

SPHERICALLY SYMMETRIC ACCRETION NEAR THE EDDINGTON LIMIT

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ABSTRACT

Steady, spherically symmetric accretion onto a compact object near the Eddington limiting luminosity is considered for an optically thick, ionized hydrogen plasma. Solutions assuming LTE are treated in detail, with non-LTE flows being briefly discussed. In contrast to previous treatments, we find that the spectrum peak for free-fall type flows is likely to be in the X-ray regime and not at optical or UV energies. There also exist settling type solutions for which these latter type spectra are more realistic. An analysis of convection and Rayleigh-Taylor instabilities is also given. There appear to be a number of different regimes in the solutions which depend rather sensitively on the boundary conditions. The different solutions have been classified in a way that should prove useful both for building detailed models, and for a further analysis of the stabilities of these flows.

Subject headings: stars: accretion — X-rays: binaries

I. INTRODUCTION

One of the most promising mechanisms for explaining galactic X-ray sources is accretion of matter onto compact objects. For a neutron star or a black hole, small rates of mass accretion $\dot{M}$ ($\sim 10^{-10}$ to $10^{-8} M_{\odot} \text{ yr}^{-1}$) can result in X-ray luminosities of the observed values of 10^{36} – $10^{38} \text{ ergs s}^{-1}$ (Gursky 1973). For white dwarfs, the accretion rates for similar luminosities are much larger (by a factor between 10 and 1000 depending on whether steady nuclear fusion is taking place or not). If the compact object is a member of a binary system, then the accretion rates needed are modest compared to the inferred transfer rates in other binary systems (van den Heuvel 1975), and they could be supplied by a stellar wind even if the primary star does not overflow its Roche lobe (Davidson and Ostriker 1973). An important role in accretion models is played by the so-called critical luminosity or Eddington limit at which radiation pressure balances gravity. For a fully ionized hydrogen plasma with Thomson scattering providing the dominant opacity, this luminosity for spherically symmetric flow is given, independent of optical depth, by

$$L_{\text{edd}} = \frac{4\pi G M c}{\kappa_{\text{es}}} \approx 1.3 \times 10^{38} \left(\frac{M}{M_{\odot}} \right) \text{ ergs s}^{-1}, \quad (1)$$

where $\kappa_{\text{es}} = 0.40 \text{ cm}^2 \text{ g}^{-1}$ is the Thomson scattering opacity. This luminosity is attained for an accretion rate

$$\dot{M}_{\text{edd}} = L_{\text{edd}} / \phi c^2, \quad (2)$$

where ϕ is the efficiency for converting rest mass into radiation. One might suppose that $\dot{M}$ could approach but never exceed $\dot{M}_{\text{edd}}$. In fact, there does appear to be a luminosity cutoff at around the value of L_{edd} corresponding to $M \approx M_{\odot}$ with a class of sources

whose luminosities cluster about this value (Gursky 1973; Margon and Ostriker 1973).

There exist models which predict luminosities nearly equal to L_{edd} . In the so-called self-excited wind models (Arons 1973; Basko and Sunyaev 1973; Davidson and Ostriker 1973), X-rays from a compact object in a binary system drive a wind emitted by the non-degenerate primary star. The wind in turn drives the X-ray source. Recently, however, doubts have been raised as to whether such a mechanism will work because of the small capture rate of the compact object for the matter driven from the primary (Alme and Wilson 1974; McCray and Hatchett 1975). Disk models with near critical luminosities have been discussed by Shakura and Sunyaev (1973), Mészáros and Rees (1975), Pringle and Webbink (1975), and Bath (1977). For these models, while the luminosity is $\sim L_{\text{edd}}$, the disk accretion rate may be $> \dot{M}_{\text{edd}}$ with the excess accretion flux being blown away by radiation pressure as it reaches the inner regions of the disk. As has been pointed out by Zel'dovich, Ivanova, and Nadezhim (1972) and by Ruffini and Wilson (1973), $\dot{M}$ may be greater than $\dot{M}_{\text{edd}}$ for spherically symmetric flow as well, if the excess energy can be carried away by neutrino cooling.

Steady, spherically symmetric accretion near L_{edd} has been considered by various authors. Let us define the fractional difference of the luminosity at infinity from the Eddington limit as $\epsilon_{\infty} = (L_{\infty} - L_{\text{edd}}) / L_{\text{edd}}$. Shakura (1974) has treated optically thin, supersonic flow onto a neutron star for $\epsilon_{\infty} \ll 1$. One expects, however, that for the low velocities which result near L_{edd} , the flow will become optically thick well before reaching the surface of the compact object. Optically thick flows onto neutron stars have been considered by Maraschi, Reina, and Treves (1974) for $\epsilon_{\infty} \ll 1$, where gas pressure was neglected, and by Shapiro and Salpeter (1975) for $\epsilon_{\infty} > 0.25$, where the structure of

the shock at the surface was treated in detail. Kafka and Mészáros (1976) have treated optically thick accretion while neglecting gas pressure onto black holes for $\epsilon_\infty \ll 1$. DeGregoria (1974) has considered LTE flow onto white dwarfs for $\epsilon_\infty > 0.6$. For all of these models, the flow is taken as being supersonic far from the surface of the compact objects.

In this paper, we carry out a more detailed study of steady, spherically symmetric accretion onto white dwarfs and neutron stars for $\epsilon_\infty \ll 1$. Our work differs from that mentioned above in that (a) we treat flows which are subsonic everywhere, as well as flows that become supersonic, and (b) we include the effects of gas pressure, which can significantly modify the flow in certain cases.

The most important assumptions we make are: (i) The flow is steady and spherically symmetric. While a spherical model may not give an adequate representation of a real X-ray source, the equations are reasonably tractable, and we believe that some of the important features of the spherical case will have more general application. (ii) The luminosity is fractionally less than L_{edd} ($\epsilon_\infty \ll 1$). (iii) The gas consists of ionized hydrogen, and it can be treated as a hydrodynamical fluid. (iv) The dominant opacity is that for Thomson scattering. (v) Heating and cooling time scales are short compared with flow time scales. (vi) The fluid is optically thick to scattering and absorption so that LTE holds. (vii) Possible magnetic field effects are ignored. (viii) The flow velocity is assumed to be nonrelativistic, and general-relativistic effects are ignored. (ix) The mass of the accreting gas is assumed to be negligible compared to that of the compact object. (x) The effects of steady nuclear fusion are treated only in an approximate way. (xi) Neutrino losses are neglected.

Sufficiently far from the surface of the compact object, the flow must be optically thin. Here the assumption of LTE is clearly invalid. The presence of a strong central source of radiation does more than just transfer momentum to the gas, but may profoundly affect the flow by preheating it, as was originally considered by Shvartsman (1971). More recent calculations by Buff and McCray (1974) and by Ostriker *et al.* (1976) have found that preheating may limit the luminosity to a value well below L_{edd} if $\phi \geq 0.1$ for a blackbody type spectrum. Towards the end of this paper the assumption of LTE will be relaxed, and the behavior of solutions including preheating effects will be considered. We will ignore, however, the question of whether preheating would indeed halt the flow, and only treat the region interior to where this could occur.

The results of this study show that, as opposed to the predictions of previous papers, the spectral peak for optically thick accretion for free-fall type flows is likely to be in the X-ray regime and not at UV or optical energies. However, there exist settling solutions which have a spectral maximum in the latter energy range. Both types of flow appear marginally stable to both convection and Rayleigh-Taylor instabilities.

In § II, the basic equations we use are derived and

the various approximations used are discussed. Section III presents the solutions found in the case of LTE flows. In § IIIb a detailed semianalytic derivation of the different types of flow is given. Because of the wide variety of possible solutions, this part is somewhat lengthy and can be skimmed by the casual reader. Section IV treats non-LTE flow and in particular, the effects of preheating. A simplified stability analysis of the solutions is carried out in § V, while § VI discusses the results and conclusions of this paper.

II. BASIC EQUATIONS

a) Assumptions

In the model, ionized hydrogen gas surrounding a compact object flows radially inward under the influence of gravity. In moving toward the surface of the compact object r_* , the gas is heated as its gravitational potential energy is converted into thermal and bulk kinetic energy. One expects low velocities and therefore high densities to result from the near balance between gravity and radiation pressure. We find that the gas becomes optically thick for $r \gg r_*$, and that for a wide range of parameters local thermodynamic equilibrium (LTE) can be presumed to hold.

We have used a hydrodynamic treatment of the plasma, considering the hydrogen and thermal radiation as a single fluid. To treat the flow hydrodynamically, the particle and photon collision time scales and mean free paths must be shorter than the flow time scale, $t_f \approx r/|v|$, and length scale $\lambda_f \approx r$. We find that, for the solutions treated, the velocities are low enough and the densities are high enough for the particles to be treated as a simple fluid, even if one considered only large-angle Coulomb scattering as the mechanism coupling them together. Where the flow is optically thick and $|v| \ll c$, the length scale and time scale constraints are also met for the photons.

Spherically symmetric flow has been assumed, as would apply for a compact object embedded at rest (or moving subsonically) in a dense cloud with little angular momentum. Even in a binary system, if mass is accreted from the stellar wind of the primary, its angular momentum may be small enough to preclude the formation of a disk (Shapiro and Lightman 1976). The gas in the wind will form a bow shock about the compact object, with the exact flow pattern being sensitive to the Mach number and to the cooling. Hunt (1971) has shown that for low Mach numbers and for $\gamma = 5/3$ the flow may be almost spherically symmetric interior to the shock. For large densities, however, cooling may change the nature of the shock, such that it “stands off” the compact object by only a small amount (Illarionov and Sunyaev 1975).

