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EFFECTIVE TEMPERATURES OF LATE-TYPE STARS: THE FIELD GIANTS FROM K0 TO M6

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ABSTRACT

Angular diameters from lunar occultation are combined with infrared photometry to determine effective temperatures, T_{eff}, for K0–M6 giants. The relations between T_{eff} and color temperature, MK spectral type, $V - K$ color, and $I(104) - L$ color are derived. The principal result is a general increase in T_{eff} for the cooler spectral types compared to previous calibrations. Throughout the temperature range studied, we obtain excellent agreement with recent model atmosphere computations.

Subject headings: occultations — photometry — stars: atmospheres — stars: diameters — stars: late-type

I. INTRODUCTION

The effective temperature of a stellar surface is a measure of the total energy, integrated over all wavelengths, radiated from a unit of surface area. Since its value is fixed by the luminosity and radius, it is readily calculated for theoretical stellar models, and as one of the coordinates of the H-R diagram, it plays a central role in discussions of stellar evolution. Most observations, however, provide spectroscopic or photometric indicators of temperature that are only indirectly related to the effective temperature. Direct determinations of effective temperature—which can be used to calibrate excitation and color temperatures and which therefore are essential to comparisons of theory and observation—can be made only when we have actual knowledge of the angular size of the stellar disk.

Angular diameters of late-type stars have been in chronically short supply. For stars cooler than the Sun the T_{eff} calibrations of Johnson (1966) and Dyck, Lockwood, and Capps (1974) were based on only eight and seven angular diameters, respectively. The list of measurements included Mira variables which may have variable diameters, a currently disputed diameter (α Her), diameters of large or possibly large uncertainty, and diameters obtained by new, difficult interferometric techniques. The list is, to say the least, heterogeneous, and as a consequence, it has been difficult to exploit these meager angular diameter data.

Now, 5 years after the work of Dyck *et al.*, the situation is somewhat improved. The occultation technique of angular diameter measurement, pursued vigorously by several groups, has produced at least 94 measurements of at least 62 stars. The multiple measurements for 19 stars are highly consistent and verify the magnitude of the published error estimates. With this improvement in the angular diameter data base, it appears timely to reexamine cool-star effective temperatures. In this paper we present a T_{eff} calibration for field giants in the range early K to middle M.

In the present study we have chosen to use solely the occultation angular diameters. This decision was motivated by several factors: first, the high-quality and large volume of occultation data; second, the desirability of a homogeneous data set; third, uncertainties about the precision of the new interferometric techniques, which must operate near their theoretical limit to reach even a small selection of bright giants.

Angular diameters have been obtained from publications and, in a few cases, by private communication in advance of publication. We are currently engaged in a program to obtain infrared photometry, near-infrared color temperatures, and photometric spectral classifications for all cool stars of measured angular diameter. The occultation angular diameter list grows steadily, and it is apparent that our photometric program will never be completely up to date. Nevertheless, the available data already represent a clear improvement over those used in previous empirical cool-star effective temperature calibrations.

We report here the results for 31 giant stars of luminosity classes III and II in the spectral range G9–M6. In contrast, Dyck *et al.* had only three such

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stars. Thirteen of these stars have two or more (up to seven) independent occultation diameter measurements. Although diameters of several supergiants are available, we consider it best to exclude them from an otherwise pure giant calibration. There is considerable uncertainty as to the intrinsic colors and hence reddening corrections and total fluxes of supergiants. Furthermore, there is evidence that scattering in the circumstellar region and/or sphericity effects due to the extended atmosphere may lead to angular diameters which are not representative of the stellar photosphere (Tsuji 1978*b*). We have also excluded Mira variables. Our list is essentially one of non-variable stars, as only five of the stars are known to vary, and the available evidence suggests that the ranges of variation of their bolometric fluxes do not exceed 15%.

There have already been some indications that an upward revision of the temperature scale for cool stars is required by the observations. Several of the papers reporting occultation measurements have pointed out that the new effective temperatures for individual stars are higher than the values given for their spectral types by existing calibrations. Hayes (1978) has reviewed the occultation diameters available in 1977 and has derived an effective temperature scale significantly hotter than that of Johnson (1966) in the range M0–M4. The present study has considered all the material available to Hayes and has supplemented it with additional diameter measurements, new reductions of some of the earlier measurements, and the derivation of individual bolometric magnitudes for all program stars, in many cases based on new infrared photometry. Our results confirm that the effective temperatures of M stars are higher than previously believed; in fact, the temperature scale for M stars indicated by the best data now available is significantly hotter even than Hayes's scale. On the other hand, our results for K stars are in good agreement with previous scales. A discussion of these results and some of their implications, especially as they relate to the calibration of multicolor photometric systems, has been given by Wing and Ridgway (1979).

II. INTERCOMPARISON OF ANGULAR DIAMETER MEASUREMENT TECHNIQUES

In this section we present the existing evidence for the consistency of occultation diameters with interferometric diameters.

For the M supergiant α Sco the occultation result is 41 ± 2 milli-arcsec (Evans 1957, five independent measurements); the Pease (1931) result is 40 milli-arcsec (no error estimate); the speckle result (Gezari, Labeyrie, and Stachnik 1972) is 42 ± 2 milli-arcsec; and the amplitude interferometer result (Braunstein 1978) is 41.9 ± 2.9 milli-arcsec. No error analysis is required in order to recognize that these results are remarkably consistent.

The α Sco example is a relatively favorable case for the interferometric techniques since the diameter is approximately twice the theoretical resolution "limit" for those methods. It is a relatively unfavorable case for the occultation technique. Because of the large diameter, the Fresnel fringe pattern, normally used to determine local lunar slope, is not available. Fortunately, the repeated occultation measurements minimize the importance of this missing information.

The M3 III star μ Gem has an occultation diameter of 12.2 ± 0.25 milli-arcsec (Ridgway, Wells, and Joyce 1977) based on seven observations. The amplitude interferometer result, based on 11 measurements, is 13.9 ± 1.2 milli-arcsec (Braunstein 1978).

The bright giant α Tau, which has been measured by Pease (1931) and Currie *et al.* (1976), is currently undergoing a long series of occultations. This star will provide another valuable cross-check.

The regularly occulted star α Leo (B7 V) has an intensity interferometer diameter (Hanbury Brown, Davis, and Allen 1974) of 1.32 milli-arcsec. This star is within reach of the occultation technique (Ridgway 1977) but would push the method to near its limit. The next occultation is in 1980. Unfortunately, in the range ~3–12 milli-arcsec (where the occultation technique is most productive), there is no prospect of a cross-check with another technique until a variable-baseline, multitelescope interferometer comes into operation (see Koechlin 1978).

