

PULSATIONAL MODE-TYPING IN LINE PROFILE VARIABLES.

I. FOUR BETA CEPHEI STARS

ALFREDO J. CAMPOS AND MYRON A. SMITH

Department of Astronomy and McDonald Observatory, University of Texas at Austin

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ABSTRACT

High-dispersion Reticon observations of the Si III $\lambda 4567$ line of four β Cephei-type variables, γ Pegasi, β Cephei, δ Ceti, and σ Scorpii, are modeled in detail through their pulsation cycles. It is found that a radial pulsation mode can easily reproduce the radial velocity amplitude, changes in line asymmetry, and uniform line width observed in these four stars. These and other criteria cannot be matched with a nonradial mode. Thus all four stars are primarily radial pulsators. This conclusion is in agreement with a colorimetric mode-typing diagnostic recently employed by Stamford and Watson and suggests that radial pulsation plays the dominant role in the observed spectral/photometric variations in most members of this class. Future papers will investigate possible nonradial mode candidates.

Evidence for shocks or moving shells is also presented in visual-line data of σ Sco and in an ultraviolet line of β Cep. This evidence and other data cited suggest that transient hydrodynamic motions are common in β Cep stars. Equivalent width variations (5–20%) are present in all four β Cep stars investigated.

A comparison of old and recent observations indicates that smooth, secular period changes, as opposed to discontinuous ones, are probably occurring in β Cep and δ Ceti; values of $\dot{P}/(P)^2 \approx 0.2$ are indicated for both. No secular period changes can be discerned from our data in γ Peg and σ Sco.

Subject headings: line profiles — stars: β Cephei — stars: pulsation

I. INTRODUCTION

One of the most usual and prominent hallmarks of β Cephei variables is the periodic change of their spectral line profiles. Ample evidence (see reviews by Struve 1955; Percy 1967; Mel'nikov and Popov 1975; Lesh and Aizenman 1976, 1978) suggests the presence of a complicated variety of spectral variations which cannot necessarily be explained by a single pulsational model. Yet, where there is diversity, one also has a handle on the type of pulsation involved. This diversity will be exploited in this series of papers on line profile analysis of stably periodic stars to determine whether their dominant pulsation modes are radial or nonradial.

A continuing astrophysical dilemma is the lack of plausible theoretical mechanisms that account for β Cep variability. An important constraint can be imposed on this undiscovered mechanism, if indeed the pulsation modes in these stars can be classified as radial or nonradial. Such a categorization tells us much about the regions of the stellar interior in which the pulsations are likely to be excited. One dares not classify modes by comparing the observed periods directly with theoretical ones, because a large overlap exists between radial and nonradial mode periods (Lesh and Aizenman 1974; Osaki 1975; Cox 1976). Therefore, three alternative tests have been put for-

ward to differentiate radial from nonradial modes in these stars. They are as follows: (1) evaluating the color-to-light amplitude ratio (the ratio is large for radial modes [Stamford and Watson 1978a]), (2) searching for shocks or moving shells associated with large amplitude radial pulsations (Odgers 1956; Smith 1979), and (3) modeling the detailed variation of line profiles through the cycle (Osaki 1971; Stamford and Watson 1977; Smith 1977, 1979).

In this paper we adopt the third of the above procedures to classify the dominant mode in four β Cep stars as radial or nonradial. In doing so we will draw upon experience accrued in modeling the related "53 Persei variables" (Smith 1978a, 1979; Buta and Smith 1979). The line profiles of the latter group differ from those in our β Cep sample in two important ways. First, in the case of the 53 Persei variables, very little radial velocity change accompanies a change in line shape. Second, in the 53 Persei sample surveyed so far, these stars tend to have smaller rotational velocities. It will be shown in this paper that increases in rotational velocity are critical to understanding why certain β Cep stars exhibit profile variations and why a few do not.

The stars analyzed in this initial survey were chosen principally because of observing convenience. They are γ Pegasi, β Cephei, δ Ceti, and σ Scorpii. Of these four, only one, σ Sco, shows a variety of complications

(multi periodicity, a van Hoof effect (Struve, McNamara, and Zeberg 1955), and strong shocks in the spectra). Two stars, β Cep and σ Sco, are known to be in binaries. While excellent ephemerides have been computed for all four stars from light and radial velocity curves, modern, photoelectric, line-profile observations are available so far only for β Cep and γ Peg (Smith and McCall 1978a). The β Cep observations were carried out by Goldberg, Walker, and Odgers (1974), who determined that line asymmetries were present every half cycle, in contradiction to a low-dispersion report by Struve *et al.* (1953). Unfortunately, the Goldberg *et al.* data were reproduced on a small scale and in analog form, so that it is difficult to subject them to the numerical scrutiny needed for profile modeling. All in all, we believed the situation warranted the acquisition of new, high signal-to-noise ratio, photoelectric profiles, which could then be compared with both radial and non-radial pulsational velocity fields. In § IV it is shown that for all four β Cep stars considered, the profile variations can easily be fit with radial, but not nonradial, pulsation theory. Evidence will also be presented for the existence of shocks or “bouncing shells” in the optical lines of γ Peg and σ Sco and in an ultraviolet line of β Cep. This paper actually represents the first attempt to model fully the line profiles of β Cep stars in detail around the pulsation cycle; it is hoped that other observers will follow suit. Paper II will deal with a pair of δ Scuti stars. Papers III and IV will treat a couple of other β Cep stars which have complicated period structures.

II. OBSERVATIONS

Most of the observations utilized in this study were obtained with the 1024-element coude Reticon (Vogt, Tull, and Kelton 1978) on the 107 inch (2.7 m) telescope of McDonald Observatory. The signal-to-noise ratio in these profiles is typically 100 to 200. The dispersion used, $2 \text{ \AA mm}^{-1}$, allowed a spectral resolution over two diodes of $100 \text{ m\AA}$ and a spectral range of $50 \text{ \AA}$ in the blue spectral region. Typical exposure times were 20–25 minutes. A second source of profile data became available in early 1979, when the commissioning of a new Reticon system on the coude spectrograph of the Struve 82 inch (2.1 m) telescope permitted additional spectra to be obtained of σ Sco. The observing parameters here were basically the same as on the 107 inch (2.7 m) Reticon system, except that signal-to-noise ratios of about 200 were typical, and the pixel geometry allowed $3 \frac{1}{3}$ diodes per $100 \text{ m\AA}$ resolution element. Third, in 1977–1978, a large number of $3 \text{ \AA mm}^{-1}$ photographic spectra were obtained of β Cep on the 82 inch (2.1 m) coude spectrograph. These were used to construct an up-to-date radial velocity curve. Fourth, all data for γ Peg have been taken from Smith and McCall (1978a). For that work, a coude scanner was used to give about the same resolution and signal-to-noise ratio as for the Reticon

data just mentioned. The γ Peg data consisted of a set of 10 minute resolution observations of the Si III $\lambda 4567$ profile through $1 \frac{1}{2}$ pulsation cycles. Finally, *Copernicus* spectra of the Si III $\lambda 1109.97$ line in β Cep were obtained on 1978 September 23–24. These observations were made with the U1 phototube-grating configuration, which allowed a sampling interval of $21 \text{ m\AA}$ and a resolution of $50 \text{ m\AA}$. A “triple scanning mode routine” permitted 40 points, or $0.9 \text{ \AA}$, to be covered in these observations. As these profiles were obtained over a number of pulsation cycles, it was decided to co-add the data and thereby enhance the signal-to-noise ratio ($=50$). The price paid is a somewhat degraded spectral resolution. Our UV observations will be displayed with the original sampling interval so as to preserve the information content in the original data. Finally, by making multiple observations of the interstellar H_2 line at $\lambda 1110$ in another star, we are able to estimate the radial velocity measurement precision for these UV observations, $\pm 1 \frac{1}{2} \text{ km s}^{-1}$.

All optical observations were centered on the Si III $\lambda 4552$, 4567 , and 4574 triplet. Since these lines arise from a common multiplet, any small differences in their shapes are a consequence of saturation in the stronger lines. It was determined that saturation effects are already important for the $\lambda 4552$ line and render this line less than ideal for modeling purposes. In the same observations we found that the noise in the weak $\lambda 4574$ lines is excessively large, and this line was eliminated from modeling too. Therefore, all profile modeling was confined to the $\lambda 4567$ line. This line is well suited to modeling and has a clear continuum defined for a few $\text{\AA}$ on the other side. Both $\lambda 4552$ and $\lambda 4567$ were used for radial velocity measurements.

