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THE STABILITY OF A ROTATING UNIVERSE

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ABSTRACT

We present an exact cosmological solution of Einstein's equation which has expansion, shear, and rotation. The source of this geometry is a fluid which has not been thermalized. The model tends asymptotically to Gödel's cosmos—thus making our solution a previous era of Gödel's universe.

Subject headings: cosmology — relativity — rotation

I. INTRODUCTION

The great majority of relativistic cosmological models treat the energy content of the Universe as an idealized perfect fluid with density ρ and pressure p. More realistic attempts to investigate the galactic fluid near the unavoidable singularity have tried to incorporate viscous effects by assuming some ad hoc forms of dissipative processes (Murphy 1973; Misner 1968). Recent investigations (Belinskii and Khalatnikov 1976) have shown that the ultimate behavior of such models is a very sensitive function of the viscosity coefficients η and ξ on the energy density. Thus, by assuming a large spectrum of the functions $\eta(\rho)$ and $\xi(\rho)$ one deals with a variety of universes, some of which may more closely conform to reality than the idealized Friedmann-like cosmologies. Such generalization from the usually accepted highly symmetric cosmological models is intimately related to a suggestion by means of which global properties of our universe may vary with time. From a somehow arbitrary initial stage, in which the arbitrarily inhomogeneous galactic fluid may have shear and/or rotation, the cosmos could evolve toward our present almost homogeneous and isotropic era through some dissipative processes (Misner 1968). Although there is no strong observational support to this speculation, it has the advantage of limiting the need for an explanation of the initial conditions. Consequently one could hope to avoid completely any discussion of initial conditions, and this would place cosmology on a much sounder base.

In the same vein one should study off-equilibrium configurations of the galactic matter. In the present paper we intend to start a systematic study of some special models in which the thermal equilibrium of the galactic fluid has not been attained and so a stage occurs in which there is heat exchange between parts of the fluid. The model we study here represents an expanding universe which has shear and rotation. As the universe expands, the total amount of heat exchanged decreases and finally vanishes for very large values of the time. The galactic content tends toward a perfect fluid configuration. The expansion and the shear also tend to zero at $t \to \infty$, but the vorticity remains constant along the whole history of our universe. As for the gravitational energy content, it is an admixture of constant electric part $E_{\mu\nu}E^{\mu\nu}$ and a magnetic part $H_{\mu\nu}H^{\mu\nu}$ which decreases as the universe expands and finally vanishes after an infinite lapse of time. Thus, our model can represent a "previous" nonstationary stage of Gödel's cosmos.

II. THE MATTER CONTENT

Let V^μ represent the velocity of the matter, and let us select a coordinate system comoving with the fluid. We set $V^\mu = \delta_0{}^\mu$. The most general expression for the energy-momentum tensor can be set into the form

$$T_{\mu\nu} = \rho V_\mu V_\nu - p h_{\mu\nu} + q_{(\mu} v_{\nu)} + \pi_{\mu\nu} \tag{1}$$

in which $h_{\mu\nu} \equiv g_{\mu\nu} - V_\mu V_\nu$, $\pi_{\mu\nu}$ is the anisotropic pressure, and q^μ is the heat flux (four-vector). These quantities obey the conditions

$$q_\mu V^\mu = 0\,, \tag{2a}$$

$$\pi_{\mu\nu} g^{\mu\nu} = 0\,, \tag{2b}$$

$$\pi_{\mu\nu} V^\mu = 0\,, \tag{2c}$$

$$\pi_{\mu\nu} = \pi_{\nu\mu}\,. \tag{2d}$$

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The cosmic fluid will have, in general, an expansion $\theta = V^{\alpha}{}_{\|\alpha}$, a shear

$$\sigma_{\mu\nu} = \tfrac{1}{2}h_{\mu}{}^{\lambda}h_{\nu}{}^{\epsilon}V_{(\lambda\|\epsilon)} - \tfrac{1}{3}\theta h_{\mu\nu}\,,$$

a rotation

$$\omega_{\mu\nu} = \tfrac{1}{2}h_{\mu}{}^{\lambda}h_{\nu}{}^{\epsilon}V_{[\lambda\|\epsilon]}\,,$$

and an acceleration $a^{\alpha} = V^{\alpha}{}_{\|\lambda}V^{\lambda}$, where the double bar ($\|$) means covariant derivative.

