

STARSPOTS AS THE CAUSE OF THE INTRINSIC LIGHT VARIATIONS IN RS CANUM VENATICORUM TYPE STARS

JOEL A. EATON

Laboratory for Astronomy and Solar Physics, NASA Goddard Space Flight Center

AND

DOUGLAS S. HALL

Dyer Observatory, Vanderbilt University

Received 1978 June 5; accepted 1978 August 10

ABSTRACT

We have used a theoretical model of a binary system in which one of the components has spots on its surface to interpret the light curves of three well-observed RS CVn type systems, RS CVn, SZ Psc, and RT Lac. The first two of these are detached while the third is presently semidetached. We find that by using this spot model it is possible to fit in detail both the out-of-eclipse complications and the distorted shapes of the secondary eclipses in the light curves of these stars and that the spot model gives significantly better results than any competing hypothesis for the origin of the light curve peculiarities.

The surface activity in RS CVn type stars has many of the properties of solar activity. These objects undergo true spot cycles, cycles in which the number of spots and with it the spotted star's luminosity varies, on time scales only somewhat longer (2-3 times) than the 11 year solar cycle. Individual spots are significantly smaller than the spotted star's disk and have lifetimes of less than 1 year. They seem to extend over a sizable range of latitude, at least 15° in both hemispheres for RS CVn and RT Lac. The spots are cooler than the surrounding photosphere, by at least 400 K for all three stars and 1200 K for one, and seem to be associated with the magnetic fields inferred from the polarization of radio emissions of flares and from chromospheric line emission. The surface activity of these stars, though, appears to be more highly organized than that of the Sun. The spatial distributions of spots indicate a global organization of the magnetic field structure and the mechanism producing it. Regions of enhanced mass loss, analogous to solar coronal holes, occur around the longitude of minimum spot concentration. Variations in the rate of migration in longitude of the spot distribution indicate that the rotation rate of the sources of the field depends on the phase of the spot cycle. These properties are analogous to the persistent organization and nonuniform rotation of the Sun's magnetic sector structure, but in the RS CVn type stars the ordering encompasses the distribution of active regions as well as the background magnetic field structure.

The system RT Lac is especially interesting in that both components are peculiar. The more massive star is spotted and goes through the typical RS CVn type activity cycles, but it appears either to be covered uniformly with dark spots or, more likely, to have an extreme ultraviolet excess for its spectral type. The less massive star has a variable radius which can change by $\sim 20\%$ over an interval of a year; around 1916 this component expanded from a well-detached to a contact component!

Subject headings: stars: chromospheres — stars: eclipsing binaries — stars: variables —
 Sun: activity

I. INTRODUCTION

The problem of the complications in the light curves of the RS Canum Venaticorum type binaries (Hall 1976) has lead to three competing explanations. The earliest of these was the spot hypothesis in which a long-lived grouping of starspots is distributed unevenly on the surface of one of the components of such a system. This idea was first advanced in modern times by Kron (1947) to explain the light curve complications of AR Lac; it has been elaborated upon more recently

by Hall (1972) in discussing the full range of peculiarities and photometric cycles displayed by RS CVn. The importance of this latter work is that it adopts the spot hypothesis in the context of a well organized panoply of stellar surface activity analogous to that of the Sun. Although it provides the easiest way to fit all the light variations and gives a reasonable analogy to the solar activity cycle, the spot model for a long time failed to gain widespread acceptance. This was because a given light variation can usually be fitted by a large range of spot distributions, especially when the light

curve complications are small as was the case with AR Lac. Perhaps this point is illustrated well by the superficial similarity of the spot-produced light curve of ϵ UMa (Molnar 1975) to that of a β Lyrae type binary.

The second competing theory, first advanced by Catalano and Rodono (1967, 1969, 1974), accounts for the light curve complications through the extinction of the light of the hotter component by circumstellar material. In this model longitudinal asymmetries in the optical cross section of a ring of material revolving synchronously about the hotter star produce the well-known wave distortion of the light curve. The ring was pictured as being tilted with respect to the orbital plane; its precession was supposed to give rise to the migration of wave minimum through the light curve. This model, of course, is a logical extension of the inference that rings of low continuum cross section exist about the primaries of certain Algol type binaries (see, for example, Plavec 1968; Batten 1973). Although Oliver (1975) has discredited it by showing that the typical RS CVn type binary is not large enough to have the kind of ring required and by pointing out other such inconsistencies, the model may still have its proponents. A recent variation of this approach is the eccentric circumstellar rings of Castle (1977) for which apsidal motion would account for the migration. Actually, the concept of circumstellar material can be generalized to the form of a spatially variable extended atmosphere or circumstellar shell. This may be thought of as spots which extend above the photosphere.

The third general mechanism is pulsation of one of the components with the same period as that of the orbit. Perhaps the most promising version of this mechanism, due to Popper (1977), accounts for the light variation through a nonradial standing wave in the cooler star's outer envelope. This is coupled with an enhanced stellar wind to produce mass loss and orbital period changes. In some respects this approach is similar to the spot model; for example, both feature magnetic fields. It differs in that for the spot model the star's geometry remains constant in time while the standing-wave hypothesis must have an asymmetrical distortion of the surface which migrates about the star.

Let us now enumerate the photometric complications observed in RS CVn systems and the time scales of their occurrence. The most conspicuous property of such systems is a distortion wave in the light curve of up to a couple of tenths of a magnitude amplitude. A typical wave is nearly sinusoidal and has a period almost exactly equal to the system's orbital period. The wave tends to migrate through the light curve toward decreasing orbital phase, usually with a migration period on the order of a decade. This behavior has been documented best for RS CVn by Catalano and Rodono (1974) and for RT Lac by Hall and Haslag (1976). Migration is apparent in other systems described recently by Hall, Richardson, and Chambliss (1976), but the available data are not so complete and the nature of the migration is not so well established for these. In at least one system, SS Boo, it seems the direction of the migration has reversed and the wave

at present is migrating toward increasing orbital phase (Hall and Henry 1978).

The wave amplitudes fluctuate in cycles with time scales of decades, as has been documented best for three cases: RS CVn (Hall 1972; Catalano and Rodono 1974), RT Lac (Haslag 1976), and SS Boo (Hall and Henry 1978). A further important result of the study of these peculiar stars is that the phasing of the wave migration varies smoothly throughout the wave-amplitude cycles, with no evidence of discontinuous jumps at the times of wave-amplitude minimum. This is very similar to the behavior of the changes in position of the solar magnetic sector structure which also has migrated smoothly with respect to the solar photosphere throughout at least the last five sunspot cycles (Svalgaard and Wilcox 1975), a similarity we return to later.

Although the distortion waves are the dominant photometric peculiarity of these systems, they are by no means the only ones. Apparently random changes in the general brightness of components is known to occur with time scales at least as short as the binary period. Keller and Limber (1951) obtained photometry of RS CVn during the central phases of primary eclipse when only the K0 IV cooler component was visible. They detected changes in brightness of up to 0.014 mag between successive orbital cycles and a change of 0.075 mag over four cycles. The yellow photometry of Chisari and Lacona (1965) and that of Catalano and Rodono (1967) also revealed similar changes in level during mid-primary eclipse over periods of a few orbital cycles. The star RT Lac has exhibited even greater intrinsic variability. Haslag (1976) found evidence in the visual photometry of Merrill and the photographic photometry of Wachmann that during the interval 1929–1932 the star changed its brightness outside eclipse by ~ 0.2 mag over a time scale of ~ 100 days. Also, many RS CVn type systems appear to have ultraviolet (Hall 1976; Rhombs and Fix 1977) or infrared (Hall, Montle, and Atkins 1975; Milone 1976) excesses for their spectral types. Three of these systems have been studied recently by Rhombs and Fix (1976), who argue that the ultraviolet excesses are the result of free-free emission of a very hot ($\sim 10^6$ K) plasma.

In the present paper we will address the question of photometric complications observationally. Solutions of the light curves of three appropriate systems will be used to differentiate between the mechanisms for producing a distortion wave. We are approaching this work from the point of view of the spot hypothesis. In the next section we describe the model used to calculate light curves for a system with one spotted component. In the third section we apply that model to the systems RS CVn, SZ Psc, and RT Lac, finding that indeed spots are the best explanation of the optical light curve peculiarities. In the fourth section we identify the properties of the spots in these systems and discuss the nature of the surface activity they imply. The final section expands upon these findings and identifies ways in which they may be extended by future observations.

II. APPROACH TO LIGHT CURVE SOLUTION

In order to study the nature of the distortion waves in these systems, we have chosen to use synthetic light curves in fitting the light variation. The light curve calculations have been carried out using the Roche model geometry (cf. Kopal 1972). Basic elements used are defined by Eaton (1975). We are specifically assuming that the light curve complications are produced by *starspots*; even if they are not, our ability to fit light curves under this hypothesis will give useful information about whether we can discredit other mechanisms which mimic the effect of spots on the surface brightness distributions of the binary components. Our basic light curve calculation computer program has been modified to include starspots statistically. In this approach individual spots are assumed to be much smaller than a surface element used in the calculation, about 2×10^{-4} of the spotted star's surface area. The effect of the spots on the light curve is enhanced by putting in a lot of these small spots rather than a few great big ones. We have decided to adopt as simple a distribution of spots as possible and to incorporate into it the symmetries of the system's geometry unless they are clearly violated. The model adopted is illustrated in Figure 1. Only the component eclipsed at secondary minimum is taken to be spotted. At a given position on this component (star 2), spots having effective temperature T_{spot} cover a fraction A of the surface. Thus a fraction $(1 - A)$ of the surface is at the normal photospheric temperature for that position on the component. The spots are localized in both latitude and longitude. In latitude they are present only in the ranges $\beta_{\text{min}} \leq \beta \leq \beta_{\text{max}}$,

both above and below the equator. That is, for $|\beta| > \beta_{\text{max}}$ and $|\beta| < \beta_{\text{min}}$, $A = 0$. This restriction is designed to reflect the symmetry about the equator set up by rotation. In practice a particularly simple longitudinal dependence of A has been adopted:

$$A = B + C \left[\frac{1 + \cos(\lambda - \lambda_0)}{2} \right] + Df(\lambda_1, \Delta\lambda), \quad (1)$$

where

$$f = 1, \quad \text{if } \left(\lambda_1 - \frac{\Delta\lambda}{2} \right) \leq \lambda \leq \left(\lambda_1 + \frac{\Delta\lambda}{2} \right)$$

$$= 0, \quad \text{otherwise.}$$

Here, λ is the longitude measured from the substellar point in the direction of orbital motion. Also, λ_0 is the longitude at which the spot concentration is maximum for a purely sinusoidal wave; it is related to the phase of the minimum of the photometric wave θ_m as $\lambda_0 = 180^\circ - \theta_m$. Further, A is constrained to be in the range $0 \leq A \leq 1$. This sinusoidal distribution is adopted to reflect the basically sinusoidal shape of the distortion waves in observed light curves. The $f(\lambda_1, \Delta\lambda)$ term is included to let us add or subtract spots around longitude λ_1 to allow for the departures from a purely sinusoidal spot distribution. In this approach with the surface brightness distribution set up to model more realistically the surface of the actual star, it is possible to determine the contrast term C and the background term B by fitting the entire light curve of a well-observed system. Further, with proper color curves