If the opacity stems mainly from Thomson scattering (as is true for the temperature-density regime we will primarily be interested in), then for LTE to be valid it is necessary that the effective optical depth, τ^* , across a scale height r be greater than unity, where $\tau^* = (3\tau_{\text{es}}\tau_{\text{ab}})^{1/2}$ (see Felten and Rees 1972). The scattering optical depth is given by $\tau_{\text{es}} = \kappa_{\text{es}}\rho r$. For the

absorptive optical depth, $\tau_{ab} = \kappa_{ab}\rho r$, the free-free opacity will be used, $\kappa_{ff} \approx 6 \times 10^{22}(\rho/\text{g cm}^{-3}) \times (T/\text{K})^{-7/2} \text{ cm}^2 \text{ g}^{-1}$ (Novikov and Thorne 1973), as this is the dominant absorptive opacity for nearly all of the regime of interest. (If ρ increases rapidly inward, τ_{es} and τ_{ab} are roughly equal to the true optical depths as evaluated by integrating $\kappa\rho$ in from infinity.) We find that

$$\tau^* \approx (3\tau_{es}\tau_{ff})^{1/2} \approx 3 \times 10^{11} \left(\frac{\rho}{\text{g cm}^{-3}} \right)^{3/2} \left(\frac{r}{\text{cm}} \right) \left(\frac{T}{\text{K}} \right)^{-7/4}. \quad (3)$$

The time scales for heating and cooling have been assumed to be much shorter than the flow time scale, with the radiation and gas thus being in thermal equilibrium. This assumption was found valid whenever $\tau^* > 1$ for the LTE flows, and it appears to hold in general. For those solutions where preheating was considered in the $\tau^* < 1$ regime, thermal equilibrium also was found to apply (see § IV for the treatment of preheating and the relevant time scales).

In this section, LTE will be assumed to be valid everywhere, even if $\tau^* < 1$. The resulting solutions will be given in § III. In § IV, the assumption of LTE is dropped. The solutions found there are shown to be qualitatively the same as the flows found using LTE.

In this paper, we will only consider in detail flows for which the velocity at large radii is subsonic, and would tend to zero were the solution extended to infinity. For these solutions, the velocity was found to be always $\lesssim (2GM\epsilon_\infty/r)^{1/2}$, which is its free-fall value in the presence of a constant fractional luminosity difference ϵ_∞ . The value of $|v|/c$ is therefore much less than unity, justifying our nonrelativistic treatment. General-relativistic effects are of the order of $GM/c^2 r \approx 0.15(M/M_\odot)(10^6 \text{ cm}/r)$. Corrections of this order may be significant for neutron star flows near r_* , but they are not important for white dwarf solutions.

b) Hydrodynamic Equations

There are three basic fluid dynamical equations expressing conservation of mass, momentum, and energy. For optically thick steady, spherically symmetric flow, they take the form:

$$\dot{M} = -4\pi r^2 v \rho = \text{constant}, \quad (4)$$

$$\frac{v dv}{dr} + \frac{1}{\rho} \frac{dP}{dr} = -\frac{GM}{r^2}, \quad (5)$$

and

$$\frac{v dE}{dr} + \frac{v P d\rho^{-1}}{dr} = -\frac{1}{4\pi r^2 \rho} \frac{dL_d}{dr}, \quad (6)$$

where the energy per unit mass E and the pressure P are given by the sum of the contributions from the gas and the radiation:

$$E = E_g + E_r = 3kT/m_p + aT^4/\rho \quad (7)$$

and

$$P = P_g + P_r = 2\rho kT/m_p + aT^4/3. \quad (8)$$

The luminosity L_d is that measured in the comoving frame and in the diffusion approximation is given by

$$L_d = -\frac{16\pi r^2 c a T^3}{3\kappa_{es}\rho} \frac{dT}{dr}. \quad (9)$$

In the above equations, the velocity is negative, and m_p is the proton rest mass. By combining equations (4), (5), and (8), ρ can be eliminated from the momentum equation, giving

$$\frac{dv}{dr} = \frac{\left[\frac{2k}{m_p} \left(\frac{2T}{r} - \frac{dT}{dr} \right) - \epsilon \frac{GM}{r^2} \right] m_p v}{m_p v^2 - 2kT}. \quad (10)$$

The terms in T and dT/dr come from dP_g/dr , and we have defined $\epsilon(r)$ as the fractional difference in L_d from the Eddington limit,

$$L_d = L_{\text{edd}}(1 - \epsilon). \quad (11)$$

L_d is in general a function of r as the infalling matter interacts with the emergent radiation. The variable $|\epsilon|$ was found to be much less than unity almost everywhere for the solutions considered. Whenever the effective gravitational attraction $\epsilon GM/r^2$ is small compared to the first term in the numerator of equation (10), or whenever $m_p v^2 < 2kT$, the gas pressure may be important. It is clear from equation (10) that $m_p v^2 = 2kT$ is a singular point for the flow; the velocity here is equal to the isothermal sound speed. For physically interesting solutions, the numerator must also vanish at this point r_c , which is called a critical point. The velocity and temperature at $r = r_c$ are v_c and T_c . Solutions with zero, one, and two critical points were found. The flow is subcritical or supercritical depending on whether $m_p v^2 < 2kT$ or $m_p v^2 > 2kT$. If the flow were optically thin, these regimes would essentially correspond to subsonic or supersonic flow. For optically thick flow, the true sound speed is of the order of $(2kT/m_p + aT^4/3\rho)^{1/2}$ which may be much greater than the isothermal sound speed. The flow may be supercritical but highly subsonic.

Bernoulli's equation expressing the net energy flux can be obtained from equations (5) and (6), and takes the form

$$\dot{M} \left(\frac{v^2}{2} + \frac{5kT}{m_p} + \frac{4}{3} \frac{aT^4}{\rho} - \frac{GM}{r} \right) - L_d = \text{constant}, \quad (12)$$

where $5kT/m_p$ and $4aT^4/3\rho$ are the gas and radiation enthalpies, respectively.

It is useful to define two new luminosities in addition to L_d . An observer at rest with respect to the compact object would observe that radiation energy is being dragged inward by the gas at a rate

$$L_c = 4E_r \dot{M}/3 = -4\pi r^2 v (4aT^4/3), \quad (13)$$

where we define L_c as the convective luminosity. The net outward flow of energy by photons is given by L_n ,

$$L_n = L_d - L_c. \quad (14)$$

For L_n to be positive, the photons must diffuse outward faster than the inward motion of the gas. In terms of L_n , Bernoulli's equation becomes

$$\dot{M} \left(\frac{v^2}{2} + \frac{5kT}{m_p} - \frac{GM}{r} \right) - L_n = \text{constant}. \quad (15)$$

Evaluating this far from the compact object and at r_* gives the constant, since $L_{n\infty}$ has the form

$$L_{n\infty} \approx \dot{M} \left(\frac{GM}{r_*} + \frac{v_\infty^2}{2} + \frac{5kT_\infty}{m_p} - W_* \right), \quad (16)$$

where the term W_* takes into account the fact that there may be some energy retained by the accreted gas at r_* , and that there may be an internal source of energy, such as steady fusion of the accreted hydrogen ($W_* = v_*^2/2 + 5kT_*/m_p - L_{n*}/\dot{M}$).

One expects virtually all of the kinetic energy and a large part of the thermal energy to be released as radiation near r_* on a length scale much less than r_* . This was found to be the case in the treatment of giant and supergiant stars with neutron star cores by Thorne and Żytkow (1975) and in the time-dependent treatment of optically thick accretion onto compact objects by Okuda and Sakashita (1977). The behavior of the accretion flow near r_* was found to be essentially isothermal in both studies. Here the density increased inward in an isothermal atmosphere, and the radiation thermal energy was squeezed out and carried outward by L_d . Both L_d and L_c became very small interior to this region. The gas thermal energy may also be squeezed out when the flow is highly degenerate, as then the gas enthalpy scales as $E_t + O[kT(kT/E_t)]$, where E_t is the Fermi energy (Thorne 1977). The flow will be continuous near r_* if at this point it is subcritical. On the other hand, if the flow is supercritical here, an isothermal radiative shock may exist, allowing the gas to go subcritical with the subsequent flow again being nearly isothermal.

To find the exact value for W_* for a particular flow requires a detailed analysis of the surface region of the compact object. We will consider three cases for the possible behavior at this point, calculating flow solutions in the exterior region only. The case treated in greatest detail is $W_* = 0$, i.e., no net energy flow out of or into the compact object. We will also consider results for full retention of the gas thermal energy ($W_* = 5kT_*/m_p$). To study the effect an energy source would have on the flow, we will also treat the case where steady fusion of the accreted hydrogen is taking place with $W_* = -0.007c^2$ (Thorne and Żytkow 1975).

The boundary conditions for accretion flows with M and r_* given are specified by W_* , $\dot{M}$, and the values of v and T far from the compact object. Continuous solutions are then sought, as is discussed below, with the additional requirement that $|v| \rightarrow 0$ as $r \rightarrow \infty$.

For the solutions presented here, the terms in equation (16) for the gas kinetic and thermal energy at infinity are negligible, so that Bernoulli's equation takes the final form

$$L_d = \dot{M} \left[\frac{v^2}{2} - GM \left(\frac{1}{r} - \frac{1}{r_*} \right) + \frac{5kT}{m_p} + \frac{4}{3} \frac{aT^4}{\rho} - W_* \right]. \quad (17)$$

It follows then that, as long as $\dot{M}(v_\infty^2/2 + 5kT_\infty/m_p)$ turns out to be negligible, v and T may be specified at any point r , from which the flow can be followed inward and outward. (Actually, the terms $\dot{M}v^2/2$ and $\dot{M}5kT/m_p$ were found to be almost always negligible.)

We now introduce dimensionless variables. Let ϵ_0 ($\epsilon_0 \ll 1$) represent the accretion rate as defined by

$$\dot{M} = (1 - \epsilon_0)\dot{M}_{\text{EDD}} = (1 - \epsilon_0)L_{\text{edd}}r_*/GM. \quad (18)$$

By this definition, $\dot{M}_{\text{EDD}} = 4\pi cr_*/\kappa_{\text{es}}$ which will equal $\dot{M}_{\text{edd}}$ defined in equation (2) only if $\phi = GM/r_*c^2$. Far from the compact object, L_n is given in terms of ϵ_0 and W_* by

$$L_{n\infty} \approx L_{\text{edd}}(1 - \epsilon_0)(1 - r_*W_*/GM) = (1 - \epsilon_\infty)L_{\text{edd}}. \quad (19)$$

We will use the dimensionless variable $\omega_* = r_*W_*/GM$ to evaluate the importance of W_* . Generally, $\omega_* \ll 1$ and $\epsilon_0 \approx \epsilon_\infty$. However, if $0.007c^2r_*/GM \gtrsim 1$, as is the case for white dwarfs, $\epsilon_0 \approx 1$.

