# SPECTROSCOPIC STUDIES OF O-TYPE STARS. VII. ROTATIONAL VELOCITIES $V \sin i$ AND EVIDENCE FOR MACROTURBULENT MOTIONS 

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#### Abstract

Rotational velocities are derived from coudé spectrograms of 205 O-type stars in the northern and southern hemispheres. The velocity distribution for main-sequence stars is bimodal, containing a slow group with a $V \sin i$ peak near $100 \mathrm{~km} \mathrm{~s}^{-1}$ and a fast group with a $V \sin i$ peak near 300 km $\mathrm{s}^{-1}$. The distribution of type III giants shows fewer of the high-velocity stars, and the type I supergiants are all distributed around a single peak near $V \sin i=100 \mathrm{~km} \mathrm{~s}^{-1}$. A number of Oe stars are found in the rapidly rotating group, by analogy with Be stars. The peak of the velocity distribution of the giants and supergiants does not shift as much to slower velocities as would have been expected from the peak for the main-sequence stars and stellar evolution leading to an increasing radius, with angular momentum conserved in shells. The detailed shapes of the distributions do not agree well with a purely rotational interpretation of the broadening, especially for the giants and supergiants. These facts, and the absence of any very sharp-lined supergiants, suggest that an additional parameter-turbulence-is probably necessary in evolved O-type stars.


Subject headings: stars: early-type - stars: rotation - turbulence

## I. INTRODUCTION

One of the most notable characteristics of the visible spectra of the O-type stars is the absence of sharp absorption lines. Lines of various elements, including $\mathrm{H}, \mathrm{He}_{\mathrm{I}}, \mathrm{He}_{\text {II, }}$ C III, N III, $\mathrm{O}_{\text {III, }}$ and Si Iv, can be identified; but they are usually quite broad and shallow. When equivalent widths are measured, principally $\mathrm{He}_{\text {I }} \lambda 4471$ and He is $\lambda 4541$, the spectra can be arranged in an orderly sequence of spectral types in which the widths change in a very systematic way (Conti and Alschuler 1971, hereafter Paper I). Furthermore, the measured line strengths agree quite well with those predicted by non-LTE model atmospheres (Auer and Mihalas 1972) characterized by effective temperatures in the range 30,000 to $50,000 \mathrm{~K}$, and surface gravities between $\log g=3.3$ and 4.0 (Conti 1973). The profiles of the lines, however, appear to be broader than those computed for hydrostatic plane-parallel atmospheres.
The two most commonly discussed mechanisms which may account for the broadened profiles are stellar rotation and atmospheric motion of some sort: micro- or macroturbulence. Attempts to compute the shapes of narrow line profiles broadened by rotation indeed predict profiles very much like those actually observed. However, there are reasons to suspect that rotation may not be the sole culprit in the O stars. The most disturbing fact is the lack of clear evidence of an

[^0]O star with sharp lines, as would be expected from a rotating star seen pole-on. Presumably, out of a large enough sample of stars, we should see at least a few nearly pole-on, but we still observe considerable broadening in all of them. This additional broadening could be due to large-scale atmospheric motions occurring everywhere on the stellar surface. Such motions seem rather plausible, for the more luminous stars especially, where high luminosity and larger radius make the balance between radiation pressure and gravity more precarious. Rosendahl (1970) has shown, in fact, that among B- and A-type supergiants, macroturbulence is an important contributor to overall line width.

Early attempts by Slettebak (1956) to estimate what a line broadened by macroturbulence would look like also produced profiles that appear quite similar to those observed. It thus seems difficult to decide which mechanism is the dominant cause of the observed profiles. The purpose of this paper is to measure observed profiles in a large number of O and Of stars and see what rotational velocities ( $V \sin i$ ) are implied if rotation is the sole source of broadening. The more significant question of the contribution of turbulence to the line broadening is at present being investigated in detail for a few stars and will be reported upon subsequently. In § II, we discuss the basis on which theoretical profiles were constructed. We present the observational data in § III. For a set of standard stars, line profiles are derived to compare with the theoretical ones. For most stars, visual eye estimates of $V \sin i$ are then given from comparison with the standards. In $\S$ IV, we analyze the $V \sin i$ distribution for the $O$ stars and discuss the results.

## II. THEORY

Historically, the determination of $V \sin i$ has been approached in several similar ways. In the first method, the sample of stars to be measured is divided into groups, each having similar spectral types and luminosities. The sharpest-lined star in each group is assumed to be not influenced by rotation, and its lines are used as zero-velocity profiles. These profiles are then broadened mathematically with various degrees of sophistication, and an assortment of other effectscontinuum limb darkening, gravity darkening, differential rotation, electron scattering, etc.-sometimes approximated.

Two significant problems detract from the usefulness of this approach for the O stars. The first problem, as discussed above, is that no O stars have really sharp lines. Two of the sharpest are 10 Lac and HD 57682, both of which have been used. However, even their profiles do not perfectly match zerovelocity models, and probably have some additional broadening. The second difficulty is that both these stars are about the same spectral type and luminosity class (O8 or O9, V or III, according to various authors). Consequently, even if these stars are indeed nonrotators, results derived from them would be reliable only for other stars of similar spectral types. As one proceeds to earlier spectral types or brighter luminosity classes, both the relative strengths and intrinsic profiles of the lines used for measurement change drastically. If these intrinsic changes are ignored, serious errors may result in rotational velocities inferred from measured profiles.

A second, more modern approach starts with theoretical profiles computed from good model atmosphere calculations. The most sophisticated calculations begin with intensity profiles, divide the apparent stellar disk into a large number of small areas, Doppler shift the profile by an amount determined by the projected velocity of that area, then sum over the disk to derive the resulting flux profile. In this way, the center-to-limb variations of both the line strength and the profile are explicitly accounted for. This can be done for those spectral types and luminosity classes for which one has available intensity profiles, and for sufficient velocities to build a large homogeneous grid of broadened profiles. This was the approach taken by Stoeckley and Mihalas (1973) for lines represented mostly in B-type stars.

