THE ASTROPHYSICAL JOURNAL, 221:635–644, 1978 April 15

THE RADII AND TEMPERATURES OF CLASSICAL CEPHEIDS

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Received 1977 June 14; accepted 1977 October 18

ABSTRACT

Radius and temperature determinations of classical Cepheids have been assembled from the literature and plotted along with theoretical relationships in (log P, log R)- and (log P, log T_e)-diagrams. Different methods of radius determination give significantly different results. At shorter periods, the slope in the (log P, log R)-diagram is probably either 0.7 or 1.0. Arguments are presented for both interpretations, but the former value seems the more likely. In either case there are probably substantial systematic errors in many Wesselink determinations. From the temperatures it is seen that, for periods less than 12 days, the observed and theoretical blue edges agree quite well, but for longer periods the observed blue edge is cooler and has a steeper slope. This discrepancy (also observed in the Small Magellanic Cloud) cannot be removed by using any of the alternative temperature calibrations in the literature. Beat Cepheids are not confined to either the red or the blue edge of the instability strip.

Subject headings: stars: Cepheids — stars: pulsation

I. INTRODUCTION

Because the phenomenon of stellar pulsation is fairly well understood, classical Cepheids have often been used as a means of testing various aspects of our theoretical understanding of stellar structure. A particular example of this is the comparison of masses derived from luminosities and stellar evolution theory with those derived from luminosities, temperatures, and stellar pulsation theory.

Recently a large number of new data on Cepheids have been published, especially new reddenings and radius determinations. Therefore it is appropriate to reexamine the question of how well the relevant theories account for the observed properties of these stars. Most observational results essentially determine one or more of three basic properties: the pulsation period, the effective temperature, and the radius. It is thus appropriate to examine these quantities and not the mass, which is never observed directly, or the luminosity, which is not often obtainable. The results summarized in the following sections are displayed in the period-radius or period-temperature plane. This representation has distinct advantages over the commonly used temperature-luminosity or mass-luminosity plane in that the plotted points represent, as directly as possible, the actual observed quantities. (Furthermore, the pulsation periods are almost always known to high precision, so that errors in the data are confined to the vertical axis alone.)

This paper will consider five different types of radius determinations: (1) surface-brightness measurements combined with radial velocities; (2) Wesselink radii; (3) radii obtained from the ratio of the fundamental and first-overtone periods of double-mode Cepheids ("beats"); (4) radii determined from the phase of a secondary bump in radial-velocity curves; and (5) radii derived from luminosities and temperatures of stars whose distances are known. The temperatures are derived from mean $B - V$ colors. None of the data presented here are derived by using a period-color or period-luminosity relationship of any type.

Sections II and III present observed radii and temperatures with comments on the sources of the data. Section IV gives a derivation of period-radius and period-temperature relationships based upon theoretical calculations of stellar pulsation and evolution. Section V discusses the properties of Cepheids in the light of the results of the earlier sections.

II. RADIUS DETERMINATIONS

In Table 1 are presented all the radius determinations which I have been able to locate. They are discussed in the following subsections. For each set of data the radius has been plotted as a function of the period. The solid line in each figure represents the center of the theoretical instability strip, as derived in § IV. In some cases a dashed line is also plotted which shows the position of overtone pulsators.

a) *Surface-Brightness Measurements* (Fig. 1)

Barnes *et al.* (1977) have determined the variation of angular diameter with phase for nine Cepheids by using an empirical relationship between surface brightness in the V magnitude wavelength region and the $V - R$ color index. This was then combined with the radial-velocity curve to determine the actual variation of radius with phase. Their paper compares their procedure with the Wesselink method.

b) *Wesselink Radii* (Fig. 2)

Wesselink radii have been determined with various degrees of refinement by many people. The uncertainties and sources of error have been discussed by Evans (1976), Karp (1975*b*), and Parsons (1972), among others. Most of the data used here are taken from Evans (1976), Woolley and Carter (1973), Fernie (1968), or Thompson (1975)—in that order of preference. Other determinations are from Milone (1971) for SU Cassiopeiae, Svolopoulos (1960) for DT Cygni, and Parsons (1972) for SU Cygni. Typical uncertainties quoted for these results are ± 0.04 for log *R*. In order to obtain as homogeneous a set of radii as possible, I have relied heavily upon the determinations of Evans and of Woolley and Carter. However, in all cases where any two of the above authors have more than two stars in common, the systematic difference in their radii is no larger than 0.04 in log *R*. The largest individual difference is 0.20 between the values of Thompson and of Fernie for *l* Car. (The value of Woolley and Carter, used here, lies halfway between these extremes.)

c) *Double-Mode Cepheids* (Fig. 3)