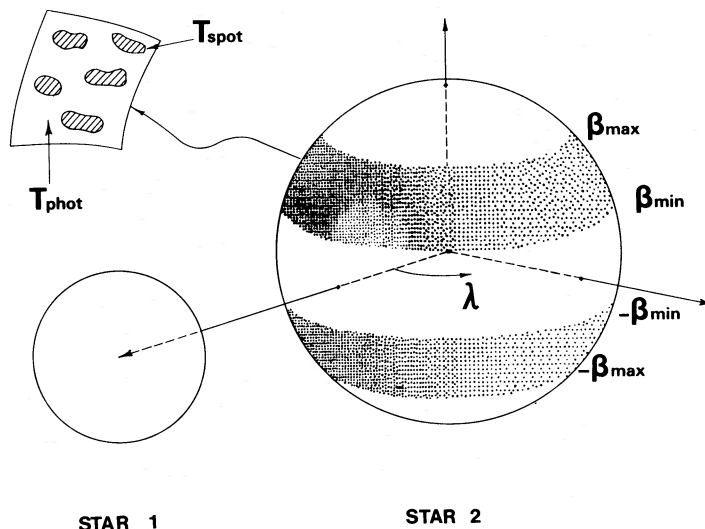


FIG. 1.—The geometry of the model spot distribution. The system is pictured in a Cartesian coordinate system centered on star 2. The three orthogonal axes point, respectively, toward star 1, in the direction of orbital motion of star 1, and perpendicular to the orbital plane. The angle λ , measured from the first of these axes in the orbital plane, is the longitude. The angle β , measured perpendicular to the orbital plane, is the latitude. The binary illustrated is seen at phase 0.39 and has the following geometrical properties: $i = 75^\circ$, $r_1 = 0.167$, and $r_2 = 0.333$. Spots in this case are restricted to lie in the latitude range $15^\circ \leq |\beta| \leq 45^\circ$ with the longitude of maximum spot concentration at $\lambda_0 = 0^\circ$. The insert at the upper left, which shows a small element of the surface, illustrates the nature of the spottedness. At a given position on star 2 a certain fraction of the surface is covered with spots of effective temperature T_{spot} . The surrounding photosphere has an effective temperature T_{phot} given by the usual laws of gravity darkening and reflection effect.

it will be possible to determine the spot temperatures relative to the underlying photosphere. To fix the temperature of the spots, all that is needed is observations shortward and longward of the Planck-maximum wavelength for the spot temperature. For bright spots, this means ultraviolet and optical observations; for dark spots, optical and near-infrared observations are required.

In calculating the theoretical light curves, emergent flux has been related to effective temperature through Johnson's (1966) calibration of the *UBVRI* system. In that calibration each spectral class was assigned an effective temperature, bolometric correction, and *UBVRI* colors. For a given spectral type, emergent flux at the star in band *i*, denoted $\mathcal{F}_i$, is given by

$$-2.5 \log \mathcal{F}_i(T_{\text{eff}}) = -10 \log T_{\text{eff}} - \text{B.C.} \\ + (m_i - V) + \text{const.} \quad (2)$$

Here, B.C. is the intrinsically negative bolometric correction and $(m_i - V)$ is the color corresponding to the ratio of the flux in the *i*-band to the flux in the *V* band. The constant in this equation is chosen to give $\mathcal{F}$ appropriate units. It is necessary to know the constant only if the absolute flux at the Earth is to be calculated; here, we can adjust the distances of the stars, so it is not important. We note that for the present work the temperatures quoted will be on Johnson's scale for main-sequence (luminosity class V) stars.

III. LIGHT CURVE SOLUTIONS

In carrying out this investigation we have chosen to concentrate on three stars: RS CVn, SZ Psc, and RT Lac. These systems are relatively well observed and illustrate well the range of problems and photometric activity characterizing the RS CVn class. The complete eclipses of all three systems make the geometrical solutions of their light curves relatively determinate.

RS CVn has been observed quite well in yellow light. For this star there exist nearly complete published optical light curves obtained during both minimum and maximum of the wave-amplitude cycle. We refer to the unfiltered blue observations of Keller and Limber for 1949 and the yellow observations of

Chisari and Lacona for 1963 and 1964 and of Catalano and Rodono for 1965 and 1966. Further, for this system the wave at maximum amplitude is nearly sinusoidal, making a solution obtained using the spot distribution of equation (1) realistic.

SZ Psc, on the other hand, is the least well observed of the three. Indications are that the distortion wave is migrating very rapidly or is changing position in the light curve sporadically, a situation which makes the combination of observations obtained more than a few months apart misleading. Partial light curves are available for 1957 and 1974 (Jakate *et al.* 1976) and for 1976 (Eaton 1977). Also, observations have recently been obtained for 1978 by Scaltriti and by Eaton. None of these light curves provides anything near complete coverage of secondary eclipse. There exist, however, observations of this star in the near-infrared which provide a measure of the temperature of the spots on the cooler component.

The third system, RT Lac, is one of the most complicated binaries known. Although it is a member of the RS CVn group as defined by Hall (1976), Popper and Ulrich (1977) have pointed out that its dimensions make it atypical in several respects, viz., a mass ratio considerably different from unity and *both* components lying above the main sequence. Important for the purpose of our paper, however, is the fact that its photometric properties are quite typical and, moreover, that it has been observed extensively enough for these properties to have been defined especially well.

Table 1 identifies the photometry used to study each system. The light curve fitting discussed here was done by adjusting the geometrical and photometric parameters until a good fit to the entire light curve was obtained. The quality of a fit was judged by comparing the computed and observed light curves graphically. This is the procedure which some light-curve-solution practitioners would refer to as subjective parameter adjustment, as no numerical procedure was used to decide what parameter changes to make. In practice, though, this approach is expected to give adequate accuracy for the purposes of the present study. This is because the three systems have highly determinate light curves which are well defined at critical phases. Also, the dependence of the light curves on the various

TABLE 1
OBSERVATIONAL MATERIAL USED IN THIS STUDY

Star	Light Curve	Epoch	Colors	Source
RS CVn.....	1964	1964.4	Yellow	Chisari and Lacona 1965
	1949	1949.3	Blue*	Keller and Limber 1951
SZ Psc.....	1974	1974.8	<i>UBVRI</i>	Jakate <i>et al.</i> 1976
	1976	1976.9	<i>VI</i> †	Eaton 1977
RT Lac.....	1965	1965.8	<i>UBV</i>	Hall and Milone‡
	1968-1969	1969.0	<i>UBV</i>	Hall and Milone‡

* Unfiltered 1P21.

† The original data, presented on the instrumental system, has been transformed to the standard system of Iriarte *et al.* 1965.

‡ Photometry, chiefly by Hall and Milone, has been combined into normal points by Hall and published by Milone 1977.

elements is phase dependent in a way that allows for a good separation of the effects of changes in geometry and changes in surface brightness distribution. In adjusting the elements to obtain a fit, we were guided by calculations of the effect on the light curve of changing individual elements. Hence, the solutions result from a differential corrections procedure; in it we were able to incorporate realistic constraints on the ranges of the elements.

The geometrical elements are based primarily on fitting the eclipse contact angles, the shape of primary eclipse, and, to a lesser degree, the temperature difference between the components and the photometric ellipticity. Thus, the geometry is determined mainly from primary eclipse and from the relative eclipse depths (with the effects of spots allowed for). The surface brightness distributions were determined from fitting the out-of-eclipse light variation and the shape of secondary eclipse. These remarks apply specifically to the work done for RS CVn and SZ Psc. Since the light variation of RT Lac is more nearly continuous, slightly different criteria were used in obtaining the solution for it. They will be discussed in a later subsection.

The quoted uncertainties for the elements are our estimates of just how much each element can be adjusted without violating some restriction imposed by the light curve. This is strictly true for the geometrical elements, but the uncertainties of the elements describing the spot distributions require some further explanation. Obviously, the elements $\beta_{\min}\beta_{\max}$ and C —similarly $\Delta\lambda$ and D —are not independent. The uncertainties in C and D for RS CVn in 1964 are the amounts that these quantities can be varied while maintaining reasonable ranges in β and $\Delta\lambda$. However, the uncertainties for RS CVn in 1949 and for RT Lac represent the range over which a fit can be obtained with the adopted ranges in β .