The approach taken in the present paper is really a compromise between these two methods. In order to derive a grid of results to adequately represent spectral types O9.5 to O5, and luminosity classes V, III, and I, it was necessary to begin with theoretical profiles. Since Auer and Mihalas (1972) did not provide intensity profiles, we began with their calculated $f l u x$ profiles (of a nonrotating star) and broadened them, using a computer program. The program is essentially a digital version of the usual technique of dividing a uniform, circular stellar disk into a number of strips, finding the velocity of each strip, shifting the profile by that velocity, scaling by the fractional area of the
strip, then summing over the disk. The main difference is that this method will give slightly too much weight to contributions from very near the limb, since the flux profile is stronger than an intensity profile would be. Consequently, the highest velocities will be expected to influence the final profile a bit too much, making it slightly shallower and wider. It will be shown that this effect did indeed show up, but only at the highest velocities, and even then only slightly.

Since no one spectral line is very suitable over the entire range of stars to be measured, it was decided to model four lines and use as many of these as possible in the final velocity determination. The lines used were $\mathrm{H} \gamma \lambda 4340$, Не I $\lambda 4388$ and 4471, and Не II $\lambda 4541$. The strengths and profiles of these lines change noticeably over the spectral type grid, but at least one is always intrinsically narrow and symmetrical. The nature of the changes is the following. The hydrogen line, $\mathrm{H} \gamma$, weakens going from O 9 to O 5 , but not drastically. However, on the main sequence, $\mathrm{H} \gamma$ has very pronounced wings, due to Stark broadening. The line becomes much narrower at higher luminosities (lower gravities); He I $\lambda 4388$ is strong and symmetrical at O9, but becomes very weak and is difficult to measure earlier than O7. He i $\lambda 4471$ also becomes weaker at higher temperatures, and it too is most useful near 09. This line has a forbidden component at $\lambda 4470$, which is strong enough at type O 9 V to produce a noticeable asymmetry in the blue wing. This component is produced by a quadratic Stark effect, and weakens very rapidly with increasing luminosity and temperature. He in $\lambda 4541$ is very weak at O9, but strengthens rapidly, and is quite strong earlier than O7. Being a hydrogenic ion, He II is Stark affected, and has broad wings on the main sequence, but is narrow and symmetric in giants and supergiants.

In order to test how well the computer program worked, two spectral lines were broadened by several velocities, and the resulting profiles were compared to other results for the same lines. Flux profiles of the two He I lines $\lambda 4026$ and $\lambda 4471$ were taken from non-LTE model atmospheres with $T_{\text {eff }}=27,500 \mathrm{~K}$ and $\log g=$ 4.0 (Stoeckley and Mihalas 1973). Velocities of 10, 25, $50,100,150,200,300$, and $400 \mathrm{~km} \mathrm{~s}^{-1}$ were used, and the resulting profiles plotted (Fig. 1). The corresponding rotational broadened profiles given by Stoeckley and Mihalas were plotted on the same axes to allow easy comparison. The profiles are absolutely indistinguishable for velocities below $200 \mathrm{~km} \mathrm{~s}^{-1}$. At higher velocities, our profiles have very slightly shallower cores and deeper wings, but even at 400 $\mathrm{km} \mathrm{s}^{-1}$ the difference is only about $2 \%$ of the continuum.

Flux profiles for the four lines from non-LTE model atmospheres of Auer and Mihalas (1972) were chosen to represent spectral types $09.5, \mathrm{O} .5$, and 06.5 ( $T_{\text {eff }}=30,000,35,000$, and $40,000 \mathrm{~K}$ ), and luminosity classes I and $\mathrm{V}(\log g=3.3,4.0)$. Each line was broadened by several velocities, the maximum being $300 \mathrm{~km} \mathrm{~s}^{-1}$. The profiles were plotted, the full width at half-central intensity was measured, and its relationship to velocity was plotted. The plots for luminosity


Fig. 1.-Comparison of rotationally broadened profiles computed by the program used in the present work (solid line), and that used by Stoeckley and Mihalas 1973 (dots). The zero-velocity profile was He I 4026 for a plane-parallel atmosphere with $T_{\text {off }}=27,500 \mathrm{~K}$ and $\log g=4.0$. The differences at high velocities are due to the explicit treatment of limb darkening used by Stoeckley and Mihalas, as explained in the text.
class III were produced by linear interpolation, point by point, between those for classes I and V. Intermediate spectral types were estimated visually on the graphs.

## iII. REDUCTION OF OBSERVATIONAL MATERIAL

The observed line profiles were obtained from the same set of $16 \AA \mathrm{~mm}^{-1}$ Lick coudé plates previously described in Paper I. In addition, we had available a number of $18 \AA \mathrm{~mm}^{-1}$ coudé spectrograms of southern stars obtained at the $60 \mathrm{inch}(1.5 \mathrm{~m})$ telescope of the Cerro Tololo Observatory. We first chose a group of 37 (northern) stars to be standard velocity O stars. We attempted to include as wide a range of velocities and spectral types as possible as well as all of Slettebak's (1956) standards, avoiding where possible known double-lined spectroscopic binaries. The selected stars were then analyzed in detail as follows.

Sections of the spectra spanning the four lines, along with the clear plate, the intensity strip calibration pattern, and the wavelength comparison spectrum, were traced on the microdensitometer at the High Altitude Observatory in Boulder. The digital reduction, which was the same for each spectrum, was performed on the University of Colorado CDC 6400 computer and proceeded as follows. An intensity (H \& D) calibration curve was constructed at about $\lambda 4420$ by fitting a quadratic to the relationship between relative intensity and microdensitometer digital output. A quadratic was also fitted to the entire clear tracing, and this function was subtracted, point by point, from the spectrum, while still in density units. The spectrum was then converted to the relative intensity scale, using the H \& D function. From the microfilm output of this spectrum, those of the four spectral lines which were present in sufficient strength to be measured were chosen, their positions noted, and segments of clear continuum identified. An attempt
was made to choose the same continuum locations for each spectrum, but sometimes other absorption lines, noise, or plate flaws precluded this. A quartic polynomial was fitted to the length of continuum specified, and the entire spectrum was normalized to this fit. The normalized line profiles from $\pm 10 \AA$ of line center were output on microfilm, and various pieces of information, including the measured equivalent widths, were printed. These equivalent widths were compared with those previously derived from the same plates in Paper I. No systematic differences were found, ensuring that the final output was reliable. No digital smoothing of the data was attempted. The instrumental profile was small on the velocity scales used here.

