_E = -16\pi^3\Gamma \int_E^{\phi(0)} dE' p(E') f(E') , \qquad (8)$$

with $\Gamma = 4\pi G^2 m^2 \ln \Lambda$, where G is the gravitational constant, m is the stellar mass, and $\ln \Lambda$ is the usual logarithmic cutoff factor. Equations (7) and (8) for the energy space flux are equivalent to equation (2.29) of Hénon (1961).

Following the algorithm outlined in Paper I the distribution function is first advanced in time, with the potential held fixed, by use of the Fokker-Planck equation (6). The potential is then advanced by iterated application of the integral form of Poisson's equation,

$$\phi(r) = \frac{4\pi G}{r}\int_0^r dr' r'^2 \rho(r') + 4\pi G \int_r^\infty dr' r' \rho(r') , \qquad (9)$$

where ρ is the mass density given by,

$$\rho(r) = 4(2)^{1/2}\pi m \int_0^{\phi(r)} dE[\phi(r) - E]^{1/2} f(E) . \qquad (10)$$

The potential recomputation procedure is subject to the constraint that f remain a fixed function of the independent variable q after each application of equation (9). It can be seen from equation (5) that this condition ensures that the distribution of particles in phase space is not altered as the potential is adjusted.

The problem of numerical nonconservation of energy noted in Paper I was greatly alleviated by the adoption of the finite difference form for the Fokker-Planck equation suggested by Chang and Cooper (1970). This scheme takes into account the quasi-steady-state nature of the

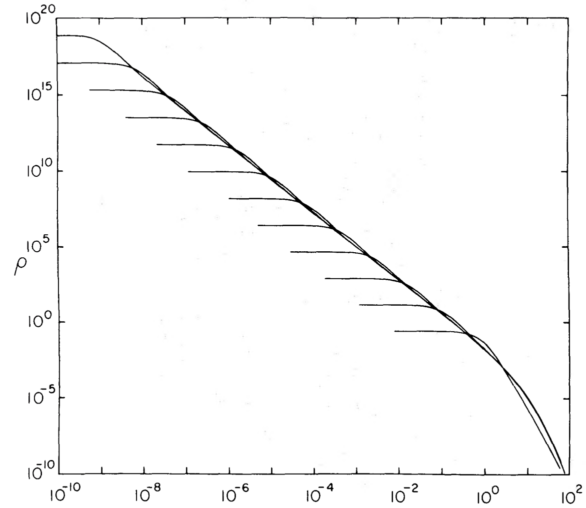

FIG. 1.—Evolution of the cluster density profile. The central density increases in time while the radius of the central core decreases. The curves correspond, in order of increasing central density, to the epochs specified by $x_0 = 6.00, 9.05, 10.58, 11.69, 12.46, 12.97, 13.31, 13.53, 13.67, 13.82$, and 13.85, where $x_0 \equiv 3\phi(0)/v_m^2(0)$. (The time dependence of the scaled escape energy x_0 is shown in Fig. 4) Note that the time interval between the first two epochs is about 80% of the time to complete core collapse.

distribution function in the energy differencing. With its adoption the secular energy drift rate was reduced by better than a factor of 100. As in Paper I, Crank and Nicholson semi-implicit time differencing was employed.

III. NUMERICAL RESULTS FOR CORE COLLAPSE

Plummer's model was chosen as the initial cluster model for an evolutionary computation, as in the study reported in Paper I. A 100 zone energy mesh was used in the computation, with the mesh spacing determined by an auxiliary function of energy as in Paper I. A logarithmic radial mesh was adopted with eight points per decade. The time step was continuously adjusted so that the central density increase per step was 2.5%. The collapse of the cluster core was followed over a period during which the central density increased by a factor of $10^{19.6}$. In contrast, the density growth for the two-dimensional computation reported in Paper I was only 10^3. The present computation required about 5 hours of CPU time on the Smithsonian Astrophysical Observatory VAX-11/780.