There is now considerable literature on the nature of double-mode Cepheids (see Stobie 1977). Radii and masses can be computed, since the fundamental and first-overtone periods differ in their dependence upon mass and radius. The radii presented here are taken from Stobie (1977) and are derived from the simplest theoretical determination of the period dependence. Other, more complex, models give different results (cf. Cogan 1977; Cox *et al.* 1977; Faulkner 1977). The variable TU Cassiopeiae is a special case. Faulkner (1977) has shown that the second overtone is also present in the light curve. From the fundamental and

TABLE 1
RADII OF CEPHEIDS

Star	log P	log R: Surface Brightness	log R: Wesselink	log R: Cluster	log R: *Beats* or Bumps
SU Cas	0.290		1.28	1.38	
TU Cas	0.330				{*1.15*, *1.35*}
DT Cyg	0.398		1.34		
U TrA	0.410				*1.23*
VX Pup	0.479				*1.29*
RT Mus	0.489		1.43		
EV Sct	0.490			{1.39, 1.53}	
AP Vel	0.495				*1.29*
SZ Tau	0.498			1.58	
BK Cen	0.502				*1.31*
Y Car	0.561				*1.36*
AX Vel	0.565				*1.36*
UX Car	0.566		1.57		
RT Aur	0.571	1.39	1.45		
AG Cru	0.584		1.57		
SU Cyg	0.585		1.61		
BF Oph	0.609		1.76		
BQ Ser	0.630				*1.43*
V Vel	0.641		1.71		
T Vul	0.647	1.58	1.72		
CE Cas b	0.651			1.63	
V482 Sco	0.656		1.69		
T Vel	0.666		1.71		
S Cru	0.671		1.69		
CF Cas	0.688			1.66	
V381 Cen	0.706		1.61		
CE Cas a	0.711			1.69	
V386 Cyg	0.721				1.53
UY Per	0.730			1.55	
δ Cep	0.730	1.61	1.76		1.58
V Cen	0.740		1.58		
VY Per	0.743			1.62	
R Cru	0.765		1.56		
CS Vel	0.771			1.61	
RV Sco	0.783		1.90		
V367 Sct	0.799			1.70	*1.59*
AW Per	0.810		1.85		
AT Pup	0.824				1.66
T Cru	0.828		1.72		
U Sgr	0.829	1.59	1.79	1.75	1.65

TABLE 1—*Continued*

Star	log P	log R: Surface Brightness	log R: Wesselink	log R: Cluster	log R: *Beats* or Bumps
V636 Sco	0.832		1.78		
BG Vel	0.840				1.63
η Aql	0.856	1.70	1.84		1.67
R Mus	0.876				1.67
W Sgr	0.881		1.83		1.69
VY Cyg	0.895				1.69
RX Cam	0.898				1.63
W Gem	0.898	1.68	1.81		
U Vul	0.903		1.78		1.70
DL Cas	0.903			1.82	
S Sge	0.923		1.78		1.72
V500 Sco	0.969				1.73
S Nor	0.989		1.79	1.83	
β Dor	0.993		1.90		1.74
ζ Gem	1.007	1.84	1.83		
TW Nor	1.033			1.80	
VX Per	1.037			1.73	
RX Aur	1.065		1.77		1.85
SZ Cas	1.134			1.81	1.90
TT Aql	1.138		1.86		
TX Cyg	1.168				1.99
X Cyg	1.214	2.00	2.00		
CD Cyg	1.232		1.96		
Y Oph	1.234		1.90		
VY Car	1.278			2.13	
WZ Sgr	1.339			2.21	
T Mon	1.432	2.26	2.16		
AQ Pup	1.475		2.09	2.16	
ℓ Car	1.551		2.17		
U Car	1.588		2.34		
RS Pup	1.617		2.38	2.29	
SV Vul	1.654		2.22		
S Vul	1.825			2.59	

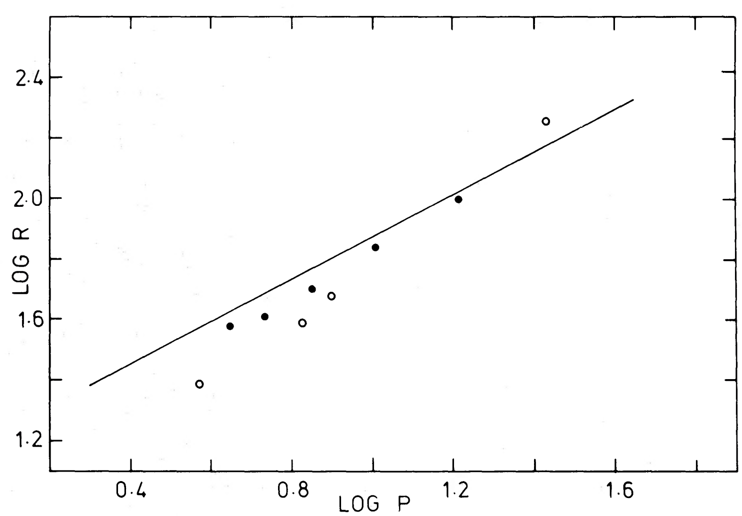

FIG. 1.—Radii of Cepheids derived by the surface-brightness method versus period. *Open circles*, stars with photometric companions. *Line*, theoretical instability strip.