Results of the light curve fitting are given in the following three subsections.

a) The Solution for RS Canum Venaticorum

We have chosen to study the light curves for two epochs, 1964 and 1949, concentrating on 1964. The light curve for 1964 is defined by the yellow observations of Chisari and Lacona (1965). This is an important epoch in two respects. First, the light curve for it is essentially complete. Examination of the individual observations, which have been published in tabular form, allows one to conclude that Chisari and Lacona's normal points provide a realistic representation of the light variation and that the shape of secondary eclipse is reliably defined. Second, the photometric wave is of relatively large amplitude and happens to be well placed in the light curve for discriminating between different mechanisms of producing such waves. In the 1964 light curve the wave minimum is located near phase 0.34, implying that spots should lead to a considerable limb-to-limb variation in the cooler star's surface brightness during secondary eclipse. This would lead to a large asymmetry of

TABLE 2
LIGHT CURVE SOLUTION FOR RS CANUM VENATICORUM

Spot-independent parameters:

$$\begin{aligned} q &= 1.0^* \\ i &= 87^\circ 0 \pm 0^\circ 2 \\ r_1 &= 0.117 \pm 0.003, r_2 = 0.235 \pm 0.003 \\ g_1 &= g_2 = 0.32^* \\ x_1 &= 0.5^*, x_2 = 0.7^* \\ T_1 &= 6500 \text{ K}^* \text{ (from the spectral type)} \end{aligned}$$

Properties of the spotted star in 1964:

$$\begin{aligned} 0^\circ \pm 10^\circ \leq |\beta| &\leq 40^\circ \pm 10^\circ \\ \lambda_0 &= 58^\circ \pm 10^\circ, B = 0.0 \pm 0.1, C = 0.62 \pm 0.15 \\ \lambda_1 &= 155^\circ \pm 10^\circ, \Delta\lambda = 40^\circ, D = -0.3 (+0.15, -0.0)^\dagger \\ T_2 &= 4685 \text{ K} \pm 25 \text{ K}, T_{\text{spot}} = 0 \text{ K}^* (\leq 4200 \text{ K}) \end{aligned}$$

Parameters changed to fit the 1949 light curve:

$$\begin{aligned} \lambda_0 &= 240^\circ, B = 0.0 \pm 0.1, C = 0.2 \pm 0.05 \\ \lambda_1 &= 325^\circ, \Delta\lambda = 40^\circ, D = 0.3 \pm 0.1 \end{aligned}$$

* Adopted values.

† This value is subject to the restriction that $A \leq 1$ at each longitude.

secondary eclipse, something which would not be predicted by the other two hypotheses.

An initial fit to the 1964 light curve was obtained for a purely sinusoidal spot distribution. This gave an adequate representation of the amplitude of the wave and the depths and shapes of the eclipses. Slight deviations of the fit near phase 0.07 indicated that a slightly modified spot distribution was required. Removal of some of the spots centered on the meridian at that phase resulted in a satisfactory fit. The solution adopted is given in Table 2, and the resulting light curve is shown in Figure 2. Several conclusions about the spot distribution may be drawn.

First, the spots must extend over a large range in latitude. If $C = 1$, it is necessary that the spots extend over at least 17° of latitude in each hemisphere. We note that it is not possible to fit the light curve with a single spot confined to the polar regions, a possibility we explored because Oskanyan *et al.* (1977) concluded that spots at high latitudes could explain the light variations of BY Dra. This is a consequence of the large ($\sim 87^\circ$) orbital inclination and large (~ 0.2 mag) amplitude of the wave in 1964. In fact, to fit the out-of-eclipse variation with polar dark spots ($C = 1, T_{\text{spot}} = 0 \text{ K}$), the spots would need to extend from the poles downward to latitude $\beta_{\min} = 22^\circ$. This is ruled out by both the shape and depth of the 1964 secondary eclipse. The unspotted equatorial region would give too little longitudinal surface brightness variation to fit the distortion of secondary eclipse; its high surface brightness would lead to a secondary eclipse too deep by about 0.06 mag. In our adopted solution practically none of the area of the unspotted polar regions is eclipsed. This leads to the large displacement of secondary eclipse required by the observations. We cannot conclude that the spots extend all the way to the equator. However, if they do not do so, an increase in the contrast term C would be required. Since C must be less than unity, it follows that there are limits to how much $\beta_{\min}$ can be adjusted.

With the longitudinal distribution taken as sinusoidal, the contrast term C follows directly from the

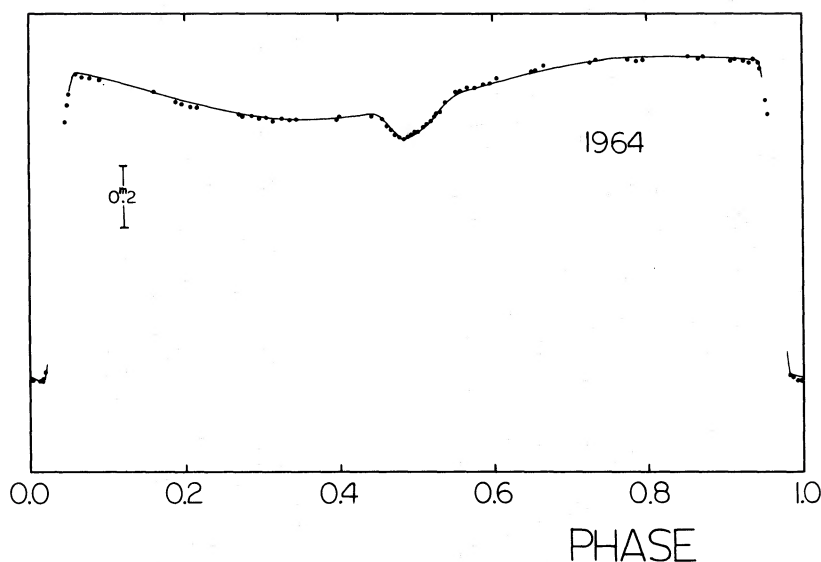


FIG. 2.—The fit of the light curve of RS CVn for 1964. The yellow normal points of Chisari and Lacona are plotted as dots in this graph of magnitude versus orbital phase. The solid curve is the theoretical representation of the 1964 elements of the solution given in Table 2.

amplitude of the distortion wave. The background term B derives from adjusting B , C , and T_2 appropriately to fit the depth of secondary eclipse. The 1964 secondary is consistent with no background spots. Actually, the quantity B as we have defined it is a rather special background. There could also be a background of spots covering the star uniformly; at any given wavelength this is equivalent to changing the temperature of star 2 and so cannot be ruled out or proved applicable by observations in one passband.

In this work we have taken the spots to be cooler than the surrounding photosphere and, further, to emit no light. The first of these assumptions is difficult to test for a light curve at one wavelength only. It is possible, however, to do so for the 1964 RS CVn data by studying the eclipse depths, provided only that we exclude the possibility of *both* dark and bright spots. Bright spots give too great a depth to the secondary eclipse and may thus be discounted. Similarly, an upper limit may be placed on the temperature of the dark spots. If $C = 1$ and the spots extend from pole to pole, a spot temperature lower than 4200 K is required to give the observed wave amplitude. By restricting the latitude range of the spots to $-40^\circ \leq \beta \leq +40^\circ$ as we have found reasonable, spots no hotter than 4070 K can be tolerated.

Let us now compare the solution derived for the spot hypothesis with those for other possible mechanisms for producing the distortion wave. The significant discriminants for this are the system's color variations and the details of secondary eclipse. One of the competing hypotheses involves pulsation of one of the components. This will lead to changes in the shape and temperature of the variable component. The dashed line in Figure 3 illustrates the result of changing the surface brightness—hence the relative luminosity—of star 2 enough with phase to give the observed out-of-

eclipse variation. This is just a first approximation to a radial pulsation in which the radius amplitude is small while the temperature variations are large. Clearly, the calculated eclipse fits badly in both shape and depth. In a nearly adiabatic pulsation star 2 would be smallest when it is hottest, leading to an increase in the ratio of radii through secondary eclipse. However, such an effect would tend to displace the deeper parts of secondary eclipse to *greater* phase, making the agreement between the radial pulsation light curve and the observations even worse than that obtained with the dashed line in Figure 3.

A further problem with pulsations is that the periods of the observed light variations are much longer than expected for stars with the dimensions of the components of RS CVn type systems. A typical value of the pulsation constant $Q = P(\bar{\rho}/\bar{\rho}_\odot)^{1/2}$ for β Cephei stars is $Q \leq 0.04$ day (Lesh and Aizenman 1978), for δ Scuti variables it is $Q \approx 0.04$ day (Allen 1973), and for the long-period variables it is $Q \approx 0.056$ day (Allen 1973). In fact, the largest Q listed by Allen is 0.16 day for W Virginis variables. For the cooler component of RS CVn this range of Q leads to pulsation periods only about one-quarter day to one day. Now it might be argued that since RS CVn type waves have essentially the same periods as their binary orbits, they are produced by tidally driven oscillations. However, this seems to be inconsistent with the lack of a stable phase relation between the orbit and the wave.

So far we have discussed radial pulsations. The possibility of nonradial pulsations, i.e., standing wave patterns which rotate at close to the rate of orbital motion, seems to be a more reasonable approach. The most promising of such pulsations seems to be the nonradial g -modes (see, e.g., Van Horn 1978). Given enough radial nodes, these modes can have long enough periods to satisfy the observations. However,

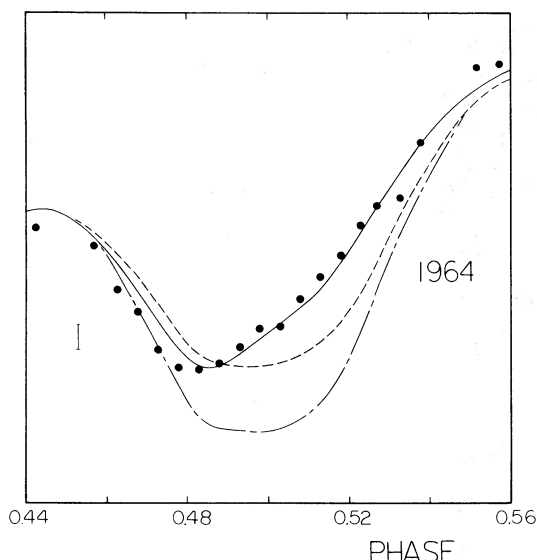


FIG. 3.—The details of the secondary eclipse of RS CVn in 1964. Here we see in detail the quality of the fit obtained for the phase range 0.44–0.56. This range of phase illustrates well the differences between light curves computed for the different mechanisms of producing the out-of-eclipse distortion wave. All three of the mechanisms considered give the same wave amplitude, but they differ significantly in their ability to fit the details of secondary eclipse. The dots are the normal points of Chisari and Lacona as in Fig. 2; a bar 0.01 mag high is given on the left side of the illustration to indicate the magnitude scale. The solid curve, computed for the spot hypothesis, represents the 1964 elements of Table 2. The other two light curves are computed for models in which star 2 is not spotted. For the dashed curve the brightness of star 2, L_2 , has been made to vary with phase in such a way as to reproduce the out-of-eclipse distortion wave. It is apparent that this curve does not fit the shape of secondary eclipse adequately. The dash-dotted curve was computed for a constant L_2 but variable L_1 : Star 1 was made to vary enough in brightness to give the distortion wave. Clearly, the details of secondary eclipse are not fit by this hypothesis either.

it is difficult to get the required amplitudes of light variation without considerable distortion of the star or large temperature variations over its surface. The absence of any anomalies in the times of eclipse contacts for the 1963–1966 light curves required the distortion of the star to be low, as indeed would be expected for a large number of radial nodes. Catalano and Rodono (1969) and later Rhombs and Fix (1976) found that for RS CVn the wave has the same color as the cooler star; but if pulsations gave large temperature variations as required by the large amplitude, the wave should have been much *bluer* than the cooler star. Also, the very large amplitude of the light variation appears difficult to explain with nonradial pulsations. In the β Cephei variables (Lesh and Aizenman 1978), some of which *may* pulsate nonradially, the amplitudes are rarely greater than 0.1 mag while for the white dwarfs (Van Horn 1978) they are generally 0.3 mag or less. Yet for RS CVn, if the cooler star is a nonradial pulsator, the amplitude must be 0.6 mag.

