

OBSERVATIONAL TESTS OF THEORIES OF CONTACT BINARIES

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Received 1978 September 26; accepted 1979 January 25

ABSTRACT

Two recent theories of the structure of contact binaries are subjected to a wide variety of observational tests. One theory, according to which a W UMa system at or near zero-age cannot achieve thermal equilibrium and so undergoes thermal relaxation oscillations about a state of marginal contact, appears to be in substantial agreement with observational data. In particular, a previously admitted difficulty is to some extent overcome by the discovery of several possible examples of W UMa systems in the broken-contact phase of these oscillations. The second theory, according to which a W UMa system can achieve thermal equilibrium and does so through the development of a contact discontinuity at the inner contact surface of the secondary, appears, on the other hand, to be in substantial disagreement with the observational data. But since several of the disagreements may no longer apply when this theory achieves its final form—evolutionary sequences are at present indeterminate—it should not yet be regarded as contradicted.

Subject headings: stars: binaries — stars: W Ursae Majoris

I. INTRODUCTION

According to Lucy (1976), Flannery (1976), and Robertson and Eggleton (1977), zero-age W UMa systems are obliged to undergo periodic thermal relaxation oscillations (“TRO” theory) about a state of marginal contact as a result of being unable to achieve thermal equilibrium. Such oscillations arise because contact configurations in thermal equilibrium are secularly unstable in the relevant mass range; they persist because noncontact equilibrium configurations are inaccessible. Each of these oscillations comprises a contact phase and a semidetached phase. During the contact phase, the thermal contact of the components is essentially perfect and the mass ratio q (< 1 , by definition) decreases. During the semidetached phase, thermal contact is essentially nonexistent and q increases.

According to Shu, Lubow, and Anderson (1976), on the other hand, zero-age W UMa systems can and do achieve thermal equilibrium. These authors argue that contact configurations in thermal equilibrium and with either convective or radiative common envelopes are possible if discontinuities (“DSC” theory) are introduced at the inner contact surface. Two such models were subsequently computed by Lubow and Shu (1977).

Our primary purpose in this paper is to compare the predictions of these radically different theories with observational data. But first we report and comment on some alleged theoretical difficulties.

Lubow and Shu (1977) object that the TRO theory

postulates oscillations about a nonexistent equilibrium configuration. In fact, as indicated above, and as the originator of this theory should evidently have stressed, the thermal relaxation oscillations are about the Biermann-Thomas (1972) equilibrium configurations, in which shallow contact is assumed to limit thermal contact (i.e., energy exchange) to the degree required to maintain the inequality of the adiabatic constants of the components’ convective envelopes. The secular instability (Hazlehurst 1975) of these configurations accounts for the onset of the oscillations, and the existence of barriers (Lucy 1976) that render noncontact equilibrium configurations inaccessible accounts for the persistence of the oscillations. The end result is a finite-amplitude periodic oscillation analogous to the limit cycle of a Van der Pol oscillator.

In a recent paper, Shu, Lubow, and Anderson (1979) imply that the TRO theory, even if basically right, must be modified to incorporate a contact discontinuity during the contact phase of the relaxation oscillations. This follows from their claim that, even in the absence of thermal equilibrium, the appropriate equations and boundary conditions demand the presence of a contact discontinuity. Their proof of this claim depends, however, on regarding the masses of the components as boundary conditions to be imposed on the next model in an evolutionary sequence; in fact, the new masses are eigenvalues (Lucy 1976; see also Robertson 1979).

An apparently fundamental objection to the DSC theory has been raised by Hazlehurst and Refsdal (1978). They argue that the convective envelope of the less massive component, which, according to this

theory, is bounded above by a contact discontinuity at the inner contact surface, cannot be in thermal equilibrium and will, in consequence, heat up until the discontinuity vanishes. A detailed rebuttal of this criticism has been offered by Shu *et al.* (1979).

In the belief, therefore, that neither theory has been eliminated by purely theoretical arguments, we attempt in the rest of this paper to decide on the relative merits of the two theories by subjecting their predictions to observational tests. In § II, we report photometric analyses of three binaries whose components appear to have the poor thermal contact predicted by the TRO theory for some phases of the thermal relaxation oscillations. A variety of other tests are discussed in § III, and our conclusions are stated in § IV.

II. SYSTEMS WITH POOR THERMAL CONTACT

The prediction of intervals when the components of a W UMa system are not in good thermal contact is of crucial importance in testing the TRO theory since this entails marked departures from a light curve of EW-type. Moreover, during a part of these intervals, the predicted structure—a semi-detached system with neither component in thermal equilibrium—is not a structure predicted by the DSC theory. Accordingly, we devote this section to the testing of these predictions.

a) Expectations

Because the vast majority of main-sequence binaries with periods < 0.45 days have EW light curves (see Lucy 1976, Fig. 4; Eggen 1967), the TRO theory's prediction of intervals when a W UMa system's light curve should depart significantly from EW-type seems at first sight to be a serious difficulty (Lucy 1976; Lubow and Shu 1977). But the calculations of Robertson and Eggleton (1977) reveal that, for the small ratios typical of W UMa systems, the fraction of time spent out of contact is rather small; consequently, only a few percent of W UMa systems are in fact expected to show departures from EW light curves. Our present ignorance of the existence of such systems is therefore not a strong argument against the TRO theory, and the discovery of just a few examples would be a strong argument for that theory.

Because this important frequency argument of Robertson and Eggleton rests only on a limited number of calculations, it is perhaps as well to note that it also follows from the explanation of the relaxation oscillations given by Lucy (1976). During the semidetached phase, q increases because the more massive component (the "primary") is losing mass through L_1 , the inner Lagrangian point, as it attempts to achieve thermal equilibrium by expanding to its zero-age main sequence (ZAMS) radius; the characteristic time scale for this phase is therefore the thermal time scale of the primary. During the contact phase, on the other hand, q decreases because of the secondary's expansion on finding itself with an overly compressed interior; the characteristic time scale for this phase is therefore the thermal time scale of the second-

ary. Accordingly, since a star's thermal time scale depends strongly on mass ($\tau \propto M^\alpha$, with $\alpha \approx -3.5$ for $M \approx M_\odot$), systems with small mass ratios are expected to spend most of their time in contact and to have EW light curves. Moreover, systems with the predicted departures for EW light curves should preferentially be of large mass ratio.

The intervals when departures from EW light curves are expected are (1) the semidetached phase, when physical as well as thermal contact is broken; and (2) at the beginning and end of the contact phase, when the degree of physical contact is so slight that energy exchange between the components is significantly inhibited. In both these circumstances, the light curve is expected to tend toward EB-type because the ending or diminution of the energy exchange between the components removes the effect maintaining the near equality of the depths of minima that characterizes EW light curve (Lucy 1968*b*). Binaries with such departures from EW light curves, but which are nevertheless in the domain of the period-color diagram dominated by conventional W UMa binaries, are therefore candidates for systems caught in the predicted intervals of poor thermal contact. When the light curve of a candidate is analyzed, the suggested interpretation requires (1) that poor thermal contact be confirmed by the difference in the effective temperatures of the components; and (2) that the binary should be found to be either a semidetached system or a contact system with a common envelope that is shallow. As a measure of this latter property, we will use the degree of contact parameter f , defined by

$$f = \frac{\Omega - \Omega_i}{\Omega_0 - \Omega_i}, \quad (1)$$

where Ω , Ω_i , and Ω_0 are the potentials of the common photosphere and of the inner and outer contact surfaces, respectively.

b) Candidates

We now report photometric solutions for three binaries that appear to be W UMa systems whose components are not in good thermal contact. These solutions are simultaneous least-squares fits to V and B light curves according to the procedures developed by Wilson and Devinney (1971) and later generalized in numerous ways. In this investigation, a recently added capability to model the first-order effects of departures from Planckian emission was included. For the gravity-darkening exponent, we take $g \equiv 4\beta = 0.32$ (Lucy 1967); for the limb-darkening coefficients, we use the results of Carbon and Gingerich (1969).

i) AK Herculis

AK Her ($P = 0.42$ days; $B - V \approx 0.53$) is an unusual W UMa binary with a long history of photometric observations but with few spectroscopic observations. Its numerous peculiarities have most recently been discussed by Woodward and Wilson (1977, hereafter WW), who analyzed two independent

sets of photometric data. Assuming physical contact, they found the degree of contact to be small ($f \approx 20\%$) and the thermal contact to be poor ($T_1 - T_2 \approx 1000$ K); the WW solutions therefore suggest that AK Her be identified as being at the beginning or the end of the contact phase of the relaxation oscillation. But since only contact solutions were investigated by WW, the possibility that it is a mass-transferring, semidetached system is not out of the question.

