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SUBSOLIDUS CONVECTION IN THE MANTLES OF TERRESTRIAL PLANETS

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INTRODUCTION

Each of the terrestrial planets, Mercury, Venus, Earth, Moon, and Mars, is undoubtedly evolving toward a state of eventual quiescence, when the gravitational potential energy made available by accretion and differentiation (principally core formation), and the heat supplied by the decay of radioactives (and the energy from other possible sources), have been lost from the interior (Kaula 1975). The rate at which a planet evolves toward its inevitable fate is determined largely by its original allocation of energy sources and the efficiency of the mechanisms which transfer heat from its interior to its surface. The most efficient of these processes may be heat transport by solid state mantle convection. Our main purpose in this review is to discuss the possible dominant role of this cooling mechanism in the past and present thermal states of the terrestrial planets.

There is, we should point out, no observational evidence that absolutely requires subsolidus convection in the interiors of any of the terrestrial planets except the Earth (Phillips & Ivins 1979). In the case of our own planet, a solid mantle and a plate tectonic structure together provide incontrovertible evidence of past and present solid state mantle convection. The moving plates themselves are an integral part of the mantle convection system (Turcotte & Oxburgh 1967). However, plate tectonics is irrefutable proof of subsolidus convection only in the uppermost mantle of the Earth, and just as we cannot prove the occurrence of convection in the interiors of the other terrestrial planets, we cannot demonstrate that convection is taking place or has occurred throughout the entirety of the Earth's mantle. In fact, the issue of shallow vs deep mantle convection in the Earth is much debated in the current literature (see, for example,

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Tozer 1972a and McKenzie & Richter 1976, who argue that convection associated with plate motions does not penetrate below 700 km, and Sammis et al 1977, O'Connell 1977, and Davies 1977, who take the opposite view).

Because the surfaces of Mercury, Moon, and Mars show no evidence of plate tectonics (Solomon 1978), convection in their interiors is largely inferred from their geometrical and dynamical figures. In a long series of papers, Runcorn (1962, 1967a, 1975) repeatedly points out that the nonhydrostatic geometrical and dynamical ellipticities of the Moon, together with the larger value of the former, can be viewed as evidence of present day convection in the lunar interior. The density beneath the bulge toward the Earth would have to be smaller than the density beneath the lunar limb to reconcile the larger geometrical ellipticity with the smaller dynamical value. Such density differences could readily be associated with temperature differences between rising and descending regions of lunar convection. However, recent numerical calculations of the dynamical ellipticity of the Moon due to finite amplitude solid state convection (Cassen, Young & Schubert 1978) show that convection could be the cause of the nonhydrostatic dynamical figure only if the lunar lithosphere were capable of resisting global scale deformation. Lithospheric inhomogeneities and surface loads then could contribute substantially to the disequilibrium of the dynamical figure (Melosh 1975). In addition, Kuckes (1977) showed that an elastic lithosphere as thin as 100 km could support the stresses associated with the disequilibrium, regardless of the state of the deeper lunar interior. Runcorn used closely related arguments to support convection in the other terrestrial planets. He suggested that the long wavelength undulations in the Earth's geoid reflect density variations associated with convection (Runcorn 1967b). Most recently, he proposed that convection in Mercury could lead to the departures from hydrostatic equilibrium apparently required by Mercury's resonant state of rotation (Runcorn 1977).

Another geometrical indication of interior convection comes from the offset of the center of figure from the center of mass in planets; Venus, Earth, Moon, and Mars are known to possess such offsets. The most straightforward and reasonable interpretation of these offsets is that they arise from a first harmonic variation in crustal thickness (Kaula et al 1972). Thus the offsets imply global differentiation of a crust, and, according to Lingenfelter & Schubert (1973), a mantle convective system at the time of crustal formation to accumulate crust preferentially in the hemisphere above the region of descending flow.

In our view, the global characteristics of internal mass distributions, as reflected in geometrical and dynamical figures and gravitational fields, argue convincingly for past or present convection in the interiors of the other terrestrial planets. Other more debatable lines of evidence are described in detail by Phillips & Ivins (1979).

Aside from the occurrence of plate tectonics on Earth, the laboratory demonstration that rocks creep while solid is the strongest reason we have for believing in the relevance of solid state convection to the interiors of the planets. Under certain conditions of temperature T, pressure p, stress, and strain rate, we know quantitatively how strain rate depends on T, p, and stress (see Heard 1976 and Carter 1976 for recent summaries of the laboratory data). Unfortunately, all the conditions in planetary interiors, especially the very low strain rates characteristic of mantle convection, cannot be reproduced in the laboratory. Under conditions relevant to the deep interior of the Earth and the interiors of the other planets, rocks may be sufficiently resistant to deformation that subsolidus creep cannot occur.

In terms of the extrapolation of the laboratory creep behavior of rocks to the temperatures and pressures encountered in planetary interiors, the deep mantles of Earth and Venus present greater uncertainties than do the interiors of Mercury, Moon, and Mars, because of the major silicate structural changes that occur only in the mantles of the larger planets. As an example, Tozer (1972a) and McKenzie & Weiss (1975) have asserted that the breakdown of spinel to a denser assemblage at a depth of about 650 km in the Earth provides an effective rheological barrier which prevents upper mantle convection from extending to much greater depth. On the other hand, Sammis et al (1977) argued that the change in effective viscosity across this phase change must be so small that it could not seriously impede whole mantle convection. Inferences of a nearly uniform viscosity for the Earth's mantle from glacial rebound observations (Cathles 1975, Peltier 1976) support the latter view.

While the ability of rocks to creep is a necessary prerequisite for convection in the planets, it does not guarantee that the process will actually occur. Other crucial factors include the amount and distribution of energy sources as reflected in the superadiabatic temperature gradient, the direct driver of convective circulation. Buoyancy forces tending to drive convection must be able to overcome the viscous forces tending to resist motion. We need, finally, a theoretical statement for the onset of instability, i.e. a way of quantitatively evaluating the net outcome of the force competition. Stability criteria are available for fluids confined to a plane layer or spherical shell, heated from below or from within containing phase changes, viscosity variations, and chemical compositional gradients. In general, when reasonable estimates of thermal, mechanical, and rheological properties are used to evaluate these stability criteria, it is found that solid state convection should occur in the interiors of the terrestrial planets. This exercise was first carried out for the Earth's upper mantle by Pekeris (1935), Knopoff (1964), and Tozer (1965a) using the stability criterion for a constant viscosity plane fluid layer heated from below. Schubert, Turcotte & Oxburgh (1969) developed the stability criterion for a viscously stratified fluid layer heated from below and applied it to the entire mantles of Earth, Venus, Mars, and Moon. This stability calculation and others we discuss later support vigorous whole mantle convection in the planets. On the other hand, an adverse chemical compositional stratification in any of the terrestrial planets would be a strong deterrent against convection in that planet (Richter & Johnson 1974). Also, since adiabatic temperature gradients are especially uncertain in the lower mantles of the large planets Earth and Venus, whole mantle convection in these bodies could have been or could now be retarded by subadiabatic deep mantle conditions (Sharpe & Peltier 1979).

In summary, there are three major reasons for believing that the thermal and dynamical states of all the terrestrial planets are controlled by the solid state convective transport of heat. First and most important is the proof of convection in the Earth's upper mantle provided by plate tectonics. Second is the demonstration of the subsolidus creep deformation of rocks in the laboratory. Third are the theories which predict vigorous whole mantle convection based on reasonable and generally accepted values of material properties. Since the last two reasons are based on extrapolation and theory, and therefore subject to obvious weaknesses, we should recognize that solid state convection could be a phenomenon unique to the Earth's upper mantle, its occurrence therein entirely dependent on the presence of volatiles (water in particular), for example.

We do not intend to make the question of the existence of whole mantle convection in the Earth and the other terrestrial planets the central theme of this review. Indeed, we have long advocated the importance of whole mantle convection in planetary interiors (Schubert, Turcotte & Oxburgh 1969). Our main purpose here is to discuss the physical aspects of whole mantle convection, assuming that it occurs, and to describe how it will control the thermal evolution of a planet. Further, we do not intend to produce a comprehensive review of the many specific models of convection in the Earth's upper mantle. Recent reviews by Oxburgh & Turcotte (1978) and Richter (1978) have concentrated on shallow mantle convection in the Earth; we emphasize the role subsolidus whole mantle convection could play in the terrestrial planets.

RELEVANT PHYSICAL PROPERTIES OF PLANETARY MANTLES

Before beginning our discussion of convection in the planets we give a brief summary of what we know about the relevant physical parameter values. Thermal expansivity α, thermal diffusivity κ, thermal conductivity k, density ρ, and specific heat at constant pressure c_p are all known probably to within factors of two or three for the Earth's upper mantle (Turcotte & Oxburgh 1972, McKenzie, Roberts & Weiss 1974). We do not expect the values of these parameters to be very different for the mantles of the other planets or for the lower mantle of the Earth and therefore we adopt $\alpha = 3 \times 10^{-5}\text{K}^{-1}$, $\kappa = 0.01$ cm^2/s, $k = 0.01$ cal/cm s K, $\rho = 3.3$ g/cm^3, $c_p = 0.25$ cal/g K as representative for general discussions of convection in any planetary mantle.

Certain situations require more careful specification of one or more of these parameters. For example, due to phase changes and compressibility, the average density of the Earth's mantle increases with depth to a value of about 5.5 g/cm^3 at the core-mantle interface. Such a density stratification in itself may not significantly influence mantle convection, but an associated effect of compression, namely the adiabatic increase of temperature with depth, is of prime importance. The mantle temperature gradient must exceed the adiabatic temperature gradient over at least a portion of the mantle if convection is to occur. The magnitude of the adiabatic temperature gradient $\alpha g T/c_p$ (g is the acceleration of gravity) increases with depth due to the increase of T with depth, an effect which would tend to concentrate convection in the upper mantle if the increase were to become significant. Changes in α with depth, e.g. an increase in α by a factor of two or three, would directly influence the adiabatic temperature gradient. Whereas α tends to increase with T, it probably tends to decrease with p, leaving its depth dependence in a mantle uncertain due to these competing effects.

In regions of a planetary mantle where there are phase transitions, the effective c_p may increase due to the latent heat of the transformations. The phase change contribution to c_p in these regions is probably no larger than the specific heats of the individual phases (Schubert, Yuen & Turcotte 1975). The changes in density associated with phase transitions can result in effective thermal expansivities in two-phase regions one to two orders of magnitude larger than the ordinary values of α of individual phases (Schubert, Yuen & Turcotte 1975).

Table 1 gives values of g, the acceleration of gravity at the surface of a

Table 1 Characteristic parameter values for the planets

Planet	$g(\mathrm{cm/s^2})$	$T_s(°\mathrm{C})$	$D(\mathrm{km})$	Ra	Nu	Di
☿	370	100	640	10^4	2	10^{-1}
♀	890	500	3000	10^7	20	1
⊕	980	0	3000	10^7	20	1
☾	160	0	1740	10^5	5	10^{-1}
♂	375	0	2030	10^6	10	10^{-1}

planet, T_s the surface temperature, and D the thickness of the mantle, for Mercury ☿, Venus ♀, Earth ⊕, Moon ☾, and Mars ♂. Surface gravity values are well known. Surface temperatures are 0°C except for ☿ and ♀, for which the appropriate values are about 100°C for ☿ (Cuzzi 1974) and about 500°C for ♀ (Marov 1972). The value of D for the planets is somewhat uncertain, except of course for ⊕. The moment of inertia of the Moon is so close to that of a homogeneous sphere (Williams et al 1974, Blackshear & Gapcynski 1977) that if the Moon has an iron core its radius is limited to about 500 km (Kaula et al 1974, Dainty et al 1974). We used the entire radius of the Moon for its value of D. The observed mass and radius of Mars, and the value of its moment of inertia inferred from observations of J_2 and the assumption of hydrostatic equilibrium (Reasenberg 1977) constrain models of its internal density structure. The value of D for Mars assumes that the planet has a core whose radius is 0.4 times the planetary radius (Reynolds & Summers 1969, Anderson 1972, Binder & Davis 1973, Johnston & Toksöz 1977). Observations of the mass and radius of Mercury and Venus constrain models of their internal density structures. The value of D used for ☿ is based on the assumption of a core with radius 0.75 times the planetary radius (Siegfried & Solomon 1974, Gault et al 1977). The value of D for ♀ is based on the planet's similarity to Earth (Toksöz & Johnston 1977, Ringwood & Anderson 1977). The uncertainties in the values of D for most of the terrestrial planets are not significant for our purposes. Table 1 also contains approximate values of several dimensionless parameters which are important in assessing the vigor of convection. These will be discussed later.

The most uncertain and yet most important of the mantle properties relevant to convection are the rheological properties and the radiogenic heat source concentrations. One must admit to order of magnitude(s) uncertainties in these properties, especially when the terrestrial planets other than Earth are considered. The uncertainties and the overriding importance of mantle rheology and energy sources are discussed in the following two sections.

Mantle Rheology

Of all the thermodynamic and mechanical parameters influencing mantle convection, the rheology of the mantle is perhaps the most important. There are a number of excellent and relatively recent papers on the rheological behavior of rocks (Stocker & Ashby 1973, Weertman & Weertman 1975, Heard 1976, Carter 1976).

Subsolidus deformation of rocks is due to the motions of either point defects, such as vacancies and interstitial atoms, or line defects such as screw or edge dislocations. The volumetric diffusion of point defects through mineral grains results in flow called Nabarro-Herring creep. The surface diffusion of point defects in grain boundaries produces deformation known as Coble creep. The glide motion of dislocations yields a deformation referred to by that name, while the ability of dislocations to both climb and glide results in deformation generally referred to as dislocation creep.

Nabarro-Herring and Coble creep give linear or Newtonian constitutive equations (stress $\tau \propto$ strain rate e). Since diffusion is a thermally activated process the viscosity μ for Nabarro-Herring or Coble creep is temperature and pressure dependent according to

$$\mu \propto \exp\left(\frac{E^* + pV^*}{\mathrm{R}T}\right), \tag{1}$$

where R is the universal gas constant and E^* and V^* are the activation energy and activation volume, respectively, for the relevant diffusion process (Nabarro 1948, Herring 1950, Coble 1963). It is uncertain whether Nabarro-Herring or Coble creep can occur in a planetary mantle. Indeed, these creep mechanisms have never been identified in laboratory deformations of rocks. If they occur at all, they might govern deformation in a mantle at low stresses. It is possible to choose reasonable values of E^*, V^*, and other microscopic parameters which influence diffusion to yield reasonable estimates of mantle viscosity (Gordon 1965, Tozer 1965a, Turcotte & Oxburgh 1969a).

