

CEPHEID MASSES FROM OBSERVATIONS AND PULSATION THEORY*

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ABSTRACT

Cepheid masses are calculated using radii measured recently by the Baade-Wesselink and the related Barnes *et al.* methods. They are compared to masses which use only the well observed fundamental mode period, Π_0 , and an approximate effective surface temperature, T_e . These latter masses, called theoretical masses, result from the solution for the four unknowns M , R , L , Q_0 of four equations: evolutionary mass-luminosity (M - L) law for blue looping yellow giants, the period-mean density (Π_0 - M - R - Q_0) relation, the variation of the pulsation constant Q_0 with the unknowns, and the definition equation for T_e . To ensure the proper calibration of these theoretical masses, the effects of a new distance scale and new temperatures on evolutionary theory and pulsation masses are discussed. It is found that pulsation masses using observed Π_0 , L , and colors (to give the mean T_e) now agree much better with evolutionary and theoretical masses, but the Wesselink and the Barnes *et al.* radii imply lower average masses for 69 cases. This discrepancy can be alleviated and the agreement of evolution, theoretical, and pulsation masses not destroyed by using recently proposed helium-rich surface layers in theoretical Cepheid models which increase Q_0 as well as the bump, beat, and the Wesselink and Barnes *et al.* radius masses.

Subject headings: stars: Cepheids — stars: pulsation

I. INTRODUCTION

Masses of Cepheids are of great interest because they can verify the accuracy of stellar evolution calculations in the yellow giant region of the Hertzsprung-Russell diagram. Normally, stellar masses are measured by gravitational interaction effects between two or more stars, but the use of appropriate observations of periods (Π), luminosities (L), colors (to give a mean T_e), and light and velocity curves, plus evolution and pulsation theory, allows new and different mass determinations. Unfortunately, these other kinds of masses, determined by as many as six different methods, do not always agree very well with one another. These mass anomalies were first pointed out by Christy (1968) and Stobie (1969) for Cepheids with bumps in their light and velocity curves, by Cogan (1970) and Rodgers (1970) for those with good L and T_e values, and by Petersen (1973), King *et al.* (1975), and later Stobie (1977), for double-mode or beat Cepheids. The theoretical mass, first discussed in detail by Cox *et al.* (1977), agrees closely with the evolutionary mass and therefore allows a comparison of any mass determination with the evolutionary theory mass for any Cepheid using only the observables Π and an approximate T_e .

Mass anomalies have been discussed in detail by many, such as Fricke, Stobie, and Strittmatter (1972*b*), Iben and Tuggle (1972*a*), Stobie (1974), Schmidt (1976), and Cox *et al.* (1977). Five reasons (Davis 1977) have been advanced for the general conclusion that, for a given Cepheid, the various masses based on

one or more aspects of pulsation theory are systematically lower than the evolutionary theory masses. Early indications by Madore (private communication) are that the Cepheid spectroscopic binary star mass for SU Cyg is much closer to the evolutionary mass than the lower masses using pulsation theory. Possible causes of the various mass anomalies are: significant mass loss in red or yellow giant stages (Christy 1968), even though evolution theory seems to firmly exclude more than 10% loss (Lauterborn, Refsdal, and Weigert 1971) if the blue loops are to occur; errors in the distance scale (Iben and Tuggle 1972*a*) which place the Cepheids too close, make them too faint intrinsically, and therefore give them radii and masses too small; errors in the temperature scale (King *et al.* 1975) which make the Cepheids too hot and therefore give them radii and masses too small; errors in the composition or its material properties such as the opacity (Fricke, Stobie, and Strittmatter 1972*a*; Vemury and Stothers 1978) which give too small a pulsation constant, Q ; and the possibly erroneous assumption that the structure is homogeneous (Cox *et al.* 1977), making Q too small.

While all of these causes are still possible, and maybe pulsation theory is even wrong, in this paper I propose that the three equally important causes are errors in distance and temperature scale, and the previous lack of consideration of inhomogeneous composition models. Indeed, the new Hyades cluster distance of Hanson (1977) now seems to imply an increase of 0.26 mag in the distance modulus to those several open clusters with Cepheids. The more accurate way of getting T_e from each multicolor observation and good model atmospheres, as used by Pel

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(1978), has generally reduced the T_e by sometimes several hundred kelvins. Finally, chemically inhomogeneous yellow giant envelopes increase Q by up to 10%, allowing pulsation mass increases as large as 20% and doubling or tripling masses based on light-curve bump phases (giving R) or observed double-mode Cepheid period ratios.

It appears that the double-mode Cepheids are the key to Cepheid masses and compositions. Only the very accurately known period ratio of the fundamental and first overtone pulsation needs to be directly interpreted by well known linear theory calculations. At present, the only successful interpretation of double-mode period ratios in terms of evolution theory masses requires surface helium enrichment.

The observational data used here are in six recent publications. These tabulations of these very difficult observations are very useful. Balona (1977) has given Wesselink radius data for 52 well observed classical Cepheids, 35 of which have accurate T_e values from Pel (1978). Fourteen more can be given approximate T_e values by use of the mean unreddened colors of Dean, Warren, and Cousins (1978) and the Kraft (1961) relation between $(B - V)_0$ and $\log T_e$. Evans (1977) tabulates radii for 14 Cepheids for which three have T_e values from Pel and nine have unreddened colors from Dean, Warren, and Cousins. For ζ Gem, I have used the two proposed Scarfe (1977) values instead of the Evans one. Counting double determinations, 69 radii are obtained, some of which are also tabulated by Cogan (1978), but I have not used any of his data directly. Barnes *et al.* (1977) have given the radius for nine Cepheids using V , $V - R$, and radial velocity curve data. Masses are computed for these cases even though their accuracy has not yet been increased to its full potential.

All the theoretical periods used in this paper are based on a linear pulsation theory computer program similar to the one described by Castor (1971). There seems to be no question that theoretical full amplitude (nonlinear) periods agree to within a few percent with

linear theory ones according to King *et al.* (1973). Therefore, linear theory periods (and Q values), needed in this paper only for the fundamental and first overtone modes, can be directly equated to observed periods. The purpose of this paper is to present for the first time detailed mass data based on Wesselink method radii, as well as masses from three other methods revised due to distance, temperature, and inhomogeneous composition improvements. The conclusion is that all mass anomalies are largely eliminated, but more accurate Wesselink radii and their correction factors for surface area effects are needed. The following sections discuss, in turn, current evolutionary, theoretical, pulsation, and Wesselink method masses. The final section gives mass comparisons for six Cepheids for which at least four mass determination methods are applicable.

II. EVOLUTIONARY THEORY MASSES

The list of 13 Cepheids of Sandage and Tammann (1969) for calibrating the period-luminosity-color relation can be expanded to 20 by using recent data by van den Bergh (1977a). These are given in Table 1 together with their fundamental mode periods, luminosities, and T_e values. I assume here, and discuss it later, that the pulsation of SU Cas is in the fundamental mode. Absolute visual magnitudes from these two lists have been decreased by 0.26 to allow for the new distance scale, but the T_e values are from the Kraft relation and the same unreddened intensity means of $(B - V)_0$ as given by Sandage and Tammann or van den Bergh. Bolometric corrections and reddenings are those used by Sandage and Tammann and van den Bergh. The distance of SU Cas (Schmidt 1978a) is very uncertain, and UY Per, VY Per, VX Per, and SZ Cas may be in the King 4 or Czernik 8 clusters rather than the h and χ Persei association (Turner 1977).