The most striking feature of the core collapse observed in the present study is the development of a self-similar density profile as seen in Figure 1. Here the evolving density profile has been plotted every 160 time steps. By the end of the computation a power law extending over eight decades of radius has developed. This result is shown more quantitatively in Figure 2*a* where the evolu-

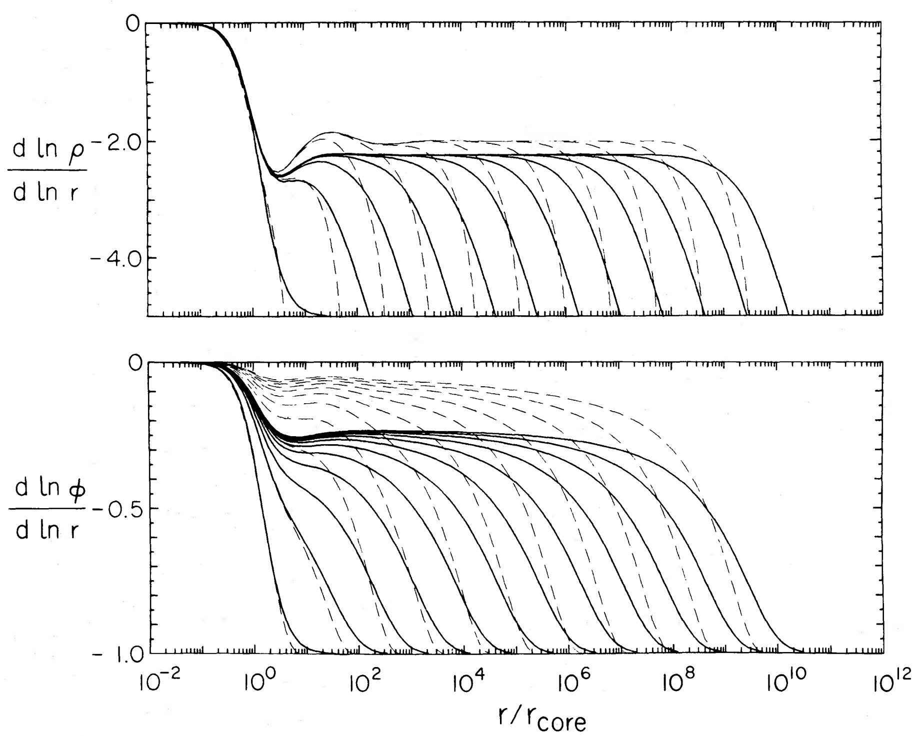

FIG. 2.—The logarithmic gradient profiles of (*a*) density and (*b*) gravitational potential. The abscissa is radius-scaled by the core radius, $r_{core} \equiv [3v_m^2(0)/4\pi\rho(0)]^{1/2}$. The solid curves represent the evolving profiles from the present computation (at the same epochs as in Fig. 1), while the dashed curves are from the King (1966) sequence. Since the core radius of the evolving cluster decreases in time, the solid curves extend to larger values of r/r_{core} at later epochs. The development of power-law structure is indicated by the constancy of $d \ln \rho/d \ln r$ and $d \ln \phi/d \ln r$ over many decades of radius at late times.

tion of the logarithmic density gradient $d \ln \rho / d \ln r$ is illustrated. In order to demonstrate the homologous nature of the evolution of the central structure the profiles have been plotted as a function of $r/r_{\rm core}$, where the core radius is defined by

$$r_{\rm core} = \left[\frac{3v_m^2(0)}{4\pi\rho(0)}\right]^{1/2} . \quad (11)$$

Here $\rho(0)$ is the central density and $v_m^2(0)$ is the central velocity dispersion. Since the core radius decreases in time, the profiles extend to larger values of $r/r_{\rm core}$ at later times. At the endpoint of the present computation the density profile is closely approximated by a power law $\rho \propto r^{-\beta}$ with $\beta = 2.23$ for $10 \leq r/r_{\rm core} \leq 10^8$. This finding confirms the conjecture of Lynden-Bell (1975) that the cluster density profile evolves homologously during late stages of core collapse. The value for β obtained here is in excellent agreement with the value of 2.21 found by Lynden-Bell and Eggleton (1980). Although both studies model the same physical situation, the stellar dynamical method used here is not mathematically equivalent to the fluid dynamical approach of Lynden-Bell and Eggleton. Thus the agreement of the results is gratifying.