Hard copies of the profiles were made, and the full widths at half-intensity were measured. Using the spectral types and luminosity classes assigned by Conti and Leep (1974), we chose the appropriate theoretical half-width versus velocity curves. The implied rotational velocity was then derived independently for each line measured.

As might be expected, the measurements derived from several lines of a given star never agreed perfectly. Therefore weights were assigned to each measurement; and the weighted mean and standard deviation were determined. Weights of either 1,2 , or 3 were assigned somewhat subjectively, based on the intrinsic shape and strength of the line, and an opinion of the reliability of the measurement. The weights used are presented in Table 1. The mean rotational velocities and the standard deviations are contained in Table 2.

In Figure 2, the measurements from individual lines are plotted against the weighted mean. The individual measurements from all four lines scatter about the mean, indicating random errors.

Column (5) of Table 2 lists the velocities adopted by Slettebak (1956) from measurements of up to seven lines. Figure 3 is a plot of those 21 stars. At low velocities ( $V \sin i<150 \mathrm{~km} \mathrm{~s}^{-1}$ ) the new values appear to be systematically slower, by perhaps 30 km $\mathrm{s}^{-1}$. The agreement appears to be better, though, at higher velocities. Most of this change can probably be understood in terms of arguments presented earlier, as

TABLE 1
Weights Used in the Calculations of the Mean Rotational Velocities of Standard Stars

|  |  | V | III | I |
| :---: | :---: | :---: | :---: | :---: |
| O9, 08. | H $\gamma$ | 1 | 2 | 3 |
|  | 4388 | 3 | 3 | 3 |
|  | 4471 | 3 | 3 | 3 |
|  | 4541 | 1 | 1 | 1 |
| 07, 06. | H $\gamma$ | 1 | 2 | 3 |
|  | 4388 | 3 | 2 | 2 |
|  | 4471 | 3 | 3 | 3 |
|  | 4541 | 1 | 2 | 2 |
| O5, O 4. | H $\boldsymbol{\gamma}$ | 1 | 2 | 2 |
|  | 4388 | 1 | 1 | 1 |
|  | 4471 | 1 | 1 | 1 |
|  | 4541 | 1 | 3 | 3 |

Table 2

| Star | Spectral <br> Type | V sin i | $\sigma$ | Slettebak | Star | $\begin{gathered} \text { Spectral } \\ \text { Type } \end{gathered}$ | $V \mathrm{sin} \mathrm{i}$ | $\sigma$ | Slettebak |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Spectra | traced, but | not incl |  | standards |
| 46202 | 09V | 15 | 9 |  | 201345 |  | 54 | 32 |  |
| 57682 | 09V | 17 | 12 | 20 | $\lambda$ Ori B | B0.5V | 76 | 10 | 75 |
| AE Aur | 09.5V | 27 | 11 | 20 | 190429B | 09.5III | 88 | 25 |  |
| 10 Lac | 08III | 32 | 14 | 20 | 191201 |  | 84, 63 |  |  |
| 48099 | 96.5 V | 49 | 18 | 75: | 193322 | 08.5III | 94 | 40 |  |
| 46149 | 08.5V | 51 | 20 |  | $\mu \mathrm{Col}$ | 09V | 97 | 20 |  |
| $\lambda$ Ori A | 08III( $(\mathrm{f})$ ) | 52 | 8 | 75 | $\theta^{1}$ Ori C | 07Vp | 98 | 17 | 115: |
| S Mon | 08III( (f) ) | 63 | 14 | 90 | 192639 | 07.5IIIf | 103 | 16 | 120 |
| 190864 | 07III( (f)) | 69 | 8 | 100 | 34656 | 07I ( (f)) | 106 | 18 | 90 |
| 1 Ori A | 08.5III | 71 | 18 | 140 | 15137 |  | 106, 178 |  |  |
| T C Ma | 09 I | 75 | 7 | 115 | 14633 | ON8.5V | 111 | 50 |  |
| 193514 | 07.5III(f) | 81 | 11 | 120 | 46150 | 05.5((f)) | 118 | 25 | 140 |
| 47432 | 09.5I | 86 | 14 | 105 | 17603 | 08.5I(f) | 119 | 16 |  |
| 9 Sge | 08If | 88 | 13 | 110 | 9 Sgr | 04 ( (f)) | 128 | 34 | 140 |
| o Ori | 09.5 V | 89 | 8 |  | 42088 | 06.5 V | 136 | 13 |  |
| 46223 | 05 ( (f)) | 95 | 11 | 140 | 53975 | 07.5 V | 147 | 29 |  |
| 210809 | 09 I | 101 | 13 |  | 19820 | 09 III | 148 | 20 |  |
| $\delta$ Ori | 09.5 I | 109 | 12 | 140 | 216532 | 09.5 V | 168 | 22 |  |
| 48279 | ON8V | 123 | 22 |  | 165921 |  | 200, 208 |  |  |
| 190429A | 04 f | 130 | 5 | 170 | 14442 | 06ef | 273 | 7 |  |
| 29 CMa | 08.5If | 135 | 9 | 145 | 68 Cyg | 08V | 274 | 48 |  |
| $\theta^{2}$ Ori A | 09 V | 147 | 24 | 165: | 46485 | 07.5 | 300 | 42 |  |
| 175754 | 08III( $(\mathrm{f})$ ) | 161 | 13 |  | 52533 | 08.5 V | 307 | 21 |  |
| 163892 | 09 III | 169 | 11 |  |  |  |  |  |  |
| 165052 | 07 | 172 | 12 |  |  |  |  |  |  |
| $\xi$ Per | 07.5 I | 200 | 10 | 210 |  |  |  |  |  |
| $\zeta$ Pup | 04ef | 208 | 10 |  |  |  |  |  |  |
| $\lambda$ Cep | 06ef | 214 | 21 |  |  |  |  |  |  |
| 175876 | 07 | 246 | 27 |  |  |  |  |  |  |
| 192281 | 05.5(ef) | 270 | 5 | 290 |  |  |  |  |  |
| 41161 | 08V | 280 | 10 | 300 |  |  |  |  |  |
| +60513 | 07.5 | 298 | 12 |  |  |  |  |  |  |
| 13268 | 07 | 305 | 19 |  |  |  |  |  |  |
| 46056 | 08V(e) | 328 | 23 |  |  |  |  |  |  |
| 217086 | 06.5 | 341 | 8 |  |  |  |  |  |  |
| $\zeta$ Oph | 09V(e) | 351 | 18 | 350 |  |  |  |  |  |
| 14434 | 06.5 (ef) | 400 | 25 |  |  |  |  |  |  |

Slettebak used 10 Lac and HD 57682 as sources of "zero-velocity" profiles.