Evolutionary masses (M_{ev}) are evaluated from the Becker, Iben, and Tuggle (1977) evolutionary tracks and their mass-luminosity formula in the instability

TABLE 1
EVOLUTION, THEORETICAL, AND PULSATION MASSES: NEW DISTANCE SCALE

Cepheid	Period(d)	$T_{eff}(K)$	$L_{OBS}/L_{\odot}$	$M_{EV}/M_{\odot}$	$Q_{TH}(d)$	$M_{TH}/M_{\odot}$	$R_{TH}/R_{\odot}$	$L_{TH}/L_{\odot}$	$R_{OBS}/R_{\odot}$	$Q_{PUL}(d)$	$M_{PUL}/M_{\odot}$
SU Cas	1.950	6599	955	4.84	0.0364	4.87	24.1	974	23.8	0.0364	4.73
EV Sct	3.090	6113	1148	5.09	.0375	5.32	33.1	1351	30.5	.0379	4.27
CEb Cas	4.480	6138	1995	5.91	.0384	6.24	44.0	2426	39.8	.0390	4.80
CF Cas	4.870	5895	1820	5.77	.0387	6.11	45.9	2249	41.3	.0395	4.62
CEa Cas	5.140	5943	2188	6.06	.0389	6.31	48.0	2541	44.5	.0394	5.18
UY Per	5.360	6088	2512	6.29	.0389	6.64	50.1	3054	45.4	.0396	5.12
CV Mon	5.380	5943	2344	6.18	.0390	6.43	49.7	2722	46.1	.0395	5.28
VY Per	5.530	6162	2818	6.49	.0390	6.83	51.6	3401	47.0	.0396	5.32
CS Vel	5.900	5895	1778	5.73	.0393	6.61	53.0	3003	40.8	.0417	3.38
V367 Sct	6.290	6313	3467	6.87	.0393	7.45	57.6	4661	49.7	.0403	5.04
U Sgr	6.740	6162	3890	7.09	.0396	7.41	59.9	4580	55.2	.0401	5.97
DL Cas	8.000	5801	3715	7.00	.0403	7.32	66.1	4372	60.9	.0410	5.92
S Nor	9.750	5731	4467	7.36	.0410	7.80	76.1	5521	68.4	.0420	5.93
TW Nor	10.790	5572	2884	6.53	.0415	7.81	80.8	5566	58.1	.0454	3.48
VX Per	10.890	5943	5888	7.93	.0413	8.57	84.1	7809	73.0	.0426	5.96
SZ Cas	13.620	6015	8511	8.77	.0421	9.53	99.8	11542	85.7	.0436	6.45
VY Car	18.930	5309	11220	9.45	.0442	9.15	118.9	9938	126.3	.0436	10.70
T Mon	27.020	5224	18621	10.85	.0460	10.29	152.5	15318	168.1	.0451	13.26
RS Pup	41.380	5373	22387	11.40	.0483	12.62	209.9	32500	174.2	.0510	8.03
SV Vul	45.040	5018	30200	12.37	0.0493	11.86	214.7	25850	232.0	0.0484	14.45

strip for $Y = 0.28$ (see eq. [3]). One can note, as discussed by Iben (1974), that higher Y , like 0.38, will give evolutionary masses about 20% smaller. This higher interior Y , which, because of other general astrophysics arguments not discussed here, is not likely, must always be kept in mind to alleviate perhaps a small mass anomaly.

III. THEORETICAL MASSES

A Cepheid mass can be derived using only the fundamental mode pulsation period and an approximate T_e , obtained from the Pel (1978) list or from the Dean, Warren, and Cousins (1978) unreddened mean colors and the Kraft relation:

$$\log T_e = 3.886 - 0.175 (\langle B \rangle_0 - \langle V \rangle_0). \quad (1)$$

An intensity mean for B and V seemed best to represent the T_e for theoretical nonpulsating models as discussed by many, including Cox and Davis (1975). This theoretical mass (M_T , M_{TH} in tables, or $M(T)$ on figures) depends almost entirely on the evolution and pulsation theories which produce the yellow giant mass-luminosity law in the instability strip, the period-mean density law, and the variation of the pulsation constant with M , R , and L . If the equation

$$L = 4\pi R^2 \sigma T_e^4 \quad (2)$$

defining T_e is used, one has four equations ([2], [3], [4], and [5]) for the four unknowns M , R , L , and Q_0 . The Becker-Iben-Tuggle mass-luminosity equation used here is:

$$\begin{aligned} \log \frac{L}{L_0} = & 0.46 - 41(Z - 0.02) + 6.6(Y - 0.28) \\ & + [3.68 + 21(Z - 0.02) \\ & - 4.5(Y - 0.28)] \log \frac{M}{M_0}, \end{aligned} \quad (3)$$

and the period-mean density relation is

$$Q_0 = \Pi_0 \left[\frac{M/M_0}{(R/R_0)^3} \right]^{1/2} \quad (4)$$

with the Faulkner (1977) form for

$$\begin{aligned} \log Q_0 = & a_0 + b_0 F^2 + c_0 F^3 + d_0 F^4 \\ & + e_0 M/M_0 + E^3(f_0 + g_0 B) \end{aligned} \quad (5)$$

and

$$F = \log \frac{M/M_0}{R/R_0}$$

$$E = 12.5(3.96 - \log T_e - 0.1 \log R/R_0),$$

$$B = 3.33(\log M/M_0 - 0.29 \log L/L_0 + 0.2).$$

Table 2 gives the fundamental and first overtone mode constants for the Cox, King, and Stellingwerf (1972) period data for $X = 0.602$, $Y = 0.354$ from Faulkner (1977) and for inhomogeneous 4, 5, 6, 7, and 8 $M_\odot$ models with $X = 0.23$, $Y = 0.75$ from the surface to 70,000 K; $X = 0.46$, $Y = 0.52$ between 70,000 and 80,000 K; and $X = 0.70$, $Y = 0.28$ at all deeper levels. While this helium enrichment has been uniformly used here, others which are deeper with smaller Y discussed by Cox, King, and Hodson (1978) may be even more appropriate for masses less than 7 $M_\odot$, and no enrichment at all is possible above 8 $M_\odot$. Since the enrichment is not well known now, the possible opacity uncertainties are not very significant. The Cox, King, and Stellingwerf data represent typical homogeneous models. Q_0 values range from 1% to 10% larger for all the inhomogeneous models from 3.5 to 15.0 $M_\odot$ and from 5000 to 7000 K at evolution theory luminosities. The differences are largest at low M/R where Q_0 and its change with M/R are also largest.

This theoretical mass for all Cepheids has the dependence

$$M_T \sim \Pi^{0.40} T_e^{1.40}, \quad (6)$$

indicating that if T_e is as poorly known as 10% or 600 K in 6000 K, the theoretical mass is known better than 15%. Table 1, with Q_0 , R , and L values also, shows that indeed M_{ev} and M_T are very close, with the ratio M_T/M_{ev} being 1.05 ± 0.06 . For the 16 Cepheids in Tables 3 and 4 for which the same luminosities but new improved reddenings and cooler T_e values can be obtained, M_T/M_{ev} can be evaluated. Errors for theoretical masses relative to evolutionary masses are generally less than 15%. The means of M_T/M_{ev} are 1.02 ± 0.07 for Table 3 (homogeneous) masses and 0.99 ± 0.07 for Table 4 (inhomogeneous) masses. The Table 4 inhomogeneous model data are plotted in Figure 1.