In order to generate cosmological models, phenomenological relations between the dynamical and kinematical quantities have been proposed through extrapolation of the behavior of certain experimentally accessible fluids. For instance, an expression relating the anisotropic pressure to the shear through a viscosity coefficient, $\pi_{\mu\nu} = \lambda\sigma_{\mu\nu}$ has been (Misner 1968) widely used as a phenomenological description of a gas of neutrinos, for instance, in certain models of the universe. Use of expressions $p' = p + B\theta$ borrowed from fluid mechanics, which has the consequence of changing the pressure p by adding to it a term proportional (through a viscosity coefficient B) to the expansion θ, has been made (Murphy 1973; Belinskii and Khalatnikov 1976). Finally a relation has been proposed which involves the heat flux q^{μ} and the acceleration a^{μ} by means of a temperature-like function, although very few (if any) explicit cosmological models have been developed which use this hypothesis. Here, instead of assuming any type of such ad hoc relations, we try to follow another method which we will explain next.

From the acceleration a^{μ} and the vorticity vector $\omega^{\tau} = \tfrac{1}{2}\eta^{\alpha\beta\rho\tau}\omega_{\alpha\beta}V_{\rho}$ we construct the vector $\eta^{\alpha} = \eta^{\alpha\beta\mu\nu}a_{\beta}\omega_{\mu}V_{\nu}$. The vector η^{α} is not only orthogonal to the velocity V^{α}, it is also orthogonal to both the acceleration and the vorticity vectors. Using these three vectors a^{α}, ω^{α}, and η^{α}, we can construct a basis in the local 3-dimensional rest space orthogonal to V^{α}. We will call this set the kinematical basis (K-basis, for short). Due to property (2a) we can develop the current q^{α} in the K-basis and write

$$q^{\mu} = \eta a^{\mu} + \psi\omega^{\mu} + \varphi\eta^{\mu}{}_{\alpha\beta\lambda}a^{\alpha}\omega^{\beta}V^{\lambda}\,. \tag{3}$$

This decomposition, when the K-basis is available, has the advantage that it does not assume any phenomenological relation. Furthermore, one is still free to impose extra conditions on q^{α}. For instance, if we set as usual $q_{\mu} = \kappa h_{\mu}{}^{\lambda}(T_{|\lambda} + Ta_{\lambda})$, then a direct inspection relates the coefficients of the q^{μ} expansion on the K-basis to the temperature T. If the temperature is constant in the 3-dimensional rest space orthogonal to V^{α}, then we can identify the coefficient η of the expansion to the temperature T, in which case $\psi = \varphi = 0$.

Here we will try to exploit this decomposition and present a specific way to generate off-equilibrium models.

Let us make a final remark on the K-basis. Unless one knows some properties of the geometry, one cannot justify the possibility of the construction of such basis.

It is worthwhile, however, to mention some example in which one can deal with such basis. The simplest case occurs when the vorticity vector ω^{α} is a Killing vector. In this case,

$$\omega_{\alpha\|\beta} + \omega_{\beta\|\alpha} = 0\,,$$

and thus the vorticity vector is divergence-free: $\omega^{\alpha}{}_{\|\alpha} = 0$.