In Nabarro-Herring creep, grains change their shape but not their nearest neighbors. Thus the mechanism is fundamentally non-steady, since the grains would continue to elongate with sustained deformation (Weertman 1968, Green 1970). It is possible that grains can also slide past one another, changing their neighbors and altering their shapes (by diffusion) only insofar as it is necessary to maintain continuity (Green 1970, Ashby & Verrall 1973). This grain boundary sliding mechanism is believed responsible for the phenomenon of superplastic flow in metals. Superplastic creep can be linear or nonlinear according to whether the grain size is independent of or dependent on the stress. Twiss (1976) has discussed the possibility that superplastic creep might occur in the mantle.

While a form of diffusion creep may govern deformation in the mantle at low stresses, the motion of dislocations by glide and climb may control creep in the mantle at intermediate stresses and over a wide range of stress. Dislocation creep is known to govern the steady state deformation of olivine at high temperature and pressure and at laboratory strain rates. The experimental data on the creep of olivine below about 2 kbar differential stress are consistent with the power law

$$e = \frac{B_n}{T} \exp\left\{\frac{-(E^* + pV^*)}{RT}\right\} \tau^n. \tag{2}$$

Kohlstedt & Goetze (1974) and Kohlstedt, Goetze & Durham (1976) have determined the values of the power law exponent n and the activation energy E^*; they find $n = 3$ and $E^* = 125$ kcal/mol. The T^{-1} dependence multiplying the exponential has not been resolved experimentally; it has only been inferred on theoretical grounds (Weertman 1970; this results in some uncertainty in determining a value of B_3 from the data). The activation volume V^* for the creep of olivine has recently been determined by Ross, Avé Lallement & Carter (1978). They give V^* values between 10.6 and 15.4 cm^3/mol with a mean value of 13.4 cm^3/mol. This is near the theoretical value of about 11 cm^3/mol appropriate if O^{2-} ion diffusion is rate-controlling.

A number of theoretical arguments have indicated that V^* should have such a low value. Sammis et al (1977) estimated V^* using Keyes' (1963) relation between activation energy and activation volume, together with the measured value of E^* for olivine and values of elastic constants either determined in the laboratory or inferred from seismic data. They predicted that at low pressure, V^* would lie between about 8 and 11.5 cm^3/mole, depending on T. Another estimate of V^* can be made from the connection between E^*, V^*, and melting temperature (Weertman 1970). From estimates of melting temperature and its pressure derivative, Sammis et al (1977) derived $V^* \approx 11.5$ cm^3/mole. Models of temperature and flow in the Earth's upper mantle using the olivine flow law (2) have yielded geophysically more reasonable results when low values of V^* (about 15 cm^3/mole) were used to calculate the effective viscosity (Froidevaux & Schubert 1975, Schubert, Froidevaux & Yuen 1976, Schubert et al 1978).

If the above constitutive relation also describes the deformation of olivine at the much slower mantle strain rates, then it is probably the relevant rheological law for at least the Earth's upper mantle, since there is abundant seismologic and petrologic evidence that the upper mantle is predominantly olivine of the approximate composition $(Mg_{0.9}Fe_{0.1})_2SiO_4$. The similarities in the results of optical and electron transmission microscopy studies of both mantle-derived and laboratory-deformed olivine crystals (Raleigh 1968, Phakey, Dollinger & Christie 1972, Goetze & Kohlstedt 1973) lend support to this conclusion. Further, since pressures throughout most of the mantles of the smaller terrestrial planets are not higher than those encountered in the Earth's upper mantle, the olivine flow law may be relevant throughout these mantles rather than just in their upper portions. The major phase changes encountered in the Earth's mantle, and presumably in Venus' as well, preclude the direct applicability of olivine flow laws to the lower mantles of these planets.

It should also be noted in connection with the issue of the applicability of the above power law constitutive relation to flow in the Earth's mantle, that deviatoric stresses in the mantle are expected to be smaller than kilobars on the basis of variations in the gravity field (Kaula 1963a, Lambeck 1976).

Since laboratory measurements of creep in rocks are not directly applicable to the mantles of the terrestrial planets, it is desirable to infer mantle creep laws directly from geophysical and geological observations. The vertical motions of the Earth in response to the redistribution of surface loads (e.g. by deglaciation) offer such a possibility although the time scale of these motions (10^3–10^4 yr) is much shorter than geologic time. The observations of such movements cannot presently distinguish among the various linear and nonlinear creep laws that have been proposed for the Earth's mantle. One may also hope that characteristics of tectonic plates, e.g. their velocities, and other plate tectonic data, e.g. heat flow and topography vs age, will distinguish between linear and nonlinear mantle creep laws. However, temperature and flow models of the upper mantle show that this is not the case (Froidevaux & Schubert 1975, Schubert, Froidevaux & Yuen 1976).

Thus, at present, we must hypothesize a mantle flow law based on theoretical mechanisms of deformation or extrapolate empirical flow laws from laboratory circumstances to conditions relevant to the mantle. Mantle temperatures and pressures are accessible in the laboratory, but relevant mantle strain rates, between about 10^{-16} s^{-1} and 10^{-12} s^{-1} are not. Laboratory strain rates are in the range 10^{-8} to 10^{-3} s^{-1}.

Although we cannot establish which flow law is relevant to a planetary mantle, it may not be essential for us to do so. All mechanisms of creep deformation have a dependence of viscosity (or effective viscosity) on T and p given by (1). According to model calculations of flow and temperature in the Earth's mantle, the dependence of effective viscosity on temperature and pressure far outweighs any influence of a possible nonlinear connection between stress and strain rate (Froidevaux & Schubert 1975, Schubert, Froidevaux & Yuen 1976, Yuen & Schubert 1976, Parmentier, Turcotte & Torrance 1976). Later in this review we discuss in more detail the overwhelming influence of the temperature dependence of the viscosity on the thermal histories of planets cooling by subsolidus mantle convection. Thus we will no longer be concerned with which deformation law is applicable to the mantle. Rather, we will employ the concept of a viscosity or effective viscosity (stress/twice the strain rate) which is T and p dependent according to (1).

If activation energy and activation volume were constants, then viscosity would tend to decrease with depth due to the increase of temperature and increase with depth due to the increase of pressure. In a large planet, the temperature effect would tend to predominate at shallow depths where the temperature increase is especially pronounced and the pressure is relatively small, and the pressure effect would take over at sufficiently large depth where the temperature would tend to increase relatively slowly along an adiabat. The net result would be a viscosity which had a minimum somewhere in the upper mantle, depending on the actual values of E^* and V^*. This is undoubtedly the situation, qualitatively at least, with the Earth's asthenosphere, the region of viscosity minimum, serving to decouple the rigid surface plates from the underlying mantle (Froidevaux & Schubert 1975, Schubert, Froidevaux & Yuen 1976). The inference from glacial rebound studies is that the viscosity of the Earth's mantle does have a minimum in the upper mantle, but that there is at best only a modest increase of viscosity with depth throughout the entire mantle; a Newtonian viscosity of 10^{22} poise is consistent with the glacial rebound observations (Cathles 1975, Peltier 1976). Weertman (1978) has argued that, because of the small strains involved, glacial rebound may be a transient creep phenomenon, in which case the rheological inferences therefrom would not pertain to steady creep on geologic time scales. However, we basically adopt the view of mantle viscosity inferred from glacial rebound data.

How then are we to reconcile the inference of nearly uniform mantle viscosity for the Earth with the viscosity dependence on T and p given in (1)? We described just above how μ would tend to increase in the Earth's lower mantle if the temperature rose along an adiabat. The pressure dependence of μ would dominate in the lower mantle and viscosity would increase with depth at a rate inconsistent with glacial rebound inferences. The answer lies in the variations of E^* and V^* with depth and across phase transitions. It is possible to construct theoretical estimates of the depth dependences of E^* and V^* and the changes in these quantities across phase transitions which result in little increase in viscosity with depth in models of the Earth's mantle (Sammis et al 1977). A major factor in limiting the increase in viscosity with depth is an inferred decrease in V^* with depth to values of only 4–6 cm^3/mol in the Earth's lower mantle (Sammis et al 1977, O'Connell 1977). Viscosity changes across any of the major upper mantle phase transitions are inferred to be less than an order of magnitude (Sammis et al 1977). There is no inconsistency in an adiabatic, nearly uniform viscosity mantle even though viscosity may be strongly temperature and pressure dependent according to (1).

Mantle Heat Sources

The average heat flow through the Earth's surface, about 1.5 μcal/cm^2 s, has been generally believed to originate mainly in the mantle from the decay of radioactive elements uranium, thorium, and potassium (Turcotte & Oxburgh 1972, Oxburgh & Turcotte 1978). Further, it has been argued that it is reasonable to assume a nearly steady state balance between mantle and crustal heat production and heat flow through the Earth's surface if convection is the fundamental mode of heat transport (Tozer 1965a, Turcotte & Oxburgh 1972). If this is the case, then the average rate of heat generation in the Earth's mantle and crust is 0.084×10^{-13} cal/cm^3 s (Oxburgh & Turcotte 1978). A number of recent studies have indicated, however, that these ideas may require revision; in particular, even with convective heat transport in the Earth's mantle, the average rate of mantle and crustal heat production may be considerably less than 0.084×10^{-13} cal/cm^3 s. In fact, Sharpe & Peltier (1978) have shown that it is possible to construct a reasonable thermal history, including cooling by subsolidus mantle convection, in which the Earth model, devoid of mantle radioactivity, evolves to a state consistent with present observations of surface heat flow (and other constraints). Although the Earth is probably not completely devoid of mantle radioactivity, it is highly possible that the average mantle concentration of radioactives is smaller, by as much as a factor of two, than the value indicated by the present surface heat flow.

The Moon is the only other terrestrial body for which we have measurements of surface heat flux. The two values of heat flow reported by Langseth, Keihm & Peters (1976) are 0.5 μcal/cm^2 s and 0.38 μcal/cm^2 s. Detailed analyses of topographic effects indicate that 0.33 μcal/cm^2 s, rather than 0.38 μcal/cm^2 s, might be more representative of the regional heat flux at the Taurus-Littrow site (Langseth, Keihm & Peters 1976). If a heat flux intermediate between the two in situ determinations, i.e. about 0.4 μcal/cm^2 s, is representative of the mean lunar surface heat flow (Langseth, Keihm & Peters 1976 argue that this may be the case), then the average heat source concentration throughout the Moon, assuming a steady state relation between production and loss, is about 0.07×10^{-13} cal/cm^3 s, similar to the value computed for the Earth's crust and mantle under the steady state assumption. Langseth, Keihm & Peters (1976) in fact employ the steady state approach to estimate uranium concentration in the Moon from the assumed mean surface heat flow. The steady state assumption is highly questionable, as we have noted, and the average lunar radiogenic heat source concentration can be much less than the value given above.

There are several reasons why the heat produced by radioactivity in a planet's interior may be substantially less than the heat flowing through its surface. First, primordial heat, i.e. energy of accretion and gravitational potential energy released in core formation, can contribute to the surface heat loss. Schubert, Cassen & Young (1979a) have quantitatively modelled the cooling of terrestrial planets without any sources of energy other than an initial thermal energy resulting from accretion and core formation. Even with heat removal from the mantle by vigorous convection, there is enough thermal energy remaining after 4.5 billion years of cooling to supply as much as a third of the presently observed surface heat flows on Earth and the Moon. Similar conclusions have been reached in recent studies by Cassen et al (1979), Stevenson (1979), and Stevenson & Turner (1979). Second, there is a fundamental lag between the decay of the surface heat flux on a cooling terrestrial planet and the decay of the heat flux into the base of the lithosphere (Schubert, Cassen & Young 1979a). This is due to the formation and thickening of the lithosphere. At any time in the cooling history of a planet there is more heat flowing through the surface than flowing into the base of the lithosphere. Even if convection in the mantle is in a quasi-steady state, the heat flow from the mantle is not in balance with the heat flow through the surface. The physical mechanism which accounts for the lag between "internal heat production" (heat from the mantle) and surface heat flux is the thickening of a rigid lithosphere, i.e. the tendency of the lithosphere to supplement the decaying mantle heat flux by feeding on the internal thermal energy of the mantle. Third, according to Daly & Richter (1978), reasonably vigorous convection may not be able to remove heat sufficiently rapidly from the central region of a system with decaying heat sources for surface heat flow to be in equilibrium with the instantaneous heat production. For all these reasons, the use of present day surface heat flux observations to infer the total concentration of radiogenic heat sources in a planet on the basis of a presumed steady state thermal balance must be viewed with caution.

Our discussion so far has emphasized the difficulty in using surface heat flow data to estimate the internal radiogenic heat content of a planet. Although these difficulties indicate that surface heat flow derived estimates of radioactive concentrations may be overly generous, there is no question that radioactive decay contributes substantially to planetary heat flow. Abundances of radioactive elements have been measured in rocks from the surfaces of Earth, Venus (Vinogradov, Surkov & Kirnozov 1973, Surkov 1977, Keldysh 1977), and the Moon (LSPET 1972, 1973). For Mercury and Mars we have only equilibrium condensation models of the solar nebula (see, for example, Lewis 1972, Grossman & Larimer 1974) with which to guess the radioactive elemental abundances.

In general, it is clear that radioactives have been concentrated in surface rocks by the processes which led to their differentiation; radioactive elements are concentrated in the partial melts of silicates which migrate toward the surface to form crustal rocks. From the known exponential decrease of radioactives with depth in the Earth's continental crust (Birch, Roy & Decker 1968, Lachenbruch 1968), Oxburgh & Turcotte (1978) have estimated that 9% of the Earth's heat flow originates there, leaving 0.076×10^{-13} cal/cm^3 s to be accounted for by radioactives in the mantle if the steady state production vs loss model is assumed. Oxburgh & Turcotte (1978) compare this latter value of mantle heat production with values that can be inferred from measurements of radioactivity in lavas and estimates of the percentage of melting of upper mantle source rocks, which presumably produced the lavas. The upper mantle sources of the lavas are estimated to have heat production rates between 2 and 4 times smaller than the surface heat flow derived average. Oxburgh & Turcotte (1978) also consider radiogenic heat production rates based on measurements of element abundances in xenoliths presumably originating in the Earth's upper mantle. The heat production rates in xenoliths are also generally less than the surface heat flow derived average. Oxburgh & Turcotte (1978) call upon prior episodes of partial melting to deplete the radioactives in the source rocks of abyssal tholeiites, thereby reconciling the low values of heat production in these lavas with the "expected" mantle-wide average. Perhaps these lower values of radiogenic heat production are more representative of the Earth's mantle than the surface heat flux derived value for the reasons stated above.