Similarly, the circumstellar-material hypothesis cannot account for the observed light variations. If the

wave is indeed produced by a ring of material absorbing the light of the hotter star, secondary eclipse has the shape given by the dot-dashed curve in Figure 3. This curve is obviously too deep and not displaced enough in phase. The calculated curve is again an approximation. In producing it we have made the relative luminosity of the hotter star, L_1 , vary in phase in just the right way to give the out-of-eclipse wave in the light curve. That is, we have ignored any eclipses of the cooler star by the ring. Now in reality any circumstellar material around the hotter star would tend to eclipse the cooler star, and including this effect in the calculations would make the computed secondary eclipse deeper still and would make its duration longer than that of the primary. Neither of these effects fits the observations.

Although the rather rudimentary circumstellar material model fails badly, there is a slightly different approach which is worth more consideration. This is circumstellar material around the cooler star to which we alluded in the Introduction. Such a model can be considered a spot model in which the spots are located above the photosphere. But, again, the observations rule it out. Material above the photosphere would indeed give a variation in surface brightness across the star's disk. But parallax would reduce its effect in giving a distorted secondary eclipse. The light curve would again look more like the dashed curve in Figure 3. Also, material above the photosphere of the cooler star would lead to varying asymmetries of primary eclipse which are not observed. So, we conclude that the characteristic light curve complications of RS CVn are best explained by *spots*—regions of low-brightness on the surface of the cooler component.

The timing of the wave-amplitude cycle has been defined by Hall (1972). The blue observations of Keller and Limber (1951) for 1949 fall near an amplitude minimum in this cycle. Within the framework of the spot model an amplitude minimum has three possible interpretations: (1) The number of spots reached a minimum, as one might anticipate in analogy with the solar sunspot cycle. (2) The spots became distributed more uniformly in longitude. (3) Enough extra spots formed at longitudes of low spot concentration to eliminate the longitude dependence. The first interpretation would predict an increase in the mean light level outside eclipse at wave-amplitude minimum, the second would predict no change in mean level, while the third would predict a decrease in level.

Koch, Plavec, and Wood (1970) estimated that the 1949 observations correspond to an effective wavelength of $0.45 \mu\text{m}$. We have obtained a fit to the 1949 light curve using a blue-wavelength calibration (B of the UBV system), and this indicates that the luminosity changed between 1949 and 1964. The change was in the sense that the photospheric effective temperature remained constant while the area emitting at that effective temperature increased with a decrease in spottedness. This is just the first of our three possible interpretations of wave-amplitude minimum identified above. At this epoch the wave was rather complicated. The parameters describing it are listed at the end of Table 2. The

dominant sinusoidal component was contaminated by a region of intense extra spottedness.

But the light curve of Keller and Limber is not defined well enough in effective wavelength for these results to be definitive. Although we can say that there were definitely fewer organized spots, i.e., those concentrated between the limiting latitudes of $\pm 40^\circ$, in 1949, we cannot conclude with any certainty that the effective temperature of the surrounding photosphere was the same as in 1964. It would have been possible to determine conclusively whether there was a luminosity change associated with the wave-amplitude cycle had the wavelength dependence of the detector sensitivity been restricted sufficiently to use the hotter star as the comparison star for the cooler star. However, since the observations were made with an unfiltered 1P21 photomultiplier, the great bandwidth of the detector and its uncertainty make it very difficult to define reliably an effective wavelength to use in this approach.

b) The Solution for SZ Piscium

This star is one of the brighter members of the RS CVn class, but as yet relatively few observations of it have been made. Most of the data presently available for SZ Psc were presented in a paper by Jakate *et al.* (1976). Included there were both spectroscopy and photometry. The system was found to consist of a pair of stars with masses $1.38 M_\odot$ and $1.87 M_\odot$ having spectral types F8 V and K1 IV, respectively. Photometry was presented for two epochs, 1957 and 1974, much of the 1974 data being *UBVRI* observations obtained by Fernie. In this photometry the system appeared to be randomly variable by up to 0.05 mag. This, however, may have resulted from comparing data obtained at epochs too far apart in time. Observations for 1976 by Eaton (1977) seemed to repeat to better than 0.01 mag. These observations, although incomplete, gave sufficient coverage out of eclipse to indicate that the wave-migration rate for this system is extremely high. For the 1974.8 data of Fernie the phase of wave maximum was 0.45; for the 1976.9 data of

Eaton it was 0.95; in unpublished *uvbyI* photometry obtained by Eaton in September 1977 it was 0.65; and in the excellent *B* and *V* light curves obtained by Scaltriti (1978) between October 1977 and January 1978 it was 0.65. For each of these values an estimated upper limit to the uncertainty is ± 0.10 . These phases are consistent with a migration period of 3.9 years. No wonder the star's light curves for a given season exhibited large scatter!

We have fitted the 1974 yellow observations of Fernie to give us the geometry of the system and fitted the 1974 and 1976 (*V* - *I*) color curves to determine the temperature of the starspots. The solution is given in Table 3 and is represented in Figures 4 and 5. We note that the solution is rather uncertain. This is because secondary eclipse has yet to be observed adequately. However, the ratio of radii must be small to account for the large difference in relative brightness in yellow light. Also, we find that the system is detached, with the radius of the cooler star only 75% the radius of its Roche lobe. Our value of $r_2 = 0.30$ for the relative radius was determined basically by varying the ratio of radii with the angles of internal and external contact kept fixed. We settled on the ratio of radii which fitted the photometric ellipticity and gave reasonable photospheric effective temperatures for the spectral types of the components. The errors we quote give the amount by which this solution can be varied within observational constraints.

Let us look at the two epochs separately. In 1974 the distortion wave had relatively low amplitude. However, the general light level seemed to be lower than in 1976. Some of the difference between 1974 and 1976 may well be the result of errors in the transformation of the 1976 data. These observations have been transformed to *V* and (*V* - *I*) using the relations $\Delta V = \Delta v - 0.033\Delta(v - i)$ and $\Delta(V - I) = 1.24\Delta(v - i)$ which were derived from observations of bright standard stars on one night. They are uncertain chiefly because of uncertainties in the correction for coincident pulses used to reduce the standard star observations. The difference in level can, to within the uncertainty of 0.02 mag, be accounted for by a general background

TABLE 3
LIGHT CURVE SOLUTION FOR SZ PISCUM

$q = 1.34^*$ $i = 77.75 \pm 1.0$ $r_1 = 0.08 \pm 0.01, r_2 = 0.30 \pm 0.02$ $x_{1V} = 0.5^*, x_{1I} = 0.3^*$ $x_{2V} = 0.8^*, x_{2I} = 0.5^*$ $g_1 = g_2 = 0.32^*$ $A_{1\text{Bol}} = A_{2\text{Bol}} = 0.5^*$ $T_1 = 6100 \text{ K}^*, T_2 = 4700 \text{ K} \pm 250 \text{ K}$							
Properties of the Spots							
Epoch	T_{spot}	λ_0	<i>B</i>	<i>C</i>	λ_1	$\Delta\lambda$	<i>D</i>
1974.....	3500 K*	200°	0.15	0.08	180°	40°	0.08
1976.....	3500 K $\pm$ 400 K	35°	0.00	0.35			

* Adopted values.

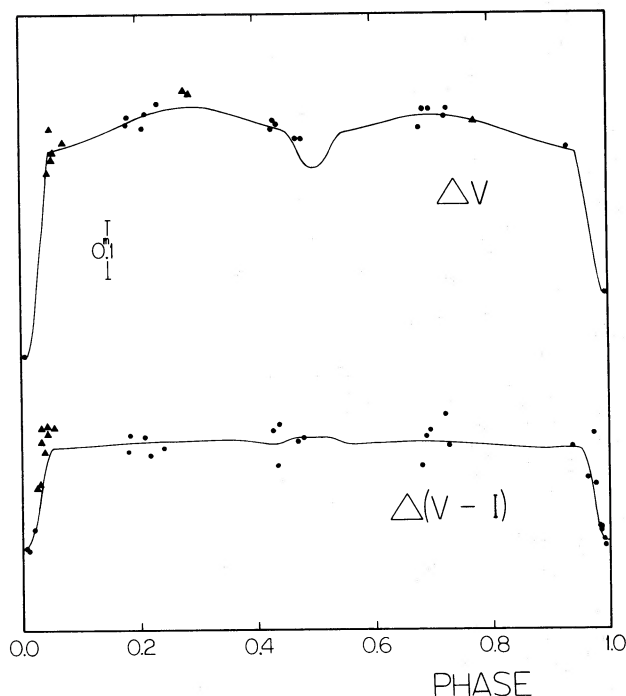


FIG. 4.—The fit of the 1974 observations of SZ Psc. Plotted as symbols are the VI observations of Fernie. Observations obtained before JD 2,442,312 are given as dots, those after JD 2,442,339 as triangles. The solid curves represent the 1974 elements given in Table 3. We note that the observations for this epoch lack the large photometric ellipticity that would have been present if the cooler component of SZ Psc were a contact component.

of spots for 1964 which was not there in either of the other two years.

