

Type I supernovae and the value of the Hubble constant

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Summary. An empirical model of the envelopes of Type I supernovae is used to determine the value of H_0 . An analysis of the composite light and colour curves establishes that near maximum brightness the radius of the photosphere increases linearly with time, over an interval of at least 20 days. This implies a strong density gradient at the photosphere. Calculated profiles of spectral lines formed by scattering in strong density gradients are used to determine the velocity of the photosphere. The derived value of the absolute magnitude at maximum brightness, $M_B = -20^m.25 \pm 0^m.31$, combined with the redshift–magnitude relation for Type I supernovae in distant galaxies, gives a value for H_0 and an internal rms error of

$$H_0 = 49 \pm 9 \text{ km/(s Mpc)}.$$

The most serious possibility of external error is associated with the redshift–magnitude relation, and is such that the best value of H_0 would fall in the range 38–49 km/(s Mpc).

1 Introduction

In this paper a simple empirical model of the envelopes of Type I supernovae is used to determine the value of the Hubble constant, H_0 . The method is basically similar to the one applied by Branch & Patchett (1973; hereafter Paper I), as a test of the thermal interpretation of the light emitted by Type I supernovae. The thermal interpretation, at least for the very peak of the light curve, now seems well established, and the supernova method now gives a value of H_0 to an accuracy which is at least comparable to that of the more traditional methods.

The model is based on composite data on Type I supernovae, as in Paper I, for two reasons. (1) At present this is the only way to obtain a global value of H_0 from supernovae. There is not enough data available on Type I supernovae in galaxies well outside the local supercluster to obtain a good value of H_0 from individual objects, and there is hardly any data available on remote Type II's. (2) If Type I supernovae are sufficiently homogeneous it may be preferable to use composite data in the expectation that random errors, uncertain reddening estimates etc, will tend to cancel. The striking spectroscopic and photometric

homogeneity of Type I supernovae (with a few conspicuous exceptions such as 1954a in NGC 4214 and 19621 in NGC 1073) is well known. The dispersion in their absolute magnitudes at maximum brightness, inferred from supernovae in remote galaxies, is only about 0.3 mag. The larger dispersions sometimes quoted may be spurious because of errors in the adopted relative distances to the nearer galaxies (Branch 1974).

2 The absolute magnitude at maximum brightness

The absolute blue magnitude and the $B-V$ colour index of an expanding, cooling photosphere of well defined radius and temperature are given by

$$M_B(t) = C_1 - 5 \log R(t) + C_2/T(t), \quad (1)$$

$$B - V(t) = C_3/T(t) + C_4 \quad (2)$$

where the values of the constants, C_i , are taken to be those found by Arp (1961) for black-bodies (Table 1). The absolute magnitude at the moment of maximum brightness, t_0 , is then given by

$$M_B(t_0) = C_1 - 5 \log R(t_0) + [B - V(t_0) - C_4] C_2/C_3. \quad (3)$$

The value of $B - V(t_0)$ is given directly by the observations, so the problem of determining $M_B(t_0)$ is just that of finding the value of $R(t_0)$.

Table 1. Photometric constants.

C_1	C_2	C_3	C_4
53.6	35,300	6900	-0.56

2.1 THE SHAPE OF THE RADIUS CURVE

The fractional change of the radius of the photosphere can be found from the observable changes in magnitude and colour, with respect to their values at t_0

$$\log R(t)/R(t_0) = -0.2 \Delta M_B(t) + \Delta B - V(t) C_2/5C_3 \quad (4)$$

with $C_2/5C_3 = 1.02$. The photometric data in Table 2 are taken from smooth curves drawn through the composite light and colour curves given by Barbon, Ciatti & Rosino (1973). The values of the radius which follow from equation (4) and the data in Table 2 are shown as filled circles in Fig. 1. Neither random errors in the data (about ± 0.05 mag in $\Delta B - V$ and ± 0.1 mag in ΔM_B) nor a reasonable error in C_2/C_3 can change the important fact that over an interval of at least 20 days, the radius increases linearly as the photosphere expands at constant velocity. The effects of changing C_2/C_3 by 10 per cent either way are shown by open circles in Fig. 1.