TABLE 2
PULSATION CONSTANT COEFFICIENTS

Models	Mode	a	b	c	d	e	f	g	σ (%)
CKS.....	F	-1.4820	0.0253	-0.0438	0.0049	0.0030	0.0006	0.0018	0.19
CKS.....	1H	-1.5852	-0.1363	-0.2881	-0.1126	0.0025	0.0030	0.0060	0.31
Inhomogeneous.....	F	-1.4952	0.0778	-0.0784	-0.0314	0.0024	0.0032	0.0866	0.15
Inhomogeneous.....	1H	-1.6151	0.3683	0.5073	0.2229	-0.0006	-0.0064	-0.0301	0.18

TABLE 3
EVOLUTION, THEORETICAL, AND PULSATION MASSES:
NEW DISTANCE SCALE AND TEMPERATURES

Cepheid	Period(d)	$T_{\text{eff}}(K)$	$L_{\text{OBS}}/L_{\odot}$	$M_{\text{EV}}/M_{\odot}$	$Q_{\text{TH}}(d)$	$M_{\text{TH}}/M_{\odot}$	$R_{\text{TH}}/R_{\odot}$	$L_{\text{TH}}/L_{\odot}$	$R_{\text{OBS}}/R_{\odot}$	$Q_{\text{PUL}}(d)$	$M_{\text{PUL}}/M_{\odot}$
SU Cas	1.950	6339	955	4.84	0.0364	4.61	23.6	798	25.8	0.0362	5.95
EV Sct	3.090	6187	1148	5.09	.0374	5.41	33.3	1434	29.8	.0381	4.01
CF Cas	4.870	5895	1820	5.77	.0387	6.11	45.9	2249	52.1	.0395	4.62
UY Per	5.360	5848	2512	6.29	.0390	6.28	49.1	2497	49.3	.0390	6.33
CV Mon	5.380	6064	2344	6.18	.0389	6.61	50.2	3012	44.3	.0398	4.75
VY Per	5.530	5943	2818	6.49	.0391	6.51	50.7	2837	50.5	.0391	6.44
U Sgr	6.740	5734	3890	7.09	.0398	6.72	57.8	3191	63.8	.0392	8.79
DL Cas	8.000	5848	3715	7.00	.0403	7.40	66.3	4553	59.9	.0411	5.68
S Nor	9.750	5502	4467	7.36	.0412	7.37	74.4	4494	74.2	.0412	7.31
TW Nor	10.790	5617	2884	6.53	.0415	7.90	81.1	5797	57.2	.0457	3.35
VX Per	10.890	5685	5888	7.93	.0415	8.06	82.2	6245	79.8	.0417	7.47
SZ Cas	13.620	5919	8511	8.77	.0422	9.32	99.0	10644	88.5	.0432	6.99
VY Car	18.930	5140	11220	9.45	.0443	8.75	116.8	8429	134.8	.0432	12.75
T Mon	27.020	5099	18621	10.85	.0462	9.95	150.4	13537	176.4	.0448	15.13
RS Pup	41.390	5309	22387	11.40	.0484	12.41	208.6	30587	178.5	.0506	8.49
SV Vul	45.100	5038	30200	12.37	0.0493	11.93	215.3	26432	230.2	0.0485	14.13

IV. PULSATION MASSES

Pulsation masses [M_Q , M_{PUL} , in tables, or $M(Q)$ on figures] as discussed by Cogan (1970), Rodgers (1970), Fricke, Stobie, and Strittmatter (1972b) (FSS), Cox, King, and Stellingwerf (1972) (CKS), and Schmidt (1976) to give only a few references, require knowledge of Π_0 , L , T_e , and the composition as it affects the Q . The L and T_e determine a photosphere radius, and then the mass comes from the period-mean density relation with its complicated Q dependence on M , R , L , and T_e . Pulsation masses, photosphere radii, and fundamental-mode Q_0 values with the new distance scale, but old T_e values, are given in Table 1.

Dean, Warren, and Cousins (DWC) discuss in detail the problem of Cepheid color reddenings due to interstellar absorption. They conclude that fitting four-color variations of Cepheids in the $(B - V, V - I)$ -plane gives more reliable reddenings than those used by previous workers using mean spectral types and observed $B - V$ colors. Possibly the spectral types are afflicted by the proposed helium enrichment. The net result is that the purely photometric unreddened colors are larger and the T_e values considerably smaller than obtained before. Nevertheless the Pel T_e data imply an even lower T_e than the DWC

unreddened colors give when the Kraft $[(B - V)_0, \log T_e]$ -relation (eq. [1]) is used. DWC interpret this as Pel having even less reddening. Neither the model atmospheres on which the Kraft relation is based nor those used by Pel to interpret his colors include any surface enrichment of helium. The Pel and DWC data are plotted in Figure 2 together with the Kraft line of equation (1).

Evolution, theoretical, and pulsation masses for 16 Cepheids using the T_e from these new DWC colors and the Kraft relation, together with the latest periods and revised luminosities, discussed above, are given in Table 3. For U Sgr and S Nor, the Pel T_e values are used instead of the T_e from the unreddened DWC colors. Table 3 masses can be compared with those given in Table 1 with the new distance but older T_e values. The larger distance scale increases masses over the older FSS data (not given here), and now the cooler T_e values increase them about the same amount again. The ratios M_Q/M_{ev} for the new T_e values are plotted in Figure 3. The mean M_Q to M_{ev} ratio was 0.70 (Stobie 1974), is now 0.84 ± 0.17 (Table 1) with the new distance scale, and is 0.97 ± 0.25 (Table 3 and Fig. 3) with new values of both distance scale and T_e .

I have ignored two small points here. First, our smaller reddening gives about 0.1 or 0.2 mag less

TABLE 4
EVOLUTION, THEORETICAL, AND PULSATION MASSES: NEW DISTANCE SCALE AND TEMPERATURES,
INHOMOGENEOUS MODEL Q_0 VALUES

Cepheid	Period(d)	$T_{\text{eff}}(K)$	$L_{\text{OBS}}/L_{\odot}$	$M_{\text{EV}}/M_{\odot}$	$Q_{\text{TH}}(d)$	$M_{\text{TH}}/M_{\odot}$	$R_{\text{TH}}/R_{\odot}$	$L_{\text{TH}}/L_{\odot}$	$R_{\text{OBS}}/R_{\odot}$	$Q_{\text{PUL}}(d)$	$M_{\text{PUL}}/M_{\odot}$
SU Cas	1.950	6339	955	4.84	0.0375	4.55	23.1	760	25.8	0.0367	6.12
EV Sct	3.090	6187	1148	5.09	.0390	5.31	32.2	1342	29.8	.0398	4.36
CF Cas	4.870	5895	1820	5.77	.0409	5.97	43.9	2062	52.1	.0415	5.11
UY Per	5.360	5848	2512	6.29	.0413	6.13	46.9	2280	49.3	.0408	6.92
CV Mon	5.380	6064	2344	6.18	.0410	6.46	48.1	2763	44.3	.0420	5.27
VY Per	5.530	5943	2818	6.49	.0413	6.35	48.5	2595	50.5	.0408	7.04
U Sgr	6.740	5734	3890	7.09	.0423	6.54	55.0	2889	63.8	.0408	9.52
DL Cas	8.000	5848	3715	7.00	.0429	7.20	63.1	4114	59.9	.0435	6.35
S Nor	9.750	5502	4467	7.36	.0442	7.15	70.3	4010	74.2	.0436	8.18
TW Nor	10.790	5617	2884	6.53	.0445	7.66	76.7	5178	57.2	.0484	3.78
VX Per	10.890	5685	5888	7.93	.0444	7.82	77.7	5584	79.8	.0441	8.35
SZ Cas	13.620	5919	8511	8.77	.0452	9.04	93.6	9514	88.5	.0459	7.89
VY Car	18.930	5140	11220	9.45	.0481	8.44	109.3	7384	134.8	.0459	14.40
T Mon	27.020	5099	18621	10.85	.0502	9.59	140.7	11836	176.4	.0475	17.01
RS Pup	41.390	5309	22387	11.40	.0525	11.98	195.4	26833	178.5	.0539	9.64
SV Vul	45.100	5038	30200	12.37	0.0535	11.51	201.5	23134	230.2	0.0517	16.01