The distributions of radioactives in the mantles of the terrestrial planets are completely unknown. The processes, which concentrate radioactive elements in crustal rocks and lead to a stratified distribution of radioactives in the continental crust of the Earth, might be expected to produce similar upward concentrations of heat sources in a planetary mantle; in the extreme case, these processes could completely deplete a mantle in heat producing elements. Whole mantle convection, on the other hand, would tend to homogenize the heat sources if a quasi-steady circulation had sufficient time to be established.

We have already mentioned that the gravitational potential energy made available during accretion (Hanks & Anderson 1969, Mizutani, Matsui & Takeuchi 1972, Wetherill 1976, Weidenschilling 1976, Safronov 1979, Kaula 1979) and core formation are important energy sources for terrestrial planets. We are reasonably certain that all the terrestrial planets with the possible exception of the Moon have metallic cores. Core formation probably occurred very early in the histories of Earth and Mercury (Solomon 1979). By analogy with Earth, core formation would also be expected to occur early in Venus' evolution. Because of the overwhelming energy release during core formation, the Earth's core must have formed prior to the oldest known rocks presently at the surface; this would require core formation within the first 750 million years of the Earth's history (Moorbath, O'Nions & Pankhurst 1975). Other lines of evidence which suggest early core formation in the Earth include remanent magnetism in 2.7 billion year old rocks with paleointensities comparable to the Earth's present field intensity (Hanks & Anderson 1969) and the radiogenic nature of Pb in the average crust and mantle (Vollmer 1977). Core formation in the Earth is likely to have been a catastrophic or runaway process (Ringwood 1960, Tozer 1965b, Ringwood 1975) and the energy released could easily overshadow the contributions of the other sources mentioned above. For the Earth, core differentiation is estimated to have raised the temperature of the planet 2000°C (Birch 1965, Tozer 1965b).

Solomon (1977, 1979) argues that core formation in Mercury is likely to have occurred prior to 4 billion years ago because of the absence of tensional tectonic features in the old cratered terrain. The heavily cratered terrain on Mercury is estimated to be at least 4 billion years old by analogy with the probable age of the lunar highlands and the end of heavy bombardment in the inner solar system. Core formation in Mercury would lead to such a large increase in the planetary radius that tensional cracks would be expected to occur in the ancient cratered terrain if it existed prior to core formation. A global system of lobate scarps on Mercury indicates that the surface has been subjected to horizontal compressive stress throughout much of its history, consistent with a thermal evolution dominated by cooling from a hot initial state. On the other hand, the dominance of tensile features in the martian surface constrains the amount that Mars may have cooled over geologic time and suggests a relatively late core formation in that planet (Solomon & Chaiken 1976). Core segregation in Mercury could have raised the temperature by about 700°C (Solomon 1979, Toksöz, Hsui & Johnston 1978); the average temperature rise due to core formation in Mars is estimated to be about 200°C to 300°C (Solomon 1979, Toksöz & Hsui 1978).

The Moon shows neither the compressional features displayed by Mercury nor the tensional ones characteristic of Mars. However, the absence of these features in itself constrains the lunar thermal history; Solomon & Chaiken (1976) and Solomon (1977, 1979) have argued, on the basis of this evidence, that the early formation of a lunar core is not likely. However, there are other persuasive arguments indicating that the Moon may have formed a core very early in its history. These include the presence of remanent magnetization in a 4 billion year old lunar sample and the apparently global remanent magnetization of the entire lunar crust and the ancient farside highlands in particular (Runcorn 1976). In any case, the temperature increment associated with the formation of a small lunar core is only about 10°C (Solomon 1979), negligibly small for it to have any direct impact on thermal history. It is generally agreed however that the Moon was hot early in its history, at least in its outer several hundred kilometers because of the requirement of early differentiation of its crust (Toksöz & Johnston 1977). In the case of the Moon, accretional heating is the dominant early source of energy (Kaula 1979), not core formation, but the net result is that the Moon also undergoes a cooling history, at least in its outer regions. Solomon & Chaiken (1976) would argue that the lunar interior on the other hand has had to heat with geologic time by radioactive decay to offset the cooling of the outer parts so that no net lithospheric compression or tension would result.

Thus, for the Earth, Mercury, and perhaps Venus, core formation may have been the single event controlling early thermal evolution. Core formation in Mars may not have played such an important role early in martian history, while core formation in the Moon probably had negligible thermal consequences. For the Moon, accretional heating is probably the important early energy source. Thermal history calculations (Schubert, Cassen & Young 1979a, Cassen et al 1979, Stevenson 1979, Sharpe & Peltier 1979, Stevenson & Turner 1979) show that the energy released in core differentiation or accretion could still be escaping through the surfaces of the Earth and Moon (and perhaps those of the other terrestrial planets as well) in amounts competitive with heat originating from radioactive decay. Other energy sources which are generally believed to be of minor importance are tidal dissipation (Kaula 1963b, 1964, Burns 1976, Kaula & Yoder 1976, Peale & Cassen 1978) (however, see Turcotte, Cisne & Nordmann 1977 for a contrary view), short-lived radionuclides such as Al^{26} (however Runcorn 1976 has suggested that Al^{26} may be an important heat source for the Moon), and joule heating by solar wind driven planetary electrical induction currents (Sonett, Colburn & Schwartz 1975, Herbert, Sonett & Wiskerchen 1977).

DIMENSIONLESS PARAMETERS AND THEIR SIGNIFICANCE

The equations governing mantle convection are the equations of conservation of mass, momentum, and energy, the equation of state, and constitutive equations for the rheological and thermal parameters. Standard nondimensionalization procedures applied to these equations show that the behavior of a convecting system is governed by relatively few dimensionless combinations of parameters. The number of these dimensionless ratios (and their specific forms) is not unique, but depends, in particular, on the degree to which one simplifies the state and constitutive equations. If, as is often done, one assumes that the mantle is a Boussinesq fluid (one whose density can be assumed constant for all purposes except the calculation of buoyancy forces) with a constant Newtonian viscosity (and constant values of other relevant thermal and mechanical parameters) then there are only two dimensionless ratios that determine the form of mantle convection; these are the Rayleigh number Ra

$$\mathrm{Ra} = \frac{\alpha g \Delta T D^3}{\kappa \nu}, \tag{3}$$

and the Prandtl number

$$\mathrm{Pr} = \frac{\nu}{\kappa}. \tag{4}$$

With ν, the kinematic viscosity, equal to 10^{21} cm^2/s and $\kappa = 10^{-2}$ cm^2/s, we get $\mathrm{Pr} = 10^{23}$. The effectively infinite value of the Prandtl number implies that inertial forces are unimportant in planetary mantles, and that only pressure forces, buoyancy forces, and viscous forces need be included in the equations of motion. Once such a simplification is made, the Prandtl number no longer explicitly appears in the equations and the Rayleigh number is the single parameter governing the nature of convection. Ra is the ratio of the rate at which buoyancy forces do work on the flow to the rate at which energy dissipation occurs. The form of the Rayleigh number given in (3) is for a fluid layer with no heat sources across which a temperature difference ΔT is applied. Actually, only the temperature difference in excess of the value associated with adiabatic compression of the fluid can drive convection, and ΔT should be so interpreted. In classical fluid dynamical situations, the adiabatic increase of temperature is usually unimportant, but for planetary mantles it can be as large as 1000°C to 2000°C. In addition to heating from below, internal heat sources can drive convection in a fluid layer. In this case, Ra takes the form

$$\mathrm{Ra} = \frac{\alpha g Q D^5}{k\kappa\nu}, \tag{5}$$

where Q is the constant rate of internal heat production per unit volume.

The Rayleigh number must exceed a critical value $\mathrm{Ra_{cr}}$ before convection can occur in a fluid layer. $\mathrm{Ra_{cr}}$ can be determined by a linearized stability analysis; its exact value depends on particular forms of boundary conditions (e.g. isothermal vs adiabatic boundaries, rigid vs stress-free boundaries), the geometry of the convecting region (e.g. spherical vs plane), and whether heating is from below, or internal, or a combination of both. For convection in spherical shells, the ratio of outer to inner radius also influences the exact value of $\mathrm{Ra_{cr}}$ (Chandrasekhar 1961). Calculations have shown that $\mathrm{Ra_{cr}}$ is $O(10^3)$ for all the many different circumstances under which convection may occur (see, for example, Chandrasekhar 1961). Convection becomes more vigorous as the Rayleigh number increases beyond the critical value. Ra is the essential dimensionless ratio which describes the character of a convecting system, even when more complicated equation of state and constitutive relationships introduce additional parameters.

Table 1 includes estimates of the Rayleigh numbers for the mantles of the terrestrial planets. These values of Ra were calculated using $\nu = 10^{21}$ $\mathrm{cm^2/s}$ and a ΔT based on a 0.1 K/km superadiabatic gradient across the entire mantle. Other parameter values have been given in the previous section. The Rayleigh numbers for all the terrestrial planets, except perhaps Mercury, are so large compared with the critical value of $O(10^3)$, that based on linear stability theory one would expect convection in their mantles. Rayleigh numbers for the Earth and Venus are so large compared to $\mathrm{Ra_{cr}}$ that one would expect quite vigorous present day mantle convection. Convection would be expected to be less vigorous for Mars and still less vigorous for the Moon. For Mercury, the estimate of Ra is sufficiently small that convection if it is taking place at all in that planet's mantle would be expected to be rather weak (Cassen et al 1976). Such simple evaluations of Ra and comparisons with $\mathrm{Ra_{cr}}$ led Pekeris (1935), Knopoff (1964), and Tozer (1965a) to conclude that convection could be expected for the Earth's upper mantle. Schubert, Turcotte & Oxburgh (1969) determined the critical Rayleigh number for a viscously stratified fluid and concluded that convection was likely in all the terrestrial planets.

Cassen & Reynolds (1973, 1974) analyzed the stability of lunar temperature profiles established by radioactive and accretional heating and concluded that solid state convection would play an important role in lunar thermal history (see also Turcotte & Oxburgh 1969b).

Estimates of the Nusselt numbers Nu for the planetary mantles are also given in Table 1. Nu is the ratio of the heat flux carried by convection and conduction to the heat flux that would be carried by conduction alone if the system were subjected to the same temperature difference across its boundaries. At the onset of convection Nu = 1. Nu is much greater than unity for a vigorously convecting system. The values of Nu in the table were based on the values of Rayleigh number and a relation between Nu and Ra which has support from experimental, theoretical, and numerical studies. The form of this relation is

$$\mathrm{Nu} = b\,\mathrm{Ra}^{\beta}, \tag{6}$$

where b and β are constants. If one argues, following Priestley (1954), that the heat transport through a fluid layer heated from below at high Rayleigh number is independent of overall layer thickness, because such heat transport is controlled by conduction through thin thermal boundary layers, then it is straightforward to show from the definitions of Nu and Ra that $\beta = 1/3$. Slightly different values of β are predicted by experiments and numerical studies. We will discuss this in more detail later. Here we simply note that the approximate Nusselt number estimates given in Table 1 were obtained assuming

$$\mathrm{Nu} = \left(\frac{\mathrm{Ra}}{\mathrm{Ra}_{\mathrm{cr}}}\right)^{1/3}, \tag{7}$$

with $\mathrm{Ra}_{\mathrm{cr}} = 10^3$. These estimates of Nu indicate that about 10 times as much heat is being transported to the surface by convection as compared with conduction in the mantles of Earth, Venus, and Mars. Convective heat transport in the Moon may be several times the conductive heat transport, while in Mercury, convective heat transport, if it is occurring at all, should be at best comparable to conductive heat flow.

The final dimensionless ratio presented in Table 1 is the Dissipation number Di

$$\mathrm{Di} = \frac{\alpha g D}{c_p}. \tag{8}$$

Di is the dimensionless ratio which determines the influence of viscous dissipation and adiabatic compression on convection, effects not explicitly incorporated in the framework of the standard Boussinesq approximation

(Peltier 1972, Turcotte et al 1974, Hewitt, McKenzie & Weiss 1975, Jarvis & McKenzie 1979). When Di is much less than unity, viscous dissipation and adiabatic compression have little influence on convection. However, increasing Di has a stabilizing effect on the flow; for sufficiently large Di the flow becomes penetrative. From the order of magnitude values of Di listed in Table 1, it is obvious that viscous dissipation should be relatively unimportant in the mantles of the smaller terrestrial planets ☿, ☾, ♂, but it may be of importance for whole mantle convection in the large planets ⊕ and ♀.

APPROACHES TO THE STUDY OF MANTLE CONVECTION

A number of different approaches are available for the study of mantle convection. One can study convection experimentally in the laboratory (Whitehead 1976) or theoretically, using various schemes to integrate the equations. Boundary layer techniques and other semi-rigorous scaling simplifications can be used to obtain solutions for convection at high Rayleigh number. Laboratory experiments are limited in their ability to simulate convection in the mantle essentially because one cannot carry out the laboratory experiments on mantle materials under mantle conditions of temperature, pressure, etc. Even if convection in ordinary viscous fluids is relevant to the mantle, laboratory experiments cannot be performed under circumstances wherein values of both Ra and Pr are comparable to those in the mantle. Many laboratory experiments that purport to be relevant to the Earth's mantle (Richter & Parsons 1975) are carried out at high Ra for large Prandtl number fluids [$Pr = O(10^4)$]. However, we do not have laboratory fluids with Pr as large as 10^{23}, a value which insures the unimportance of inertial forces in the mantle. Inertial forces may not be negligible in high Rayleigh number laboratory experiments even with large Prandtl number fluids if the relative importance of these forces scales according to the ratio Ra/Pr (Corcos, private communication, Peltier 1972) or $Ra^{2/3}/Pr$ (Elsasser, Olson & Marsh 1979) instead of 1/Pr (Oxburgh & Turcotte 1978). In the experiments of Richter & Parsons (1975), Ra/Pr varies between 1 and 10, while $Ra^{2/3}/Pr$ varies between 1 and 5.