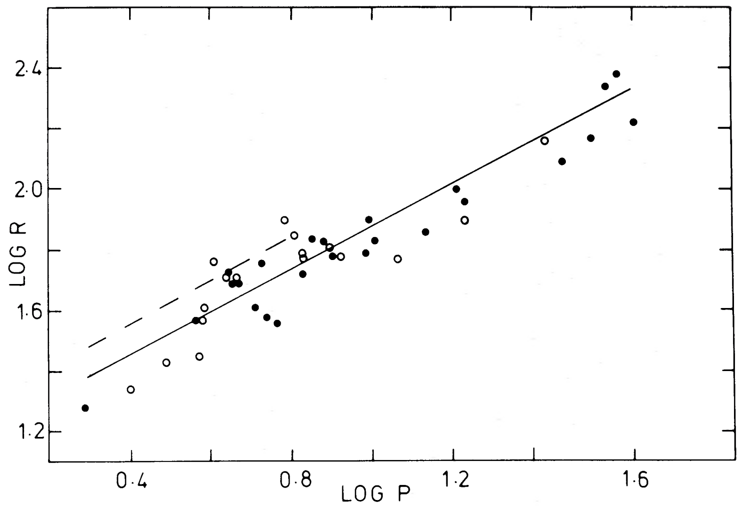

FIG. 2.—Radii of Cepheids derived by the Wesselink method versus period. *Open circles*, stars with photometric companions. *Solid line*, theoretical instability strip. *Dashed line*, first overtone.

first overtone, he derives log $R = 1.15$, and from the fundamental and second overtone, log $R = 1.35$. Both values are plotted in Figure 3.

d) *Secondary Bumps in Radial-Velocity Curves* (Fig. 3)

Christy (1968) discovered that in his nonlinear models the phase difference between the primary and secondary minima of radial-velocity curves (for periods between about 5 and 10 days) was determined by the radius of the model. A more extensive investigation (Christy 1975) showed that the phase difference, $\Delta\phi$, was given by

$$\Delta\phi = 0.24R/P \quad (1)$$

for a wide range of models. (Fricke, Stobie, and Strittmatter 1972 derived a very similar result from the models of Stobie 1969.) Using this formula and Christy's (1975) definitions of the phases, I have derived radii from the radial-velocity curves of Stibbs (1965) and Joy (1937), and also Herbig and Moore (1952), for S Sagittae; Sanford (1951) for U Vulpeculae; Shane (1958) for δ Cephei; Schwarzschild, Schwarzschild, and Adams (1948) for η Aquilae; and Struve (1945) for V386 Cygni, VY Cygni, and TX Cygni. These data include most of the stars examined by Fricke *et al.* but also include a number of others. For three stars on their list (S Muscae, S Normae, and ζ Geminorum), the bump in the light curve did not

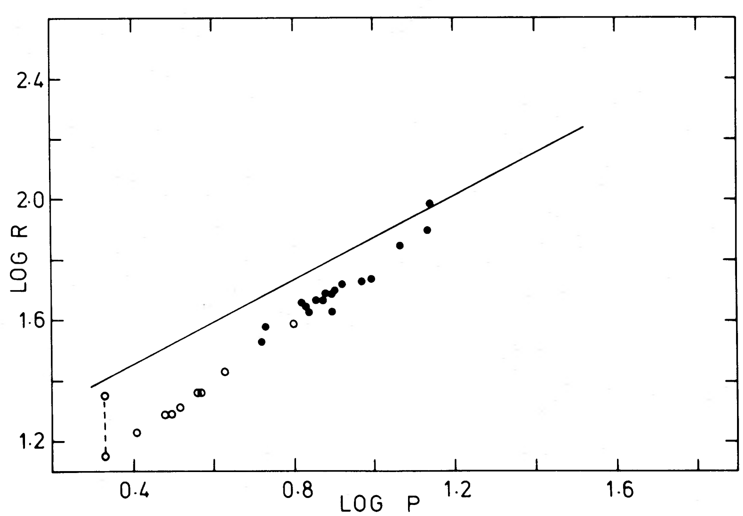

FIG. 3.—Radii of double-mode Cepheids (*open circles*) derived from period ratios and radii of Cepheids derived from secondary bumps on their radial-velocity curves (*filled circles*) versus period. *Line*, theoretical instability strip.