The O-type standards adopted give very good coverage for all spectral types, and all velocities. Low velocities, from 15 to $150 \mathrm{~km} \mathrm{~s}^{-1}$, are very well covered, and it is felt that subsequent visual estimates in this regime are accurate to about $20 \mathrm{~km} \mathrm{~s}^{-1}$-the same as the standards. Although we have an adequate
number of standards covering velocities between 150 and $400 \mathrm{~km} \mathrm{~s}^{-1}$, visually determined velocities in this regime are probably somewhat less accurate. The visual differences in broad, shallow lines are rather subtle, and more difficult to measure precisely.

With a dispersion of $16 \AA \mathrm{~mm}^{-1}$, velocities of about $20 \mathrm{~km} \mathrm{~s}^{-1}$ correspond to displacements of $20 \mu \mathrm{~m}$ on the plate. Since this is comparable to the grain size of


Fig. 2.-Rotational velocity determinations from individual lines plotted against the weighted mean. The measurements from all four lines, $\mathrm{H} \gamma 4340(\mathrm{O}), \mathrm{He} \mathrm{I} 4388(\square), 4471(\mathrm{D})$, and He II $4541(\bullet)$ scatter about the line slope $=1$, indicating no systematic errors.

IIa-O, it is felt that the standard deviations less than about $30 \mathrm{~km} \mathrm{~s}^{-1}$ may be attributed to noise in the data. This suspicion is strengthened by inspection of the plates for those stars showing the smallest errors ( $\sigma<10 \mathrm{~km} \mathrm{~s}^{-1}$ ). These spectrograms are wider, and their tracings are considerably smoother than those having errors in the $20-30 \mathrm{~km} \mathrm{~s}^{-1}$ range.
For the rest of the O stars, including the southern ones, visual eye estimates of $V \sin i$ were made in the usual fashion by comparison with the standard stars


Fig. 3.-Comparison of present rotational velocities with those of Slettebak (1956); the new scale is about $30 \mathrm{~km} \mathrm{~s}^{-1}$ slower than the old one. Main sequence $=\bigcirc$, giants $=\nabla$, supergiants $=$ - .
of Table 2. The comparison was made on the Boller and Chivens microcomparator of the SommersBausch Observatory of the University of Colorado. The slight difference in dispersion between the Tololo and Lick spectrograms could readily be accounted for by changing the scale on the comparator. Experiments in repeatability of a derived $V \sin i$ indicated this was not a problem. The adopted values of $V \sin i$ for all 205 stars are listed in Table 3.

## IV. ANALYSIS OF THE ROTATIONAL VELOCITIES

The analysis was begun by plotting the data for all O stars as a frequency histogram. The velocity interval was chosen to be $25 \mathrm{~km} \mathrm{~s}^{-1}$, as this is very nearly the expected accuracy of the measurements. All 205 O-type stars are plotted in Figure 4-both the standards and those determined visually. Double-line spectroscopic binaries were plotted twice if the lines were well enough resolved to measure. A visual inspection of Figure 4 reveals the following features:

1. The distribution is asymmetrical. There are very few stars with velocities less than $50 \mathrm{~km} \mathrm{~s}^{-1}$, and there is a substantial "tail" to large velocities.
2. There is a maximum in the distribution at about $100 \mathrm{~km} \mathrm{~s}^{-1}$, a minimum near $200 \mathrm{~km} \mathrm{~s}^{-1}$, and a smaller secondary maximum near $300 \mathrm{~km} \mathrm{~s}^{-1}$.

Chandrasekhar and Munch (1950) have indicated a way to analytically find a "best fit" for such a histogram, and to derive a true distribution of rotation velocities from the observed distribution. Essentially, one assumes that the intrinsic rotation velocities of the sample of stars, $x$, are distributed according to a probability function $f(x)$. The observed projected