To investigate the possibility that AK Her is in fact a broken-contact system, we analyzed the Woodward observations assuming a semidetached configuration, with star 1 (the primary in terms of mass, luminosity, and temperature) constrained to fill its Roche lobe. Such solutions, done in mode 4 (semidetached mode—see Leung and Wilson 1977), reveal the secondary to be slightly overcontact, however; the semidetached assumption does not therefore lead to an acceptable model. Accordingly, our final solution assumes physical contact (i.e., $\Omega_1 = \Omega_2$), and thus was done similarly, but not identically to those of WW (mode 3). The solution is given in Table 1, where adjusted parameters can be identified by their attached probable errors, and it confirms the conclusions of WW: the degree of contact is small ($f \approx 10\%$) and the thermal contact is poor ($T_1 - T_2 \approx 370$ K). (The marked reduction in $T_1 - T_2$ compared to the WW solutions can be traced to changes in β and the limb-darkening coefficients; WW allowed them to adjust and obtained anomalous values.)

(Following the example and procedure of WW, the light curves used to obtain the solutions discussed above were corrected for the observed asymmetry of about 0.03 mag amplitude. But, in contrast to WW, third light was not allowed to adjust; instead it was adopted from Eggen 1967).

In the course of this investigation, we found that the bolometric albedos A_1 and A_2 are crucial fitting parameters. Specifically, with conventional values for convective envelopes, a fit satisfactory to even casual inspection is impossible; the albedos were therefore allowed to adjust to their separate "best fit" values. For AK Her, the result was a factor 8 reduction in the weighted sum of squares of the residuals, and this is the solution given in Table 1. Similar but somewhat smaller improvements were found for W Crv and RW PsA.

ii) *W Corvi*

W Crv ($P = 0.39$ days; $B - V \approx 0.71$) has such noticeably different eclipse depths that the EB designation in the *General Catalogue of Variable Stars* is undoubtedly appropriate. Nevertheless, W Crv finds itself among W UMa systems in the period-color diagram and is isolated from the short-period β Lyrae systems; it is therefore a promising candidate for identification as a W UMa system caught in a phase of poor thermal contact. Unfortunately, there are several difficulties that preclude a definitive analysis of this system. The only published photoelectric light curves (Dycus 1968) have an unusually large scatter, and no spectroscopic observations exist. The eclipses are partial, or at least not demonstrably complete; consequently, it is not obvious which component is eclipsed at primary minimum. Finally, the light curves have asymmetries much larger than those of AK Her, so that we must again resort to the unjustified procedure of removing the asymmetries by adding \sin (phase) correction terms. For W Crv, the semiamplitudes of these corrections are 0.060 mag and 0.086 mag in V and B , respectively.

In the preliminary solutions, we assumed a semi-

TABLE 1
PARAMETER VALUES*

PARAMETER	AK Her	W Crv		
		I	II	RW PsA
i	$80^{\circ}81 \pm 0^{\circ}27$ p.e.	$87.0 \pm 1^{\circ}3$ p.e.	$86^{\circ}04 \pm 0^{\circ}53$ p.e.	$77^{\circ}45 \pm 0^{\circ}37$ p.e.
g ($= 4\beta$).....	0.32	0.32	0.32	0.32
T_1 (polar) (K).....	6400	5600	5600	5600
T_2 (polar) (K).....	6033 ± 17	4982 ± 23	4937 ± 16	5325 ± 9
A_1	-0.82 ± 0.08	-0.10 ± 0.45	0.50	0.26 ± 0.26
A_2	0.86 ± 0.09	2.85 ± 0.30	0.50	1.83 ± 0.25
Ω	2.2980 ± 0.0041	3.271 ± 0.040	3.272 ± 0.041	3.405 ± 0.014
$\mathcal{M}_2/\mathcal{M}_1$	0.2331 ± 0.0014	0.757 ± 0.029	0.807 ± 0.026	0.813 ± 0.011
$(L_1)_V/(L_1 + L_2)_V$	0.8283 ± 0.0008	0.698 ± 0.005	0.696 ± 0.006	0.606 ± 0.006
$(L_1)_B/(L_1 + L_2)_B$	0.8337 ± 0.0009	0.715 ± 0.006	0.714 ± 0.006	0.615 ± 0.007
x_V	0.65	0.69	0.69	0.69
x_B	0.79	0.81	0.81	0.81
l_V^{\dagger}	0.0072	0.0000	0.0000	0.0000
l_B^{\dagger}	0.0048	0.0000	0.0000	0.0000
f (%).....	10 ± 3	17 ± 9	35 ± 9	7 ± 3
$\Sigma w r^2$ †	0.1978	0.3661	... †
Ω_i	2.3131	3.343	3.429	3.439
Ω_0	2.1656	2.913	2.975	2.982

* Those with attached probable errors were adjusted by least squares.

† In units of the sum of the V or B fluxes from the binary components at phase 0.25.

‡ This quantity is useful only for intercomparison of solutions for the same binary.

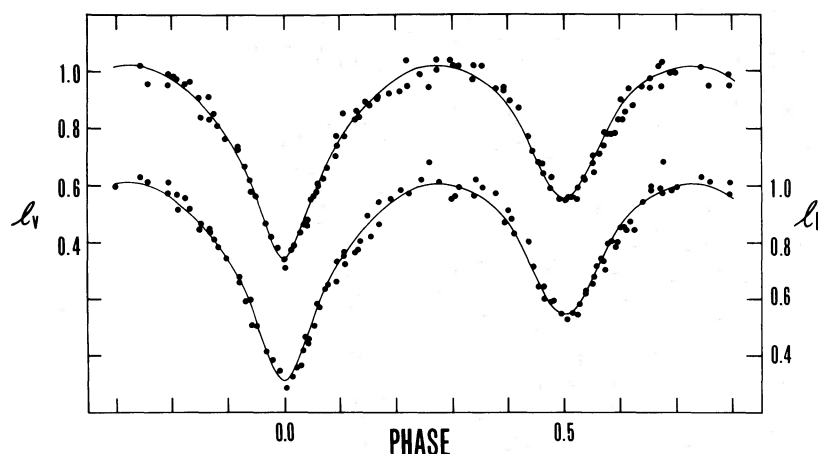


FIG. 1.—Light curves of W Crv. Dots are individual observations of Dycus (1968) after removal of asymmetries (see text). Solid lines are theoretical curves corresponding to solution I in Table 1.

detached structure (mode 4) with the primary constrained to fill its Roche lobe. But since we again found the secondary to be overcontact, the final solutions (with and without constrained albedos) in Table 1 assume physical contact. Throughout this analysis, we assumed, as did Dycus (1968), that the primary eclipse is of the larger, more massive component. Also, in the absence of contrary information, third light was taken to be zero.