The value of $R(t_0)$ is obtained by multiplying the velocity of the photosphere and the reciprocal of the slope of the radius curve. Over the 20-day interval centred on t_0 , the reciprocal of the slope and its rms internal error are taken to be 16.4 ± 1.0 days. An error of 10 per cent in C_2/C_3 would cause an additional error in the slope of about 10 per cent.

Table 2. Photometric data.

$t - t_0$	-10	-5	0	5	10	15	20
$\Delta M_B(t)$	0.75	0.10	0.00	0.15	0.50	1.00	1.70
$B - V(t)$	-0.40	-0.30	-0.15	0.00	0.15	0.30	0.50

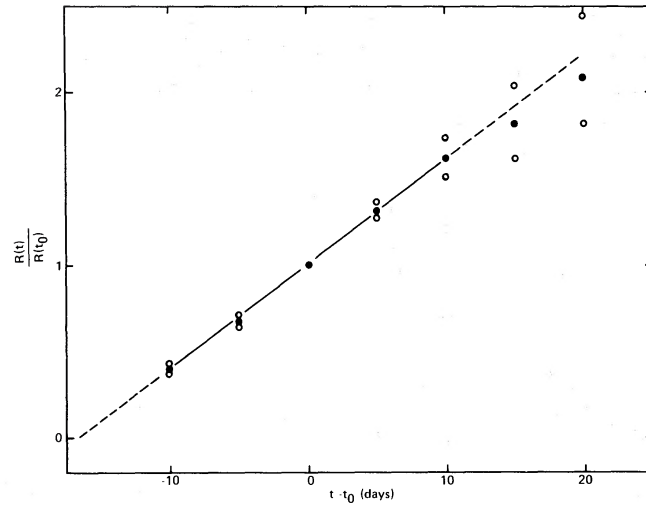


Figure 1. The radius curve for Type I supernovae (filled circles). Open circles show the effect of changing the value of C_2/C_3 by 10 per cent.

If we assume that the linear radius curve can be extrapolated down to a negligibly small radius then the rise time, t_0 , is also 16.4 days, but the result for $R(t_0)$ does not depend on the validity of the extrapolation.

2.2 THE DENSITY GRADIENT AT THE PHOTOSPHERE

The linearity of the radius curve leads to a conclusion regarding the density gradient at the photosphere. Consider a differentially expanding envelope in which the physical quantities of interest can be expressed in terms of powers of the radius, r , and the time. Specifically, the dependence of velocity, temperature, and density on radius and time is written as

$$V(r, t) \propto V_0 r t^{-1} \quad (5)$$

$$T(r, t) \propto T_0 r^{-p} t^{p-s} \quad (6)$$

$$\rho(r, t) \propto \rho_0 r^{-n} t^{n-3} \quad (7)$$

where subscript zeroes refer to values of the physical quantities at the photosphere at t_0 . Each mass element moves at constant velocity, with $T \propto r^{-s}$ and $\rho \propto r^{-3}$, whereas at any fixed time the envelope has $V \propto r$, $T \propto r^{-p}$, and $\rho \propto r^{-n}$. From the assumption that the opacity can be written

$$k = k_0 T^m \rho^q \quad (8)$$

and the definition of the optical depth

$$\tau(r, t) = \int_r^\infty k(r, t) \rho(r, t) dr \quad (9)$$

it follows that the radius of the photosphere varies as

$$R(t)/R(t_0) = (t/t_0)^x \quad (10)$$

where

$$x = \frac{m(p-s) + (q+1)(n-3)}{mp + n(q+1) - 1} \quad (11)$$

with the constraint that

$$mp + n(q + 1) - 1 = 3 k_0 \rho_0 R(t_0)/2 > 0. \quad (12)$$

The linearity of the radius curve and the homogeneity require that in each Type I supernova the temperature and density structure and the opacity law are such that $x \approx 1$. Most of the ways of arranging this seem either physically implausible or require quite specific values of several parameters. A more satisfactory solution requires a strong density gradient, $n \gg 1$, and $q > -1$. Near t_0 the opacity source is probably electron scattering, so the absolute values of m and q should not be large. In the special case of $m = q = 0$, x is given by $(n - 3)/(n - 1)$.

A strong density gradient provides a simple explanation of the linear radius curve and to some extent the homogeneity as well, since $x \approx 1$ for a range of temperature structures and opacity laws. It also vindicates the initial assumption that the photosphere is well defined, and it simplifies the analysis of the spectral lines.