Each Reticon observation was accompanied immediately by a flat field lamp and Fe-Ne hollow-cathode comparison-line observation. The stellar and comparison observations were divided by the lamp observation to remove diode-to-diode gain differences and (to first order) vignetting effects across the field. The comparison lines utilized for the radial velocity measurements were the Fe I $\lambda 4552.544$ and $\lambda 4568.789$ lines. Radial velocities were measured by overlaying the comparison and spectral scans, measuring the differences in line positions, and applying heliocentric and diurnal velocity corrections. The errors achieved were $\pm 1 \text{ km s}^{-1}$ (rms). The radial velocity measurement criterion employed in this procedure was the centroid of the half-power point of the spectral lines. This criterion is to be preferred as a measure of a star's systemic velocity, because it minimizes distortions on the profile imposed by asymmetries (upper and lower halves of profile move in opposite directions) and by atmospheric shocks (visible only in upper portion of profile), because it is insensitive to the spectroscopic resolution employed, and because it uses a large portion of the profile to determine the radial velocity. In the limit of small pulsation velocities, the lines remain symmetric, and all measurement criteria give

TABLE 1
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β Cephei						δ Ceti			
Visual			Ultraviolet						
Obsn. No.	JD. 2440000.0+	V_r (km s ⁻¹)	Obsn. No.	JD. 2440000.0+	V_r (km s ⁻¹)	Obsn. No.	JD. 2440000.0+	V_r (km s ⁻¹)	$V_{r\text{orb}}$
10682	3350.7004	-29.9	1	3775.2749	-20.5	1086	3747.8107	+11.6	
10683	3350.7163	-34.8	2	3775.2832	-24.9	1087	3747.8343	+12.2	
10684	3350.7274	-30.9	3	3775.3443	-27.6	1089	3747.8781	- 1.0	
10685	3350.7364	-27.4	4	3775.3526	-24.3	1116	3748.8857	- 0.2	
10686	3350.7455	-24.1	5	3775.4013	+ 4.6	1117	3748.9107	+ 5.6	
10687	3350.7622	-12.9	6	3775.4103	+ 6.0	1129	3779.8189	+ 0.1	
10689	3350.7823	- 2.1	7	3775.4193	+ 6.2	1149	3780.7383	+ 2.8	
10690	3350.7920	+ 2.7	8	3775.4749	-25.7	1152	3780.8021	+ 3.2	
10691	3350.8031	+ 8.5	9	3775.4818	-29.5				
10692	3350.8142	+13.3	10	3775.4908	-33.0				
10693	3350.8240	+12.5	11	3775.5367	-27.0				
10694	3350.8365	+ 7.8	12	3775.5492	-21.0				
10695	3350.8517	- 3.5	13	3775.5582	-14.5	388	3205.9468	+36.6	+29.6
10696	3350.8670	-13.1	14	3775.6061	+ 8.2	391	3205.9913	-18.3	-25.3
10697	3350.8906	-28.4	15	3775.6186	+ 2.0	574	3385.6145	+43.4	+50.4
10698	3350.9045	-34.6	16	3775.6811	-32.6	608	3386.6201	+46.7	+51.7
10699	3350.9170	-29.6	17	3775.6943	-34.2	639	3387.6126	+44.7	+47.7
10725	3353.7475	-30.1	18	3775.7429	-19.4	1432	3950.9627	-30.2	-28.2
10726	3353.7607	-33.3	19	3775.7506	-14.2	1433	3950.9787	-38.8	-36.8
10727	3353.7725	-32.6	20	3775.7589	- 9.4	1434	3950.9946	-45.4	-43.4
10729	3353.8135	-11.9	21	3775.8256	- 5.1	1435	3951.0120	-41.3	-39.3
10730	3353.8281	- 4.7	22	3775.8325	-11.5	1436	3951.0273	-28.4	-26.4
10731	3353.8406	+ 4.2	23	3775.8957	-35.1	1459	3951.8911	+31.8	+32.8
10732	3353.8614	+12.5	24	3775.9047	-32.1	1460	3951.9077	+ 3.6	+ 4.6
10733	3353.8892	+ 4.6	25	3775.9651	- 0.2	1461	3951.9223	-12.1	-11.1
10754	3355.7725	+13.0	26	3775.9742	+ 4.7	1462	3951.9355	-20.8	-19.8
10755	3355.7843	+10.1	27	3776.0346	-17.2	1463	3951.9508	-32.6	-31.6
10756	3355.7954	+ 3.0	28	3776.0429	-23.2	1464	3951.9640	-41.0	-40.0
10757	3355.8093	- 9.4	29	3776.1033	-29.7	1465	3951.9786	-43.3	-42.3
10758	3355.8246	-15.6	30	3776.1124	-26.4	1480	3980.9582	+27.6	+37.6
10759	3355.8371	-25.2	31	3776.1728	+ 6.3	1500	3981.9227	+ 0.5	+ 8.5
10760	3355.8510	-33.7	32	3776.1818	+ 5.8	1501	3981.9380	+13.0	+21.0
10761	3355.8718	-29.7	33	3776.2422	-27.8	1502	3981.9553	+26.3	+34.3
10762	3355.8913	-21.9	34	3776.2506	-31.0	1534	3983.8462	-41.0	-36.0
10763	3355.9093	-13.6	35	3776.3110	-22.4	1535	3983.8601	-33.4	-28.4
10764	3355.9309	- 1.6	36	3776.3200	-16.4	1536	3983.8739	-21.5	-16.5
10765	3355.9517	+ 9.8	37	3776.3804	+ 2.5	1542	3983.9406	+37.1	+42.1
10766	3355.9684	+ 9.4	38	3776.3888	- 0.7	1543	3983.9538	+41.1	+46.1
991	3700.9791	- 8.0							
1001	3701.9797	-32.6							
1017	3703.9637	+ 4.9							
1148	3780.7173	- 6.2							
1771	4083.9461	-23.2							

the same radial velocity. However, it should be borne in mind in the discussion below that because most earlier radial velocities were obtained from photographic plates, with undefined measurement criteria, our double amplitudes ($2K$) will not necessarily coincide with earlier determinations. Fortunately, the differences happen to be small in all program stars, except possibly σ Sco, where a 10% difference is found.

A Journal of Observations is given for reference in Table 1. This table includes the type of observing mode, heliocentric corrected Julian Dates, and radial velocities, and, when necessary, velocities corrected for binary motion (Fitch 1967).

III. RADIAL AND NONRADIAL MODES: EFFECTS ON LINE PROFILES

a) The Line-Profile-Modeling Algorithm

Two computer programs and a model atmosphere were employed to compute line profiles broadened by radial/nonradial pulsation, rotation (V), and radial-tangential macroturbulence (M , Gray 1976). A $T_{\text{eff}} = 20,000$ K, $\log g = 3.5$ atmosphere was taken from Kurucz, Peytremann, and Avrett (1974) and used to calculate line profiles by the first program, ABUND. A microturbulence value of 3 km s^{-1} appropriate to τ Sco (B0.2 V) was assumed for all calculations (Smith

and Karp 1978). The program ABUND computes 20 intensity line profiles at equally spaced μ -points across the disk. These profiles are stored on tape and used by a second program, NONRAD, to integrate across the stellar disk the net line-of-sight Doppler motions due to the three velocity fields. For this task, 3600 grid points on the disk are used. The final result is a velocity integrated flux profile which reflects the effects of these velocities, as well as limb darkening, departures from LTE, and instrumental broadening. Note in this approach that any differential atmospheric velocities along a path length are ignored. In our actual profile-fitting process, symmetric phase profiles are utilized to obtain the best (V, M) combination. The asymmetry and radial velocity ranges fix the pulsation amplitude. The rotational velocity is held constant through the cycle, and M is adjusted to give the best fit. The stability of M for all cases becomes a reasonable "goodness of fit" parameter.

All calculations were performed on the CDC 6400/6600 computer of the University of Texas.

b) Radial Pulsation Effects

Since differential atmospheric motions are provisionally neglected in our radial pulsation (RP) model, one may write the radial velocity of a point on the stellar disk as follows:

$$V_{RP} = A_R \mu \cos \phi, \quad (1)$$

where A_R is the radial pulsation velocity, μ is the cosine of the angle between the observer substellar point and an arbitrary point on the disk, and ϕ is the pulsation phase (re $\phi = 0.0$ at velocity maximum). Although this model is simple, its successful application indicates that differential expansion effects are not significant, except when atmospheric shocks become important.

Each modeling parameter has some effect on the computed profiles. Macroturbulence acts merely as a convolving function, forcing the profile into the shape of an inverted exponential function. As is well known,

rotation broadens the profile into a U-shaped function. Without pulsation, both functions produce a symmetric profile. A radial pulsation field alone, computed at $\phi \approx 0.0$ or 0.5 , introduces a small asymmetry, no change in the line width, but a large change in radial velocity. However, the interesting effects are observed when rotation and pulsation are combined, as will be shown.

Figure 1 shows what happens to a representative profile at velocity maximum ($\phi = 0.0$) when the rotational velocity is changed and the amplitude A_R is held constant. As expected, when $V \equiv V_{\text{rot}} \sin \mathcal{J}$ increases, the line broadens; the radial velocity becomes more difficult to measure and increases only very slightly. The important and surprising change is that the line becomes more asymmetric. The degree of asymmetry, normalized with respect to the full width at half-maximum (FWHM), appears to go through a maximum when $V \approx A_R$. As V increases to even larger values, difficulties in placing the continuum and noise in the observed profile will cause it to evolve to a broad symmetric profile. Hence in rapid rotators the asymmetry is likely to go unnoticed.

Figure 2 shows what happens when V is fixed and A_R is increased. The line width remains essentially unchanged, but both the radial-velocity amplitude and degree of asymmetry increase. This is because the core is moved to longer wavelengths more than the wings are. The cancellation of the two shifts is incomplete, and the line becomes asymmetric, so that the radial velocity is seen indeed to depend upon the measurement criterion chosen.

There are two more key points to keep in mind about the effects of radial pulsation on profiles. The first is that a number of modeling experiments in the range $5 \leq V \leq 50 \text{ km s}^{-1}$ demonstrate that the measured radial-velocity amplitude is almost linear with A_R . For our B-star models this relation can be expressed by the equation

$$2K = 1.5 \left(\frac{V}{10} \right)^{0.06} A_R. \quad (2)$$

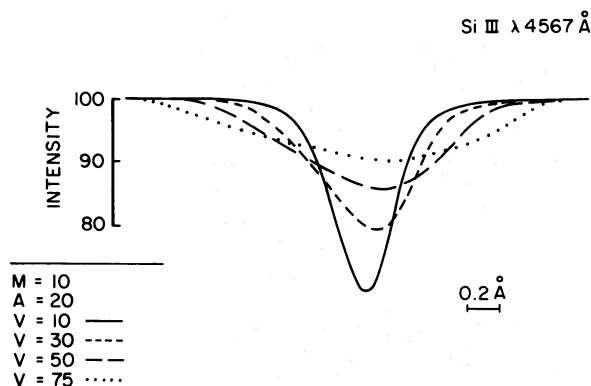


FIG. 1.—The effect of keeping A_R constant and of increasing the rotational velocity on a $\lambda 4567$ line profile computed at $\phi = 0.0$ (uniformly expanding atmosphere assumed).