Not surprisingly, in view of the light curve (see Fig. 1), our solutions confirm the poor thermal contact of the components: we find that $T_1 - T_2 \approx 650$ K, significantly larger than the differences found for AK Her and RW PsA. The degree of contact ($f \approx 25\%$) is also larger than the values found for the other two candidates, but it is less well determined and is, in any case, still modest. Taken at face value, our solutions suggest that, in the context of TRO theory, the poor thermal contact of W Crv be attributed to the system's being near the beginning or the end of the contact phase. But because of the aforementioned difficulties with the photometric analysis, we do not regard the possibility that this system is in the broken-contact phase as having been eliminated (see § II*d*). In this regard, the photometric mass ratio $q \approx 0.78$ is of interest since, as noted earlier (§ II*a*), systems with large mass ratios are predicted to be more readily caught in the broken-contact phase.

iii) RW Piscis Austrini

RW PsA ($P = 0.36$ days; $B - V \approx 0.75$) is in many ways similar to W Crv: they are almost coincident in the period-color diagram; their photometric mass ratios are nearly identical (see Table 1); they have light-curve asymmetries of similar character, though the amplitude of the distortion is much less for RW PsA; finally, the eclipse depths for RW PsA are also unequal (see Fig. 2), though the difference is not as striking as it is for W Crv (see Fig. 1). Fortunately, in contrast to W Crv, accurate photoelectric light curves exist (Chambliss 1970). The eclipses are seen

to be partial, and both Chambliss (1970) and Koch, Plavec, and Wood (1970) have commented on the abnormally large amount of (apparent) differential reflection, which is plainly noticeable without analysis.

As with AK Her and W Crv, preliminary solutions were obtained assuming a semidetached configuration. But since we once again found the secondary to be overcontact, the final solution given in Table 1 assumes physical contact. Because of the small amplitude of the light-curve distortions, these were not removed prior to the analysis. Also, for the final solution, third light was set to zero after finding negative values when it was allowed to vary—this is a somewhat surprising result since Chambliss states that a faint visual companion was measured along with the variable.

From the solution in Table 1, we see that poor thermal contact is confirmed ($T_1 - T_2 \approx 275$ K) and that the degree of contact is slight ($f \approx 7\%$). Again, therefore, in the context of the TRO theory, interpretation as a system at the beginning or the end of the contact phase is indicated.

In the above analysis, we assumed, as we did for W Crv, that the primary eclipse is that of the larger, more massive component.

c) Implications

For each of these binaries, the poor thermal contact of their components is well established: we find $\Delta T \equiv T_1 - T_2$ to be 370 K for AK Her, 650 K for W Crv, and 275 K for RW PsA. Lucy's (1976) model in its broken-contact phase has ΔT in the range 0–880 K. Also for comparison, we note that Rucinski (1974) finds that the average ΔT is -280 ± 30 K for W UMa systems of W-type and is 80 ± 25 K for those of A-type. (In Binnendijk's [1970] terminology, W UMa systems, whose primary minima are due to the eclipse of the more massive component, are A-type; the rest are W-type.) With regard to physical contact, however, our results are less definitive: shallow contact is indicated for all three systems but because of their

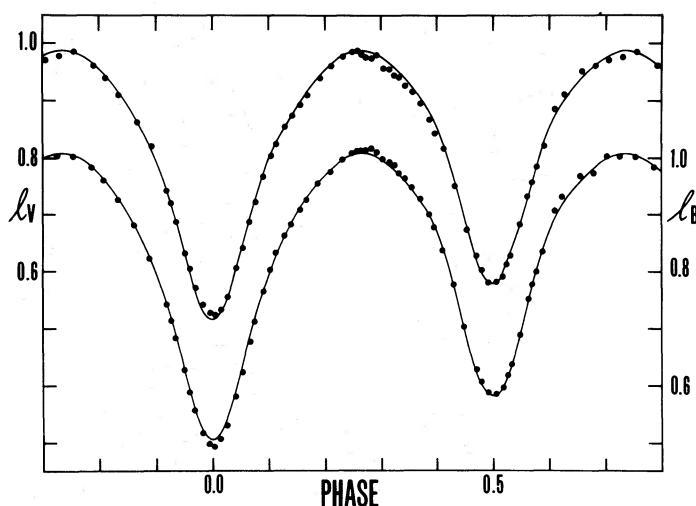


FIG. 2.—Light curves of RW PsA. Dots are normal points formed from the observations of Chambliss (1970). Solid lines are theoretical curves based on parameters in Table 1.

light curve distortions we are not confident that semi-detached configurations have been disproved.

In addition to similarities with regard to their degrees of physical and thermal contact, these three binaries have light-curve distortions of similar character, though of different amplitudes. Moreover, they all have unexpectedly large $\cos(\text{phase})$ terms in their light curves leading to the anomalous, yet similar values obtained for the albedos of their components, viz. $(A_1, A_2) = (-0.8, +0.9)$ for AK Her, $(-0.1, +2.9)$ for W Crv, and $(+0.3, +1.8)$ for RW PsA. Taken together, these similarities suggest that they are members of a coherent subclass of W UMa systems for which, because of their tendency toward light curves of β Lyrae type, we suggest the designation B-type systems. In view of their evident importance for testing theoretical ideas, more examples should be sought.

Because the poor thermal contact of these three B-type systems is well established, a major drawback of the TRO theory—the apparent absence of such systems—has now been overcome; and this theory's prediction of their existence is much to its credit. Nevertheless, *contact* binaries whose components are in poor thermal contact are no embarrassment to the DSC theory since, for that theory also, inhibited energy exchange is an obvious consequence of a sufficiently shallow common envelope.

d) Light-Curve Distortions

The light-curve distortions of AK Her, W Crv, and, to a lesser extent, RW PsA do not at present allow us to obtain their definitive photometric elements. This is especially unfortunate because, as we have just noted, merely establishing the poor thermal contact of their components does not yield a decisive test of the two theories. Had we proved that even one of these binaries has the semidetached structure predicted by the TRO theory for the broken-contact phase of the relaxation oscillations, however, a strong case could

have been made that the TRO theory was decisively confirmed. In fact, we will now argue that these hitherto unwelcome distortions are in themselves strong evidence that these binaries are indeed examples of the predicted nonequilibrium, semidetached configurations; we will show that both the character and the approximate amplitudes of the observed distortions can be understood on this hypothesis.

A semidetached binary of the kind predicted by the TRO theory is in the process of transferring mass from its primary, which fills its Roche lobe, to its secondary, which lies within its Roche lobe. This mass transfer does not result in the formation of a gaseous ring or an accretion disk, however. Because R_2 , the radius of the secondary, is predicted to be only fractionally less than R_2^* , the radius of its Roche lobe, the gas stream that leaves the primary at L_1 will collide directly with the photosphere of the secondary at some point on its trailing hemisphere. (In Lucy's [1976] calculation, for example, $R_2 > 0.86R_2^*$ throughout the semidetached phase.) The kinetic energy thermalized in this impact should therefore enhance the light-curve maximum that follows primary minimum, since the secondary is the eclipsing star at this minimum. Interestingly, the higher maximum follows primary minimum for both AK Her and W Crv, as well as for the somewhat similar systems TY Men (Naqvi and Grønbech 1976) and AG Vir (Binnendijk 1969).

In addition to deducing the general character of the distortion, its amplitude is also readily estimated. Using the details given by Lucy (1976, Table 1) for the semidetached phase, we can compute, as functions of time, the mass-transfer rate and the potential drop from L_1 to the surface of the secondary. From these quantities, we find that the accretion luminosity is typically 7% and, at its greatest, 13% of the total luminosity. Thus, if all this is radiated from the trailing hemisphere of the secondary, the maximum following primary minimum will be enhanced relative to the

other maximum by up to 0.25 mag, with the typical enhancement being 0.14 mag. Observed values (from V light curves) are: 0.03 mag for AK Her, 0.10 mag for W Crv, and 0.06 mag for both TY Men and AG Vir. The agreement is gratifying, and it is perhaps not necessary at this stage to chase after factors of 2. Nevertheless, we may note that the predicted effect will of course be reduced if the accretion luminosity is radiated from an area of the secondary's surface that is not confined to the trailing hemisphere. (It is obvious from the light curves that there is no well-defined hot spot. Presumably, therefore, thermalization occurs below the secondaries' photospheres.) Moreover, in this circumstance, the accretion luminosity begins to affect the secondary's surface boundary condition and, since the change is in the sense of more energy being radiated from the photosphere, the result will be a larger radius and a corresponding drop in the accretion luminosity.

