FLUCTUATION THEORY OF THE MASS FLUX FROM THE STARS

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(Received 14 October, 1978)

Abstract. The mass flux from a star is adopted to result from a fluctuation of the photosphere, which is not in complete thermal equilibrium. Because of the large difference between the dynamical and thermal relaxation times, its state can be approximated by a partial equilibrium. Using a theory of thermodynamic fluctuations, the mass flux is expressed in a correlation function of gravitational perturbations of the photosphere. A hypothesis is proposed that the susceptibility to these perturbations, if normalized to the available thermal energy, is the same for all stars. Its value is obtained by considering the upper limit to the mass flux. This results in a mean mass loss of $L^{3/2}(R/M)^{9/4}/G^{7/4}$, where the symbols have their common meaning. The result is compared to empirical data on the mass flux from some 50 stars of various luminosities and luminosity classes. With a possible exception for late-type (super) giants the agreement is good, in many cases within a factor 2.

1. Introduction

Mass fluxes from stars may well be a general phenomenon. Early type supergiants have mass losses up to 10^{-5} $M_{\odot}y^{-1}$ (Barlow and Cohen, 1977) and a main-sequence B-star like τ Sco loses almost 10^{-8} $M_{\odot}y^{-1}$ (Lamers and Rogerson, 1978). Red giants and supergiants have typical mass losses of 10^{-8} $M_{\odot}y^{-1}$ and 10^{-6} $M_{\odot}y^{-1}$, respectively (Reimers, 1975). These values clearly correlate with the luminosity of the stars. For luminosities below $10^3 L_{\odot}$ the evidence for mass fluxes is indirect only, except in the case of the sun with its loss of some 10^{-14} $M_{\odot}y^{-1}$ (Hundhausen, 1972). The evidence consists of a temperature inversion above the photosphere, which points to dissipation in an outward stream of particles.

If mass fluxes are a general phenomenon, they may well have a common origin. There is an intriguing scaling of the loss from red (super) giants to that from the Sun (Fusi-Pecci and Renzini, 1976). The process may not even depend on whether or not there is an outer convective zone producing acoustic waves, as an M- and A-supergiant have similar luminosities and mass losses [α Ori: $5 \times 10^4 L_{\odot}$ and 1×10^{-6} $M_{\odot}y^{-1}$, Reimers (1975); α Cyg: $1 \times 10^4 L_{\odot}$ and 7×10^{-7} $M_{\odot}y^{-1}$, Barlow and Cohen (1977)]. Significantly, Cannon and Thomas (1977) have shown that *any* stellar atmosphere is unstable to gravitational (acoustic or pulsational) perturbations and that these perturbations lead to an outward flux of matter, which is fed by the thermal energy of the photosphere. Also, it may not depend on the steady acceleration mechanism of this flux, either by radiation pressure (e.g., Lucy and Solomon, 1970; Castor *et al.*, 1975) or by expansion of a corona (e.g., Hearn, 1975). Significantly again, Żytkow (1974) has noted that one encounters serious problems in explaining the mass flux as a *stationary* phenomenon.

Astrophysics and Space Science **61** (1979) 205–216. 0004–640X/79/0611–0205$01.80

In this paper the idea is worked out that the process originates in a *fluctuation* about the quasi-equilibrium reached in the stellar atmosphere. 'For the few cases which have been observed in detail, the evidence is strong that the fluctuations in mass flow are as significant as the very fact of flow existing at all'*. These cases comprise the Sun (e.g., Hundhausen, 1972) and some hot stars (York *et al.*, 1977; Lamers *et al.*, 1978). On top of a 'stationary' wind one observes irregularly enhanced emissions ('puffs') on time scales of an hour, typically. In the case of the solar wind, the latter appear to be associated with high-speed streams, whereas the 'stationary' wind in reality undergoes large variations on a time scale of a week. The stochastic nature of the mass flux stresses that it is due to non-equilibrium processes and this certainly complicates the theory of the phenomenon.