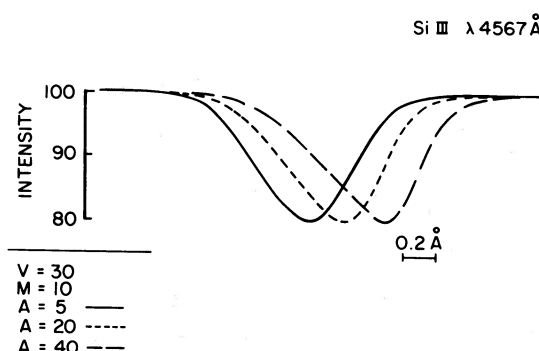


FIG. 2.—The effect on $\lambda 4567$ of keeping the rotational velocity constant and of increasing the radial pulsational amplitude.

This coefficient, ~ 1.55 , is to be compared to the "classical" geometrical coefficient, $24/17 = 1.41$ and Parsons's (1972) modified coefficient 1.31, for differentially expanding atmospheres of δ Cepheids. The use of ~ 1.55 is to be recommended in any future application of a modified Baade-Wesselink analysis to β Cep stars. The second effect of radial pulsation to be stressed is that although it produces opposite profile asymmetries near $\phi = 0.0$ and 0.5 , the line width (FWHM) is virtually constant through the cycle.

The finding that increased rotation can enhance asymmetries runs counter to the classical view that rotation and pulsational velocity fields can be treated as convolving functions. It will be recalled that in general the convolution theorem holds only when the two broadening functions to be combined are independent of one another. In fact, RP and rotational velocities each have highly ordered geometries and are correlated across the disk, albeit in a complicated way. For example, the locus of isovelocity on the stellar disk is a simple geometrical figure, an ellipse (Duval and Karp 1978). As a result, the convolution theorem cannot be used to combine these two broadening functions. In fact, application of the theorem to them even gives incorrect *qualitative* results (Smith and Gray 1976; Stamford and Watson 1977). The realization that it is the *combined effect of rotation and pulsation velocities* that produces asymmetries will enable us to understand in § IV how radial pulsation can produce virtually no profile variations in sharp-lined stars like γ Peg and large asymmetries in more rapid rotators like σ Sco.

c) Nonradial Pulsation Effects

A large body of literature has been compiled to explore the types of profile variations produced by nonradial pulsation (NRP). Nonradial pulsations are synonymous with periodic traveling or standing waves. The NRP velocity field is described by three algebraic terms. For short-period (β Cephei-like) oscillations the first (radial) term dominates at the stellar surface. For an arbitrary element residing at polar coordinates (θ, ψ) (referred to the rotational axis), the radial velocity an observer sees from an inclination $\mathcal{I}$ is given by:

$$V_{\text{NR}} = -A_{\text{NR}} P_{\ell}^m(\cos \theta) \text{Re}\{\exp[i(m\psi + \sigma t)]\} \times [\cos \mathcal{I} \cos \theta + \sin \mathcal{I} \sin \theta \cos(\psi - \Phi)], \quad (3)$$

where A_{NR} is the nonradial, pulsational velocity amplitude, P_{ℓ}^m is the associated Legendre polynomial, m and ℓ are surface quantum numbers, σ is the pulsation frequency, and t is the elapsed time since phase zero. The nonradial phase is defined as $\Phi = (\sigma t + m\psi)$, such that $\Phi = 0.0$ corresponds to radial velocity minimum (Osaki 1971). Thus, in discussing a radial velocity curve, Φ and ϕ differ by 0.5 cycles. (Throughout this series of papers the radial/nonradial phase conventions will be kept separate by lower/upper case symbols.) Finally, one obtains the total effect of V_{NR} on a

line profile by integrating it, V , and M over the stellar disk.

The effect of the nonradial mode order ℓ on profiles will be investigated in Paper III. For now let us consider the effect of the mode index m , where $|m| \leq \ell$. The most widely studied modes (see, e.g., Osaki 1971; Smith and McCall 1978; Unno *et al.* 1979) are the sectorial modes (equatorial traveling waves), for which $|m| = \ell$. Such modes give blue-displaced cores and depressed lines at $\Phi = 0.00$, symmetric narrow lines at $\Phi = 0.25$, "blue asymmetries" at $\Phi = 0.50$, and broad lines at $\Phi = 0.75$. Thus, traveling waves cause modulations both in line width and asymmetry. These two attributes vary 90° out of phase with one another. Moreover, the symmetry properties of equation (3) dictate that profiles at $\Phi = 0.25$ (or 0.75) $\pm \delta\Phi$ are mirror images if $\delta\Phi \leq 0.25$. (One can obtain a good qualitative feel for *why* the rotational and pulsational velocity fields interact as they do to produce these shapes by consulting Fig. 2 of Smith and McCall 1978b.) An increase in A_{NR} accentuates the profile distortions until $A_{\text{NR}} \approx V$, a point evidently not reached in most variables. The effects decrease to zero with inclination. The remarkable property of profile distortions imposed by nonradial pulsation is that the wavelength shifts of core and wing nearly cancel. Then, because of our FWHM-centroid measurement criterion, only a very small radial velocity change will be measured. Finally, only $m < 0$ modes (prograde with rotation) will be considered herein, as these are observed to be most active in the 53 Persei variables (Smith 1978a) and probably in most β Cep stars that have visible nonradial modes (e.g., Jerzykiewicz 1978).

Standing waves ($m = 0$) have a totally different effect on the line profile. Consider the simplest example, that of an observer looking down on the pole of an $\ell = 1, m = 0$ pulsator. The axisymmetric oscillation he witnesses will closely mimic radial pulsation, because nearly the entire visible hemisphere is moving toward or away from the star's center at the same time. Therefore the profile variations through the cycle will be indistinguishable from the radial pulsation case already cited. For various ℓ values and observer-inclinations, the amplitude of the radial velocity and asymmetries will be considerably reduced, but the qualitative effect remains much the same. In the foregoing it will be argued that "radial pulsation" alone is consistent with the profile variations presented in this study. Strictly speaking, one cannot altogether eliminate a nonradial $m = 0$ solution, because its effects on profiles are so similar to RP. However, there are a few physical reasons for believing the $m = 0$ solution is implausible. First, at a random inclination, the standing wave amplitude needed to produce the observed $2K$ and asymmetries will usually be considerably larger than the radial one (exception: pole-on orientation, $\ell = 1$). Thus we prefer to adopt the mode-type which has the smaller amplitude, as the observed variability range of the β Cep group is already small in the first place. Second, it will be shown

below that all *four* stars examined show a dominant "radial mode" at the surface. It is almost inconceivable that all four of them have $m = 0$ active and are observed pole-on. Third, one would expect $m \neq 0$ modes to be excited along with $m = 0$ in some cases, and their profile variations would signal their NRP origin; they are not seen. To sum up, the $m = 0$ nonradial solution is a conceivable, but very implausible, solution based on several considerations.

The intermediate standing-wave solutions satisfy the condition $0 < |m| < \ell$ and are called tesseral modes. In brief, these modes produce profile distortions intermediate between the $m = 0$ and $|m| = \ell$ cases. Thus, moderate variations both in line shape and in radial velocities are produced. Finally, note from equation (3) that for certain latitude belts on the star $P_\ell(\cos \theta) < 0$. Therefore, the profile variations can appear to move in the retrograde sense with time. Modeling results obtained from radial, nonradial-sectorial, and nonradial-tesseral modes will be discussed with particular regard to observations of the star β Cep itself.

IV. RESULTS

a) γ Pegasi

i) General

Gamma Pegasi (B2 IV) is a small-amplitude β Cep having a peak-to-peak photometric variation, Δm_p , of 0.017 mag (Jerzykiewicz 1970), a total radial-velocity variation, $2K$, of $6.9 \pm 0.1 \text{ km s}^{-1}$ (McNamara 1953; Smith and McCall 1978*b*) and a single period $P = 0.15175020 \pm 4$ days (Smith and McCall 1978*a*).

Several studies have been unable to unearth evidence for a secular $\dot{P}$ change in the last few decades (van Hoof 1970; Sareyan, Valtier, and Le Contel 1975; Smith and McCall 1978*b*; Lane and Percy 1979). No line profile variations occur during the cycle, except for weak features in the wings. These have been ascribed to the presence of a moving shell (or possibly a shock) in the atmosphere (Smith and McCall 1978*a*).

ii) Mode-Typing

In order to test our radial-pulsation line-broadening algorithm, it was decided to start with a good candidate for radial pulsation. Using spectroscopic and color-to-light amplitude arguments, respectively, Smith and McCall (1978*b*) and Stamford and Watson (1978*a*) have argued that γ Peg is a radial pulsator. It is now easy to test this hypothesis. We have selected four quarter-phase profiles of the 22 equally spaced observations made by Smith and McCall (1978*b*) around the cycle. It was quickly determined that both nonradial sectorial and tesseral modes could be ruled out, as they introduce profile distortions (unobserved) and do not produce the observed radial velocity changes. On the other hand, as Figure 3 shows, a radial mode solution with $A_R = 5.0 \pm 0.1 \text{ km s}^{-1}$ works well. It will not produce a change in shape and will reproduce the observed radial velocity amplitude (Fig. 4). Note that, as required, "no" asymmetries are produced because both velocities A_R and V are small. To arrive at this solution, it was necessary to adopt a constant value of $V = 4.5 \text{ km s}^{-1}$ through the cycle; the macro-turbulence M was varied by only $\pm \frac{1}{2} \text{ km s}^{-1}$. This consistency suggests an excellent "goodness of fit" and