In addition to having unequal maxima, three of these binaries (AK Her, TY Men, and AG Vir) have asymmetric primary minima, the sense of the asymmetries being in each case such that there is excess light at positive phases. Because the amount of light received from the trailing hemisphere of the secondary is increasing with time at phase zero, an asymmetry in the observed sense will result if the energy radiated from the trailing hemisphere exceeds that from the leading hemisphere. It seems likely, therefore, that this aspect of the light-curve distortions can also be understood on the basis of our hypothesis concerning the nature of these binaries.

(In view of these qualitative successes, one might hope that a theory of these light-curve distortions will also explain the distortion giving the anomalous albedos reported earlier for AK Her, W Crv, and RW PsA.)

The above discussion shows that eclipsing binaries in the broken-contact phase of the relaxation oscillations predicted by the TRO theory should have light-curve distortions that are similar in character and amplitude to the distortions observed for AK Her and W Crv, binaries that were selected as possibly being in this evolutionary phase not because their light curves were distorted but because the unequal depths of their minima indicated poor thermal contact. Thus, despite the failure of our photometric analyses to demonstrate the hypothesized semidetached structure, we suggest that these binaries may indeed be examples of the predicted mass-transferring, semi-detached systems. We believe, therefore, that a not implausible case has been made for the confirmation of a very distinctive prediction of the TRO theory.

In order to put this interpretation of these binaries beyond doubt, our model of the light-curve distortions must be developed to the point where it can be incorporated into the light-curve analyses. If such a development resulted in a detailed fit to the distortions and to a demonstration of semi-detached structure, our interpretation would be confirmed.

Assuming the correctness of our interpretation of these binaries as semidetached systems transferring

mass on the thermal time scale of their primaries, we now ask: Can such configurations be accommodated by the DSC theory? The only possibilities that suggest themselves are that they represent rapid evolutionary phases that immediately precede or immediately follow the slow, nuclear-time-scale evolution of equilibrium contact binaries with appropriate discontinuities. Because the ratio of the thermal to the nuclear time scales is ~ 0.002 for $M \approx M_{\odot}$, these explanations would only be tenable if they were being offered to explain $\sim 0.2\%$ of well-studied W UMa systems. Since we have identified four such systems (including TY Men and AG Vir) and there are roughly fifty W UMa systems with photoelectric light curves, we conclude that these explanations are not tenable. Accordingly, if these systems have indeed been correctly interpreted, they would appear to constitute strong evidence against the DSC theory.

e) *Period Changes*

If these B-type systems are indeed in the broken-contact phase of the relaxation oscillations, their orbital periods should be decreasing with $P/|\dot{P}| \sim 10^7$ yr (Lucy 1976).

The results of our search for this effect are briefly as follows: For AK Her, orbital motion about one or two other stars masks the predicted effect (Schmidt and Herczeg 1959; Binnendijk 1961; WW). For both W Crv and RW PsA, on the other hand, the timing data yield marginal, but certainly not convincing evidence for the predicted changes. Although more data should be collected, decisive results may be elusive since time-dependence of the light-curve distortions will produce spurious period fluctuations.

III. ADDITIONAL TESTS

In this section, we discuss a variety of additional tests that offer the prospect of discriminating between the two theories.

a) *Period-Color Relation*

Eggen's (1961, 1967) period-color relation for W UMa systems has hitherto played a crucial role in testing theories of their structure. The inadequacy of Lucy's (1968a) earlier work was exposed by its failure to reproduce this relation, and most subsequent investigations (see Lucy [1976] for references) have therefore had this as their aim and sole measure of success: indeed, the claim by Lubow and Shu (1977) that the DSC theory is in satisfactory agreement with observations is based only on its rough accord with this relation. But since the TRO theory also yields models over the full range of observed colors (Lucy 1976; Flannery 1976), it too overcomes the principal failing of Lucy's (1968a) original models and may therefore also be described as being in rough accord with the period-color relation. Accordingly, this relation does not provide a means of discriminating between these rival theories, at least not on the basis of the limited number of models calculated to date.

b) Degree of Contact

For a given composition, the DSC theory defines a unique model for a zero-age contact binary once the three fundamental parameters—the total mass \mathfrak{M}_t , the total angular momentum H , and the mass ratio q —have been specified. For fixed \mathfrak{M}_t and q , therefore, variation of H yields a one-dimensional family of models with surface potentials that span the permitted range whose limits are the potentials of the inner and outer contact surfaces. Accordingly, as Lubow and Shu (1977) have pointed out, the DSC theory does not predict any preference for shallow contact.

The TRO theory, on the other hand, does predict such a preference since marginal contact is the state about which the oscillations occur. More specifically, on the basis of an uncertain generalization of the calculations of Robertson and Eggleton (1977), which in a particular case (their Fig. 3) follow the relaxation oscillations until they are nearly periodic, we conclude that f is restricted to be $\lesssim 0.3$ for W UMa systems undergoing such oscillations. This restriction should therefore apply for much of their main-sequence lifetimes since the oscillations die out and overcontact develops only after substantial core evolution (Lucy 1976; Robertson and Eggleton 1977). Thus the TRO theory predicts that many W UMa systems should have $f < f_M$, where $f_M \approx 0.3$. Moreover, the results of Robertson and Eggleton (1977) also suggest that the time-averaged degree of contact in the contact phase is $\approx 1/2 f_M$; consequently, \bar{f} , the average degree of contact for a sample of systems in this phase, should be ~ 0.15 .

The observational situation is that many W UMa systems are indeed found to have shallow common envelopes. In particular, there is strong evidence that this is a general property of W UMa systems of W-type, a result first found by Lucy (1973), who analyzed the light curves of 16 systems with spectroscopic mass ratios, and confirmed by Rucinski (1973), who obtained information on the degrees of contact for 28 systems. Several others (Wilson and Devinney 1973; Whelan, Mochnacki, and Worden 1974; Whelan, Worden, and Mochnacki 1973; Berthier 1975; Wilson and Biermann 1976) have also published solutions that support this conclusion. In fact, to the best of our knowledge, no modern photometric solutions are in conflict with the conclusion that the W-type systems have shallow common envelopes.

In order to emphasize the reliability of this crucial result, we add the following remarks: (1) The various computer programs used in obtaining this result do reveal significant overcontact for other types of eclipsing binary—specifically, for the A-type W UMa systems (§ IIIc) and for certain early-type contact binaries (§ IIIg). (2) Shallow contact is not a spurious consequence of either the gravity-darkening exponent or the bolometric albedo used by both Lucy and Rucinski since this result persists when these quantities are also solved for in the least-squares analyses (Wilson and Devinney 1973; Berthier 1975; Twigg 1978).

The above discussion shows that the shallow contact

of the W-type systems is firmly established; consequently, this is undoubtedly a result that theories of contact binaries must explain. As we have already noted, the DSC theory has not yet yielded an explanation, but the TRO theory has. This latter theory predicts that contact remains shallow during the contact phase of a relaxation oscillation; the shallow contact of the W-type systems is therefore explained if we identify them with this phase. Previously (§ II), we have called attention to a new subclass of W UMa binaries—the B-type systems—that we have plausibly identified with the broken-contact phase of the relaxation oscillations.

In addition to providing strong qualitative support the TRO theory, the data on the degrees of contact of W-type systems can also be used for a quantitative test. On the assumption that the calculations of Robertson and Eggleton (1977) are not untypical, we have concluded that a sample of systems in the contact phase of their relaxation oscillations should be such that $f_M \approx 0.3$ and $\bar{f} \approx 0.15$. Because we have identified the W-type systems as being in this phase, we now compare these predictions with the observational data for such systems. Lucy's (1973) investigation yields a sample of 13 W-type systems (SW Lac is excluded because of the remarkable variability of its light curve) from which we obtain the estimates $f_M = 0.25$ and $\bar{f} = 0.07$. Rucinski's (1973) investigation also yields a sample of 13 systems (SW Lac is again excluded) from which we obtain the estimates $f_M = 0.3$ and $\bar{f} = 0.15$. These results clearly represent significant additional support for the TRO theory. Nevertheless, this test should be pursued further since both the theoretical calculations and the light curve analyses can be improved and extended. (The light curve aspect of this program is in fact well under way [Twigg 1978].)