2.3 SPECTRAL LINES AND THE VELOCITY OF THE PHOTOSPHERE

The lines are assumed to be formed above the photosphere by pure scattering. Kirshner & Kwan (1974) have argued that in an envelope expanding with velocity proportional to radius, the velocity of the photosphere is given by the wavelength of the absorption minimum of a weak line ($\tau \leq 1$) or from the red edge of the absorption trough of a stronger line. Scattering profiles calculated by the escape-probability method (Castor 1970) show that in a strong density gradient the situation is somewhat different. Over a fairly wide range of optical depths and density gradients the velocity of the photosphere is given by the absorption minimum (Fig. 2). Weak lines in sufficiently strong gradients have smaller displacements to shorter wavelengths. The concept of the red edge of an absorption line is not very useful.

The velocity of the photosphere is evidently best found from the absorption minimum of a moderately strong line. I think that the best line for this purpose is the one used in Paper I: the absorption feature which appears near 6125 Å until about 25 days after maximum brightness, identified as the red $\lambda\lambda 6347, 6371$ doublet of Si II. This feature

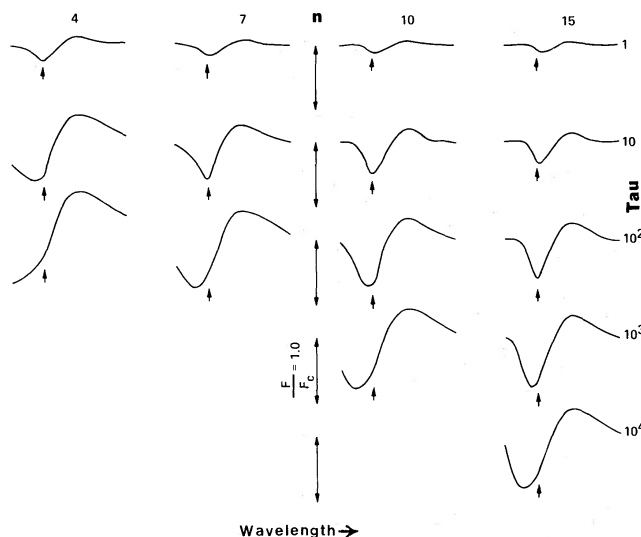


Figure 2. Scattering profiles for various density laws ($\rho \propto r^{-n}$) and optical depths at the photosphere. For each profile the emission peak occurs at the rest wavelength, and the wavelength shift corresponding to the velocity of the photosphere is shown by a short vertical arrow.

appears to be free from blending, it does not show significant wavelength shifts, and it is moderately strong.

The identification of this feature (Pskovskii 1969; Mustel 1971; Paper I) has been based on the grounds of wavelength and the strength of the Si II doublet in spectra of novae near maximum light and of supergiants. A further, more quantitative test is possible. If the density gradient at the photosphere is strong, the lines will be formed near the photosphere under similar physical conditions. The number of absorbers falls off rapidly above the photosphere, while the ionization increase caused by the decreasing electron pressure is partially cancelled by the increasing dilution of ionizing radiation. A reasonable first estimate of the relative line strengths to be expected is given by the LTE line strengths at the photosphere, with solar abundances. The relative strengths of the leading lines of some ions of interest are shown in Figs 3–5, for a range of temperatures and densities encountered by the photosphere during the first 30 days after maximum brightness. The temperature is derived from equation (2). The density ($N_e(t_0) = 2.7 \times 10^{10} \text{ cm}^{-3}$) is found by assuming that at t_0 the electron scattering optical depth at the photosphere is 3 (rather arbitrarily, to allow for thermalization of the scattered radiation) and that the value of n is 10. The density is then assumed to fall off with time as $(t/16.4)^{-3}$.