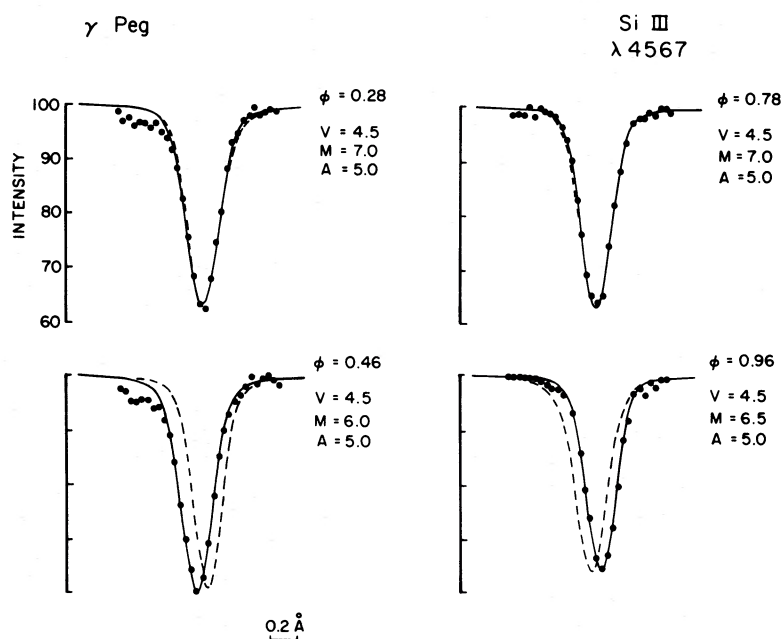


FIG. 3.—Selected (quarter-phases) photoelectric observations of $\lambda 4567$ in γ Peg. Solid line is the radial pulsation fit with the parameters indicated; dashed line represents the "no pulsation" profile.

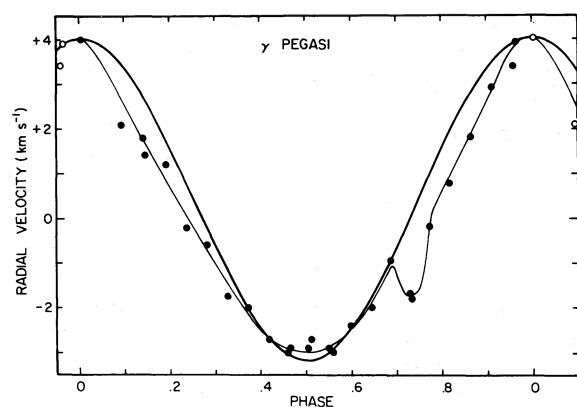


FIG. 4.—Radial velocity curve of “parent” $\lambda 4567$ line of γ Peg. Thin line: smoothed fit to observations (dip corresponds to contamination by “shock” component); thick line: sinusoid resulting from simple radial pulsation theory.

lends credence to the radial mode solution for this star. Accordingly, we can be satisfied that the radial model can be applied to other radial pulsation candidates.

iii) Radial Velocity Waveform

Figure 4 shows the comparison of the observed and predicted radial velocity curve. The dip in the former near $\phi = 0.75$ is “real” and results from the shell feature distorting the centroid wavelength position of the “parent” line, not this phase. It is irrelevant to our analysis. Notice otherwise that the observations do not conform to the predicted sinusoidal waveform predicted from our model. This same discrepancy occurs with β Cep and σ Sco and is always in the sense that the negative velocity lobe is longer in duration and more rounded than the positive lobe. There are several reasons to expect such departures from sinusoidality, differential atmospheric expansion, and a change in the optical depth scale with pulsation phase being among the most likely. Still, any differential expansion effects do not appear to alter the FWHM of observed profiles, even in larger amplitude pulsators.

b) β Cephei

i) General

Beta Cephei (B1 III) is the prototype of this variable class. It exhibits an amplitude $\Delta m_v = 0.0365$ mag (Gray 1970). Struve *et al.* (1953) determined a single period of 0.1904844 days, but Gray (1970) concluded that the period had discontinuously changed to 0.1904881 ± 8 days in the late 1960s (see § IVb[iii]). Struve *et al.* (1953) found no profile variations using 32 Å mm^{-1} plates, but did find a variable $2K = 18\text{--}46 \text{ km s}^{-1}$. However, they did not conclude that a second period was present. Fitch (1967) found a 10.893 day velocity modulation which could be due to binary

motion ($2K = 6.6 \text{ km s}^{-1}$), but neither Gray’s nor our data are compatible with modulated $2K$ or γ -velocities.

As already mentioned, Goldberg, Walker, and Odgers (1974) published a set of high-time-resolution $\lambda 4552$ triplet observations through the cycle and showed that asymmetries were present at $\phi \approx 0.0$ and 0.5 . Unfortunately, the reproduction of their data does not allow a judgment of whether line width variations are present. Based on the Goldberg, Walker, and Odgers (1974) data, Smith (1977) concluded that a mixture of radial and nonradial modes was present. However, since this scheme would imply the presence of two modes having exactly equal periods and phases, an amazing coincidence, Smith had reservations about his own conclusion. Therefore it was decided to investigate the spectral behavior of β Cep firsthand. Our results that this star is a radial mode pulsator indeed modify Smith’s earlier conclusion.

ii) Mode-Typing

Our initial suspicion was that β Cep possesses only a nonradial mode, and so our initial attempts to model five Reticon observations near quarter-phase points, utilized a representative NR $-m = \ell = 2$ mode. The observations (Fig. 5) show asymmetries, but essentially no width variations. The nonradial model employed was incapable of reproducing these characteristics. Moreover, the large radial velocities determined from our data (see below) could not be mimicked with a realistic A_{NR} amplitude. Next, we considered tesseral modes, which produce both changes in shape and radial velocity. A $\ell = 2$, $m = -1$ mode was chosen as representative. This attempt failed too in that the width changes generated by this mode are still much larger than those observed for a given radial velocity amplitude. Our third attempt employed a combined radial-nonradial mode solution ($P_{NR} = P_R$) with constructively interfering phases. However, as the modeling progressed, it was repeatedly found necessary to decrease A_{NR} in order to produce better agreement with the shape of the line. Even small values ($A_{NR} = 2 \text{ km s}^{-1}$) produced line width changes incompatible with the data and line cores that were too narrow. Ultimately the nonradial-radial solution degenerated into a pure radial solution, our fourth attempt. The satisfactory results of this solution are shown in Figures 5 and 6. Note that the observed line shapes, as well as the radial velocity amplitude, can easily be fitted. Our fitting parameters are $V = 30$, $M = 9 \pm 1$, and $A_R = 28 \text{ km s}^{-1}$. The errors in the observed radial velocity curve from both Reticon and photographic data are $\pm 2 \text{ km s}^{-1}$. The radial velocity errors and macroturbulence “adjustments” (Table 2 below) needed for a perfect fit are slightly larger than those found for γ Peg and δ Cet. These small inconsistencies appear to scale roughly with pulsational amplitude and do not affect the conclusion that this star pulsates radially, a conclusion also arrived at by Stamford and Watson (1978a).

TABLE 2
SUMMARY OF VELOCITY MODELING PARAMETERS (km s^{-1})

Star	V	M	A	Mode Type	Comments
γ Pegasi.....	4.5 ± 0.5	6.5 ± 0.5	5 ± 0.1	Radial	Shocks in visual
β Cephei.....	30 ± 1	9 ± 1	28 ± 1	Radial	Shocks in UV
δ Ceti.....	11 ± 0.5	8 ± 0.5	10 ± 0.5	Radial	
σ Scorpii.....	47 ± 3	10 ± 2	68 ± 2	Radial (+NR?)	Shocks in visual and UV

NOTE.—All quoted errors are internal and do not reflect possible cycle-to-cycle variations present outside our data string.

iii) Waveform and Periods

Figures 6 and 7 show virtually identical radial velocity curves for visual and ultraviolet data. In both cases, one finds $2K \approx 45 \text{ km s}^{-1}$ (see also Hutchings and Hill 1977). Also, the shape of the radial velocity curve in both figures departs somewhat from the predicted sinusoid (solid line) in the same sense as the curve for γ Peg does.

It was noticed that the radial velocity errors could be minimized over the total 2 yr baseline during which our data were accumulated by assuming a pulsational period larger than Gray's (1970) value of 0.1904881 ± 8 days. Using Gray's and our own epochs, one finds

a "period" of 0.1904916 ± 9 days. By way of comparison, a "period" of 0.1904918 days can be derived from the epochs of Lane and Percy (1979) and Gray (1970). Either of the recent values differs by 3σ from the value Gray found, and suggests that the period has not remained constant since Gray's epoch. Thus, these recent "period" values imply that the period of the star β Cep does not simply increase in discrete jumps after all. Actually, Struve *et al.* (1953) derived positive secular terms $\dot{P}$ and $\ddot{P}$ from theirs and previous data. Their predicted period for our 1977 epoch is 0.1904921 days, in remarkable agreement with the mean Gray-Campos-Smith value for the 1970s. Thus, we believe that the values of Struve *et al.* of $\dot{P}$ and $\ddot{P}$ are generally

