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THE EXTRAGALACTIC DISTANCE SCALE. I. A REVIEW OF DISTANCE INDICATORS: ZERO POINTS AND ERRORS OF PRIMARY INDICATORS

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ABSTRACT

The principal primary, secondary, and tertiary extragalactic distance indicators are reviewed. The zero points and errors of the *primary* distance indicators, (I) novae, (II) Cepheids, and (III) RR Lyrae and HB stars, are determined as follows:

(I) Novae, 15 days past maximum, $\langle M_{15}\rangle_{pg} = -5.5 \pm 0.18$, from 15 galactic novae with expansion and/or interstellar line parallaxes (Appendix A).

(II) Cepheids at mean period $\log P_0 = 0.8$, $\langle M_v\rangle(P_0) = -3.60 \pm 0.15$ with $\langle(B - V)_0\rangle(P_0) = 0.65$ from zero-age main-sequence fits in galactic clusters, secular parallaxes, and two photometric methods independent of the Hyades modulus (Appendices B, C).

(III) RR Lyrae *ab*-type variables ($P > 0.42$d, $\Delta S > 5$), $\langle M_v\rangle = +0.8 \pm 0.15$, with $\langle(B - V)_0\rangle = +0.20$ from statistical parallaxes and zero-age main-sequence fits in globular clusters (Appendix D).

Two subsidiary indicators are considered: (IV*a*) AB supergiants calibrated by the BCD method in the h + χ Per cluster for which $\mu_0 = 12.01 \pm 0.30$ (on the revised Hyades zero point), and (IV*b*) eclipsing binaries calibrated by the Gaposchkin method applied to seven galactic stars, according to Dworak.

Subject headings: cosmology — stars: Cepheids — stars: luminosities — stars: novae — stars: RR Lyrae

I. INTRODUCTION

Primary extragalactic distance indicators are classes of objects which (1) are measurable in nearby galaxies, (2) have a small cosmic dispersion about a stable mean, and (3) can be calibrated in our Galaxy by fundamental geometric methods. The main primary indicators are (I) *novae*, calibrated by expansion parallaxes and subsidiarily by interstellar line intensities, (II) *Cepheids*, calibrated via the period-luminosity-color (*P-L-C*) relation by zero-age main-sequence (ZAMS) fits of open clusters to the Hyades, by secular parallaxes, and by photometric methods independent of Hyades modulus, and (III) *RR Lyrae* variables (and horizontal branch stars), calibrated from statistical parallaxes, and subsidiarily ZAMS fits of the nearest globular clusters. Lesser known but potentially powerful indicators which also qualify in principle as primary include (IV*a*) *supergiants of spectral types B and A*, calibrated via the λ_1–D–ϕ_b parameters by the Barbier-Chalonge-Divan (Chalonge and Divan 1953, 1973) (BCD) method in the Per I (h + χ) cluster, also referred to the Hyades, and (IV*b*) *eclipsing binaries*, calibrated by the Gaposchkin (1938, 1940) method with respect to nearby main-sequence eclipsing stars. Neither of these methods has yet reached its full potential precision or the ease of application of methods (I) to (III).[1]

Tradition notwithstanding, distances derived from Cepheids calibrated in open clusters deserve no greater weight than the others. The unending discussions, revisions, and rediscussions of the *P-L*, *P-L-C*, *P-L-A*[2] relations make the point clear. Because of possible effects of age and chemical evolution on *any* indicator, it is risky to rely primarily or exclusively on any one indicator, while it is unlikely that *all* are affected in the same sense and amount by evolutionary differences between galaxies. Rather than rely entirely on a select few so-called "precision-indicators," the basic philosophy of "spreading the risks" will be adopted here.

Because primary indicators have a very short range of applicability hardly in excess of 1 Mpc for most, recourse must be had to secondary and tertiary indicators which have greater range but which are not amenable to absolute calibration in our galactic neighborhood.

Secondary indicators include (V) the brightest red giants in Population II systems, in particular globular clusters, (VI) the brightest star clusters, and especially globular clusters, (VII) the brightest nonvariable blue stars in Population I systems, (VIII) the brightest red supergiant variables, (IX) the brightest blue supergiant variables (Hubble-Sandage stars), and (X) the largest H II regions, either of all types or, preferably, restricted to the remarkable ring or "bubble" type. Indicators

[1] Other possible indicators (W Virginis variables, supernova remnants, etc.; see van den Bergh 1977) are of still lower accuracy and currently do not qualify as primary indicators.

[2] Period-luminosity-amplitude.

(V) and (VI) are calibrated from cluster variables in our Galaxy, assuming them to have the same mean absolute magnitude as the RR Lyrae field stars in our neighborhood; indicators (VII)–(X) are calibrated in the Magellanic Clouds and other galaxies of the Local Group, because it is doubtful that the brightest or largest in our Galaxy are within the restricted radius of optical observation.

Secondary indicators have a potential range in excess of 30 Mpc; however, their current domain of validity may be less than 10 Mpc because of increasing susceptibility to systematic errors beyond.

Tertiary indicators including (XI) the isophotal diameters and (XII) total magnitudes of the galaxies themselves can be calibrated by means of galaxies whose distances are known from primary and secondary indicators, provided some other, distance-independent index of absolute diameter and luminosity, such as morphological type and/or luminosity class, is available. The last two methods can be extended further without recourse to independent luminosity indices in the special case of the first-ranked (or first five) galaxies in groups or clusters.[3]

Most of the "first-ranked" secondary and tertiary distance indicators depend on stochastic processes defining the extrema of a distribution function and as such, are sensitive to statistical size-of-sample effects which must be calibrated out through some appropriate empirical or theoretical correlation with the absolute magnitude of the parent galaxy. The latter is derived by successive approximations or from independent luminosity indicators (cluster population or richness class in the case of first-ranked galaxies).

Several of the secondary indicators, in particular the brightest globular clusters and largest H II regions, are close to the current limits of detection and measurement, and may be subject to large accidental and systematic errors. Among the tertiary indicators, photographic diameters visually estimated are also subject to systematic errors which have too often been ignored. (Metric and photometric diameters properly defined and measured have not yet been used to any significant extent; these will be discussed in a later paper.)

Another systematic error that has been often neglected because it is difficult to estimate is the effect of internal extinction in other galaxies. In the present study the extinction $A(i)$ calculated in the *Second Reference Catalogue of Bright Galaxies* (RC2) (de Vaucouleurs, de Vaucouleurs, and Corwin 1976, eq. 24) will be used as a guide. $A(i)$ is the magnitude difference between the observed and *face-on* magnitude $B_T(i) - B_T(0)$; the absorption-free magnitude may be ~ 0.17 mag brighter on the average (except for dust-free ellipticals). Because of this extinction, objects on the far side of the equatorial plane tend to be underrepresented or absent in samples selected by apparent magnitude, in particular when the brightest or brighter examples of a kind are selected. In the present series of papers a correction $\delta m_B(i) = [A(i) + 0.17]/2$ (and $\delta m_v \approx 3\delta m_B/4$) was applied to disk population stars, such as Cepheids, except when located in outlying arms where $\delta m/2$ was used as in M31; no correction was applied to objects such as novae or globular clusters predominantly associated with the spheroidal component or selected by apparent magnitude or both. The maximum corrections are ≤ 0.2 mag.