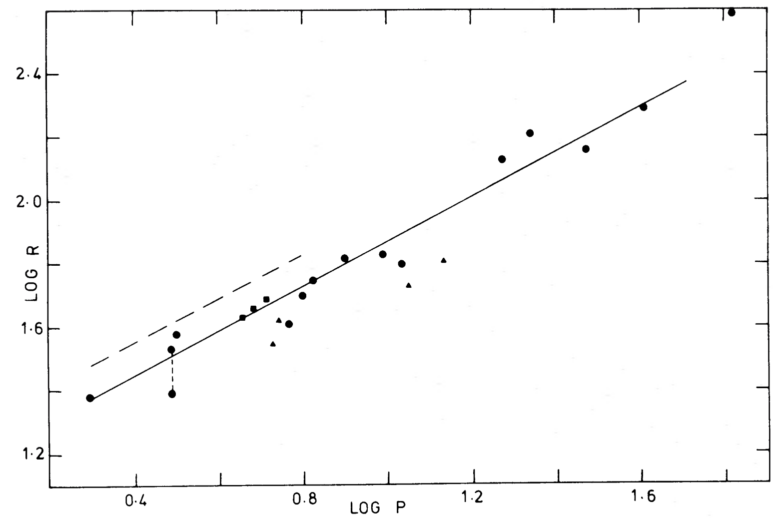

FIG. 4.—Radii of Cepheids derived from luminosities and temperatures versus period. Two points connected by a dashed line are results for EV Sct using the two different moduli in Table 2. *Squares*, members of NGC 7790. *Triangles*, members of Per OB1. *Solid line*, theoretical instability strip. *Dashed line*, first overtone.

TABLE 2

DISTANCE MODULI OF CEPHEIDS

Star	Membership	Distance Modulus Publ.	Distance Modulus Adj.	Reference
VY Car	Car OB1	11.57	11.84	Turner (1977)
SU Cas	-	7.5	7.8	Racine (1968)
SZ Cas UY Per VX Per VY Per	Per OB1	11.4	11.3	Crawford *et al.* (1970)
CE Cas a CE Cas b CF Cas	NGC 7790	12.50	12.76	Becker and Fenkart (1971)
DL Cas	NGC 129	11.20	11.46	Becker and Fenkart (1971)
S Nor	NGC 6087	9.57	9.83	Becker and Fenkart (1971)
TW Nor	Lyngå 6	10.8	11.5	van den Bergh and Harris (1976)
RS Pup	-	11.25	11.25	Havlen (1972)
AQ Pup	Pup OB1	12.28	12.54	Becker and Fenkart (1971)
U Sgr	M 25	8.90	9.16	Becker and Fenkart (1971)
WZ Sgr	Sgr OB4	11.7	12.0	Klare (1967)
EV Sct	NGC 6664	10.33 11.03	10.59 11.29	Becker and Fenkart (1971) Kraft (1961)
V367 Sct	NGC 6649	11.15	11.55	Madore and van den Bergh (1975)
SZ Tau	NGC 1647	8.70	8.96	Becker and Fenkart (1971)
CS Vel	Ruprecht 79	12.25	12.70	Harris and van den Bergh (1976)
S Vul	Vul OB2	13.12	13.38	Tsarevskii (1971)

seem well enough defined to permit determination of the radius.

e) *Cepheids at Known Distances* (Fig. 4)

The most direct means of determining the radius of a star is from its luminosity and effective temperature. This requires knowing the distance, and for only a few Cepheids (mostly in galactic clusters or associations) are the distances known. Cepheids for which an independent distance modulus has been determined are listed in Table 2. Membership for many of these is well established, but for some, especially those in associations, the membership is less certain. The table also gives the distance modulus taken from the cited references and an adjusted modulus which I have used. In making these adjustments I have considered two factors.

1. All distance moduli dependent upon the Hyades (all but RS Puppis and Per OB1) have been increased to fit Hanson's (1975) distance modulus of the Hyades of 3.29.

2. In instances where true moduli were derived from apparent moduli by using values of A/E_{B-V} other than 3.0 for early-type main-sequence stars, the true modulus has been recomputed by using this value.

The purpose of these adjustments is not to derive what might be claimed as definitive values, but rather to arrive at as consistent a set of distances as possible, without attempting an exhaustive review of the stellar distance scale.

Although radii determined in this manner depend very little upon the pulsating behavior of the stars, the variability must be considered in deciding how to define a mean color index from which the temperature can be derived. I have used the intensity mean $\langle B\rangle - \langle V\rangle$, since the theoretical models of Karp (1975*a*) and of Cox and Davis (1975) indicate that this mean is closest to the color index of the equilibrium model. When possible, intrinsic colors have been derived from the references cited in § III. I have also used data from Stobie (1977) for V367 Scuti; Harris and van den Bergh (1976) for CS Velorum; and Fernie (1970) for S Vulpeculae. Temperatures have been computed from Flower's (1977) temperature scale.