III. BOLOMETRIC FLUXES

Bolometric fluxes are usually obtained by applying a bolometric correction, tabulated as a function of spectral type, to the measured visual magnitude. This procedure can, however, lead to unacceptably large errors, especially for cool stars, for which the bolometric correction is large and very sensitive to the spectral type. One problem is that the spectral types of stars used in previous determinations of bolometric corrections have not been known with sufficient precision and may even differ systematically from the modern scale. Another problem is that there may exist appreciable intrinsic scatter in the relation between bolometric correction and spectral type, in which case we cannot be sure that a particular star whose angular diameter has been measured has the normal bolometric correction for its spectral type. Furthermore, since it is our aim to derive a new fundamental calibration of the effective temperature scale, we wish to make that scale independent of previous calibrations. For all of these reasons we have chosen to obtain bolometric fluxes for each program star directly and independently, from photometry in the critical spectral regions where their flux distributions peak.

Photometry in the standard *JHKLM* filters has been obtained at the KPNO 1.3 m telescope. The results of this photometric program are collected in Table 1. The program stars are identified by Bright Star or HD number; other designations for each star are given in Table 3 (see § V). Whenever available, we have used the *JKLM* magnitudes of Johnson *et al.* (1966) and the *K* magnitudes of Neugebauer and Leighton (1969) in equal weight with our own photometry in determining mean fluxes for each star.

TABLE 1

BROAD-BAND PHOTOMETRY

BS/HD	J	H	K	L	M	$\Delta(V-K)$	ΔK
224	+1.75	+0.98	+0.84	+0.71	+0.98	+0.02	
284	+2.60	+1.76	+1.57	+1.38	+1.67	+0.17	
601	+2.32	+1.51	+1.30	+1.12	+1.41	+0.15	
867	+0.11	−0.80	−1.05	−1.21	−0.97	+0.03	0.06
29051	+3.00	+2.08	+1.84	+1.70	+1.87	+1.18	
2286	−0.81	−1.69	−1.86	−2.02	−1.76	−0.03	
2938	+2.13	+1.35	+1.18	+1.03	+1.30	+0.08	
3095	+3.65	+2.98	+2.90	+2.83	+3.08	(+0.01)	
75156	+2.94	+2.18	+1.88	+1.71	+2.11	−0.01	0.12
3779	+2.72	+2.04	+1.90	+1.82	+2.01	(+0.28)	
3950	+1.50	+0.69	+0.52	+0.40	+0.67	+0.13	
3980	+1.88	+1.19	+1.05	+0.92	+1.15	−0.01	
4127	+2.11	+1.25	+1.01	+0.90	+1.18	+0.36	
4432	+2.10	+1.33	+1.16	+1.08	+1.31	+0.15	
4471	+2.57	+2.12	+2.04	+1.97	+2.06	(−0.14)	
4902	+1.18	+0.32	+0.14	−0.02	+0.27	+0.26	
5150	+1.75	+0.92	+0.70	+0.58	+0.87	−0.17	
5301	+1.62	+0.76	+0.55	+0.41	+0.65	+0.45	0.13
5622	+2.39	+1.65	+1.48	+1.35	+1.61	(+0.04)	
5824	+2.73	+2.15	+2.01	+1.88	+1.84	(−0.05)	
6861	+1.08	+0.10	−0.20	−0.41	−0.10	+0.94	
6913	+0.95	+0.42	+0.38	+0.30	+0.42	(−0.17)	
7023	+0.77	−0.17	−0.47	−0.71	−0.35	+0.49	0.15
7150	+1.58	+1.03	+0.91	+0.83	+1.01	(+0.14)	
176124	+2.15	+1.27	+1.01	+0.83	+1.21	+0.07	
7776	+1.45	+0.98	+0.88	+0.80	+0.95	(+0.04)	
7900	+1.83	+0.98	+0.77	+0.59	+0.95	+0.26	
8318	+2.29	+1.42	+1.20	+1.06	+1.34	+0.27	
8698	+0.39	−0.47	−0.66	−0.80	−0.53	+0.36	
8834	+1.13	+0.39	+0.20	+0.06	+0.38	+0.04	
9047	+1.22	+0.35	+0.12	−0.07	+0.25	−0.09	

To complete the data needed to specify the flux distributions, *UBVRI* photometry has been collected from Johnson *et al.* (1966), Barnes, Evans, and Moffett (1978), and J. C. Golson (private communication). A few N magnitudes have also been obtained from the literature. The total fluxes have been computed by integration under the flux curves. The integration has been carried to wavelength $\lambda = 0$ and $\lambda = \infty$ by extrapolation from each extreme of the available photometry. In a few cases the photoelectric photometry was incomplete, and we have extrapolated the U magnitude or interpolated R and/or I magnitudes on the basis of Johnson's (1966) colors. The numerical integration through the filters is based on the central wavelengths and calibrations tabulated by Johnson (1966). A trapezoidal integration was found adequate for the filters U through J. For the region *KLMN* it was necessary to apply weighting factors to allow for the shape of the Planck function.

Some of the fainter stars show evidence of interstellar reddening. We have used $\Delta(V - K)$, tabulated in Table 1, as an indicator of color excess; it is the difference between the observed $V - K$ color and the mean color for the star's spectral type according to Johnson's (1966) tabulation. Whenever possible, we have used spectral types from our narrow-band photometry, as discussed in § IV; when it has been necessary to employ classifications determined spectroscopically, the resulting values of $\Delta(V - K)$ have been placed in parentheses. We have not simply equated $\Delta(V - K)$ to the excess $E(V - K)$ due to interstellar reddening, since some intrinsic scatter is likely to exist in the relation between $V - K$ and spectral type as a result of differences in luminosity or metallicity [note that some values of $\Delta(V - K)$ are negative], and since most of the program stars are so nearby that they are unlikely to be reddened. Rather, we set $E(V - K) = \Delta(V - K)$ only when it exceeded 0.2 mag; otherwise, we set $E(V - K) = 0$. Van de Hulst reddening curve 15 (Johnson 1968) was then used to compute the corrections to the flux at each filter. These corrections were usually small. In 21 cases no correction was required, and for only three of the stars were the total fluxes changed by more than 15%. The final column in Table 1 shows ΔK, the extreme range in observed K magnitude, for the four stars which appear to be variable from the infrared photometry. The maximum observed range, 0.15 mag, corresponds to a ~15% variation in bolometric flux, and hence to a ~4% change in T_{eff} (assuming constant radius). In the worst case, the uncertainty in T_{eff} resulting directly from stellar variability is about ±140 K. The star ψ Vir (BS 4902) is a known variable of small amplitude (Barnes and Moffett 1978) but has shown no evidence of variability in the infrared.

IV. NARROW-BAND PHOTOMETRY

A primary use of the effective temperatures derived from angular diameter determinations is in the establishment of a (spectral type, T_{eff})-relation. A second purpose is the calibration of photometric indices of color temperature. For these reasons we have given particular attention to acquisition of narrow-band photometry on an eight-color system which gives both spectral types and blanketing-free color temperatures for late-type stars (Wing 1971; White and Wing 1978).