Figure 2*a* also shows logarithmic density gradients from the King (1966) sequence of dynamical cluster models, for comparison with the present evolutionary results. For the first two epochs shown, there is good agreement between the evolving density gradient profile and a corresponding profile from the King sequence, out to a radius in each case which contains about half the cluster mass. However, at later epochs, the evolving density gradient profile cannot be fitted by a King profile beyond a radius of about 10 $r_{\rm core}$, which contains a decreasing fraction of the total cluster mass. Unlike the King model profiles, the density profile of the evolving cluster does not approach the profile of an infinite isothermal sphere as the radius of the cluster becomes large relative to the core radius.

Figure 2*b* illustrates the evolution of the logarithmic gradient of the gravitational potential. It can be seen from this figure that a power-law potential profile $\phi \propto r^{-\alpha}$ is also developing with an index $\alpha = 0.23 = \beta - 2$. Thus over several decades of radius the structure of the cluster is closely approximated by a singular polytrope of index $n = \beta/\alpha = 9.7$.

Also shown in Figure 2*b* are potential gradients from the King sequence. In the limit of an infinite isothermal sphere, $d \ln \phi / d \ln r = 0$, which is clearly inconsistent with the evolutionary results. Again, the evolving profile cannot be fitted with a King profile beyond a radius of about 10 $r_{\rm core}$. Examination of the late-epoch velocity dispersion (kinetic temperature) profile indicates that within a radius of 10 $r_{\rm core}$, $v_m^2(r)$ varies by only 25%, while for larger radii the gradient steepens and is characterized by $d \ln v_m^2 / d \ln r = -0.23$. Thus as core collapse proceeds to completion the central isothermal region of the cluster shrinks to zero radius and mass.

Figure 3 illustrates the evolution of the distribution function, which is plotted versus $x \equiv 3[\phi(0) - E]/v_m^2(0)$.

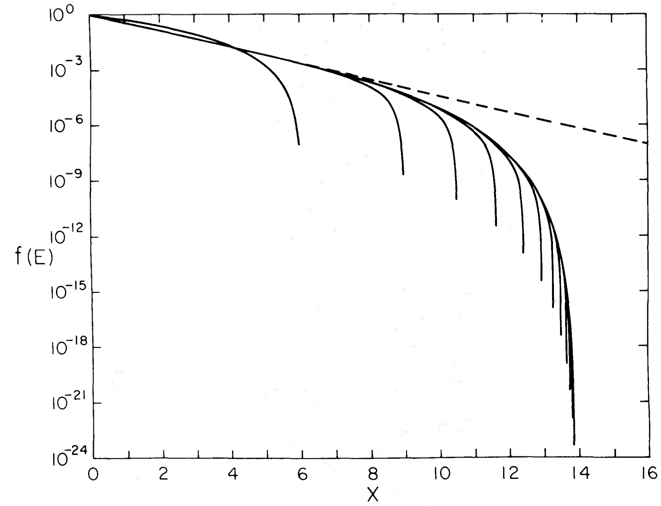

FIG. 3.—The evolution of the normalized stellar distribution function. The abscissa is scaled energy, $x = 3[\phi(0) - E]/v_m^2(0)$. The solid curves represent the same epochs as in Fig. 1, and are normalized so that $f = 1$ at $x = 0$. The dashed line is the normalized Maxwellian $f_{\rm M} = \exp(-x)$. Note that the curves for the late epochs, which extend beyond $x = 9$, all begin to deviate from the Maxwellian near this point. This behavior is predicted by the gravothermal instability picture.