The 1976 observations of Eaton (1977) suffer from poor phase coverage, yet they are useful in letting us estimate spot temperatures. Unfortunately, for this epoch there are no observations on the shoulders of primary eclipse to be used in establishing the eclipse depth. The $(V - I)$ color of the wave as inferred from this photometry provides limits on the temperature of the spots. For this diagnostic color the fact that the wave is *blue* means that the spots must be hotter than 2600 K ($T_2 - T_{\text{spot}} \leq 2100$ K). In the present solution the out-of-eclipse fit yields a spot temperature near 3500 K ($T_2 - T_{\text{spot}} = 1200$ K). However, this temperature makes the system a bit too red at mid-primary eclipse, as is apparent in Figure 5. Making the spots hotter improves the situation slightly, but it is possible that the discrepancy is caused merely by an error in the transformation of the data. If this is the case, the color of the wave should be bluer, requiring slightly hotter spots to fit it. The spots would need to be about 3700 K, only a few hundred degrees hotter than in the previous estimate. To recapitulate, the data fix the spot temperature at around 1200 K cooler than the temperature of the unspotted photosphere, and we estimate that this value is certain to at least 400 K. Clearly, $BVRI$ light curves with complete phase coverage would fix the spot temperature even more reliably.

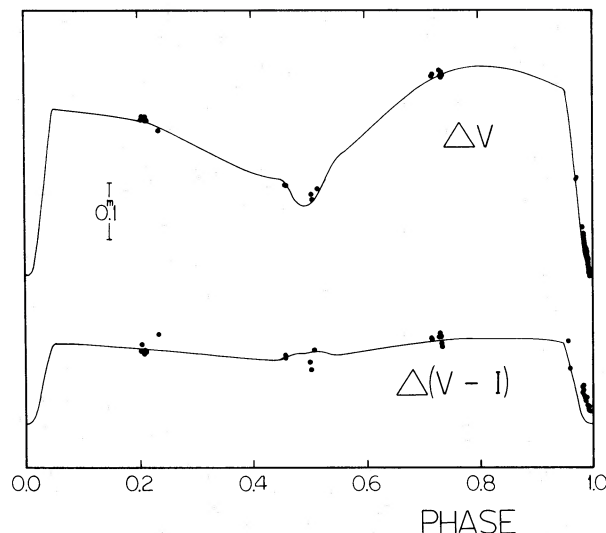


FIG. 5.—The partial light curve of SZ Psc for 1976. The observations of Eaton, transformed to VI on the Johnson $UBVRI$ system, are plotted as dots. The solid curves represent the 1976 elements of Table 3. Note that the wave changed significantly between 1974 and this epoch. The amplitude increased, and the phase of maximum brightness changed from 0.45 to 0.95. Also note that the computed $(V - I)$ color is too red during primary eclipse. This may be the result of an error in the transformation of the data to the standard system as discussed in the text.

c) The Solution for RT Lacertae

The binary RT Lac is one of the truly remarkable variable stars of any class. Not only does it exhibit the full range of light curve variability associated with the RS CVn group, but it also shows evidence of having a component with changing relative radius and variable ellipticity. Knowledge of its light variations rests upon an extensive body of data. Reasonably complete photoelectric light curves, based on standardized UBV observations by Hall and Milone (Milone 1977), exist for the two epochs 1965 and 1968–1969. Over 60 light curves, some going back to 1890, allow the evolution of the star's light variation to be studied over an interval of 85 years. All of this material has been systematically studied in a thesis by Haslag (1976) which was summarized by Hall and Haslag (1976) and part of which is presented graphically by Shore and Hall (1978).

The system has shown remarkable changes in its light curve over the years. Perhaps the most striking of these was the significant change in the ellipticity which occurred around 1916–1920. Prior to that epoch the A_2 coefficient in the Fourier decomposition of the out-of-eclipse light was smaller, i.e., was closer to zero, than about -0.03 . Since then the A_2 coefficient has remained between -0.10 and -0.07 with the exception of a brief time around 1929 when it was again small. This is behavior reminiscent of the W UMa type variable SW Lac which also has had large ellipticity variations over relatively short intervals of time (Bookmeyer 1965). Such rapid and extreme changes would appear to represent changes in the distribution

of surface brightness on one of the components—changes in what is commonly called gravity darkening. This inference derives from the consideration that changes in the structure of a component's envelope are expected to occur on the envelope's thermal time scale, orders of magnitude longer than the time scales of the light curve variations. We note, though, that light curve changes in W UMa type systems have at various times been interpreted as changes in system geometry (see the solutions for SW Lac by Ruciński 1973 and by Lucy 1973). In the case of RT Lac, radius variations are indicated by changes in the durations of the eclipses. Haslag found that at epochs for which the ellipticity was low—essentially the time prior to 1916—both eclipses had similar duration ($\phi_e \approx 0.07$). On the other hand, he found that, at epochs of high ellipticity, secondary eclipse seemed to have had a slightly longer duration ($\phi_e \approx 0.08$) while primary eclipse may have had a still longer duration ($\phi_e \approx 0.10$). As we shall see later on, an increase in eclipse duration of this magnitude requires an increase of at least 20% in one of the relative radii.

This star has been followed through nine wave-migration cycles (~ 9.5 yr per cycle) and through two complete wave-amplitude cycles (~ 30 yr per cycle). Remarkably, the migration in phase of the minimum of the wave does not seem to have been affected by the changes in ellipticity. Although the rate of migration is not constant and is correlated with the wave-amplitude cycle, a smooth migration continued through two epochs of nearly vanishingly small wave amplitude. In other words, after epochs of minimum wave amplitude the star apparently reestablished the wave minimum at a phase which could be predicted successfully from the phasing of the wave preceding the amplitude minimum.

Information on the spectrum of this system is important in that it is inconsistent with the colors of the components. Joy (1931) found the star to be a double-lined spectroscopic binary and derived masses $1.0 M_\odot$ and $1.9 M_\odot$, respectively, for the components of earlier and later spectral type. A redetermination of the orbit by Popper, as reported by Popper and Ulrich (1977), gave masses $0.6 M_\odot$ and $1.5 M_\odot$. Milone (1977) has classified the spectra of the components G9 IV and K1 IV. We note that the more massive component is the later in spectral type. It is difficult to believe that this conclusion could be a result of a misclassification, yet the colors of the components contradict it. The more massive star is decidedly bluer in both $(B - V)$ and $(U - B)$; the system becomes bluer at primary minimum when the more massive star is in front and redder at secondary minimum when the more massive star is in eclipse. In visual light, secondary eclipse (the shallower one) is the eclipse of the more massive star, making that star the component of lower mean surface brightness. So the more massive star has a later spectral type and lower surface brightness but a bluer color than its companion.

This discrepancy between surface brightness and color can be reconciled by making a large fraction of the more massive star's disk dark. One way to accomplish this is to concentrate the light to the center of the

disk as would happen with the extreme limb darkening of a highly extended atmosphere (Cassinelli and Haisch 1974). But this possibility is excluded by the shapes and durations of the eclipses. Working with *UBV* light curves rectified for both the usual proximity effects and the wave complication (see Milone 1977 for the procedures used to handle the observations), we find that the ratio of radii must be close to unity and that the luminous portion of the more massive star's disk cannot be significantly smaller than the dark portion. This means that the dark area must be spread out in a way that does not concentrate the bright area to the disk center; some sort of dark spots are indicated. Now there is another possible solution to this dilemma to which the spot solution may eventually converge. It is possible that what we are dealing with is a peculiar energy distribution, an ultraviolet excess which is much more extreme than that in the typical RS CVn type binary. It will be difficult to differentiate between this possibility and a model in which the more massive star is relatively hotter but covered uniformly with dark spots.

In applying the synthetic light curve method to RT Lac we have adopted the geometrical elements of Milone's (1977) Russell model solution as an initial approximation. For this work the more massive, spotted star is star 2 while the less massive component is star 1. We are assuming for simplicity that only star 2 is spotted. This seems to be a realistic approach since the wave is associated with star 2 (Milone 1977) and since the chromospheric Ca II emission of star 1 is significantly weaker than that of star 2 (Oliver 1974). We admit, however, that the system was about 5% fainter in 1968–1969 than in 1965 at mid-secondary eclipse when star 1 was contributing most of the light. This seems to require some change of the brightness of star 1, which one might associate with surface activity. We adopt the mass ratio $q = m_2/m_1 = 2.5$ following Popper's radial-velocity work. The assumed spectral type G9 V for star 1 leads to the effective temperature $T_1 = 5300$ K. The difference in the $(U - V)$ color between mid-primary and mid-secondary eclipses fixes the temperature difference at $(T_2 - T_1) = 415 \text{ K} \pm 40 \text{ K}$.

The variations in ellipticity give valuable constraints on the nature of the solution. There are two ways to account for the changes from low to high ellipticity. The variations may all be associated with star 2 or they may be caused chiefly by star 1. The first of these two possibilities is intellectually somewhat less than satisfying since we have no convincing mechanism for producing the enhanced photometric ellipticity after 1916. We note, though, that this approach would make the system similar to LY Aur (Eaton 1978) for which the expected light curve solution fails badly in reproducing the observed A_2 coefficient. In the case of RT Lac, prior to 1916 the photometric ellipticity would have been contributed chiefly by star 1. With the mass and light ratios of this system the low observed A_2 coefficient gives the restriction $r_1 \leq 0.25$, making the system detached prior to 1916. On the other hand, it is possible that the changes in ellipticity

were caused by a change in the relative radius of star 1. An apparent increase in the period occurred at about the same time as the ellipticity change of 1916. This would be explained naturally by a transfer of mass from the less to the more massive component (star 1 to star 2), a situation requiring star 1 to be a contact component with $r_1 \approx 0.29$. We note that RT Lac is the only RS CVn binary showing a long-term period increase in its ($O - C$) curve. The increase in eclipse durations at the time of the ellipticity increase indicates the radius of one of the components increased. The fact that the ellipticity change did not seem to affect the wave migration argues that it was the radius of star 1, not that of star 2, which changed. The change in eclipse durations ($\Delta\phi_e \approx 0.01$) can be accounted for by a difference of $\sim 20\%$ in the relative radius of star 1. This is consistent with the change, $r_1 \leq 0.25$ to $r_1 \approx 0.29$, needed to give the increases in ellipticity and period. So the ellipticity and eclipse-duration changes can be satisfied by either (1) a model in which the system is always detached but for which the radius and gravity darkening of star 2 are variable or, more plausibly, (2) a model in which the radius of star 2 remains constant but in which the radius of star 1 is variable over surprisingly short intervals of time.