Now, from the definition of the Riemann tensor we have

$$V_{\alpha\|\beta\|\lambda} - V_{\alpha\|\lambda\|\beta} = R_{\alpha\mu\beta\lambda}V^{\mu}\,.$$

Antisymmetrizing in the indices α, β, γ and using the identity

$$R^{\mu}{}_{\alpha\beta\lambda} + R^{\mu}{}_{\beta\lambda\alpha} + R^{\mu}{}_{\lambda\alpha\beta} = 0\,,$$

we obtain the so-called constraint equation (Ellis 1971)

$$\omega^{\alpha}{}_{\|\alpha} = \omega^{\alpha}a_{\alpha}\,.$$

Thus, if the vorticity vector generates an isometry, we have $\omega^{\alpha}a_{\alpha} = 0$.

In the case we present here we will take the vorticity vector in the z-axis, that is, $\omega^{\alpha} = (0, 0, 0, \Omega)$, where, as we will see, Ω does not depend on the Z-coordinate.

Besides this, our metric does not depend on the Z-coordinate, too. Thus we obtain $(-g)^{-1/2}[(-g)^{1/2}\omega^{\alpha}]_{|\alpha} = 0$, and thus by means of the constraint equation, the vorticity and the acceleration vectors are orthogonal.

III. THE GEOMETRY

In 1949 K. Gödel published the first cosmological model generated by a solution of the modified Einstein equations in which a cosmological repulsive term ($\Lambda g_{\mu\nu}$) has been added. The congruence of the geodesics $V^{\alpha} = \delta_0{}^{\alpha}$ has no shear, no expansion, no acceleration but presents a constant rotation of the matter relative to the compass of inertia. After this discovery, many attempts have been made to construct more general solutions which, besides rotation, should present expansion and/or shear.

Let us describe our model by setting

$$ds^2 = dt^2 + 2A(x, t)dydt + 2M(x, t)dzdt - B(x, t)dy^2 - C(x, t)dz^2 - F^2(t)dx^2 . \tag{4}$$

Choosing a comoving frame we can set the fluid velocity V^α to have the values $V^0 = 1$, $V^i = 0$ $(i = 1, 2, 3)$. In the present paper we limit our discussion to the case in which the rotation is in the z-direction by assuming $M = 0$. Our perturbations of the rotating Gödel model leaves the vorticity vector in the z-direction. We will make some comments on this later on. Further, we set $C(x, t) = H^2(t)$ in which C depends only on the t-parameter. The acceleration vector a^α is given by

$$a^\alpha = \left(\frac{A\dot{A}}{A^2 + B}, 0, \frac{-\dot{A}}{A^2 + B}, 0\right),$$

and the components of the vorticity vector are $\omega^\alpha = (0, 0, 0, \Omega)$ where

$$\Omega = \frac{A'}{2FH}\frac{1}{(A^2 + B)^{1/2}}$$

and $A' = \partial A/\partial x$, $\dot{A} = \partial A/dt$. From formula (3) we obtain the covariant components of the heat flux in the kinematical basis:

$$q_\alpha = \left(0, \frac{\varphi}{2}\frac{\dot{A}A'}{A^2 + B}, \eta\frac{B\dot{A}}{A^2 + B}, -\psi H^2\Omega\right).$$

The total amount of heat is given by the norm $L = q^\alpha q_\alpha$. We analyze the case in which there is no anisotropic pressure; that is, we set $\pi_{\mu\nu} = 0$.

Let us choose a class of locally stationary observers represented by the tetrad vectors $e^A{}_{(\alpha)}$ in which (α) signifies the tensor indices and A $(= 1, 2, 3, 4)$ signifies the tetrad indices given by

$$e^0{}_{(0)} = 1 ; \quad e^1{}_{(1)} = F ; \quad e^2{}_{(2)} = (A^2 + B)^{1/2} ; \quad e^3{}_{(3)} = H ; \quad e^0{}_{(2)} = A .$$