The curves are plotted for the same epochs as in Figure 1, and are normalized so that $f = 1$ at $x = 0$. Also shown in Figure 3 is the normalized Maxwellian $f_{\rm M}(x) = \exp(-x)$. The curves for the later epochs are well approximated by $f_{\rm M}$ for $x \lesssim 9$ but drop below the Maxwellian for larger x. As noted by Lynden-Bell and Eggleton (1980) this behavior finds a natural explanation in the gravothermal instability picture of Lynden-Bell and Wood (1968). According to this picture a sequence of spatially truncated isothermal models becomes thermodynamically unstable when the scaled escape energy $x_0 = 3\phi(0)/v_m^2(0)$ exceeds a critical value at which the entropy, $S = -\int d^3r\, d^3v\, f \ln f$, has a local maximum. As shown in Figure 5, the King (1966) sequence has an entropy maximum (at fixed mass and energy) at $x_0 = 9.3$.[1] The evolution of a cluster is therefore expected to diverge from the King sequence for $x_0 \gtrsim 9$. Thus for $x \gtrsim 9$, the distribution function should drop below the Maxwellian, as seen in Figure 3. The power-law structure observed in the density and potential profiles indicates that the distribution function should exhibit power-law behavior, $f \propto E^p$, for energies corresponding to $x \gtrsim 9$, with p related to the polytropic index $n = 9.7$ by $p = n - \frac{3}{2}$. Examination of the evolution of the logarithmic gradient $d \ln f / d \ln E$ indicates the development of such a power law with $p \approx 8$, near the end of the computation.

Figure 4 shows the time dependence of x_0 found in the present evolutionary computation. The time unit used here is the initial value of the mean relaxation time for the cluster, $t_{\rm rh}$, as defined by Spitzer and Hart (1971). It can be

[1] The cluster structure parameter x_0 is related to King's similar parameter W_0 by $x_0 \approx W_0 + \frac{1}{2}$ for $W_0 \gtrsim 8$.

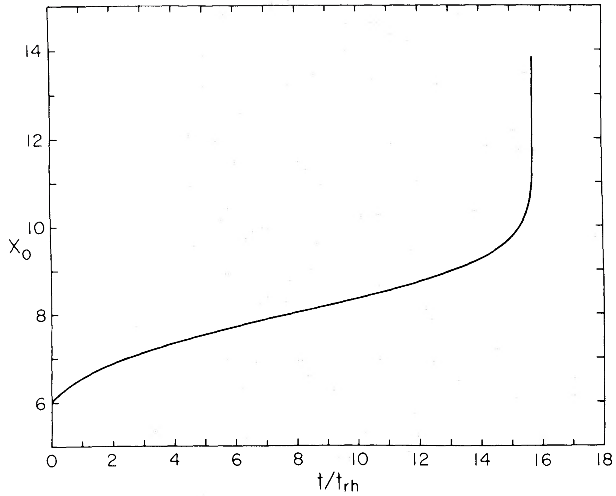

FIG. 4.

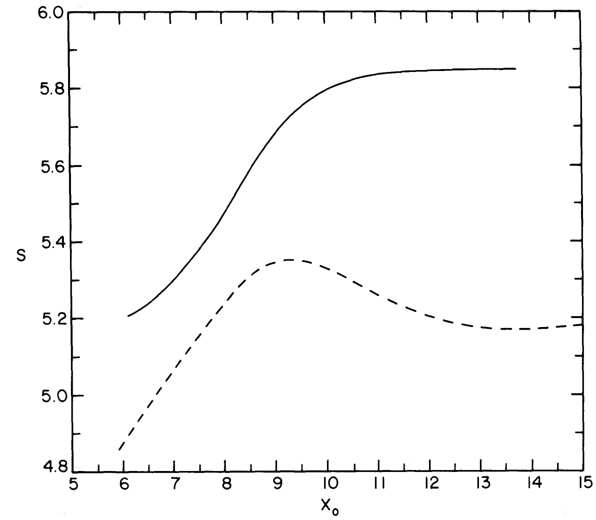

FIG. 5.

FIG. 4.—The evolution of the scaled escape energy $x_0 \equiv 3\phi(0)/v_m^2(0)$. Note the sharp upturn in x_0 for $x_0 \gtrsim 9$, which apparently indicates the onset of the gravothermal instability. The scaled escape energy tends to a limiting value of $x_0 \approx 13.9$ as core collapse proceeds to completion.