Table 3

| Star | Spectral Type | $V \sin i$ | Star | Spectral Type | $\mathrm{V} \sin \mathrm{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 108 | 07If | 100: | 46202 | 09V | 15 |
| 1337 | 09.5111 | 130 | 46223 | 05((f)) | 95 |
| 5005 | 05.5((f)) | 95 | 46485 | 07.5:* | 300 |
| 5005C | 09V | 250 | 46573 | 07.5V((f)) | 95 |
| 12323 | ON9V | 100 | 46966 | 08.5 V | 60 |
| 12993 | 0N6.5V | 130 | 47129 | 07.5III (f) | 75 |
| 13268 | 07: | 305 | 47432 | 09.5I | 86 |
| 14434 | 06.5(ef) | 400 | 15 Mon | 08III( (f)) | 63 |
| 14442 | 06ef | 273 | 48099 | 06.5 V | 49 |
| 14633 | ON8.5V | 111 | 48279 | ON8V | 123 |
| 14947 | 05.5f | 130 | 52266 | 09.5 V | 285 |
| 15137 | 09.5 III | 106, 178 | 52533 | 08.5 V | 307 |
| +60498 | 09.5V | 145 | 53667 | B0.5III | 85 |
| +60501 | 06.5 V | 200 | 53975 | 07.5 V | 147 |
| 15558 | 05(f) | 145 | 54662 | 07III | 70 |
| 15570 | 04 f | 130 | 29 CMa | 08.5If | 135 |
| 15629 | 05( $(\mathrm{f})$ ) | 130 | $\tau \mathrm{CMa}$ | 091 | 75 |
| +60513 | 07.5: | 298 | 57682 | 09 V | 17 |
| 16429 | 09.51 | 140 | 60848 | 08Ve | 162 |
| 16691 | 05f | 150 | 5 Pup | 04ef | 208 |
| 17505 | 06(f) | --- | 68450 | 09.7Ib* | 60 |
| 17520 | 09V | 90 | 69106 | 05 | 328 |
| 17603 | 08.51(f) | 119 | 73882 | 08.5V* | 160 |
| +60586 | 0811 I | 115 | 74194 | 08.5Ib(f)* | 170 |
| +60594 | 09V | 330 | 75211 | 09Ib* | 110 |
| 18326 | 07V | 170 | 75222 | 09.71ab* | 85 |
| 19820 | 09III | 148 | 75759 | 09V | 50,50 |
| 24431 | 09V | 75 | 76968 | 09.7 Ib * | 80 |
| $\xi$ Per | 07.5I | 200 | 93028 | 09V* | 60 |
| $\alpha$ Cam | 09.51 | 85 | 93128 | 03V((f))* | 130 |
| AE Aur | 09.5V | 27 | 93129A | 03If* | 120 |
| 34656 | 071( (f)) | 106 | 93130 | $06 \mathrm{III}(\mathrm{f}) *$ | 180 |
| 35921 | 09.5: | 100 | 93160 | $06 \mathrm{III}((\mathrm{f}))$ * | 180 |
| $\delta$ Ori | 09.51 | 109 | 93204 | $05 \mathrm{~V}(\mathrm{f}))^{*}$ | 130 |
| $\lambda$ Ori A | 08III ( f ) ) | 53 | 93205 | 03V((f))* | 150 |
| $\lambda$ Ori b | B0.5v | 76 | 93206 | 09 Ib | --- |
| 36879 | 07.5III | 200 | 93222 | 07III((f))* | 65 |
| $\theta^{1}$ Ori C | 07Vp | 98 | 93250 | $03 \mathrm{~V}(\mathrm{f})$ )* | 70 |
| $\theta^{2}$ Ori A | 09v | 147 | -59 2600 | 06V((f))* | 140 |
| 1 Ori | 08.5III | 71 | -59 2603 | 07V((f))* | 310 |
| $\sigma$ Ori | 09.5 V | 89 | E303308 | 03V((f))* | 170 |
| $\zeta$ Ori | 09.51 | 110 | 93403 | 05III(f)* | 270 |
| ${ }_{\mu} \mathrm{Col}$ | 09V | 97 | 93521 | 09V | 400 |
| 39680 | 08Ve | 122 | 93843 | 05III (f)* | 120 |
| 41161 | 08 V | 280 | 94963 | 06.5III(f)* | 90 |
| 42088 | 06.5V | 136 | 96670 | 08p* | 150 |
| 45314 | OBe | --- | 96917 | 08.5Ib (f)* | 130 |
| 46056 | 08 V (e) | 328 | 97166 | 07.5III((f))* | 130 |
| 46149 | 08.5V | 51 | 97253 | $05.5111(f) *$ | 105 |
| 46150 | 05.5((f)) | 118 | 101131 | 06V* | 170 |


| Star | Spectral Type | $v \sin i$ | Star | Spectral Type | $\mathrm{V} \sin \mathrm{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 101190 | 06V((f))* | 120 | 164438 | 09 III | 65 |
| 101205 | 07III((f))* | 330 | 164492 | 07I | 65 |
| 101298 | 06V((f))* | 86 | 9 Sgr | 04((f)) | 128 |
| 101413 | 08V* | 140 | 165052 | 07: | 172 |
| 101436 | 06.5V* | 270 | 165921 | 07 | 200,208 |
| 105056 | 09.71ae* | 40 | 166546 | 09III | 55 |
| 112244 | 08.5Iab (f)* | 70 | 166734 | 07.5If | 150 |
| $\theta$ Mus B | 09III* | 110 | 16 Sgr | 09111 | 105 |
| 114886 | 09II-III* | 270 | 167659 | 07I ( (f)) | 80 |
| 115071 | 09V | 280 | 167771 | 07I (f) | 80 |
| 120678 | Ope | 350 | 167971 | 07.5If | 105 |
| 124314 | 06V((f))* | 300 | 168075 | 06.5III ( (f)) | 90: |
| $\delta \mathrm{Cir}$ | 07.5III((f))* | 140 | 168076 | 04((f)) | 111 |
| 135591 | 07.5III( $(\mathrm{f})$ )* | 65 | 168112 | 05.5((f)) | 90 |
| 148546 | 091a* | 110 | 168504 | 08III | 100 |
| 148937 | 0.65 f | 200 | 169582 | 05.5(f) | 99 |
| $\mu$ Nor | 09.71ab* | 85 | 171589 | 07.5V((f)) | 91 |
| 149404 | 091a* | 45 | 175754 | 08III( (f)) | 161 |
| 5 Oph | 09.5 V (e) | 351 | 175876 | 07: | 246 |
| 150135 | 06.5V((f))* | 300 | +223782 | 071II ( $(\mathrm{f})$ ) | 120 |
| 150136 | 05III(f)* | 170 | 186980 | 08III ( $(\mathrm{f})$ ) | 70 |
| 150958 | 06.5Iaf* | 110 | 9 Sge | 08If | 88 |
| 151003 | 09II* | 115 | 188209 | 09.51 | 70 |
| 151515 | 07II(f)* | 115 | E227018 | 06.5III | 60 |
| 151804 | 08If | 50 | 190429A | 04 f | 130 |
| 152003 | 09.71ab* | 115 | 190429B | 09.51 II | 88 |
| 152147 | 09.7Ib* | 80 | 190864 | 071II((f)) | 69 |
| 152218 | 09.5IV* | 130 | 191201 | 09 III | 84,63 |
| 152219 | 09.5III* | 250 | 191612 | 07.5III(f) | 110 |
| 152233 | 06(f) | 130 | 192281 | 05.5(ef) | 270 |
| 152246 | 09III-IV* | 280 | 192639 | 07.5III (f) | 103 |
| 152247 | 09.5II-III | 110 | 193322 | 08.5III | 94 |
| 152248 | 07Ib(f)p* | <100: | 193443 | 09 III | 110 |
| 152249 | 0C9.5I* | 105 | 193514 | 07.5III(f) | 81 |
| 152386 | 06 f | 160 | 195592 | 09.51 | 60 |
| 152408 | 08If | 130: | 199579 | 06.5 III | 110 |
| 152424 | OC9.5I | 85 | 201345 | ON9.5V | 54 |
| 152623 | 07V((f))* | 120 | 202124 | 09.51 | 60 |
| 152723 | 06.5III(f)* | 130 | 68 Cyg | 08 V | 274 |
| 153919 | 06f | 140 | 206267 | 06 | --- |
| 154368 | 09.5Iab* | 85 | 207198 | 091 | 70 |
| 154811 | OC9.7Iab* | 110 | 207538 | 09.5V | 15 |
| 155806 | 07.5IIIe | 115 | 14 Cep | 08.5III | 120 |
| 155889 | 09IV* | 40 | 19 Cep | 09 I | 75 |
| 157857 | 07: (f) | 110 | 210809 | 09I | 101 |
| 159176 | 07V+07V | 170 | $\lambda$ Cep | 06ef | 214 |
| 162978 | 08.5III( $(\mathrm{f})$ ) | 50 | 10 Lac | 08111 | 32 |
| 163758 | 06.5If* | 95 | 215835 | 05.5 | 100 |
| 163800 | 07I( (f)) | 55 | 216532 | 09.5v | 168 |
| 163892 | 09III | 169 | 216898 | 09V | 80 |
|  |  |  | 217086 | 07.5: | 341 |
|  |  |  | 218915 | 09.51 | 110 |
|  |  |  | +602522 | 06.5IIIef | 240 |
|  |  |  | 225160 | 08I(f) | 95 |