Rigorous numerical modelling of highly nonlinear mantle convection is probably not feasible at present, because such convection is likely to be fully three-dimensional and perhaps time-dependent. Nevertheless, there has been a great deal of effort expended in constructing two-dimensional numerical models of convection in the Earth's mantle (see e.g. Torrance & Turcotte 1971a,b, Richter 1973a, Turcotte, Torrance & Hsui 1973, McKenzie, Roberts & Weiss 1974, Parmentier, Turcotte & Torrance 1976). The emphasis on two-dimensional numerical models must largely be attributed to the relative ease of carrying out such calculations. Only if convection in the Earth were confined to its upper mantle would two-dimensional models of convection have potential relevance. If convection extends throughout the Earth's mantle it should be represented by spherical shell models. Even if convection were restricted to the Earth's upper mantle, strictly two-dimensional models of it have questionable relevance since such solutions may be unstable to perturbations in the third dimension, i.e. even shallow upper mantle convection may be three-dimensional. Busse (1967) has shown that two-dimensional convection in a layer of infinite Prandtl number fluid becomes unstable when the Rayleigh number exceeds 2.26×10^4. Most estimates of Ra even for the upper mantle exceed this value. The authors of two-dimensional numerical models of mantle convection have been among the strongest advocates of shallow mantle convection in the Earth (McKenzie, Roberts & Weiss 1974, Richter 1978). The depth of convection in the Earth's mantle is a topic of much current debate and we will discuss it in more detail in a later section.

Basic fluid dynamical calculations of finite-amplitude thermal convection in spherical geometry have been carried out by Hsui, Turcotte & Torrance (1972) and Young (1974). Numerical models of the thermal states of planetary interiors based on computations of convection in spheres and spherical shells have been constructed by Turcotte et al (1972), Young & Schubert (1974), Cassen & Young (1975), Schubert & Young (1976), and Schubert, Young & Cassen (1977). These computations have been restricted to axisymmetric solutions and their relevance to convection in planetary interiors can only be determined by testing their stability to nonaxisymmetric perturbations. Recent studies indicate that axisymmetric modes of convection may in fact be unstable to nonaxisymmetric perturbations. Busse's (1975) stability analysis of axisymmetric convection in spherical geometry shows that axisymmetric modes of convection which are symmetric about an equatorial plane are generally unstable to nonaxisymmetric perturbations for Rayleigh numbers near the critical value. The one exception to this conclusion is the lowest order even axisymmetric mode, which is stable to nonaxisymmetric perturbations. Zebib, Schubert & Straus (1979) have studied heated from below convection in a spherical shell the size of the Earth's mantle for Rayleigh numbers up to ten times critical. The critical motion may be axisymmetric but it is not symmetric about an equatorial plane. At a given supercritical Rayleigh number, it is possible to calculate axisymmetric solutions that are symmetric about an equatorial plane, and ones that do not possess such symmetry. Zebib, Schubert & Straus (1979) have shown that the former are unstable to the latter and, further, that the latter are unstable to nonaxisymmetric perturbations at Rayleigh numbers near the critical value. However, these axisymmetric solutions that are not symmetric about the equator can be stable to azimuthal perturbations for Rayleigh numbers which are not too close to the critical value.

The pattern of axisymmetric convection at the onset of instability is often compared with physical observations which suggest convection in the planets (see, for example, Runcorn 1977, Elsasser, Olson & Marsh 1979). However, these studies of finite amplitude convection in spherical geometry show that the form of convection at the onset of instability may have little relevance to the patterns of vigorous convection in planetary mantles. Not only must we question the relevance of the linearized axisymmetric flows to actual motions in the planets, but instability of axisymmetric finite amplitude convection to general three-dimensional perturbations (Busse 1979) would obviously also preclude the relevance of detailed characteristics of even these nonlinear solutions. The same criticism also applies, of course, to nonlinear two-dimensional models of convection. It is possible however if interest is confined to the average characteristics of convection, e.g. mean heat flux or temperature, that the axisymmetric (or two-dimensional) solutions could give results similar to those of the more complex fully three-dimensional motions. It would be fortunate if this were the case since numerical modelling of three-dimensional convection at very high Rayleigh number is a formidable task. If one adds to the burden of carrying out high Rayleigh number three-dimensional convection calculations by incorporating realistic equation of state and rheological behavior, e.g. a nonlinear stress-rate of strain connection, a temperature- and pressure-dependent effective viscosity, etc, then the effort will be totally beyond our capabilities for some years to come. These difficulties argue for the development of simplified theoretical approaches to the description of mantle convection, a point forcefully argued by Tozer (1972a). The very large uncertainties in rheological properties and heat source content of the mantles of the terrestrial planets makes a simplified treatment of convection necessary for the systematic investigation of parameter variations.

Boundary layer theories of high Rayleigh number convection represent one important type of simplified theoretical description of the phenomenon. They are based on observations of the form of steady two-dimensional convection of a constant viscosity Boussinesq fluid layer heated from below at high Rayleigh number. As Ra increases, convection tends to take the form of a nearly isothermal core region (or an adiabatic one if the adiabatic temperature gradient is not negligible) with horizontal thermal boundary layers at the upper and lower boundaries and vertical plumes adjacent to the lateral boundaries. The core region of the convection cell is at the average temperature of the upper and lower boundaries and the temperature differences between the boundaries and the fluid interior occur across the horizontal boundary layers. Heat transport across these boundary layers is by the process of conduction. Buoyancy forces in the vertical plumes drive the circulation of the convection cell. Turcotte & Oxburgh (1967) developed this two-dimensional boundary layer model of mantle convection and showed, if the upper and lower boundaries are isothermal, stress-free surfaces, that the maximum thickness of the thermal boundary layer δ is

$$\delta = 7.38 \ \mathrm{Ra}^{-1/3} D, \tag{9}$$

and the overall heat transport q is

$$q = 0.167 \ \mathrm{Ra}^{1/3} \left(\frac{k\Delta T}{D}\right), \tag{10}$$

where D is the thickness of the fluid layer and ΔT is the temperature difference across the layer. Equation (10) shows that the Nusselt number is given by

$$\mathrm{Nu} = 0.167 \ \mathrm{Ra}^{1/3}. \tag{11}$$

Thus as the Rayleigh number increases, the boundary layers become thinner and harder to resolve by direct numerical solution techniques. Corcos (private communication) has found that the numerical factors in (9)–(11) need revision to correct an error in the details of the original boundary layer solution by Turcotte & Oxburgh (1967).

The scaling and planform assumptions that enter the development of a boundary layer theory place potentially serious limitations on the ability of the theory to properly describe high Rayleigh number mantle convection. Assumptions such as two-dimensionality and steady state can only be verified by direct numerical or experimental tests; the difficulties involved in performing these checks motivate the formulation of a boundary layer theory in the first place. Boundary layer theories which can cope with three-dimensional, time-dependent flow, or nonlinear, temperature- and pressure-dependent creep behavior are not presently available.

In certain instances, it may suffice to know some average characteristic of a convecting system, e.g. the mean heat flux. It is then possible to use a relation such as (6) to infer the heat flux from the average properties of a convecting region. This power law relation between Nusselt number and Rayleigh number is known from laboratory experiments and theoretical and numerical calculations to characterize the heat flux from a vigorously convecting system ($Ra \gg Ra_{cr}$) in a variety of circumstances. Tozer (1967, 1972a,b, 1974) made extensive use of it throughout his papers discussing the importance of subsolidus creep in regulating the temperatures of the terrestrial planets. Sharpe & Peltier (1979) strongly advocated its utility for studying the thermal histories of the planets. The power law relation was used by Kaula (1979) and Stevenson & Turner (1979) to investigate thermal history models of the Earth and by Cassen et al (1979) to model the thermal evolution of the Moon. Schubert, Cassen & Young (1979a,b) have used the relation to develop cooling history models of the terrestrial planets. Most of the evidence supporting relation (6) applies to a fluid layer heated from below which is transporting a steady quantity of heat. The data from laboratory experiments fit (6) quite well (Rossby 1969, Chu & Goldstein 1973, Garon & Goldstein 1973). Values of b and the power law exponent β depend somewhat on the particular fluid used in the experiment (i.e. on the Prandtl number of the fluid) and on the Rayleigh number range studied; b is generally $O(10^{-1})$ and β is about 0.3. This value of β is consistent with the value 1/3 which comes from various scaling arguments and boundary layer theories (Priestley 1954, Kraichnan 1962, Howard 1966, Turcotte & Oxburgh 1967, Long 1976), and the $O(10^{-1})$ value of b is consistent with the form of the power law relation given in (7) since Ra_{cr} is $O(10^3)$. There are numerous other experimental, theoretical, and numerical studies which support relation (6) for the fluid layer heated from below; Busse's recent review (1978a) provides a detailed discussion of many of these. Relation (6) appears to adequately describe the steady heat transport through a vigorously convecting layer heated from below. However, the values of b and β depend on boundary conditions, on Prandtl number, and weakly on the Rayleigh number itself.

Because the mantles of the planets are not simply fluid layers heated from below in steady state, it is important to expand the justification of relation (6) to more general situations, including internally heated convection and transient convection. Schubert, Cassen & Young (1979a) show that the experimental data of Kulacki & Nagle (1975) and Kulacki & Emara (1977) for steady convection in an internally heated layer insulated at the bottom are in good agreement with the power law (6) (with Ra based on the temperature difference across the layer); these experiments imply $b = 0.23$ and $\beta = 0.29$ (Schubert, Cassen & Young 1979a). Convection in spherical shells with internal heating and insulated lower boundaries was studied numerically by Young & Schubert (1974), Schubert & Young (1976), and Schubert, Young & Cassen (1977). These papers presented thermal models of Mars, Earth, and the Moon from which one can calculate the amounts by which the average temperatures of the convecting mantles exceed the base temperatures of the lithospheres. Log-log plots of these excess temperatures ΔT against $\mathrm{Ra}/\mathrm{Ra}_{\mathrm{cr}}$ reveal approximate power law dependences of ΔT on $\mathrm{Ra}/\mathrm{Ra}_{\mathrm{cr}}$ which are consistent with (6) and imply $\beta \approx 0.3$.

Cassen & Young (1975) analyzed the situation in which steady convection is driven both by internal energy sources and an imposed constant temperature differential across the boundaries. They found that the heat flux through the bottom boundary is approximately a linear function of the strength of the internal energy sources, with negative slope. This is the expected result for steady state, as long as the flux through the top of the layer is independent of heat source content, as is assumed in (6).

It is also possible to test the applicability of (6) to transient convection. Kulacki & Nagle (1975) and Kulacki & Emara (1977) also studied the response of the internal temperature of their convection cell (again, volumetrically heated and insulated at the bottom) to step changes in the heating rate. They measured the time required for a system, initially with a uniform temperature, to come to steady convective equilibrium after a step increase in heating and the time for the system in equilibrium with a given heat source concentration to return to a constant temperature after the heat was turned off. Schubert, Cassen & Young (1979a) compared these data with an analytic solution of the heat conservation equation made possible by relation (6); they found good agreement between the theoretical prediction and the experimental results.

The experiments of Booker (1976) and Booker & Stengel (1978) with Bénard convection in a variable viscosity fluid show that temperature-dependent viscosity has a relatively minor effect on the Nu-Ra relation when the Rayleigh number is based on the viscosity corresponding to the mean temperature of the layer. Even this minor effect can be accounted for by writing

$$\mathrm{Nu} = b'\left(\frac{\mathrm{Ra}}{\mathrm{Ra}_{\mathrm{cr}}}\right)^{\beta} \tag{12}$$

and using the actual value of the critical Rayleigh number for the variable viscosity situation. Booker & Stengel (1978) suggest $b' = 1.49$. Finally, numerical calculations of convection in non-Newtonian fluids (Parmentier, Turcotte & Torrance 1976, Parmentier 1978) suggest that relation (6) holds for even more complicated rheologies.

Thus the power law Nusselt number–Rayleigh number relation (6) can be used with some confidence to study the thermal balance of planetary interiors. However, although the experimental and theoretical justifications for such a law are numerous, there are presently no data available to rigorously validate its use under the circumstances required for general studies of convection in planetary mantles. The application of the power law to studies of mantle convection involves the extension of a principle beyond its rigorously defendable region of validity with the expectation of at least qualitatively correct results.

DEPTH OF MANTLE CONVECTION

One of the major issues facing mantle convection theorists is whether convection is confined only to the upper regions of the Earth's mantle or extends throughout. Proponents of shallow mantle convection in the Earth argue that the major upper mantle structural transformations should restrict convection to the upper mantle. Since similar phase transformations could occur in the mantles of Venus and perhaps Mars, the issue of shallow vs whole mantle convection applies to these planets as well. However, pressures in the mantle of Mercury and throughout the Moon are sufficiently low that if convection occurs in these bodies it should be of the whole mantle type.

McKenzie and Richter have been among the strongest advocates of shallow mantle convection. Throughout their papers (Richter 1973a,b, 1977, 1978, McKenzie, Roberts & Weiss 1974, McKenzie & Weiss 1975, McKenzie & Richter 1976, McKenzie 1977, Richter & McKenzie 1978) they argue that the spinel-postspinel phase change should act as a barrier, restricting convection associated with plate motions to the upper 650–700 km of the mantle (they do not preclude a separate convective circulation in the lower mantle). Their numerical models describe two-dimensional flows in plane fluid layers. These authors are not alone in constructing such models of convection in the Earth's mantle; in fact, the major effort to model convection in the Earth's mantle has been the development of two-dimensional, plane layer models (see, for example, Torrance & Turcotte 1971a,b, Houston & De Bremaecker 1975, Parmentier, Turcotte & Torrance 1976). McKenzie and Richter point out that the predominance of compressional focal mechanisms in deep earthquakes between depths of 500 and 700 km and the absence of earthquakes at depths greater than 700 km (Isacks & Molnar 1971) support the view that the spinel-oxide phase change is a barrier to convection.

Although the compressional nature of deep earthquakes indicates that descending slabs meet some resistance at the 650-km phase change, this does not necessarily imply the inability of a slab to penetrate the phase transition. Schubert, Yuen & Turcotte (1975) have shown that, while the spinel-oxide transition may exert an upward body force on a descending slab due to the depression of the phase boundary within the slab, the downward body forces, due to the negative buoyancy of the cold slab and the upward distortion of the olivine-spinel phase boundary within the slab, are overwhelming and readily drive the slab through the 650-km phase transition. The absence of earthquakes below 700-km depth may indicate only that we have not detected earthquakes any deeper. Also cessation of earthquake activity within the slab below a certain depth does not imply that the slab itself ceases to exist below that depth. An alternative explanation for this behavior is that the upper layers of the slab have become sufficiently heated by the time they reach depths in excess of about 700 km that earthquakes can no longer occur. There is, in addition, a growing body of seismic travel time data (Julian & Sengupta 1973, Jordan & Lynn 1974, Engdahl 1975, Dziewonski, Hager & O'Connell 1977) providing evidence for lateral heterogeneity in the Earth's lower mantle; such heterogeneity may be associated with the temperature differences of a deep mantle circulation.