Actually a mass flux in itself proves the absence of thermal equilibrium in the photosphere. Likewise the generality of mass fluxes proves the generality of non-equilibrium at the interface star–interstellar medium, which underlies the scheme discussed by Pecker *et al.* (1973). This state of non-equilibrium, however, can be approximated by a *partial* equilibrium, if there is a relaxation time much less than the time to reach equilibrium itself (e.g., Landau and Lifshitz, 1969). In that case equilibrium methods (thermodynamics) may be used to describe the factual non-equilibrium, which of course is an important advantage. Fortunately this happens to be the case, as the time determining the dynamics at the stellar surface

$$\tau_d = \left(\frac{4/3\pi R^3}{GM}\right)^{1/2} \tag{1}$$

is in general much less than the thermal (Kelvin–Helmholtz) time for reaching equilibrium, given by

$$\tau_t = \frac{GM^2}{RL}; \tag{2}$$

where, as usual, G is the gravitational constant and L, M and R are the luminosity, mass and radius of the star, respectively.

In what follows we first integrate elements from the thermodynamic fluctuation theory, as given by Becker (1961), in the problem of mass fluxes (Section 2). This will allow us to express the mass flux in a correlation function of perturbations of the quasi equilibrium in the photosphere. The advantage gained by this formal restatement of the problem is the clear physical meaning of the correlation function, which gives a hold for the heuristic treatment of the perturbation in Section 3. We conclude with a discussion of the theory in the light of empirical data on the mass flux (Section 4).

2. Thermodynamic Fluctuations

We have to define a thermodynamic function, which we denote by λ, to describe the state of partial equilibrium associated with a certain mass flux $\dot{M}$. We choose the

* Quotation from a letter by A. B. Underhill to the author.

dimensionless parameter (cf. Williams, 1967)

$$\lambda = \frac{GM}{LR}\dot{M}. \tag{3}$$

If P is the total power generated in the stellar interior, one obtains from the energy balance that $\lambda = (P - L)/L$. This makes it clear that λ is the excess $(P - L)$ in non-thermal (gravitational) power, associated with the mass flux, normalized to the thermal power L. Complete thermalization implies $L \uparrow P$ and thus $\lambda \downarrow 0$; incomplete thermalization about partial equilibrium implies $\lambda \downarrow \bar{\lambda}$. For a fluctuating quantity x (like λ or $\dot{M}$ or any other), we denote by $\bar{x}$ its r.m.s. value

$$\bar{x} = \langle x^2 \rangle^{1/2}, \tag{4}$$

where the brackets denote a time average. Obviously, our problem is to calculate $\bar{\lambda}$ rather than λ.

Following Landau and Lifshitz (1969), or Becker (1961), we propose that $\dot{M}$, and thus λ, varies erratically in time according to the Langevin equation

$$\dot{\lambda} = -\lambda/\tau_t + A(t), \tag{5}$$

which defines a stochastic perturbation function $A(t)$ with the dimension of a frequency. One recalls that the Langevin equation is useful in describing fluctuation phenomena in the absence of equilibrium, like the brownian motion. In the absence of $A(t)$, λ would decay to zero according to $\exp(-t/\tau_t)$ – with the thermal relaxation time, as it should. However, the presence of perturbations prevents such a decay to zero and stops it at the level $\bar{\lambda}$. $A(t)$ oscillates around zero with an extremely unsteady amplitude but with a rather steady time constant of the order of τ_d. This corresponds to gas motions in the (sub)photosphere, which all develop on the dynamical time scale, but for which the strength depends on randomly varying circumstances. As $\bar{\lambda}$ is a normalized power fluctuation, $\bar{A}$ is the normalized strength of the gravitational perturbations, which represent an average power of $\bar{A}$ units of gravitational energy or $\bar{A}(GM^2/R)$. We recall that these perturbations are fed by the thermal power L of the star. As long as the perturbations are small, one may adopt that the response is linear, so that

$$\bar{A}(GM^2/R) = \chi L \quad \text{or} \quad \bar{A} = \chi/\tau_t. \tag{6}$$

For stars, where the physical cause of the mass flux is the same, the susceptibility χ may be the same. This leads to the *hypothesis* that

for all stars χ has the same value. (7)

We shall return to the behaviour of $A(t)$ and the value of χ in the next section.