*Spectral type due to Walborn

- Luminosity type uncertain

Notes

| 108 | P Cygni profiles |
| :--- | :--- |
| 1337 | SB2 |
| 15137 | Double lines partially resolved |
| 17505 | Unresolved double lines |
| 19820 | SB2 |
| 1 Ori | SB2 |
| 42088 | 90 from another plate |
| 45314 | Helium lines filled in |
| 46223 | 109 from another plate |
| 46485 | 07Vn(e)* |
| 46966 | 40 from another plate |
| 47129 | SB2 |
| 15 Mon | same star as S Mon |
| 29 C Ma | SB2 |
| 75759 | SB2 |
| 93205 | SB2 |
| 93206 | Unresolved double lines |
| 93403 | SB2 |
| 93521 | Very broad lines |
| 120678 | Sin |


| 148937 | P Cygni profiles |
| :---: | :---: |
| 151804 | P Cygni profiles |
| 152218 | SB2 |
| 152248 | Only H\%, H $\gamma$ visible |
| 152249 | SB2 |
| 152386 | P Cygni profiles |
| 152408 | P Cygni profiles. Absorption components match ~130 but all lines appear P Cygni |
| 157857 | 06.51II(f)* |
| 163800 | 07III((f))* |
| 164492 | 07.5III((f))* |
| 165052 | 06.5V((f))* |
| 165921 | Double lines |
| 175876 | 06.5III(f)* |
| 191201 | SB2 |
| 206267 | Unresolved SB2 |
| 14 Cep | SB2, both sets of lines seem about equally wide |
| 215835 | SB2 |
| 217086 | 07V* |



Fig. 4.-Distributions of rotational velocities for mainsequence, giant, and supergiant stars. The Oe and Oef stars are represented by the shaded areas. The shaded supergiant at $40 \mathrm{~km} \mathrm{~s}^{-1}$ is HD 10505609.7 Iae (Walborn classification).
velocities $y=x \sin i$ are then distributed according to

$$
\phi(y) d y=\int_{i=0}^{\pi / 2} \int_{x=0}^{\infty} f(x) \sin i d x d i .
$$

The distribution of intrinsic velocities is assumed to have the form

$$
f(x)=\frac{1}{\sqrt{ } \pi}\left[\exp -\left(x-x_{1}\right)^{2}+\exp -\left(x+x_{1}\right)^{2}\right]
$$

in which case

$$
\begin{aligned}
& \phi\left(y, x_{1}\right) \\
& \quad=\frac{y}{\sqrt{ } \pi} \int_{y}^{\infty} \frac{\exp -\left(x-x_{1}\right)^{2}+\exp -\left(x+x_{2}\right)^{2}}{x\left(x^{2}-y^{3}\right)^{1 / 2}} d x .
\end{aligned}
$$

As Chandrasekhar and Munch showed, the moments of the observed and intrinsic distributions are related, and those measured moments (the mean and the mean square velocity) allow both $f(x)$ and $\phi\left(y, x_{1}\right)$ to be computed-though not without considerable effort. The procedures suggested by Chandrasekhar and Munch were applied to our data, and the two distributions computed and plotted on the same scale as the histogram.

We first tried computing a single distribution function for the entire set of data. The agreement with the observed distribution was miserable, not matching the histogram well at all. In an effort to improve upon this situation, we considered four factors: the validity of the assumed intrinsic distribution; the influence of other broadening mechanisms; misinterpretation of unresolved binaries; and the possibility of a multiple distribution. The assumed $f(x)$ appears to reproduce
very well the observed distributions for $\mathrm{B}, \mathrm{Be}, \mathrm{A}$, and F0-F2 stars, as shown by Chandrasekhar and Munch (1950) and subsequent workers. There is no obvious reason to expect something different for the $O$ stars. Broadening due to processes like macroturbulence or stellar winds would not explicitly involve the inclination angle. In this case the integrations over $i$ involved in the derivation of $\phi\left(y, x_{1}\right)$ would be inappropriate. We believe that this effect is present, and some insight into its importance was gained in this study. This will be discussed shortly. We traced the spectra of all broad-lined stars and derived velocities from measured profiles. Three stars, HD 165921, HD 191201, and HD 15137, were found to have double lines, but it turned out that the eye estimates of the others had been rather accurate. The revised velocities did not change the observed distribution at all. The crucial realization-which led to the apparent solution to this dilemma-was that trying to fit a single distribution to the entire data set is inappropriate.