We define the dimensionless radius, velocity, and temperature as x , y , and t , where the units of length, velocity, and temperature used are given (cf. Cassinelli and Castor 1975) by

$$l \equiv \frac{m_p^2(GM)^{3/2}(\kappa_{\text{es}}a)^{1/2}}{k^2} \approx 1.23 \times 10^{16} \left(\frac{M}{M_\odot} \right)^{3/2} \text{ cm}, \quad (20)$$

$$v \equiv \frac{k}{m_p(GM\kappa_{\text{es}}a)^{1/4}} \approx 1.04 \times 10^5 \left(\frac{M}{M_\odot} \right)^{-1/4} \text{ cm s}^{-1}, \quad (21)$$

and

$$\tau \equiv \frac{k}{m_p(GM\kappa_{\text{es}}a)^{1/2}} \approx 1.31 \times 10^2 \left(\frac{M}{M_\odot} \right)^{-1/2} \text{ K}. \quad (22)$$

These units are related by $v^2 = GM/l = k\tau/m_p$. In terms of x , y , and t , equations (10), (9), and (17) become

$$\frac{dy}{dx} = y \left(\frac{4t}{x} - 2 \frac{dt}{dx} - \frac{\epsilon}{x^2} \right) (y^2 - 2t)^{-1}, \quad (23)$$

$$\frac{dt}{dx} = \frac{3x_*(1 - \epsilon)c(1 - \epsilon_0)}{4yx^4t^3v}, \quad (24)$$

and

$$\epsilon = \epsilon_\infty - (1 - \epsilon_0)x_* \left(\frac{y^2}{2} - \frac{1}{x} + 5t \right) + \frac{4t^4 y x^2 \nu}{3c}. \quad (25)$$

The density is given from the continuity equation as

$$\rho = -\frac{x_*(1 - \epsilon_0)c}{y x^2 l_{\text{es}} \nu}. \quad (26)$$

The flow is supercritical or subcritical for the new variables depending upon whether $y^2 > 2t$ or $y^2 < 2t$.

c) Numerical Method

Given ϵ_∞ , $(1 - \epsilon_0)x_*$, and ν/c (i.e., M , $\dot{M}$, and a combination of r_* and W_*), the flow can be solved from equations (23)–(26), if y and t are given for some value of x . A Runge-Kutta scheme with self-correcting step size was used. Calculations starting from x_* and integrating outward and from $x \gg x_*$ and integrating inward were performed for a wide range of y and t at either point. To explore solutions with critical points, integrations were started at x_c going both inward and outward, using l'Hospital's rule to evaluate dy/dx initially. Flows with critical points were considered in detail for $W_* = 0$, where for a given x_c , the values of y_c and t_c were found by setting the numerator and the denominator of equation (23) equal to zero and solving the resulting polynomial. For a given critical point x_c ,

there are two choices for the slope dy/dx . For optically thin flow, these correspond (cf. Bondi 1952; Parker 1963) to the choice between stellar wind type solutions (subsonic at r_* and supersonic at infinity) and supersonic accretion flow (subsonic at infinity and supersonic at r_*). We have used the slope corresponding to the smaller velocity beyond x_c . Figure 1 is a plot of y_c as a function of x_c , for several values of ϵ_0 , for $W_* = 0$. For the lower branch of critical velocities, t_c is less than the blackbody temperature defined by $L_d = 4\pi r^2 \sigma T_{\text{BB}}^4$. For $t_c < t_{\text{BB}}$, it follows from the diffusion equation that the change in the radiation energy density over a scattering mean free path would be greater than a factor of order unity and hence non-physical (numerical integration outward of these critical points leads to $t \rightarrow 0$ just beyond x_c). Only solutions along the upper branch will be considered. Except for these flows, only subcritical solutions will be considered.

III. RESULTS FOR LTE SOLUTIONS

a) Numerical Results

The most extensive set of calculations was carried out for typical neutron star values of mass and radius $M = M_\odot$ and $r_* = 10^6$ cm ($x_* = 8.13 \times 10^{-11}$ and $\nu/c = 3.46 \times 10^{-6}$), and for $\epsilon_0 = 0.1, 0.003, 0.002$, and $W_* = 0$. No solutions continuous through the region $x \approx x_*/\epsilon_0$ were found for $\epsilon_0 = 0.002$, suggesting that there is an upper limit to the luminosity

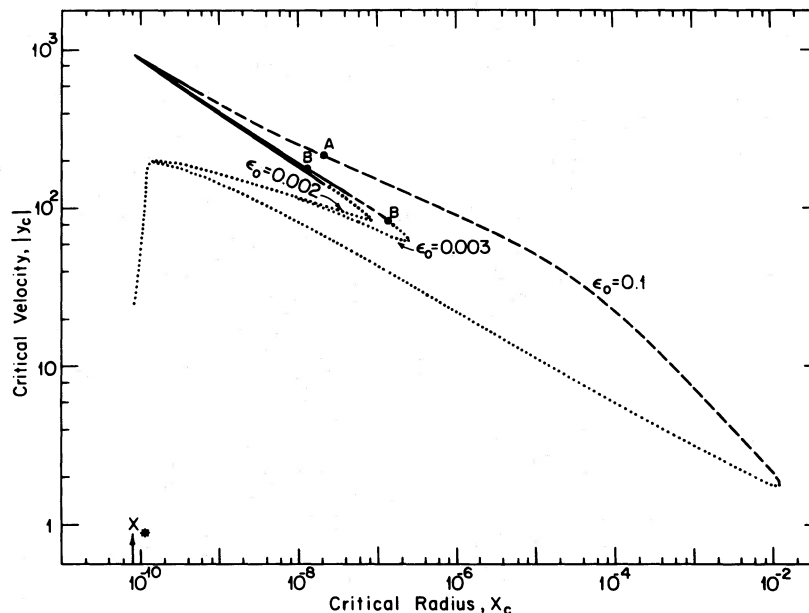


FIG. 1.—Critical velocity y_c as a function of critical radius x_c for various accretion rates ϵ_0 , with $M = M_\odot$, $r_* = 10^6$ cm, and $W_* = 0$ ($\nu/c = 3.46 \times 10^{-6}$, $x_* = 8.13 \times 10^{-11}$, $\epsilon_0 = \epsilon_\infty$). Points A and B show the critical points for adiabatic solutions near t_{min} for $\epsilon_0 = 0.1$ and 0.003 , respectively. For critical points along the dashed curves transitions from supercritical to subcritical and vice versa occur as the gas flows in. Along the solid curves only supercritical to subcritical transitions take place. Solutions along the dotted curve on the lower branch have the behavior that upon outward integration $t \rightarrow 0$ just beyond x_c , as is also the case for flows on the dotted upper branch for $\epsilon_0 = 0.003$. The solutions along the dotted upper branch for $\epsilon_0 = 0.002$ have complex values of dy/dx at x_c .

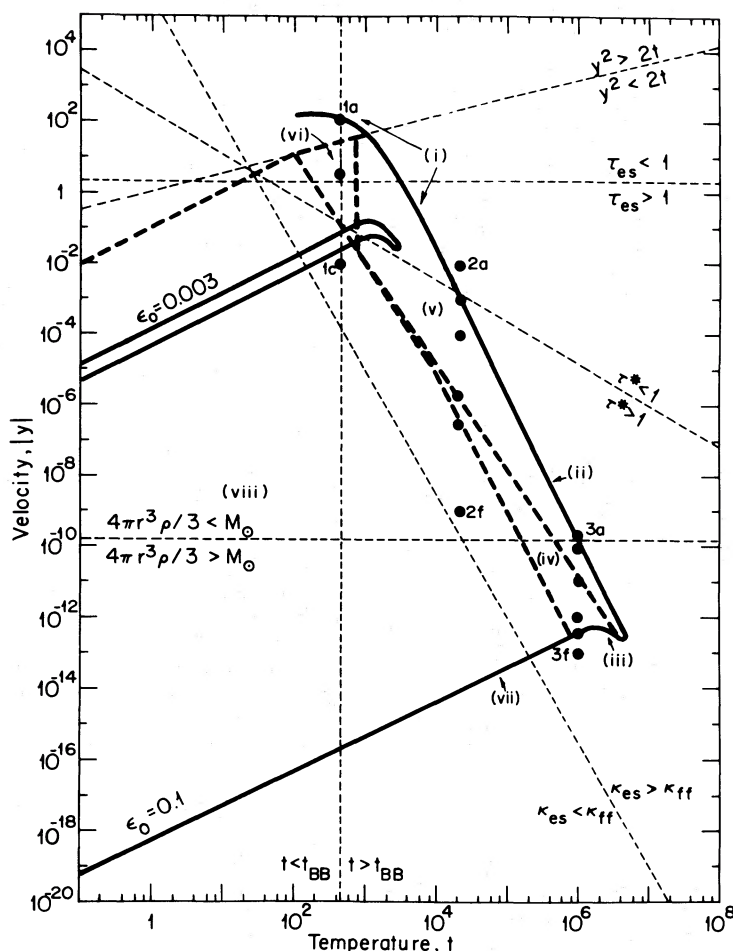


FIG. 2.—Boundaries in the (t, y) -plane at $x = 10^{-5}$ determining the character of subcritical and transcritical accretion flows for the neutron star model with $M = M_\odot$, $r_* = 10^6$ cm, and $W_* = 0$ for the accretion rates $\epsilon_0 = 0.003$ and 0.1 . The light dotted curves mark the transitions at $x = 10^{-5}$ between regimes of interest for several parameters for $\epsilon_0 = 0.1$. Solutions for $\epsilon_0 = 0.1$ which are continuous from $x = 10^{-5}$ to x_* pass through either the heavy solid curve labeled $\epsilon_0 = 0.1$ [which is divided into regions (i)–(iii) and (vii)] or the areas bounded by the heavy dashed curves labeled $\epsilon_0 = 0.1$ and the heavy dashed curves [regions (iv)–(vi) and (viii)]. The points labeled alphabetically 1a–1c, 2a–2f, and 3a–3f show where the solutions given in Figs. 3–5 pass through $x = 10^{-5}$.

caused by gas pressure effects, as will be discussed later. The boundaries in the (t, y) -plane separating the different regions of flow at $x = 10^{-5}$ are shown in Figure 2. The behavior of the solutions passing through the regions labeled (i)–(viii) in Figure 2 is discussed below (see § IIIb). Figures 3–5 give the velocity and temperature as a function of radius for flows with $\epsilon_0 = 0.1$ which pass through the labeled points 1a–1c, 2a–2f, and 3a–3f in Figure 2. The value of ϵ as a function of radius for several of these curves is given in Figure 6.