This photometric system yields two-dimensional spectral types for stars later than K3. The temperature classes, based on the strength of the TiO absorption near 7100 Å, are on the MK system and have far higher internal accuracy than is achievable by photographic classification. The luminosity classes, on the other hand, are based on the CN strength, which, although very sensitive to luminosity, also depends upon the chemical composition.

The color temperature provided by the eight-color photometry is obtained by passing a blackbody curve through calibrated measurements of the flux in the continuum near 8000 and 10,500 Å. These color temperatures are much less sensitive to molecular and atomic line blanketing than any broad-band color-temperature indicator such as $V - R$ or $R - I$; furthermore, the effects of absorption by weak CN lines on the raw eight-color continuum magnitudes have been removed, to first order, by the reduction procedure (see White and Wing 1978). The color temperatures are tied to the absolute flux distribution calculated from a model for Vega by Schild, Peterson, and Oke (1971). Since recent observations of Vega by Hayes and Latham (1975) and by Tüg, White, and Lockwood (1977) have confirmed the adopted slope for Vega in the near-infrared, it does not seem likely that any significant change in the color-temperature scale will be required in the future.

The eight-color data have been obtained in part from a compilation by Wing (1978) and in part from new observations made at KPNO and Lowell Observatory. The results of this part of our photometric program are collected in Table 2. The $I(104)$ magnitude is measured at 10,400 Å and is zero for Vega; T_c is the color temperature. The spectral types are from the eight-color photometry, with the following exceptions: (1) Stars lacking TiO bands cannot be classified on the eight-color system; types in the range G9–K3 have therefore been taken from the literature and are enclosed in parentheses. The sources of the photographic spectral types were, in order of preference, Jaschek, Conde, and Sierra (1964) and Hoffleit (1964). (2) We have preferred MK luminosity classifications (based on atomic line ratios) to those based

TABLE 2

RESULTS FROM NARROW-BAND PHOTOMETRY

BS/HD	n	$I(104)$	T_c	Sp	Notes
224	3	+2.22	3810	K4.8 III	
284	2	+3.16	3400	M2.4 III	
601	3	+2.86	3410	M3.3 III	
867	8	+0.75	2610	M5.9 IIIab	1
29051	2	+3.52	3130	M1.1 III	2
2286	9	−0.28	3380	M3.3 IIIab	3
2938	2	+2.63	3710	M0.0 III	
3095	2	+4.06	4260	(gK3)	
75156	4	+3.49	3390	M3.3 III	
3779	1	+3.18	4070	(K3 III)	
3950	9	+1.99	3570	M1.7 IIIab	3
3980	4	+2.33	3920	K4.2 III	
4127	2	+2.63	3460	M1.8 IIIa	1
4432	2	+2.59	3840	K4.5 III	
4471	5	+2.95	4740	(G9 III)	
4902	2	+1.76	3380	M2.7 IIIa	1
5150	2	+2.22	3500	M2.1 IIIa	1
5301	8	+2.17	3470	M1.5 III	4
5622	1	+2.86	3780	K4.8 III	5
5824	2	+3.16	4250	(K3 III)	
6861	2	+1.78	2740	M4.1 III	
6913	2	+1.37	4610	(K2 III)	
7023	5	+1.48	2660	M5.2 III	
7150	2	+1.96	4450	(K1 III)	5
176124	4	+2.76	3120	M4.3 III	
7776	3	+1.80	4810	(K0 II–III)	6
7900	3	+2.36	3540	M2.1 III	
8318	2	+2.85	3420	M3.4 III	
8698	2	+0.92	3480	M2.0 IIIa	3
8834	3	+1.63	3680	M1.5 III	5
9047	2	+1.83	3070	M4.6 IIb	1

NOTES.—(1) Luminosity classification from Yamashita 1967. (2) Spectral type abnormally early for observed color. (3) Luminosity classification from Morgan and Keenan 1973. (4) Spectral type observed to vary from M1 to M3. (5) Luminosity class III from the La Plata catalog (Jaschek *et al.*) used in preference to eight-color class II. Probably a strong-CN giant. (6) For discussion of spectral type see Ridgway *et al.* 1977 and Evans and Fekel 1979. CN strength indicates luminosity class II.

on CN strength, when they differ. The eight-color photometry cannot, for example, distinguish between a class II giant of normal CN strength and a strong-CN class III giant, but these stars should appear differently on the MK system. For some stars, therefore, we have listed the temperature class obtained photometrically together with a published MK luminosity class. Notes concerning the classifications of individual stars are indicated in the last column.

V. EFFECTIVE TEMPERATURES

The effective temperature is found from the relation

$$F = \left(\frac{\phi}{2}\right)^2 \sigma T_{\rm eff}{}^4 , \qquad (1)$$

where F is the apparent bolometric flux received at the Earth, σ is the constant of the Stefan-Boltzmann law, and ϕ is the angular diameter. In convenient units,

$$T_{\rm eff} = 2341(F_{\rm bol})^{1/4}/(\phi')^{1/2} , \qquad (2)$$

where $F_{\rm bol}$ is in units of 10^{-8} ergs cm^{-2} s^{-1} and ϕ' is in milli-arcsec. The quantity ϕ' is the "true" angular diameter; i.e., the limb darkening has been allowed for.

Most occultation angular diameters are quoted for a uniform disk and/or a fully darkened disk. The fully darkened diameters are typically larger than the uniform disk diameters by a factor of approximately 1.13. Ridgway, Wells, and Carbon (1974) have found that full darkening is reasonable for some typical cases. To treat a wider range of cases, Ridgway, Wells, and Joyce (1977) computed limb-darkening correction factors based on the limb-darkening predictions of Carbon and Gingerich (1969). The corrections vary in magnitude from 1.02 (near-infrared) to 1.19 (blue). We have used the same technique to derive limb-darkening corrections from the model intensities of Johnson (1974), Manduca, Bell, and Gustafsson (1977), and Manduca (1979).

One known source of error has not been treated. Equations (1) and (2) are based on the assumption of a well-defined stellar radius. In fact, for cool giant and supergiant stars, the atmosphere is so extended that the apparent photospheric radius will be wavelength dependent because of variations in opacity. (This is independent of the limb-darkening differences, which *have* been approximately treated.) In particular, the stellar "radius" at the 1.6 μm opacity minimum will be less than in the visible spectral region. Presumably, the radius near the peak of the energy distribution (1.6 μm) is physically most relevant. In the particular case of μ Gem (Ridgway, Wells, and Carbon 1974) we have estimated from model computations that the radius variation through the visible and near-infrared will be $\sim 3\%$, implying an ambiguity of ~ 50 K in the corresponding values of $T_{\rm eff}$. Eventually, this effect should be taken into account. We have not attempted to correct for it here because of the lack of pertinent parameters in most published models, the lack of cool-star models in general, and uncertainty about the actual radii of giant stars.