A straightforward calculation gives, for the contracted Riemann tensor R_{AB}, in the tetrad frame, the values:

$$R_{00} = \frac{\ddot{F}}{F} + \frac{\ddot{H}}{H} - \frac{1}{4}\frac{A^{12}}{\gamma F^2} - \frac{\dot{F}}{F}\frac{A\dot{A}}{\gamma} + \frac{1}{2}\frac{\ddot{\gamma}}{\gamma} - \frac{1}{4}\frac{\dot{\gamma}^2}{\gamma^2} - \frac{A}{\gamma^{1/2}}\left(\frac{\dot{A}}{\gamma^{1/2}}\right)^{\cdot} - \frac{\dot{A}^2}{\gamma} - \frac{1}{4}\frac{A^{12}}{F^2\gamma} - \frac{\dot{H}}{H}\frac{A\dot{A}}{\gamma};$$

$$R_{01} = \frac{1}{2F}\frac{\dot{\gamma}'}{\gamma} - \frac{1}{4F}\frac{\dot{\gamma}\gamma'}{\gamma^2} - \frac{1}{F}\frac{\dot{A}A'}{\gamma} + \frac{1}{2}\frac{AA'}{\gamma}\frac{\dot{F}}{F^2} - \frac{1}{2}\frac{A\dot{A}'}{F\gamma} - \frac{1}{2}\frac{\dot{F}}{F^2}\frac{\gamma'}{\gamma} + \frac{1}{4}\frac{AA'\dot{\gamma}}{\gamma^2} - \frac{1}{2}\frac{\dot{H}}{H}\frac{AA'}{F\gamma};$$

$$R_{02} = -\frac{1}{2F^2}\left(\frac{A'}{\gamma^{1/2}}\right)' - \frac{A}{\gamma^{1/2}}\frac{\ddot{H}}{H} + \frac{1}{2}\frac{\dot{\gamma}}{\gamma^{3/2}}\frac{\dot{F}A}{F} - \frac{A}{\gamma^{1/2}}\frac{\ddot{F}}{F} + \frac{1}{2}\frac{\dot{H}}{H}\frac{A\dot{\gamma}}{\gamma^{3/2}};$$

$$R_{03} = 0;$$

$$R_{11} = -\frac{\ddot{F}}{F}\left(1 - \frac{A^2}{\gamma}\right) - \frac{1}{2}\frac{A'^2}{F^2\gamma} + 2\frac{A\dot{A}}{\gamma}\frac{\dot{F}}{F} + \frac{1}{2F^2}\frac{\gamma''}{\gamma} - \frac{1}{4F^2}\frac{\gamma'^2}{\gamma^2} + \frac{A^2}{\gamma}\frac{\dot{F}}{F}\frac{\dot{H}}{H} - \frac{1}{2}\frac{\dot{F}}{F}\frac{\dot{\gamma}}{\gamma} - \frac{1}{2}\frac{A^2\dot{F}}{F}\frac{\dot{\gamma}}{\gamma^2} - \frac{\dot{F}}{F}\frac{\dot{H}}{H};$$

$$R_{12} = -\frac{1}{H\gamma}\frac{d}{dt}\left(\frac{H}{F}A'\gamma^{1/2}\right) + \frac{\gamma'}{\gamma^{3/2}}\frac{1}{FH}\frac{d}{dt}(AH);$$

$$R_{13} = 0;$$

$$R_{22} = -\frac{1}{2}\frac{\ddot{\gamma}}{\gamma} + \frac{1}{4}\frac{\dot{\gamma}^2}{\gamma^2} + \frac{A\ddot{A}}{\gamma} - \frac{1}{2}\frac{A\dot{A}\dot{\gamma}}{\gamma^2} + \frac{\dot{A}^2}{\gamma} - \frac{1}{2}\frac{A'^2}{F^2\gamma} + \frac{1}{2F^2}\frac{\gamma''}{\gamma} - \frac{1}{4F^2}\frac{\gamma'^2}{\gamma^2} + \frac{A^2}{\gamma}\frac{\ddot{F}}{F} - \frac{1}{2}\frac{\dot{F}}{F}\frac{\dot{\gamma}}{\gamma} + \frac{A\dot{A}}{\gamma}\frac{\dot{F}}{F} + \frac{A^2}{\gamma}\frac{\ddot{H}}{H}$$
$$- \frac{1}{2}\frac{A^2}{\gamma^2}\frac{\dot{F}}{F} + \frac{A\dot{A}}{\gamma}\frac{\dot{H}}{H} - \frac{1}{2}A^2\frac{\dot{H}}{H}\frac{\dot{\gamma}}{\gamma^2} - \frac{1}{2}\frac{\dot{H}}{H}\frac{\dot{\gamma}}{\gamma};$$