The numerical solutions passing through $x = 10^{-5}$ and continuous to x_* can be classified into two categories: (1) solutions found by inward integration for initially subcritical values of y and t , and (2) solutions found by integration outward from x_* , outward and inward from a critical point within $x = 10^{-5}$, and inward from a critical point beyond $x = 10^{-5}$. The

difference in direction of integration stems from the existence of growing modes which for one direction of integration were convergent but for the opposite were divergent. The first type of solution corresponds to the regions (iv)–(vi) and (viii) in Figure 2 for $\epsilon_0 = 0.1$ and to the area within the heavy solid curve labeled $\epsilon_0 = 0.003$ for that value of ϵ_0 . The second type of flow is given by the heavy solid curves for $\epsilon_0 = 0.1$ [regions (i)–(iii) and (vii)] and 0.003 . In Figures 3–5, the arrows give the direction of convergent integration. For $x \gtrsim 10^{-5}$, the direction of integration for solutions without a critical point was found to be unimportant.

Subcritical solutions which at $x = 10^{-5}$ pass outside the regions labeled (i)–(viii) for $\epsilon_0 = 0.1$ or outside the heavy solid curve labeled $\epsilon_0 = 0.003$ in Figure 2 for that value of ϵ_0 have the feature, as shown in curve 2a of Figure 4 and curve 3f of Figure 5, that upon inward

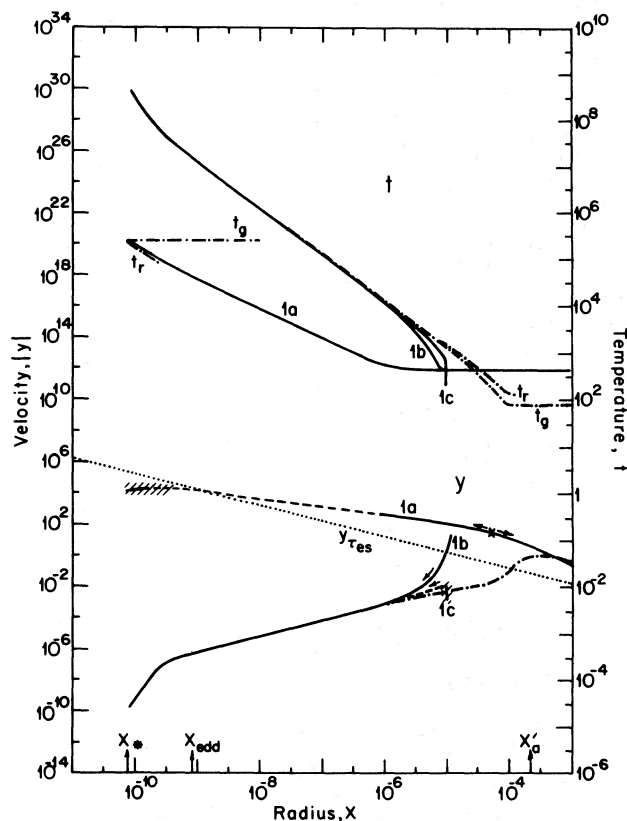


FIG. 3.—Velocity and temperature profiles for solutions passing through the points labeled 1a–1c in Fig. 2, and for the non-LTE free-fall and settling solutions (dot-dashed curves). For the non-LTE flows both t_g and t_r are shown for the regions calculated, which for the free-fall solution consists of the region $x < 8.1 \times 10^{-9}$. The non-LTE velocity for the free-fall solution is indistinguishable from that for curve 1a. Curves 1b and 1c are continued outward until $|dy/dx| = 10^3|y|/x$. The cross on the velocity curve 1a marks a critical point. The convection and Rayleigh-Taylor unstable regions for LTE flow are given respectively by the dashed and crosshatched portions of the velocity curves. The effective accretion radius x_a' , as evaluated for t at $x = 10^{-5}$, is shown, as is the velocity y_{res} , above which $\tau_{es} < 1$ (dotted curve).

integration $|dy/dx| \rightarrow \infty$ at some point at which $y^2 = 2t$. Flows above the curve $y^2 = 2t$ in Figure 2 and not on the heavy curve of region (i) either require y to be much larger than that for free-fall at infinity or show $|dy/dx| \rightarrow \infty$ at some point. These types of flows will not be considered further. Solutions passing to the left of t_{BB} in Figure 2 are nonphysical.

Physical solutions which are continuous from $x = 10^{-5}$ to x_* can also be categorized by their behavior far from the compact object. The three basic types of solutions are: (1) optically thin and supercritical (supersonic), free-fall flows which become optically thick well before r_* [region (i)]; (2) optically thin and subcritical (subsonic), settling flows which also become optically thick before reaching r_* [found in regions (v), (vi), and (viii)]; and (3) flows which are everywhere optically thick, are subcritical at infinity,

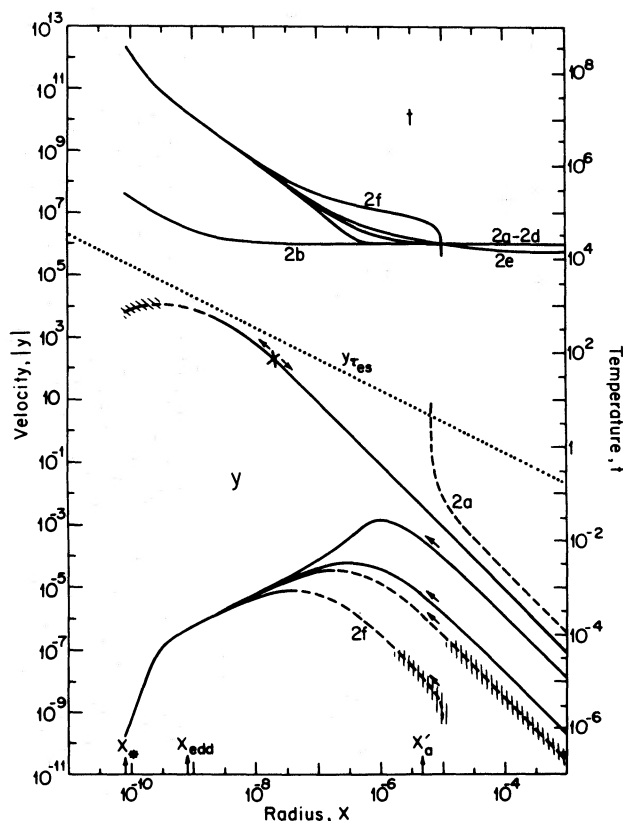


FIG. 4.—Velocity and temperature profiles for solutions passing through the points labeled 2a–2f in Fig. 2. The cross on the velocity curve 2b marks a critical point. Curves 2a and 2f are continued inward and outward, respectively, until $|dy/dx| = 10^3|y|/x$. The curves are labeled alphabetically unless shown explicitly otherwise. The solution for curve 2a is isothermal. The convection and Rayleigh-Taylor unstable regions are given respectively by the dashed and crosshatched portions of the velocity curves. The curve y_{res} (dotted curve) and the point x_a' have the same meaning as discussed in the caption to Fig. 3.

and may have a supercritical region close to the surface of the compact object, which may or may not include r_* [found in regions (ii)–(viii)]. These latter flows, which are optically thick everywhere, may be used for the inner regime of models of compact objects embedded in extended and supermassive stars or dense clouds.

We are assuming in this section that equations (23)–(26) can be used to describe the flow everywhere. As will be shown later, where a more accurate treatment of optically thin and non-LTE flows will be given, the basic results found by this treatment are qualitatively correct.

b) Approximate Analytic Results

In this section, we will discuss in detail the behavior of the neutron star flows as classified by Figure 2 for $\epsilon_0 = 0.1$ and $W_* = 0$. Results for a white dwarf model with $M = M_\odot$, $r_* = 5.6 \times 10^8$ cm (Vauclair

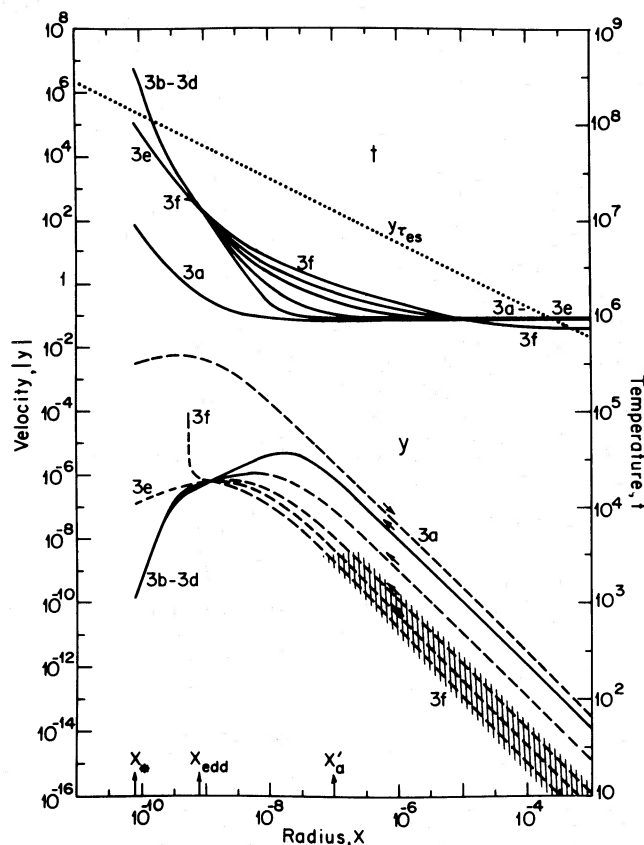


FIG. 5.—Velocity and temperature profiles for solutions passing through the points labeled 3a–3f in Fig. 2. Curve 3f is continued inward until $|dy/dx| = 10^3|y|/x$. The curves are labeled alphabetically unless shown explicitly otherwise. The convection and Rayleigh-Taylor unstable regions are given respectively by the dashed and crosshatched portions of the velocity curves. The curve y_{res} (dotted curve) and the point x'_a have the same meaning as discussed in the legend to Fig. 3.