At one time it was argued that the thermodynamic properties of a phase change would inhibit convective motions from occurring across the major phase transitions of the Earth's upper mantle (Knopoff 1964, Verhoogen 1965). The stability analysis of a fluid layer with a univariant phase transition heated from below (Schubert, Turcotte & Oxburgh 1970, Busse & Schubert 1971, Schubert & Turcotte 1971, Peltier 1972) clarified the physics of the stabilizing (latent heat release) and destabilizing (phase boundary distortion caused by advection of ambient temperature) effects of an exothermic phase change and showed that the olivine-spinel phase change in the presence of a negative temperature gradient could enhance deep mantle convection. Richter's (1973c) finite-amplitude numerical calculations of convection with a univariant phase change supported the conclusions of the stability analysis. The destabilizing character of the olivine-spinel phase change is dramatically illustrated by the elevation of the phase change in the descending slab. This phase boundary elevation in the descending slab provides an important driving force for mantle convection (Schubert & Turcotte 1971, Turcotte & Schubert 1971, Griggs 1972, Schubert, Yuen & Turcotte 1975).

Solid-solid phase transitions in the Earth are divariant in nature and a linear stability analysis of such an exothermic phase transformation (Schubert, Yuen & Turcotte 1975) shows that the destabilizing effect of an enhanced effective coefficient of volume expansion in the two-phase region can dominate the stabilizing effect of an enhanced adiabatic temperature gradient in the transition region (Ringwood 1972, Tozer 1972a). The destabilizing effect of phase boundary distortion also occurs for finite amplitude motions through exothermic divariant phase changes.

It is presently not clear whether the spinel-oxide phase transition is exothermic or endothermic (Liebermann, Jackson & Ringwood 1977). Schubert, Yuen & Turcotte (1975) discuss the consequences of an endothermic spinel-oxide phase change, since this possibility distinguishes the behavior of the 650-km phase transition from that of the 400-km phase change. An endothermic spinel-oxide phase change would offer some resistance to mantle convection (this would help explain the compressional nature of deep earthquakes, while an exothermic behavior could not), but not enough to terminate the descent of lithospheric plates into the deep mantle. It seems reasonable to conclude, on the basis of the above discussion, that thermodynamically, upper mantle phase changes do not necessarily confine convection to the upper mantle. On the contrary, they may promote whole mantle convection and provide an important driving force for plate motions.

Another way of discussing phase changes in the Earth's upper mantle as barriers to convection is by hypothesizing a dramatic increase in viscosity across them. McKenzie & Weiss (1975) assert that an increase in activation energy of 2 eV/mol (≈ 46 kcal/mol) across the 650-km phase change leads to such a large increase in viscosity below 650 km as to confine convection to the upper mantle. Such a large change in activation energy across the spinel-oxide phase transition could increase the viscosity by a factor of 10^5 across it. Tozer (1972a) also suggests an increase in viscosity by a factor of 10^5 or 10^6 across the spinel-oxide phase transition which could confine convection to the upper mantles of all terrestrial bodies with radii larger than some value between about 3000 and 6000 km. Such enormous increases in viscosity across the spinel-oxide phase change, however, seem in conflict with the inference of a uniform mantle viscosity from glacial rebound data (Cathles 1975, Peltier 1976). Also, a systematic relation betwen activation energy and oxygen ion packing predicts an increase in activation energy across the spinel-oxide phase change of only several kcal/mol implying a viscosity increase of no more than one order of magnitude (Sammis et al 1977). The upper mantle phase transitions, in particular the spinel-oxide phase change, should not necessarily act as rheological barriers to whole mantle convection.

Instead of relying on sudden changes in viscosity across phase transitions to limit convection to the upper mantles of large terrestrial planets similar to Earth and Venus, one could hypothesize a gradual increase in viscosity with depth associated mainly with the large increase in pressure in the lower mantle. The inference of uniform viscosity in the Earth's mantle from glacial rebound observations also argues against this possibility. In addition, a decrease of activation volume with increasing depth in the Earth's mantle would make it unlikely that the pressure effect would increase viscosity substantially (Sammis et al 1977, O'Connell 1977).

Two theoretical studies emphasize that only unreasonably large mantle viscosity stratifications could preclude whole mantle convection. Schubert, Turcotte & Oxburgh (1969) considered the stability of the mantles of the terrestrial planets assuming a viscosity that increased exponentially with depth. If the depth of the convecting layer D is large compared with the scale height of the viscosity increase h, then the Rayleigh number, based on the entire thickness of the mantle and the minimum viscosity ν_s,

$$\mathrm{Ra} = \frac{\alpha g}{\kappa \nu_s}\left(\frac{\Delta T}{D}\right) D^4, \tag{13}$$

would have to exceed either $23(D/h)^4$ (for a stress-free upper surface) or $30(D/h)^4$ (for a rigid upper surface) for whole mantle convection to occur (Schubert, Turcotte & Oxburgh 1969). This condition for the onset of convection in the entire viscously stratified mantle can be compared with the criterion for onset of instability in a shallow, constant viscosity upper mantle of thickness D_u and viscosity ν_s across which the destabilizing temperature rise is ΔT_u; that criterion is

$$\mathrm{Ra} = \frac{\alpha g}{\kappa \nu_s}\left(\frac{\Delta T_u}{D_u}\right) D_u^4 > \begin{array}{l} 1707 \text{ (rigid upper surface)} \\ 1101 \text{ (stress-free upper surface).} \end{array} \tag{14}$$

For the constant viscosity shallow upper mantle layer to be more unstable than the entire viscously stratified mantle (assuming the same destabilizing temperature gradient for both configurations, i.e. $\Delta T/D = \Delta T_u/D_u$) equations (13) and (14) show that

$$\begin{aligned} \frac{D}{h} &\geqq \left(\frac{1101}{23}\right)^{1/4} \frac{D}{D_u} \quad \text{(stress-free upper surface)} \\ & \left(\frac{1708}{30}\right)^{1/4} \frac{D}{D_u} \quad \text{(rigid upper surface).} \end{aligned} \tag{15}$$

Thus, before shallow mantle convection would be more likely than whole mantle convection, at least on the basis of linear stability theory, the viscosity increase across the mantle would have to exceed the value given by

$$\begin{aligned} \frac{\nu}{\nu_s} &= \exp\left(2.63\,\frac{D}{D_u}\right) \quad \text{(free-surface upper boundary)} \\ &= \exp\left(2.74\,\frac{D}{D_u}\right) \quad \text{(rigid upper boundary).} \end{aligned} \tag{16}$$

If we use $D/D_u = 30/7$ for the Earth, then according to equation (16), the viscosity increase across the mantle would have to exceed about 10^5 before shallow mantle convection would be the preferred mode. Davies (1977) recently arrived at the same conclusion from a linear stability analysis of a mantle separated into two constant viscosity layers. A viscosity increase of 10^5 across the Earth's mantle is wholly inconsistent with glacial rebound data.

Other physical properties of a mantle that could conceivably restrict the depth of convection are chemical compositional stratification and an enhanced lower mantle adiabatic temperature gradient. Both these possibilities are rather speculative. Even a very small chemical compositional stratification would be very effective in limiting convection to the upper mantle of a planet (Richter & Johnson 1974). Although Anderson (1977) argues that it is likely for the Earth's mantle to be compositionally stratified, our knowledge of the mantle is consistent with its being compositionally homogeneous (Wang & Simmons 1972, Liebermann & Ringwood 1973, Davies 1974, Watt, Shankland & Mao 1975).

If the scaling and boundary layer arguments already discussed indeed characterize high Rayleigh number convection, then they can be used, together with measured characteristics of the Earth's convection, e.g. plate velocities, surface heat flow, etc, to shed light on whether shallow or deep convection in the Earth's mantle is more likely. One possibility is to use Equation (7) or Equation (11) [they are essentially identical for approximate calculations if Ra_{cr} is $O(10^3)$] to estimate the heat flux from convecting layers of different depth and compare these estimates with the observed average heat flow at the Earth's surface. With $D = 3000$ km, a temperature gradient of 0.1 K/km and other parameter values given previously, Equation (7) predicts a heat flux of about 0.3 μcal/cm^2 s. This estimate is only about a factor of 5 less than the Earth's mean heat flux; it can be adjusted upward to better agree with observation by reasonable changes in the parameter values entering Equation (7), e.g. one could have taken a somewhat larger value of temperature gradient. With $D = 700$ km, on the other hand, the estimated heat flux would be about an order of magnitude smaller (for the same temperature gradient), and it would be more difficult to reconcile such a reduced estimate with observations.

An estimate of the boundary layer thickness from Equation (9), or the essentially equivalent form

$$\delta = \left(\frac{\mathrm{Ra}_{cr}}{\mathrm{Ra}}\right)^{1/3} D, \tag{17}$$

can be compared with an oceanic lithosphere thickness of about 100 km. For $D = 3000$ km, and a temperature gradient of 0.1 K/km, Equation (17) gives $\delta = 100$ km, while for $D = 700$ km and the same temperature gradient, one gets $\delta = 170$ km, a somewhat poorer estimate of average oceanic lithosphere thickness.

The scaling arguments can actually be used to derive an estimate of D from observations alone (Elsasser, Olson & Marsh 1979). This requires the introduction of a velocity scale which can be obtained by equating heat transport by horizontal advection with heat transfer by vertical conduction in the thermal boundary layer at the top of the convecting region. If u is the velocity scale, then this procedure gives

$$u = \frac{\kappa}{D}\frac{D^2}{\delta^2} = \frac{\kappa}{D}\frac{\delta}{D}\frac{\mathrm{Ra}}{\mathrm{Ra}_{\mathrm{cr}}}, \tag{18}$$

where we have used Equation (17) to introduce the Rayleigh number. To solve for D, eliminate δ using Equation (17), ΔT using Equation (7), and find

$$D = u\left(\frac{\mathrm{Ra}_{\mathrm{cr}}\, k\nu}{\alpha g \kappa q}\right)^{1/2} \tag{19}$$

With $u = 4$ cm/yr (the average plate speed), $\mathrm{Ra}_{\mathrm{cr}} = 10^3$, $k = 0.01$ cal/cm s K, $\nu = 10^{21}$ cm^2/s, $\alpha = 3 \times 10^{-5}$ K^{-1}, $g = 10^3$ cm/s^2, $\kappa = 10^{-2}$ cm^2/s, and $q = 1.5 \times 10^{-6}$ cal/cm^2 s, Equation (19) gives $D = 6000$ km, an estimate that is more consistent with whole mantle convection than it is with shallow mantle convection.

Another observation consistent with deep mantle convection is the length scale of the largest tectonic plates on Earth. It is not immediately obvious how the sizes of plates should relate to an underlying convective pattern that may be three-dimensional and time-dependent, but conventional wisdom would suggest that the length scales of the plates should be comparable to the depth of the convecting system. It is, of course, much easier to reconcile the size of the Pacific plate, for example, with whole mantle convection than with shallow upper mantle convection.

An alternative explanation of the relation between plate scale and mode of convection relies on the strong temperature dependence of mantle viscosity. The thermal boundary layer of a mantle convection system is also a rheological boundary layer because viscosity is exponentially dependent on the inverse absolute temperature. The rheological and thermal boundary layer at the Earth's surface, the lithosphere, is effectively rigid on geologic time scales and this rigidity may tend to prevent subduction at the relatively young ages that would be expected on the basis of convection in ordinary viscous fluids. The numerical experiments of Parmentier & Turcotte (1978) indicate that this effect results in two-dimensional convection cells with large aspect ratios.

Related to the problem of understanding the sizes of the large plates is the difficulty of explaining the range of different plate sizes in a single convecting system, shallow or deep (Richter 1978).

The observed angles of subducting slabs also seem to be more readily understandable by the deep mantle circulation models of Hager & O'Connell (1978). These authors used observed plate motions and geometries to calculate the viscously induced flow driven in spherical shell models of the Earth's mantle by the prescribed surface velocities. Dip angles inferred from streamline patterns could be readily reconciled with the observed subduction angles of the plates only for deep circulation models.

Schubert & Turcotte (1971) argued against a shallow mantle circulation by suggesting that an excessively large pressure gradient might be required to drive the return flow beneath the plates. Limits on the magnitude of such a pressure gradient could be set by the slope of the ocean floor. More recent two-dimensional boundary layer calculations of shallow mantle return flow beneath oceanic plates (Schubert et al 1978) indicate that shallow return flows can be driven by much smaller pressure gradients than previously thought, thus weakening the constraint that ocean floor topography places on models of mantle convection.

There are strong arguments in favor of whole mantle convection in the Earth. If there is convection in the interiors of the smaller planets, ☿, ☾, ♂, then it should be of the whole mantle variety. Thus more emphasis needs to be placed on the construction of models of convection in spherical shells, rather than simply on modelling two-dimensional convection in plane layers.

EFFECTS OF MANTLE CONVECTION ON THE THERMAL AND MECHANICAL STATE OF A PLANETARY INTERIOR

In this section we discuss, in general terms, the thermal and mechanical state of the interior of a planet in which the mantle is in a vigorous state of convection. We describe the planet's internal structure at one instant, leaving to the following section a discussion of how the interior evolves with time. Since vigorous mantle convection is likely to be fully three-dimensional and perhaps time-dependent, we cannot describe the interior temperature and velocity fields in detail. However, calculations (see e.g. Schubert, Young & Cassen 1977) show that the average properties of a convecting mantle, in particular the spherically averaged temperature profile, may be quasi-steady even when convection is basically unsteady. We may also hope that the spherically averaged temperature profile, for example, is reasonably well-determined by axisymmetric models even if convection is nonaxisymmetric.