Since the minimum near $200 \mathrm{~km} \mathrm{~s}^{-1}$ appears to be real, we divided the data into a "slow" group with $V \sin i<200 \mathrm{~km} \mathrm{~s}^{-1}$, and a "fast" group with $V \sin i \geq 200 \mathrm{~km} \mathrm{~s}^{-1}$. The computed distribution functions were found to be in much better agreement with the histograms. The intrinsic distribution for the fast group is very nearly Gaussian, with a peak near $350 \mathrm{~km} \mathrm{~s}^{-1}$ and a $1 / e$ full width of $138 \mathrm{~km} \mathrm{~s}^{-1}$. Perhaps not coincidentally, this is almost exactly the distribution observed for Be stars (Massa 1975).

Additional insight into the nature of the velocity distribution can be gained by plotting to histograms for different luminosity classes separately, as is done in Figure 4. It is seen that the characteristic peak near $100 \mathrm{~km} \mathrm{~s}^{-1}$ is present in all three groups, but the behavior at low and high velocities differs systematically. The main-sequence group shows both the widest range of velocities and the clearest evidence for a bimodal distribution. The Oe stars (Frost and Conti 1976) are generally the most rapidly rotating $O$ types. Among the giants, there are fewer stars at both ends, and only a hint of a bimodal distribution. The supergiants also show few stars at low velocities and none at all faster than $225 \mathrm{~km} \mathrm{~s}^{-1}$.

One very conspicuous feature in Figure 4 is the failure of the computed functions to reproduce the general shape of the observed distribution. For the giants and supergiants especially, the histogram is skewed, with a steep rise at the low-velocity end and a slower descent toward higher velocities. The fits computed from the moments of the histograms are skewed in the opposite sense, with a more gradual increase and a steeper descent. We feel that the most natural explanation for this discrepancy is that nonrotational "macroturbulent" motions have contributed enough broadening to distort the low-velocity end of the observed distribution.

To demonstrate the plausibility of this hypothesis, the following experiment was performed. We took the observed rotational velocity for the main-sequence stars, and calculated the apparent rotational velocity that would result by a doubling of the radius, and the


Fig. 5.-A, observed main-sequence distribution; B, distribution calculated from A, assuming doubling of the radius and onset of macroturbulence with characteristic velocity of $V^{*}=20 \mathrm{~km} \mathrm{~s}^{-1} ; \mathrm{C}$, observed supergiant distribution.
onset of macroturbulence with a velocity parameter of $20 \mathrm{~km} \mathrm{~s}^{-1}$. The calculation was performed using the half-width of the He I line 4471 for O 9 I. Doubling the radius reduces the rotation velocity by one-half. Turbulence of $20 \mathrm{~km} \mathrm{~s}^{-1}$ produces a half-width about equal to rotation of $30 \mathrm{~km} \mathrm{~s}^{-1}$. The final half-width is calculated as $w_{f}=\left(w_{\text {rot }}{ }^{2}+w_{\text {tur }}\right)^{1 / 2}$, and the implied rotation velocity is determined from its value. In Figure $5 a$, the observed main-sequence distribution is plotted. The distribution resulting from the above calculation is plotted in Figure $5 b$, and the observed supergiant distribution in Figure 5c. The fine details do not (and would not be expected to) agree closely, but the gross features of the observed distribution are reproduced quite well. As anticipated, the turbulence has eliminated the slowest rotators, while expansion has eliminated the fastest ones. The majority of stars
are clustered between 50 and $100 \mathrm{~km} \mathrm{~s}^{-1}$, with the distribution trailing off smoothly to zero at 225 km $\mathrm{s}^{-1}$. Significantly, the turbulence has modified the low-velocity end of the distribution, so that it is now skewed toward higher velocities. This is exactly the feature that the purely rotational functions could not reproduce in Figure 4. Although this is not claimed to be hard proof, it is highly suggestive that the type of mechanism affecting the broadening of supergiant lines is similar to what we propose.

Finally, in Figure 6 the distribution is resolved into both spectral type and luminosity class. The features which stand out in this representation are: (1) the concentration of slowest velocities at later spectral types; (2) the $200 \mathrm{~km} \mathrm{~s}^{-1}$ gap later than type 06 ; (3) the preponderance of main-sequence stars in the highvelocity group; (4) a possible minimum velocity increasing toward earlier spectral type, and possibly being different for the three luminosity classes; and (5) the absence of rapid rotators of class O3 and O4.

Some plausible interpretation of these features will be discussed in the following section.

## V. DISCUSSION

The main-sequence sample represents the youngest stars, whose rotation has been least affected by evolutionary or mass loss effects. This group shows the widest range of velocities, from near zero to a few values in excess of $400 \mathrm{~km} \mathrm{~s}^{-1}$. This distribution presumably reflects the distribution of angular momentum with which these stars formed. We are somewhat at a loss to understand the apparent existence of a bimodal distribution of rotational velocities for the main-sequence O stars. About onethird of the rapid rotators have been previously identified as Oe stars (Frost and Conti 1976), but not all rapid rotators have yet shown emission lines. It is


Fig. 6.-Rotational velocities of individual stars plotted as a function of both spectral type and luminosity class; $O=$ main sequence (V), $\mathrm{V}=$ giants (III), $-=$ supergiants (I).
quite possible that all rapidly rotating $O$ stars will show emission at some time, as might be inferred from the recent shell episode of $\zeta \mathrm{Oph}$ (Niemelä and Méndez 1974). Possibly a continuation of these events could force a star to lose sufficient angular momentum to move from relatively rapid to slower rotational velocity. However, without differential rotation in the star itself, the angular momentum loss required seems excessive.

As O stars age, the observed broadening would be expected to change, due both to direct evolutionary effects and to possible accompanying changes in the structure of the stars and their atmospheres. We propose a scenario which may help us understand some of the observed characteristics of the distribution of rotational velocities of the evolved O-type stars. The hope is to present a physically plausible mechanism which helps explain the observations while making few ad hoc assumptions.