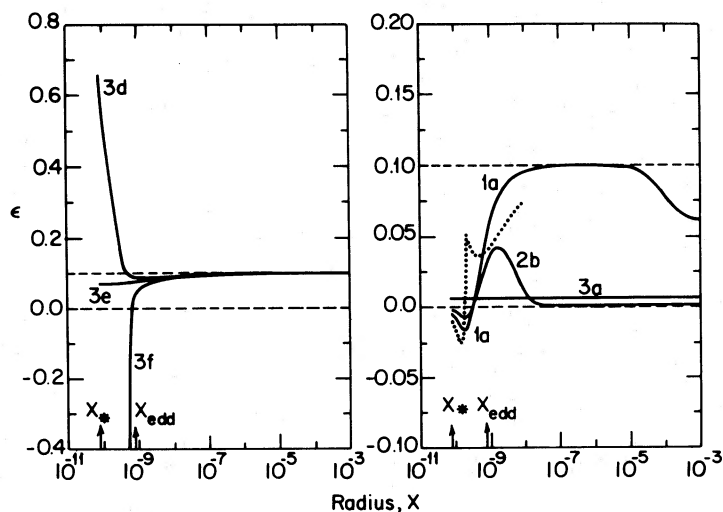


FIG. 6.— ϵ profiles for solutions passing through the points labeled 3d–3f, 1a, 2b, and 3a in Fig. 2 (solid curves), and for the non-LTE free-fall solution (dotted curve). The behavior of ϵ as a function of radius for regions (iv)–(vi) and (viii) on Fig. 2 and for the non-LTE settling solution is as given by curve 3d here.

1971), and $\epsilon_0 = 0.1$ will also be briefly mentioned, as will the effect of $W_* \neq 0$ for the flows with $\epsilon_\infty = 0.1$. Each of the subsections (i)–(viii) describes the flow passing through the correspondingly labeled region in Figure 2.

Before going on, let us define here the relevant length scales used to describe the different flow regimes. In the standard polytropic accretion solutions, as given originally by Bondi (1952), there are two length scales: the accretion radius $r_a \approx GM/a_\infty^2$ ($x_a \approx 1/t_\infty$), where a_∞ is the gas sound speed at infinity, and r_* . For flows near L_{edd} ($\epsilon_\infty \ll 1$), there are several additional important length scales. The effective gas accretion radius, and the radiation accretion radius, are given by $r_a' \approx \epsilon_\infty r_a$ ($x_a' \approx \epsilon_\infty/t_\infty$), and $r_{ar} \approx GM\rho_\infty/aT_\infty^4$ ($x_{ar} \approx \rho_\infty t_{\text{es}}/t_\infty^4$) respectively. Furthermore, we define the effective radius of the compact object $x_{\text{eff}} = x_*(1 - \epsilon_0)/(1 - \epsilon_\infty)$, and the Eddington radius $x_{\text{edd}} = x_*(1 - \epsilon_0)/\epsilon_\infty$.

i) Free-Fall Flow

These solutions on the heavy solid curve are supercritical and optically thin far from the compact object, and have the largest velocities of any flows considered. Curve 1a in Figure 3 shows the velocity and temperature profile typical of these flows, while curve 1a in Figure 6 gives the behavior of ϵ . The general variation of these quantities with radius is easily understood, and can be divided into three regimes. The flow is initially subcritical (subsonic) at infinity, with t and ρ constant and y varying as x^{-2} . In the outer regions, $L_c \lesssim L_{\text{edd}}\epsilon_\infty$, from which it follows with equation (25) that $\epsilon \approx \epsilon_\infty$. The momentum conservation equation with $\epsilon = \epsilon_\infty$ is then equivalent to replacing M by an effective mass $M\epsilon_\infty$ and ignoring radiation pressure. The gas goes supercritical at roughly the effective accretion radius x_a' , and interior to this point y becomes nearly equal to its free-fall value for the effective mass $M\epsilon_\infty$, $y = -(2\epsilon_\infty/x)^{1/2}$. L_c increases inward in the free-fall regime until by $x \approx x_{\text{edd}}$ it causes ϵ to become negative, decelerating the gas. The flow at this point becomes nearly adiabatic, the solution for $x < x_{\text{edd}}$ being described by the adiabatic flow for $t_* \approx t_{\text{min}}$ (see § IIIb[ii]). For our neutron star model, τ_{es} becomes of order unity at x_{edd} .

The effect of a nonzero W_* on the flow is slight, as any thermal energy retained or fusion energy released is much smaller than the gravitational potential energy ($\omega_* \ll 1$, $\epsilon_0 \approx \epsilon_\infty$).

For a white dwarf, the solutions for $W_* = 0$ are the same except now for the same luminosity at infinity a much larger accretion rate, and hence a large value for ρ , is needed. The flow may be thick to scattering even at x_a' and remain so out to the boundary of the cloud that the white dwarf is embedded in. The effect on the flow if $W_* \neq 0$ is quite different now. Fusion releases more energy than that from the gravitational potential ($\omega_* \gg 1$), and the accretion rate for a given luminosity at infinity, ϵ_∞ , is thus reduced ($\epsilon_0 \approx 1$). The flow for $x \gtrsim x_*$ behaves as though $W_* = 0$ and $x_* = x_{\text{eff}}$. For $\epsilon_\infty = 0.1$, $\epsilon_0 =$

0.968 and $x_{\text{edd}} = 0.68x_*$. The free-fall flow in this case continues down to the surface of the white dwarf and is optically thin for $x \gtrsim 20x_*$.

The solutions in regions (ii)–(viii) for $W_* \neq 0$ are affected in the same way as for the free-fall flows. Neutron star models are only slightly changed, while for white dwarfs, fusion leads to the various regimes found for $x \gtrsim x_*/\epsilon_\infty$ now extending inward, and down to x_* for $\epsilon_\infty = 0.1$.

For both neutron star and white dwarf free-fall solutions, $\tau_* \lesssim 1$ down to x_* . The more detailed non-LTE treatment of these free-fall flows given later shows, however, that the nature of these solutions is unchanged.

ii) Adiabatic Flows

Solutions passing through this region of the heavy curve in Figure 2 are optically thick to scattering, and nearly adiabatic everywhere, with $P \gg \rho^{4/3}$ and $P_r \gg P_g$. For these flows the gravitational potential energy of the gas is transformed into radiation thermal energy (L_c). The point 2b roughly gives the division between these and free-fall flows. For flows with $t \gtrsim 10^5$ at $x = 10^{-5}$, $\tau_* > 1$ everywhere. Curve 2b in Figure 4 and curve 3a in Figure 5 show the behavior of y and t , while curves 2b and 3a in Figure 6 give ϵ .

For the flow to be adiabatic, it follows from equation (6) that $|dL_c/dx| \gg |dL_d/dx| = L_{\text{edd}}|d\epsilon/dx|$. This requires L_d to be nearly constant, but not ϵ if $L_c \gg L_{\text{edd}}|\epsilon|$. Setting L_d constant in the equation (9) and using $P_r \propto \rho^{4/3}$ results in the following solution for t and y :

$$t = t_* \left(\frac{x_*}{x} + b \right) (1 + b)^{-1}, \quad (27)$$

$$y = y_* \left(\frac{x_*}{x} \right)^2 \left(\frac{x_*}{x} + b \right)^{-3} (1 + b)^3, \quad (28)$$

where b is a constant generally much less than unity. For $|\epsilon| \ll 1$ and $\epsilon_0 \ll 1$, the value of y_* in terms of t_* is given, using equation (25), by

$$y_* \approx -3c/4x_*^2 t_*^4 \nu. \quad (29)$$

It follows that $x_{ar} \approx x_*/b$.

The value of b can be found by use of equations (23) and (25). From the momentum conservation equation, ϵ for adiabatic, radiation-pressure-dominated flow is given by

$$\epsilon \approx 8 \left(\frac{kT_*}{m_p} \right) \left(\frac{r_*}{GM} \right) - \left(\frac{v dv}{dr} \right) \left(\frac{r^2}{GM} \right). \quad (30)$$

Substituting equations (27)–(30) into equation (25) for $W_* = 0$ and ignoring terms in y^2 and dy/dx , we find

$$b \approx \epsilon_0 - 8t_* x_*. \quad (31)$$

In dropping terms in y^2 and in dy/dx we are assuming that y is small [$|y| \ll (2\epsilon_0/x_*)^{1/2}$] and that $|y dy/dx| \ll \epsilon_0/x^2$. This holds true for the adiabatic flows.

The solutions in region (ii) of Figure 2 consist of adiabatic flows as described by equations (27)–(31), for the temperature range at x_* , $t_{*max} \gtrsim t_* \gtrsim t_{*min}$. At either temperature extreme, $\epsilon \approx \epsilon_0$ at x_{edd} and the inequality $|dL_c/dx| \gg L_{edd}|d\epsilon/dx|$ breaks down (for adiabatic flows, $L_c \approx \epsilon_0 L_{edd}$ at x_{edd}). Here the exterior flows are given by the free-fall solutions [region (i)] for $t_* \approx t_{*min}$, and the nonadiabatic and isobaric solutions [regions (iii) and (vii)] for $t_* \approx t_{*max}$. The values of t_{*min} and t_{*max} follow from equation (30) and are given by

$$t_{*min} \approx \frac{1}{2x_*^{3/8}\epsilon_0^{1/2}} \left(\frac{c}{v}\right)^{1/4} \quad (32)$$

and

$$t_{*max} \approx \epsilon_0/8x_*. \quad (33)$$

These adiabatic solutions are simply those for the subcritical branch of polytropic solutions as given by Bondi (1952), with $\gamma = 4/3$ ($x_{edd} \approx x_a$). Usually the subcritical branch is ignored, because the temperature necessary for gas pressure alone to counterbalance gravity would be high enough for neutrino losses to be excessive ($T_* \approx 10^{12}$ K for a neutron star and $T_* \approx 10^9$ K for a white dwarf). In our case, it is radiation pressure ($P_r \propto T^4$) that allows much smaller temperatures.

iii) Nonadiabatic Flow

Accretion flows passing through this portion of the heavy solid curve in Figure 2 are subcritical everywhere. These flows are nonadiabatic far from the compact object, and join onto the adiabatic solution for $t_* \approx t_{*max}$ just before x_{edd} (see curve 3e in Fig. 5 for the behavior of y and t , and curve 3e in Fig. 6 for ϵ). In the outer regime ($x \geq x_{edd}$), L_c is negligible ($\epsilon = \epsilon_0$), and $P \approx P_g > \epsilon_0 GM\rho/r$. The gas flows inward with constant t and ρ ($y \propto x^{-2}$) until the point x_a . Interior to x_a , t and y vary with radius roughly as $x^{-1/5}$ and $x^{-11/5}$, respectively, until the flow reaches x_{edd} , by which point L_c has become large enough for the flow to behave adiabatically ($P \approx P_r \approx GM\rho/r$).

iv) Gas-Pressure-dominated Settling Solutions

Solutions passing through this region are subcritical everywhere and generally behave far from the compact object like flows passing through region (iii). The flows now, however, become adiabatic (with $P \approx P_r$) before x_{edd} with $b \lesssim \epsilon_0$ (i.e., the adiabatic regime with $t \propto x^{-1}$ and $y \propto x$ begins well before x_{edd}). If the flow were adiabatic down to x_* , it would reach there with $t_* \approx t_{*max}$. (See curves 2e, 3d, and 3d in Figs. 4, 5, and 6, respectively, for typical behavior of y , t , and ϵ .) The flows along the curve separating regions (iv) and (v) have no nonadiabatic outer regime.