$$R_{23} = 0,$$

$$R_{33} = -\frac{\ddot{H}}{H} + \frac{\dot{H}}{H}\frac{A\dot{A}}{\gamma} - \frac{\dot{H}}{H}\frac{\dot{F}}{F} + \frac{A^2}{\gamma}\frac{\dot{F}}{F}\frac{\dot{H}}{H} + \frac{A}{\gamma^{1/2}}\left(\frac{\dot{H}}{H}\frac{A}{\gamma^{1/2}}\right)^{\cdot} + \frac{A^2}{\gamma}\frac{\dot{H}^2}{H^2} - \frac{1}{2}\frac{\dot{\gamma}}{\gamma}\frac{\dot{H}}{H},$$

in which $\gamma = A^2 + B$.

The fact that R_{03} vanishes implies $T_{03} = 0$.

In the tetrad frame we have

$$T_{03} = q_3 V_0 = -\psi H^2 \Omega = 0\,;$$

and thus we conclude that the heat flux q^α rests in the plane orthogonal to the vorticity vector.

The non-null components of the energy-momentum tensor in the tetrad frame are given by

$$T^0{}_0 = \rho\,; \qquad T^0{}_1 = \frac{\varphi}{2F}\frac{\dot{A}A'}{A^2 + B}\,; \qquad T^0{}_2 = \eta\frac{B\dot{A}}{(A^2 + B)^{3/2}}\,; \qquad T^1{}_1 = T^2{}_2 = T^3{}_3 = -p\,.$$

Now, the above energy-momentum tensor and Einstein's equations imply the condition $R_{12} = 0$. This gives

$$\frac{d}{dt}\left[\frac{H}{F}A'(A^2 + B)^{1/2}\right] - \frac{(B' + 2AA')}{(A^2 + B)^{1/2}}\frac{1}{F}\frac{d}{dt}(AH) = 0\,. \tag{5}$$

If we set

$$B = (m - 1)A^2\,, \tag{6}$$

where m is a constant, we obtain from equation (5)

$$\frac{A'}{A} = \frac{1}{m^*}HF\,, \tag{7}$$

in which m^* is an arbitrary constant.

A remarkable property of geometry (4) is that the condition of proportionality (6) [and, for the general case, the additional requirement $C(x, t) = \mu M^2(x, t)$] ensures separability of the variable $A(x, t)$ [correspondingly also for $C(x, t)$]. We will call Gödel-like any geometry of form (4) in which condition (6) (and its generalization form) applies.

Writing $A(x, t) = A_1(x)A_2(t)$, we can integrate equation (7) immediately to obtain

$$A(x, t) = A_0 e^{cx} A_2(t) \tag{8}$$

in which A_0 and C are arbitrary constants. Furthermore, we obtain the result that the product HF is a constant:

$$HF = m^* c\,.$$

The conditions $T_{11} = T_{22} = T_{33}$ imply $R_{11} = R_{22} = R_{33}$. These give rise to the equations

$$2(m - 1)\left[\frac{\ddot{F}}{F} + \frac{\dot{F}}{F}\frac{\dot{A}}{A} - \frac{\dot{F}^2}{F^2}\right] - \frac{(2m - 1)}{2}\frac{C^2}{F^2} = 0\,, \tag{9}$$

$$(m - 1)\left[\frac{\ddot{A}}{A} + \frac{\dot{F}^2}{F^2} - \frac{\dot{A}}{A}\frac{\dot{F}}{F} - \frac{\ddot{F}}{F}\right] + (2m - 1)\frac{C^2}{F^2} - 2\frac{\dot{F}^2}{F^2} = 0\,. \tag{10}$$