Finally, because all data are affected by galactic extinction, it is important to calculate it from a consistent expression of its overall distribution as a function of galactic latitude *and* longitude, rather than depend on its precarious estimate from often marginally significant color excesses in individual objects. Sandage (1973) and Sandage and Tammann (1974*a*, *b*) assume $A_B \approx 0$ at $|b| \geq 50°$ and a modified "cosec b" law, independent of l, at $|b| < 50°$. This implies that the Sun is at the common apex of two opposite dust-free cones 80° in aperture centered at the galactic poles. This strange model, inspired by the low color excesses of *stars* near the poles, is contradicted by all available data on galaxy counts and galaxy colors (de Vaucouleurs 1978). In the present series of papers, galactic extinction A_B as well as fully corrected galaxy magnitudes B_T^0, colors $(B - V)_T^0$ and diameters ($\log D_0$) will be taken from RC2 which should be consulted for details of definitions, formulae, and procedures. The expressions for $A_B(l, b)$ and $R = A_V/E(B - V)$ adopted in RC2 (eqs. [23], [29], and [32]) are consistent with $\langle A_B \rangle = 0.20$ mag and $\langle E(B - V) \rangle = 0.046$ mag at the galactic poles, in agreement with several independent estimates (Holmberg 1974; Kron and Guetter 1976; de Vaucouleurs 1978).

The objectives of the present series of papers are (1) to review the basic calibrations of the primary indicators, revise their zero points where appropriate, and provide realistic estimates of their systematic and random errors (Paper I); (2) to derive revised distance moduli for members of the Local Group from a combination of primary and other indicators (Paper II); (3) to introduce new or modified versions of some secondary indicators, in particular brightest stars, brightest clusters, and largest H II rings (Paper III); (4) to derive from (2) and (3) preliminary estimates of the distances of the M81, M101, and Sculptor groups and of some nearby field galaxies (Paper IV); (5) to calibrate the luminosities and diameters of galaxies as tertiary indicators via a composite index of absolute luminosity and diameter for Sb–Sm spirals, combining morphological type T and luminosity class L (Paper V); (6) to derive from (5) distances to Sc and late-type galaxies in the field, in the S-cloud of the Virgo cluster, and in the Grus cloud (Paper VI); and (7) to use the deduced distances of ~ 480 spirals within ~ 40 Mpc to derive a preliminary mean value of the local Hubble ratio (Paper VII), with allowance for the supergalactic anisotropy of the redshift law.

[3] Other possible indicators such as diameters of ring structures in SB(r) galaxies, Type I and Type II supernovae, and H I line parameters will not be considered here, pending the results of programs in progress for their reevaluation and calibration.

II. ZERO POINTS OF THE PRIMARY INDICATORS

a) *Novae* (Indicator I)

i) *Novae at Maximum, M_0 (log t_3) Relation*

The most common application of novae as extragalactic distance indicators is through the correlation between absolute magnitude at maximum M_0 and the decay rate parameter $\log t_3$ (McLaughlin 1945) or $\log t_2$ (Schmidt-Kaler 1957). The slope of McLaughlin's relation,

$$M_0 = b \log t_3 - a\,, \tag{1a}$$

is well established, but the zero point based first on 11 and now on 15 galactic novae is more uncertain. Various estimates, collected in Table 3 in Appendix A, are in the range $10.9 \leq a \leq 11.8$, and $2.3 \leq b \leq 2.5$. Excluding very fast and very slow novae, the linear approximation holds best in the interval $1.0 \leq \log t_3 \leq 2.0$, where the range of M_0(pg) is a minimum; at $\langle \log t_3 \rangle = 1.5$ the range of $M_0(1.5) = 1.5b - a$, is from -7.45 to -8.05. The adopted values, consistent with the revised zero point of § II*a*(ii), give

$$M_0(\mathrm{pg}) = 2.4 \log t_3 - 11.3 \tag{1b}$$

and $M_0(1.5) = -7.7$, with an estimated mean error $\sigma_0 = 0.18$ mag in the useful range of $\log t_3$. This relation is adequate for systems such as the Magellanic Clouds with few well-observed novae that are neither very fast nor very slow (Buscombe and de Vaucouleurs 1955; Ardeberg and de Groot 1973). For richer, well-patrolled systems, such as M31 (Arp 1956; Rosino 1964, 1972), a nonlinear relation valid over a greater range of $\log t$ is preferable (Schmidt-Kaler 1957; Rosino 1972). However, except perhaps in M31, observations of extragalactic novae are often fragmentary, the apparent magnitudes at maximum m_0 and decay rate parameters are seldom well-determined, and distance moduli derived from the $M_0(\log t_3)$ relation are often uncertain.

The standard deviation of individual values of M_0 from equation (1) is $\sigma_1' = 0.60$ mag for 15 calibrating galactic novae and $\sigma_1'' = 0.35$ mag from the corresponding equation for the best observed Magellanic novae (Paper II); the total mean error of the zero point $\langle M_0(1.5)\rangle$ at $\langle \log t_2 \rangle \approx 1.5$ is $\sigma_0 = \sigma_1'/(15-2)^{1/2} = 0.18$ mag, and the additional error introduced by application of equation (1) to Magellanic novae in the interval $1.0 < \log t_3 < 2.0$ is $\sigma_1 = \sigma_1''/\sqrt{n} \approx 0.18$ (LMC) to 0.25 (SMC) (or 0.15 on the mean modulus of both Clouds) for a total error $\sigma = (\sigma_0^2 + \sigma_1^2)^{1/2} =$ 0.25–0.30 (or 0.23 on mean modulus).

ii) *Novae at 15 Days, M_{15} Method*

A better method relies on the magnitudes 15 days past maximum, m_{15}, when all galactic novae, irrespective of type, have the same mean absolute magnitude $\langle M_{15} \rangle = -5.2$ (visual) (Buscombe and de Vaucouleurs 1955) or -5.6 (visual) and -5.86 (photographic) (Schmidt-Kaler 1957). The zero point depended initially on 11 galactic novae with distances from expansion parallaxes (seven novae) and interstellar line-strengths (eight novae). With more recent data on Nova Puppis 1942 (Payne-Gaposchkin 1957; Schmidt-Kaler 1957) the average maximum M_0 for the group of fastest novae ($t_3 \approx 8$ d) is raised by -0.3 mag (Young *et al.* 1976) and brought into closer agreement with the three other groups of slower novae for a revised $\langle M_{15} \rangle = -5.5 \pm 0.18$ (pg). The revised data for 15 novae are collected in Table 4 in Appendix A. The method has been most successfully applied to the Magellanic Clouds where $\langle m_{15} \rangle = +13.4$ (Paper II); in M31, the M_0–$\log t_2$ method is probably to be preferred, because $\langle m_{15} \rangle \approx 19$ is uncomfortably close to the faint limit of existing surveys (Arp 1956; Rosino 1964, 1972) where photometric data are more scanty and systematic errors may be significant.

The standard deviation of individual values of M_{15} from $\langle M_{15} \rangle$ is $\sigma_1' = 0.6$ mag for 11 galactic novae; that of m_{15} from $\langle m_{15} \rangle$ is $\sigma_1'' = 0.40$ mag for the six best observed Magellanic novae; and $\sigma_1'' = 0.60$ mag for the 21 best observed novae in M31 (Paper II). The mean error of the zero point $\langle M_{15} \rangle$ is $\sigma_0 = \sigma_1'/\sqrt{11} =$ 0.18 mag; a distance modulus derived from n well-observed extragalactic novae has a total mean error $\sigma = (\sigma_0^2 + \sigma_1^2/n)^{1/2}$, or 0.24 mag for the mean modulus of the Clouds.

b) *Cepheids* (Indicator II)

i) *P-L Relation at Mean Light*

The difficulties with period-luminosity-color (*P-L-C*) relations for Cepheids at mean and maximum light used as distance indicators by Sandage and Tammann (1968, 1969, 1971) have already been discussed by many authors (Butler 1971, 1976; Madore 1976; van den Bergh 1977; Clube and Dawe 1977). Because of these difficulties, little confidence can be placed in the *P-L* relation until its shape and dependence on color and/or amplitude have been more securely established, especially for the longer periods.

ii) *Mean Luminosity at Standard Period, $M\langle P_0 \rangle$ Method*

A better way to use the Cepheid criterion is to to compare directly galactic and extragalactic Cepheids near the mean period $\langle \log P_0 \rangle = 0.8$ of the calibrating stars (de Vaucouleurs 1955; Sandage 1962). This procedure avoids the extrapolation errors due to uncertainties in the shape of the *P-L* relation; only the mean slope of the relation near $\langle P_0 \rangle$ is used for interpolation to the reference period. Reducing the 13 calibration cepheids of Sandage and Tammann (1969, Table 5) to a common $\log P_0 = 0.8$ via the slope of the *P-L* relation (2.8 in V) with proper allowance for the dependence of R on $(B-V)_0$ and A_v (see Appendix C) and equal weight for the 13 stars gives $\langle M_v \rangle (P_0) = -3.58$, on the old Hyades zero point ($\mu_0 = 3.03$).