Interior to x_{edd} , the nature of these solutions change, and the flow becomes highly nonadiabatic with $t \propto x^{-2}$ and $y \propto x^5$. In this region, $\epsilon \propto x^{-1}$, and is of the order of $\frac{1}{2}$ at x_* . Such behavior was found independent of ϵ_0 for all inwardly calculated flows which were continuous to x_* . Analytically we find by

substituting $y \propto x^{-\alpha}$ and $t \propto x^{-\beta}$ into equations (23)–(25) (with $y^2 \ll 2t$, $5t \ll 1/x$, $\epsilon_0 \ll 1$, and $\epsilon \lesssim 1$),

$$t \approx \frac{x_*}{36x^2}, \quad (34)$$

$$y \approx -\frac{3c}{8x_*^2 t_*^4 v} \left(\frac{x}{x_*}\right)^5, \quad (35)$$

and

$$\epsilon \approx x_*/2x. \quad (36)$$

This agrees with the numerical results to within factors of order unity. The high temperatures and densities for this solution stem from the large value of the effective gravitational attraction $\epsilon GM/r^2$ which must be balanced by gas pressure.

v)–vi) Radiation-Pressure-dominated Settling Solutions

The second important class of optically thin solutions pass through regions (v) and (vi). These flows are radiation-pressure-dominated, and generally have small values of L_c ($\epsilon \approx \epsilon_0$) at large radii. For solutions in region (v) (which are subcritical everywhere), the gas flows in with constant t and ρ ($y \propto x^{-2}$), until x_a . Interior to x_a , $|y|$ decreases [initially as $x^{-2} \exp(-\epsilon_0/2tx)$], with the flow rapidly matching onto the adiabatic solution described for gas-pressure-dominated settling solutions. Curves 2c in Figure 4 and 3b in Figure 5 show the typical behavior for y and t , where ϵ varies as curve 3d in Figure 6.

Flows which pass through region (vi) have values of $x_a' > 10^{-5}$ as evaluated using t at $x = 10^{-5}$ due to their low temperatures. Outward integration from $x = 10^{-5}$ results in $|dy/dx| \rightarrow \infty$, at some point x' where $y^2 = 2t < \epsilon/2x'$, as is shown by curve 1b in Figure 3. Interior to $x = 10^{-5}$ the solutions are similar to those for region (v).

vii)–viii) Isobaric Flow

Initially for these solutions, $P \approx P_g > \epsilon_0 GM\rho/r$ and $|dt/dx| \gg t/x$. The flow proceeds inward at constant P_g ($y \propto t$), and nearly constant x until $dt/dx \approx -t/x$. This occurs at roughly the boundary between these regions [(vii) and (viii)] and either region (iii), (iv), or (vi). The flow thereafter is similar to that of the bordered region. Curves 1c in Figure 3 and 2f in Figure 4 show typical solutions for y and t , while ϵ varies as curve 3d in Figure 6.

In § IV, we will discuss the behavior of flows where the assumptions of LTE and $\tau_{es} > 1$ are relaxed. Aside from the breakdown of these assumptions, there are several other major regimes inconsistent with our treatment. For high temperatures, neutrino losses can no longer be ignored. From Zel'dovich and Novikov (1971), the loss rate due to neutrinos for a nondegenerate gas is roughly $dU_\nu/dt \approx 5 \times 10^{18} (T/10^9 \text{ K})^3 \times \exp(-11.9 \times 10^9 \text{ K}/T) \text{ ergs s}^{-1} \text{ cm}^{-3}$ for $T \lesssim 6 \times 10^9 \text{ K}$. This loss rate equals L_{edd} (for $M = M_\odot$) from a volume x_*^3 at $t_* \approx 2 \times 10^7$ for our neutron star model and at $t_* \approx 5 \times 10^6$ for our white dwarf

model. For a neutron star, temperatures higher than this are found for all nonadiabatic inner regime solutions and for adiabatic solutions with t_* near t_{*max} for $\epsilon_0 \gtrsim 0.01$. Temperatures for the flows onto a white dwarf are in general much lower, with neutrino losses being small for all solutions considered. Solutions may also be inconsistent due to the neglect of the gravitational potential of accreting gas itself ($\int 4\pi r^2 \rho dr > M$) for small velocities (high densities) far from the compact object (see Fig. 2).

c) $\epsilon_0 \rightarrow 0$

When ϵ_0 is very small, the nature of the solutions changes. This is clear for adiabatic solutions, since when $\epsilon_0 < \epsilon_{0min}$, where

$$\epsilon_{0min} \approx 3 \left(\frac{c}{v} \right)^{1/6} x_*^{5/12}, \quad (37)$$

$t_{*min} > t_{*max}$ (we will assume for now that $W_* = 0$). For our neutron star model, $\epsilon_{0min} \approx 0.002$. Solutions for $\epsilon_0 = 0.003$ which are continuous from $x = 10^{-5}$ to x_* and subcritical at infinity were found only along or within the heavy curve labeled $\epsilon_0 = 0.003$ shown in Figure 2. Solutions on the upper and lower branches of the heavy curve behave like flows in regions (iii) and (vii) for $\epsilon_0 = 0.1$, with the inner adiabatic flows tending to t_{*min} and t_{*max} , respectively. Solutions interior to the heavy curve behave like flows in regions (iv), (v), and (viii) for $\epsilon_0 = 0.1$. There is no free-fall regime due to the small value of the net acceleration (i.e., gas pressure is always important), and all flows except those for $t_* \approx t_{*min}$ are subcritical everywhere, and even these are only briefly, and then just barely, supercritical near x_{edd} , with $|y| \ll (2\epsilon_0/x)^{1/2}$.

For $\epsilon_0 = 0.002 \lesssim \epsilon_{0min}$, no continuous solutions through $x \approx x_{edd}$ were found with y less than or equal to the free-fall value for $\epsilon = \epsilon_0$ at large radii. What appears to occur for flows with $\epsilon_0 < \epsilon_{0min}$ is that, as the gas flows in subcritically from infinity, its kinetic energy always increases until $y^2 = 2t$, as the numerator of equation (23) does not change sign, either $4tx - 2x^2 dt/dx > \epsilon$ or $\epsilon < 0$ everywhere (for solutions with larger ϵ_0 continuous down to x_* the flow reaches x_a' before $y^2 = 2t$). The effect that a nonzero W_* would have upon these flows, for the same ϵ_∞ , is expected to be small as x_{edd} , before which the discontinuities take place, is still larger than x_* . It must be mentioned that for the $\epsilon_0 = 0.003$ and $W_* = 0$ flows, $\tau^* < 1$ for much of the solution. As gas pressure seems to be very important for these flows, a more detailed non-LTE analysis for small ϵ_0 should be made to determine if gas pressure does actually limit the luminosity.

IV. NUMERICAL SOLUTIONS FOR NON-LTE FLOWS

a) Preheating

We will consider again the behavior of the solutions which were found to be optically thin far from the compact object. Here we will drop the assumptions that the flow is in LTE and that it is optically thick,

and treat the outer regime more accurately. Three temperatures are useful in treating non-LTE flows. The first two are defined by

$$E_g = 3kT_g/m_p \quad (38)$$

and

$$E_r = aT_r^4/\rho, \quad (39)$$

and are a measure of the gas thermal energy and the radiation energy density, respectively. Since for $\tau_{es} \gtrsim 1$ the thermal energy is primarily in the radiation field ($E_r \gg E_g$), the mean photon energy in the regime $\tau^* < 1$ is given by the temperature t_{*} at the radius where they were emitted ($\tau^* \approx 1$). The energy density U_r and pressure P_r of the radiation are now given in terms of the comoving angle mean intensity and the second moment of the intensity, respectively, with $L_c = 4\pi r^2 v(U_r + P_r)$. If $\tau_{es} \lesssim 1$, $U_r = P_r = L_d/4\pi r^2 c$. L_d is defined as the net luminosity in the comoving frame as before. For $x \gg x_{edd}$, $L_n \approx (1 - \epsilon_\infty)L_{edd} \approx L_d$ (we will assume $W_* = 0$ for now).

If the flow for $x \gg x_{edd}$ could be described by a Bondi type polytropic solution, then the effects of radiation pressure could be included if M were replaced by $\epsilon_\infty M$. The presence of a strong source of radiation, however, does more than simply slow the flow via radiation pressure; it may significantly preheat it as well. The net effect of preheating on the flow is very sensitive to the efficiency and to the spectrum of the source, and may in fact prevent steady flow entirely (cf. Buff and McCray 1974; Ostriker *et al.* 1976). For free-fall flows, we will consider only the regimes interior to r_a' , where $m_p v^2 \gg kT_g$, and ignore the problem of whether preheating would halt the flow before this point or not.

The gas temperature for $\tau^* \lesssim 1$ is assumed to be a function of the parameter $\xi = 4\pi U_r c m_p / \rho$ only (cf. Tarter, Tucker, and Salpeter 1969). For $\xi \gtrsim 10^4$, $T_g \approx T_r$ due to inverse Compton heating, while for $\xi \lesssim 10^4$, $T_g \approx 10^4$ K due to cooling (see, e.g., Buff and McCray 1974). (In this section we will consider the gas to include impurities which are important for determining the temperature structure, but will ignore their direct effects on the hydrodynamics.) For free-fall type solutions, the limit $\xi \gg 10^4$ will be treated, where $T_g = T_r$ for $\tau^* \lesssim 1$, and $T_g = T_r$ for $\tau^* \gtrsim 1$. The subcritical solutions have such high densities that ξ remains less than 10^4 everywhere.