A solution of this set of equations can be found by setting $F = 1$. Then the value of the constant m is $\frac{1}{2}$ and the function $A_2(t)$ is given by $A_2 = \theta_0 t + 1$, where θ_0 is a constant. The diagonal Einstein equations are

$$\rho + \Lambda = \tfrac{1}{2}C^2\,, \tag{11}$$

$$p - \Lambda = \tfrac{1}{2}C^2\,. \tag{12}$$

We remark that we can have solution in which the cosmological constant Λ vanishes. In this case we have

$$p = \rho = \tfrac{1}{2}C^2\,. \tag{13}$$

If Λ does not vanish but pressure is null, we obtain

$$\rho = -2\Lambda = C^2\,. \tag{14}$$

For the general case, with an equation of state $p = \epsilon\rho$, we have

$$\rho = -\frac{2}{1 - \epsilon}\Lambda\,, \tag{15}$$

$$\Lambda = -\frac{C^2}{2}\frac{1 - \epsilon}{1 + \epsilon}\,. \tag{16}$$

Finally, the off-diagonal components relates the coefficients φ, η with geometry. One finds

$$\varphi = 1\,, \tag{17}$$

$$\eta = 0\,. \tag{18}$$

This completes the set of Einstein's equations. The heat flux q^α is in the direction orthogonal to the plane generated by the vorticity and the acceleration. The total amount of heat and the expansion θ are given by

$$L = \frac{L_0}{(\theta_0 t + 1)^2}\,, \tag{19}$$

$$\theta = \frac{\theta_0}{\theta_0 t + 1}\,. \tag{20}$$

We can now recognize the constant θ_0 as the value assumed by the expansion at the origin of time. The constant C which appears in the expression for $A_1(x)$ can be written in terms of the ratio of the total amount of flux to the initial value of the expansion, that is, $C^2 = -L_0\theta_0^{-2}$.

A simple inspection of these results shows that, as time goes on, our model evolves in the direction of more equilibrium by slowing down the heat exchanged among its parts. Asymptotically the fluid tends to eliminate its irregularities and move in the direction of a perfect fluid behavior. Correspondingly, anisotropy also tends to disappear. Indeed the tetrad components of the shear σ_{AB} are

$$\sigma^1{}_1 = \sigma^3{}_3 = -\tfrac{1}{2}\sigma^2{}_2 = -\frac{\theta_0}{3(\theta_0 t + 1)}\,.$$

Thus, the above properties show that the ultimate fate of our model is a static rotating configuration, that is, Gödel's model. The very fact that Gödel's model is a limiting situation which our model can have, shows that Gödel's model is stable for a given set of perturbation in the direction of the vorticity. It is interesting to compare this result with that obtained by Silk (1970) some years ago. Silk has shown that Gödel's model is stable for perturbations in the plane orthogonal to the vorticity ω^α and unstable for perturbations in the direction of ω^α. However, Silk obtained this result by limiting the perturbations of the matter to those which do not change the perfect fluid conditions, that is, to fluctuations only of the density and the pressure. The result obtained by Silk does not hold if the perturbations include a small change of the perfect fluid behavior.