As repeatedly emphasized by Tozer (e.g. 1967, 1972a, 1974), the temperature-dependence of the viscosity of rocks [see Equation (1)] is the single most important factor controlling the thermal and mechanical state of a planetary interior. The main variation of temperature and viscosity in a planet at a given time in its evolution is between the cold rigid lithosphere and the underlying mantle. This thermomechanical state is illustrated by the model temperature vs depth profiles shown in Figure 1 for the present day Moon (Turcotte et al 1972, Schubert, Young & Cassen 1977, Cassen et al 1979). These average lunar temperature profiles are representative of the average thermal structures of the mantles of the smaller terrestrial planets. The average temperature profiles show that the lunar interior is divided into essentially two regions, one just below the surface in which the temperature increases from its value at the surface to its interior value, and the other, the underlying interior in which the mean temperature is essentially constant. The outer region wherein most of the temperature rise occurs is a thermal boundary layer, the lithosphere. Convection in the deep interior maintains the average temperature constant with depth. The lithosphere or thermal boundary layer forms because the average temperature in the deep interior cannot be maintained throughout the planet; the interior temperature must decrease with proximity to the surface to match the low value of surface temperature. Heat transfer through the lithosphere is by conduction. On a one

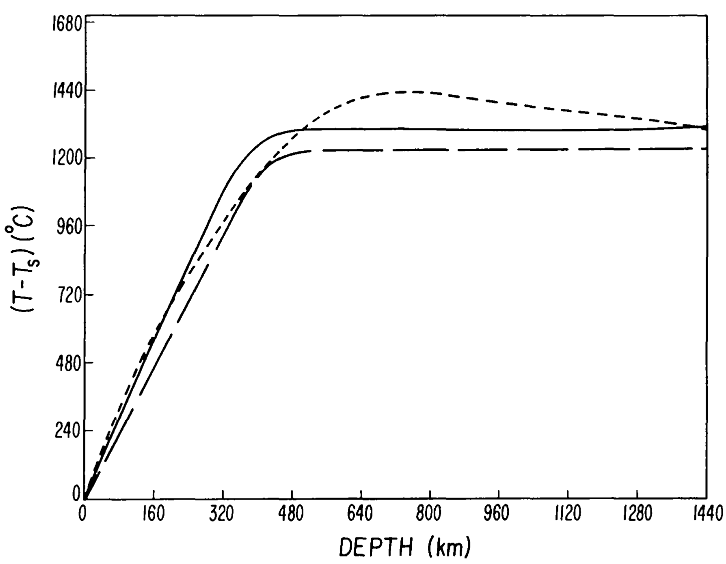

Figure 1 Average lunar temperature profiles based on calculations of finite amplitude convection in spherical geometry. Solid curve (Schubert, Young & Cassen 1977), short-dashed curve (Turcotte et al 1972), long-dashed curve (Cassen et al 1979).

plate planet like the Moon, all the internal heat must be conducted through a relatively thick lithosphere to the surface. The lithospheres in the models of Figure 1 are about 300-km thick. Superimposed on the spherically averaged quasi-steady temperature profiles, such as those shown in Figure 1, are temperature variations due to the spatial dependence of the convective state. A number of numerical calculations show that such temperature differences between hot ascending regions and cold descending ones are of the order of hundreds of degrees (Hsui, Turcotte & Torrance 1972, Turcotte et al 1972, Young & Schubert 1974).

The average viscosity in the Moon is essentially determined by the mean temperature profile because the increase of pressure with depth is too small to have much of an influence on viscosity. Also, no major silicate phase transformations can occur under the low pressures in the lunar interior. In the lithosphere, the temperature is so low that the material is effectively rigid on a geologic time scale. (It is generally accepted that subsolidus creep is negligible at temperatures below about 800°C). Thus the lithosphere is also a rheological boundary layer. In the deeper interior, the average viscosity is essentially constant, reflecting the uniformity in mean temperature. The viscosities corresponding to the temperature profiles shown in Figure 1 are about 10^{21} cm^2/s. The deep interior is convecting rather vigorously, for such a viscosity corresponds to a Rayleigh number about five-hundred times critical.

There is a relatively thin transition region just below the base of the lithosphere in which the viscosity decreases rapidly with depth from its effectively infinite value in the lithosphere to its constant value in the deep interior. The temperature profile also adjusts in this region, changing its character from a profile with a monotonic increase with depth to one which is constant with depth. This transition region could be referred to as the lunar asthenosphere, although there is no pronounced viscosity minimum in the lunar models.

Although the temperature-dependence of the viscosity is responsible for the basic thermomechanical structure just described, the very nature of this structure allows one to construct relevant models of the mean temperature profile in a planet's interior using calculations of convection in fluid spheres and spherical shells with constant viscosity. It is only necessary to combine a thermal conduction calculation in an outer, rigid, spherical shell with a temperature calculation in an inner convecting, constant viscosity, fluid spherical shell. However, one must be careful to maintain an internal consistency in the model; the temperature in the deep interior must correspond to a viscosity value (on the basis of an acceptable rheological law) similar to the one used in the constant viscosity convection calculation, and the temperature at the base of the lithosphere must not allow significant creep to occur, also on the basis of the same rheological law (Schubert, Young & Cassen 1977). When modelling the thermal evolutions of the planets, lithosphere thickness and deep temperature are functions of time, and adapting constant viscosity convection calculations to a basically time-dependent situation would be more complex, if possible at all.

The lunar temperature profiles shown in Figure 1 extend all the way to the center of the Moon because the models do not include a core. The effect of a core on the mean temperature profile in a mantle would depend on the heat flux through the core-mantle boundary as illustrated by the temperature profiles in Figure 2. The figure shows spherically averaged quasi-steady temperature profiles from Schubert & Young (1976) for models of constant viscosity, internally heated convecting fluid shells with rigid, conducting, internally heated outer shells. The dimensions of the shells are those of the Earth's lithosphere and underlying mantle and the total heat through the surfaces of the models matches that through the Earth's surface. The viscosity of the convecting interiors is 10^{25} cm^2/s, probably too large to be representative of the Earth's mantle, so the actual temperatures are not expected to represent those in the Earth's interior. In one case, the heat flux entering the mantle from the core is zero and the temperature profile is flat near the core-mantle boundary. In the other case, about 13% of the surface heat flux emanates from the core, and there is a large rise in temperature near the core-mantle boundary

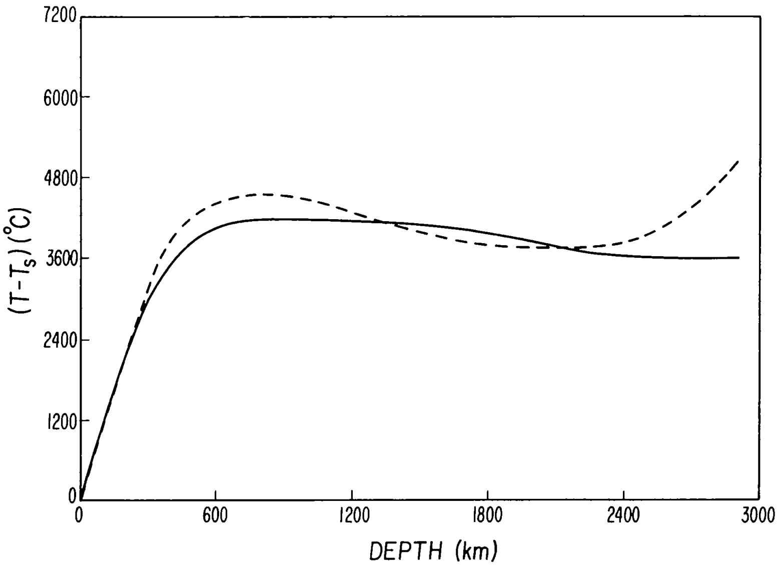

Figure 2 Average temperature profiles in an internally heated, constant viscosity fluid undergoing convection in a spherical shell the size of the Earth's mantle (Schubert & Young 1976). Solid curve (adiabatic lower boundary), dashed curve (heat flux into the lower boundary).

through a lower mantle thermal boundary layer. In this model, the mean temperature profile consists of three regions, two boundary layers, one near the surface and the other near the core-mantle interface, and an interior region in between with reasonably uniform temperature. The distinctive character of the thermal boundary layers and the interior isothermal region would presumably be more readily apparent if it were possible to carry out the numerical calculations with much smaller viscosities, more representative of the Earth's.

None of the mean thermal profiles shown in Figures 1 and 2 include the adiabatic increase of temperature with depth. For the smaller terrestrial planets like the Moon, the temperature rise due to adiabatic compression is small, and a convecting interior would be nearly isothermal, with upper and lower boundary layers as appropriate. For the larger planets, Earth and Venus, the adiabatic temperature increase with depth is substantial. Mean temperature profiles in these planets would consist of a nearly adiabatic interior region with surface and core-mantle thermal boundary layers. It is possible that the effects of adiabatic compression in the large planets could modify the thermal profiles even more substantially by leading to penetrative convection (Peltier 1972, Turcotte et al 1974, Jarvis & McKenzie 1979).

It is usually assumed that the mean temperature in the interior of a convecting mantle would lie along an adiabat, since such is the case for a constant viscosity fluid. However, it can also be argued that a convecting mantle should have nearly uniform viscosity because enhanced convection in regions of relatively low viscosity, for example, would tend to remove heat more efficiently from the region thereby reducing its temperature and raising the viscosity. Tozer (1967) asserts that viscosity will be constant along an adiabat, in which case there is no difficulty in reconciling an adiabatic temperature distribution with a constant viscosity. However, since viscosity is determined by the rheological parameters E^* and V^*, together with T and p, and the adiabatic temperature gradient depends on α, g, T, and c_p, it is not clear that there should be a connection between these thermodynamic and rheological parameters which insures that μ remains constant on an adiabat. Sammis et al (1977) and O'Connell (1977) have shown that it is possible for an adiabatic mantle to have nearly uniform viscosity because of the likely decrease of V^* with pressure.

We have mentioned several times that high Rayleigh number convection in the planets may be basically time-dependent. The numerical calculations of finite amplitude axisymmetric convection in spherical shells with internal heating by Schubert & Young (1976) and Schubert, Young & Cassen (1977) show that convection is unsteady even at modestly supercritical Rayleigh numbers. If convection in the planets is indeed unsteady, it may have important geophysical consequences. Jones (1977) argued that the possible intermittency of high Rayleigh number convection in the Earth's mantle may explain certain temporal variations in the geomagnetic field by modulating conditions at the core-mantle boundary Recent papers by Busse (1978b) and Walzer (1978) emphasize the potential tectonic importance of time-dependent models of mantle convection.

Effects of Mantle Convection on Thermal History

The profound effect that solid state convection can have on the thermal evolution of a planet is most dramatically illustrated by thermal histories which involve cooling from initial high temperature states. Cooling is the ultimate destiny of any planet, but it may have dominated the entire thermal histories of the larger terrestrial bodies, Earth and Venus, following early core formation. The same may be true of Mercury, as we have already discussed. The gravitational potential energy released upon core formation in the Earth, and perhaps Venus, could have been sufficient to melt these planets. Efficient convection in the molten state would remove some of this energy quite rapidly, but at some point in its cooling, the mantle would solidify, and subsequent cooling would be controlled mainly by subsolidus convection. Thermal history models in which subsolidus convective cooling from a hot initial state governs the evolution of a planet have recently been studied by Schubert, Cassen & Young (1979a), Sharpe & Peltier (1978), and Stevenson & Turner (1979).

The temperature-dependence of mantle viscosity is the single most important factor controlling the thermal evolution of a cooling planet (Tozer 1967, 1972a, 1974). It acts as a thermostat to regulate the mantle temperature. Initially, when the planet is hot, mantle viscosity is low, and extremely vigorous convection rapidly cools the planet. Later in its history, when the planet is relatively cool, its viscosity is higher and more modest convection cools the planet at a reduced rate. This is illustrated in Figure 3, which shows average mantle temperatures as functions of time for Earth models cooling from initial temperatures between 1500 and 3000°C (Schubert, Cassen & Young 1979a). When the planet is initially hot, there is an extremely rapid reduction in mantle temperature very early in the cooling history. This is followed by a much more gradual decrease in temperature over most of the lifetime of the planet. An initial temperature of 3000°C is reduced to 2690°C after only 10 Myr of vigorous convective cooling. The temperature is only 2060°C after 100 Myr and by 500 Myr it has fallen to 1670°C. Cooling between 500 Myr and 4.5 Gyr reduces the temperature by only an additional 360°C. Cooling is self-regulated through the dependence of μ on T.

The temperature after 4.5 Gyr of convective cooling is extremely in-sensitive to the initial temperature; for a starting temperature of 1500°C, the temperature after 4.5 Gyr is 1275°C, while for an initial temperature of 3000°C, T after 4.5 Gyr is 1307°C. Convection reduces the initial 1500°C temperature difference between the models to only 32°C after 4.5 Gyr. Thus, subsolidus convection rapidly cools a hot planet to a temperature determined essentially by the rheology of the mantle alone; the temperature in the interior of a planet has no memory of its initial value after convective cooling over geologic time.

Once the interior temperature of a planet reaches the value determined by its rheology, there is very little further change in temperature even after billions of years of convective cooling. This is shown by the Earth models in Figure 3, especially the one with an initial temperature of 1500°C. It is even more dramatically illustrated by the evolution of average temperature in the lunar thermal history model also shown in Figure 3 (Schubert, Cassen & Young 1979a). The initial temperature of 1300°C is reduced by only 96°C after 4.5 Gyr of convective cooling. Except for a relatively short period of time when the temperature of a planet's mantle may decrease substantially during the early stages of cooling, a planet cools mainly by thickening its lithosphere; the underlying mantle temperature decreases relatively slowly. For the Moon model of Figure 3, cooling produces a thick lithosphere; it reduces the temperature beneath this lithosphere only slightly. The temperatures of the Moon and Earth models are nearly the same after 4.5 Gyr despite the difference in radii of these planets. This is because mantle temperature is determined mainly

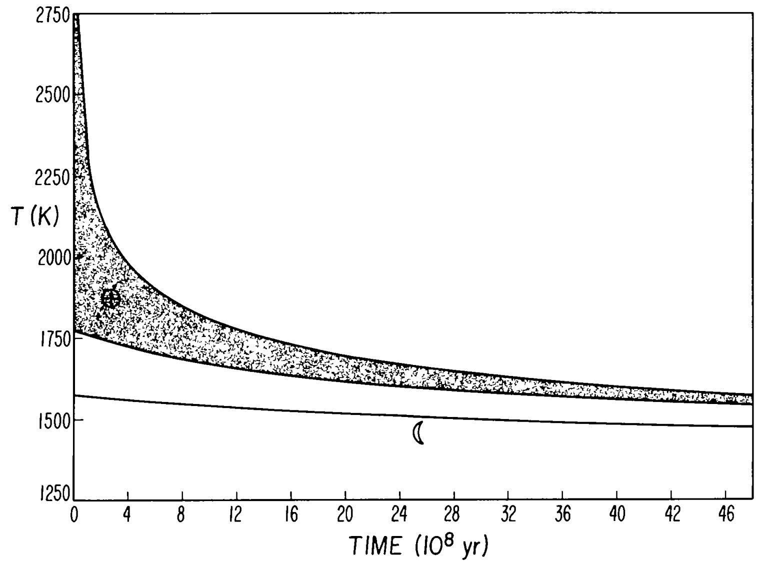

Figure 3 Average mantle temperature vs time for cooling history models of Earth and Moon (Schubert, Cassen & Young 1979a). The shaded region includes Earth models with initial temperatures between 1500 and 3000°C.

by rheology, independent of planetary size. The size of a planet influences the cooling history mainly through its effect on lithosphere thickness. Smaller planets generally have thicker lithospheres. Because average mantle temperature is principally fixed by rheology, the temperatures in planetary mantles are essentially independent of lithosphere thickness. If fact, calculations show that mantle temperatures for models that have no lithospheres at all are the same as temperatures for models that include lithospheric growth (Schubert, Cassen & Young 1979a).