FIG. 5.—Entropy $S \equiv -\int d^3r\, d^3v\, f \ln f$ as a function of scaled escape energy x_0. The solid curve is the result of the present evolutionary computation, while the dashed curve shows the variation of entropy along the King (1966) sequence, scaled to the mass and energy of the evolutionary model. Note the entropy maximum which occurs along the King sequence at $x_0 = 9.3$.

seen from this figure that for $t \lesssim 13\ t_{rh}$ ($x_0 \lesssim 9$), x_0 increases approximately linearly in time, while for $t \gtrsim 13\ t_{rh}$ ($x_0 \gtrsim 9$) there is a sharp upturn in x_0. This transition in the behavior of x_0, which appears to indicate the onset of the gravothermal instability, occurs when about 80% of the time to complete core collapse has passed. The parameter x_0 is found to tend to a limiting value of about 14 as the core collapse proceeds to completion. This is in reasonable agreement with the asymptotic value of 15.3 obtained by Lynden-Bell and Eggleton (1980). Thus the depth of the cluster potential well, scaled by the central velocity dispersion, does not increase to arbitrarily large values as core collapse proceeds.

The onset of the upturn in x_0 can be seen in the results reported in Paper I. Similar behavior occurs in the study of Marchant and Shapiro (1980), where x_0 reaches a value of 11.6 at the end of the computation. This agreement between the results of previous two-dimensional calculations and those presented here supports the validity of the use of a one-dimensional Fokker-Planck approach in the study of core collapse.

The interpretation of the present results in terms of the gravothermal instability picture is supported by the evolution of the cluster entropy S. Figure 5 shows the evolution of S parametrized by x_0. As expected on the basis of the second law of thermodynamics and the monotonicity of $x_0(t)$, the curve obeys the constraint $dS/dx_0 \geq 0$. Also shown in Figure 5 is the variation of S along the King sequence, scaled to the mass and energy of the evolutionary model. For $x_0 \leq 8$ the two curves are roughly similar, while for larger x_0 there is a significant divergence. Although the entropy continues to increase in the present calculation, a local entropy maximum occurs along the King sequence at $x_0 = 9.3$. The occurrence of an entropy maximum is apparently a general feature of truncated isothermal sequences, independent of the details of the truncation procedure, as has been discussed by Lynden-Bell and Wood (1968). The divergence of the two curves in Figure 5 for $x_0 \gtrsim 8$ evidently indicates the onset of the gravothermal instability in the present calculation.

Besides the behavior of the entropy with increasing x_0, there are other indications that the late stages of core collapse are properly identified with the gravothermal instability. Figure 6 shows the dependence of the core collapse rate, $\xi \equiv t_r(0)\, d \ln \rho(0)/dt$, on the scaled escape

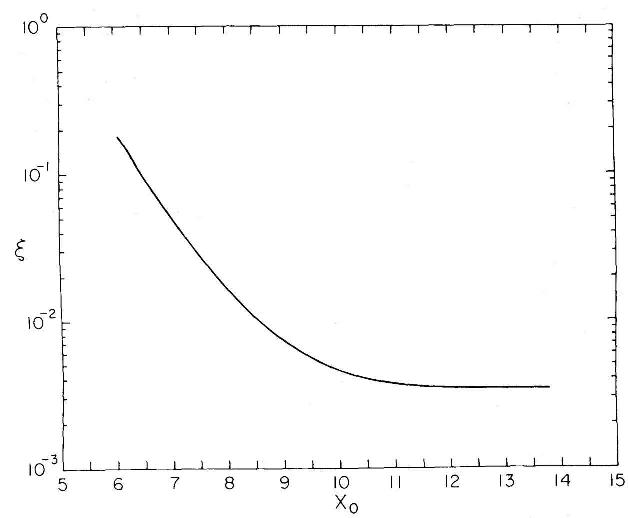

FIG. 6.—Core collapse rate $\xi \equiv t_r(0)[d \ln \rho(0)/dt]$ as a function of scaled escape energy x_0. Note the transition from exponential decrease to flat behavior near $x_0 = 9$, which indicates the onset of the homologous phase of core collapse.