The limb-darkening correction factors have been used to convert published diameters to true (limb-darkened) diameters. Since Ridgway, Wells, and Joyce (1977) noted the usefulness of the correlated formal error derived in the usual differential corrector reduction, we have adopted it as the error estimate even when the investigator suggested a larger error on subjective grounds. The resulting limb-darkened angular diameters and their errors are collected in Table 3. In this table, n is the number of independent angular diameter measurements, references to which are given in the following column. Table 3 also lists the total fluxes $F_{\rm bol}$, discussed earlier, in units of 10^{-8} ergs cm^{-2} s^{-1}. Note that $F_{\rm bol}$ is related to the apparent bolometric magnitude by

$$m_{\rm bol} = -2.5 \log F_{\rm bol} + 8.50 , \qquad (3)$$

where we have used a flux of 2.52×10^{-5} ergs cm^{-2} s^{-1} to correspond to $m_{\rm bol} = 0.0$. With this choice of zero point, solar-type stars have bolometric corrections of about -0.14 mag and $V = m_{\rm bol}$ near spectral type F0.

The angular diameters and total fluxes are readily combined in equation (2) to obtain the stellar effective temperatures, also tabulated in Table 3. The diameter errors are carried explicitly. Observational errors in the photometry are generally negligible since $T_{\rm eff} \propto F^{1/4}$, but any error in photometric calibration will enter systematically. (For the spectral types considered here, the H and K magnitude calibrations are the most crucial.)

We have estimated the significance of possible errors in the absolute calibration by reference to a recent semiempirical calibration by Hayes (1979). With Hayes's calibration we have repeated the reductions. The net result is a uniform decrease in $T_{\rm eff}$ of almost exactly 1% over the entire range of spectral types studied here. While a fundamental calibration of infrared stellar photometry is certainly needed, there is no indication at present that it will lead to a large change in the current results.

VI. EFFECTIVE TEMPERATURE AND COLOR TEMPERATURE

Twenty of the angular diameters in Table 3 have errors which lead to uncertainties of less than 250 K in the effective temperature. For this subset of data, $T_{\rm eff}$ is plotted in Figure 1 against T_c, the color temperature from the eight-color photometry. Note that 12 of these diameters each have two or more (up to seven) independent measurements.

Stars of type M ($T_{\rm eff} \lesssim 4000$ K) lie systematically above the dashed line representing $T_{\rm eff} = T_c$. A solid curve has been fitted to the data by hand to characterize this trend. In constructing this curve, we have attempted to satisfy the following conditions: (1) it should pass through the center of gravity of the numerous excellent results near M2 and M3 ($T_{\rm eff} \approx$ 3500–3700 K); (2) it should represent the results for the two coolest stars, 45 Ari and BS 7023; and (3) it should tend asymptotically to the condition $T_{\rm eff} = T_c$

TABLE 3

INTEGRATED FLUXES, ANGULAR DIAMETERS, AND EFFECTIVE TEMPERATURES

Star	BS	HD	F_{bol}	φ' (milli-arc-sec)	n	Ref.	T_{eff}(°K)
δ Psc	224	4656	121.	4.75 ±1.13	1	1	3560 ±400
	284	5820	48.5	3.16 ±0.16	1	7	3480 ± 90
	601	12479	64.3	2.65 ±0.51	1	2	4070 ±380
45 RZ Ari	867	18191	433.	10.2 ±0.2	3	3,4	3350 ± 30
		29051	55.1	3.01 ±0.15	1	7	3680 ± 90
μ Gem	2286	44478	1170.	13.7 ±0.3	7	4	3710 ± 40
74 Gem	2938	61338	82.3	2.97 ±0.29	1	6	4090 ±200
1 Cnc	3095	64960	23.2	2.31 ±0.66	1	1	3380 ±450
		75156	35.6	4.13 ±0.32	3	4,6,7	2810 ±110
6 Leo	3779	82381	59.9	3.51 ±0.28	1	7	3480 ±140
π Leo	3950	86663	150.	4.88 ±0.28	3	7,8,9	3710 ±110
31 Leo	3980	87837	108.	3.55 ±0.22	2	1,10	3930 ±120
46 Leo	4127	91232	95.9	6.22 ±1.22	1	11	2940 ±280
87 Leo	4432	99998	87.4	3.71 ±0.35	2	12,13	3720 ±170
υ Leo	4471	100920	68.5	2.60 ±0.35	2	9,14	4180 ±270
ψ Vir	4902	112142	200.	5.85 ±0.18	3	4,5,15	3640 ± 60
82 Vir	5150	119149	116.	4.34 ±0.25	1	7	3690 ±110
	5301	123934	152.	3.97 ±0.17	3	2,4,6	4120 ± 90
ν Lib	5622	133774	64.7	2.85 ±0.41	1	7	3930 ±270
42 Lib	5824	139663	50.6	2.44 ±0.33	1	16	4000 ±260
	6861	168574	253.	3.55 ±0.39	1	16	4950 ±590
λ Sgr	6913	169916	290.	4.29 ±0.33	1	17	4660 ±180
	7023	172816	274.	9.03 ±0.17	2	7,18	3170 ± 30
ξ^2 Sgr	7150	175775	160.	3.79 ±0.41	2	3,8	4280 ±230
		176124	71.5	2.85 ±0.92	1	2	4030 ±610
β Cap	7776	193495	188.	3.18 ±0.15	3	4,9,19	4860 ±110
υ Cap	7900	196777	116.	4.72 ±0.52	2	1,20	3540 ±190
47 Cap	8318	207005	72.4	3.16 ±0.68	1	16	3840 ±390
λ Aqr	8698	216386	442.	8.21 ±0.44	1	21	3750 ±100
φ Aqr	8834	219215	199.	5.44 ±0.89	1	11	3770 ±300
	9047	224062	174.	6.30 ±0.40	1	9	3390 ±110

REFERENCES.—(1) de Vegt 1976. (2) Africano *et al.* 1976. (3) Africano *et al.* 1975. (4) Ridgway *et al.* 1977. (5) Evans *et al.* 1977. (6) White 1978*a*. (7) Ridgway *et al.* 1979. (8) Vilas and Lasker 1977. (9) Africano *et al.* 1978. (10) Glass and Morrison 1976. (11) Poss 1971. (12) Dunham *et al.* 1974. (13) White 1978*b*. (14) Jacoby and Price 1978. (15) Walker 1975. (16) Harwood *et al.* 1975. (17) Nather 1972. (18) Africano *et al.* 1977. (19) Fekel 1978. (20) Dunham *et al.* 1973. (21) Nather *et al.* 1970.

at high temperatures. The trend to $T_{eff} = T_c$ may be justified by appeal to other calibrations for earlier spectral types, to model atmosphere results (see below), or to empirical studies of the Sun and solar-type stars.

The curve from Figure 1 is reproduced in Figure 2 with the balance of the occultation angular diameter stars. It is clear that these low-weight determinations, whose formal errors lie in the range ± 250–700 K, are of no value in the calibration of the effective temperature scale.