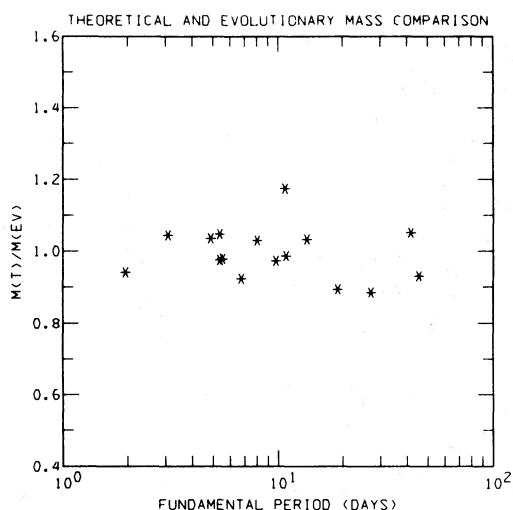


FIG. 1.—The ratio of the theoretical mass (M_T) to evolutionary theory mass (M_{ev}) is plotted versus period for 16 Cepheids with well known fundamental pulsation mode periods and luminosities and at least an approximate T_e value. The initial homogeneous composition for the evolution theory calculations is $X = 0.70$, $Y = 0.28$. The mean M_T/M_{ev} is 0.99 ± 0.07 .

absorption, if $A_V = 3.0(B - V)$, and therefore the Cepheid intrinsic luminosities may be up to 20% smaller. This may decrease M_{ev} by perhaps 5% only; but since $R \sim L^{0.5}$ and $M_Q \sim R^{2.44}$, M_Q may be 20% or more smaller near $7 M_\odot$. This mass decrease may not be appropriate if the entire open cluster containing

a calibrating Cepheid is moved farther away and all of its stars made brighter because of the reduced absorption. I here assume that the reddening corrections apply only to the Cepheid, as they are actually measured, and that the cluster distance is unaffected. Second, the T_e obtained from the DWC colors and the Kraft relation may be a few percent too large according to Figure 2. Now $R \sim T_e^{-2}$ and $M_Q \sim R^{2.44}$, so a few percent smaller T_e can result in a 10% or so larger M_Q . These errors tend to cancel but do produce a smaller pulsation mass.

As will be indicated later, the Wesselink and Barnes *et al.* radii indicate much lower Cepheid masses than the theoretical masses. Various systematic errors can be blamed, maybe for good reasons, but here I propose that the anomaly can be alleviated with the larger Q_0 values which result when there are considerably helium-enriched outer convection zones. This helium enrichment can nicely solve the bump and beat Cepheid mass anomalies, and, in spite of a possible layer instability and possible atmosphere abundance problems in the spectrum, it seems likely to actually exist according to Cox, Michaud, and Hodson (1978).

Table 4 gives the evolutionary, theoretical, and pulsation masses using the larger Q_0 values from the inhomogeneous envelopes. If they are to be believed for bump Cepheids with Π_0 between 5.5 and 13 days, we must believe them here because five of these 16 Cepheids also have light-curve bumps. The small mass anomalies that remained after increasing distances and decreasing T_e values are all gone now, and the pulsation masses are, at last, too large by about 10%.

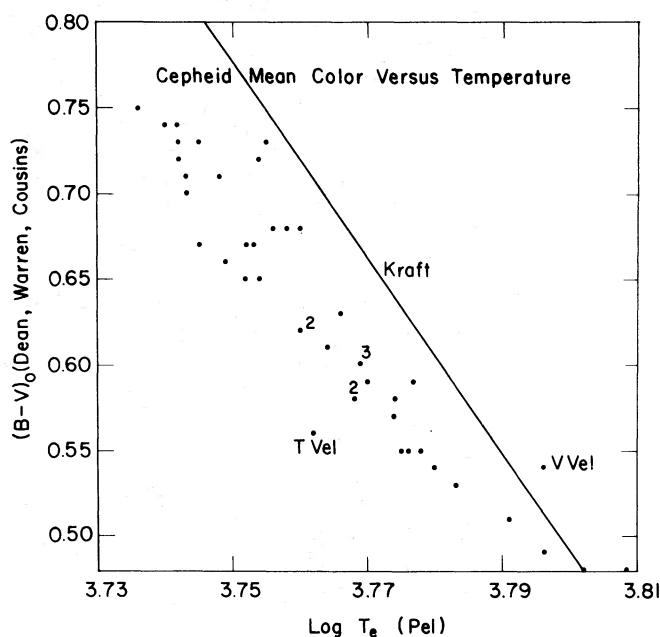


FIG. 2.—The Dean, Warren, and Cousins (1978) unreddened $(B - V)$ colors, based on the Schaltenbrand and Tammann (1971) intensity means, are plotted versus the Pel (1978) effective temperatures for 44 Cepheids. At three points there are represented two or even three values. The two largest deviants are labeled. The Kraft relation is also shown. The lack of agreement of the points and the line can be due to an error in the Kraft relation, errors in the Pel reddening corrections and T_e determinations, or an error in the reddening correction procedure of Dean *et al.*

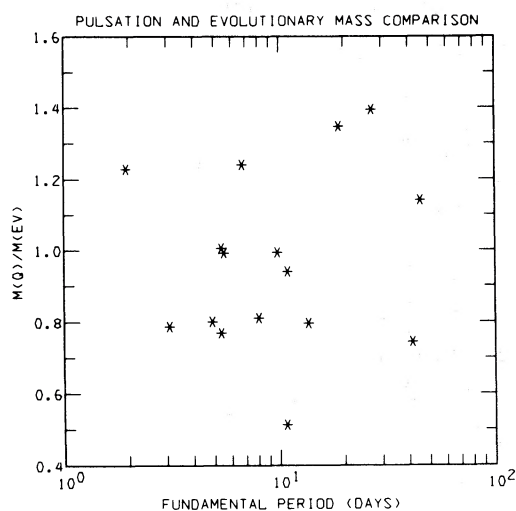


FIG. 3.—The ratio of the pulsation mass (M_Q) to evolution theory mass (M_{ev}) is plotted versus period for 16 Cepheids with well known fundamental pulsation mode periods, luminosities, and T_e values. The new larger distance and smaller T_e values are used. The initial composition for the evolution calculations is $X = 0.70$, $Y = 0.28$, and the homogeneous CKS models give Q_0 values. The mean M_Q/M_{ev} is 0.97 ± 0.25 .

Actually, for periods longer than about 13 days ($8 M_\odot$) these larger Q_0 values should not be used, and the pulsation masses should be as in Table 3 and Figure 3, because the surface helium is not likely to be enriched during the short lifetimes for massive Cepheids. If the new smaller Schmidt (1978b) distances for those open clusters with Cepheid members is verified, by more of his β photometry, or if the reddening corrections to the absolute magnitude are smaller as discussed above, they may make the pulsation masses somewhat smaller again and very close to the evolution and theoretical masses. Main-sequence fitting of the Cepheid clusters to the Hyades may also reduce the distances (van den Bergh 1977b) and masses.

Figure 4 shows a plot of M_Q/M_{ev} for the 16 Cepheids in Table 4. The mean of this ratio is now 1.07 ± 0.27 .

It is comforting to note that the Table 4 masses, as well as those revised masses for bump and beat Cepheids discussed by Cox *et al.* (1977) and Cox, Michaud, and Hodson (1978), do not exceed the limits of 3.5 – $18 M_\odot$ for Cepheids indicated by evolution theory. Many of the FSS bump masses and all the Petersen (1973) and King *et al.* (1975) beat masses, which assume homogeneous compositions, were less than the lower-limit blue-loop mass of 3.5 – $4.0 M_\odot$.