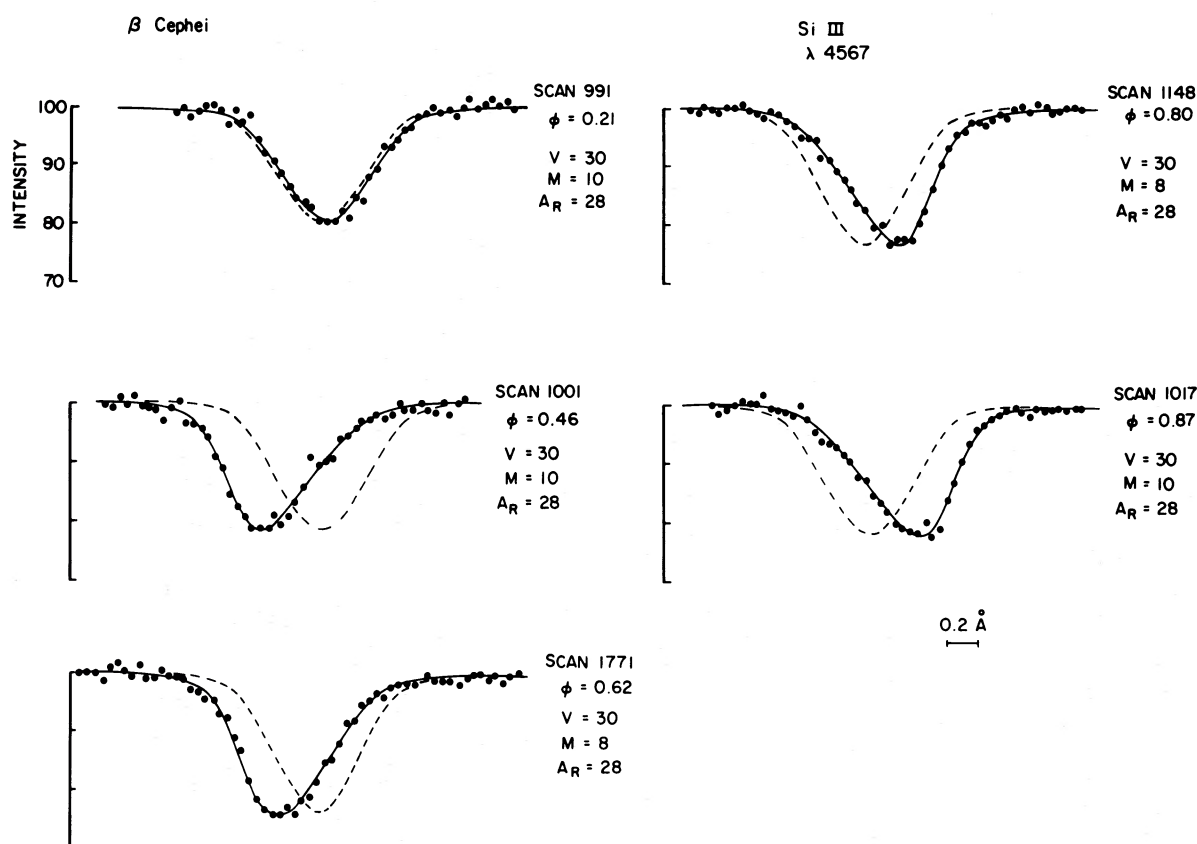


FIG. 5.—Selected Reticon profiles of $\lambda 4567$ in β Cep modeled with radial pulsation

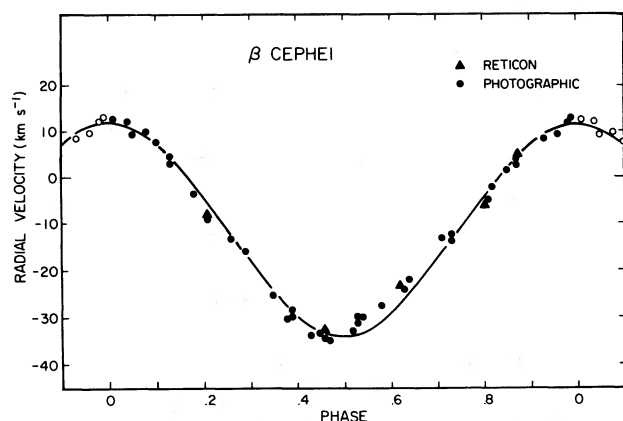


FIG. 6.—Radial velocity curve for β Cep from Reticon and photographic 1977–1978 $\lambda\lambda 4552, 4567$ data. Solid curve is a sinusoid.

valid, though it is probably time to reevaluate their precise values. It should be emphasized that if P is changing continuously for β Cep, then our “period” or any other obtained merely by comparing two epochs, is invalid because the true period must be defined as an instantaneous quantity.

c) δ Ceti

i) General

Delta Ceti (B2 IV) is the third singly periodic variable in our sample. This star has a small amplitude, with values $\Delta m_v = 0.025$ (van Hoof 1968), and $2K = 12.6 \text{ km s}^{-1}$ (McNamara 1955). Jerzykiewicz (1965, 1971) determined period values of 0.1611366 ± 3 and 0.1611380 days, respectively, from extensive photometric work. Earlier reports (van Hoof 1968) that the period of this star is constant appear to be erroneous (Lane 1977).

ii) Mode-Typing

An examination of the Reticon spectra (Fig. 8) distributed around the cycle betray the existence of small asymmetries (hitherto unreported), but no detectable line width variations. On the assumption that δ Ceti has a pulsational amplitude intermediate between those of γ Peg and β Cep, we employed another radial model. This solution quickly resulted in an excellent fit. The parameters derived were $V = 11$, $M = 8 \pm 0.5$, and $A_R = 10 \pm 0.5 \text{ km s}^{-1}$. As before tesseral and sectoral modes could not reproduce the width constancy and radial velocity changes. Once again this radial pulsation conclusion is in agreement with the colorimetric work of Stamford and Watson (1978a). In some ways the fits in Figure 8 are the most perfect fits in our study. Note in Figure 9 that the radial velocity deviations from the theoretical curve are $\pm 1 \text{ km s}^{-1}$, i.e. the measurement accuracy. The only puzzle is why this radial velocity curve is the only one conforming to a sinusoidal shape.

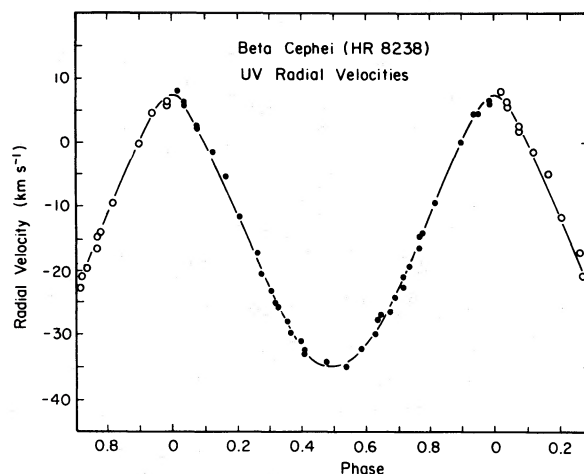


FIG. 7.—Radial velocity curve of Copernicus $\lambda 1110$ data for β Cep. Solid curve is a smooth fit to data.

iii) Period

Our observations require that the period of δ Ceti be lengthened over the 1955–1965 epochs. This is the same conclusion recently arrived at by Lane (1977), who noticed a lengthening of this star’s period over the last few decades. There are three explanations for this behavior. First, the star’s radial velocity may be modulated by binary motion ($P \approx 50 \text{ yr}$; Lane 1977). However, this explanation would contradict several period measurements made earlier in the century. In addition, binary motion might modulate radial velocities but not profile phases; our data show that the latter are not modulated. Second, the period changes could be intrinsic to the star and occur in discrete jumps. However, it has just been mentioned that recent data challenge the best case (β Cep) for a discrete period change, making this alternative less likely. Finally, the period change may be smooth, as Lane herself prefers. She suggests $P = 2.2 - 2.9 \times 10^{-5}$, and $\dot{P} = 1.1 \times 10^{-10}$ for secular change coefficients. We point out that indeed $\dot{P} = 2.68 \times 10^{-5}$ fits our 1978 epoch arrival well. It can also be added that the period estimated from McNamara’s (1955) and our own epochs is 0.1611372 ± 6 , well in excess of McNamara’s value. By way of comparison, the “period” derived from the McNamara (1955) and Lane (1977) epochs is 0.1611376 days, i.e., the same value as we find. There can be no doubt that the period has increased, probably smoothly, over the last several years.

Finally, note that Struve *et al.* (1953) and Lane (1977) compute ratios $\dot{P}/(P)^2 = 0.2$ for β Cep and δ Ceti. This coincidence leads us to suspect that Lane’s interpretation of a secular increase in period is correct and, further, to wonder whether $\dot{P}/(P)^2 \approx 0.2$ has a special significance for some stars in this evolutionary state and position on the H-R diagram.