Let us turn now to the gravitational energy, that is, to the term $\tfrac{1}{2}(E^{\mu\nu}E_{\mu\nu} + H^{\mu\nu}H_{\mu\nu})$. The unique non-null components of the conformal Weyl tensor are (in the tetrad frame).

$$C_{0101} = C_{0202} = -\tfrac{1}{2}C_{0303} = \tfrac{1}{2}C_{1212} = -\tfrac{1}{2}C_{1313} = -\tfrac{1}{2}C_{2323} = \tfrac{1}{6}C^2$$

and

$$C_{1220} = -2C_{1330} = -\frac{\theta_0 C}{\theta_0 t + 1}\,.$$

From this we obtain the electric and magnetic parts

$$E_{\mu\nu} = -C_{\mu\rho\nu\lambda}V^\rho V^\lambda; \qquad H_{\alpha\beta} = \tfrac{1}{2}\eta_{\alpha\mu}{}^{\rho\sigma}C_{\rho\sigma\beta\lambda}V^\mu V^\lambda$$

which have the following nonzero components:

$$E^1{}_1 = E^2{}_2 = -\tfrac{1}{2}E^3{}_3 = \tfrac{1}{6}C^2\,,$$

$$H^2{}_3 = -\frac{C\theta_0}{\theta_0 t + 1}\,.$$

The magnetic gravitational energy slows down as time goes on. Only the stable (constant) electric part survives, which appears as a typical Gödel-like behavior.

IV. CONCLUSION

In the present paper we have exhibited an exact cosmological solution of Einstein's equation which has expansion, shear, and rotation. The energy content is represented by a fluid in an off-equilibrium situation which has not been thermalized. As the future of our model is Gödel's solution, it may be thought of as a previous stage of Gödel's universe. This shows that there are perturbations in the direction of the vorticity of the Gödel cosmos which are stable. This contradicts a general feeling on perturbations of rotating cosmological models—suggested by a work of Silk which says that the effect of rotation is to stabilize the model only in the plane of rotation.

The apparent contradiction is solved once one recognizes the limitation of Silk's analysis which assumes, as is usual, that the energy content does not change its perfect fluid behavior during the perturbation era. This, of course, is a limitation on the spectra of perturbations, so one cannot generalize Silk's results. Indeed, this is made clear by the solution we have exhibited in the present paper.

APPENDIX

THE EFFECT OF NONTHERMAL EQUILIBRIUM IN EINSTEIN'S COSMOS

We have shown in this paper that the stability condition is a sensitive function of the perfect fluid condition of matter in a Gödel universe. In order to gain some insight of the general behavior, we will give here a short analysis of situation in Einstein's static and homogeneous model. The perturbation of the equation of conservation of energy implies

$$(\delta\rho)^{\cdot} + \rho\delta\theta + \delta q^{\alpha}{}_{\|\alpha} = 0\,; \tag{A1}$$

and for the perturbed expansion factor $\delta\theta$ we find

$$(\delta\theta)^{\cdot} - (\delta a^{\alpha})_{\|\alpha} = -\tfrac{1}{2}\delta\rho\,. \tag{A2}$$

From the spatial components of the energy-momentum conservation law we obtain

$$\rho\delta a_{\alpha} \approx \delta\dot{q}_{\alpha} \tag{A3}$$

in which ρ is the density of energy of the background (ρ = constant). From these equations we obtain

$$(\delta\rho)^{\cdot\cdot} + \rho(\delta\theta)^{\cdot} + (\delta q^{\alpha}{}_{\|\alpha})^{\cdot} \approx 0\,. \tag{A4}$$

Now

$$(\delta q^{\alpha}{}_{\|\alpha})^{\cdot} = \delta\dot{q}^{\alpha}{}_{\|\alpha}\,.$$

This last step depends on the diagonal character of Einstein's geometry and on the conditions $\delta q^0 = 0$. We obtain

$$(\delta\rho)^{\cdot\cdot} - \tfrac{1}{2}\rho\delta\rho \approx 0\,.$$

Thus we conclude that, contrary to Gödel's model, the instability property of the density of matter in Einstein's universe is not altered by the introduction in the perturbation fluid of a small quantity of heat flux—although in this case the presence of off-diagonal terms in the energy-momentum tensor drastically changes the basic properties of the model.

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