V. WESSELINK RADIUS MASSES

It is possible to obtain masses from radii measured from the Baade (1926)–Wesselink (1946) method in a way very similar to the way one gets pulsation masses. The radius depends here on the proper phasing of the observed light and velocity curves and their accuracy. Another problem is whether the velocity curve obtained from spectral lines in the high stellar atmosphere

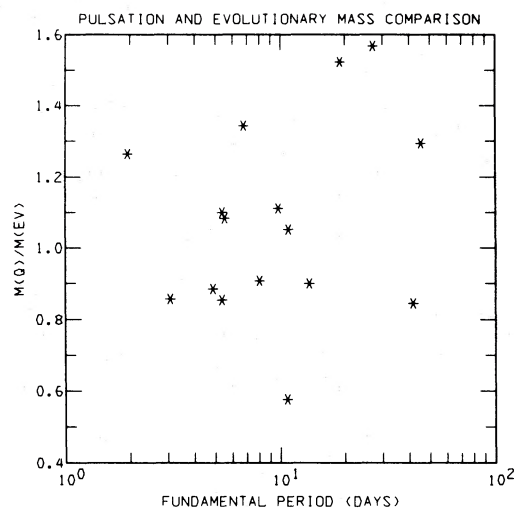


FIG. 4.—The ratio of the pulsation mass (M_Q) to evolution theory mass (M_{ev}) is plotted versus period for 16 Cepheids with well known fundamental pulsation mode periods, luminosities, and T_e values. The new larger distance and smaller T_e values as well as Q_0 values from the surface enriched helium envelope models are used. The initial composition for the evolution calculations is $X = 0.70$, $Y = 0.28$. The mean M_Q/M_{ev} is 1.07 ± 0.27 .

accurately gives the changes in photosphere radius between two phases of equal color and equal T_e . Further, it is important that one properly correct the observed surface-averaged radius changes to true radius changes. These problems have been discussed by Parsons (1972) and many others, including Cox and Wing (1974) and Cox and Davis (1975).

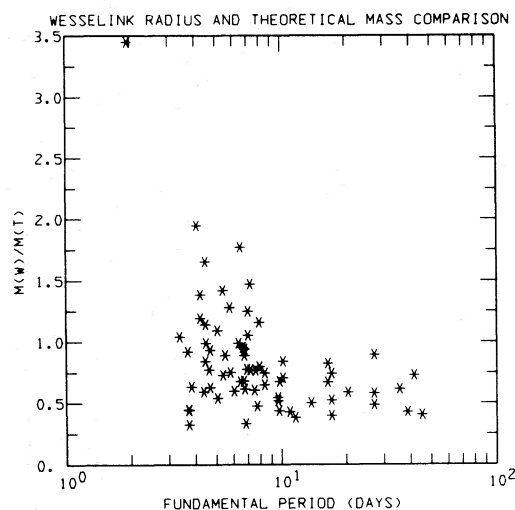


FIG. 5.—The ratio of the Wesselink radius mass (M_W) to theoretical mass (M_T) is plotted versus period for 69 cases with measured fundamental pulsation mode periods, T_e values, and Wesselink method radii. The assumed composition is homogeneous with $X = 0.70$ and $Y = 0.28$ for the evolution calculations, and $X = 0.602$, $Y = 0.354$ for the CKS envelope Q_0 values, all of which enter into the theoretical mass determinations. M_W/M_T has mean values of 0.93 below 10 days and 0.60 above.

TABLE 5
THEORETICAL AND WESSELINK RADIUS MASSES

Cepheid	Period(d)	$T_{\text{eff}}(K)$	$Q_{\text{th}}(d)$	$M_{\text{th}}/M_{\odot}$	$R_{\text{th}}/R_{\odot}$	$L_{\text{th}}/L_{\odot}$	$R_{\text{wes}}/R_{\odot}$	$Q_{\text{wes}}(d)$	$M_{\text{wes}}/M_{\odot}$	$L_{\text{wes}}/L_{\odot}$
U Aql	7.024	5708	0.0399	6.79	59.4	3318	64.6	0.0394	8.50	3919
FF Aql	4.471	5986	0.0385	6.02	43.3	2134	43.2	0.0385	5.97	2120
							45.5	0.0382	6.88	2352
V496 Aql	6.807	5502	0.0400	6.37	57.0	2630	36.4	0.0452	2.13	1074
η Aql	7.177	5612	0.0401	6.69	59.9	3146	54.2	0.0409	5.17	2578
							69.0	0.0393	9.86	4178
V Car	6.696	5644	0.0398	6.56	57.0	2918	49.2	0.0411	4.49	2173
UX Car	3.682	6146	0.0379	5.76	37.9	1813	36.8	0.0380	5.32	1709
ER Car	7.719	5520	0.0404	6.74	62.7	3227	46.7	0.0435	3.24	1791
V Cen	5.494	5958	0.0390	6.51	50.5	2845	48.4	0.0393	5.81	2611
XX Cen	10.954	5644	0.0415	8.00	82.2	6073	58.4	0.0456	3.45	3062
RY CMa	4.678	5881	0.0386	5.99	44.5	2091	37.2	0.0400	3.77	1464
R Cru	5.826	5881	0.0393	6.55	52.5	2912	47.1	0.0401	4.94	2348
S Cru	4.690	5936	0.0386	6.07	44.7	2199	43.6	0.0388	5.66	2088
T Cru	6.733	5526	0.0399	6.38	56.6	2645	55.4	0.0401	6.03	2532
AG Cru	3.837	6253	0.0379	6.00	39.4	2104	33.2	0.0391	3.80	1491
β Dor	9.842	5532	0.0412	7.46	75.2	4684	64.6	0.0427	5.08	3458
W Gem	7.920	5760	0.0403	7.22	65.3	4156	60.0	0.0410	5.78	3506
							69.0	0.0400	8.36	4637
R Mus	7.510	5806	0.0401	7.14	63.0	3995	52.0	0.0418	4.36	2718
S Mus	9.660	6000	0.0408	8.27	77.3	6859	59.9	0.0434	4.33	4114
							61.0	0.0431	4.53	4267
S Nor	9.754	5502	0.0412	7.37	74.5	4496	53.4	0.0451	3.26	2312
BF Oph	4.068	5860	0.0383	5.63	39.9	1662	50.4	0.0376	10.96	2650
AP Pup	5.084	5840	0.0389	6.14	47.2	2289	48.8	0.0387	6.73	2451
RV Sco	6.061	5888	0.0394	6.67	54.1	3109	44.4	0.0410	4.01	2096
v636 Sco	6.797	5538	0.0399	6.43	57.1	2712	47.2	0.0417	3.96	1854
S Sge	8.382	5594	0.0406	7.09	67.1	3903	56.7	0.0422	4.62	2785
							60.0	0.0416	5.32	3119
U Sgr	6.745	5734	0.0398	6.72	57.8	3195	55.3	0.0401	5.98	2925
W Sgr	7.595	5760	0.0402	7.10	63.3	3904	57.3	0.0410	5.47	3197
X Sgr	7.012	6176	0.0397	7.55	61.8	4916	63.1	0.0396	7.99	5125
Y Sgr	5.773	5663	0.0394	6.20	51.1	2375	56.0	0.0388	7.95	2853
AP Sgr	5.058	5760	0.0389	6.01	46.7	2120	36.8	0.0410	3.27	1319
BB Sgr	6.637	5557	0.0399	6.39	56.2	2663	55.2	0.0400	6.11	2571
R TrA	3.389	6029	0.0377	5.42	35.2	1451	35.8	0.0376	5.66	1498
S TrA	6.323	5682	0.0396	6.46	54.8	2770	54.6	0.0396	6.40	2749
T Vel	4.640	5780	0.0387	5.83	43.8	1893	39.7	0.0394	4.50	1556
V Vel	4.371	6253	0.0383	6.33	43.5	2564	35.8	0.0397	3.78	1733
AH Vel	4.227	6065	0.0383	5.99	41.8	2092	44.6	0.0380	7.16	2381
							47.0	0.0378	8.30	2644
BG Vel	6.924	5676	0.0399	6.70	58.6	3157	53.3	0.0407	5.23	2609
ϵ Gem	10.154	5594	0.0413	7.67	77.4	5190	67.6	0.0426	5.43	3959
							72.4	0.0419	6.45	4541
RT Aur	3.728	6187	0.0379	5.84	38.4	1910	24.7	0.0419	1.91	791
							28.2	0.0404	2.63	1031
							28.0	0.0405	2.58	1016
U Car	38.756	5352	0.0479	12.24	200.0	29028	140.1	0.0538	5.30	14249
I Car	35.535	4937	0.0480	10.58	179.8	16985	146.9	0.0511	6.55	11343
SU Cas	1.949	6339	0.0364	4.61	23.6	797	35.2	0.0372	15.89	1770
s Cep	5.366	6113	0.0389	6.68	50.3	3123	44.6	0.0397	4.87	2457
							57.0	0.0384	9.48	4014
X Cyg	16.387	5287	0.0435	8.59	106.8	7885	91.0	0.0454	5.78	5725
							99.0	0.0443	7.10	6776
CD Cyg	17.071	5417	0.0436	9.03	111.5	9469	85.7	0.0469	4.75	5595
T Mon	27.020	5099	0.0462	9.95	150.4	13537	111.1	0.0508	4.85	7383
							120.0	0.0494	5.78	8613
							144.0	0.0467	8.92	12403
Y Oph	17.122	5754	0.0433	9.82	115.4	12908	102.6	0.0445	7.29	10210
							79.0	0.0483	3.92	6053
AW Per	6.463	5991	0.0395	7.01	57.2	3735	49.3	0.0407	4.75	2770
							70.0	0.0389	12.41	5585
RS Pup	41.388	5309	0.0484	12.41	208.6	30585	183.7	0.0501	9.09	23721
RZ Vel	20.398	5572	0.0443	10.07	128.8	14158	104.1	0.0469	5.95	9243
T Vul	4.436	6113	0.0384	6.18	43.5	2342	40.9	0.0388	5.22	2066
							52.0	0.0378	10.23	3341
SV Vul	45.100	5038	0.0493	11.93	215.3	26432	146.2	0.0564	4.89	12184
RX Aur	11.610	5801	0.0417	8.50	87.1	7601	59.0	0.0464	3.28	3488
TT Aql	13.740	5439	0.0426	8.33	95.3	7034	72.0	0.0460	4.19	4014