In integrating (5) we follow a method developed by Einstein and Hopf (cf. Becker, 1961). We take a time τ such that

$$\tau_d \ll \tau \ll \tau_t, \tag{8}$$

within which λ has changed its value many times but $\bar{\lambda}$ has hardly changed. With the abbreviation

$$\int_0^\tau \mathrm{d}t\, A(t) = B_0, \tag{9}$$

we have for the change of λ after such a time τ

$$\lambda_1 - \lambda_0 = -\tau/\tau_t \lambda_0 + B_0. \tag{10}$$

More in general, with

$$\int_{j\tau}^{(j+1)\tau} \mathrm{d}t\, A(t) = B_j, \tag{11}$$

where j is an integer, we can write

$$\lambda_{j+1} - \lambda_j = -\tau/\tau_t \lambda_j + B_j; \tag{12}$$

or, using

$$\gamma = 1 - \tau/\tau_t, \tag{13}$$

$$\lambda_{j+1} = \gamma\lambda_j + B_j. \tag{14}$$

The last equation is a recursion formula relating the final value λ_n to the initial value λ_0 through the intermediate values $\lambda_1, \lambda_2, \ldots, \lambda_{n-1}$. We can eliminate these intermediate values by multiplying the equation for λ_j with γ^{n-j} and then adding the newly found equations as

$$\lambda_n = \gamma^n\lambda_0 + (\gamma^{n-1}B_0 + \gamma^{n-2}B_1 + \cdots + B_{n-1}). \tag{15}$$

If we square λ_n and consider the time average, we have $\langle\lambda_n^2\rangle = \langle\lambda_0^2\rangle = \langle\lambda^2\rangle$, whereas $\langle B_i B_j\rangle = \langle B^2\rangle$ if $i = j$ and 0 if $i \neq j$, since the B_j are assumed to be statistically independent. This results in

$$\langle\lambda^2\rangle(1 - \gamma^{2n}) = \langle B^2\rangle \frac{1 - \gamma^{2n}}{1 - \gamma^2}, \tag{16}$$

which after substitution of $\gamma^2 = 1 - 2\tau/\tau_t$ leads to

$$\langle\lambda^2\rangle = \tau_t \frac{\langle B^2\rangle}{2\tau}. \tag{17}$$

It should be noted, that B as given by (11) depends on τ. In the following section we shall relate $\langle B^2\rangle/(2\tau)$ to the stochastic perturbation function $A(t)$ using the definition

$$\langle B^2\rangle = \int_0^\tau \mathrm{d}t \int_0^\tau \mathrm{d}t'\, \langle A(t)A(t')\rangle. \tag{18}$$

3. Photospheric Perturbations

Because of the stochastic nature of $A(t)$, one expects that the time correlation $\langle A(t)A(t')\rangle$ depends only on the time difference $u = t' - t$. Moreover, this function

$\langle A(t)A(t+u)\rangle$ will have an appreciable value only for time differences u of the order of τ_d, whereas for larger values of u it will tend to zero. If the perturbations can be studied as individual events ('puffs'), $\langle A(t)A(t+u)\rangle$ gives the time evolution of the associated mass flux. In Figure 1 we show the mass flux in a solar wind perturbation as given by the product of flow speed and density observed by a satellite. One 'puff' stands out clearly and its time evolution can be followed during 3 days before another event takes over to determine the flux: the decay seems to be exponential. Taking into account that some velocity dispersion will cause a broadening of the phenomenon in time when observed at 1 a.u., this is empirical evidence for a functional behaviour like

$$\langle A(t)A(t+u)\rangle = \langle A^2\rangle \exp(-u/\tau_d). \tag{19}$$

Our calculation will not depend much on the adopted functional form of the decay. As shown by Becker (1961), any decay of the autocorrelation of $A(t)$ with the