From 14 Cepheids of periods $3 < P < 50$ d in 12 associations (only seven are in common with the Sandage-Tammann list) van den Bergh (1977) derives $\langle M_v \rangle = -1.18 - 2.90 \log P$ and $(\langle B \rangle - \langle V \rangle)_0 = 0.27 + 0.48 \log P$, which for $\log P_0 = 0.8$ gives

$\langle M_v \rangle(0.8) = -3.50$ in close agreement (for details see Appendix C).

A common zero-point correction of -0.26 mag will reduce the mean $\langle M_v \rangle(0.8) = -3.54$ to the revised Hyades zero point ($\mu_0 = 3.29$) adopted here (Appendix B):

$$\langle M_v^c \rangle(0.8) = -3.80 \pm 0.15 .$$

The zero point based on cluster Cepheids only is rather uncertain due to possible systematic errors in ZAMS fits (van den Bergh 1977; see Appendix B). Fortunately, other, independent methods are now available.

iii) *Zero Point of P-L Relation from Secular Parallaxes*

Improvements in the fundamental system and in the precision of proper motions of the brighter Cepheids have in recent years rekindled hopes of deriving statistically significant secular parallaxes for the bright Cepheids (Jung 1968, 1970; Heck and Jung 1975; Clube and Dawe 1977). From a carefully selected sample of 33 nearby Cepheids ($r \leq 1$ kpc) corrected for galactic extinction after Fernie (1967*a*), Jung (1970) derived by the method of maximum likelihood systematic corrections to the *P-L* relation adopted by Sandage and Tammann (1968); the mean correction is $(m - M) = -0.4 \pm 0.4$ (m.e.) for Jung's "scale 3" which "is more positive by 0.10, than would have resulted from the application of Sandage and Tammann's relations, uncorrected for extrinsic reddening," indicating a correction $\Delta M = +0.3 \pm 0.4$ to the zero point of the Sandage-Tammann (1968) *P-L* relation. From Table A1 of that paper $\langle M_v \rangle(0.8) = -3.64$ and $\langle M_B \rangle(0.8) = -3.12$, leading to the corrected zero points $\langle M_v^c \rangle(0.8) = -3.34$, and $\langle M_B^c \rangle(0.8) = -2.82$, both with a mean error of 0.4 mag.[4]

iv) *Zero Point of P-L-C Relation from Fernie's Calibration*

Fernie (1967*b*) has calibrated the form and the zero point of the *P-L-C* relation by a theoretical procedure independent of open clusters or secular parallaxes. It relies on a period-radius relation (Fernie 1968) derived from the average radii of the Cepheids calculated by the Baade-Wesselink method. From the definition of bolometric magnitudes M_b and inserting several empirical relations between radius, effective temperature, and period and between $M_b - M_v$ and color index, Fernie obtained for the *P-L-C* relation

$$\begin{aligned} M_v = &-2.54 - 2.79 \log P + 2.79(B-V) - 1.55(B-V)^2 , \\ &\pm .15 \quad \pm .07 \qquad\qquad \pm .08 \qquad\qquad \pm .06 \end{aligned} \tag{2a}$$

where the zero point depends on the adopted solar values $M_b(\odot) = 4.72$, $T_e(\odot) = 5800$ K, but the other coefficients are determined from observations of 43 Cepheids. The color term is given by the mean color-period relation (Fernie 1967*a*)

$$\langle (B - V)_0 \rangle = 0.24 + 0.49 \log P . \tag{3}$$

Fernie compared absolute magnitudes calculated through equations (2a) and (3) with those of eight Cepheids in open clusters or binary systems and modified the constants of equation (2a) to remove systematic differences; the revised equation

$$M_v = -2.55 - 2.85 \log P + 2.73(B-V) - 1.60(B-V)^2 , \tag{2b}$$

or, by substitution of equation (3),

$$M_v = -1.99 - 1.89 \log P - 0.38(\log P)^2, \tag{2c}$$

were subsequently used by Fernie and Hube (1968) to derive distances to a large number of galactic Cepheid (see § II*b*[v] below). However, this forces the relation to fit the few calibrating cluster Cepheids, and reintroduces the Hyades zero point.

Here we consider only the independent zero point defined by (2a) for $\log P_0 = 0.8$,

$$\langle M_v \rangle(0.8) = -3.63 \pm 0.23 ,$$

with $\langle (B - V)_0 \rangle(0.8) = 0.632$ from (3).

v) *Zero Point from the Barnes-Evans Relation*

Barnes, Evans, and Parsons (1976) have proposed a method of deriving *distances* to the Cepheids (rather than absolute magnitudes as in other methods) based on the Barnes-Evans (1976) relation

$$F_v = 4.2207 - 0.1\, V_0 - 0.5 \log \phi' \tag{4}$$

between the V-band surface brightness and apparent diameter ϕ' (in milliseconds) of stellar disks, if V_0 is the corrected apparent V magnitude. The zero point is fixed by assuming $M_v(\odot) = 4.73$; however, it agrees closely with an independent calibration from supergiants observed with the intensity interferometer.

This relation was applied by Barnes *et al.* (1977) to derive diameters of Cepheids which follow the empirical relation

$$\begin{aligned} F_v = &\ 3.966 - 0.390\,(V - R) . \\ &\pm .005 \quad \pm .019 \end{aligned} \tag{5}$$

The variations in ϕ' derived from (4) and (5) are then compared with variations in linear radius derived from the velocity curve, assuming 1.31 for the projection and limb darkening correction factor. A great strength of the method is that it is insensitive to interstellar extinction.

Distances of nine Cepheids ($0.56 \leq \log P \leq 1.43$) so derived average 0.956 ± 0.066 of those on the Fernie-Hube scale (1968). The latter is defined by equations (2b) and (2c)[5] which, for $\log P_0 = 0.8$, give

[4] Note that the implied $(B - V)_0 = 0.52$ is slightly inconsistent with the color-period relation $(B - V)_0 = 0.37 + 0.264 \log P$ or $\langle (B - V) \rangle(0.8) = 0.58$ adopted in the same paper.

[5] Note that Fernie and Hube (1968) calculate extinction by the $R = 3$ rule.

$\langle M_v \rangle(0.8) = -3.75$. Reduction to the Barnes *et al.* scale leads to

$$\langle M_v \rangle(0.8) = -3.65 \pm 0.15 \,.$$

The estimated errors listed by Barnes *et al.* correspond to an average mean error $\sigma_1(5 \log r) = 0.386$ mag per star, or 0.13 for the mean of nine stars. With allowance for uncertainties in the constants of eq. (4) and (5), the total mean error of this zero-point determination is at least 0.15 mag.