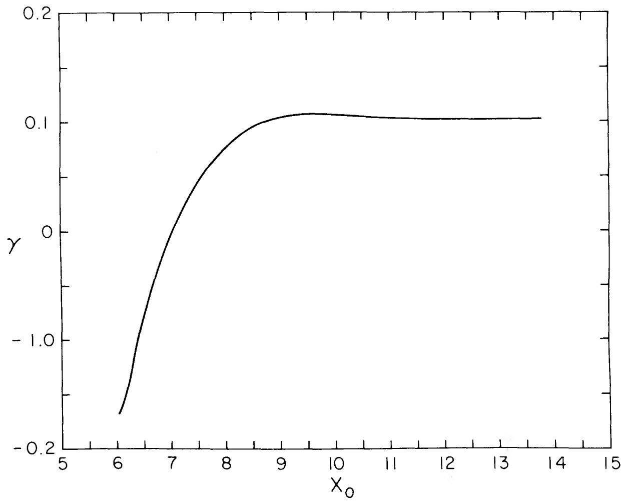

FIG. 7.—The quantity $\gamma = d \ln v_m^2(0)/d \ln \rho(0)$ as a function of scaled escape energy x_0. Note the flattening of the curve for $x_0 \gtrsim 9$. During the late homologous phase of core collapse, $v_m^2(0) \propto \rho(0)^{0.10}$.

energy x_0. As noted in Paper I this rate tends to an asymptotic value as x_0 becomes large. The transition from decreasing to flat behavior in the $\xi(x_0)$ curve occurs near the critical value of $x_0 = 9$. Beyond this point the rate of collapse of the core apparently depends only on the central density and velocity dispersion, which determine the central relaxation time, and not on the structure of the rest of the cluster, which determines x_0. This result does not agree with Heggie's (1979) finding that ξ should decrease approximately exponentially with increasing x_0, for all values of x_0. This discrepancy is discussed in § IV.

It was not clear from the results of Paper I to what extent the asymptotic behavior found there for the core collapse rate was determined by numerical error. As discussed above, the adoption of the Chang and Cooper (1970) finite difference form for the Fokker-Planck equation in the present study has greatly reduced the level of numerical nonconservation of energy. A number of numerical experiments indicate that the flattening of the $\xi(x_0)$ curve in Figure 6 is real. Thus the present study both provides justification for the assumption by Lynden-Bell and Eggleton (1980) of the asymptotic constancy of ξ and provides an estimate for the asymptotic value of this parameter, $\xi \approx 3.6 \times 10^{-3}$.

Additional evidence for the identification of the late stages of core collapse with the gravothermal instability is provided by the evolution of the quantity $\gamma \equiv d \ln v_m^2(0)/d \ln \rho(0)$, which is illustrated in Figure 7. Physically γ specifies the equation of state of the core and is related to the usual adiabatic index $\Gamma = d \ln p/d \ln \rho$ by $\gamma = \Gamma - 1$. Core collapse at constant energy corresponds to $\gamma = 0.2$ (Lynden-Bell 1975). Smaller values of γ indicate a "softer" collapse. For $x_0 \lesssim 7$, γ is negative, which indicates the core initially cools as the cluster evolves. This is the sort of behavior expected for a thermodynamic system of positive specific heat with a temperature profile which decreases outward. However, for $x_0 \gtrsim 7$, γ is positive, which is consistent with a negative specific heat. For $x_0 \gtrsim 9$ the $\xi(x_0)$ curve flattens out and γ approaches an asymptotic value of $\gamma \approx 0.10$, in excellent agreement with the result of Marchant and Shapiro (1980). This is another indication of a transition in the behavior of the core collapse corresponding to $x_0 \approx 9$. As the scaled escape energy becomes larger than this value, the behavior of the core becomes independent of the structure of the remainder of the cluster.

Given the asymptotic constancy of the parameters ξ and γ, it is possible to describe the late stages of core collapse with a simple homological model as has been discussed by Lynden-Bell (1975). One result from such a model is a relation between τ, the time remaining to complete core collapse, and the present central relaxation time,

$$\tau = [(1 - \tfrac{3}{2}\gamma)\xi]^{-1}\, t_r(0)\,,$$
$$\tau = 330\, t_r(0). \qquad (13)$$

IV. DISCUSSION

As noted in Paper I there is a puzzling discrepancy between the present results and those obtained by Heggie (1979). While the present study indicates that the collapse of a cluster core becomes asymptotically independent of the structure of the remainder of the cluster, Heggie (1979) finds that the behavior of the core collapse depends on x_0 even for large x_0. Thus the present results are consistent with the homologous core collapse picture of Lynden-Bell (1975), while Heggie's results are not.