Similarly, the recent V -band light curves can be satisfied by either a detached solution in which the gravity darkening of star 2 is abnormally large or by a semi-detached solution in which star 1 is the contact component. Both of these possibilities give computed light curves which are unable to reproduce some observed property of the system. Neither can account directly for the discrepancy between the spectral types and the color difference, but the solution in which the system is presently detached has far more serious limitations. In it the large gravity darkening of star 2 needed to give the V -band ellipticity makes the ellipticity term in the color curves considerably larger than observed. This in itself might not be too objectionable. A physically unexplained large gravity darkening having been postulated, there is no reason not to change further the wavelength dependence of the gravity-darkening law. However, there is another, more serious, objection concerning the geometrical elements needed to fit the eclipses. In order to reproduce the eclipse depths and durations, we found it necessary to make the relative radius of star 1 larger than its pre-1916 upper limit. On the other hand, the semidetached solution with more nearly normal gravity darkening fits the visual light curve satisfactorily, gives a much better reproduction of the color variation, and is consistent with the light curve changes. This solution is given in Table 4 and represented in Figure 6. The geometrical elements were derived from fitting the 1968–1969 V light curve only. The fit of the ($U - V$) color curve was again not ideal in that the computed distortion wave was not quite blue enough preceeding secondary eclipse, but it was significantly better than the fit for the detached case.

Although the initial solution for RT Lac was derived from the 1968–1969 light curve, an adequate fit to the 1965 observations has been obtained with appropriate

TABLE 4
LIGHT CURVE SOLUTION FOR RT LACERTAE

$q = 2.5^*$ $i = 89^\circ 0 \pm 0^\circ 5$ $r_1 = 0.295 \pm 0.01, r_2 = 0.270 \pm 0.01$ $x_{1V} = 0.82^*, x_{2V} = 0.7^*$ $0^\circ \leq \beta \leq 90^\circ$ $T_{\text{spot}} = 0 \text{ K}^*$	
Elements for 1968–1969	
$g_1 = g_2 = 0.4 \pm 0.3$ $T_1 = 5300 \text{ K}^*, T_2 = 5715 \text{ K} \pm 50 \text{ K}$ $\lambda_0 = 180^\circ \pm 10^\circ, B = 0.4 \pm 0.05, C = 0.15 \pm 0.03$ $\lambda_1 = 170^\circ \pm 10^\circ, \Delta\lambda = 40^\circ, D = 0.2$	
Elements for 1965	
$g_1 = 0.3 \pm 0.3, g_2 = 0.4^\dagger$ $T_1 = 5300 \text{ K} \pm 20 \text{ K}, T_2 = 5715 \text{ K} \pm 50 \text{ K}$ $\lambda_0 = 70^\circ \pm 10^\circ, B = 0.45 \pm 0.02, C = 0.20 \pm 0.02$	

* Adopted values.

† Chosen to agree with the 1968–1969 value.

changes in only the surface-brightness parameters. We note that changes for both components were needed: the less massive component's gravity darkening changed enough to reproduce the level at mid-secondary eclipse. In this solution on the average *half* of the surface ($4\pi r^2$) of the spotted component is covered with dark spots, as opposed to about 22% for the cooler component of RS CVn. However, if the background is ignored, the fraction covered is closer to 17%.

In the solution given we have chosen to let the spots range from pole to pole. This was to avoid reducing the duration of secondary eclipse by letting the spots concentrate at the equator and to keep the large background term B as numerically small as possible. But we can now restrict the extent of the spot distribution somewhat, although the results here will not be as definitive as were those for RS CVn. This is because the wave in RT Lac is of lower amplitude than the wave in RS CVn and because the large background needed to fit the color discrepancy complicates the analysis. In fact, there are two ways to handle this background. It may be taken to be uniform, that is, to extend from pole to pole, or taken to be restricted in latitude in the same way as the longitudinally ordered spots. In this latter case, spots must not extend to the equator. This is a firm conclusion based on the observed eclipse durations. Letting the spots extend to the equator but not to the poles would reduce the duration of secondary eclipse, but our Roche model analysis indicates that secondary has about the same duration as primary. So, if all the spots are located in latitude bands as for the model illustrated in Figure 1, these bands must be located partway between the equator and poles. Also, for them to give the right effect on the light curve, they must extend over at least 40° of latitude in each hemisphere. Spots extending from $\beta_{\text{min}} = 10^\circ$ to $\beta_{\text{max}} = 50^\circ$ suffice to reproduce the observations, giving essentially the same computed

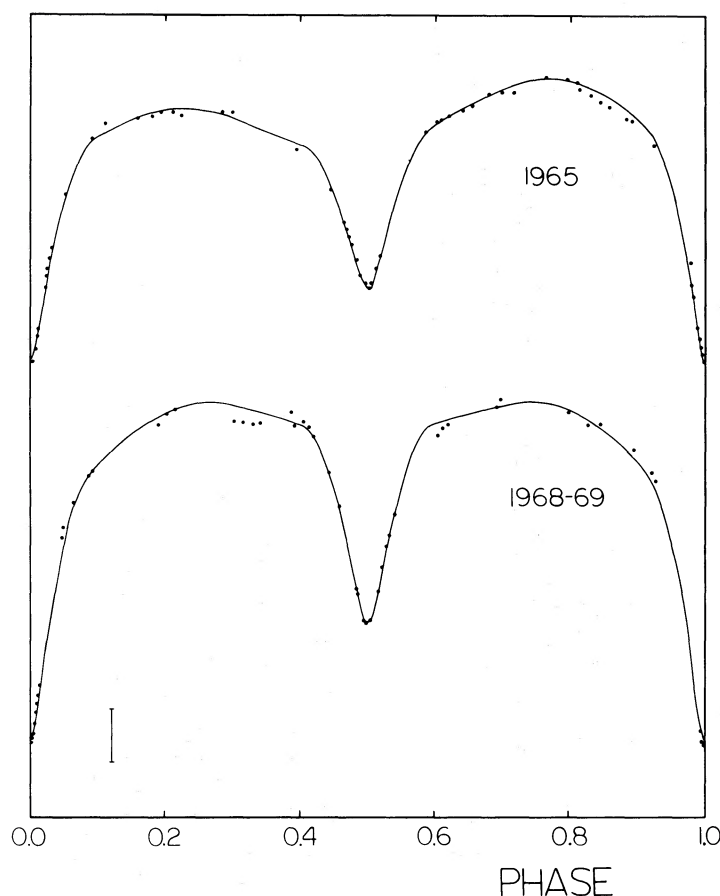


FIG. 6.—The light curves of RT Lac for 1965 and 1968–1969. The V -band normal points formed by Hall and presented by Milone (1977) are plotted as dots. The solid curves are the theoretical light curves computed for the elements of Table 4. A vertical bar 0.2 mag high is given in the lower left corner to indicate the amplitude of the brightness variations. Note that the magnitude scale is the same for both years; no arbitrary shift of the zero point for the 1965 theoretical light curve relative to that for the 1968–1969 theoretical light curve was needed to fit the difference in general level of the two observed light curves.

light variation as the quoted solution. Now, it is possible that the large background is not related to the spots that produce the wave or that it is just an artifact of an abnormal spectral energy distribution as we suggested previously. In that case the wave-producing spots would have to extend over only 13° of latitude in both hemispheres. There is a fair amount of leeway in where to place their latitude band, but again eclipse durations suggest that it should not be concentrated at the equator. We actually prefer this suggestion because it accounts naturally for the difference between the color and spectral type of the more massive component. However, further observations will be needed to test its validity.

IV. RS CANUM VENATICORUM TYPE SURFACE ACTIVITY

Our most important conclusion is that spots are necessary to fit the optical light curves of RS CVn type binary systems. In their observable properties these starspots appear to be the same phenomenon as the spots on the Sun. The similarity includes the probable sizes, lifetimes, and temperatures of the spots. It extends to how the spots are distributed on the stellar

disk. And, finally, it encompasses the long-term evolution of the starspot distributions.

As with the Sun, the individual spots must be considerably smaller than the stellar disk. Since eclipses provide only a limited ability to resolve spatial variations of stars' surface brightness, this conclusion is based on rather circumstantial evidence. In conducting this study we found that very large individual spots will not reproduce the light-curve waves satisfactorily. Instead, the variation of brightness seems better fitted by a continuously varying number of *small* spots, the assumption built into our model. This is precisely the conclusion Vogt (1975) reached from trying to fit the light variation of BY Dra. The restriction of the latitude extent of the spots on RS CVn in 1964 is further evidence that the spots are considerably smaller than the disk. The photometry of the RS CVn type systems with complete occultations at primary eclipse the brightness of the spotted components to be measured. In all these systems small but finite (~ 0.01 mag) variations from cycle to cycle are detected. In the context of the spot model these variations would represent the production or dissolution of in-

dividual spots or small groups of spots. The variations then suggest the individual spots cover at most 1% of the area of the stellar disk and so should have radii less than one-tenth the disk radius.

As well as can be determined from the present data, these starspots have lifetimes not substantially different from those of sunspots. The temporal variations of the spot distributions and variations during mid-primary eclipse suggest that spots are continually being formed and destroyed. RS CVn is the best system in which to study this process because of the extensive photometry available for it (Catalano and Rodono 1974). In that system the amplitude of the wave changed little over 6 years. Yet the spotted component's brightness changed detectably over a few cycles. This suggests that the spots are continually decaying and being formed on time scales less than 1 year. The fact that the *detailed* shape of the wave varies from year to year (Ludington 1978) provides further evidence that spot lifetimes are shorter than 1 year.