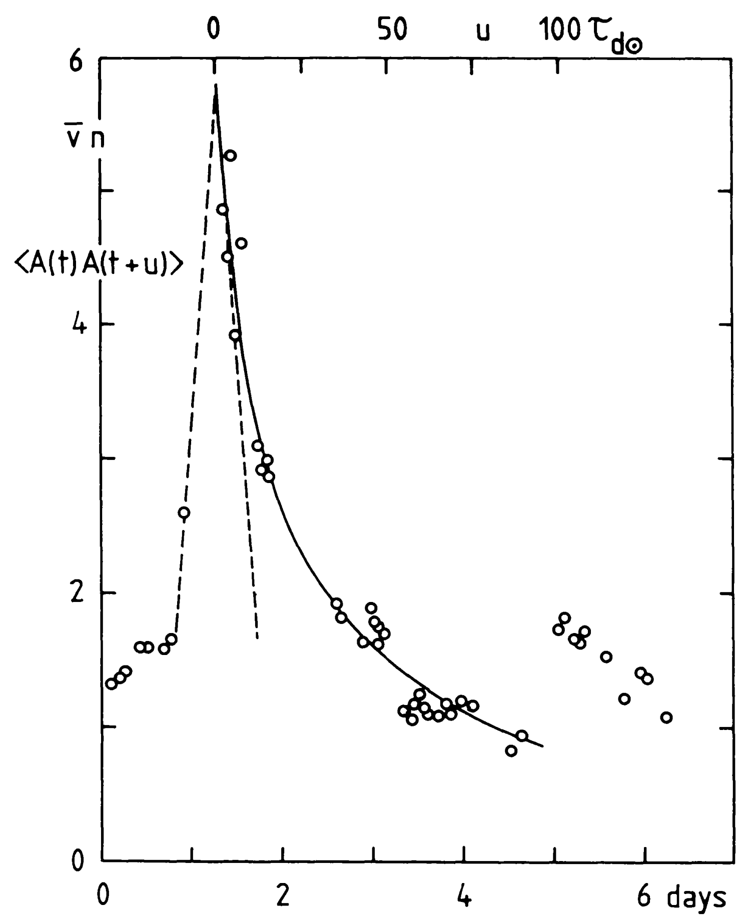

Fig. 1. Sketch of the correlation $\langle A(t)A(t+u)\rangle$ of photospheric perturbations $A(t)$ in the Sun as a function of the time difference u (full line; the abscissa is in arbitrary units). This correlation is proportional to the flux in a single event of mass loss. The data are the product of the average flow speed $\bar{v}$ and density n in a transient solar wind disturbance measured by the Vela 3 satellite (adapted from Hundhausen, 1972; his Figure 5.16). There is an intrinsic broadening of the peak due to differences in the time of flight of particles to the Earth, suggested by the dotted lines. The probable half time of $\langle A(t)A(t+u)\rangle$ is a few $\tau_{d\odot}$ and the decay seems to be exponential.

characteristic time τ_d will yield the result below. To make the simple form (19) invariant for time reversal ($u \to -u$), we replace u by $|u|$.

In evaluating (18) we introduce, besides the time difference $u = t' - t$, the time sum $v = t' + t$. We can then write

$$\begin{aligned} \langle B^2 \rangle &= \frac{1}{2}\int_0^{2\tau} dv \int_{-\tau+|\tau-v|}^{\tau-|\tau-v|} du\, \langle A(t)A(t+u)\rangle \\ &\simeq \tau \int_{-\infty}^{\infty} du\, \langle A^2\rangle \exp(-|u|/\tau_d) \\ &= 2\tau\tau_d\langle A^2\rangle. \end{aligned} \tag{20}$$