The four independent determinations of the Cepheid zero point are collected in Table 7; the adopted unweighted mean is

$$\langle M_v \rangle(0.8) = -3.60 \pm 0.15$$

with $\langle (B - V)_0 \rangle = +0.65 \pm 0.03$.

c) RR Lyrae Variables and HB Stars (Indicator III)

The median absolute magnitude of the RR Lyrae stars from statistical parallaxes has been in a state of flux ever since Mineur's (1944) discovery of the discrepancy between the zero points of the *P-L* relations of the short and long period "Cepheids." Early revisions (Parenago 1954; Pavlovskaya 1953) indicated values as low as $M_{pg} = +0.5 \pm 0.3$, while later studies (Woolley *et al.* 1965; van Herk 1965) gave $M_{pg} \approx +0.8 \pm 0.2$ (with $\text{Pg} - \text{Pv} \approx +0.2$), and recent discussions (Clube and Jones 1971) suggest values as high as $M_v = +1.3 \pm 0.2$, which seem to bracket the possible range. The latest results (Hemenway 1975) are near the middle of this range with $\langle M_v \rangle = +0.70$ for *ab* types of all periods and $\langle M_v \rangle = +0.83$ for $P \geq 0.42$ d (see Appendix D).

The second major method for the absolute calibration of RR Lyr depends on C-M diagrams of globular clusters whose main sequence, corrected for differential line blanketing, are fitted to the Hyades zero-age main sequence, or to the H-R diagram of nearby stars from trigonometric parallaxes. There are obvious difficulties and questionable assumptions in this approach (see, e.g., Kraft 1972), but the results appear to agree with the statistical methods within ± 0.2 mag (see, e.g., Plaut 1965).

The values of $M_v(\text{RR})$ given by Kukarkin (1974) in 86 globular clusters lead to $\langle M_v \rangle(\text{RR}) = +0.86$. For the present paper the values

$$\langle M_v \rangle(\text{RR}) = +0.80 \pm 0.15$$

and

$$\langle (B - V)_0 \rangle(\text{RR}) = +0.20 \pm 0.05$$

may be adopted as representative of high $-\Delta S$ stars (see Appendix D) and appropriate to the calibration of extragalactic globular clusters as distance indicators (Paper III). This is the main use of RR Lyrae stars, since they cannot yet be observed much beyond the Magellanic Clouds.

From the range of suggested $\langle M_v \rangle(0.3 \text{ to } 1.3)$ and their quoted mean errors the zero-point uncertainty may be estimated at $\sigma_0 = 0.15$ mag; from the dispersion in M_v among the galactic globular clusters the internal mean error of a distance modulus derived from the adopted $\langle M_v(\text{RR}) \rangle$ and $\langle m_v(\text{RR}) \rangle$ in an external galaxy or cluster is at least 0.15 mag, but including the unavoidable photometric errors in crowded star fields at faint magnitudes ($\langle m \rangle \approx 19.5$ in the Magellanic Clouds), $\sigma_1 = 0.25$ mag is probably more realistic.

The adopted zero points of the three primary indicators are collected in Table 1. *A posteriori* comparison (Paper II, Table 12) will show that they are mutually consistent within ± 0.15 mag or better.

Table 2 allows a comparison of zero point mean error σ_0 and observational dispersion σ_1 for the three primary indicators. It is clear that none is significantly superior to the others in zero point accuracy; but, in combination with equal weights, they may ultimately (when $\sigma_1/\sqrt{n} \ll \sigma_0$) define the zero point of the extragalactic distance scale with a m.e. $\sigma_0(\mu_0) \leq 0.10$ mag i.e., distances to the nearest galaxies correct within $\pm 15\%$ (3 σ).

TABLE 1

ADOPTED ZERO POINTS OF PRIMARY INDICATORS

Indicator	Method	Zero Point
I. Novae..........	$\langle M_{15} \rangle$	−5.50 (pg)
II. Cepheids........	$\langle M \rangle(\log P_0 = 0.8)$	−3.60 (*V*)
		−2.95 (*B*)
III. RR Lyrae.......	$\langle M \rangle(P > 0.42\ d)$	+0.80 (*V*)
		+1.00 (*B*)

TABLE 2

ZERO-POINT UNCERTAINTIES AND OBSERVATIONAL DISPERSION OF PRIMARY INDICATORS

Indicator	Method	σ_0	σ_1	N_0*
I. Novae..........................	*a*) $M_0(\log t_3)$	0.18	0.5	8
	b) M_{15}	0.18	0.5	8
II. Cepheids........................	*a*) Mean *P-L* relation	0.2:	0.25	2
	b) $\langle M(P_0) \rangle$	0.15	0.25	3
III. RR Lyrae stars	*a*) $\langle M(P_0) \rangle$	0.15	0.25	3
and HB stars..................	*b*) $\langle M_v \rangle = \langle M_v \rangle(\text{RR})$	0.20	0.3	3
IV*a*. AB supergiants..................	$\lambda_1 - D - \phi_b$ (BCD)	0.30	0.4:	2
IV*b*. Eclipsing variables................	Gaposchkin's	0.3:	0.6:	4

*N_0 = minimum number of objects needed for $\sigma_1/\sqrt{N} \leq \sigma_0$.

d) *AB Supergiants* (Indicator IVa)

Until recently B and A type supergiants were not acceptable as primary indicators because two-parameter classification schemes such as the MK spectral and luminosity classes or Hγ line intensities are poor discriminants of absolute magnitudes for the brightest supergiants and calibration problems are severe (*cf.* discussions by Fehrenbach 1958; Blaauw 1963; Heck 1976). For example, luminosity class Ia generally applies to any AB supergiant brighter than about -6.5. This imprecision is much reduced by the three-parameter spectral classification system of Barbier, Chalonge, and Divan (BCD system) which unambiguously defines spectral type and absolute luminosity in terms of the Balmer discontinuity (λ_1, D) and gradient of the blue continuum ϕ_b (Chalonge and Divan 1953, 1973), all of which can be measured precisely. The absolute calibration $M_v = f(\lambda_1, D)$ for $M_v < -6$ depends only on the adopted modulus of h + χ Per and, consequently, on the Hyades zero point (Appendix B). Recently it has become possible to apply the BCD system to stars brighter than $m_v \approx 11.5$ in the Magellanic Clouds to obtain directly the differential modulus $\Delta\mu_0$ between each Cloud and h + χ Per (note that the reddening is corrected individually for each star from ϕ_b, but $\Delta\mu_0$ is independent of the assumed value of the intrinsic gradient $\phi_b{}^0$). The zero point adopted by Divan is the distance modulus $\mu_0 = 11.8 \pm 0.3$ (p.e.) for h + χ Per after Johnson *et al.* (1961), which rests on a "tie-in with the nearby stars via the Hyades" for which $\mu_0 = 3.0 \pm 0.1$ (p.e.) was adopted. Two corrections are needed: (i) $+0.26$ to reduce to the new Hyades modulus (Appendix B), (ii) -0.05 to correct for the assumption $R = 3$ in calculating A_v from $E(B - V) = 0.56$. Thus $\mu_0{}^c(\mathrm{h} + \chi) = 11.8 + 0.21 = 12.01$, and the total correction to the Divan scale is $\delta\mu_0 = +0.21$.