Near t_0 the calculated strengths are sensitive to the adopted temperatures and the assumption of LTE ionization, because most elements are several times ionized. (Knowledge of the temperature is more important in this context than for the determination of $M_B(t_0)$.) The calculated line strengths would imply changes in the spectrum during the first few days after t_0 which are not observed. It is not clear that the present model applies at $t - t_0 = 30$ days, because by then the observed spectroscopic and photometric properties have

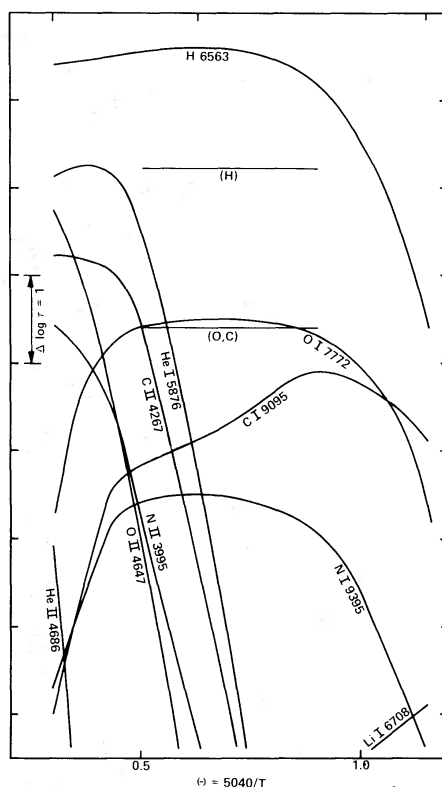


Figure 3. Calculated optical depths for various lines, on a log scale. Tic marks at the top represent $t - t_0 = 0, 15$, and 30 days. Horizontal lines represent the optical depths to electron scattering when electrons are provided by hydrogen or by oxygen and carbon.

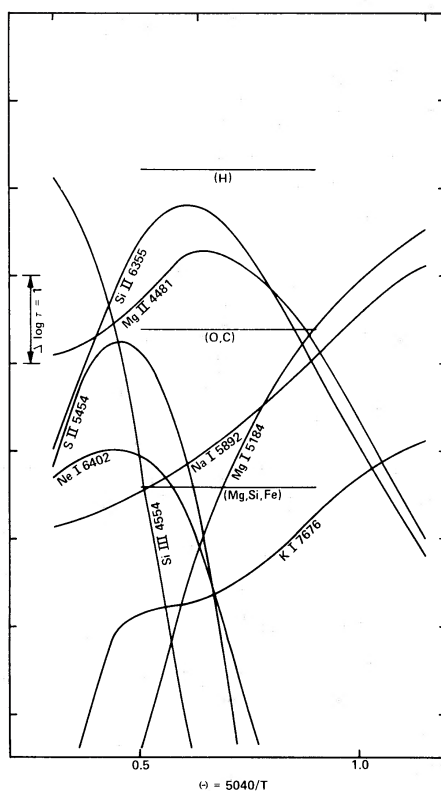


Figure 4. Like Fig. 3. The optical depth to electron scattering when electrons are provided by magnesium, silicon, and iron is also shown.

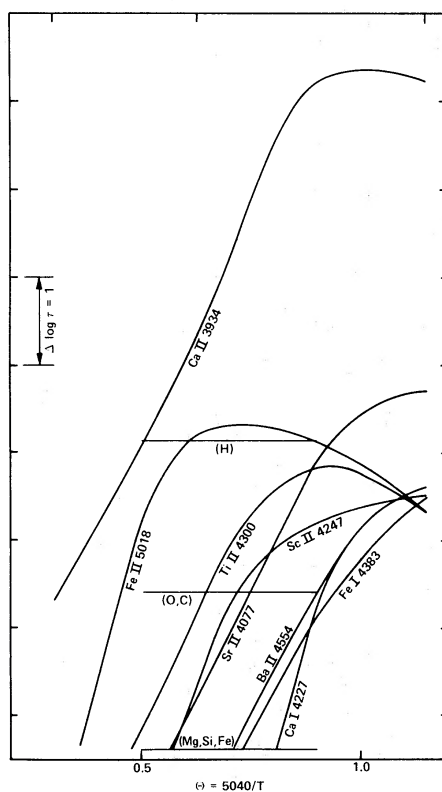


Figure 5. Like Fig. 4.

undergone significant changes. But at intermediate times, near $t - t_0 = 15$ days, these qualifications do not apply and the calculated line strengths can probably be taken fairly seriously. In Figs 3–5 the horizontal lines show the optical depth to electron scattering for $t - t_0 = 15$ days, assuming that $n = 10$. The upper horizontal line in each diagram is for solar abundances, in which case the electrons come from hydrogen and only H and Ca II lines should be strong. The Ca II lines are observed but the H lines are not. If hydrogen is deficient, as it evidently is, and electrons come from carbon and oxygen or from magnesium, silicon, and iron (lower horizontal lines), we should see also strong lines of Fe II, Si II, and Mg II. The $\lambda 4481$ line of Mg II may appear, but if so it is blended with Fe II lines (Paper I). The prediction that the Si II doublet should be strong is not very model-dependent, and therefore provides further support for the identification.