The replacement of the integration limits by $\pm\infty$ is justified by the fact that $\langle A(t)A(t + u)\rangle$ is virtually zero at $u = \tau \gg \tau_d$. Combining (20) with (17) and (6) we obtain

$$\bar{\lambda} = \chi(\tau_d/\tau_t)^{1/2}. \tag{21}$$

Let us now consider the case that $\tau_d \uparrow \tau_t$. In this situation any dynamical perturbation becomes resonant with its thermal accommodation, i.e., the photosphere is practically released from the star and the mass flux reaches its maximum possible value. The upper limit to the mass flux for *any* star is given by $\lambda \simeq 1$ (Williams, 1967; Thomas, 1973). Note that this λ refers to a physical (normal) average and not to the r.m.s. value $\bar{\lambda}$. As derived from the Langevin equation (5), the normal average of λ differs from $\bar{\lambda}$ and decays with the thermal time scale; the author is indebted to R. J. Takens for his analysis of this point. This formal inconsistency can possibly be removed by starting from a mathematically different but physically similar proposition. By identifying $\bar{\lambda}$ with the physical average λ, we adopt the scaling (21) to hold for λ also. The upper limit $\lambda \simeq 1$ implies that the susceptibility

$$\chi \simeq 1, \tag{22}$$

entirely in agreement with hypothesis (7). It should be noted, that by having $\tau_d \uparrow \tau_t$, we violate the linearity of the response adopted in (6) and the integration condition (8). The validity of (21) in the neighbourhood of this limit may thus be questioned. Our final result is of an extreme simplicity: namely,

$$\bar{\lambda} \simeq (\tau_d/\tau_t)^{1/2}. \tag{23}$$

In order to facilitate a comparison with empirical data on mass fluxes, we substitute in (23) the definitions of $\bar{\lambda}$, τ_d and τ_t given by the first three equations to obtain

$$\bar{\dot{M}} \simeq L^{3/2}(R/M)^{9/4}/G^{7/4}. \tag{24}$$

In this last equation we have left out the factor $(4/3\pi)^{1/4}$, which like χ is close to 1. As one can define τ_d and τ_t also with other numerical constants of the order of unity than we have done, we expect (24) to be uncertain by a factor 2.

4. Discussion

Let us first calculate the r.m.s. mass flux from the Sun: (24) yields 2.547×10^9 kg s^{-1} or $4.06 \times 10^{-14} M_\odot y^{-1}$. In the normal (low-speed) wind of 3.2×10^5 m s^{-1} at 1 a.u. from the Sun, one counts on the average 9×10^6 protons m^{-3} (Hundhausen, 1972). This amounts to a flux of 1.4×10^9 kg s^{-1}, which should be increased by the flux in high-speed streams. Thus the theoretical value is fairly close to the actual flux.

Using $\overline{\dot{M}}_\odot = 4.06 \times 10^{-14} M_\odot y^{-1}$ as a unit, we calculate the mass flux from stars, for which the luminosity, mass and radius are known. Table I lists $\log (\overline{\dot{M}}/\overline{\dot{M}}_\odot) = 3/2 \log (L/L_\odot) - 9/4 \log (M/M_\odot) + 9/4 \log (R/R_\odot)$ with values according to Allen (1973). In Figure 2 we show the dependence on the luminosity of the stars, which, as remarked in the Introduction, should clearly correlate with the mass flux. *The dependence is not linear and it is different for different luminosity classes.* On the main sequence (label V), where $M \propto L^{0.290}$, and $R \propto L^{0.208}$, the mass flux scales according to

$$\overline{\dot{M}}/\overline{\dot{M}}_\odot = (L/L_\odot)^{1.32}, \tag{25}$$

as can be verified by inserting these power laws in (24). For giants (label III) one has a stronger dependence, roughly with L^2. For supergiants (label I) the temperature appears to be an important parameter: if it is below 5000 K, the dependence for giants is roughly followed, and if it is above 10 000 K, the mass flux is roughly independent of L.