The errors calculated by Johnson *et al.* are external errors from all sources including zero point; the latter are much reduced with the improved value for the Hyades and for R and the zero point error in $\delta\mu_0$ reduces essentially to the transfer error in $\delta\mu_0(\mathrm{h}\chi\text{-Hya})$ (i.e., ZAMS fit) estimated by Johnson *et al.* to be ~ 0.2 (p.e.) on the average, i.e., $\sigma_0 \approx 0.30$ mag. The internal mean error of the BCD method in its application to AB supergiants in other galaxies is estimated at $\sigma_1 \approx 0.4$ mag (0.25 from 10 LMC stars, 0.55 from 8 SMC stars). Then the total m.e. of the modulus derived from, say, $n \approx 9$ stars is $\sigma = (\sigma_0{}^2 + \sigma_1{}^2/n)^{1/2} = 0.33$ mag.

e) *Eclipsing Binaries* (Indicator IV*b*)

Distance moduli of eclipsing binaries can be derived by S. Gaposchkin's method (1938, 1940) modified by Dworak (1974). It applies best to main-sequence stars and rests on a combination of Kepler's third law, the Stefan-Boltzmann radiation law, Eddington's mass-luminosity relation, and the relation between the absolute and relative dimensions of the components. It requires estimates of the effective temperatures and bolometric corrections from spectra or colors; the zero point rests on an assumed value for the bolometric absolute magnitude of the Sun ($M_b = +4.66$). An application to the seven nearest eclipsing binaries with good trigonometric parallaxes is said to verify the validity of the basic relations and zero point calibration (Dworak 1974). Published information is insufficient to reliably evaluate the systematic and accidental errors of this method; however, from examples of applications to the Magellanic Clouds and M31 (Paper II), the internal accidental mean error of the modulus derived from one binary is $\sigma_1 \approx 0.6$ mag; it will be assumed that $\sigma_0 \simeq 0.3$ mag (Table 2).

Indicators IV*a* and IV*b* are currently less reliable than the others; they will be used with half-weight in deriving distance moduli of Local Group galaxies from primary indicators (Paper II).

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APPENDIX A

ABSOLUTE MAGNITUDES OF GALACTIC NOVAE

Various determinations of the constants a, b of McLaughlin's relation $M_0 = b \log t_3 - a$ are collected in Table 3. Absolute magnitudes at maximum M_0 and 15 days past maximum M_{15} are collected in Table 4 for 15 galactic novae with distances derived from expansion parallaxes or interstellar line intensities ($N\pi$ = number of determinations). The original values of M_0 given in various systems (v, pg, V, B) are as far as possible reduced to the photographic system for convenience of comparison with novae observed in the Magellanic Clouds or M31. The M_{15}

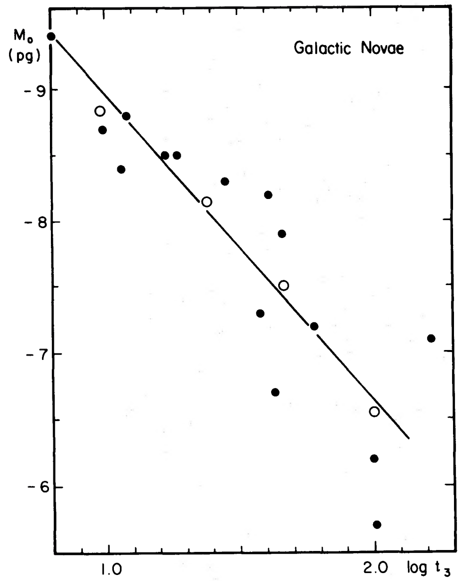

FIG. 1*a*.—McLaughlin's relation for galactic novae. Revised absolute photographic magnitudes M_0(pg) of 15 calibrating novae versus log t_3. *Circles*, mean points.

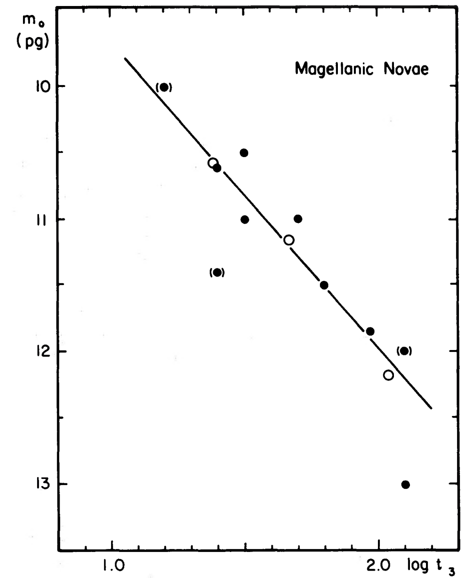

FIG. 1*b*.—McLaughlin's relation for Magellanic novae. Revised apparent photographic magnitudes m_0(pg) of 10 novae versus log t_3. *Circles*, mean points.

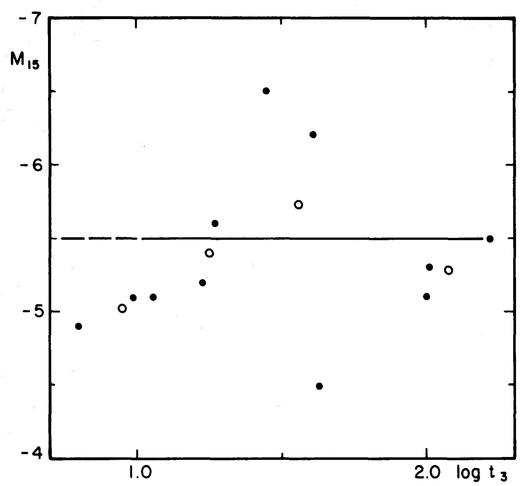

FIG. 2.—Absolute magnitudes M_{15} of 11 galactic novae 15 days past maximum. *Circles*, mean points. Adopted mean $\langle M_{15} \rangle = -5.5$ is essentially independent of t_3.

values are presumed to be applicable to any system in the absence of definitive data on the intrinsic colors of novae 2 weeks past maximum ($\sim 0.0 \pm 0.2$ from fragmentary data).

The McLaughlin relations for these revised data are shown in Figures 1*a* and 1*b* for the Galaxy and the Magellanic Clouds. Both are consistent with $M_0(\text{pg}) = 2.4 \log t_3 - 11.3$ within mean errors $\sigma_1' = 0.60$ mag (including calibration errors) for galactic novae and $\sigma_1'' = 0.35$ mag for Magellanic novae.

The constancy of M_{15} and independence of log t_3 are shown in Figure 2. The scatter from the adopted mean $\langle M_{15} \rangle = -5.5 \pm 0.18$ (m.e.) is consistent with $\sigma_1' = 0.60$ (galactic) and $\sigma_1'' = 0.40$ (Magellanic). Novae in M31 will be discussed in Paper II, Appendix A.

TABLE 3

CONSTANTS OF MCLAUGHLIN'S RELATION

Source	Data	b	$-a$	$M_0(1.5)$	M_{15}	Remarks
1955, Buscombe and de Vaucouleurs	11 bright galactic novae	2.3	−10.9	−7.45	−5.20	1
	25 novae in Sgr-Sco-Oph	2.3	(+4.4)	(+7.85)	(+10.10)	2
	10 novae in Magellanic Clouds	2.3	(+7.7)	(+11.15)	(+13.40)	2
1957, Schmidt-Kaler	11 bright galactic novae	2.5	−11.8	−8.05	−5.86	
1977, This paper	15 bright galactic novae	2.3	−11.25	−7.8	−5.55	3
	10 novae in Magellanic Clouds	2.4	(+7.45)	(+11.05)	(+13.40)	2
	Adopted	2.4	−11.3	−7.7	−5.5	

REMARKS.—(1) After McLaughlin 1945. (2) Apparent magnitudes. (3) ZP revised.