Several stars require special comments. The distance modulus of RS Puppis is based solely upon Havlen's (1972) study of the surrounding nebula, since Eggen (1977) has shown that the apparent association Pup OB3 is not an actual physical grouping. Two distance moduli for NGC 6664 have been given, since they differ so greatly. Several stars which have had distance moduli derived by other authors have been excluded. The distance of Polaris derived by Fernie (1965) requires the observed color of the secondary to be corrected for the scattered light of the variable. The resulting distance is very sensitive to the assumed reddening. Eggen (1977) has shown that the stars thought to be associated with l Carinae do not have similar distances, so this variable was also excluded. A distance to δ Cephei has been obtained by Vitrichenko *et al.* (1974) on the basis of δ Cephei's being a physical companion. However, this is not certain, and, since whether it is included or not makes no appreciable difference to the overall distribution, I have not used it.

The distances of the stars in Per OB1 are derived from β-photometry of h and χ Persei and are not tied to the Hyades. Furthermore, Crawford (1973) states that the correction for interstellar absorbtion which he uses is equivalent to $R = 3.2$. This accounts for the smaller adjusted distance modulus in Table 2. The resulting radii in Figure 4 are surprisingly small: it would be necessary to increase the distance modulus of Per OB1 by about 0.6 mag to bring them in line with the other stars. This may be evidence that they are not members of the association.

III. TEMPERATURE DETERMINATIONS

Although mean colors are available for many Cepheids, reliable reddenings have been harder to obtain. This situation has recently been improved by the work of Parsons and Bell (1975), Pel (1977), and Dean, Warren, and Cousins (1977). Reasons for preferring these photometric reddenings over the older spectroscopic ones have been discussed by Dean *et al.* and by Canavaggia, Mianes, and Rousseau (1975). I have derived intrinsic colors by using the reddenings of Dean *et al.*, which are mean values for all photometric reddening determinations, and the $\langle B\rangle - \langle V\rangle$ mean colors of Schaltenbrand and Tammann (1971). Stars which Pel (1977) or Janot-Pacheco (1976) considers to have photometric companions have been excluded. Temperatures have been obtained from the intrinsic colors by using the calibration of Flower (1977), who has reviewed the problem of temperature calibration of $B - V$ colors. For $\log T_e < 3.720$ he uses a linear fit,

$$\log T_e = 3.859 - 0.175\,(B - V)\,, \qquad (2)$$

to the results of van Paradijs (1973). For bluer colors his scale gives temperatures higher than equation (2). The derived temperatures are plotted against period in Figure 5.

Temperatures have also been computed for Cepheids in the Small Magellanic Cloud, using a reddening of 0.03. Data are from Eggen (1977), Madore (1975), and Gascoigne (1969). Temperatures have been computed from Bell and Parson's (1972) calibration,

$$\log T_e = 3.887 - 0.222\,(\langle B\rangle - \langle V\rangle)\,, \qquad (3)$$

which represents results of model atmospheres for which the metal abundance, [*A*/H], is −0.6. These results are plotted in Figure 6.

IV. THEORETICAL RELATIONSHIPS

To compare the data with theoretical calculations of stellar pulsation and evolution, one can derive a theoretical period-radius relationship for the center of the instability strip and a period-temperature

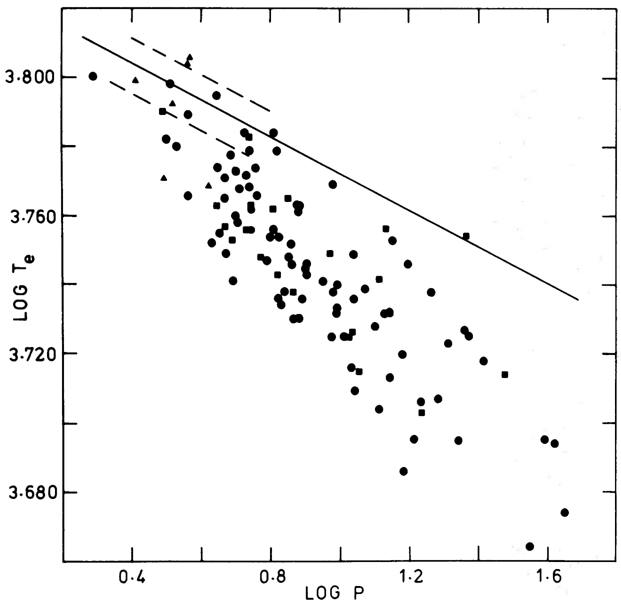

FIG. 5.—Effective temperatures of Cepheids derived from $\langle B\rangle - \langle V\rangle$ intensity means versus period. *Triangles*, double-mode Cepheids. *Squares*, stars for which $E > 0.5$ mag. *Solid line*, theoretical blue edge. Dashed lines show theoretical region of first-overtone instability.