Table 5 and Figure 5 give the theoretical Wesselink method masses [M_w , M_{wes} in the tables or $M(W)$ on the figures] using the CKS homogeneous model Q_0 values according to the Faulkner formula. The Balona-Pel and Evans-Pel data for 35 Cepheids and the Evans-Pel data for W Gem are given first. Next come the Scarfe ζ Gem values. The Balona-DWC and Evans-DWC are intermingled (again in alphabetic order) for

14 more Cepheids. Finally, the two Cepheids not listed by Balona are given. The scatter is very much larger for these 69 cases (53 Cepheids) than implied by the better than 10% radius accuracy (or 25% mass accuracy) implied by Balona (1977). The average ratio M_w/M_T is 0.93 below a period of 10 days and 0.60 above. Thus a mass loss, or at least a mass anomaly, is implied.

TABLE 6
THEORETICAL AND WESSELINK RADIUS MASSES
INHOMOGENEOUS MODEL Q_0 VALUES

Cepheid	Period(d)	T_{eff} (K)	Q_0 (d)	$M_W/M_\odot$	$R_W/R_\odot$	$L_W/L_\odot$	$R_{\text{Wes}}/R_\odot$	Q_{Wes} (d)	$M_{\text{Wes}}/M_\odot$	$L_{\text{Wes}}/L_\odot$
U Aql	7.024	5708	0.0425	6.60	56.5	2999	64.6	0.0412	9.26	3919
FF Aql	4.471	5986	.0404	5.89	41.6	1965	43.2	.0401	6.48	2120
---	---	---	---	---	---	---	45.5	.0396	7.39	2352
V496 Aql	6.807	5502	.0427	6.19	54.0	2362	36.4	.0451	2.12	1074
η Aql	7.177	5612	.0427	6.50	56.8	2834	54.2	.0432	5.78	2578
---	---	---	---	---	---	---	69.0	.0409	10.67	4178
V Car	6.696	5644	.0424	6.38	54.2	2635	49.2	.0434	5.01	2173
UX Car	3.682	6146	.0396	5.65	36.5	1686	36.8	.0395	5.75	1709
ER Car	7.719	5520	.0432	6.54	59.4	2894	46.7	.0455	3.54	1791
V Cen	5.494	5958	.0412	6.36	48.3	2603	48.4	.0412	6.38	2611
XX Cen	10.954	5644	.0445	7.76	77.7	5426	58.4	.0485	3.90	3062
RY Cma	4.678	5881	.0407	5.85	42.6	1918	37.2	.0421	4.18	1464
R Cru	5.826	5881	.0415	6.39	50.1	2655	47.1	.0422	5.49	2348
S Cru	4.690	5936	.0407	5.93	42.9	2021	43.6	.0405	6.18	2088
T Cru	6.733	5526	.0426	6.20	53.7	2378	55.4	.0423	6.72	2532
AG Cru	3.837	6253	.0397	5.88	38.0	1958	33.2	.0411	4.20	1491
β Dor	9.842	5532	.0442	7.23	71.0	4182	64.6	.0453	5.71	3458
W Gem	7.920	5760	.0429	7.02	62.0	3750	60.0	.0433	6.46	3506
---	---	---	---	---	---	---	69.0	.0418	9.15	4637
R Mus	7.510	5806	.0426	6.95	59.9	3613	52.0	.0444	4.91	2718
S Mus	9.660	6000	.0435	8.04	73.4	6185	59.9	.0462	4.92	4114
---	---	---	---	---	---	---	61.0	.0460	5.14	4267
S Nor	9.754	5502	.0442	7.15	70.3	4012	53.4	.0474	3.59	2312
BF Oph	4.068	5860	.0403	5.50	38.3	1529	50.4	.0382	11.30	2650
AP Pup	5.084	5840	.0411	5.99	45.1	2093	48.8	.0403	7.31	2451
RV Sco	6.061	5888	.0417	6.50	51.6	2832	44.4	.0435	4.50	2096
V636 Sco	6.797	5538	.0426	6.24	54.1	2439	47.2	.0439	4.38	1854
S Sge	8.382	5594	.0434	6.89	63.6	3502	56.7	.0447	5.19	2785
---	---	---	---	---	---	---	60.0	.0441	5.96	3119
U Sgr	6.745	5734	.0423	6.54	55.0	2892	55.3	.0422	6.63	2925
W Sgr	7.595	5760	.0428	6.90	60.2	3526	57.3	.0433	6.12	3197
X Sgr	7.012	6176	.0420	7.37	59.0	4485	63.1	.0413	8.71	5125
Y Sgr	5.773	5663	.0418	6.04	48.6	2153	56.0	.0405	8.65	2853
AP Sgr	5.058	5760	.0412	5.86	44.6	1934	36.8	.0431	3.62	1319
BB Sgr	6.637	5557	0.0425	6.22	53.3	2398	55.2	0.0422	6.79	2571
R TrA	3.389	6029	0.0395	5.32	34.0	1349	35.8	0.0390	6.07	1498
S TrA	6.323	5682	.0421	6.29	52.1	2507	54.6	.0417	7.06	2749
T Vel	4.640	5780	.0408	5.69	41.9	1732	39.7	.0413	4.97	1556
V Vel	4.371	6253	.0401	6.20	41.9	2376	35.8	.0419	4.21	1733
AH Vel	4.227	6065	.0402	5.86	40.2	1934	44.6	.0392	7.63	2381
---	---	---	---	---	---	---	47.0	.0388	8.75	2644
BG Vel	6.924	5676	.0425	6.51	55.7	2851	53.3	.0430	5.83	2609
ϵ Gem	10.154	5594	.0442	7.44	73.2	4639	67.6	.0452	6.12	3959
---	---	---	---	---	---	---	72.4	.0444	7.24	4541
RT Aur	3.728	6187	.0396	5.73	37.0	1776	24.7	.0444	2.14	791
---	---	---	---	---	---	---	28.2	.0427	2.94	1031
---	---	---	---	---	---	---	28.0	.0428	2.89	1016
U Car	38.756	5352	.0520	11.81	187.3	25473	140.1	.0570	5.95	14249
I Car	35.535	4937	.0522	10.19	167.9	14817	146.9	.0538	7.27	11343
SU Cas	1.949	6339	.0375	4.55	23.1	760	35.2	.0359	14.78	1770
δ Cep	5.366	6113	.0410	6.53	48.2	2868	44.6	.0418	5.39	2457
---	---	---	---	---	---	---	57.0	.0395	10.04	4014
X Cyg	16.387	5287	.0470	8.30	100.2	6944	91.0	.0482	6.53	5725
---	---	---	---	---	---	---	99.0	.0472	8.05	6776
CD Cyg	17.071	5417	.0470	8.73	104.7	8359	85.7	.0499	5.37	5595
T Mon	27.020	5099	.0502	9.59	140.7	11836	111.1	.0533	5.33	7383
---	---	---	---	---	---	---	120.0	.0523	6.47	8613
---	---	---	---	---	---	---	144.0	.0498	10.16	12403
Y Oph	17.122	5754	.0466	9.51	108.7	11459	102.6	.0474	8.26	10210
---	---	---	---	---	---	---	79.0	.0516	4.47	6053
AW Per	6.463	5991	.0418	6.84	54.6	3404	49.3	.0430	5.31	2770
---	---	---	---	---	---	---	70.0	.0398	12.98	5585
RS Pup	41.388	5309	.0525	11.98	195.4	26832	183.7	.0534	10.33	23721
RZ Vel	20.398	5572	.0478	9.73	121.1	12503	104.1	.0500	6.78	9243
T Vul	4.436	6113	.0403	6.04	41.8	2163	40.9	.0405	5.71	2066
---	---	---	---	---	---	---	52.0	.0385	10.62	3341
SV Vul	45.100	5038	.0535	11.51	201.5	23134	146.2	.0587	5.29	12184
RX Aur	11.610	5801	.0446	8.25	82.4	6802	59.0	.0495	3.74	3488
TT Aql	13.740	5439	0.0459	8.06	89.7	6231	72.0	0.0488	4.71	4014