In this subsection, we have pointed out that the shallow contact of the W-type systems can be explained—to some degree even quantitatively—by the TRO theory, and we have interpreted this achievement as strong evidence in favor of that theory. However, since the discovery of shallow contact was the direct stimulus of this theory (Lucy 1976), the legitimacy of citing this result as evidence in its favor might be questioned. This objection would certainly be valid if the TRO theory were in any sense an *ad hoc* construct to explain shallow contact—as it would be, for example, if free parameters were chosen to achieve this result. But as this is not so, the shallow contact of the W-type system is unquestionably legitimate as evidence in favor of the TRO theory. Moreover, even were this not so, it would still be evidence against the DSC theory in its present form.

c) A-Type Systems

The DSC theory, as noted above, allows models of zero-age contact binaries to be constructed with any degree of contact. The TRO theory, on the other hand, allows only shallow contact at and near zero-age, restricting deeper contact to later evolutionary phases when the primary's expansion results in the existence

of a stable thermal-equilibrium configuration (Lucy 1976; Robertson and Eggleton 1977). Accordingly, since the A-type systems are generally found to be markedly overcontact (Mochnacki and Doughty 1972*a, b*; Lucy 1973; Rucinski 1973; Wilson and Devinnay 1973), their evolutionary status provides a test of the two theories.

This test has in fact already been discussed in detail by Wilson (1978); we will therefore merely summarize those aspects of his work that are directly relevant to this investigation. Selecting systems because of their accurately determined parameters—especially their photometric mass ratios—Wilson compiles a sample of eight A-type systems. (This sample confirms, incidentally, the deeper contact of these systems, since it yields the estimates $f_M = 0.74$ and $f = 0.41$ [see § III*b*].) For each of these systems, he computes a_1 , the semimajor axis of the relative orbit, as a function of \mathfrak{M}_1 , the mass of the primary, and finds *in every case* that, for reasonable values of \mathfrak{M}_1 , a_1 substantially exceeds a_2 , the size of the semimajor axis computed under the assumption that the components have zero-age radii. Wilson thus demonstrates that *all* eight systems are significantly evolved, and he therefore concludes that unevolved A-type systems probably do not exist.

This discussion shows that the evidence on the evolutionary status of the A-type systems agrees with the prediction of the TRO theory. Moreover, the absence of unevolved overcontact systems is not accounted for by the DSC theory.

Parenthetically, we might note here the results obtained when Wilson's (1978) procedure is applied to our three B-type systems (§ II*c*). We find that $a_1/a_2 \approx 1.22$ for AK Her, 1.09 for W Crv, and 1.00 for RW PsA. Since the corresponding ratios for Wilson's eight A-type systems range from 1.18 to 1.70 and average 1.42, we see that the B-type systems are markedly less evolved. This result is consistent with our claim (§ II) that the B-type systems are in a poor thermal contact phase of the relaxation oscillations.

d) Mass Ratios

The DSC theory, as already noted, has three fundamental parameters, \mathfrak{M}_1 , H , and q . Thus, for given \mathfrak{M}_1 , and for H in the appropriate range, a model for a zero-age contact binary can be constructed for any value of q . Accordingly, the DSC theory appears to play no role in determining $\phi_0(q)$, the probability density function (PDF) for the mass ratios of zero-age W UMa systems; this function is therefore determined solely by the way in which the formation mechanism populates (\mathfrak{M}_1, H, q) -space.

The TRO theory, on the other hand, has only two fundamental parameters, \mathfrak{M}_1 and H ; mass ratio is not a fundamental parameter in this theory since it is time-dependent and the character of its periodic variability is governed completely by \mathfrak{M}_1 and H —in particular, therefore, the time-averaged mass ratio $\bar{q} = \bar{q}(\mathfrak{M}_1, H)$. Accordingly, the TRO theory does play a role in determining $\phi_0(q)$, which reflects in fact the combined result of three effects: (1) the way in which

the formation mechanism populates (\mathfrak{M}_1, H) -space; (2) the way in which the function $\bar{q}(\mathfrak{M}_1, H)$ maps (\mathfrak{M}_1, H) -space onto the interval (0, 1) of the real line; and (3) the way in which q oscillates about \bar{q} .

Effects (1) and (2) determine $\chi_0(\bar{q})$, the zero-age PDF for \bar{q} . In fact, if $\Psi_0(\mathfrak{M}_1, H)$ is the PDF describing the distribution of zero-age systems in (\mathfrak{M}_1, H) -space, we may readily show that

$$\chi_0(q) = \int \Psi_0(\mathfrak{M}_1, H) \left(\frac{\partial \bar{q}}{\partial H} \right)^{-1} d\mathfrak{M}_1, \quad (2)$$

where H is the solution of the equation $\bar{q}(\mathfrak{M}_1, H) = \bar{q}$. The function $\phi_0(q)$ may then be obtained by taking the third effect into account. However, if the available calculations are not untypical, the variations of q during the relaxation oscillations are not large; consequently, to a good approximation,

$$\phi_0(q) = [\chi_0(\bar{q})]_{\bar{q}=q}. \quad (3)$$

Because Ψ_0 is not known, these equations cannot be used to predict ϕ_0 . Nevertheless, since the formation mechanism presumably acts in ignorance of the function $\bar{q}(\mathfrak{M}_1, H)$, the PDF Ψ_0 will not in general be large where $(\partial \bar{q} / \partial H)^{-1}$ is small; the PDF ϕ_0 should therefore be small for mass ratios such that $\partial \bar{q} / \partial H$ is large. According to Lucy (1976), this is true for $\bar{q} \approx 1$, and he therefore attributed the paucity of W UMa systems with $q \gtrsim 0.6$ to this effect. (The crucial factor determining the sensitivity of \bar{q} to H is the dependence of the separation on q ; see Lucy 1976, Fig. 2, and the associated discussion.)

So far we have discussed the zero-age PDF $\phi_0(q)$. Of more importance for testing these theories, however, is $\phi(q)$, the PDF for a complete sample of W UMa systems in the solar neighborhood, and this function will obviously differ from $\phi_0(q)$ primarily in consequence of evolutionary changes in the mass ratios. Because this is discussed in detail later (§ III*e*), we merely note here that the DSC theory cannot yet be used to compute such evolutionary changes, and so it makes no prediction about $\phi(q)$. The TRO theory, on the other hand, predicts that as long as the relaxation oscillations continue, \bar{q} will decrease with time. Thus, since this evolutionary effect reinforces the zero-age effect discussed earlier, the TRO theory clearly predicts that $\phi(q)$ should be small for $q \lesssim 0.6$.