Mantle viscosity vs time for the cooling history models of Figure 3 are shown in Figure 4. The dramatic increase in viscosity early in the cooling of the Earth models reflects the rapid decrease in T. The viscosity for the Moon model increases only by about one order of magnitude during 4.5 Gyr because of the small decrease in T over geologic time. At $t = 4.5$ Gyr, ν is substantially the same for ⊕ and ☾ (ν for the highest temperature Earth model is 4.1×10^{21} cm^2/s and ν for ☾ is 4.8×10^{22} cm^2/s) reflecting the approximate equality of temperature calculated for the mantles of these planets.

The calculations of Schubert, Cassen & Young (1979a) include the thickening of a lithosphere as a planet cools. They assumed that litho-

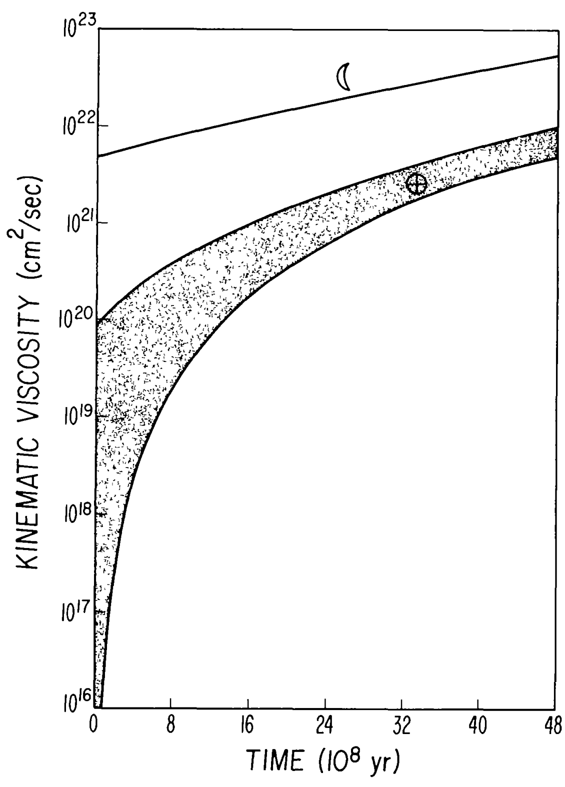

Figure 4 Mantle viscosity vs time for the cooling history models of Earth and Moon shown in Figure 1 (Schubert, Cassen & Young 1979a). Viscosity curves for Earth models with initial temperatures between 1500 and 3000°C lie in the shaded region.

spheres grow from an initial thickness of only 100 m; the growth is extremely rapid during the very earliest stages of cooling, as shown in Figure 5. The growth rate is more modest throughout most of a planet's history. The lithosphere thickness vs time curves of Figure 5 correspond to the cooling histories of Figure 3. Lithospheres thicken to 1 km at $t = 10^4$ yr for the Moon model and 35 Myr for the highest temperature Earth model. Ten-kilometer-thick lithospheres are formed after only 1 Myr for ☾ and 280 Myr for ⊕. At $t = 4.5$ Gyr lithospheres have thickened to 225 km for ⊕ and 550 km for ☾. The growth curves in Figure 5 assume that lithospheric thickening on the planets is unimpeded by other processes. During the initial growth, vigorous convection may preclude the formation of a competent lithosphere. A higher rate of impacting objects during this time than at present may also slow the accumulation of a competent lithosphere. Thus, while the figure shows that nearly a third of a billion years is required to form a ten-kilometer thick lithosphere for the hot Earth model, the time is probably closer to a billion years, in agreement with other lines of evidence suggesting a very thin terrestrial lithosphere during its first billion years of evolution (Wetherill 1972). Except for the hot Earth model, lithosphere growth is so rapid during these early cooling stages that the impediments to lithosphere formation probably make

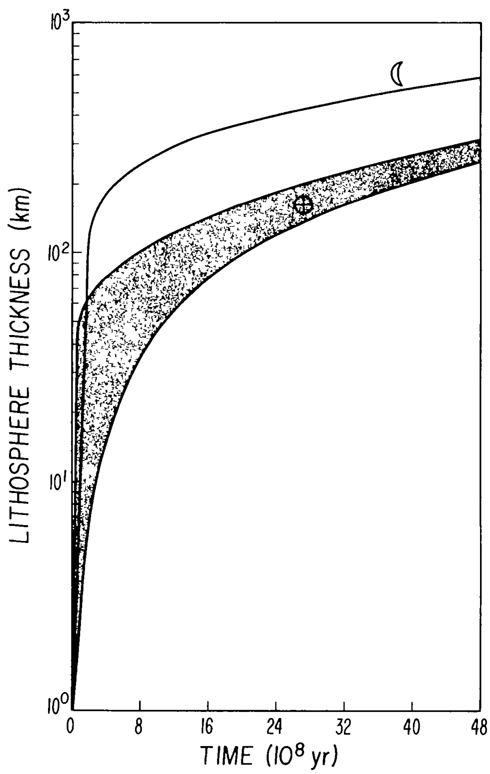

Figure 5 Lithospheric growth in the Earth and Moon models of Figure 1 (Schubert, Cassen & Young 1979a).

little difference to its thickness after 4.5 Gyr. On Earth, constraints on the growth of a lithosphere continue through the present. Plate tectonics severely limits the lithosphere thickness beneath the ocean basins. However, the estimates of lithosphere thickness in Figure 5 should be relevant to the lithosphere beneath continental shields. Since the Moon shows no evidence of plate tectonics the estimates of lithosphere thickness should be directly applicable.

Why we have plate tectonics on Earth with the continual creation and destruction of oceanic lithosphere is still an open question. When we are asked to explain why there is no plate tectonics on Mars, Mercury, or the Moon we are tempted to say that these planets have thicker lithospheres which are more resistant to breakup in the style of Earth tectonics. However, one may wonder if such tectonics occurred on these smaller terrestrial planets when their lithospheres were thinner only to have the evidence obliterated by meteoritic bombardment. We have no evidence from the present geologic surface features of these planets that plate tectonics ever occurred on them while very ancient cratered surfaces have survived. Kaula (1975) suggested that all the terrestrial planets did in fact pass through a stage of plate tectonics in their evolutions, but that

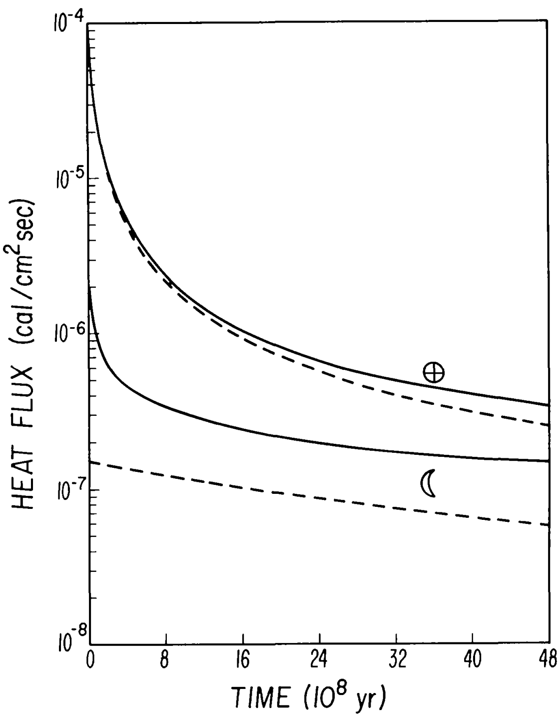

Figure 6 Temporal decay of surface heat flow (solid curves) and heat flow into the base of the lithosphere (dashed curves) for the cooling history models of the Moon and the Earth (for a 3000°C initial temperature) shown in Figure 1 (Schubert, Cassen & Young 1979a).

this stage occurred too early and too rapidly on Mercury and the Moon for any traces of it to remain.

The temporal behavior of surface heat flux q_s and heat flux into the base of the lithosphere q_c can be deduced from the cooling history calculations of Schubert, Cassen & Young (1979a). These quantities are shown in Figure 6 for the hot Earth model and the lunar model discussed in previous figures. The values q_s and q_c for the Earth model undergo dramatic early decreases due to the rapid decay of extremely vigorous convection followed by more gradual decreases throughout most of geologic time. Because of the rapid early thickening of the lunar lithosphere, q_s decreases markedly for the Moon model; q_c for the Moon model decreases slowly over the entire thermal evolution because convection is not very vigorous at the relatively low initial temperature of the model.

A significant result of these calculations is the lag between the decay of the surface heat flux and the decay of the heat flux into the base of the lithosphere. This is entirely due to the formation and thickening of the lithosphere. At any time in the cooling history of a planet there is more heat flowing through the surface than is flowing into the base of its lithosphere (assuming a competent lithosphere that is not a part of the mantle convection system). Even if convection in the mantle is in a quasi-steady state, the heat flow from the mantle is not in balance with the heat flow through the surface. The physical mechanism which accounts for the lag between "internal heat production" (heat from the mantle) and surface heat flux is the tendency of the lithosphere to supplement the decaying heat flux from the mantle by feeding on the internal energy of the mantle.

Daly & Richter (1978) recently addressed the issue of whether there is a balance between surface heat flux and instantaneous internal heat sources. On the basis of numerical calculations of convection in a two-dimensional box with decaying radiogenic heat sources, they concluded that the surface heat flux exceeds the instantaneous internal heat production even for convection with initial Rayleigh number as large as 10^6. They attribute this to the fact that conduction across closed streamlines must still play a role in removing heat from the core region of a convecting system with decaying internal heat sources. This source of nonequilibrium between internal heat production and surface heat flux is distinct from the one associated with lithospheric thickening. It is also not clear how relevant Daly & Richter's (1978) result is for the real Earth, since Rayleigh numbers much larger than 10^6 have probably characterized the Earth's mantle throughout most of its evolution.

Thus the use of present day surface heat flux observations to infer the total concentration of radiogenic heat sources in a planet on the basis of a presumed steady state thermal balance must be viewed with caution.

The surface heat flux after 4.5 Gyr of cooling without internal heat sources calculated for the Earth is still 0.35 μcal/cm^2 s (Schubert, Cassen & Young 1979a), a significant fraction of the actual present day mean surface heat flux of 1.5 μcal/cm^2 s (Oxburgh & Turcotte 1978). The Moon's surface heat flux after 4.5 Gyr of convective cooling from a modest initial temperature with no internal heat sources is calculated to be 0.15 μcal/cm^2 s (Schubert, Cassen & Young 1979a), nearly 1/3 to 1/2 of the two measured lunar surface heat flow values (Langseth, Keihm & Peters 1976). Thus, a rather substantial fraction of the present day heat flow from a planet could be attributed to primordial heat, still another reason to exercise caution in estimating the present day concentration of radiogenic heat sources in a planet from surface heat flux observations. Sharpe & Peltier (1978) and Stevenson & Turner (1979) have also recently concluded that whole mantle convection in the Earth driven solely by primordial heat content could persist for the age of the Earth and that cooling could be a significant contribution to the present day terrestrial surface heat flux. The convective lunar thermal history models of Cassen et al (1979) also exhibit a surface heat flux in excess of that attributable to radioactive heat sources.

Figures 7 and 8 show Ra and Nu vs time for the cooling history models of Schubert, Cassen & Young (1979a) already discussed in the previous

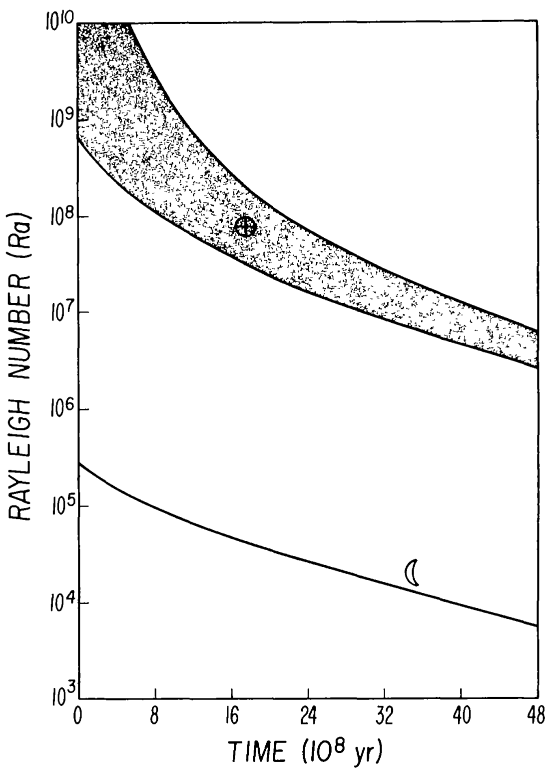

Figure 7 Decrease of Rayleigh number with time in the cooling history models of Earth and Moon shown in Figure 1.

figures. During their early histories, the Earth and Moon models are highly supercritical; even the present day values of Ra for ⊕ and ☾ indicate a vigorously convecting mantle for ⊕ and modest convection in the Moon. The present day estimate of Nu in ⊕ indicates that about 10 times as much heat is being transported to the surface by convection as compared with conduction; for the Moon model the factor is only about two.