relationship for the blue edge. Results of pulsation calculations can be represented approximately by

$$\log P = \log Q_0 + 1.5 \log R - 0.5 \log M - A \log (M/R) \quad (4)$$

for the pulsation period in the fundamental mode,

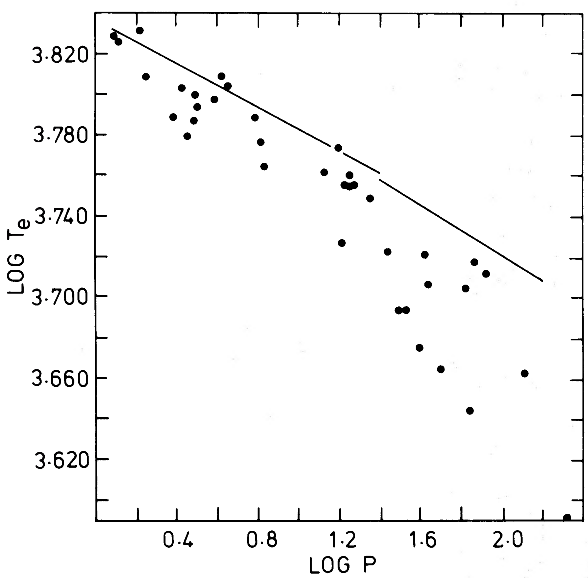

FIG. 6.—Effective temperatures of Cepheids in the Small Magellanic Cloud derived from $\langle B\rangle - \langle V\rangle$ intensity means versus period. *Lines*, theoretical blue edge.

and

$$\log T_e = b - a \log L + \log T_{e\odot} \quad (5)$$

for the fundamental blue edge in the temperature-luminosity plane. Stellar evolution calculations provide a mass-luminosity relationship,

$$\log M = \gamma + \beta \log L \,. \quad (6)$$

Good fits to Iben and Tuggle's (1975) more detailed form of these relationships can be obtained by adopting the following coefficients (for a composition of $X = 0.70$, $Y = 0.28$, $Z = 0.02$):

$$\log Q_0 = -1.635\,,$$
$$A = 0.25\,,$$
$$\gamma = -0.10\,,$$
$$\beta = 0.256\,,$$
$$a = 0.045\,,$$
$$b = 0.183\,.$$

Combining equations (4), (5), and (6) gives an equation of the form

$$\log R = z + s \log P \quad (7)$$

for the blue edge, where

$$s = \frac{1}{3/2 + A - (1/2 + A)2\beta/(1 + 4a)} \quad (8)$$

and

$$z = s\{(1/2 + A)[\gamma + 4b\beta/(1 + 4a)] - \log Q_0\}\,. \quad (9)$$

To transform this into an equation for the center of the instability strip, we must determine the width of the strip in the period-radius plane. Inspection of Figure 5 reveals that the observed width is about 0.04 in log T_e at a given period. We need to compute $(\partial \log R/\partial \log T_e)_P$ to obtain the width in the period-radius plane. This is the same quantity as $\partial z/\partial b$ from equation (9), and its numerical value is 0.43. Thus, if we replace b by $b - 0.02$ in equation (9), the constant in equation (7) will be decreased by 0.1, and the resulting equation,

$$\log R = 1.17 + 0.70 \log P\,, \quad (10)$$

represents the center of the instability strip. Since the radius depends only very weakly upon the temperature at a given period, knowing the exact width of the strip is not very important. In fact, even the position of the strip (the value of b) is not critical: the theoretical period-radius relationship is determined primarily by the theoretical mass-luminosity relationship. (For example, if the slope β is set equal to zero, then one obtains $s = 0.57$.)

A similar procedure may be carried out to obtain a theoretical period-temperature relationship for the blue edge,

$$\log T_e = \zeta + \sigma \log P . \qquad (11)$$

The result is

$$\sigma = \frac{-a}{(3/2 + A)(2a + 1/2) - (1/2 + A)\beta} \qquad (12)$$

and

$$\zeta = -\sigma[\log Q_0 + (3/2 + A)b/2a - (1/2 + A)(\gamma + \beta b/a)] + \log T_{e\odot} . \qquad (13)$$

With the same numerical values as before, one obtains

$$\log T_e = 3.826 - 0.054 \log P . \qquad (14)$$

The slope σ is primarily a function of a, the slope of the blue edge in the temperature-luminosity plane, but it also depends upon β. (If $\beta = 0$, then $\sigma = -0.44$.)