The observational situation for W UMa systems with spectroscopic mass ratios is shown in Figure 3. This histogram is constructed from the data compiled by Binnendijk (1970), except that $q = 0.54$ for W UMa (Worden and Whelan 1973), $q = 0.41$ for U Peg (Worden, Whelan, and Rucinski 1979); the systems HD 101799 (Sisteró and Castore de Sisteró 1974), AE Phe (Duerbeck 1977), CC Com (Rucinski, Whelan, and Worden 1977), and BV Dra (Batten and Hardie 1965) have been added; and the two determinations for VW Cep have been averaged. From this histogram, we see that the observational evidence indicates that ϕ is roughly a factor 4 smaller in the interval (0.6, 1.0) than in the interval (0.3, 0.6). Moreover, there can be little doubt as to the reality of this effect, since selection

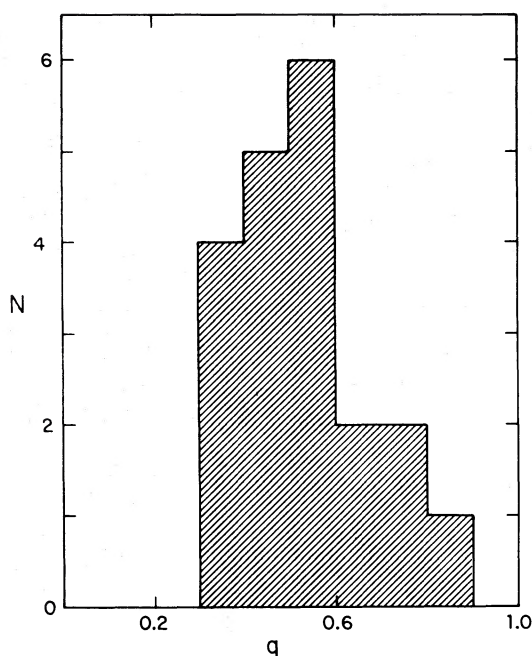


FIG. 3.—Histogram showing distribution of spectroscopic mass ratios of W UMa binaries

effects favor systems with $q \approx 1$: their probability of discovery as eclipsing binaries is markedly greater than for systems with $q \approx 0.3$, say, as is the ease with which the lines of the secondary can be seen and measured. We conclude, therefore, that these data are in agreement with the qualitative prediction of the TRO theory.

As we have noted, the DSC theory makes no prediction as yet concerning $\phi(q)$. Shu *et al.* (1976) have speculated, however, that there might be secular shifts in mass ratio following formation and that such an effect is responsible for the predominance of systems with $q \approx 0.5$. But until this is demonstrated, the small mass ratios of W UMa systems can only be cited as evidence in favor of the TRO theory.

e) Evolution

The DSC theory has not yet been used to investigate the nuclear evolution of W UMa systems. In part, this no doubt results from the expectation that zero-age models should explain the main properties of these binaries. But the absence of such calculation may also be the result of a nontrivial difficulty associated with computing evolutionary sequences according to this theory, a difficulty that is a direct consequence of there being three, not two, fundamental parameters. Because of this, when \mathfrak{M}_i is specified and an appropriate choice made for H , a unique model is not obtained: instead, a one-dimensional family of models may be generated by varying the third fundamental parameter q within its permitted range. But since this is true also for nonhomogeneous models, a one-dimensional family of models is available after each

time-step in an evolutionary calculation, and the DSC theory does not yet specify which of these models is to be chosen. This indeterminacy will presumably be eliminated by the inclusion of secular terms, a development already hinted at by Shu *et al.* (1976) in their speculations about secular shifts in mass ratio (see § III*d*). Nevertheless, as yet, the DSC theory makes no predictions about the evolutionary changes of W UMa systems.

The TRO theory, on the other hand, has been used to investigate the evolution of W UMa systems (Webbink 1976; Robertson and Eggleton 1977). Although accurate calculations are not practicable because of the need to follow an enormous number of relaxation oscillations, the main evolutionary effect—assuming conservation of mass and angular momentum—is well established and readily understood: as long as stable, thermal-equilibrium configurations do not exist, models evolve to smaller mass ratios in order to allow for the evolutionary expansion of the primary while still maintaining marginal contact. The TRO theory therefore predicts that evolved W UMa systems should typically have smaller mass ratios than zero-age systems.

The possibility of testing this prediction follows from the TRO theory's observationally supported interpretations of the A- and W-type systems. According to this theory, the A-type systems have evolved significantly and have thereby reached a state of stable thermal equilibrium (§ III*c*); the W-type systems, on the other hand, have evolved relatively little and are therefore still compelled to undergo relaxation oscillations (§ III*b*). Thus the predicted evolutionary effect implies that the A-type systems should, on an average, have the smaller mass ratios.

Rucinski (1973, 1974) has in fact already emphasized that the A-type systems have the smaller mass ratios, this being one of the several characteristics in which they differ significantly from the W-type systems. Specifically, he finds $0.08 \leq q \leq 0.54$ for A-type systems and $0.145 \leq q \leq 0.88$ for W-type systems. The observational evidence is therefore in agreement with the evolutionary effect predicted by the TRO theory. (The upper limit to the mass ratios of the A-type systems should in fact be revised down to 0.37. Two independently determined photometric mass ratios of RZ Tau agree closely: Mochnacki and Doughty [1972*b*] obtain $q = 0.38 \pm 0.01$; Wilson and Devinney [1973] obtain $q = 0.372 \pm 0.005$.)

f) Continuity

The DSC theory does not recognize a fundamental distinction between radiative and convective common envelopes; zero-age models can in fact be constructed in either case. The probability density function describing the distribution of zero-age contact binaries in (\mathfrak{M}_i, H, q) -space should therefore vary smoothly across the surface on which the transition from convective to radiative common envelopes occurs.

To put this prediction into a testable form, we must first consider the photometric characteristics of the

models. These characteristics are largely determined by the gravity-darkening law,

$$T_{\text{eff}} \propto g^{\beta}, \quad (4)$$

where T_{eff} and g are the local effective temperature and gravity, respectively, and where $\beta = 0.25$ (von Zeipel 1924) if the common envelope is radiative, but $\beta \approx 0.08$ (Lucy 1967; see also Osaki 1970) or $= 0.00$ (Anderson and Shu 1977), if it is convective. Because this formula implies little variation of T_{eff} over the surfaces of contact binaries built according to the DSC theory, the predicted light curves, for both radiative and convective common envelopes, are of W UMa-type, i.e., rounded maxima, and minima of nearly equal depth (see, e.g., Anderson and Shu 1977, Fig. 3). Accordingly, the above prediction implies that W UMa systems should not end when common envelopes cease to be convective.

The TRO theory, on the other hand, does postulate a fundamental distinction between radiative and convective common envelopes. This theory assumes, following Lucy (1968a), that radiative common envelopes cannot adjust to mechanical equilibrium; the variation of T_{eff} over the surface of an early-type contact binary is therefore not expected to follow von Zeipel's law, and the predicted light curves will not in general be of W UMa-type. Accordingly, the TRO theory does predict that W UMa systems should end when common envelopes cease to be convective.

Observationally, it is of course a well-known characteristic of the W UMa systems that their spectral types are F or G, and this was cited by Lucy (1968a) as evidence for their having *convective* common envelopes. Moreover, there is evidence (Lucy 1976) that the ending of the W UMa systems coincides with the beginning of the short-period β Lyrae systems, i.e., of systems whose light curves have rounded maxima, and minima of *unequal* depth, and this coincidence is readily interpreted as being due to the ending or severe diminution of energy exchange between the components in consequence of (1) a convective common envelope no longer being possible and (2) the inability of a radiative common envelope to adjust to mechanical equilibrium when $q \neq 1$.

These remarks show that the observations are consistent with the prediction of the TRO theory. Nevertheless, they are not necessarily inconsistent with the DSC theory. This theory predicts the continuation of W UMa systems to spectral types earlier than F0 and periods longer than ~ 0.5 days only on the assumption that in the relevant mass range binaries of small enough angular momentum are actually formed. Thus the ending of the W UMa systems is explained, and a contradiction with the DSC theory avoided, if we postulate that only binaries that are well detached at zero-age are formed for $M_1 \gtrsim 2.5 M_{\odot}$. (The qualification of being *well* detached is necessary, since otherwise they are likely to evolve quickly into contact, thus forming the unobserved continuation of the W UMa systems.)