In treating T_g as only a function of ξ we have assumed that the heating and cooling time scales are much shorter than the flow time scale, $t_f = r/|v|$. For the free-fall solutions, inverse Compton scattering is the dominant heating process. Using the fact that $|v| \lesssim (2GM\epsilon_\infty/r)^{1/2}$, it follows that the ratio of the inverse Compton heating time scale $t_{compt} \approx m_e c^2 T_g / \kappa_{es} m_p T_r c U_r$ to t_f is roughly

$$\frac{t_{compt}}{t_f} \lesssim \left(\frac{r}{10^{12} \text{ cm}} \right)^{1/2} \left(\frac{\epsilon_\infty M_\odot}{M} \right)^{1/2} \left(\frac{T_g}{T_r} \right) \left(\frac{L_{edd}}{4\pi r^2 c U_r} \right). \quad (40)$$

Thus for $r \lesssim 10^{12}$ cm, $t_{\text{comp}}/t_f \ll 1$. The region treated for free-fall flow lies well within this radius. For the settling solutions, as $\xi < 10^4$ we have assumed $T_g = 10^4$ K initially. Free-free emission is in fact sufficient to overcome adiabatic heating and to allow the thermal structure to be treated in the static approximation. This follows from the ratio of the free-free cooling time scale $t_{\text{ff}} \approx 3kT_g/\epsilon_{\text{ff}}$, where $\epsilon_{\text{ff}} \approx 10^{-3}\rho T_g^{1/2}$ ergs s $^{-1}$ is the free-free cooling rate, to t_f being everywhere $\ll 1$ for the settling solutions.

b) Free-Fall Flows

Consider the behavior of free-fall solutions. If L_c , as well as the gas enthalpy, is negligible, the solution for L_n can be expressed as (Cassinelli and Castor 1973)

$$L_n = L_{\text{edd}}(1 - \epsilon_\infty) \exp [-(1 - \epsilon_0)(r_*/r)]. \quad (41)$$

Shakura (1974) has considered supercritical flow onto neutron stars ignoring E_g , P_g , and L_c altogether ($L_n = L_d$). He finds that for $W_* = 0$, the flow is simply free-fall for the effective mass $\epsilon_\infty M$ until $r \approx r_{\text{edd}}$. As is clear from equation (41), L_d would begin to decrease significantly at this point, with the result being that the velocity becomes greater than $(2\epsilon_\infty GM/r)^{1/2}$. This treatment is not valid at this point, though. Using the free-fall value for v at r_{edd} , we find that $|v/c| \approx 0.5\epsilon_\infty(M/M_\odot)^{1/2}(10^6 \text{ cm}/r_*)^{1/2}$, and $\tau_{\text{es}} \approx 2(M/M_\odot)^{-1/2}(r_*/10^6 \text{ cm})^{1/2}$. As $U_r \approx 3P_r \approx \tau_{\text{es}}L_d/4\pi r^2 c$ for $\tau_{\text{es}} \gtrsim 1$, L_c is roughly equal to $\epsilon_\infty L_{\text{edd}}$ at r_{edd} and cannot be ignored. For a white dwarf, τ_{es} may be greater than unity throughout the free-fall regime, with y and t_r given as for the LTE flow. For a neutron star, $\tau_{\text{es}} \approx 1$ at r_{edd} , with $t_r \approx t_{\text{BB}}$ at that point. In either case, L_c must be retained in the calculations for $r \lesssim r_{\text{edd}}$. We will use the approximation $L_c \approx L_d[2v(1 + \tau_{\text{es}}/2)c]$ when $L_c \lesssim \epsilon_\infty L_d$ giving

$$L_d \approx L_n \left[1 - \frac{2v}{c} (1 + \tau_{\text{es}}/2) \right] \approx L_{\text{edd}}(1 - \epsilon), \quad (42)$$

where L_n is given by equation (41). This value for ϵ is used in the calculations for accretion onto a neutron star until, in the regime $\tau_{\text{es}} \gtrsim 1$, it agrees to within 10% with the LTE value in equation (25) with the $t = t_r = t_{\text{BB}}$ (matching was found to occur for $\tau_{\text{es}} \approx 5$ for $\epsilon_0 = 0.1$, with the gas thermal energy term in eq. [25] being negligible). Interior to this point, equation (25) with $t = t_r$ was used to calculate ϵ , while the diffusion equation was used to follow t_r .

For our neutron star model with $\epsilon_0 = 0.1$ and $W_* = 0$, the free-fall flow is optically thin to absorption ($\tau^* < 1$) down to x_* . Because interior to this point the gas is expected to be nearly isothermal, we set $t_r \approx t_{\text{r}} \approx t_{* \text{min}}$. The results for numerical calculations starting at $10x_{\text{edd}}$ with $y = -(2\epsilon_0/x)^{1/2}$ and $t_g = t_{* \text{min}}$ are shown in Figures 3 and 6. The value for t_r is shown interior to the matching point for ϵ . In general, what was found for this and smaller values of ϵ_0 (but still larger than $\epsilon_{0 \text{min}}$) was that the behavior of t_r and y is weakly affected by the matching at $\tau_{\text{es}} \approx 1$ or by the behavior of t_g . For $\epsilon_0 = 0.1$, the

flow is highly supercritical everywhere, and t_g is not significant. The solutions show that instead of L_d decreasing inward, with a corresponding increase in velocity, what actually occurs is that L_d increases (ϵ decreases), and may become greater than L_{edd} ($\epsilon < 0$). The primary effect that the non-LTE value of t_g has on the flow is in its modification of the behavior of ϵ . The larger $4t_g x - 2x^2 dt_g/dx$ becomes, the greater is the value of ϵ . (The bump in the curve for $\epsilon > 0$ is due to our poor fitting at $\tau_{\text{es}} \approx 5$.)

For $\epsilon_0 = 0.1$, $T_{\text{r}} \approx 3 \times 10^7$ K, corresponding to hard X-ray photons. The parameter $Y = 4kT_{\text{r}}\tau_{\text{es}}^2/m_e c^2$ is a measure of the interaction strength between photons and gas for $\tau_{\text{es}} > 1$ (Felten and Rees 1972). For $Y \gg 1$, photons interact to the extent that they retain a Planck spectrum even if the system is thin to absorption. This is the case for our neutron star model, with the result being that the expected spectrum should be Wien [$I_\nu \propto (h\nu/kT_{\text{r}})^3 \exp(-h\nu/kT_{\text{r}})$] for energies near kT_{r} . For $\epsilon_0 < 0.1$, τ^* at x_* remains less than unity until $\epsilon_0 = 0.01$, with T_{r} increasing to 10^8 K. For smaller ϵ_0 , the point for $\tau^* \approx 1$ occurs at larger radii, and T_{r} decreases. For white dwarfs, the results are similar. For $\epsilon_0 = 0.1$, the point for $\tau^* \approx 1$ is found at $2x_*$, with $T_{\text{r}} \approx 10^6$ K.

c) Settling Flows

For subcritical solutions, τ_{es} and τ^* quickly become greater than unity interior to x'_a . We will use the approximation $T_g = (10^4 + T_{\text{r}}\tau^*)/(\tau^* + 1)$ for these flows and consider the regime $x \lesssim x'_a$. For $\tau_{\text{es}} < 1$, we use $\tau^* = 0$ and $\epsilon = \epsilon_0$, and only keep track of y and t_g . At $\tau_{\text{es}} = 1$, t_r was set equal to t_{BB} , and it was calculated, thereafter, from equation (24). The quantity ϵ was calculated for $\tau_{\text{es}} \gtrsim 1$ from equation (25) with $t = t_r$. In calculating τ^* , T_r was used. Figure 3 shows the dependence of y , t_r , and t_g on radius. The behavior of ϵ follows that for curve 3d in Figure 6. The numerical solution for $\epsilon_0 = 0.1$, $W_* = 0$, was calculated by integrating inward with $y = -0.41$ ($v \approx -a_\infty/30$) at $x = 10^{-3} \approx x'_a$. The resulting flow is qualitatively similar to the flows in region (v). The value of T_{r} is roughly 10^5 K; and since $Y \ll 1$, the spectrum will be that of a modified blackbody (Felten and Rees 1972). For a white dwarf, the results again remain similar to the LTE solutions.

d) $W_* \neq 0$

If one considers $W_* \neq 0$, the observed effect for neutron star flows would be small, as it would for a subcritical flow onto a white dwarf. For a supercritical flow onto a white dwarf, the flow would be essentially the same as for the LTE flow with fusion, that is, free-fall down to x_* . Throughout the free-fall region $\tau^* < 1$. At x_* , however, the flow is not likely to be isothermal now, and T_{r} thus depends upon the nature of the flow interior to that treated here.

V. STABILITY

We have considered only the possibility of convective Rayleigh-Taylor instabilities. In our crude

analysis, which aims at giving order-of-magnitude estimates, we have neglected the effect caused by the motion of the plasma, but have found though that whenever the system is unstable with a static criterion, the e -folding time is generally much less than the flow time.

The criterion for convective instability is

$$\left. \frac{d\rho}{dr} \right|_{\text{ad}} < \frac{d\rho}{dr}, \quad (43)$$

where $(d\rho/dr)|_{\text{ad}} = (\rho/\Gamma P)dP/dr$. If the system is unstable, then an element of fluid displaced adiabatically in pressure equilibrium outward will accelerate outward due to buoyancy. We will approximate the convective velocity v_c at r by considering a displacement from $r/2$ to r , and using $v_c \approx (\delta\rho GM/\rho r)^{1/2}$, evaluated at r . The quantity $\delta\rho$ is the difference between the ambient density and that found by integrating $(d\rho/dr)|_{\text{ad}}$. The e -folding time for convection is then given by $t_c \approx r/v_c$. The thermal behavior of the fluid is determined by the radiation diffusion time scale $t_d \approx 4\pi r^3 U_r/L_d$, and the gas-radiation interaction time scale $t_{\text{gr}} \approx kT/m_p c U_r [(kT\kappa_{\text{es}}/m_e c^2) + \kappa_{\text{ff}}]$, where free-free absorption and inverse Compton effects are included for the latter. For the diffusion time scale, the length scale of the displaced fluid element is assumed to be of order r . If $t_{\text{gr}} \ll t_c \ll t_d$, the radiation and the gas in the displaced element behave as a single fluid, and the motion is adiabatic with $\Gamma = \Gamma_1$. If, on the other hand, t_d and t_{gr} are both much less than t_c , then the displacement is highly nonadiabatic and convection would not be important (Joss, Ostriker, and Salpeter 1973). These two were the only regimes found to be important. In calculating v_c , we used $\Gamma = \Gamma_1$ with the flow being unstable whenever ϵ increases outward (entropy increasing inward). The neutron star model results for $\epsilon_0 = 0.1$ and $W_* = 0$ show that for the LTE flows, efficient convective instabilities occur for solutions passing through regions (ii)–(iv), (vii), and (viii). For the free-fall flows, the solutions were found to be unstable with $t_c \gtrsim t_d \gg t_{\text{gr}}$. Solutions passing through regions (v) and (vi) are stable. The solutions found in § IV are stable for the isothermal free-fall flow or show an instability near x_a' with $t_c \gg t_d \gg t_{\text{gr}}$ for the subcritical flow. All instabilities occur with $t_f \gg t_c$.