In these diagrams the region of first-overtone instability can also be obtained. From Iben and Tuggle's (1975) results, one finds $\log P_{\rm H} \approx \log P_F - 0.13$, and the boundary of the first-overtone blue edge is given by $b_{\rm H} = b_F + 0.015$. (These approximate relationships are applicable only to shorter periods [$\log P_F \lesssim 0.8$], since the first-overtone blue edge bends over toward lower temperatures, and the ratio of fundamental to first-overtone period is not strictly constant.) In the period-radius plane, the center of the first-overtone region will be displaced, at a given observed period, by

$$\Delta \log R = 0.13s + 0.01 + \frac{0.015}{2}\left(\frac{\partial \log R}{\partial \log T_e}\right)_P = 0.10 . \qquad (15)$$

In the period-temperature plane, the first-overtone blue edge is shifted by

$$\Delta \log T_e = 0.13\sigma + 0.015\frac{\partial \zeta}{\partial b} = 0.007 . \qquad (16)$$

The low-temperature limit of first-overtone instability (i.e., the fundamental blue edge) will also be shifted, owing to the change in period, by

$$\Delta \log T_e = 0.13\sigma = -0.009 .$$

Thus the region of first-overtone instability overlaps that of fundamental instability in the period-temperature plane.

The equations derived in this section have been plotted in Figures 1–6. Formulae containing quadratic terms exist as refinements to equations (4) and (5). However, the differences that including these terms would make would be almost undetectable in the figures. The numerical values used for the lines in the figures are the same as those given in this section, with the exception of Figure 6. For that figure, b has been increased by 0.01 and the mass-luminosity relationships of Cogan (1976) have been used, both to account for the heavy-element abundance of $Z = 0.005$ in the Small Magellanic Cloud.

V. INTERPRETATION OF OBSERVED RADII AND TEMPERATURES

a) *Radii*

Inspection of Figures 1–4 shows (as others have repeatedly pointed out) that there are substantial disagreements among the various radius determinations, especially for periods less than about 12 days. For longer periods the agreement is fairly good, except that the Wesselink radii seem to be somewhat smaller than the others.

At shorter periods the data suggest two possible interpretations: (1) the period-radius relationship has a slope of about 0.7, characteristic of the cluster Cepheids; or (2) the slope is near 1.0, as given by the beat Cepheids and the bump radii. Unfortunately, the other results do not clearly support either possibility. The surface-brightness radii support the second interpretation, primarily on the strength of the radius of RT Aurigae. But if one treats the supposed binary members with suspicion, then the remaining data support the first interpretation.

The evidence of the Wesselink radii is more confusing. The upper envelope of the distribution has a hump between 4 and 9 days, while the lower envelope is nearly parallel to the theoretical line. Since the hump is not evident in the other data, it is most likely due to some peculiarity of the Wesselink method. (This peculiarity has led some authors—e.g., Parsons 1972 and Evans 1976—to derive a slope of $s \approx 0.5$. In view of the overall distribution of points in Figure 2, it seems doubtful that this result is very meaningful. Schmidt 1971 also has concluded that some Wesselink radii may be too large.) If one considers only those stars with periods less than 9 days, one obtains a slope similar to that of the beat Cepheids, although the actual values of $\log R$ are larger by about 0.15. But if one assumes that the radii between 4 and 9 days are in error, then the few remaining short-period stars fit the first interpretation fairly well, lying on the extension of the long-period distribution. Thus, depending upon how one wishes to interpret the peculiar distribution of Wesselink radii, they can be used to support either interpretation.

After this paper was originally submitted for publication, Dr. T. G. Barnes brought to my attention a paper by Balona (1977) in which he determines radii of about 50 Cepheids by using a refinement of the Wesselink method. His period-radius diagram differs from Figure 2 in that the hump is not obvious, and there is less evidence for a steep slope at short periods.

If the second interpretation is the correct one, then some explanation for the cluster Cepheids must be found. Errors in the temperature alone can be excluded, since the required correction would be very large. The alternative is a systematic error in the distance modulus of the shorter-period, but *not* the longer-period, Cepheids. Furthermore, accepting the second interpretation requires the mass-luminosity relationship to have a slope of $\beta = 0.6$ for masses less than about 7 $M_\odot$.

Adopting the first interpretation requires an explanation for the beat Cepheids and the bump radii. Although there is apparently no problem with the latter, Faulkner (1977) has discussed some difficulties associated with the beat radii—especially the disagreement of the two determinations for TU Cassiopeiae. In addition, the derived slope, $s = 1.0$, is suspicious. One can show that if one uses Q-values which depend only upon the ratio M/R, this slope is a necessary consequence of the fact that they all have the same period ratio. It seems fortuitous that their evolutionary history would result in just this slope. The alternative is that one or both of the periods do not depend primarily upon M/R. However, a satisfactory explanation of the beat phenomenon must also account for the fact that the double-mode Cepheids are not confined to either the high- or low-temperature side of the instability strip, as can be seen in Figure 5.