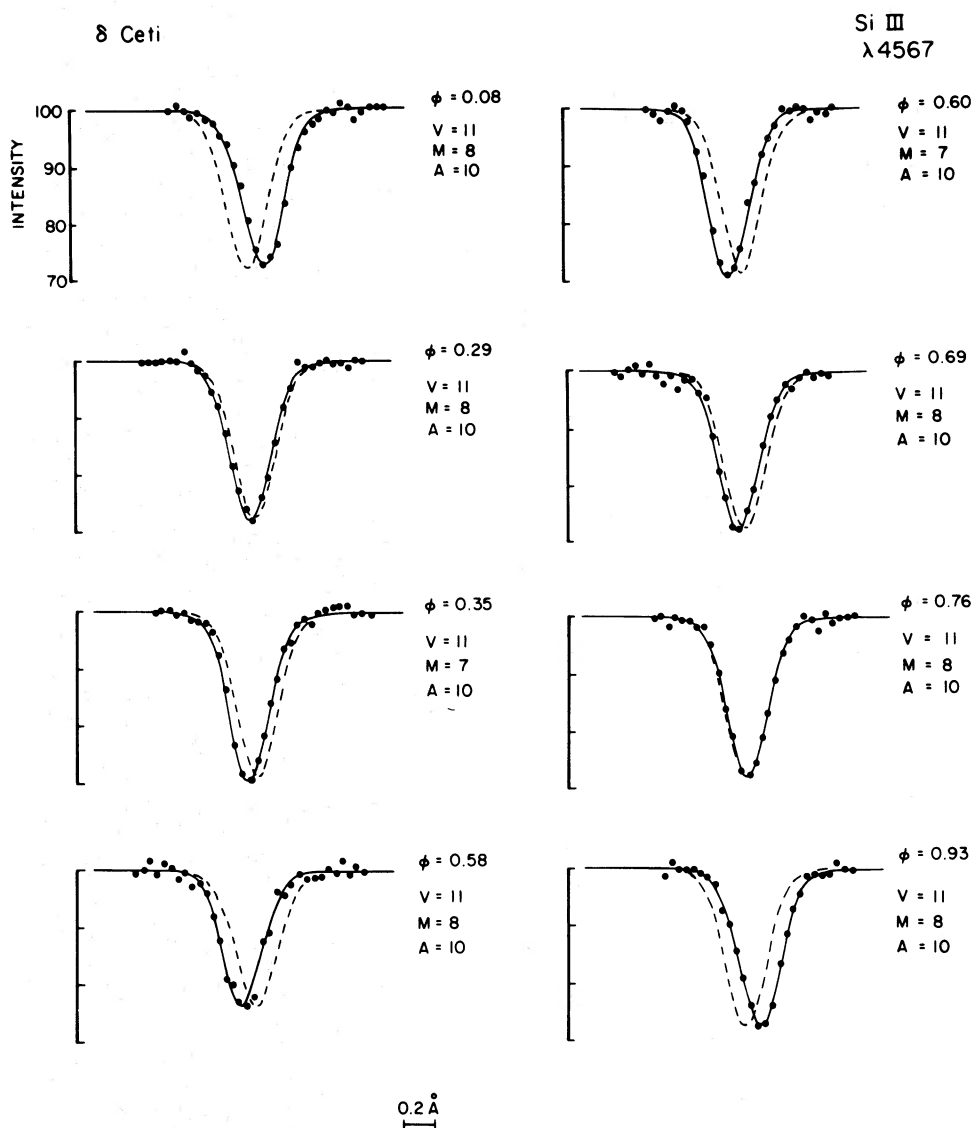


FIG. 8.—Reticon profiles of $\lambda 4567$ in δ Ceti modeled with radial pulsation. Note the excellent consistency of the macroturbulence “best fit” parameter.

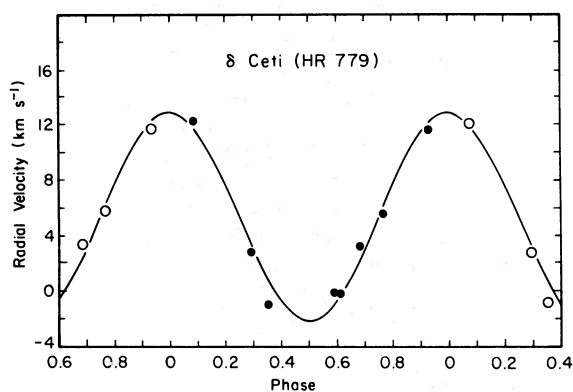


FIG. 9.—Radial velocity curve for δ Ceti ($\lambda\lambda 4552, 4567$)

d) σ Scorpii

i) General

Sigma Scorpii (B1 III) exhibits by far the most complicated spectroscopic behavior of any star in our sample. Struve, McNamara, and Zeberg (1955) found two periods, $P_1 = 0.239$ and $P_2 = 0.2468474$ days. Struve, Sahade, and Zeberg (1961) later revised P_2 to 0.2468429. Here P_2 and P_1 , in Struve's designation, refer to the profile or primary radial-velocity profile period, and the secondary velocity period, respectively. Possible secular changes in P_2 have been investigated by van Hoof (1966) and Sterken (1975). These authors found *shorter* periods than Struve's. Sterken speculated that this variation was not evolutionary, but arises from binary motion or from mass loss events.

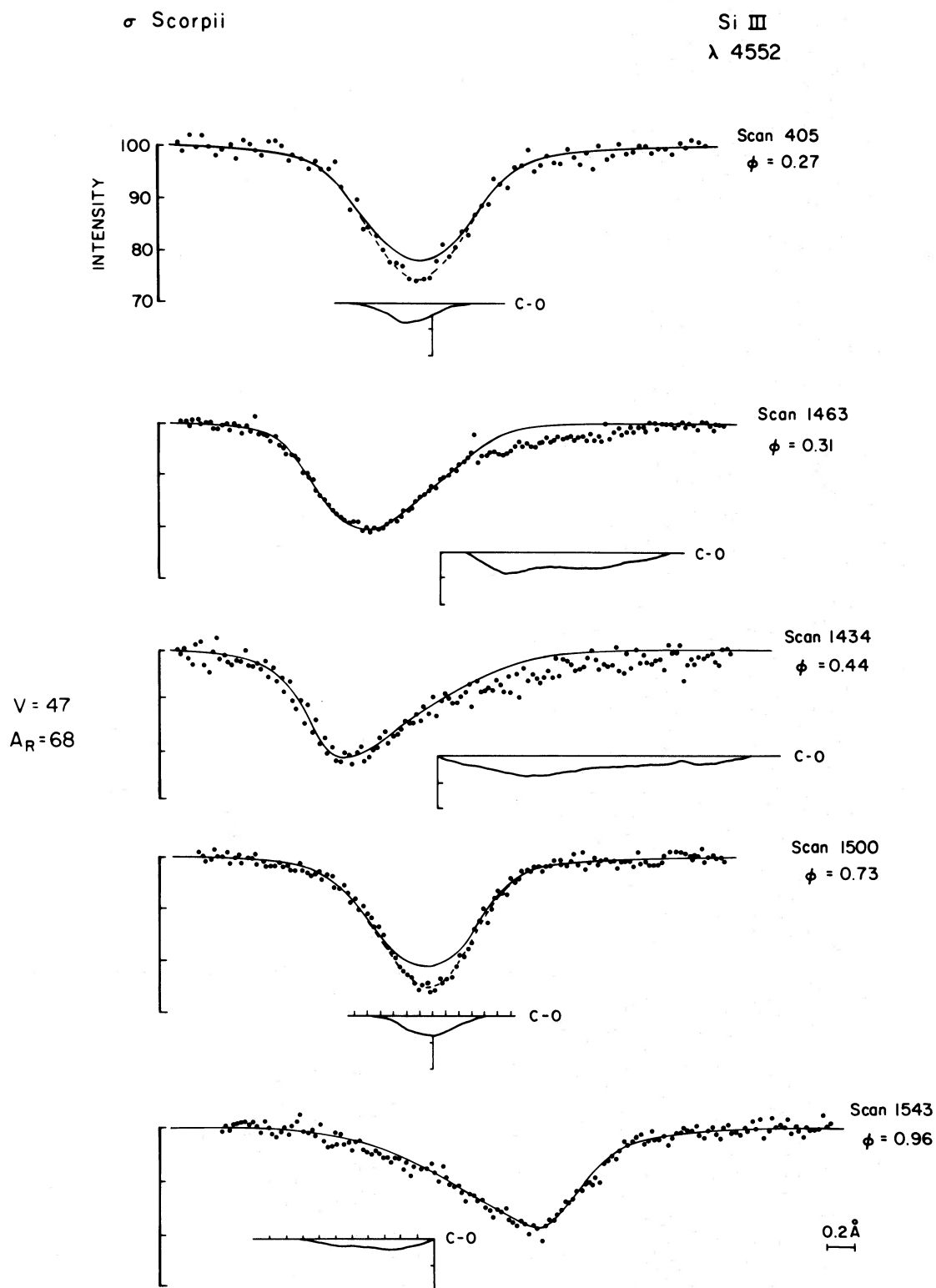


FIG. 10.—Selected Reticon profiles of $\lambda 4567$ in σ Sco modeled with radial pulsation. Note the disagreement (attributed to “shocks”) in the wings of the asymmetric profiles and of the core of scans 405 and 1500. The dot-dashed line in the latter two profiles is the smoothed fit for scan 405, which is shown also to fit scan 1500 closely. The marked vertical line near the center of the diagram denotes the systemic radial velocity of the star.

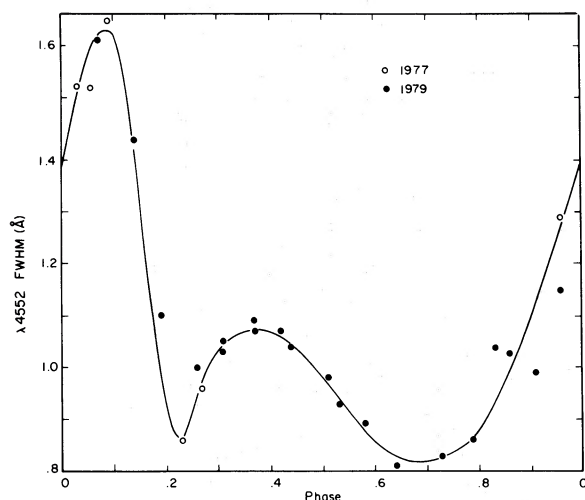


FIG. 11.—The variation in $\lambda 4452$ line width in σ Sco with phase. Reticon observations in 1977 (107 inch system [2.7 m]) are plotted separately from 1979 ones (82 inch system [2.1 m]).

Struve, Sahade, and Zeberg (1961) also found $2K_2 = 110$ and $2K_1 = 15 \text{ km s}^{-1}$. On several occasions a van Hoof effect has been found in the spectrum of σ Sco (Struve, McNamara, and Zeberg (1955) and Struve, Sahade, and Zeberg (1961). Van Hoof (1966) determined photometric amplitudes $\Delta m_{v_2} = 0.040$ and $\Delta m_{v_1} = 0.021$ mag. A beat period between P_1 and P_2 of 8.0 days is also observed. In addition to all these complications the radial velocities are modulated by a binary period of 33.13 days and $K_{\text{orb}} = 34.7 \text{ km s}^{-1}$ (Fitch 1967).