We also find that the spots are *dark* spots with effective temperatures considerably lower than the photosphere. For SZ Psc, the only object for which we could actually infer a unique spot temperature, the spots were $1200\text{ K} \pm 400\text{ K}$ cooler. Although the limits on the temperatures of spots in RS CVn and RT Lac were $\sim 500\text{ K}$ cooler than the photosphere, we expect suitable photometry to measure temperatures considerably below this limiting value. In deriving the limits for these systems, the spots were allowed to range from pole to pole and to have the maximum amount of contrast between bright and dark hemispheres. Both restrictions seem overly extreme. If the spots are restricted to rather narrow bands of latitude as in the Sun, the temperatures must be much lower than our quoted limits. We can compare these spot temperatures with those for the Sun and for dMe (BY Dra type) stars. For the Sun, spots are typically 1600 K cooler than the photosphere (Minnaert 1953); while for the BY Dra type variables they are $200\text{--}500\text{ K}$ cooler (Bopp and Espenak 1977). So we find spot temperatures for RS CVn binaries which are rather typical of other stars known to have or suspected of having solar-type starspots.

The surface distributions of spots on these objects are far from random. We have found evidence for restriction of the spots in both latitude and longitude. The restriction in latitude is mainly suggestive. The geometry of these systems is such that the eclipses give only limited resolution of the disk in the direction perpendicular to the orbital plane. We can conclude that the spots do not extend to the poles and that they probably do not extend any farther from the equator than $\beta_{\text{max}} = 40^\circ$ in the case of RS CVn. Also, we cannot say for sure whether the spots are constrained to latitude bands as in our computational model; they could be in a band that varies its mean latitude as it goes around the star. However, if the spots are so confined, the bands must be at least as wide ($\gtrsim 15^\circ$) as those on the Sun, typically $15^\circ\text{--}20^\circ$ (Kiepenheuer 1953).

The ordering of spots in longitude is determined

much more precisely. Here we are aided by the rotation of the spotted star and by the eclipses, both of which sample the spot distribution in longitude. Interpretation of these light curves indicates that the spots are distributed roughly sinusoidally in longitude. Examined in detail, however, the spot distributions are much more complicated. Again, this longitude dependence has an analogy with that of the Sun. On the Sun there are preferred longitudes for active regions which persist over many solar rotations. Losh (1939) studied sunspot numbers measured over a 34 year interval and Greenwich sunspot areas covering 18 years of time. She found an approximately $27\frac{1}{3}$ day periodicity in the sunspot numbers, indicating long-lived structure in the longitudinal distribution. Although the spot areas showed evidence for long-term ordering, that ordering was far from simple. Spots seemed to form preferentially in certain narrow ranges of longitude and to avoid forming in others. However, the distributions of spots in the northern and southern hemispheres were often out of phase by $\sim 180^\circ$ of longitude. Although other studies (e.g., Haurwitz 1968) have failed to detect ordering of sunspots on very long time scales, long-lived longitude structure is evident in other indicators of solar activity and in the background magnetic field. The Sun has been observed routinely as a star, i.e., without resolving its disk, both in the radio continuum and in the extreme ultraviolet (EUV) emission lines originating in the upper chromosphere and the lower corona. The flux in these bands shows an unmistakable rotational modulation, with the variations in the radio and EUV being highly correlated (Chapman and Neupert 1974). The emission also correlates with sunspot number; that is, emission is strongest when the relative sunspot number is highest. Also, Haurwitz (1968) found that solar proton flares are distributed nonrandomly in longitude. They seem to fit a rigidly rotating system with synodic period 27.213 days. And, finally, there is the evidence presented by Svalgaard and Wilcox (1975) that the Sun's magnetic sector structure (Svalgaard *et al.* 1975) rotates at an average rate of once per 27.0 days and has maintained its phasing over the last five solar cycles.

As yet it has not been demonstrated that these starspots are regions of intense magnetic field as are the sunspots, and it is probably not technically possible to do so. However, there is direct evidence of magnetic fields in RS CVn type systems and indirect evidence that the distribution of magnetic fields is similar to that of the spots. The direct evidence comes from observations of flares in HR 1099 (Owen, Jones, and Gibson 1976; Weiler *et al.* 1978) and UX Ari (Hjellming 1978). Circular polarization of up to 20% has been detected at various phases of these flares. This suggests the radio emission is produced by the synchrotron mechanism or is transferred through regions containing magnetic fields. So we can conclude that magnetic fields are located in the vicinities of these objects, i.e., in their coronae, and are associated with their flaring. The indirect evidence is the chromospheric activity and its interpretation through solar analogy.

Weiler (1975) has found a correlation between the phasing of chromospheric $H\alpha$ emission and the photometric waves in several RS CVn type systems. The correlation, which appears to be even better for the $L\alpha$ emission of HR 1099 (Weiler 1978), is in the sense that when the star is faint in the optical continuum, i.e., when spots are most prominent, the chromospheric emission is strongest. On the Sun such lines are emitted chiefly in the active regions associated with spots. The work of Chapman and Neupert (1974) indicates a good correlation between the two, and by analogy one would expect this association of magnetic field with active regions to hold for the RS CVn type binaries also. Hence, Weiler's observations of line strengths in RS CVn systems simply indicate that the spots are magnetic.

Based on the expected correlation of magnetic field strength with the spots, we can use the spots to determine how magnetic fields are distributed on the surfaces of RS CVn type stars and how they evolve with time. Again, a strong solar analogy emerges. From it we infer that these stars go through spot cycles like those of the Sun, have coronal holes and stellar winds, and rotate at a nonuniform rate in their regions of magnetic field generation. The observational basis of this discussion is the large number of studies done by Hall and collaborators which define for a sizable number of systems the variation of light curve wave amplitude with time, the variation of wave migration with phase of the wave amplitude cycle, and the correlation of the orbital-period fluctuations with the phasing of the wave. As mentioned in the Introduction, wave amplitude cycles have been demonstrated conclusively for two stars. RS CVn (Hall 1972) has ~ 23 year cycles while RT Lac (Hall and Haslag 1976) has ~ 30 year cycles. Our light curve work for RS CVn indicates that a lower wave amplitude means there are fewer spots rather than that the spots are less well organized in longitude. This means that the wave amplitude cycles which occur on time scales similar to that of the solar cycle are true spot cycles.

Observations of the period fluctuations have revealed a correlation of sign of the period change with the phase of the wave in the light curve (Hall 1972; Arnold, Hall, and Montle 1973; Arnold and Hall 1973; Hall, Richardson, and Chambliss 1976). These period fluctuations have been explained in terms of mass loss restricted in direction (Arnold and Hall 1973). The model for this requires the mass to be ejected from the brighter hemisphere of the spotted star, i.e., from regions of low spot concentration. On the Sun we would refer to such an area from which enhanced high-speed mass loss occurs as a coronal hole (Timothy, Krieger, and Vaiana 1975; Nolte *et al.* 1976; Sheeley, Harvey, and Feldman 1976).

Variations of wave migration rate with phase of the starspot cycle have been documented for the few of these stars for which there are adequate data. In this group are RS CVn (Hall 1972), RT Lac (Haslag 1976; Hall and Haslag 1976), and SS Boo (Hall and Henry 1978). For all three the wave migration speeds up toward the end of the spot cycle and slows down

around the time of spot minimum. This dependence on phase is reminiscent of the change in rotation rate of the solar sector structure as inferred from geomagnetic data (Svalgaard and Wilcox 1975). However, it is $\sim 180^\circ$ out of phase with this solar analogy. We note that in the solar case we are observing a structure which rotates essentially as a rigid body (Svalgaard *et al.* 1975). Hence, the variations in rotation speed cannot be attributed directly to differential rotation. The interpretation of variations of wave migration rate, then, cannot unambiguously be used to support a rotation of the surface which changes its rate with latitude. It does, however, indicate the nonuniform rotation of the magnetic structure or, alternatively, of the sources of the magnetic field.

The ordering of the surface activity in these stars seems to be both stronger and simpler than that of the Sun. For the Sun the long-lived background magnetic structure tends to be associated with four preferred longitudes. The surface of the Sun within 40° of the equator is divided roughly into four sectors of alternating magnetic polarity which are 90° of longitude wide (Svalgaard and Wilcox 1975). Similar ordering is also seen in the solar coronal holes. Wagner (1976) studied daily spectroheliograms for Fe xv $\lambda 284$ obtained over an interval of nearly 1.5 yr by the Goddard Space Flight Center OSO 7 experiment (Underwood and Neupert 1974). He found that the coronal holes, related to the interplanetary magnetic field from which the magnetic sector structure is inferred by Sheeley, Harvey, and Feldman (1976), tend to favor four orthogonal longitudes. On the active components of RS CVn systems there appears to be but *one* range of longitude at which large coronal holes form. What's more, in the RS CVn systems the long-lived magnetic structure, associated with the coronal-hole mass loss, and the distribution of active regions, inferred from the behavior of the waves, migrate in phase. On the other hand, in the Sun the distribution of active regions seems to be independent of the ordering of the background magnetic structures. The periods of these phenomena appear to be different in the case of the Sun. Synodic recurrence periods for the magnetic sector structure and the coronal hole distribution are both close to 27.0 days while the recurrence period for the sunspot distribution deduced by Losh (1939) is about $27\frac{1}{3}$ days. Moreover, it appears that active regions are carried through coronal holes by the Sun's differential rotation (Timothy, Krieger, and Vaiana 1975). North-south asymmetries of the spot distributions, such as those Losh (1939) noted for the Sun, cannot be ruled out for RS CVn type stars, especially for the ones having distortion waves of low amplitude. However, they cannot be of much importance in the case of RS CVn itself. This is because for that particular system the large amplitude of the distortion wave requires the spot distributions above and below the equator to be in phase.

V. CONCLUSION

Although there appears to be an excellent analogy between the surface activity of RS CVn type stars and

that of the Sun, there are a number of areas in which future work is needed to define the relationship better.