This possibility for avoiding a contradiction with

the DSC theory is of course testable by examining the properties of upper-main-sequence binaries. Unfortunately, the outcome is not yet certain. One of the present authors (L. B. L.), in an unpublished investigation of the period-spectrum diagram for eclipsing binaries, concluded some years ago that early-type binaries are indeed well detached at zero-age. However, among the early-type contact binaries recently investigated in some detail (see § IIIg) are three—BH Cen, V701 Sco, and V1010 Oph—whose dimensions are consistent with zero-age structures. This question of whether *zero-age* binaries exist outside the W UMa domain is clearly of importance not only for testing these two theories but also for testing theories of the formation of close binaries. The current active investigation of early-type contact systems should answer this question.

g) Early-Type Contact Binaries

According to the DSC theory, mechanical equilibrium of a radiative common envelope is possible and is in fact the expected state of such envelopes since departures from this condition should be erased on a dynamical time scale (Shu *et al.* 1976). It then follows that, for a sufficient degree of contact, the variation of T_{eff} over the surfaces of early-type contact binaries should obey von Zeipel's law—i.e., equation (4) with $\beta = 0.25$. Moreover, this conclusion applies not only to zero-age systems, whose structure has been investigated with the DSC theory, but also to evolved systems and even to a system such as SV Cen, which appears to be undergoing mass exchange on a thermal time scale (Rucinski 1976; Wilson and Starr 1976).

According to the TRO theory, on the other hand, mechanical equilibrium of a radiative common envelope is in general impossible; von Zeipel's law is therefore not expected to provide a description of the variation of T_{eff} over the surfaces of early-type contact binaries.

Because information on this question can be extracted from their light curves, we have collected in Table 2 all the relevant data obtained thus far by applying the Wilson-Devinney (1971) code to early-type contact binaries. In these photometric solutions, von Zeipel's law is assumed, but *not* with a single constant of proportionality over the entire surface of the contact binary; instead, each component has its own constant. Thus the variation of T_{eff} over that part of the surface deemed to belong to component i is given by

$$T_{\text{eff}} = T_i \left(\frac{g}{g_i} \right)^{0.25}, \quad (5)$$

where T_i and g_i are the effective temperature and the effective gravity, respectively, at the poles of component i . Of these two polar temperatures, T_1 is assigned a value on the basis of the spectral type of the primary, and T_2 is solved for as a photometric element. Of course, this procedure does in general imply an unphysical discontinuity in T_{eff} on the neck connecting the two components. Nevertheless, it has the virtue that the extra parameter does dramatically improve the

TABLE 2
PROPERTIES OF EARLY-TYPE CONTACT BINARIES

Star (1)	M_2/M_1 (2)	f (%) (3)	T_1 (K) (4)	$T_1 - T_2^0$ (K) (5)	$T_1 - T_2^c$ (K) (6)	Source (7)
AO Cas.....	1.186 ± 0.041	3.4 ± 0.7	34700	-333 ± 51	-60 ± 41	Schneider and Leung 1978
SV Cen.....	1.15 ± 0.04	67.4 ± 9.9	23000	+6947 ± 149	-125 ± 51	Wilson and Starr 1976*
V701 Sco.....	1.00† ± 0.03†	51.4 ± 0.9	20500	-86 ± 59	+0 ± 21	Wilson and Leung 1977
BH Cen.....	0.969 ± 0.015	20.6 ± 1.5	17900	+60 ± 50	+10 ± 4	Leung and Schneider 1977
UW CMa.....	0.751 ± 0.024	23.5 ± 3.8	43000	+3806 ± 984	+228 ± 26	Leung and Schneider 1978 <i>b</i>
AU Pup.....	0.644 ± 0.007	71.6 ± 3.2	9600	+533 ± 16	+158 ± 6	Leung and Schneider 1978 <i>a</i>
V1010 Oph.....	0.489 ± 0.002	18.2 ± 1.4	8200	+2529 ± 30	+94 ± 3	Leung and Wilson 1977
BV 845.....	0.399 ± 0.001	2.6 ± 2.2	8500	+3804 ± 63	+257 ± 5	Leung and Darland 1978
V535 Ara.....	0.361 ± 0.004	2.5 ± 3.6	8750	+178 ± 14	+108 ± 11	Leung and Schneider 1978 <i>a</i>
V1073 Cyg.....	0.340 ± 0.009	7.0 ± 1.5	8750	+543 ± 21	+129 ± 19	Leung and Schneider 1978 <i>a</i>
V729 Cyg.....	0.237 ± 0.018	31.0 ± 4.0	34300	+11565 ± 258	+1083 ± 238	Leung and Schneider 1978 <i>c</i>

* See also Rucinski 1976.

† Spectroscopic observations favoring this mass ratio are now available (Andersen 1978).

‡ Guess.

fit to many observed light curves (Rucinski 1974; Berthier 1975; Leung and Wilson 1977).

Because the DSC theory expects von Zeipel's law to be obeyed over the entire surface of a contact binary, it predicts that T_2 should be found to have the value $T_2^c = T_1(g_2/g_1)^{0.25}$, and this can be tested by comparing T_2^c with T_2^0 , the value given by the photometric analysis. The data for this test are given in columns (5) and (6) of Table 2. In these columns, the temperature differences $T_1 - T_2^0$ and $T_1 - T_2^c$ are tabulated since these should be insensitive to errors in T_1 ; their quoted probable errors are therefore those of T_2^0 and T_2^c , with the errors of the former coming from the least-squares analysis, and those of the latter being derived from the uncertainties in the mass ratios and the degrees of contact.

If the temperature differences $T_1 - T_2^0$ and $T_1 - T_2^c$ are in reasonable agreement, the light curve is consistent with the application of von Zeipel's law over the entire surface of the contact binary. This is evidently true for V701 Sco, BH Cen, and V535 Ara, but not for the remaining eight systems. Moreover, only for V 535 Ara is the agreement significant for the DSC theory: V701 Sco and BH Cen have $q \approx 1$, and the discontinuities that are fundamental to the DSC theory have zero magnitude when the components are identical.

Of the remaining eight systems, there are several—SV Cen, UW CMa, V1010 Oph, BV 845, and V729 Cyg—for which the temperature differences differ enormously, and yet, with the sole exception of BV 845, the probable errors of their degrees of contact indicate little doubt of their in being contact—less doubt, in fact, than for V535 Ara. For the remaining three systems, the temperature difference differ significantly but not enormously. Of these, AO Cas has $q \approx 1$ and is therefore not of much interest, but AU Pup and V1073 Cyg do have components of disparate mass.

The conclusion that seems to be indicated by these data is that von Zeipel's law does not in general hold over the entire surfaces of early-type contact binaries. Had this been found to be true, it would have been a decisive proof of the existence of the DSC theory's discontinuities, since it is their introduction that

allows radiative common envelopes to achieve the condition of mechanical equilibrium from which von Zeipel's law derives. Nevertheless, although this proof has not been found, there exist several systems—V535 Ara, AU Pup, and V1073 Cyg—whose elements show that substantial energy exchange can occur within a radiative common envelope.

(*N.B.* Eggen's [1978] photometric estimate of the spectral type of V535 Ara is A8 V; this system is therefore more or less coincident in the period-color diagram with such W UMa systems as S Ant and V401 Cyg.)

IV. DISCUSSION

In this concluding section, we discuss the status of the two theories in the light of the above observational tests.

a) DSC Theory

In discussing their models for zero-age contact binaries, Lubow and Shu (1977) reported satisfactory agreement with observations on the basis of the models' rough accord with the period-color relation (§ IIIa). In discussing a variety of further tests, however, we find this theory to be in substantial disagreement with observational data. Nevertheless, in view of the implications of the need to include secular terms to eliminate the present indeterminacy of evolutionary sequences (§ IIIe), the DSC theory should not yet be regarded as contradicted. The necessity of such terms implies that some permitted configurations may in fact evolve on thermal and not nuclear time scales, and this may eliminate the observational disagreements that result from the DSC theory's prediction of unobserved, or rarely observed, configurations. For example, the problem with the apparent absence of overcontact zero-age systems (§ IIIc) would disappear if such a configuration evolves on a thermal time scale into a system with shallow contact that then evolves on a nuclear time scale. Evidently, then, the status of the DSC theory will be greatly clarified when further theoretical work resolves this indeterminacy and brings this theory to the TRO theory's level of development.