TABLE 4

ABSOLUTE MAGNITUDES OF 15 GALACTIC NOVAE

Nova	$-M_0$	$\langle -M_0 \rangle$*	ΔM_{15}	$\langle -M_{15} \rangle$*	$\log t_3$		$N\pi$	Sources
CP Pup 1942	9.6 v	9.4	4.5	4.9	0.85	0.80	2	1, 2
	9.4 pg		...		0.75		3	3
V603 Aql 1918	8.7 v	8.7	3.6	5.1	0.90	0.99	2	1, 2
	8.8 pg		...		1.08		4	3
CP Lac 1936	8.2 v	8.4	3.3	5.1	0.95	1.06	2	1, 2
	8.6 pg		...		1.18		3	3
V446 Her 1960	8.7 B	8.8	...	...		1.08	1	4
GK Per 1901	8.4 v	8.5	3.3	5.2	1.11	1.23	1	1, 2
	8.7 pg		...		1.35		2	3
V476 Cyg 1920	8.4 v	8.5	2.9	5.6	1.20	1.27	2	1, 2
	8.8 pg		...		1.34		3	3
DK Lac 1950	7.5 pg	8.3	1.8	6.5	1.38	1.45	1	1, 2
	8.6 pg		...		1.51		3	3
DN Gem 1912	6.4 v	6.7	2.2	4.5	1.57	1.63	1	1, 2
	7.0 pg		...		1.68		2	3
V533 Her 1963	7.5 V	7.3	...	...		1.58	1	4
EU Sct 1949	8.1 v	8.2	2.0	6.2	1.60	1.61	2	1, 2
	8.3 pg		...		1.62		3	3
LV Vul 1968	7.8 B	7.9	...	...		1.66	1	4
FH Ser 1970	7.4 V	7.2	...	...		1.78	1	4
DQ Her 1934	5.3 v	6.2	1.1	5.1	2.00	2.00	1	1, 2
	6.5 pg		...		2.00		4	3
T Aur 1891	5.3 pg	5.7	0.4	5.3	2.00	2.01	1	1, 2
	6.1 pg		...		2.02		1	3
RR Pic 1925	7.3 v	7.1	1.6	5.5	2.18	2.22	1	1, 2
	7.1 pg		...		2.26		1	3

* Photographic.

SOURCE.—(1) Buscombe and de Vaucouleurs 1955. (2) Payne-Gaposchkin 1957. (3) Schmidt-Kaler 1957. (4) Arhipova and Mustel 1935.

APPENDIX B

DISTANCE MODULUS OF THE HYADES

The distance moduli of Cepheids used by Sandage and Tammann from 1968 to 1974 rest ultimately on van Bueren's (1952) modulus $\mu_0 = (m - M)_0 = 3.03$ for the Hyades from the convergent method. Recent determinations by a variety of methods reviewed by van Altena (1974) range from this low value to a high value of 3.29 from trigonometric parallaxes. A weighted mean of 12 methods gives $\langle \mu_0 \rangle = 3.16 \pm 0.03$; rejecting all values less than 3.15, van Altena proposed $\langle \mu_0 \rangle = 3.21 \pm 0.03$ as the "best present" estimate.

More recently Hanson (1975) found an even higher value 3.42 ± 0.20 from the convergent-point method and proposed $\langle \mu_0 \rangle = 3.29 \pm 0.08$ as the best mean of recent determinations (see also Heck 1976). However, a rediscussion of the Hanson (1975) convergent point solutions by McAlister (1977) shows that when stars with $m_v \leq 9.5$ are rejected, the distance modulus is decreased from 3.42 ± 0.20 to 3.18 ± 0.16. In rebuttal Hanson (private

TABLE 5

DISTANCE MODULUS OF HYADES

Source	Method	μ_0	m.e.
Hanson 1975	Convergent point, Lick data	3.42	0.20
McAlister 1977	Convergent point, Lick data corrected	3.18	0.16
Hanson 1977	Convergent point, Lick data revised	3.33	0.17
Corbin *et al.* 1975	Convergent point, meridian circles	3.19	(0.04)
Carpenter 1977	Convergent point, meridian circles	3.27	0.1:
Upgren 1974	Trigonometric parallaxes, van Vleck data (11*)	3.29	0.18
Klemola *et al.* 1975	Trigonometric parallaxes, Lick data (18*)	3.19	0.15
Klemola 1976	Trigonometric parallaxes, corrected	3.33	0.19
Anthony-Twarog and Demarque 1977	Stellar models	3.34	(0.05)
	Unweighted average	$\langle\mu_0\rangle = 3.28$	
van Altena 1974	Weighted mean of earlier data	3.16	0.03
	"Best estimate"	3.21	0.03
Hanson 1975	"Best estimate"	3.29	0.08
Hanson 1976	"Best estimate"	3.31	0.03
Adopted here		$\langle\mu_0\rangle = 3.29$	$\pm$ 0.05

communication) claims that this correction is excessive and that the distance modulus is decreased only to 3.33 $\pm$ 0.17 "when the proper correction is applied."

New solutions from meridian circles observations have been reported by Corbin, Smith and Carpenter (1975) as leading to 3.19 $\pm$ 0.04 which seemed to confirm the lower value; however, a more recent analysis of meridian-circle proper motions yields 3.27 $\pm$ 0.1: (Carpenter 1977). Effects of rotation and expansion of the cluster have also been considered as possible complications (M. S. Carpenter, private communication), but according to Hanson (1975) such effects have a negligible impact on the Hyades convergent problem.

Recent trigonometric parallaxes (Upgren 1974; Klemola *et al.* 1975, 1976) range from 3.19 to 3.33, while comparisons of stellar structure models with observation require $\mu_0 \geq 3.20$ as Faulkner (1967, 1971) first pointed out, with the latest fit indicating 3.34 $\pm$ 0.05 (Anthony-Twarog and Demarque 1977).

I am indebted to Dr. R. B. Hanson for most of these recent determinations collected in Table 4 and from which he derives an (unweighted) $\langle\mu_0\rangle = 3.31 \pm 0.03$ (m.e.). This would seem to settle the matter as far as the geometric modulus μ_0 is concerned. However, possible differences in chemical composition and line blanketing between the Hyades, an old cluster, and the more distant, younger clusters in which Cepheids are observed may complicate the matter. For example, van den Bergh (1977) has suggested that such differences might make "the [younger] cluster distance modulus too large by ~0.15 mag," which almost exactly cancels the van Altena correction (but not the Hanson correction) to the Hyades modulus. This admittedly speculative correction is not accepted by some authors, and one might even argue for an opposite correlation. Clearly, any estimate of the possible magnitude of such effects, if any, would be highly uncertain at the present time.

For an application to the zero point of the *P-L* relation (Appendix C) the mean modulus from Table 4, $\mu_0 = 3.29 \pm 0.05$, will be adopted, but the possibility of larger systematic errors in ZAMS fits to the more distant clusters including Cepheids must be kept in mind. Most authors assign probable errors of 0.1 to 0.2 mag to distance moduli so derived; however, Blaauw (1963) has estimated the total *probable* error in the distance modulus of h + χ Per to be as large as 0.23 mag relative to the Hyades and 0.29 mag relative to the Sco-Cen association. This is in agreement with Johnson *et al.* (1961), who quote differential *probable* errors of ~0.2 mag and total errors of ~0.3 mag for most clusters. A total mean error $\sigma_0'' = 0.4$ mag will be adopted here.