This is especially true in the case of defining the restrictions on the latitude distribution of spots and the variation of this distribution with phase of the starspot cycle. It seems to us that the most promising approach to this would be to obtain accurate observations of the secondary eclipses of systems in which the ratios of radii are small. Eclipses in these systems are shallow, and very painstaking photometry must be done to define the eclipse shapes accurately, but it is just these systems which give a relatively high resolution sampling of the spotted star's surface brightness distribution. Further, detecting the systematic variation in general depth of secondary eclipse with phase of the starspot cycle seems to be the straightforward way to detect drift of the spot distribution in latitude if it exists. The systems which have the most promise for such studies are RS CVn ($k \approx 0.5$), SZ Psc ($k \approx 0.3$), and AR Lac ($k \approx 0.6$). All three of these are relatively bright, and so are observable with telescopes of moderate aperture. As yet, a spot cycle has not been demonstrated for SZ Psc, but, as we have seen, its wave amplitude is variable and its rapid wave migration rate suggests that the activity cycles may be especially fast for this star.

Further work is needed in defining the spot temperatures. This means that multicolor, e.g., *BVRI*, light curves must be obtained and interpreted. Such temperature results will not only determine a further physical property of the spots, but will also restrict further the allowable range of the spots in latitude.

Another major area of uncertainty about these objects is their luminosity variations. Our admittedly

limited results for RS CVn indicate that the active component changes its luminosity—the amount of energy radiated at its surface—as it goes through the starspot cycle. Further observations are required to define how much the luminosity varies over the spot cycle of RS CVn and to document variation for other stars. Complete light curves for well-defined passbands suitable for determining the amount of energy radiated by both the photospheres and spots need to be obtained over entire spot cycles.

Accurate photometry in the infrared will be needed to investigate the infrared excesses. The large excesses that Milone (1976) inferred for many of these objects are hard to understand in the context of a source as small as the photosphere of one of the components. If these large excesses do exist, it will be important to determine what their sources are and how much of the systems' luminosity they represent.

This research was carried out over a period of several years and benefited from the aid of a number of individuals and organizations. Joel Eaton thanks J. S. Gallagher and E. C. Olson of the University of Illinois for the observing time used to obtain the 1978 observations of SZ Psc which we used to define the wave migration rate of that system. We thank the University of Wisconsin Space Astronomy Laboratory and Goddard Space Flight Center for the large amounts of computer time necessary to do the detailed light curve calculations discussed here. Also, we deeply appreciate it that a number of our colleagues sent us the results of their work in progress in advance of publication, and we thank J. D. Fix and S. M. Ruciński for sending us very constructive comments on the manuscript.

REFERENCES

- Allen, C. W. 1973, *Astrophysical Quantities* (3d ed.; London: Athlone), p. 217.
- Arnold, C. N., and Hall, D. S. 1973, *Inf. Bull. Var. Stars*, No. 843.
- Arnold, C. N., Hall, D. S., and Montle, R. E. 1973, *Inf. Bull. Var. Stars*, No. 796.
- Batten, A. H. 1973, *Binary and Multiple Systems of Stars* (Oxford: Pergamon Press), chapters 8 and 9.
- Bookmyer, B. B. 1965, *A.J.*, **70**, 415.
- Bopp, B. W., and Espenak, F. 1977, *A.J.*, **82**, 916.
- Cassinelli, J. P., and Haisch, B. M. 1974, *Ap. J.*, **188**, 101.
- Castle, K. G. 1977, *Pub. A.S.P.*, **89**, 862.
- Catalano, S., and Rodono, M. 1967, *Mem. Soc. Astr. Italiana*, **38**, 395.
- . 1969, in *Nonperiodic Phenomena in Variable Stars*, ed. L. Detre (Dordrecht: Reidel), p. 435.
- . 1974, *Pub. A.S.P.*, **86**, 390.
- Chapman, R. D., and Neupert, W. M. 1974, *J. Geophys. Res.*, **79**, 4138.
- Chisari, D., and Lacona, G. 1965, *Mem. Soc. Astr. Italiana*, **36**, 463.
- Eaton, J. A. 1975, *Ap. J.*, **197**, 379.
- . 1977, *Inf. Bull. Var. Stars*, No. 1297.
- . 1978, *Acta Astr.*, **28**, in press.
- Hall, D. S. 1972, *Pub. A.S.P.*, **84**, 323.
- . 1976, in *IAU Colloquium No. 29, Multiple Periodic Variable Stars*, Part I, ed. W. S. Fitch (Dordrecht: Reidel), p. 287.
- Hall, D. S., and Haslag, K. P. 1976, in *IAU Colloquium No. 29, Multiple Periodic Variable Stars*, Part II, ed. W. S. Fitch (Budapest: Academic Press), p. 331.
- Hall, D. S., and Henry, G. W. 1978, *Inf. Bull. Var. Stars*, No. 1404.
- Hall, D. S., Montle, R. E., and Atkins, H. L. 1975, *Acta Astr.*, **25**, 125.
- Hall, D. S., Richardson, T. R., and Chambliss, C. R. 1976, *A.J.*, **81**, 1138.
- Haslag, K. P. 1976, Masters thesis, Vanderbilt University.
- Haurwitz, M. W. 1968, *Ap. J.*, **151**, 351.
- Hjellming, R. M. 1978, private communication (talk given at the IAU Comm. 42-sponsored RS CVn Workshop, Socorro, NM, 1978 April 6–7).
- Iriarte, B., Johnson, H. L., Mitchell, R. I., and Wisniewski, W. K. 1965, *Sky and Tel.*, **30**, 21.
- Jakate, S., Bakos, G. A., Fernie, J. D., and Heard, J. F. 1976, *A.J.*, **81**, 250.
- Johnson, H. L. 1966, *Ann. Rev. Astr. Ap.*, **4**, 193.
- Joy, A. H. 1931, *Ap. J.*, **74**, 101.
- Keller, G., and Limber, D. N. 1951, *Ap. J.*, **113**, 637.
- Kiepenheuer, K. O. 1953, in *The Sun*, ed. G. P. Kuiper (Chicago: University of Chicago Press), p. 335.
- Koch, R. H., Plavec, M., and Wood, F. B. 1970, *Pub. Univ. Pennsylvania, Astr. Ser.*, Vol. 11.
- Kopal, Z. 1972, *Adv. Astr. Ap.*, **9**, 1.
- Kron, G. E. 1947, *Pub. A.S.P.*, **59**, 261.
- Lesh, J. R., and Aizenman, M. L. 1978, *Ann. Rev. Astr. Ap.*, **16**, in press.
- Losh, H. M. 1939, *Pub. Obs. Univ. Michigan*, **7**, 127.
- Lucy, L. B. 1973, *Ap. Space Sci.*, **22**, 381.
- Ludington, E. W. 1978, private communication (talk given at the IAU Comm. 42-sponsored RS CVn Workshop, Socorro, NM, 1978 April 6–7).

- Milone, E. F. 1976, in *IAU Colloquium No. 29, Multiple Periodic Variable Stars*, Part II, ed. W. S. Fitch (Budapest: Academic Press), p. 321.
- . 1977, *A.J.*, **82**, 998.
- Minnaert, M. 1953, in *The Sun*, ed. G. P. Kuiper (Chicago: University of Chicago Press), p. 722.
- Molnar, M. R. 1975, *A.J.*, **80**, 137.
- Nolte, J. T., *et al.* 1976, *Solar Phys.*, **46**, 303.
- Oliver, J. P. 1974, Ph.D. thesis, University of California, Los Angeles.
- . 1975, *Pub. A.S.P.*, **87**, 695.
- Oskanyan, V. S., Evans, D. S., Lacy, C., and McMillan, R. S. 1977, *Ap. J.*, **214**, 430.
- Owen, F. N., Jones, T. W., and Gibson, D. M. 1976, *Ap. J. (Letters)*, **210**, L27.
- Plavec, M. 1968, *Bull. Astr. Inst. Czechoslovakia*, **19**, 11.
- Popper, D. M. 1977, *Highlights of Astronomy*, Vol. 4, Part II, p. 397.
- Popper, D. M., and Ulrich, R. K. 1977, *Ap. J. (Letters)*, **212**, L131.
- Rhombs, C. G., and Fix, J. D. 1976, *Acta Astr.*, **26**, 301.
- . 1977, *Ap. J.*, **216**, 503.
- Ruciński, S. M. 1973, *Acta Astr.*, **23**, 79.
- Scaltriti, F. 1978, letter to D. S. Hall.
- Sheeley, N. R., Harvey, J. W., and Feldman, W. C. 1976, *Solar Phys.*, **49**, 271.
- Shore, S. N., and Hall, D. S. 1978, *A.J.*, submitted.
- Svalgaard, L., and Wilcox, J. M. 1975, *Solar Phys.*, **41**, 461.
- Svalgaard, L., Wilcox, J. M., Scherrer, P. H., and Howard, R. 1975, *Solar Phys.*, **45**, 83.
- Timothy, A. F., Krieger, A. S., and Vaiana, G. S. 1975, *Solar Phys.*, **42**, 135.
- Underwood, J. H., and Neupert, W. M. 1974, *Solar Phys.*, **35**, 241.
- Van Horn, H. M. 1978, paper presented at the Los Alamos/Goddard Conference, *Current Problems in Stellar Pulsation Instabilities*, to be published.
- Vogt, S. S. 1975, *Ap. J.*, **199**, 418.
- Wagner, W. J. 1976, in *IAU Symposium No. 71, Basic Mechanisms of Solar Activity*, ed. V. Bumba and J. Kleczek (Dordrecht: Reidel), p. 41.
- Weiler, E. J. 1975, Ph.D. thesis, Northwestern University.
- . 1978, private communication (talk given at the IAU Comm. 42-sponsored RS CVn Workshop, Socorro, NM, 1978 April 6–7).
- Weiler, E. J., *et al.* 1978, *Ap. J.*, in press.

JOEL A. EATON: Department of Astronomy, Pennsylvania State University, 525 Davey Laboratory, University Park, PA 16802

DOUGLAS S. HALL: Dyer Observatory, Vanderbilt University, Nashville, TN 37235