The wavelength of the absorption feature in the rest frame of a Type I supernova is found from published data on 11 objects to be $6125 \pm 15 \text{ \AA}$. Combined with the weighted wavelength of the Si II doublet, $6355 \text{ \AA}$, this gives $V = 10900 \pm 700 \text{ km/s}$.

2.4 THE RADIUS AND THE ABSOLUTE MAGNITUDE AT MAXIMUM BRIGHTNESS

The adopted values of the slope of the radius curve and the velocity of the photosphere give $R(t_0) = 1.54 \pm 0.14 \times 10^{15} \text{ cm}$. This value, and $B - V(t_0) = -0.15 \pm 0.05 \text{ mag}$ (Table 2), gives

$$M_B(t_0) = 20.25 \pm 0.31 \text{ mag}.$$

3 The value of H_0

The value of H_0 is given by

$$\log H_0 = 5 + [M_B(t_0) - C_B] / 5 \quad (13)$$

where C_B is the zero point of the relation between the apparent magnitudes of Type I supernovae at maximum brightness and the logarithm of the radial velocities of the supernova parent galaxies. The value of C_B , from nine supernovae having radial velocities in excess of 3000 km/s (Branch 1974), is $-3.7 \pm 0.3 \text{ mag}$. Then

$$H_0 = 4.9 \pm 9 \text{ km/(s Mpc)}.$$

4 Errors

The improved accuracy of this result, compared with the one given in Paper I ($40^{+25}_{-13} \text{ km/(s Mpc)}$), is due to several factors. The new composite light and colour curves (Barbon *et al.* 1973) establish the linearity of the radius curve over a wide time interval, and lead to an improved value for the slope of the radius curve. The recognition of the strong density gradient at the photosphere, combined with the improved understanding of line formation begun by Kirshner & Kwan (1974), leads to a more accurate value for the velocity.

The internal error in H_0 comes about equally from the errors in $M_B(t_0)$ and in C_B . If the limits of accuracy imposed by departures from homogeneity have not yet been reached, further reduction of the error requires (1) accurate measurements of $B - V$ over the interval $-10 < t - t_0 < 10$, to reduce the errors in both $B - V(t_0)$ and the slope of the radius curve, and (2) measurements of the peak apparent magnitudes of remote supernovae, to reduce the error in C_B .

The greatest possibility of external error in the value of $M_B(t_0)$ would be in the value of C_2/C_3 . As already mentioned, an error of 10 per cent in C_2/C_3 would produce an error of

10 per cent in $R(t_0)$, and therefore also in H_0 . Surprisingly, the greatest uncertainty of all is associated not with the astrophysical problem of determining the value of $M_B(t_0)$, but with the purely astronomical problem of finding the value of C_B . The value of C_B used here is based only on relatively distant supernovae, in order to obtain a global value for H_0 . But nearer supernovae show a larger scatter in the redshift–magnitude diagram and they give a different value for C_B (Branch 1974). This is also the case when the distances to the parent galaxies of the nearer supernovae are taken from Sandage & Tammann (1975). If all Type I supernovae, rather than just the distant ones, were used, C_B would be -3.15 mag (Barbon, Capaccioli & Ciatti 1975) and H_0 would be 38 km/s/Mpc. However the current distance estimates imply intrinsic differences of 3 or 4 mag in the absolute magnitude at maximum brightness between apparently normal supernovae (e.g. 1972e in NGC 5253 and 1974 in NGC 4414). These differences are very hard to reconcile with the thermal interpretation of supernovae light. The observational data which are needed most of all are further measurements of peak apparent magnitude for supernovae of various distances.

5 Conclusion

A simple empirical model of the envelopes of Type I supernovae near maximum brightness now gives a value for H_0 and an internal rms error of 49 ± 9 km/(s Mpc). The most serious possibility of external error is associated with the zero point of the redshift–magnitude relation for Type I supernovae. This uncertainty is such that the best value for H_0 would fall in the range 38–49 km/(s Mpc).

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