We now compare these theoretical results with the data, also shown in Figure 2, derived from various observations. We distinguish between the flux from (i)

TABLE I

Log $(\overline{\dot{M}}/\overline{\dot{M}}_\odot)$ derived from (24) and Allen's (1973) table on L, M and R with spectral class; I = supergiant, III = giant, V = Main Sequence

Sp	I	III	V
O5			7.76
B0	7.20		5.60
B5	7.43		3.83
A0	7.35		2.60
A5	7.35		1.77
F0	7.43		0.98
F5	7.73		0.53
G0	7.95	2.40	0.11
G5	7.95	2.83	−0.15
K0	8.55	3.25	−0.51
K5	9.45	3.88	−1.13
M0	10.13		−1.51
M2	10.65		−2.00
M5			−2.77

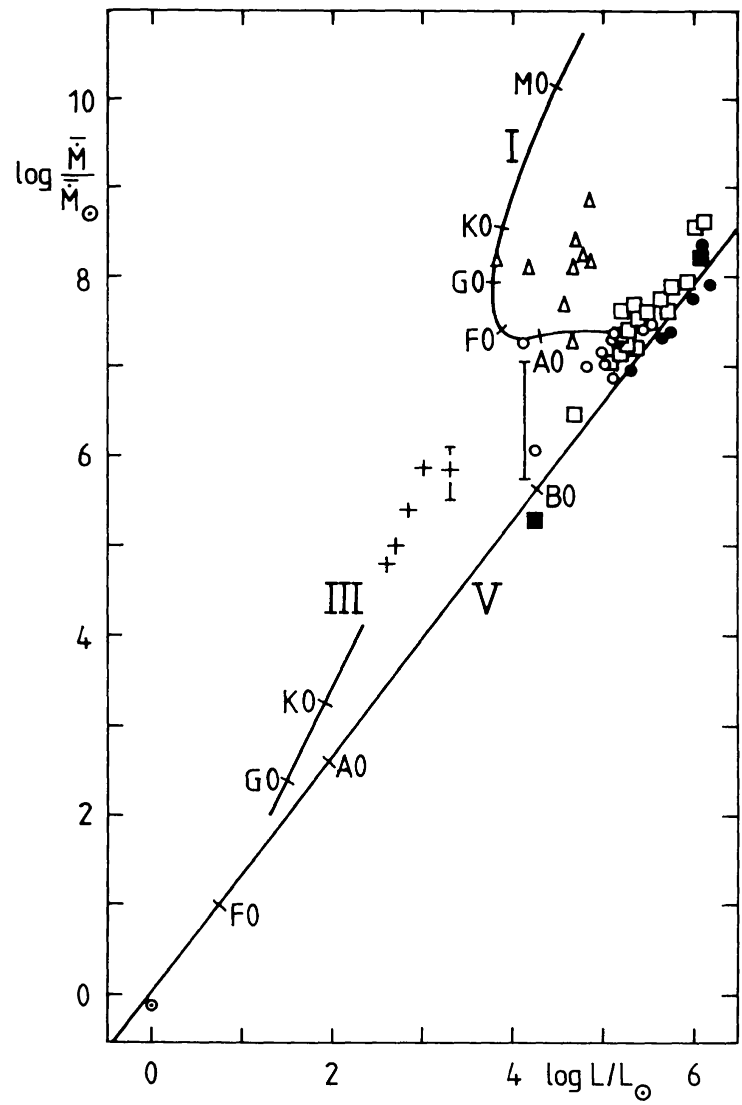

Fig. 2. Mass flux as a function of the luminosity as predicted by (24) (full lines). The labels I, III and V refer to supergiants, giants and main-sequence stars, respectively; in the latter case the straight line follows (25). Markings are given for the different spectral types. The data are due to the following authors: ●: Barlow and Cohen (1977), ■: Lamers and Morton (1976) and Lamers and Rogerson (1978), +: Reimers (1975), + with error bar: Reimers (1977), □ (B0 to B6-stars) and ○ (B7 to A5-stars): Barlow and Cohen (1977), vertical error bar: Lamers *et al.* (1978), △: Gehrz and Woolf (1971). They are discussed in the text.

main-sequence stars, including O-supergiants, (ii) giants, (iii) BA-supergiants and (iv) late-type supergiants, and will discuss these in turn.