Although the choice between the two interpretations described at the beginning of this section is not obvious, I believe that the weight of the evidence supports the first. The choice is essentially a negative one: it seems easier to believe that there are systematic errors of the required magnitude in the beat and bump radii than in the cluster and theoretical ones. Neither the Wesselink nor surface-brightness determinations seem to strongly favor either interpretation.

b) *Temperatures and the Blue Edge*

The general features of the blue edges in the period-temperature diagrams for the Galaxy and the Small Magellanic Cloud may be summarized as follows:

1. For periods of less than about 12 days ($\log P = 1.1$), the observed edge is parallel to the theoretical one, but at a higher temperature, the difference in $\log T_e$ being less than 0.005.
2. For periods greater than 12 days the observed blue edge has a slope considerably steeper than the theoretical one.
3. At both the longer and the shorter periods, the slopes of the blue edges in the two diagrams appear to be the same.

Iben and Tuggle (1972) have found that the slope of the theoretical blue edge is independent of heavy-element abundance, which is in agreement with feature (3) above. However, this observed agreement in the slopes of the blue edges is dependent upon the use of different color-temperature relations for the Galaxy and the Small Cloud, which has a lower Z. Thus the result of Bell and Parsons (1972) that the slope of the color-temperature relation varies with Z is necessary if the slopes of the blue edges in the two systems are to agree. This is in contrast to the suggestion by Iben and Tuggle (1975) that the slope of the color-temperature relations in the two systems should be the same.

The small apparent displacement of the theoretical edge from the observed one (for $\log P < 1.1$) might be explained in several ways. One possibility is that the hottest stars are first-overtone pulsators, although the absence of first-overtone stars in the period-radius diagram makes this doubtful. A more likely explanation lies in the uncertainties in the derived intrinsic colors. The tabulation of Dean *et al.* suggests that the reddenings are not known to better than 0.03, which would give an uncertainty in the temperature of 0.005. Thus the scatter of the data from this alone could extend the blue edge in Figure 5 by this amount. (In the Small Cloud this effect would be less important, since the reddening is very low.)

The observed blue edge for longer-period Cepheids presents a much more serious problem. One possible explanation is that the temperature scale is no longer valid at these longer periods. This possibility may be examined by determining the slope of the color-temperature calibration required to make the observed blue edge parallel to the theoretical one. Using the data in Figure 5, one can derive a required slope of about -0.10, compared with the value of -0.175 in equation (2). Furthermore, the alternative scales discussed by Flower (1977) have steeper slopes than the one he adopts, and would make the discrepancy even worse. From this alone it appears very unlikely that the discrepancy between the observed blue edge and the theoretical one in the Galaxy (or, by inference, in the Small Cloud) can be removed by a change in the temperature scale.

If the problem of the longer-period Cepheids does not lie with the color-temperature relationship, then it may lie in the theoretical interpretation of the data. The most obvious cause would be simply that the stability calculations for these high-luminosity stars contain some systematic error which results in the extension of the region of instability too far to the blue. Another possibility is that the instability strip becomes quite wide, because the red edge curves downward, and the few stars in the strip do not accurately define the edges at all.

VI. GENERAL CONCLUSIONS

The diagrams presented in this paper give radii or temperatures of more than 100 Cepheids. The radius data clearly show that various methods give contradictory results, especially at periods shorter than ~ 9 days. Two possible interpretations of the data are that the correct slope in the period-radius plane is 0.7 or that it is 1.0. While there are arguments favoring both interpretations, I believe that the former is preferable. In either case fairly substantial corrections to many of the Wesselink radii appear to be necessary to obtain agreement with the other results.

The period-temperature data show good agreement for periods less than 12 days. The longer-period stars exhibit a divergence from the theoretical blue edge toward lower temperatures which cannot be resolved by adopting a different color-temperature relationship. Finally, the beat Cepheids are not confined to either the high- or the low-temperature side of the instability strip.

It is a pleasure to thank a number of colleagues at Mount Stromlo, especially Don Faulkner, Alex Rodgers, and Peter Wood, for helpful comments and discussions during the preparation of this paper. The final version of this paper has benefited from the critical comments of Dr. E. G. Schmidt. The discussion in § V*a* is a result of his insistence that the second interpretation could not be easily dismissed.

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Note added in proof.—Gieren (*Astr. Ap.*, **47**, 211, 1976), using new material, has derived a Wesselink radius of SU Cassiopeiae of log $R = 1.46 \pm 0.04$. This supports the conclusion reached at the end of § V*a*.

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