If the inhomogeneous models are assumed, and the larger Q_0 values used, the masses given in Table 6 and Figure 6 are obtained. In this case the average ratio of M_W/M_T is 1.02 for periods below 10 days with the same large scatter. At longer periods the mass anomaly persists with an average ratio of 0.70.

I speculate that the ratio between pulsation velocity

and radial velocity for the longer-period Cepheids could be 1.41, not the Parsons (1972) value of 1.31 used by Balona. One can see the typical effect of this factor change by looking at the two ζ Gem cases of Scarfe in Tables 5 and 6. The Table 6 M_W is $6.1 M_\odot$ when the factor is 1.31; and it is $7.2 M_\odot$, very near the theoretical mass of 7.4, when the factor is 1.41.

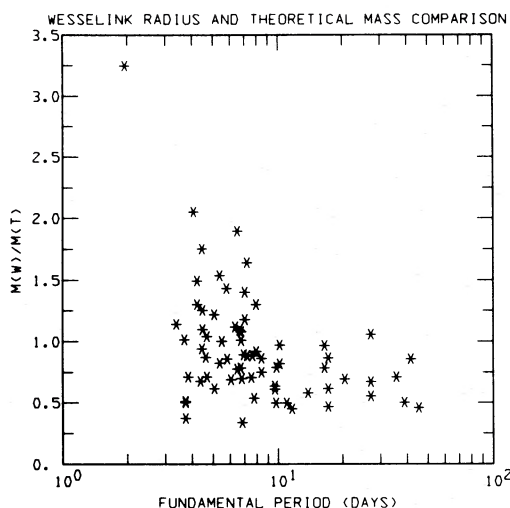


FIG. 6.—The ratio of the Wesselink radius mass (M_W) to theoretical mass (M_T) is plotted versus period for 69 cases with measured fundamental pulsation mode periods, T_e values, and Wesselink method radii. The assumed interior composition is $X = 0.70$ and $Y = 0.28$ for the evolution calculations, and the inhomogeneous envelope Q_0 values are used. M_W/M_T has mean values of 1.02 below 10 days and 0.70 above.

This factor ratio is 1.41/1.31 or 1.08 which, since $M \sim R^{2.44}/\Pi_0^{1.38}$ near $7 M_\odot$ for the inhomogeneous envelopes, implies a 20% mass increase, about as actually seen for the two ζ Gem Wesselink radius masses. The Parsons 1.31 value is well based, however.

There is a problem, nevertheless. The larger Q_0 values are not expected to apply to the more massive Cepheids, because their rapid evolution may not allow enough time for the Cox, Michaud, and Hodson (1978) Cepheid wind to enrich the convection zones. In that case, an increased velocity correction factor is largely offset, and a mass anomaly remains. As usual, this mass anomaly can be fixed by larger Y or lower Z in the theoretical mass-luminosity relation, or by a lower T_e , slightly reducing M_T .

Due to the large scatter of these Wesselink radius masses, there are some Table 6 masses lower than those that evolution theory would expect to be blue-looping. These cases, V496 Aql ($2.1 M_\odot$) and RT Aur ($2.1\text{--}2.9 M_\odot$), respectively due to Balona and both Balona and Evans, are interesting because the indi-

cated errors by Balona seem impossibly small. The largest Wesselink radius mass is $14.8 M_\odot$ for SU Cas, still within evolution theory expectations, however, but maybe too large because SU Cas is possibly an overtone pulsator.

The Barnes *et al.* (1977) radius data are used to get masses given for nine Cepheids in Table 7. These preliminary radii obviously need refinement, as also stated by authors, because one-third of the mass values here violate the expected mass range from evolution theory. The stars RT Aur (which may have a period change [Henden, private communication]) and U Sgr have too small masses, and T Mon is too massive.

The pulsation and Wesselink radius methods have a good deal in common. The first step is to get the radius. However, the pulsation mass anomaly has now disappeared, according to Pel and Lub (1978) and this paper, except for large-mass Cepheids where, as noted by Schmidt (1976), the anomaly was the worst. The anomaly for the Wesselink radius masses is possibly due to systematic errors in radii, as noted earlier by Parsons (1972). Vardya (1977) discusses the fact that selection of the proper color for the Wesselink method is important because the photosphere radius may depend on the color. Higher radius accuracy, for perhaps a dozen Cepheids, would be very helpful for this mass anomaly problem. The 30% mass anomaly for these long-period Cepheids may be due, however, to mass loss from the progenitor early B star as discussed by Sreenivasen and Wilson (1977).