APPENDIX C

ZERO POINT OF PERIOD LUMINOSITY RELATION FROM HYADES MODULUS

The zero point of the *P-L* relation defined here as $\langle M_v\rangle(\log P_0 = 0.8)$ was derived by interpolation from the absolute magnitudes of 20 galactic Cepheids in 15 clusters. The basic data were tabulated by Sandage and Tammann (1969, Table 5; 1971, Table 2) for 13 stars in eight clusters and associations and by van den Bergh (1977, Table 1) for 14 stars in 12 clusters. Only seven stars are common to the two lists. The absolute magnitudes collected in Table 6 are based on the old Hyades modulus ($\mu_0 = 3.03$) (Appendix B), the first line from Sandage and Tamann corrected, second line from van den Bergh. The Sandage-Tammann data were corrected for the effects of the second-order terms of galactic extinction in the linear approximation

$$R = A_v/E(B-V) = R_0 + a(B-V)_0 + bE(B-V)\,, \tag{C1}$$

TABLE 6

CORRECTED MAGNITUDES AND COLORS OF CEPHEIDS AT MEAN LIGHT

Star	Cluster	$\delta_1 A_v$	$(m - M)_0^c$	$\delta_2 A_v$	δM_v^c	$-M_v^c$	$-M_v^c(P_0)$	log P	$-M_B^c$	$-M_B^c(P_0)$	Remarks
SU Cas	...	0.078	7.42	0.007	0.085	2.45	(3.87)	0.290	2.05	(3.31)	1
EV Scu	6664	0.140	10.89	0.016	0.156	2.46	3.33	0.490	1.83	1.83	
						2.62	3.52	...	2.05	2.80	
CE Cas b	7790	0.133	12.40	0.105	0.148	3.06	3.48	0.651	2.44	2.81	
						3.20	3.63	...	2.64	3.00	
CF Cas	...	0.133	12.40	0.017	0.150	2.92	3.24	0.687	2.21	2.49	
						3.08	3.41	...	2.42	2.69	
CE Cas a	...	0.133	12.40	0.016	0.149	3.13	3.38	0.711	2.43	2.65	
						3.28	3.54	...	2.64	2.85	
UY Per	$h + \chi$	0.251	11.65	0.028	0.279	3.26	3.46	0.730	2.58	2.76	1
CV Mon	...	...	...	...	...	...	...	0.731	...	...	
						3.35	3.55	...	2.71	2.87	2
VY Per	h + χ	0.275	11.62	0.030	0.305	3.61	3.77	0.743	2.96	3.09	1
CS Vel	Ru 79	...	...	...	...	...	...	0.771	...	...	
						3.05	3.13	...	2.39	2.46	2
V367 Sct	6649	...	...	...	...	...	...	0.799	...	...	
						3.82	3.82		3.33	3.33	2
U Sgr	M25	0.132	8.85	0.015	0.147	3.78	3.70	0.828	3.18	3.11	
						3.93	3.85		3.38	3.31	
DL Cas	129	0.119	11.16	0.016	0.135	3.70	3.41	0.903	2.94	2.68	
						3.84	3.54		3.14	2.89	
S Nor	6087	0.056	9.78	0.007	0.063	3.97	3.44	0.989	3.22	2.75	
						4.03	3.48		3.30	2.84	
TW Nor	Lyn 6	...	...	...	...	...	...	1.033	...	...	
						3.53	(2.85)	...	2.73	(2.17)	2
VX Per	h + χ	0.140	11.76	0.017	0.157	4.18	3.52	1.037	3.48	2.89	1
SZ Cas	h + χ	0.222	11.68	0.026	0.248	4.46	3.52	1.134	3.76	2.92	1
VY Car	Car OB1	...	...	...	...	...	...	1.277	...	...	
						4.97	3.59	...	4.05	2.90	2
T Mon	Mon OB2	...	...	...	...	...	...	1.432	...	...	
						5.50	3.67	...	4.54	3.01	2
RS Pup	P3	0.132	11.17	0.020	0.152	5.80	(3.51)	1.617	4.84	(2.80)	1
SV Vul	Vul OB1	...	...	...	...	...	...	1.653	...	...	
						6.00	3.53	...	4.94:	2.88:	2

REMARKS.—(1) Star rejected by van den Bergh 1977. (2) Star added by van den Bergh 1977.

with $R_0 = 3.1$, $a = 0.25$, $b = 0.05$ after Olson (1975) which is valid for $(B - V) < 1.4$ and $E(B - V) < 1.5$. Two corrections are needed:

1. In calculating the extinction correction for open clusters many authors, including Sandage and Tammann, assume that $R = 3 =$ const.; hence the derived distance moduli of the calibrating clusters are too large by $\delta_1 A_v = (R - 3)E$ and the inferred absolute magnitudes of cluster stars, including the Cepheids, require a correction $\delta_1 M_0 = +\delta_1 A_v$ ranging from $+0.05$ to $+0.27$ mag for the eight calibrating clusters; on the average $\langle \delta_1 A_v \rangle = +0.15$ for the 13 stars.[6]

2. In correcting the cluster Cepheids for galactic extinction, Sandage and Tammann assumed that $A_v =$ const. independent of color for all stars in the cluster, while from basic theory (King 1952) one may infer as a close enough approximation for the color range of interest that

$$A_v(B - V)_0 = A_v(0)[1 - 0.01(B - V)_0] , \tag{C2}$$

so that the absolute magnitudes $M_0(V)$ of Cepheids listed by Sandage and Tammann (1971) require a second correction

$$\delta_2 M_0 = \delta_2 A_v = 0.01[(B - V)_0 + 0.2]A_v \tag{C3}$$

ranging from 0.01 to 0.03 mag and averaging $\langle \delta_2 M_0 \rangle = 0.02$ mag for the 13 stars.

The total correction, then, is $\langle \delta M_v^0 \rangle = \langle \delta_1 A_v + \delta_2 A_v \rangle = +0.17$ mag. The corrected color excess follows from

$$E^c(B - V) = A_v^c/R = (A_v + \delta_1 A_v + \delta_2 A_v)/R , \tag{C4}$$

where R is calculated through equation (C1) for the first approximation value of $(B - V)_0$; the average correction

[6] This result, which assumes $\langle (B - V)_0 \rangle = -0.15$ for the intrinsic color of B stars used in estimating E, depends little on the adopted values of the coefficients in equation (C1); Blanco's (1956) early formulation gives $\langle \delta_1 A_v \rangle = +0.13$; the latest formulation by Straižys, Sūdžias, and Kurilienė (1976) gives $\langle \delta_1 A_v \rangle = +0.19$.

TABLE 7

ZERO POINT OF CEPHEID $P-L$ RELATION ($\log P_0 = 0.8$)

Method	$-\langle M_v(0.8)\rangle$	Notes
Open clusters (20 stars, 15 clusters).....	3.80	1
Secular parallaxes (33 stars)...........	3.34	2
Fernie's method (54 stars).............	3.63	3
Barnes-Evans relation (9 stars).........	3.65	4
Adopted mean.....................	3.60	

NOTES.—(1) Mean of Sandage and Tammann 1971, corrected and van den Bergh 1977, both reduced to μ_0(Hya) = 3.29. (2) Sandage and Tamman 1968 zero-point corrected after Jung 1970. (3) Zero point from Baade-Wesselink method and photometric theory by eqs. (2a) and (3). (4) Zero point from mean correction to Fernie-Hube 1968 distance scale based on eqs. (2b) and (2c).

to the Sandage-Tammann values is $\langle\delta(B-V)_0\rangle = +0.06$ mag. and the mean total corrections to the Sandage-Tammann zero points are $\langle\delta M_v^0\rangle = +0.17$ and $\langle\delta M_B^0\rangle = +0.23$ mag.

Then the magnitudes corrected to $\log P_0 = 0.8$ are $M_v^c(0.8) = M_v^c + 2.8\ (\log P - 0.8)$ and $M_B^c(0.8) = M_B^c + 2.5(\log P - 0.08)$, where the slopes are appropriate to the Sandage-Tammann data over the range of interest. The mean values for the 13 stars used by Sandage and Tammann are $\langle M_v^c(P_0)\rangle = -3.58$, $\langle M_B^c(P_0)\rangle = -2.84$; for 11 stars in the interval $0.45 < \log P < 1.15$, $\langle M_v^c(P_0)\rangle = -3.56$, $\langle M_B(P_0)\rangle = -2.79$. The 14 stars in van den Bergh's list reduced to $\log P_0 = 0.8$ (with the slopes 2.90 and 2.42 which are appropriate to this set) give -3.50 and -2.85 in close agreement.