The most prominent characteristic of the (T_{eff}, T_c)-calibration in Figure 1 is its increasing departure from equality at cooler spectral types. Apparently, there is a source of line blanketing or continuous opacity that depresses the flux at the shortward continuum point on the eight-color system (which, for M0–M6 stars, is a filter 50 Å wide centered at 7540 Å) and causes the color temperatures to be lower than the effective temperatures. The cause of this blanketing, however, is not obvious. It is known that very cool stars show VO absorption in this filter, so that color temperatures given by the eight-color photometry are too low for stars later than about M7. However, VO cannot have an appreciable effect upon the stars of types M0–M3.

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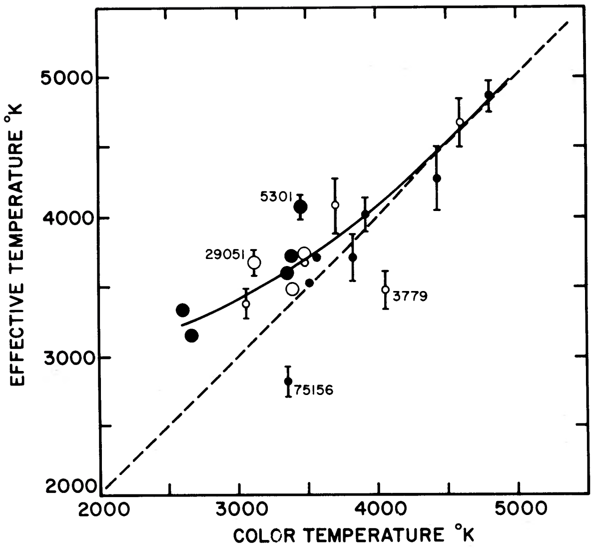

FIG. 1.—Effective temperature from Table 3 versus color temperature from Table 2. Large circles represent data with 1 σ errors $\leq$ 100 K. Small circles represent data with errors in the range 100–250 K. Error bars are shown where space permits. Filled symbols represent stars with two or more independent diameter measurements. The adopted calibration is shown by the solid line, which departs from the dashed $T_{\text{eff}} = T_c$ line at temperatures below about 4000 K. Several stars are labeled by BS or HD number.

Atomic lines in this region are virtually nonexistent. Neither H^- opacity (which decreases toward later types as the supply of free electrons is reduced) nor CN (which also decreases toward later types as the C and N atoms become more completely locked into CO and N_2 molecules) has the right temperature dependence to account for the observed displacement. TiO lines would have the correct temperature dependence, but since no TiO features have been identified in this region even in spectra as late as M7 or M8, they would not be expected to constitute an important opacity source in the M0–M3 stars.

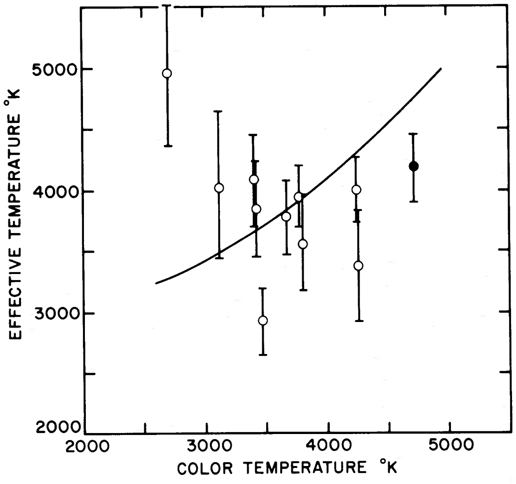

FIG. 2.—Same as Fig. 1 but for effective temperature data with 1 σ errors > 250 K. The solid line is repeated from Fig. 1.

Although we cannot uniquely identify the cause of the displacement from the $T_{\text{eff}} = T_c$ line in Figure 1, we do feel that the displacement is real. Several of the M stars (notably 45 Ari, BS 7023, μ Gem, and ψ Vir) have been measured accurately and repeatedly, and they fall many standard deviations from the $T_{\text{eff}} = T_c$ line. Quite possibly the displacement is the result of a combination of factors—residual CN absorption in the M0–M3 stars, perhaps, and the onset of VO absorption in the M5–M6 stars.

Uncertainties in the color-temperature measurements are typically only 30 or 40 K; nearly all the observational error in Figure 1 is associated with the angular diameter measurements. However, a few genuine anomalies are included here and contribute to the scatter. The M giant BS 5301 ($T_{\text{eff}} = 4120$, $T_c = 3470$ K) appears discrepant in Figure 1. Yet three independent measurements of the diameter are consistent. Observations on the eight-color system from eight nights during a 2 year interval yield a constant color temperature but a range of spectral types from M1.0 to M2.1 III. Infrared photometry on four nights during a 1 year interval shows stable colors but a significant variation in magnitude. P. C. Keenan (private communication) obtained a classification of M3 III, outside the range of the eight-color results. The star is thus definitely variable in spectral type. Furthermore, the star deviates from the mean relations between T_c and various broad-band color indices. This suggests that BS 5301 is a genuine anomaly and not a poor datum.

Another grossly discrepant point in Figure 1 is HD 75156 ($T_{\text{eff}} = 2810$, $T_c = 3390$ K). Again, three independent diameter measurements and repeated photometry confirm that this star falls 800 K (7 σ) below the solid line in Figure 1.

The color temperature and spectral type of HD 29051 as measured on the eight-color system are inconsistent. The T_c of 3130 K would normally indicate a type near M4, but the measured type is M1.1. The $V - K$ color is also too red for the measured spectral type, by the amount $\Delta(V - K) = 1.18$. Following our normal procedures, we have corrected the photometry for this amount of interstellar reddening; but this is an unusually large reddening correction for a giant in the vicinity of the Hyades, and it may be erroneous. If we suppose that the type is peculiar and the star is in fact not significantly reddened, the point in Figure 1 will be moved from approximately 2 σ above the solid line to approximately 2 σ below. Spectroscopic studies may be valuable in facilitating understanding of the anomalous character of this star.

The discrepancies we have noted (and there may be others less obvious) indicate the danger in assigning excessive weight to a single star. In spite of these hazards, the measurements of T_{eff} are good enough to establish the *mean* relation between T_c and T_{eff} and to show that the color temperatures for M stars on the eight-color system differ systematically from their effective temperatures by amounts ranging up to several hundred kelvins.

While the data illustrated in Figure 1 obviously leave much to be desired, we believe that this is nevertheless the best empirical (T_{eff}, T_c)-calibration available for this temperature range for any photometric color-temperature system. The importance of this calibration is that it will now be possible, from measurements of T_c that can be made easily and accurately on the eight-color system, to obtain good values of T_{eff} for M stars of normal composition.

VII. POSSIBLE SYSTEMATIC ERRORS IN OCCULTATION ANGULAR DIAMETER MEASUREMENTS

We have seen that the effective temperatures obtained for M giants are higher than their color temperatures measured in the near-infrared. Since it is not obvious why the color temperatures should be too low, we now consider the possibility that the effective temperatures may be too high as the result of some systematic error in the diameter measurements.