VI. DISCUSSION FOR INDIVIDUAL CEPHEIDS

In this paper each Cepheid is assumed to be pulsating in the fundamental mode. The Cepheid SU Cas has been proposed by Iben and Tuggle (1972b) and most recently by Gieren (1976) to be an overtone pulsator. Other conceivable overtone pulsators are SZ Cas, AG Cru, S Mus, X Sgr, V Vel, δ Cep, and T Vul, due to their position near the blue edges proposed by Cox and Hodson (1978). Table 8 gives the masses for the overtone pulsation case for these eight Cepheids using the inhomogeneous envelope model Q_1 values.

For SU Cas and SZ Cas, in an assumed first overtone, one can evaluate their evolution, theoretical, and pulsation masses, while for SU Cas and all the others

TABLE 7
MASSES FROM BARNES *et al.* RADII

Star	Period (d)	T_{eff} (K)	Q (d)	Mass ($M_\odot$)	Radius ($R_\odot$)	Luminosity ($L_\odot$)
RT Aur.....	3.728	6187	0.0446	2.06	24.3	766
T Vul.....	4.436	6113	0.0413	4.76	38.0	1784
δ Cep.....	5.366	6113	0.0429	4.37	40.9	2067
U Sgr.....	6.745	5734	0.0464	2.76	38.8	1440
η Aql.....	7.177	5612	0.0441	4.70	49.9	2185
W Gem.....	7.920	5760	0.0462	3.80	48.2	2263
ζ Gem.....	10.154	5594	0.0451	6.30	68.4	4054
X Cyg.....	16.387	5287	0.0471	8.19	99.7	6872
T Mon.....	27.020	5099	0.0472	18.87	183.5	20141

TABLE 8
MASSES FOR POSSIBLE FIRST OVERTONE PULSATORS

Cepheid	Period(d)	$T_{\text{eff}}(K)$	$L_{\text{obs}}/L_{\odot}$	$M_{\text{ev}}/M_{\odot}$	$Q_{\text{th}}(d)$	$M_{\text{th}}/M_{\odot}$	$R_{\text{th}}/R_{\odot}$	$L_{\text{th}}/L_{\odot}$	$R_{\text{obs}}/R_{\odot}$	$Q_{\text{pul}}(d)$	$M_{\text{pul}}/M_{\odot}$	$R_{\text{wss}}/R_{\odot}$	$Q_{\text{wss}}(d)$	$M_{\text{wss}}/M_{\odot}$	$L_{\text{wss}}/L_{\odot}$
SU Cas	1.950	6339	955	4.84	0.0279	5.19	29.4	1232	25.9	0.0284	3.67	35.2	0.0271	8.44	1770
SZ Cas	13.620	5919	8511	8.77	.0297	10.89	131.8	18865	88.5	0.0340	4.31	---	---	---	---
AG Cru	3.837	6253	---	---	.0284	6.82	50.0	3378	---	---	---	33.2	.0308	2.36	1491
S Mus	9.660	6000	---	---	.0292	9.59	101.5	11823	---	---	---	59.9	.0347	2.77	4115
---	---	---	---	---	---	---	---	---	---	---	---	61.0	.0344	2.88	4267
X Sgr	7.012	6176	---	---	.0288	8.70	80.1	8259	---	---	---	63.1	.0304	4.73	5126
V Vel	4.371	6253	---	---	.0285	7.22	55.4	4156	---	---	---	35.8	.0313	2.35	1734
δ Cep	5.366	6113	---	---	.0287	7.64	64.5	5137	---	---	---	44.6	.0311	2.99	2458
---	---	---	---	---	---	---	---	---	---	---	---	57.0	.0293	5.52	4015
T Vul	4.436	6113	---	---	0.0285	7.04	55.4	3798	---	---	---	40.9	.0303	3.19	2067
---	---	---	---	---	---	---	---	---	---	---	---	52.0	0.0288	5.93	3341

one can get the theoretical and Wesselink radius masses. From equations (2), (3), and (4) using $Y = 0.28$ and $Z = 0.02$, one can find that

$$M_T \sim \Pi^{0.44} Q^{-0.44} T_e^{1.33}, \quad (7)$$

similar to the more exact equation (6), in which the variations of Q with M , R , L , and T_e are allowed. Thus theoretical masses, assuming first overtone pulsation with $Q_1 \approx 0.7 Q_0$, will increase about 15% as actually seen in comparing Table 8 with Table 4 for SU Cas and SZ Cas. Pulsation and Wesselink radius masses will decrease for the smaller first-overtone Q as $M \sim Q^2$, or about a factor of 2. The ratio of M_w/M_T should then greatly decrease, and the overtone pulsators should be easily identified.

Considering first the two calibrating Cepheids we see that, putting SU Cas and SZ Cas as overtone pulsators, their theoretical masses are then both larger than their unchanged evolutionary values, and the agreement is worse but probably still acceptable. The SU Cas and SZ Cas pulsation masses, however, get unacceptably low if they are in the first overtone. In all these possible overtone cases, except SU Cas, X Sgr, and the larger of the two observed radii of δ Cep and T Vul, the Wesselink radius mass is smaller than the theoretical mass already. A mass decrease due to the assumption of first overtone pulsation makes the Wesselink radius masses always smaller than the theoretical masses, except for SU Cas. Thus the comparison becomes very poor. I conclude that the only possible case of an overtone pulsator is SU Cas where the evolution and

theoretical masses agree reasonably well with either pulsation mode, the pulsation mass seems to favor neither mode, whereas the Wesselink radius mass favors the overtone. If SU Cas is more luminous, as Iben and Tuggle (1927b) proposed, its pulsation mass would go up, favoring the overtone pulsation. A better luminosity and Wesselink radius for SU Cas is urgently needed.

Table 9 gives the evolution, theoretical, pulsation, bump, beat, and Wesselink radius masses for six Cepheids, including the only double-mode case in this paper, V367 Sct. These are the only cases with masses from at least four methods, not counting SU Cas and several Cepheids in FSS Table 3, for which the luminosity is not directly known. Clearly, as has been pointed out for the bump masses by FSS (1972b) and for the beat masses by Cox *et al.* (1977) and Stobie (1977), these two masses are impossibly low. Surface helium enrichment for the two bump Cepheids and one beat Cepheid in Table 9, all with masses less than 8 or 9 $M_{\odot}$, seems to be the only way to reconcile their mass determinations. The Wesselink radius data still indicate somewhat lower masses, but the picture is actually seen to be acceptable in Figure 6, at least for periods less than 10 days. While the situation still needs clarification, it appears that the masses given by stellar evolution theory are reliable. There is no reason to believe that there is any disagreement between evolution and pulsation theories as proposed by many workers such as, most recently, Saio, Kobayashi, and Takeuti (1977) and Takeuti (1978).

TABLE 9
CEPHEID MASSES

Cepheid	M_{ev}	M_T	M_Q	M_{ϕ}	M_B	M_w
U Sgr*	7.1	6.5	9.5	4.0	...	6.6
S Nor*	7.4	7.2	8.2	3.8	...	3.6
V367 Sct†	6.9	7.4	5.0	...	2.3	...
T Mon	10.8	9.6	17.0	...	...	5.3–10.2
RS Pup	11.4	12.0	9.6	...	...	10.3
SV Vul	12.4	11.5	16.0	...	...	5.3

* The bump masses (M_{ϕ}) are given by FSS based on homogeneous models.

† The V367 Sct theoretical and pulsation masses are based on the van den Bergh T_e . It is not listed by Dean, Warren, and Cousins. The homogeneous model beat mass indicated is from Cox *et al.* With inhomogeneous models the V367 Sct mass is 5.2 $M_{\odot}$.

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