Evolutionary models (for example, Simpson 1971) predict that, as a massive star evolves, there is a considerable change in the mass distribution within it. Basically, the core contracts, and the envelope expands. The models of Roxburgh (1964) for rotating upper-main-sequence stars indicate that the angular velocity is a monotonically decreasing function of radial distance, and that angular momentum is conserved in spherical shells. If this is the case, as the envelope expands, its equatorial velocity will decrease linearly. Assuming that the observed photosphere before and after expansion occupies roughly the same mass shell, the ratio of final to initial velocities would be the ratio of initial to final radii $V_{f} / V_{i}=R_{i} / R_{f}$. On this simple basis alone, one expects to see the supergiants rotating more slowly than the main-sequence stars.

It is now widely accepted both observationally (Morton 1967a, b; Smith 1970) and theoretically (Lucy and Solomon 1970; Castor, Abbott, and Klein 1975), that early-type stars are losing mass at an appreciable rate due to radiation-driven stellar winds. Such mass loss, and its associated angular momentum loss, will also slow the rotation even more as the star ages.

There is one main difficulty with this simple picture that suggests that the physics may be a bit more complicated. We see a few main-sequence stars with fairly sharp lines, indicative of rotational velocities less than $50 \mathrm{~km} \mathrm{~s}^{-1}$. We would expect to see a greater percentage of evolved stars with sharp lines, since our expectation is that aging stars slow down. However, one quite conspicuous feature of the distributions, seen clearly in Figures 4 and 6, is the absence of sharplined supergiants.

At this time it is tempting to consider the possibility of line broadening due to the type of macroturbulence discussed by many authors (see Gray 1976 for a comprehensive discussion). Although quantitative measurement of such turbulence requires higherquality observational material than that used in this study (Smith 1976), we can make some qualitative remarks about its probable presence. A possible mechanism for driving the turbulence is suggested,
after which we indicate those observations that can be more easily understood in terms of such motions.

As the rotating star passes from the core-burning hydrogen phase to the shell-burning hydrogen phase, the luminosity increases, the core contracts, and the envelope expands. As previously noted, angular momentum is conserved in shells, so the core rotates more rapidly but the envelope slows down. This enhances the radial gradient of angular velocity (increases the shear), creating a potentially unstable hydrodynamic situation. The increasing luminosity contributes to the radiative destabilization of the envelope, and eventually the static atmosphere becomes dominated by large-scale turbulence.

When line broadening is treated mathematically as a convolution of an "intrinsic" flux profile with some broadening function, it is found that macroturbulence leads to an observed flux profile that is very similar to a rotationally broadened profile. Moderate resolution and plate noise make the subtle differences difficult to detect observationally (Slettebak 1956). It may be that the absence of slowly rotating supergiants is illusory. Many of them may be rotating quite slowly indeed, but their lines are now broadened mostly by the turbulent velocity field. Turbulent broadening would not, however, affect the rapidly rotating stars as much. A main-sequence star with an equatorial velocity of $300 \mathrm{~km} \mathrm{~s}^{-1}$ would evolve into a supergiant with its photosphere rotating at between 100 and $150 \mathrm{~km} \mathrm{~s}^{-1}$. The effect of an additional broadening by turbulence would be to increase the width of the profiles only slightly, giving an apparent rotational velocity only about $20 \mathrm{~km} \mathrm{~s}^{-1}$ too high. It therefore seems likely that the absence of very rapidly rotating supergiants is real, and that this circumstance is due entirely to the increased radius.

Another observed feature which may be explained by the presence of turbulence is the apparent minimum velocity seen in Figure 6. This minimum, which seems to be present for all luminosity classes, appears to increase from near $0 \mathrm{~km} \mathrm{~s}^{-1}$ at 09 to perhaps 50 km $\mathrm{s}^{-1}$ at O3. It therefore seems plausible to suggest that the narrowest-lined stars of each spectral type are genuinely slow rotators, or are seen pole-on. The residual broadening would then be due to the tur-bulence-which appears to become more important at higher effective temperatures. This argument is not original; it was proposed by Slettebak (1956), based on a smaller sample of O stars.

There is some evidence that argues against macroturbulence being the dominant cause of the broadening in all cases. In Figure 4 the main-sequence distribution is matched very well, in both form and magnitude, by the derived rotation function. In the derivations of the formulae used to find this function, the rotation problem involves an explicit integration over inclination angles. Turbulence covering the stellar surface would have no such projection effect, and it would seem rather fortunate that a rotation function would fit a histogram of turbulent velocities so well. Furthermore, it would be difficult on hydrodynamic grounds to understand turbulence with characteristic velocities
an order of magnitude greater than the sonic velocities in the atmospheres. Also, the addition of a small amount of turbulent broadening, superposed on the rotational broadening, has been shown to change the detailed shape of the distribution in exactly the manner needed to understand the discrepancies for the giants and supergiants in Figure 4.

Our conclusions from this section may be summarized as follows. Early-type stars begin their evolutionary lives on the main sequence with a wide range of angular momentum. This range of angular momentum is reflected in the rotationally broadened spectral lines, from the very sharp and deep lines of HD 46202 and HD 57682 to the diffuse, nearly invisible shallow lines of $\zeta \mathrm{Oph}$ and HD 14434. The stars evolve off the main sequence with both their radii and luminosities increasing. The increased radius results in a slower photospheric rotational velocity. Differential rotation in the radial direction and increased luminosity destabilize the envelope, resulting in development
of large-scale macroturbulence. Among the evolved stars now rotating slowly, macroturbulence is the major broadening agent, producing a minimum width to the lines. The line profiles of rapidly rotating stars are less influenced by turbulence, and more accurately reflect the rotation.

Finally, it will be noticed from Figure 4 that many of the most rapidly rotating O stars have been classified as Oe previously. This strengthens the implication that Oe stars are to O stars as Be stars are to B stars. It will also be noticed that the Oef stars too are rapid rotators, and at least some (e.g., $\lambda$ Cep and $\zeta$ Pup) may be evolved from main-sequence Oe stars.

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