Although secular effects might well improve

dramatically the fortunes of the DSC theory, they cannot eliminate all difficulties. Specifically, the generally unconfirmed prediction (§ IIIg) that the variations of T_{eff} over the surface of early-type contact binaries should obey von Zeipel's law will not be affected. The possible difficulty (§ IIIf) resulting from the DSC theory's failure to predict that W UMa systems should end when common envelopes cease to be convective will also remain.

b) TRO Theory

In discussing the thermal relaxation oscillations, both Lucy (1976) and Robertson and Eggleton (1977) stressed the apparent absence of binaries that could be identified with the predicted broken-contact phase. But such systems may in fact exist: we have shown (§ IIb) that a new subclass of W UMa systems can be recognized—the B-type systems—whose members have components in poor thermal contact, and we have argued (§ IIc) that these systems may indeed be the predicted mass-transferring, semidetached binaries.

In addition to this possible confirmation of a very distinctive prediction of the TRO theory, we find in our discussion (§ III) of further tests that this theory is in substantial agreement with the observational data. Most of these agreements are only qualitative, however; the possibility therefore exists that disagreements will be found if these tests are increased in precision, as several can be if additional models are calculated. Specifically, additional zero-age calculations could be

used to reevaluate the TRO theory's agreement with the period-color relation (§ IIIa) and with the degrees of contact of W-type systems (§ IIIb); and evolutionary calculations could further test the interpretation of the A-type systems (§ IIIc).

However, with regard to future work, the refinement of the photometric analyses of the broken-contact candidates is potentially of greater significance. As we stressed in § II, conventional analyses of the light curves of AK Her and W Crv do not yield definitive elements because of the asymmetries of their light curves. But if our explanation (§ IIc) of the asymmetries is basically correct, a closer approach to definitive elements should be possible even without a detailed theory of this effect: specifically, the asymmetries should be corrected for in a manner that takes account of their being due to *extra* light. Proving that these binaries are mass-transferring, semidetached systems would establish the existence of a class of close binaries that the TRO theory predicted and that the DSC theory cannot readily explain (§ IIc).

We wish to thank K. C. Leung, F. H. Shu, J. Andersen, L. W. Twigg, S. P. Worden, and S. M. Rucinski for advance knowledge of their recent researches, and F. B. Wood for data from the University of Florida Eclipsing Binary Card Catalogue.

This work has been supported by the National Science Foundation under grants AST 76-08917 and AST 77-09188.

REFERENCES

- Andersen, J. 1978, private communication.
 Anderson, L., and Shu, F. H. 1977, *Ap. J.*, **214**, 798.
 Batten, A. H., and Hardie, R. H. 1965, *A.J.*, **70**, 666.
 Berthier, E. 1975, *Astr. Ap.*, **40**, 237.
 Biermann, P., and Thomas, H.-C. 1972, *Astr. Ap.*, **16**, 60.
 Binnendijk, L. 1961, *A.J.*, **66**, 27.
 ———. 1969, *A.J.*, **74**, 1024.
 ———. 1970, *Vistas in Astronomy*, Vol. **12**, p. 217.
 Carbon, D. F., and Gingerich, O. 1969, in *Theory and Observation of Normal Stellar Atmospheres*, ed. O. Gingerich (Cambridge, Mass.: MIT Press), p. 377.
 Chambliss, C. R. 1970, *A.J.*, **75**, 725.
 Duerbeck, H. W. 1977, *Acta Astr.*, **27**, 51.
 Dycus, R. D. 1968, *Pub. A.S.P.*, **80**, 207.
 Eggen, O. J. 1961, *Royal Obs. Bull.*, No. 31.
 ———. 1967, *Mem. R.A.S.*, **70**, 111.
 ———. 1978, *A.J.*, **83**, 288.
 Flannery, B. P. 1976, *Ap. J.*, **205**, 217.
 Hazlehurst, J. 1975, *Astr. Ap.*, **36**, 49.
 Hazlehurst, J., and Refsdal, S. 1978, *Astr. Ap.*, **62**, L9.
 Koch, R. H., Plavec, M., and Wood, F. B. 1970, *A Catalogue of Graded Photometric Studies of Close Binaries* (Philadelphia: University of Pennsylvania Printing Office).
 Leung, K. C., and Darland, J. J. 1978, *Pub. A.S.P.*, **90**, 97.
 Leung, K. C., and Schneider, D. P. 1977, *Ap. J.*, **211**, 844.
 ———. 1978a, *Ap. J.*, **222**, 917.
 ———. 1978b, *Ap. J.*, **222**, 924.
 ———. 1978c, *Ap. J.*, **224**, 565.
 Leung, K. C., and Wilson, R. E. 1977, *Ap. J.*, **211**, 853.
 Lubow, S. H., and Shu, F. H. 1977, *Ap. J.*, **216**, 517.
 Lucy, L. B. 1967, *Zs. f. Ap.*, **65**, 89.
 ———. 1968a, *Ap. J.*, **151**, 1123.
 ———. 1968b, *Ap. J.*, **153**, 877.
 ———. 1973, *Ap. Space Sci.*, **22**, 381.
 ———. 1976, *Ap. J.*, **205**, 208.
 Mochnacki, S. W., and Doughty, N. A. 1972a, *M.N.R.A.S.*, **156**, 51.
 ———. 1972b, *M.N.R.A.S.*, **156**, 243.
 Naqvi, S. I. H., and Grønbech, B. 1976, *Astr. Ap.*, **47**, 315.
 Osaki, Y. 1970, *M.N.R.A.S.*, **148**, 391.
 Robertson, J. A. 1979, *M.N.R.A.S.*, in press.
 Robertson, J. A., and Eggleton, P. P. 1977, *M.N.R.A.S.*, **179**, 359.
 Rucinski, S. M. 1973, *Acta Astr.*, **23**, 79.
 ———. 1974, *Acta Astr.*, **24**, 119.
 ———. 1976, *Pub. A.S.P.*, **88**, 244.
 Rucinski, S. M., Whelan, J. A. J., and Worden, S. P. 1977, *Pub. A.S.P.*, **89**, 684.
 Schmidt, H., and Herczeg, T. 1959, *Zs. f. Ap.*, **47**, 106.
 Schneider, D. P., and Leung, K. C. 1978, *Ap. J.*, **223**, 202.
 Shu, F. H., Lubow, S. H., and Anderson, L. 1976, *Ap. J.*, **209**, 536.
 ———. 1979, *Ap. J.*, **229**, 223.
 Sisteró, R. F., and Castore de Sisteró, M. E. 1974, *A.J.*, **79**, 391.
 Twigg, L. W. 1979, in preparation.
 von Zeipel, H. 1924, *M.N.R.A.S.*, **84**, 665.
 Webbink, R. F. 1976, *Ap. J.*, **209**, 829.
 Whelan, J. A. J., Mochnacki, S. W., and Worden, S. P. 1974, *M.N.R.A.S.*, **168**, 31.
 Whelan, J. A. J., Worden, S. P., and Mochnacki, S. W. 1973, *Ap. J.*, **183**, 133.
 Wilson, R. E. 1978, *Ap. J.*, **224**, 885.
 Wilson, R. E., and Biermann, P. 1976, *Astr. Ap.*, **48**, 349.
 Wilson, R. E., and Devinney, E. J. 1971, *Ap. J.*, **166**, 605.
 ———. 1973, *Ap. J.*, **182**, 539.
 Wilson, R. E., and Leung, K. C. 1977, *Astr. Ap.*, **61**, 137.
 Wilson, R. E., and Starr, T. C. 1976, *M.N.R.A.S.*, **176**, 625.
 Woodward, E. J., and Wilson, R. E. 1977, *Ap. Space Sci.*, **52**, 387 (WW).
 Worden, S. P., and Whelan, J. A. J. 1973, *M.N.R.A.S.*, **163**, 391.
 Worden, S. P., Whelan, J. A. J., and Rucinski, S. M. 1979, in preparation.

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