A possible source of these differences is Heggie's (1979) *a priori* adoption of a specific form for the gravitational potential, viz., a truncated isothermal. In the present work cluster evolution is followed by advancing in time both the distribution and the gravitational potential in a self-consistent manner. No assumption is made concerning the form of the potential. Two-body relaxation evidently drives the potential to a nonisothermal profile, a situation which is not considered by Heggie (1979).

This conclusion is supported by the recent work of Inagaki (1979), who has studied the linear stability of confined isothermal stellar systems with the same energy space Fokker-Planck equation used here. He argues that the self-gravity of clusters is crucial to the gravothermal instability and that this property is not sufficiently incorporated into Heggie's theory.

As the number of stars in the core of an evolving cluster decreases, the Fokker-Planck approximation will eventually break down. It is of interest to determine how far a cluster will evolve before this occurs. For a globular cluster containing 10^6 stars, which begins as Plummer's model, the present computation indicates that when 10^3 stars remain in the core the central density has increased by a factor of $10^{6.2}$ and the scaled escape energy is $x_0 = 12.1$. Thus the Fokker-Planck approach should be valid until the cluster is well into the homologous phase of evolution. During subsequent evolutionary phases, large angle scattering and three-body binary formation processes should play an increasingly important role in the evolution of a cluster core. Direct physical collisions between stars, binary formation by two-body tidal capture, and other dissipative interactions may also become important during late cluster evolution. No attempt was made to treat these processes in the present study.

V. SUMMARY

An evolutionary computation of core collapse in a one-component star cluster has been presented. The numerical method utilizes the energy space Fokker-Planck equation which was obtained from the energy-angular momentum space Fokker-Planck equation of Paper I by integrating over the angular momentum dependence. Thus the stellar velocity distribution is assumed to be isotropic. The gravitational potential of the cluster is treated in a self-consistent manner.

During the course of the computation the central density of the cluster increased by a factor of 10^{20} over its initial value. In the later stages of the core collapse, power-law structure developed in the profiles of the density, potential, velocity dispersion, and distribution function. The collapse of the core of the cluster is marked by two distinct phases. During the first phase, for which the scaled escape energy x_0 is less than 9, the cluster becomes increasingly isothermal. In this phase the evolution resembles the King (1966) sequence of cluster models. During the second phase of core collapse, which sets in when x_0 first exceeds about 9, the isothermal region shrinks in size. The radius of this region remains about 10 r_{core} as r_{core} decreases to zero. At larger radii the cluster density profile is characterized by a power law $\rho \propto r^{-2.23}$. A number of changes in the nature of the core collapse which occur when $x_0 \sim 9$ indicate the transition between the two phases. The core collapse rate, $\xi = t_r(0)\, d \ln \rho(0)/dt$, begins to tend to a nonzero asymptotic value when x_0 exceeds this critical value. The quantity $\gamma = d \ln v_m^2(0)/d \ln \rho(0)$, which specifies the equation of state of the core, becomes constant as x_0 exceeds 9. Finally, the entropy $S = -\int d^3r\, d^3v\, f \ln f$ continues to increase for $x_0 \sim 9$, in contrast to the $S(x_0)$ curve for the King sequence, which has a local maximum at this point.

These results are in good agreement with the gravothermal instability picture of Lynden-Bell and Wood (1968). There has recently been some disagreement concerning the extent to which the core collapse observed in detailed studies of cluster evolution corresponds to such an instability. In particular, it has been noted that unlike an enclosed system an unconfined star cluster has no global isothermal equilibrium state. Such unconfined clusters are expected to evolve in the sense of increasing central density and decreasing core radius on the basis of the evaporation model (Lightman and Shapiro 1978). However, this model does not predict the two-phase character of cluster evolution. The onset of the homologous phase, when the scaled escape energy exceeds a critical value, is an essential feature of the gravothermal instability picture.

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