Rayleigh-Taylor instabilities may occur when $\epsilon > 0$ and $d\rho/dr > 0$ or when $\epsilon < 0$ and $d\rho/dr < 0$. The time scale for this instability is roughly

$$t_{\text{RT}} \approx [(\epsilon GM/r^2)d \ln \rho/dr]^{-1/2}$$

(Chandrasekhar 1961). The neutron star solutions unstable for $\epsilon_0 = 0.1$ and $W_* = 0$ are those in regions (i), (iii), (iv), (vii), and (viii). For the free-fall solutions, $t_{\text{RT}} \approx t_f$, while for the others $t_{\text{RT}} \ll t_f$. The non-LTE free-fall flows are unstable, with $t_{\text{RT}} \approx t_f$, while the settling solutions are stable.

We find, therefore, that the solutions in regions with optically thin flow, far from the compact object, may be stable to both convection and Rayleigh-Taylor

effects, while all other solutions are unstable to one or the other (or both). The regimes of instability for the LTE solutions are shown in Figures 3–5.

Solutions may be unstable also in the hydrogen-burning regime on the surface of the compact object. For neutron stars, this regime is likely to be highly isothermal, and therefore stable (Thorne and Żytkow 1975). White dwarf solutions are much less straightforward, and periods of steady fusion may alternate with epochs of little or no fusion or fusion at a rate faster than accretion (Katz and Salpeter 1976; Sienkiewicz 1975).

VI. DISCUSSION

We have constructed numerical models of steady spherically symmetric accretion flow onto white dwarfs and neutron stars. With respect to actual X-ray sources, the conclusions we can draw are bound to be tentative, owing to the restrictive approximation of spherical symmetry, and to the lack of detailed treatment of the outer flow region. Nevertheless, the study does illustrate a number of important physical effects that will probably show up in more detailed models for accretion near L_{edd} .

Accretion flows which have small scattering optical depths far from the compact object were found to belong to either free-fall supercritical flows or subcritical settling solutions. For free-fall solutions, the gas goes supercritical near x_a' , and then flows inward with the free-fall velocity for the effective mass $\epsilon_\infty M$. Interior to x_{edd} , the behavior is adiabatic with $P_r \gg P_g$, and $|\epsilon| \lesssim \epsilon_\infty$. For the settling solutions, the flow becomes adiabatic for $x \gg x_{\text{edd}}$, and it is highly non-adiabatic interior to x_{edd} . The value for ϵ is roughly equal to ϵ_∞ for $x \gtrsim x_{\text{edd}}$, and it increases interior to x_{edd} as x^{-1} . The bulk of the potential energy of the plasma is transformed into photons in either case. Gas pressure is more important for settling solutions with the resultant density and temperature being much greater at x_* . Such high temperatures for neutron star models result in unphysical neutrino losses, but they may be allowed for flows onto white dwarfs.

Free-fall type solutions result in hard or soft X-rays, respectively, for neutron stars or white dwarfs. In the studies of spherical accretion near L_{edd} by Kafka and Mészáros (1976), DeGregoria (1974), Maraschi, Reina, and Treves (1974), and Okuda and Sakashita (1977) T_r was assumed to be equal to T_e . Flows which become optically thick to scattering for $r \gg r_*$ were thus predicted to have spectra more typical of UV energies than of X-rays. We have found for our free-fall flows that T_r is generally larger than for optically thin flows and is $\gg T_e$ at $\tau_{\text{es}} = 1$. Even for very small ϵ_∞ , the mean photon temperature may be quite high. Inverse Compton scattering does not lead to a reduction in photon energy, as whenever $\tau_{\text{es}} > 1$ there is more energy in radiation than in the gas at temperature T_r . Absorption, though, may lead in the outer regions to a cutoff in the spectrum at quite high energies. The subcritical solutions are, however, likely to emit

primarily in the UV or optical. The behavior of the spectrum of both types of flow depends critically on the outer regime which has not been treated in detail here.

A detailed study of the outer non-LTE regime should be carried out to include the effects of preheating and of the photoionization cross section. Far from the compact object, where the gas is not highly ionized ($\xi \lesssim 10^5$), the absorption opacity may be much larger than κ_{es} (Tarter and McKee 1973; Buff and McCray 1974; Hatchett, Buff, and McCray 1976). This would limit the flow to luminosities much less than L_{edd} if it occurs for $x \lesssim x_a'$, even if preheating is not important. If the gas is accreted from a stellar wind, preheating and absorption may not be serious problems, since ξ may be large everywhere (Hatchett, Buff, and McCray 1976). The nature of the apparent limiting luminosity ($\epsilon_{\infty} \approx \epsilon_{0\text{min}}$), due to the gas pressure, needs to be explored also. Any completely self-consistent model must fit boundary conditions both at infinity and at the surface of the compact object, as the behavior near x_* can be coupled through the radiation preheating to the outer boundary.

Stability needs to be considered further also. Both settling and free-fall flows appear to be stable in the simple treatment given here, however, the inclusion of a more detailed treatment of the preheating or general relativistic effects may lead to instabilities such as photon bubbles mentioned by Prendergast and Spiegel (1973). Such instabilities may lead to luminosities much greater than L_{edd} . This present study provides steady solutions which can be used as a starting point for a full stability analysis.

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REFERENCES

- Alme, M. L., and Wilson, J. R. 1974, *Ap. J.*, **194**, 147.
 Arons, J. 1973, *Ap. J.*, **184**, 539.
 Basko, M. M., and Sunyaev, R. A. 1973, *Ap. Space Sci.*, **23**, 117.
 Bath, G. T. 1977, *M.N.R.A.S.*, **178**, 203.
 Bondi, H. 1952, *M.N.R.A.S.*, **112**, 197.
 Buff, J., and McCray, R. 1974, *Ap. J.*, **189**, 147.
 Cassinelli, J. P., and Castor, J. I. 1973, *Ap. J.*, **179**, 189.
 Chandrasekhar, S. 1961, *Hydrodynamic and Hydromagnetic Stability* (Oxford: Clarendon Press), chap. 10.
 Davidson, K., and Ostriker, J. P. 1973, *Ap. J.*, **179**, 585.
 DeGregoria, A. J. 1974, *Ap. J.*, **189**, 555.
 Felten, J. E., and Rees, M. J. 1972, *Astr. Ap.*, **17**, 226.
 Gursky, H. 1973, in *Black Holes: Les Houches*, ed. B. DeWitt and C. DeWitt (New York: Gordon & Breach).
 Hatchett, S., Buff, J., and McCray, R. 1976, *Ap. J.*, **206**, 847.
 Hunt, R. 1971, *M.N.R.A.S.*, **154**, 141.
 Illarionov, A. F., and Sunyaev, R. A. 1975, *Astr. Ap.*, **39**, 185.
 Joss, P. C., Salpeter, E. E., and Ostriker, J. P. 1973, *Ap. J.*, **181**, 429.
 Kafka, P., and Mészáros, P. 1976, *Gen. Rel. Grav.*, **7**, 841.
 Katz, J. I., and Salpeter, E. E. 1976, CRSR Report 640, Cornell University.
 Maraschi, L., Reina, C., and Treves, A. 1974, *Astr. Ap.*, **35**, 389.
 Margon, B., and Ostriker, J. P. 1973, *Ap. J.*, **186**, 91.
 McCray, R., and Hatchett, S. 1975, *Ap. J.*, **199**, 196.
 Mészáros, P., and Rees, M. J. 1975, *Bull. AAS*, **7**, 242.
 Novikov, I. D., and Thorne, K. S. 1973, in *Black Holes: Les Houches*, ed. B. DeWitt and C. DeWitt (New York: Gordon & Breach).
 Okuda, T., and Sakashita, S. 1977, *Ap. Space Sci.*, **52**, 35.
 Ostriker, J. P., McCray, R., Weaver, R., and Yahil, A. 1976, *Ap. J. (Letters)*, **208**, L61.
 Parker, E. N. 1963, *Interplanetary Dynamical Processes* (New York: Wiley-Interscience), chap. 11.
 Prendergast, K. H., and Spiegel, E. A. 1973, *Comments Ap. Space Phys.*, **5**, 43.
 Pringle, J. E., and Webbink, R. F. 1975, *M.N.R.A.S.*, **172**, 493.
 Ruffini, R., and Wilson, J. 1973, *Phys. Rev. Letters*, **31**, 1362.
 Shakura, N. I. 1974, *Soviet Astr.—AJ*, **18**, 259.
 Shakura, N. I., and Sunyaev, R. A. 1973, *Astr. Ap.*, **24**, 337.
 Shapiro, S. L., and Lightman, A. P. 1976, *Ap. J.*, **204**, 555.
 Shapiro, S. L., and Salpeter, E. E. 1975, *Ap. J.*, **198**, 671.
 Shvartsman, V. F. 1971, *Soviet Astr.—AJ*, **14**, 662.
 Sienkiewicz, R. 1975, *Astr. Ap.*, **45**, 411.
 Tarter, C. B., and McKee, C. F. 1973, *Ap. J. (Letters)*, **186**, L63.
 Tarter, C. B., Tucker, W. H., and Salpeter, E. E. 1969, *Ap. J.*, **156**, 943.
 Thorne, K. S. 1977, private communication.
 Thorne, K. S., and Żytkow, A. N. 1975, *Ap. J. (Letters)*, **199**, L19.
 van den Heuvel, E. P. J. 1975, *Ap. J. (Letters)*, **198**, L109.
 Vauclair, G. 1971, in *Proc. IAU Symposium 42, White Dwarfs*, ed. W. J. Luyten (New York: Springer-Verlag).
 Zel'dovich, Ya. B., Ivanova, L. N., and Nadezhim, D. K. 1972, *Soviet Astr.—AJ*, **16**, 209.
 Zel'dovich, Ya. B., and Novikov, I. D. 1971, *Relativistic Astrophysics, Vol. 1, Stars and Relativity* (Chicago: University of Chicago Press), chap. 13.

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