(i) The pertinent data points in Figure 2 have black symbols and they are fitted, within a factor of about 2, by relation (25). The B0 V-star τ Sco has been extensively studied by Lamers and Rogerson (1978), who derived a mass flux of $(7.0 \pm 1.6) \times 10^{-9}\ M_\odot y^{-1}$, which amounts to $\log(\bar{\dot{M}}/\dot{M}_\odot) = 5.24 \pm 0.11$. The corresponding value in Table I is 5.60 and the discrepancy is a factor 2.3. Part of this discrepancy can be due to the observational fact that the star is seen on its pole, so that there is less than average gas in the line of sight contributing to the spectral lines, from which the mass flux is inferred. Lamers and Rogerson noted that τ Sco misses by far the empirical law, derived by Barlow and Cohen (1977), for B-supergiants. This underlines our result that the luminosity class is important. As O-supergiants have not yet evolved far from the main sequence, however, we may include them in class V. Indeed there is a good agreement of our main-sequence result with the data by Barlow and Cohen for these stars. This is also true for the value of log $(\overline{\dot{M}}/\overline{\dot{M}}_\odot) = 8.26 \pm 0.15$ for the O4-star ζ Pup, derived by Lamers and Morton (1976), where we have, extrapolating in Table I, a value of about 8.0.

Thus, result (25) seems to hold over a luminosity range of 10^6. This contrasts with earlier attempts to scale the loss from stars similar to the Sun to that from the Sun. For instance, de Loore (1977) refers to old calculations of his giving log $(\overline{\dot{M}}/\overline{\dot{M}}_\odot) = 1.34$ and 0.64 for F0 V- and A5 V-stars, respectively (cf. Table I). This expected turning down of the mass flux with the luminosity, based on the reduction of acoustic flux from the outer convective zone, probably does not take place. Otherwise the mass flux from τ Sco would be many orders of magnitude below its above, empirical value. There will be forms of gravitational energy different from that in acoustic waves, which perturb the state of partial equilibrium in the photosphere of those hotter stars.

(ii) The pertinent data points in Figure 2, which refer to M-giants (Reimers, 1975), are indicated by crosses. They lie quite precisely in the extension of the values calculated for GK-giants. The mass flux from the M5 II-star α^1 Her, which may not differ much from that of an M5 III-star, has been thoroughly studied by Reimers (1977). He finds log $(\overline{\dot{M}}/\overline{\dot{M}}_\odot) = 5.82 \pm 0.3$ and this value is represented in Figure 2 by a cross with error bar. Extrapolating our line III to the luminosity of this star we predict 6.0, which is within the error. As for M-giants λ, defined by (3), is of the order of 10^{-5} (Reimers, 1975), we have, by equating (23) to this value, the estimate that $M/R \simeq L^{2/5}/(\lambda^{4/5}G^{3/5})$ $\simeq 10^{22}$ kg m^{-1} or 3 $M_\odot/R_\odot$. For K-giants this quantity is thought to be about 0.2 $M_\odot/R_\odot$ (Allen, 1973). Our result raises the question whether giants of the size prescribed by their luminosity and temperature are an order of magnitude *heavier* than presently thought, or that our mass loss equation is not applicable to these stars.