We have redetermined P_2 to be 0.2468396 ± 6 days. This value is coincident with Sterken's (1975) recent values. We are in agreement with him that no evidence for a secular-period change exists yet.

ii) Mode-Typing

Reticon observations of σ Sco were made on three observing runs in 1977 and on two in 1979. On each occasion, secondary absorption features of varying strengths were observed, which we have attributed to shocks (§ IVe). These features cannot be fitted within our velocity field models and constitute the greatest uncertainty in our interpretation of any of the β Cephei-class data.

Several examples of the interference of shocks with the interpretation of the "parent line" are shown in Figure 10. The secondary absorptions show velocity variations that run 180° out of phase from the photospheric parent lines, and this behavior is probably responsible for the "double lobe" structure seen in the contours drawn by Huang and Struve (1955). At asymmetric phases these satellite features are usually distinguishable from the parent line and appear as extensions of a strengthened far wing. In such cases the parent line can be modeled out to the far wing with relative confidence. A mirror-symmetry condition

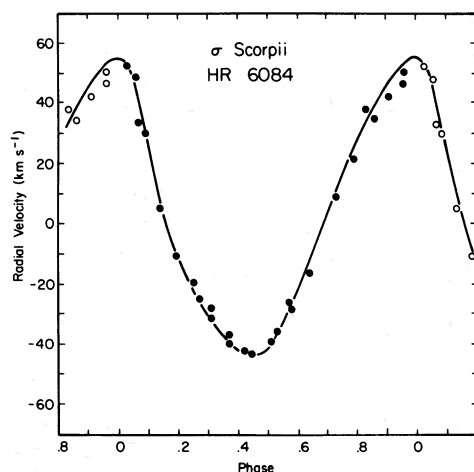


FIG. 12.—Radial velocity curve of σ Sco from 1977–1978 $\lambda\lambda 4552$, 4567 data. Solid curve is a smooth fit to the data.

does not hold well through the cycle, e.g., between phases 0.0 and 0.5. Moreover, as shown in Figure 11, the line widths (FWHM) show an extreme broadening briefly at $\phi \approx 0.1$, and to a far lesser extent near $\phi \approx 0.4$. (Remarkably, this sudden line width change is hardly detectable on the radial velocity curve shown in Fig. 12.) We believe that in our 25 minute time frames we are underresolving a short-lived shock related phenomenon. Perhaps the likeliest interpretation is that the atmosphere is suddenly becoming transparent near $\phi = 0.1$, allowing us to see at once both remnants of a new shock and a weak old one (possibly a reflection from a previous cycle) lower in the photosphere. In any case, the same phenomenon, a double "shell" or shock, was observed by Smith and McCall (see their Fig. 3b) near $\phi = 0.0$, in γ Pegasi as well. It is also interesting to note that Smith and McCall (1978a) observed the emergence of a strong new shock at $\phi \approx 0.4$ in γ Peg, at the same phase when the line width shows a secondary modulation in Figure 11.

From Figure 11 it is clear that low ℓ -order NRP, which produces a sinusoidal width variation peaking at $\phi = 0.75$, cannot provide a good representation to the profile data. Moreover, profiles near phases $\phi = 0.25$ and 0.75 (see dot-dashed lines, observations 405 and 1500 in Fig. 10) show the same line width, so that no narrow and broad-lines occur at these phases, as they must with NRP.

Since NRP cannot explain the P_2 oscillations, we have made an attempt to model the data with RP. The resulting fitting parameters, $V = 47$, $M = 10 \pm 2$, $A_R = 68 \text{ km s}^{-1}$, show the poorest agreement of any of our trials. The fitting discrepancies are most likely due to the presence of strong shocks at intermittent phases. These shocks even appear to change their strength from night to night. Some residual ambiguities in the radial velocities may also arise from the lack of an adequate binary ephemeris for modern epochs. Nonetheless, the model fits in Figure 10 show an agreement

with the estimated line asymmetries, the widths at quarter-cycle phase, and the radial velocity range. Therefore they are adequate for mode-typing purposes. It should be reiterated that the overall behavior of the profiles, particularly including the transient character of the shocks, cannot be understood at all within NRP theory. (A NRP traveling wave can produce a shock at a localized point on the stellar surface, but this should be manifested as a low-amplitude, *smoothly* modulated spectral feature through the pulsation cycle.) Finally, it can be added that our RP conclusion for the P_2 mode is once again in accord with Stamford and Watson's (1978a) colorimetric results.

Because σ Sco is excited in two frequencies, such that $P_1/P_2 = 0.97$, the P_1 oscillation must correspond to an NRP mode. This conclusion follows because period ratios resulting from even neighboring radial-overtones are substantially smaller than unity (e.g., Cox 1976). The small, radial velocity-to-light amplitude observed by Struve (1955) and others is fully consistent with its suspected nonradial nature. Unfortunately, the competition from the shock features is so strong that profile distortions due to P_1 are undetectable.

iii) Waveform

The radial velocity range, $2K_2$, is 100 km s^{-1} , close to Struve *et al.*'s (1953) value. Our observations do not show a K_1 modulation. As for γ Peg and β Cep, the radial velocity curve for σ Sco shows a broad extended negative lobe and a pointed narrow positive one compared to a sinusoid. However, the waveform for this curve is sensitive to shock strength and to the adopted radial velocity measurement criterion (cf. Huang and Struve 1955, Fig. 3a). Our half-width centroid criterion is comparatively unaffected by shocks because for most phases the satellites avoid the FWHM point on the profile. Still, we suspect the peculiar waveform observed for three out of four program stars is related to the shock, inasmuch as absorptions due to them have been observed in the same three stars in visual or ultraviolet lines.

e) Transient Atmospheric Effects

In 1956 Odgers observed BW Vulpeculae and noticed the rapid evolution at certain phases of a double-lobed structure in the core of the line profile. He suggested that this feature is caused by a shock propagating through the atmosphere. This interpretation has been considerably bolstered in this star by observations of H α emission (Odgers and Kushwaha 1960), by a photometric "standstill" at $\phi \approx 0.2$ (Lesh 1978a), and by a recent high-time-resolution movie of profile variations (Young, Snowden, and Furenlid 1979) that shows the progression of rapid transient events through the atmosphere. In addition, those stars having the strongest shocks also tend to exhibit a "van Hoof effect," which is usually taken as evidence

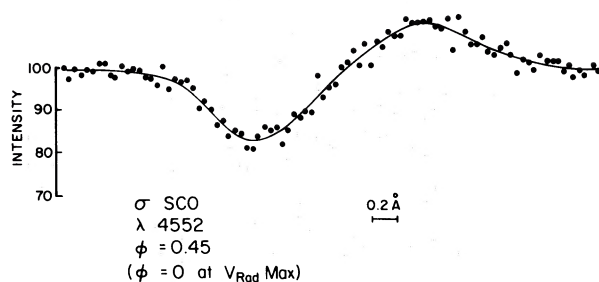


FIG. 13.—A Reticon observation of $\lambda 4552$ in σ Sco (1977 March 3) showing a strong shock/shell component in emission. (Solid line is smoothed fit to data.)

for a running wave. Shocks, or "moving shells,"¹ are manifested as secondary absorption features that are occasionally seen in emission. Le Contel (1968) and Smith and McCall (1978b) have documented a weak shock in γ Peg. In this star the radial velocity curve of the shock feature is in phase with the photospheric lines. As already noted, a strong shock is present in σ Sco; its velocities seem to be 180° out of phase with the photosphere. On 1977 March 3 the shock feature went into emission (Fig. 13) in all three Si III lines, again out of phase with the photospheric velocities. Over shorter time scales (days) the shock strengths can show detectable changes, and large variations are also evident between 1977 and 1979. An attempt will be made to place these observations in a more physical context in a later paper.

Lesh (1978b) has reported UV observations which show persistent blue asymmetries² through the cycle in a number of β Cep stars, including σ Sco. In fact, although no complete survey has been made, there does not yet seem to be a β Cep star in which this phenomenon has been found *not* to exist. In σ Sco the velocities represented by the depressed blue wing of Lesh's (1978b) data are comparable to those indicated in Figures 10 and 13 ($\sim 75 \text{ km s}^{-1}$ relative to photospheric velocity, $\sim 25 \text{ km s}^{-1}$ relative to systemic), and they occur at the same phase. It is natural to suggest that they are indicative of the same phenomenon at different atmospheric strata. There is also reason to believe that the persistent blue asymmetry in UV lines is a more sensitive indicator of these transient motions. For example, neither Goldberg, Walker, and Odgers's (1974) nor our observations show a hint of a shock or shell in visual wavelength lines of β Cep itself, but Figure 14 shows that *Copernicus* satellite observations of the Si III $\lambda 1110$ line show the same persistent asymmetry (especially in $\phi = 0.30, 0.39, 0.59, 0.70$ profiles) as noted by Lesh (1978a) in other β Cepheids. It is also appropriate to point out the persistence of a small emission feature in the line core which is present

¹ Herein the terms are used interchangeably; the choice between these interpretations awaits a more complete analysis of our data.

² This is to say that at $\phi \approx 0.5$ when one expects a blue-displaced core and a depressed red wing, one continues to observe a depressed blue wing.