Many possible sources of systematic error can indeed be identified; but they all have the effect of making the measured diameter too large, and this would make T_{eff} too low. For example, we find that ignoring the finite telescope aperture diameter, ignoring the telescope secondary obstruction, ignoring the finite electrical response of analog systems or the frequency response of boxcar averaging in counting systems, or incorrectly specifying the convergence criterion for a least-squares iteration analysis all systematically lead to an erroneously large diameter. Unfortunately, some of the occultation results in the literature are not documented adequately to permit independent evaluation of the reduction technique.

Other effects may also contribute. Barnes *et al.* comment on a possible systematic error for very small diameters. We have found that simple nonuniformities of the lunar limb can lead to an erroneously large diameter. Possible systematic effects of scintillation-type noise have not been fully explored, although an approach to the problem has been established (Knoechel and von der Heide 1978).

All of these problems merit attention, but the fact that they have been identified promises continuing improvement in the occultation method. It is also evidently very important to maintain full documentation of current occultation measurements to permit improved analysis at a future date.

The several considerations cited above all indicate that any systematic error in the occultation diameters is expected to be in the direction of erroneously large diameters and hence *low* T_{eff}. Thus errors in the measured diameters cannot explain why the effective temperatures of M stars are higher than their color temperatures, and we must conclude that the discrepancy is caused by a problem with the color temperatures. As we shall see in § VIII, our effective temperatures are also higher than previous calibrations have given for the various spectral types; since this change is in the wrong sense to be attributed to errors in the occultation diameters, it is more likely to be the consequence of errors in the previous calibrations. It is also important to realize that the possible systematic errors we are considering are relatively small compared to the uncertainties in previous empirical T_{eff} calibrations.

VIII. COMPARISON WITH PREVIOUS T_{eff} CALIBRATIONS

The most widely used effective temperature scale for late-type giants has been the one published by Johnson (1966) on the basis of broad-band multicolor photometry. Johnson did not give effective temperatures for individual stars but presented a relation between effective temperature and spectral type. This was derived by forming the mean colors for each spectral type on the *UBVRIJKLMN* system (by averaging the data for many stars grouped according to their published spectral types), using the mean $I - L$ color as the temperature index for that spectral type, and calibrating this index in terms of effective temperature through photometry of the small number (eight) of late-type stars whose diameters had been measured. The calibration stars included supergiants, a Mira variable, and even the dwarf eclipsing system YY Gem.

It has previously been shown (Wing 1967) that color temperatures measured in the 1 μm region agree very well with Johnson's (1966) scale of effective temperatures. Therefore, if the new effective temperatures differ systematically from the eight-color temperatures (as is shown in Fig. 1 to be the case), then they must also differ from Johnson's scale.

In Figure 3 the upper curve is our new calibration, transformed from Figure 1 to a function of spectral type through the well-established relation between spectral type and color temperature on the eight-color system (see Wing and Yorka 1979). The thin line below it is Johnson's (1966) calibration, which assigns substantially lower temperatures to the M stars. Kuiper's (1938) calibration (*dashed line*) was yet cooler than Johnson's.

There are several reasons why the previous studies gave lower values of T_{eff}. Kuiper (1938) based his results on a rediscussion of the radiometric measurements of Pettit and Nicholson (1928, 1933), whose temperature index was simply a comparison of all the radiation longward and shortward of approximately 1.3 μm. Such a crude color index, in stars whose spectra are cluttered by molecular bands, is virtually impossible to calibrate absolutely or to correct for absorption by the Earth's atmosphere, and it is now generally recognized that the temperatures of Pettit and Nicholson were systematically too low. Johnson's filter photometry in the atmospheric windows was easier to calibrate and resulted in higher values of T_{eff}, but his temperature scale was still dependent upon angular diameter measurements from the 1920s and 1930s. Historically, one of the seemingly best determined diameters has been that of o Cet, and the very low temperature of this star has received large weight in all previous calibrations, reducing the "mean relation" even for earlier spectral types. Since it is now

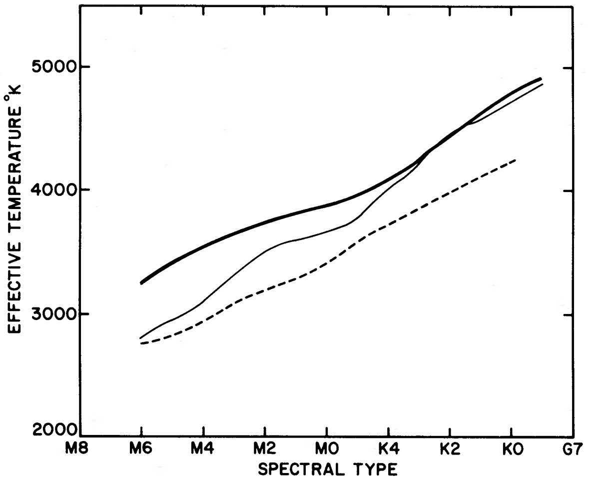

FIG. 3.—*Heavy solid line*, ($T_{\rm eff}$, T_c)-relation from Fig. 1 transformed to a ($T_{\rm eff}$, MK spectral type)-diagram. *Thin solid line*, Johnson's (1966) $T_{\rm eff}$ calibration. *Dashed line*, Kuiper's (1938) $T_{\rm eff}$ calibration.

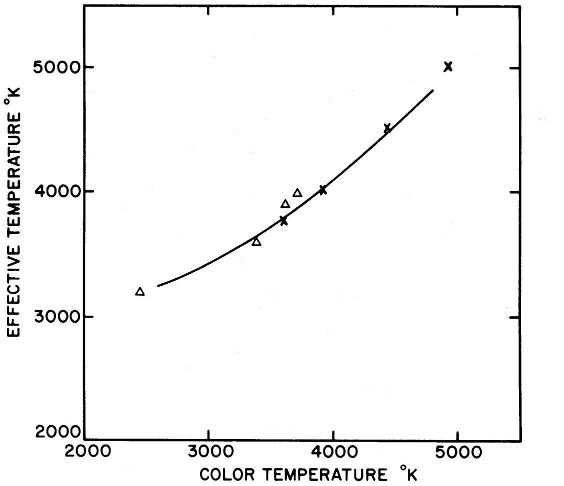

FIG. 4.—Comparison of the adopted ($T_{\rm eff}$, T_c)-relation from Fig. 1 (*solid curve*) with the results of two recent studies. The × symbols represent the ($T_{\rm eff}$, T_c)-relation for radiative models by Bell *et al.* (1976). The △ symbols indicate the effective temperatures found by Tsuji (1978*a*) for four cool giants of known T_c. This figure includes empirical, semiempirical, and theoretical results and should be interpreted with caution.

known that no unique relation between spectral type and effective temperature exists for Mira variables (Spinrad and Wing 1969), we have excluded Miras from our calibration, although occultation diameters are available for several. Another important star in previous calibrations has been α Ori. Again, it now appears unwise to include supergiants with less luminous stars since, in addition to uncertainties in their reddening and limb darkening, it is likely that they follow different ($T_{\rm eff}$, T_c)- and ($T_{\rm eff}$, MK)-relations.