The thermal history calculations already referenced in this section were all carried out using the Nusselt number – Rayleigh number power law relation given in Equation (6). Recently, Hsui & Toksöz (1978), Toksöz & Hsui (1978), and Toksöz, Hsui & Johnston (1978) reported thermal evolution computations for all the terrestrial planets except Earth in which they incorporated solid state convective heat transport by numerically solving the equations of motion and heat transfer for axisymmetric convection in spherical geometry of a Newtonian fluid with temperature-dependent viscosity. Their models evolve from arbitrary temperature profiles determined mainly by accretional heating and they simulate core formation, melting, and upward differentiation of radioactives as well as solid state convection. For the Moon, Toksöz, Hsui & Johnston (1978) predict a present day thermal state which involves solid state convection

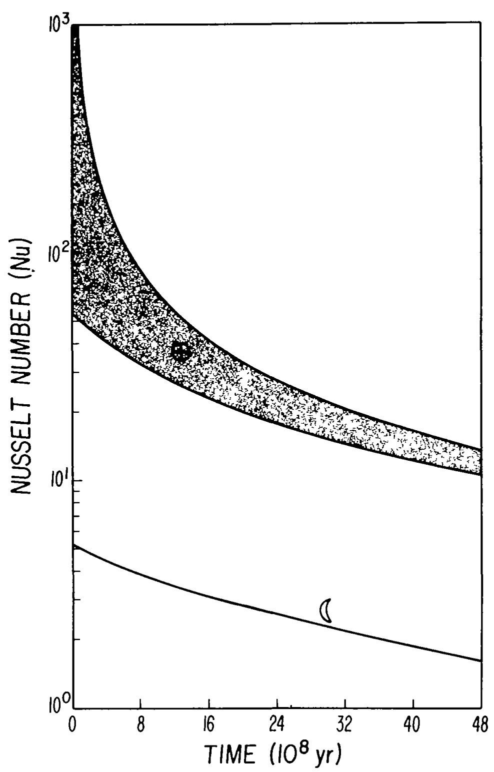

Figure 8 Decrease of Nusselt number during the evolution of the Earth and Moon for the thermal history models of Figure 1.

below a depth of 800 km. However, their mean temperature profile does not look very much like the typical convection profiles shown in Figure 1. This may result from their assumption of rather extensive differentiation of the outer volume of the Moon early in its history with accompanying efficient upward differentiation of radioactives, leading to a present day model which has a rather thick lithosphere highly depleted in radioactives and a central convecting region undepleted in radioactives. While the deep interior temperature is probably still regulated mainly by the rheology of the convecting material, the thermal profile in the outer part of the Moon has been strongly influenced by processes other than convection, e.g. initial conditions and upward differentiation of radioactives. Cassen et al (1979) also concluded that the thermal state of the lunar lithosphere is sensitive to the efficiency of heat source redistribution while that of the deep interior depends primarily on rheology.

For Mercury, Toksöz, Hsui & Johnston (1978) concluded that solid state mantle convection would cease about 2 billion years after formation of the planet. While this is in agreement with the cooling history calculation of Schubert, Cassen & Young (1979a), our previous discussion and other models of Mercury's internal thermal state show that convection in Mercury's mantle at present cannot be ruled out. However, present day convection in Mercury's mantle should at best be rather weak (Cassen et al 1976). Toksöz & Hsui (1978) and Toksöz, Hsui & Johnston (1978) calculate present day Mars models which involve convection beneath a lithosphere two hundred kilometers thick. The simple cooling history model of Schubert, Cassen & Young (1979a) produces a martian lithosphere about 300 km thick at present.

Hsui & Toksöz (1978) asserted that the size of a planet is more important than any other factor in controlling thermal evolution, whereas we, and Tozer (e.g. 1972a), have emphasized the importance of mantle rheology. We would agree with Hsui & Toksöz (1978) that the size of a planet is important, to the extent that planetary size limits the occurrence of convection; objects which are too small may not be convecting. For planetary bodies sufficiently large to be convecting, rheology, not size, will control the deep temperature. Size, however, will determine the thickness of the lithosphere.

Mantle Convection and Core Freezing

Mantle convection is so efficient at cooling a planet that it can readily lead to core freezing. This was first demonstrated quantitatively by Young & Schubert (1974) and Schubert & Young (1976), who calculated temperatures in convecting, constant viscosity, internally heated fluid models of the mantles of Mars and Earth. Schubert & Young (1976) showed that the temperature at the core-mantle boundary would lie

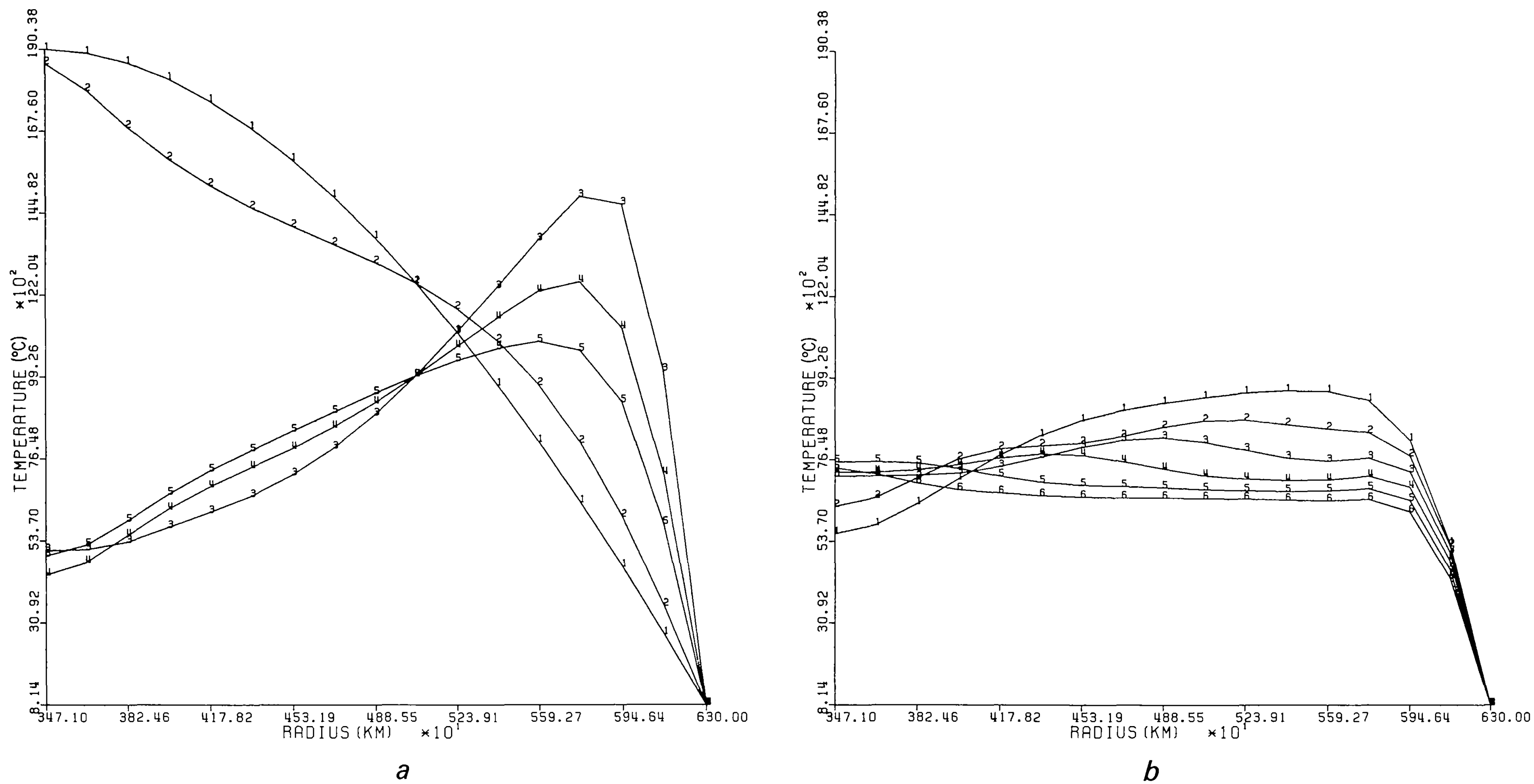

Figure 9 (*a*) Rapid decay of temperature in a model of the Earth's mantle (Schubert & Young 1976, Schubert, Cassen & Young 1979b) which cools from a hot initial conduction profile (curve 1). The profiles with successively higher numbers correspond to later times. The time between successive profiles is only 5×10^{-3} times the conduction time across the Earth's mantle (about 3×10^{11} yr). The large temperature drop early in the evolution of the model requires only 3×10^{9} yr. The numerical calculations are for $Ra/Ra_{cr} = 100$. (*b*) The continued evolution of the temperature profiles of Figure 9(*a*). Curve (1) on this figure is the temperature profile 5×10^{-3} conduction times after curve (5) on Figure 9(*a*).

significantly below the iron melting point if the Earth's mantle viscosity were less than 10^{24} cm^2/s. Cassen et al (1976) showed that convection in the relatively thin mantle of Mercury could freeze its core in a billion years or less.

If the viscosity of the earth's mantle is indeed nearly uniform with the value of 10^{22} cm^2/s, as inferred from glacial rebound data (Cathles 1975, Peltier 1976), how could the outer core still be liquid? The existence of a liquid outer core in the Earth places a significant constraint on the efficacy of subsolidus convective cooling during the thermal history of our planet. While we must find a reason why overly efficient mantle convection has not frozen the Earth's core, this problem may not exist for one or more of the other terrestrial planets if future seismic observations should reveal a solid core or if future magnetic observations should confirm the absence of a planetary field.

Schubert & Young (1976) reported that the quasi-steady, average mantle temperatures in their Earth models were established on a time scale no larger than a tenth of a conduction time across the mantle (a conduction time for the Earth's mantle is about 3×10^{11} yr). In fact, upon examining the transient development of mean temperature profiles for the Earth models in more detail, Schubert, Cassen & Young (1979b) found that only a few hundredths of a mantle conduction time is required to establish the low quasi-steady average temperatures at a Rayleigh number only 100 times the critical value. Figure 9, from Schubert, Cassen & Young (1979b), shows how quickly a hot initial conduction temperature profile is reduced by vigorous convection to relatively low temperatures throughout the mantle and in particular at the core-mantle interface for $\mathrm{Ra} = 100\,\mathrm{Ra_{cr}}$. Since we estimated the Rayleigh number for the present Earth to be about 10^7, and the Rayleigh number is likely to have been many orders of magnitude larger just after core formation (see Figure 7), the ease with which whole mantle convection could freeze the core on time scales very much less than the age of the Earth is clear.

For the Earth, there are several ways in which core solidification by subsolidus convective cooling can be prevented. One way is to have a significant source of radioactive heating in the core. Another way is to prevent convection from reaching the lower mantle for a portion of the Earth's history, particularly during the initial period of cooling after core formation when convection should be especially vigorous. This may be accomplished by chemically or viscously stratifying the lower mantle or by hypothesizing that the lower mantle geotherm is subadiabatic. In view of both the inference from glacial rebound data that mantle viscosity is essentially uniform and the argument of Sammis et al (1977) against large viscosity jumps across the major mantle phase transitions, a subadiabatic lower mantle accessible only by some form of weak penetrative convection may be the more likely explanation. The models of Sharpe & Peltier (1978) rely on an assumed subadiabaticity of the lower mantle to prevent core solidification by solid state convective cooling.

CONCLUDING REMARKS

Whereas a decade ago subsolidus convection even in the Earth's upper mantle was a minority view, today this is no longer true. The debate usually centers on the characteristics of convection in the terrestrial planets rather than on its existence. We have made considerable progress in our understanding of mantle convection. Two-dimensional and axisymmetric three-dimensional numerical calculations incorporating the temperature- and pressure-dependence of mantle rheology and its non-Newtonian character have allowed us to gain an appreciation for the relative importance of these properties. Theoretical scaling arguments and boundary layer theories have extended our knowledge of convection to higher Rayleigh numbers than direct numerical calculations can deal with. Even so, we are far from being able to model the extremely high Rayleigh number convection likely to have occurred in all the terrestrial planets at some time in their evolutions and likely to be occurring in the larger terrestrial planets at present. Very high Rayleigh number convection in the planets is undoubtedly fully three-dimensional and probably time-dependent as well; present computing limitations probably preclude the direct numerical modelling of such convection. Our understanding of high Rayleigh number mantle convection is made difficult by both these computational barriers and the uncertainties in thermodynamic and rheologic properties of mantle materials, especially for the constituents of the lower mantles of the large planets.

Nevertheless, we need to persevere in our attempts to study mantle convection using all the approaches available to us whether theoretical, numerical, or experimental. Each of these approaches has its own set of advantages and disadvantages and all of them are worth pursuing for the different insights they provide. Rigorous fluid dynamic studies of convection should be carried out even for parameter values not directly applicable to planetary interiors because of the fundamental knowledge we gain of the convective process. Reasonable though nonrigorous modelling of mantle convection for parameter values directly relevant to the planets is also worthwhile as a way of providing some quantitative assessments of our ideas about the way convection might actually work in the planets.

Much emphasis has been placed on the building of two-dimensional models of mantle convection by the community of modellers primarily interested in the Earth. However, the evidence for shallow mantle convection vs whole mantle convection in the Earth necessitates efforts in directions other than two-dimensional numerical computations. Since whole mantle convection in the Earth and the other terrestrial planets is a highly likely form of convection, we should direct attention toward constructing models of convection in spherical geometry. Even models of axisymmetric convection in spheres and spherical shells would be a welcome addition to the literature, although their stability to nonaxisymmetric disturbances should always be determined.

Future planetary exploration will provide crucial data against which to test our ideas about mantle convection. We will learn a great deal when radar observations of Venus allow us to determine whether the surface contains any record of plate tectonic activity (Weertman 1979). Earth-based radar observations (Malin & Saunders 1977) and Pioneer Venus radar measurements (Masursky et al 1977) will contribute significantly toward this end, but definitive conclusions will require the global coverage and resolution of a Venus Orbiting Imaging Radar. Our discussion of convection in terrestrial planets has concentrated on the planets of the inner solar system. Yet the outer solar system contains bodies whose global properties, density in particular, would place them in the category of terrestrial planets. Io and Europa, two of the Galilean satellites, are examples of such bodies (Johnson 1978) and future spacecraft reconnaisance of their surfaces, shapes, gravitational and magnetic fields, etc will provide still additional experimental tests of our theories of convection. The Galilean satellites Ganymede and Callisto, which by virtue of their low densities must have water as a major constituent (Johnson 1978), may allow us to view the consequences of solid state convection in a planet with a rheology quite different from that of the silicate-dominated bodies of the solar system (Reynolds & Cassen 1979). The imminent Voyager exploration of the Galilean satellites and the planned Galileo observations of these objects may reveal new worlds whose evolutions have also been influenced, if not dominated, by solid state convection.

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