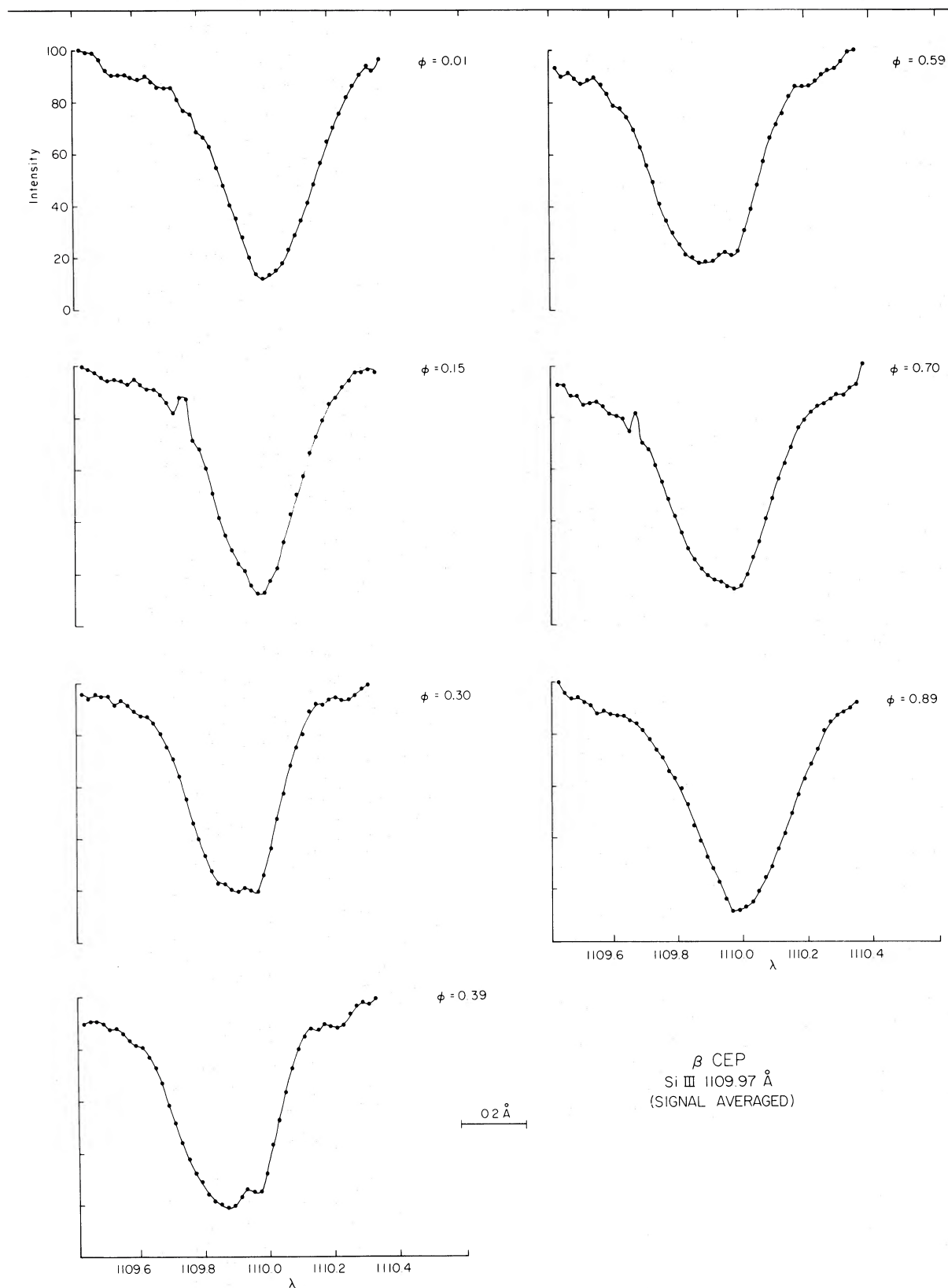


FIG. 14.—Signal-averaged *Copernicus* λ 1110 profiles of β Cep. Note depressed wing and central dimple at phases $\phi = 0.30, 0.39, 0.59$, and 0.70 .

at the same velocity during these phases. This may hint at the existence of a second shock/shell similar to that discussed by Le Contel (1968) and Smith and McCall (1978a) in γ Peg.

It is tempting to speculate that all β Cep stars without shock/shell signatures in the visual will show them in good UV data. In the meantime, in light of the behavior of BW Vul and σ Sco, let us note that there is evidence of a correlation between shock strength and radial pulsation amplitude. The shock in γ Peg is rather weak and can only be observed because the other velocity fields, especially rotation, are small. The shocks are strongest in two of the largest amplitude stars known (BW Vul, σ Sco). The picture that emerges is one in which shocks arise from the nonlinear decay of a sinusoidal waveform introduced by a *radial oscillation*. Such a phenomenon is observed in other radial pulsating variables as well (large-amplitude classical δ Cepheids and Mira-types). Stamford and Watson (1978b) have recently presented evidence for such a picture by showing that a decayed sinusoid becomes a shock and produces spectral transients similar to those observed (Goldberg, Walker, and Odgers 1971, Young, Snowden, and Furenlid 1979) in BW Vul. By way of contrast, no obvious shock features (emission, absorption satellites) have been observed in H α or in metallic lines of the NRP 53 Persei variables. However, in these stars it may be possible to attribute smoothly varying equivalent width changes to shocks (Smith 1978b).

f) Equivalent Width Variations

A commonly occurring statement in the literature is that profile variations in β Cepheids occur in such a way that the equivalent width is conserved. Figure 15 shows that 5–20% line-strength variations are actually present in all four β Cep stars investigated. It should be clarified that our measurements were made so as to avoid a “shock feature” wherever possible; otherwise the variations in σ Sco would range over 28%. It is difficult to make exclusive statements from the behavior shown in Figure 14. However, one generalization that can be made is that the line strengths of these β Cep stars go through a maximum near $\phi = 0.5$, i.e., at a time when shocks are most prominent and when differential velocities of 3 km s^{-1} (necessary to account for the strength variations [Smith and McCall 1978a]) are certainly not unexpected. There is also a tendency for lines to weaken at phases $\phi \approx 0.0$ – 0.2 . It is surprising that the equivalent width variations in γ Peg, a small-amplitude pulsator, are nearly as large as in σ Sco.

V. CONCLUSIONS

Table 2 summarizes the assigned pulsation mode-type and the velocity parameters determined in this study. The error estimates in V and A_R are internal; the error in A_R does not reflect possible cycle-to-cycle variations in the star. The errors in M reflect the

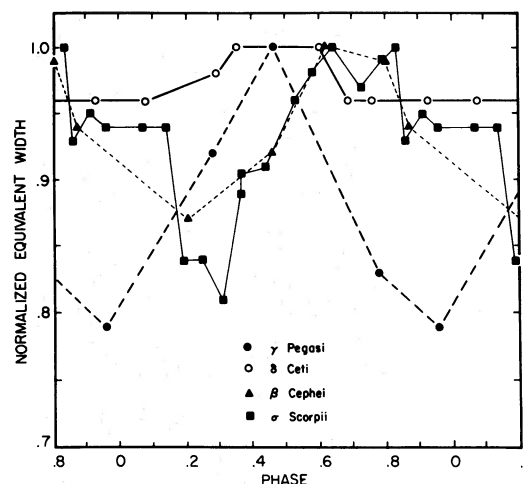


FIG. 15.—Equivalent width variations with phase of the 4 program β Cep stars.

measured variation of the macroturbulence through the cycle and may be indicative of small differential atmospheric motions.

The primary conclusions of this work can be summarized as follows:

1. An analysis of the velocity fields responsible for line profile variations in β Cep stars shows that all four program objects are radial pulsators. The alternative interpretation that the stars show very large amplitude oscillations in $m = 0$ (but not at all in the companion m -modes) is in our view not a serious alternative.
2. The conclusions favoring radial pulsation are in exact agreement with the color-to-light arguments of Stamford and Watson (1978a). Since the two methods are entirely independent, their agreement tends to lead credence to them both. Note that each technique has a weakness: (1) the analysis of profile variations can confuse RP and $m = 0$ NRP modes, and (2) non-adiabatic and geometric effects can appreciably alter the computed light-to-color amplitude ratio of a pulsating star (Buta and Smith 1979).
3. We are able to confirm Lane's (1977) conclusions concerning the smooth secular variations in β Cep and δ Ceti. Both stars exhibit $\dot{P}/P \approx 0.2$. There appears to be no evidence yet for a $\ddot{P}$ term for γ Peg and σ Sco.
4. Contrary to many earlier reports, moderate equivalent width variations appear to be the rule in β Cep stars. The appearance or nonappearance of these variations cannot be used as a RP versus NRP discriminant.
5. Features we attribute to shocks or “moving shells” are prevalent in many, perhaps all, β Cep stars. These features appear to be most prominent in strong UV lines. These features appear just after the compression phase of the pulsation cycle ($\phi \approx 0.4$) and again near $\phi = 0.0$. These shocks are probably intimately related to radial pulsation. If shocks are seen at any of the nonradially pulsating 53 Persei variables, they are

manifested only by a small equivalent width modulation (Smith 1978b).

Conclusion (1) should not be construed to mean that nonradial modes cannot also occur in β Cep stars. In fact, one is forced to the conclusion that *some* modes in stars having closely spaced frequencies (e.g., σ Sco) must be nonradial. From the foregoing we wish to speculate that a radial mode is always present in a β Cep star and that in some stars it may drive a nonradial mode(s) which *happens* to have nearly the same frequency. It may even happen that the nonradial amplitude at the stellar surface exceeds the radial mode's amplitude in certain stars.

A second speculation concerns the relationship between β Cep and related variables: considering that the 53 Persei variables surround the β Cepheids in the H-R diagram (Smith 1978a), it may be that radial

oscillations are inherently stable in B stars, whereas nonradial modes are unstable. Such a circumstance would lead to a natural delineation between these two classes of variable stars and explain why the β Cep stars have easily discovered variations with stable periods, and why the 53 Per variables do not.

This work was done in partial fulfillment of the requirements for a master's degree by A. J. C. We wish to thank Dr. Ron Polidan of Princeton for his competent assistance in obtaining *Copernicus* observations of β Cephei. It is our pleasure to thank Dr. Lee Anne Willson for helpful conversations concerning observations of shock waves in stellar atmospheres. Much of this work was carried out under NASA grant NSG-5309.

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ALFREDO J. CAMPOS: Applied Research Laboratories, Acoustical Measurements Division, University of Texas, Austin, TX 78758

MYRON A. SMITH: Department of Astronomy, University of Texas, Austin, TX 78712