IX. COMPARISON WITH MODEL ATMOSPHERE CALCULATIONS

We have seen that color temperatures measured in the 7000–11,000 Å region are lower than the best determinations of $T_{\rm eff}$ for late K and M giants (Fig. 1), and that this difference is not likely to be due to systematic errors in $T_{\rm eff}$. In this section we inquire whether color temperatures computed from model stellar atmospheres show the same trend.

For the temperature range of the K stars, direct comparison is possible with model computations by Bell *et al.* (1976). For their solar-composition models with $T_{\rm eff} = 3750$, 4000, 4500, and 5000 K, T_c was evaluated by treating the appropriate continuum fluxes as in the reduction of the eight-color photometry. The photometric continuum temperatures so determined are indicated by crosses in Figure 4, where the solid curve is our empirical ($T_{\rm eff}$, T_c)-relation from Figure 1. The agreement between theory and observation is excellent for the K and early M giants. Furthermore, the model atmosphere ($T_{\rm eff}$, T_c)-relation was found not to depend significantly upon luminosity over the range $\log g = 0.75$–3.00 covered by the models of Bell *et al.*

We have not been able to make similar direct comparisons for the M stars since model atmospheres with emitted flux distributions have not been published.[3] However, it is interesting to compare our color temperatures to the semiempirical effective temperatures obtained for several M stars by Tsuji (1978*a*). By fitting line-blanketed model atmosphere flux distributions to available photometry, Tsuji determined $T_{\rm eff}$ from the best model fits. He estimated the accuracy of the technique to be ±150–200 K and commented that his effective temperatures for M stars are substantially higher than the color temperatures usually quoted for the corresponding spectral types. The triangles in Figure 4 represent Tsuji's effective temperatures for g Her (3200 K), μ Gem (3600 K), β And (3900 K), and α Tau (4000 K) plotted against the color temperatures obtained from eight-color photometry (2495, 3380, 3645, and 3740 K, respectively [Wing 1978]). These triangles fall close to the curve representing the occultation temperatures. Thus the effective temperatures obtained by the occultation technique confirm the temperature scale for M stars indicated by Tsuji's work, and vice versa.

It is important to realize that Tsuji's (1978*a*) technique is almost completely independent of ours, since he fits only flux distribution and ignores total flux while we use total flux and ignore flux distribution. Furthermore, different stars were involved, with only μ Gem figuring in both studies. The close agreement between the triangles and the curve in Figure 4 may

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[3] Dr. John Piccirillo informs us that such calculations have recently been carried out at Indiana University by Bernat, Johnson, and Piccirillo for a series of M-star models. The opacity was treated by the opacity-sampling technique and included TiO. Fluxes at the eight-color continuum points 7545 and 10,400 Å were subjected to the same reduction procedure as the photometric data. Excellent agreement with our empirical curve was obtained throughout the interval 2500–4000 K.

also be taken as evidence for the relative completeness and correctness of Tsuji's line opacities and model techniques.

Tsuji recognized that his results for T_{eff} formed an upper envelope bounding all direct determinations of T_{eff} by all techniques for measuring angular diameter, and he suggested that systematic errors may be at fault. We now find that the best available occultation measurements are completely consistent with Tsuji's results.

Another comparison of observed and computed energy distributions has been published recently by Scargle and Strecker (1979). They used excellent new data from the Kuiper Airborne Observatory covering the range 1.25–5.5 μm with almost no breaks. These spectra were compared to a grid of energy distributions from atmospheric models computed by Johnson (1974). Unfortunately, these models use the straight-mean opacity representation, which can give large errors (Carbon 1974). For the M stars with strong molecular absorptions, there are gross differences between the observed and calculated spectra, and since the fitting was done by a blind least-squares procedure in which reddening was treated as a free parameter, Scargle and Strecker obtained some astonishing values for both reddening and temperature. Consequently, although Scargle and Strecker have published effective temperatures based on these fits, we feel that their study should be regarded as a test of the models rather than as a determination of effective temperatures.

X. CALIBRATION OF OTHER PHOTOMETRIC TEMPERATURE INDICES

Since the eight-color T_c is not ideal for some applications and may not always be available, we have also calibrated other popular temperature indices. The indices considered are $B - V$, $V - K$, and $I(104) - L$.

A plot of T_{eff} versus $B - V$ becomes a scatter diagram near $B - V = 1.6$. We do not reproduce the tight relation for this regime shown by Flower (1977). Since $B - V$ is of little use as a temperature indicator in the range of spectral type considered here, we will not examine it further.

The $V - K$ color is much more useful than $B - V$ as a temperature indicator for K and M stars. Its increased temperature sensitivity is partly due to its longer wavelength baseline, bracketing the region of the flux maximum in these stars, and partly due to the effect of TiO absorption in the V filter, which increases the $V - K$ color but decreases $B - V$. Since the strength of TiO is closely correlated with temperature for normal M giants, the presence of TiO bands in the V filter will not normally introduce scatter in the (T_{eff}, $V - K$)-diagram. However, it is important to remember that the normal (T_{eff}, $V - K$)-relation will not be valid for any star that has abnormal TiO strength for its temperature, as in the case of the globular-cluster red giants (Mould and McElroy 1978). Also, since the opacities at V and K are of different origin and in general are very different in amount, $V - K$ will be sensitive to differences in stellar temperature structure, of whatever origin.

An optimum temperature-sensitive color index would employ two filters, each encompassing little line absorption, at wavelengths of similar continuous opacity but differing temperature sensitivity to the Planck function. An index which satisfies these criteria rather well for very cool stars is $I - L$, which was recommended by Johnson (1966). To avoid the moderately strong TiO absorption in the broad-band I filter, Dyck *et al.* used $I(104) - L$, where $I(104)$ is the continuum magnitude of the eight-color system. This color is comparable, in a sense, to the criterion that Tsuji (1978*a*) used in fitting model flux distributions to observed flux distributions. In order to avoid the effects of molecular blanketing variations in the region $\lambda < 1$ μm, Tsuji gave most weight to the fits near $\lambda = 1$ μm and longward of 2 μm. To the extent that this criterion was used, Tsuji's results for T_{eff} should be closely related to the $I(104) - K$ or $I(104) - L$ color.

The relations between T_{eff} and the colors $V - K$ and $I(104) - L$, shown in Figure 5, have been determined by combining the T_{eff} versus T_c calibration curve of Figure 1 with the known relations between T_c, $V - K$, and $I(104) - L$ (see Table 4). Note that the two color indices are plotted on scales that differ by a factor of 5. The index $V - K$ is quite sensitive over the range K0–M6, changing on the average by about 0.35 mag per subclass. The index $I(104) - L$ is less sensitive ($\sim$0.07 mag per subclass), but for stars of unusual or unknown abundances it may be preferred for the reasons noted above.