(iii) B-A supergiants are indicated in Figure 2 by open squares and circles; these data are due to Barlow and Cohen (1977). Furthermore there is an error bar for the A2 I-star α Cyg due to Lamers *et al.* (1978). The data cluster along the main-sequence result, all being somewhat *higher*. This is in the direction suggested by curve I. However, the points scatter much around the constant level log $(\overline{\dot{M}}/\overline{\dot{M}}_\odot) \simeq 7.4$ predicted by (24). It should be noted, that the results of Barlow and Cohen are based on a hardly justified scaling of velocity fields to that of P Cyg, which may cause part of this scatter. Furthermore these stars do not exactly belong to the same luminosity class. It can be inferred from (24) that differences in Ia, Ia–ab, Iab and Ib give significant differences in the theoretical mass flux. Physically this means that of stars of the same temperature the heavier ones lose more mass. If this explanation is accepted, the early-type supergiants seem to obey (24).

(iv) The data on late-type supergiants in Figure 2, due to Gehrz and Woolf (1971), are indicated by open triangles. It is here that we meet really serious discrepancies between our theory and the mass flux derived from observations. In this case we have to do with infrared observations of dust shells around the cool supergiants; the various assumptions in the derivation of mass fluxes from such infrared data (in particular about the dust- and gas-distributions) may cause an error of one order of magnitude, but not more. However, only for the G8 I-star AX Sgr, which with its $\log(\bar{\dot{M}}/\bar{\dot{M}}_\odot) = 8.2$ falls right on the calculated curve, there is no discrepancy. In the other cases the fluxes are up to 3 orders of magnitude below the calculated value. The M4 I-star S Per has with $\log(\bar{\dot{M}}/\bar{\dot{M}}_\odot) = 8.8$ the largest mass flux of our sample, whereas Table I suggests that this value ought to be about 11. [An even larger discrepancy exists for the M2 I-star α Ori, which has a loss of 7.25 against, theoretically, 10.65. However, more recently Bernat (1977) has derived the higher value 8.9; for μ Cep, which is also M2 I, he found 10.0]. An intermediate case is offered by the K5 I-star RW Cep with 8.10 and 9.45, respectively. As the discrepancy seems to increase when the temperature decreases, a correlation is suggested with the evolution time from the main sequence.

Does (24) not apply to red supergiants, or are they *lighter* than expected? De Loore *et al.* (1977) have argued that the evolution of heavy, luminous stars off the Main Sequence is drastically changed if mass loss is taken into account. While losing much of their mass, the luminosity is only little reduced, so that these stars are too bright for their mass or too light for their luminosity. This means that the values we used for L, M and R of late-type supergiants, based on computations of the evolution without mass loss, probably are in error. Explanation of the above discrepancy in mass flux of 3 orders of magnitude at a given L, would imply that the error in $\log(R/R_\odot) - \log(M/M_\odot)$ is $4/9 \times 3 = 1.33$. As an example we note that for a K5-star the change of $\log(M/M_\odot)$ by 1.36 corresponds to a change of $\log(R/R_\odot)$ by 2.73 (Allen, 1973), where the net difference is 1.37. Thus it seems that a loss of, e.g., 95% of the initial mass might explain the largest discrepancy; and the loss of a smaller fraction of the mass might explain the smaller discrepancies. This would fit in the above-mentioned correlation with the evolution. However, because of a lack of more accurate data on L, M and R, we cannot quantitatively prove or disprove the validity of (24).

Having compared the predictions of our theory with empirical data on the mass flux, we conclude that the overall agreement is good, with a possible exception for late-type (super) giants. In many significant cases the agreement is within a factor of 2. Thus we find a good deal of truth in the idea that the mass-loss process is nothing but a fluctuation about the quasi equilibrium in the stellar atmosphere. Furthermore, the agreement sheds a strong light on the validity of hypothesis (7). That the solar wind and the mass flux from all other stars are, to quote Fusi-Pecci and Renzini (1975), 'basically governed by the same type of physical process, is extremely fascinating'.

Acknowledgement

This work, originating from the author's interest in the condensation of solid particles in gas streams from stars, owes much to discussions and correspondence with A. G. Hearn, H. J. G. L. M. Lamers, C. de Loore and A. B. Underhill, whereas the lively interest of S. R. Pottasch has been decisive in completing it.

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