The unweighted mean is $\langle M_v(0.8)\rangle = -3.54$, for μ_0(Hya) = 3.03, and $\langle M_v(0.8)\rangle = -3.80 \pm 0.12$ for μ_0(Hya) = 3.29 (Appendix B).

The internal mean error is calculated from the following components: (i) zero point, 0.05 in μ_0(Hya), (ii) zero point transfer, 0.10 in V (20 stars, 15 clusters, $\sigma_0'' = 0.4/\sqrt{15} = 0.10$) with the constant estimated in Appendix B, (iii) estimated uncertainty in $(B-V)_0$, 0.05. The external mean error should include possible effects of differential line blanketing as noted in Appendix B; assuming the latter to be of order ~ 0.10, the total mean error of the Cepheid zero point from open clusters will be taken as $\sigma_0 \approx 0.15$ mag.

The zero points of the *P-L* relation derived by several methods (see § II*b*) are collected in Table 7 giving an unweighted mean $\langle M_v(0.8)\rangle = -3.60 \pm 0.15$.

APPENDIX D

ABSOLUTE MAGNITUDES OF RR LYRAE VARIABLES

The mean absolute magnitude of the *ab*-type RR Lyrae variables at mean light is best determined from statistical parallaxes. For cluster variables, methods relying on main-sequence fits of the C-M diagrams of globular clusters to nearby dwarfs or subdwarfs are too heavily dependent on uncertain corrections for line blanketing and other effects, and in some cases are not independent of the Hyades modulus. For field stars, methods involving "moving group" parallaxes depend entirely on the reality of the group and on the membership of the variable, both of which are often uncertain.

Because globular cluster variables have mean periods close to 0.55 or 0.65 days (see, e.g., review papers in Philip 1972), solutions for periods $P \geq 0.42$ or 0.45 d are given special consideration in Table 8 to minimize effects of the *P-L-C* relation and exclude overtone variables (Breger and Bregman 1975).

Rejecting preliminary values (set 4) or doubtful statistical corrections (3a, 3b), the average for all periods ($\langle P\rangle \approx$ 0.45 d) from sets 2, 3, 5, and 6 is $\langle M_v\rangle = +0.70$; for the selected period interval ($P \geq 0.42$ d) the average from sets 1, 1*a*, 2*a*, 6*a* is $\langle M_v\rangle = +0.83$.

For comparison, Kukarkin (1974) adopts from a general review of all distance indicators for cluster variables, including effects of period P, metal abundance (m/H), and spectral type (Sp):

$$M_v(\mathrm{RR}) = -0.58 - 4.46 \log P\,,$$

$$M_v(\mathrm{RR}) = 1.32 + 0.38\,(\mathrm{m/H}) \quad [\text{with } (\mathrm{m/H})_\odot = 1.0]\,,$$

$$M_v(\mathrm{RR}) = 0.425 + 0.60 \quad (\mathrm{Sp} - \mathrm{FO})\,;$$

TABLE 8

MEAN MAGNITUDES OF *ab*-TYPE RR LYRAE VARIABLES FROM STATISTICAL PARALLAXES

Source	Data	$\langle M_v \rangle$	$\langle (B-V)_0 \rangle$	Set
1965 Woolley *et al.*	119 stars, $P \geq 0^d45$	0.52 ± 0.30*	0.24	1
	75 stars, $0^d45 \leq P \leq 0^d57$	0.84*		1a
1965 van Herk	164 stars, all P	0.91 ± 0.22†	0.19	2
	92 stars, $0^d42 < P < 0^d6$	0.99 ± 0.13†		2a
1977 Clube and Jones	59 stars, all P	0.88 ± 0.30		3
	same + statisical correction	(1.3)		3a
1972 Heck	same + statistical correction	(0.6)		3b
1972 Heck	102 stars, all P (preliminary)	0.50		4
1973 Heck	127 stars, all P (revised)	0.51 ± 0.20		5
1975 Hemenway	172 stars, all P	0.49 ± 0.42	(0.14)	6
	108 stars, $0^d42 < P < 0^d6$	0.82 ± 0.32		6a
1975 Oort and Plaut	27 stars, all P	...	0.21	
Mean all P		0.70		
Mean for $P > 0^d42$		0.83	0.20	

* Assuming $\langle V \rangle - \bar{m}_v = +0.1$, if $\bar{m} = (4\text{ Max} + 5\text{ min})/9$.

† From median $m_{pg} = (\text{Max} + \text{min})/2$, corrected to $B \approx m_{pg} + 0.1$, and $V = B - 0.2$ with $\langle M \rangle =$ median $+ 0.14$ (Oort and Plaut 1975); thus $\langle M_v \rangle = 0.87 + 0.14 + 0.1 - 0.2 = 0.91$ for set 2; $\langle M_v \rangle = 0.95 + 0.04 = 0.99$ for 2*a*.

and estimates the final mean errors at 0.1 mag. The values of $\langle M_v \rangle$ so calculated in 86 galactic globular clusters range from $+0.58$ to $+1.33$ ($\sigma \approx 0.15$ mag) with a general mean $\langle M_v \rangle(\text{RR}) = +0.86$.

The values adopted in the present paper are

$$\langle M_v \rangle(\text{RR}) = +0.8 \pm 0.15 \quad \text{and} \quad \langle (B-V)_0 \rangle = +0.20 \pm 0.05\,.$$

Addendum (*1977 June*).—The continuing debate and uncertainty in the zero point of RR Lyrae stars is illustrated by two recent statistical studies, both using the principle of maximum likelihood, but leading to widely different results. Heck and Lakaye (1977) derive from 123 stars ($P > 0.2$ d) a value $\langle M_v \rangle = 0.30 \pm 0.13$ with $\sigma_M = 0.57 \pm 0.09$; they confirm the metallicity-luminosity relation $M_v + (0.33 \pm 0.20) = (0.12 \pm 0.04)\Delta S$, but find little indication of a *P-L* relation. Clube and Dawe (1977) derive for the 59 *a*-type high-ΔS stars of Clube and Jones (1971) a value $\langle M_v \rangle = 1.0 \pm 0.2$ ($P = 0.56$ d, $\Delta \dot{S} = 7$); this result depends on a galactic extinction calculated from $\langle (B-V)_0 \rangle = 0.26$ and takes into account a correction to the proper motions of the reference stars. For $\Delta \dot{S} = 7$ the relation of Heck and Lakaye predicts $\langle M_v \rangle = 0.51 \pm 0.34$. Both values are within the range permitted by $\langle M_v \rangle = 0.80 \pm 0.15$ adopted here.

Addendum (*1977 December*).—In a further study of RR Lyrae *ab* stars Clube and Dawe (1978) have applied their maximum likelihood algorithm to 103 stars taken from the *RGO Bulletins* and to 130 stars supplied by Heck from the Liège files. The results are now in fair agreement; both sets suggest a mixture of two populations ($\Delta S \gtrless 3$); the high-velocity component ($\Delta S > 3$; $V_\odot \approx 180$ km s^{-1}) has $\langle M_v \rangle = +0.2$ to $+1.1$ from the RGO data, $+0.2$ to $+0.8$ from the Liège data. If the high ΔS stars ($\Delta S > 6$) are comparable to those in globular clusters and the Magellanic Clouds (as suggested by similarities in period-frequency distributions), then $\langle M_v \rangle \approx +1.0$ from the RGO data (54 stars) and $+0.7$ from the Liège data, both consistent with the adopted value.

The reader is referred to these two papers for detailed discussions of the possible sources of this persistent disagreement which confirms the wisdom, indeed the necessity, of using several *independent* indicators to form the basis of a reliable extragalactic distance scale.

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