The relations that exist between spectral type, color, color temperature, and effective temperature are given in Table 4, which is identical to the tabulation shown and discussed by Wing and Ridgway (1979). The numbers in the first four columns are well determined since they depend only upon narrow- and wide-band photometry and not upon angular diameter measurements. The effective temperature in the final column

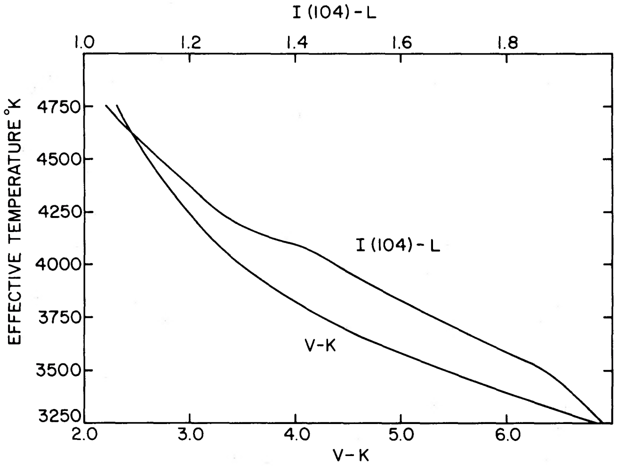

FIG. 5.—The (T_{eff}, T_c)-relation of Fig. 1 transformed to T_{eff} versus $V - K$ and T_{eff} versus $I(104) - L$.

TABLE 4

RELATIONS BETWEEN SPECTRAL TYPE, COLOR, AND TEMPERATURE FOR CLASS III GIANTS

Spectral Type	T_c[a] (K)	$V - K$	$I(104) - L$	T_{eff} (K)
G8 III.......	4900	2.20	0.97	4930
K0 III.......	4760	2.30	1.01	4790
K1 III.......	4580	2.48	1.08	4610
K2 III.......	4400	2.68	1.17	4450
K3 III.......	4200	2.96	1.29	4270
K4.0 III......	4000	3.26	1.41	4095
K5.0 III......	3860	3.52	1.49	3980
M0.0 III.....	3750	3.78	1.56	3895
M1.0 III.....	3640	4.02	1.62	3810
M2.0 III.....	3530	4.30	1.69	3730
M3.0 III.....	3400	4.64	1.76	3640
M4.0 III.....	3250	5.10	1.82	3560
M5.0 III.....	3000	5.96	1.89	3420
M6.0 III.....	2600	6.84	1.98	3250

[a] Eight-color determination.

has been added to the table as a function of T_c on the basis of the curve drawn in Figure 1; the relations between T_{eff} and spectral type, $V - K$, and $I(104) - L$ could then be read from the table for the construction of the curves shown in Figures 3 and 5. These relations apply to normal giant stars of luminosity class III.

XI. THE INCREASE IN T_{eff}

The effective temperature calibration is fundamental to many astronomical and astrophysical topics, and we cannot attempt to trace in detail the implications of the proposed increase in cool-star effective temperatures. Three general areas, however, are readily noted.

Studies related to the giant branch in the H-R diagram should now consider the implications of an increase of ~450 K in the effective temperatures of middle M stars (types M4–M6). As noted by Tsuji (1978*a*), an upward revision of cool-star temperatures may alleviate difficulties associated with the evolutionary status of very late M giants. Of equal or greater interest is the T_{eff} calibration for still cooler stars, especially the non–Mira types.

A second general area affected by a revised T_{eff} calibration is the spectroscopic determination of the chemical compositions of cool stellar atmospheres. Many studies still adopt temperatures from a (T_{eff}, spectral type)-relation (instead of deducing spectroscopic temperatures or otherwise specifying the model structure by analysis). Since it is often necessary to employ temperature-sensitive spectral features in abundance analyses, it is important that the temperature scale be free from systematic error. As a rather obvious example, an erroneous temperature scale could lead to incorrect conclusions regarding the O/C ratios of cool giants deduced from the observed strengths of their infrared H_2O bands.

Studies of the physics of stellar atmospheres also require knowledge of the temperature. A current example is Ramsey's (1977) investigation into non–LTE effects in the ionization equilibrium. Carbon (1979) has shown that an upward revision of the effective temperatures substantially reduces the apparent non–LTE effects.

XII. EXTENSIONS OF THIS WORK

The use of occultations to measure stellar angular diameters should continue. We expect that further observations will substantially improve the accuracy of the calibration and will also isolate the truly discrepant cases. Considering the vagaries of the occultation technique and the scarcity of opportunities, it is unlikely that any foreseeable observational effort would generate excessive angular diameter data. Hence the most direct extension of this work is to acquire additional observations.

Extension of the study to other spectral types is also possible. We are currently investigating the angular diameters and effective temperatures of M7–M10 stars, Mira variables, and carbon stars.

XIII. CONCLUSIONS

A new empirical calibration of cool-star effective temperatures is consistent with previous calibrations in the range K0–K3 but yields substantially higher temperatures in the range M0–M6. Our results are quantitatively consistent with recent model atmosphere studies by Tsuji.

Although the occultation technique is still developing, it is already by far the richest source of the cool-star angular diameters needed for T_{eff} calibration. Observational errors are sufficiently low that, at least when multiple determinations are available, effective temperatures of individual stars may be specified with high confidence. While the occultation technique of angular diameter measurement will always remain of rather specialized and limited application, it appears to be the best currently available technique for reaching large numbers of cool giants.

Note added in manuscript, 1979 July 16.—After preparation of the tables and figures, first results on the current series of α Tau occultations began to arrive. From the photometric spectral type of K5.7 III (Wing 1978), published photometry (Johnson *et al.* 1966), and our (T_{eff}, spectral type)-calibration, we would predict an angular diameter for α Tau of 21.3 milli-arcsec. The values 23 ± 4, and 22 (no error quoted) have been obtained by Brown *et al.* (1979). White (1979) obtained diameters of 20.4 ± 0.6 and 21.2 ± 0.6 from a two-color measurement. Beavers and Eitter (1979) report 18.4 ± 2.1 and 20.9 ± 2.2. The Kitt Peak occultation group (Ridgway 1979) has obtained seven diameter measurements: preliminary reduction yields 18.8, 19.2, 21.0, 21.4, 23.1, 23.6, and 24.0 milli-arcsec (errors ~1). The wavelength dependence of the α Tau diameter measurements deserves careful study for possible determination of limb darkening. In the present state of the analysis, the new α Tau results are consistent with the calibration of Figure 1.

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Note added in proof.—Photoelectric photometry by N. R. Stokes (*M.N.R.A.S.*, **152**, 165 [1971]) has established the variability of BS 5301, which has recently been designated ET Vir.

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